

Single Node Math Implementation with Coding

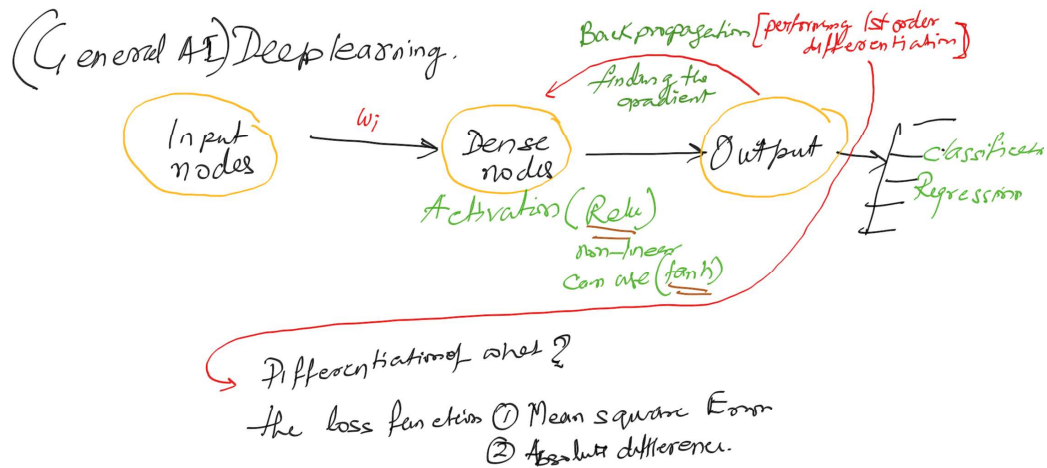
Notebook: Deep Learning

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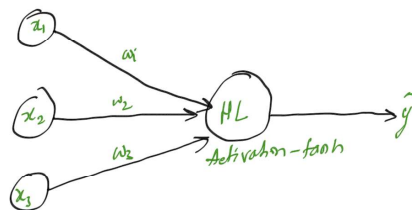
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Neural Network from Scratch



1 node NN (Implementation from Scratch)



Forward propagation Block

$$\hat{y} = f(x_1 w_1 + x_2 w_2 + x_3 w_3 + b_0) \quad \text{--- (i)}$$

$$f = \tanh$$

$$\hat{y} = \tanh\left(\sum_{i=0}^n x_i w_i\right)$$

Training Phase

Let's find out the error.

y = actual output

\hat{y} = Calculated output

$$e = (y - \hat{y}) \quad \text{--- (ii)}$$

from (i) & (ii)

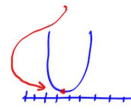
$$e = \left[y - \tanh(x w^T) \right]^2 \quad \text{--- (iii)}$$

e is also called ① Cost function
② Error function

TARGET \rightarrow MINIMIZE e

near 1st order derivative of e w.r.t w will be calculated.
... as the inflection points of the

i.e. $\frac{\partial \mathcal{E}}{\partial w} = 0$ This gives us



We need the weight value which will make the above equation almost close to 0

Since we have 3 independent inputs we can calculate the individual derivatives

$$\text{i.e.} \rightarrow \frac{\partial \mathcal{E}}{\partial w_1} \approx 0 \quad \frac{\partial \mathcal{E}}{\partial w_2} \approx 0$$

$$\frac{\partial \mathcal{E}}{\partial w_3} \approx 0$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial w_1} &= \frac{\partial}{\partial w_1} \left[y - \tanh(xw_1 + x_2w_2 + x_3w_3 + b) \right]^2 \quad \text{[Considering bias = 0, let's assume]} \\ &= \frac{\partial}{\partial w_1} \left[y - \tanh(xw^T) \right]^2 \\ &= \frac{\partial}{\partial w_1} \left[y^2 - 2y \tanh(xw^T) + \tanh^2(xw^T) \right] \\ &= \frac{\partial}{\partial w_1} (y^2) - \frac{\partial}{\partial w_1} (2y \tanh(x_1w_1 + k)) + \frac{\partial}{\partial w_1} \tanh^2(x_1w_1 + k) \end{aligned}$$

$\frac{\partial}{\partial x} (\tanh(x))$
 $= 1 - \tanh^2(x)$

$$\begin{aligned} &= 0 - 2y (1 - \tanh^2(x_1w_1 + k)) x_1 + 2 \tanh(x_1w_1 + k) (1 - \tanh^2(x_1w_1 + k)) x_1 \\ &= -2yx_1 + 2yx_1 \tanh^2(x_1w_1 + k) + 2x_1 \tanh(x_1w_1 + k) - 2x_1 \tanh^3(x_1w_1 + k) \end{aligned}$$

= from eq (i) [For computation]

$$= -2yx_1 + 2yx_1(\hat{y})^2 + 2x_1\hat{y} - 2x_1(\hat{y})^3$$

$$= 2x_1(\hat{y} - y) + 2x_1y(\hat{y})^2 - 2x_1(\hat{y})^3$$

$$= 2x_1(-y + \hat{y} + y(\hat{y})^2 - (\hat{y})^3) \quad \text{--- (iv)}$$

Similarly

$$\frac{\partial \mathcal{E}}{\partial w_2} = 2x_2(-y + \hat{y} + y(\hat{y})^2 - (\hat{y})^3) \quad \text{--- (v)}$$

$$\frac{\partial \mathcal{E}}{\partial w_3} = 2x_3(-y + \hat{y} + y(\hat{y})^2 - (\hat{y})^3) \quad \text{--- (vi)}$$

We need to reduce the error on a global level

These are my individual adjustments.

$$\Delta w \propto \frac{\partial \mathcal{E}}{\partial w}$$

i.e. $w_{1(\text{new})} = w_{1(\text{old})} - \frac{\partial \mathcal{E}}{\partial w_1(\text{old})}$

$$w_{2(\text{new})} = w_{2(\text{old})} - \frac{\partial \mathcal{E}}{\partial w_2(\text{old})}$$

$$w_{3(\text{new})} = w_{3(\text{old})} - \frac{\partial \mathcal{E}}{\partial w_3(\text{old})}$$