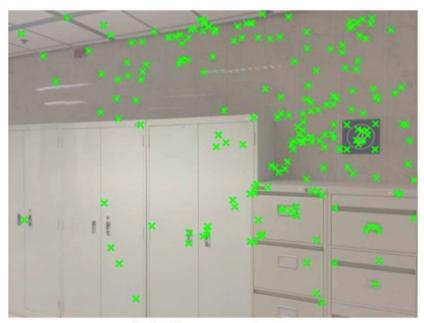
# Dual-Feature Warping-based Motion Model Estimation (ICCV2015)

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### Motivation (limitation)

1. Misalignments due to the lack of reliable keypoints

2. Distortion artifact induced by flexible warps due to insufficient corresponding information



(a) keypoints



(b) homography by keypoints

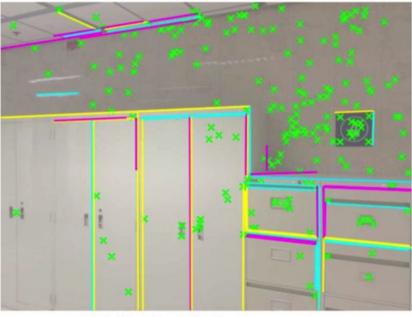


(c) result of APAP [27]

### Motivation (solution)

1. Dual-features (keypoints & lines)

2. Line preserving flexible warp



(d) dual features



(e) homography by dual features (f) our final dual-feature warp



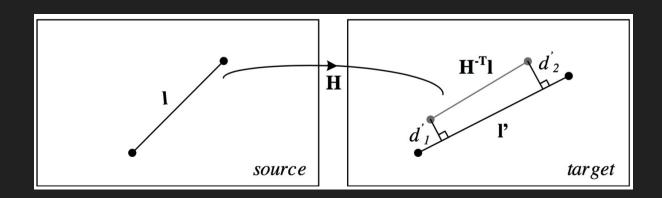
### Dual-features (keypoints & lines)

#### **Parameterization**

Endpoints:  $\hat{p}^0$ ,  $\hat{p}^1$ 

Line transformation:  $I' = \mathcal{M} \circ I$ 

Line to line distance:  $d(I,I') = \sqrt{d^2(\hat{p}^0,I') + d^2(\hat{p}^1,I')}$ 



#### **Energy function**

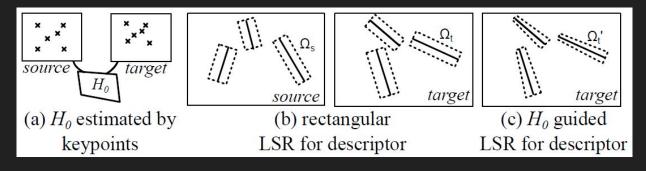
$$\widehat{\mathcal{M}} = \underset{\mathcal{M}}{\operatorname{argmin}} \left( \sum_{i} d^{2}(\hat{p}_{i}, p'_{i}) + \sum_{j} d^{2}(\widehat{l}_{j}, l'_{j}) \right)$$

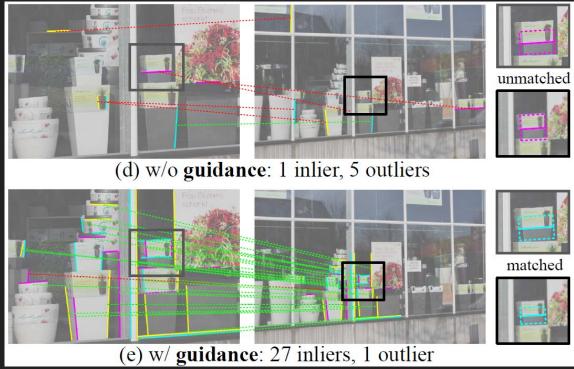
$$\hat{p}_i = \mathcal{M} \circ p_i \qquad \hat{I}_j = \mathcal{M} \circ I_j$$

## Dual-features (keypoints & lines)

#### Line detection and matching

- 1. Detection: EDLine [1]
- 2. Matching: LSR (Line Supporting Region) & MSLD [2]





#### **Dual-feature Homography Estimation**

1. Geometric Distance: 
$$\min \left( \sum_{i} \|p_i' - \hat{p}_i\|^2 + \sum_{j} \frac{\left|{I_i'}^T \hat{p}_j^0\right|^2 + \left|{I_i'}^T \hat{p}_j^1\right|^2}{a'^2 + b'^2} \right)$$

2. Algebraic Distance: 
$$\widehat{H} = \underset{H}{\operatorname{argmin}} \left( \sum_{i} \|p_i' \times Hp_i\|^2 + \sum_{j} \|I_j'^T Hp_j^{0,1}\|^2 \right)$$
$$= \underset{H}{\operatorname{argmin}} \left( \sum_{i} \|A_i h\|^2 + \sum_{j} \|B_j h\|^2 \right)$$

$$A_{i} = \begin{bmatrix} x_{i} & y_{i} & 1 & 0 & 0 & 0 & -x'_{i}x_{i} & -x'_{i}y_{i} & -x'_{i} \\ 0 & 0 & 0 & x_{i} & y_{i} & 1 & -y'_{i}x_{i} & -y'_{i}y_{i} & -y'_{i} \end{bmatrix} \quad B_{i} = \lambda_{i} \begin{bmatrix} a'_{j}u_{j}^{0} & a'_{j}v_{j}^{0} & a'_{j} & b'_{j}u_{j}^{0} & b'_{j}v_{j}^{0} & b'_{j} & c'_{j}u_{j}^{0} & c'_{j}v_{j}^{0} & c'_{j}u_{j}^{0} &$$

#### Dual-feature Homography Estimation

#### 1. Balancing

Algebraic residual:

 $||A_i h|| \sim d(Hp_i, p_i') \cdot w_i$  (i.e., the geometric distance multiplying  $w_i$ )



$$||B_i h|| = \lambda_i I_j'^T H p_j^{0,1} \sim d(\hat{p}_j^{0,1}, I_j') \cdot w_j^{0,1} \Rightarrow \lambda_i = \frac{1}{\sqrt{a_j'^2 + b_j'^2}}$$

#### Dual-feature Homography Estimation

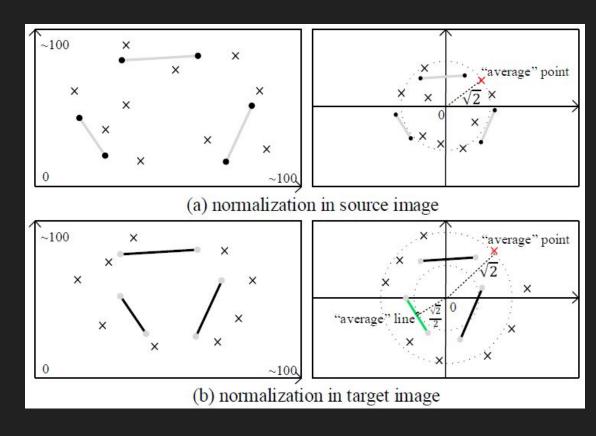
- 2. Normalization (Hartley's normalization)
- 2.1 Source image: (x, y, u, v) only points and endpoints

Points & Endpoints: "average point" is (1, 1, 1)

2.2 Target image: (x', y', a', b', c') points and lines

Point: "average point" is (1, 1, 1)

Line:  $\left[\frac{a'}{c'}, \frac{b'}{c'}, 1\right] \sim 1$  "average distance" to origin is  $\frac{1}{\sqrt{\left(\frac{a'}{c'}\right)^2 + \left(\frac{b'}{c'}\right)^2}} = \frac{1}{\sqrt{2}}$ 



#### Dual-feature Homography Estimation

3. Robust Estimation (RANSAC)

Fitting error for point: 
$$d(\hat{p}, p') = \sqrt{\|p' - \hat{p}\|^2}$$

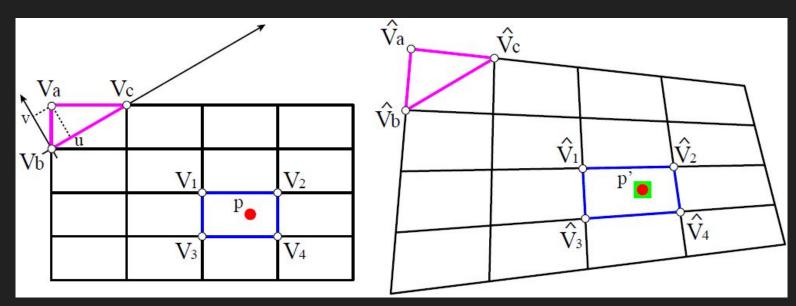
Fitting error for line: 
$$d(\hat{I}, I') = \sqrt{\frac{|I'^T \cdot \hat{p}^0|^2 + |I'^T \cdot \hat{p}^1|^2}{{a'}^2 + {b'}^2}}$$

The inliers of dual-feature will be further used for local warp estimation

#### Dual-feature Local Warps

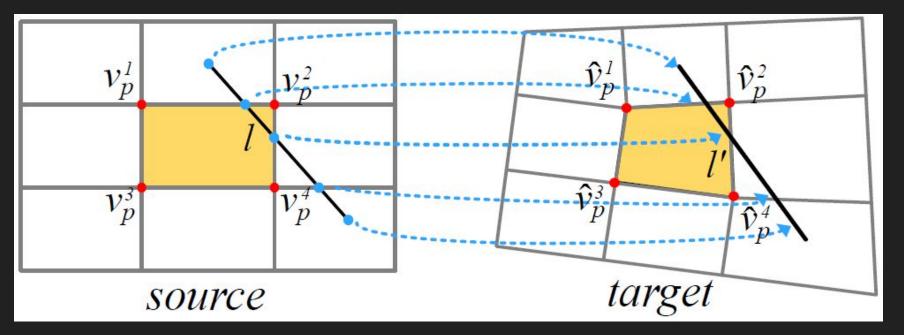
Content-preserving warp (CPW): Data term + Smoothness term

$$E_{point}(V) = \sum_{i} ||V_{p_i} w_{p_i} - p_i'||^2 \qquad E_{smoothness}(V) = \sum_{i} ||V_a - (V_b + u(V_c - V_b) + vR_{90}(V_c - V_b))p_i'||^2$$



#### Dual-feature Local Warps

Line term: 
$$E_{line}(V) = \sum_{i,k} \left\| I_j^{'T} \cdot V_{p_k} w_{p_k} / \left( \sqrt{a_j^{'2} + b_j^{'2}} \right) \right\|^2$$



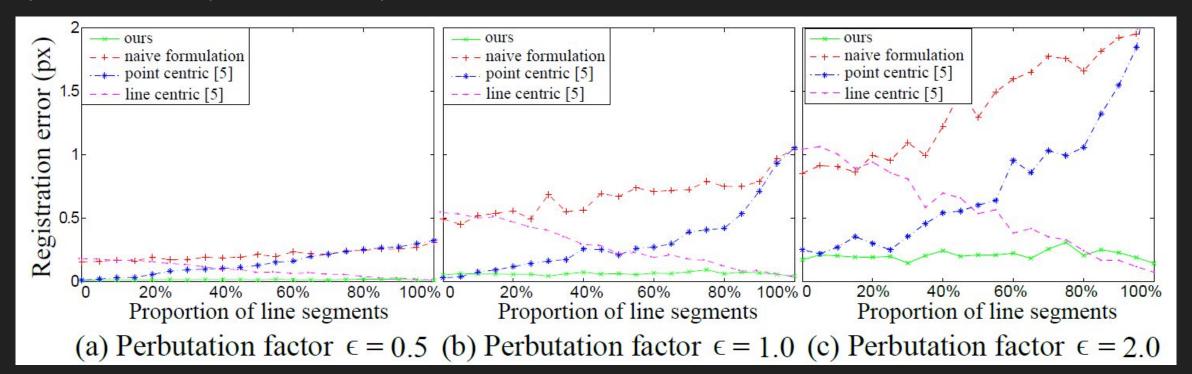
#### Dual-feature Local Warps

Total energy:  $E(V) = E_{point}(V) + E_{line}(V) + \alpha E_{smoothness}(V)$ 



#### Numerical stability

Synthesized data: perturbation on point and line features



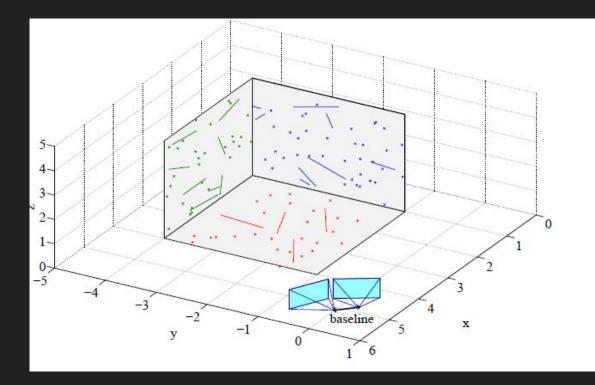
### Numerical stability

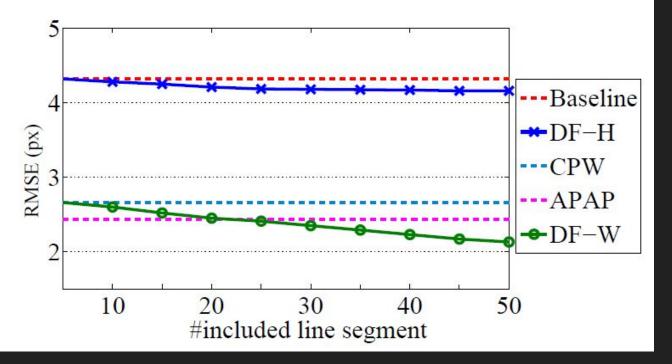
Condition number of the matrix for SVD decomposition

ratio	naive formulation	point centric	line centric	ours
0%	2.12E+6	1.04E+3	2.12E+6	1.04E+3
20%	2.28E+6	2.87E+3	1.32E+6	1.05E+3
40%	2.60E+6	5.18E+4	6.78E+5	1.12E+3
60%	3.11E+6	5.30E+4	4.20E+4	1.19E+3
80%	3.20E+6	1.34E+5	3.26E+3	1.30E+3
100%	8.75E+6	8.75E+6	1.62E+3	1.49E+3

#### Translational camera motion

Synthesized data (points & lines) + translational cameras





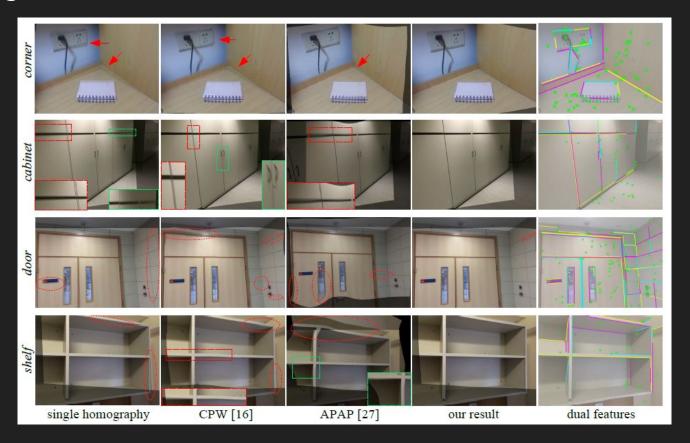
Real Images

$$EMSE(I_i, I_j) = \sqrt{\frac{1}{N}} \sum_{\pi} (1 - NCC(x_i, x_j))^2$$



model	homography model		warping-based model		
method	Baseline	DF-H	CPW	APAP	DF-W
four	12.78	6.12	7.42	6.92	2.36
door	14.47	8.31	4.89	7.37	3.50
shelf	8.62	3.04	6.28	8.76	1.54
window	9.90	6.94	7.46	5.78	4.94
cabinet	6.75	3.72	3.48	4.55	2.63
roof	4.84	4.28	5.68	7.82	2.25
desk	16.94	12.71	10.67	6.17	4.89
corner	10.02	4.34	8.67	6.84	1.44
park	21.73	12.61	16.87	11.07	8.18
car	3.08	2.77	2.65	2.07	2.13
bridge	11.37	7.70	8.47	7.95	6.60
girl	8.76	7.82	9.17	5.20	4.81
villa	16.23	13.38	7.58	6.72	5.20
road	8.17	6.38	6.48	2.28	4.59
rotation	2.57	2.28	1.37	1.12	1.06
bench	11.5	7.18	8.97	4.01	7.12

### Image Stitching (indoor environments)



#### Video Stabilization

