MATLAB example

$$-\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right] = x^2,\ 0 < x < 1,$$

$$u(0) = 0,\ u(1) = 0,$$

1. Get the closed form solution

Use subspace spanned by the following four polynomials that satisfies BC's:

```
clear
syms x
p1=x*(1-x)
p2=x*(1/2-x)*(1-x)
p3=x*(1/3-x)*(2/3-x)*(1-x)
p4=x*(1/4-x)*(1/2-x)*(3/4-x)*(1-x)
```

2. Study the errors by considering basis functions from p2 to p5

3. Changes the basis to trigonometric functions

MATLAB example

function I= eip (f, g, k, a,b,x)

Options: Use function handle to solve this integration. You need not write a subroutine, in that case.

```
%I=eip(f, g,k,a,b,x)
% This function computes the energy inner product of two functions
% f(x) and g(x), that is, it computes the integral from a to b of
% k(x)*f'(x)*g'(x). The three functions must be defined by symbolic
% expressions f, g, and k.
% The variable of integration is assumed to be x.
% The inputs a and b, defining the interval [a, b] of integration,
% are optional. The default values are a = 0 and b = 1.

% Compute the integral
I= int (k*diff(f, x)*diff(g, x),x ,a, b);
```

MATLAB example

Automate this step:
Use for loop

```
k=1+x;
K=[eip(p1,p1,k), eip(p1,p2,k), eip(p1,p3,k), eip(p1,p4,k)
    eip(p2,p1,k), eip(p2,p2,k), eip(p2,p3,k), eip(p2,p4,k)
    eip(p3,p1,k), eip(p3,p2,k), eip(p3,p3,k), eip(p3,p4,k)
    eip(p4,p1,k), eip(p4,p2,k), eip(p4,p3,k), eip(p4,p4,k)]
```

```
K =
[ 1/2, -1/30, 1/90, -1/672]
[ -1/30, 3/40, -19/3780, 3/896]
[ 1/90, -19/3780, 5/567, -41/60480]
[ -1/672, 3/896, -41/60480, 43/43008]
```

Force vector

Write a subroutine or use function handle

f = x^2;

F=[l2ip(p1,f); l2ip(p2,f); l2ip(p3,f); l2ip(p4,f)]

F =
1/20
-1/120
1/630
-1/2688

Solution

c = K\F

Estimate the errors between the true and the Galerkin- 3, 4 & 5 term solutions

c =

3325/34997

-9507/139988

1575/69994

420/34997

p=c(1)*p1+c(2)*p2+c(3)*p3+c(4)*p4

p =

(420*x*(x - 1)*(x - 1/2)*(x - 1/4)*(x - 3/4))/34997 - (9507*x*(x -1)*(x - 1/2))/139988 – (1575*x*(x - 1)*(x - 1/3)*(x - 2/3))/69994 - (3325*x*(x - 1))/34997