Mixing Model Derivation

Suppose we have some material composed of two reservoirs, (1) and (2). Then suppose we have some different material composed of those same two reservoirs, but in different proportions. Given the composition of the first material and one of its constituent reservoirs, we can calculate the excess or depletion of the constituent reservoir in the new material.
In this case, reservoir (1) is the Solar System composition without any s – process component. This can be thought of as an unknown set of materials that contributes to the modern and Archean mantles. Since it is unknown, we will substitute it out in the derivation. Reservoir (2) is the solar system s-process composition. The two materials in question, which are composed of (1) and (2) in different proportions, are the bulk silicate earth (BSE or modern mantle) and the Archean mantle.

Let $\mathrm{M}^{(1)}$ and $\mathrm{M}^{(2)}$ be the masses of the constituent reservoirs contributing to the resultant material. Then, $M = M^{(1)} + M^{(2)}$, where M is the total mass of the resultant material. Now if we want the mass of some species i in the resultant material, we have

$$M_i = X_i^{(1)} M^{(1)} + X_i^{(2)} M^{(2)} \quad [1]$$

where M_i is the mass of i in the resultant material and $X_i^{(n)}$ is the mass fraction of i in reservoir n. Then,

$$X_i = f X_i^{(1)} + (1 - f) X_i^{(2)} \quad [2]$$

where X_i is the mass fraction of i in the resultant material and the factor $f = M^{(1)}/M$. Solving for $X_i^{(1)}$:

$$X_i^{(1)} = \frac{X_i - (1-f) X_i^{(2)}}{f} \quad [3]$$

Now suppose some other material has a different compositional makeup of (1) and (2). We introduce a new factor f' representing the mass fraction of (1) in the new material such that,

$$X_i' = f' X_i^{(1)} + (1 - f') X_i^{(2)} \quad [4]$$

Substituting [3] into [4], we retrieve

$$X_i' = X_i^{(2)} + \frac{f'}{f} (X_i - X_i^{(2)}) \quad [5]$$

This is the mixing equation with mass fractions. However, we require atomic abundances (number of atoms). We can use the following conversion: $N_i = \left(X_i \cdot \frac{M}{M_{r,i}}\right) \cdot 6.0221409 \times 10^{-23}$ where N is atomic abundance and $M_{r,i}$ is the atomic weight of species i.

Then we have the mixing equation for atomic abundances:

$$N_i' = N_i^{(2)} + \frac{f'}{f}\left(N_i - N_i^{(2)}\right) \quad [6]$$

Let us introduce a new factor $F = 1 - \frac{f'}{f}$ such that $F \times 100$ represents a percent excess (if $F > 0$) or a percent depletion (if $F < 0$) of reservoir (2) in the resultant material. Substituting into [6], we get the final equation:

$$N_i' = N_i + F\left(N_i^{(2)} - N_i\right) \quad [7]$$