CS566 Fall, 2015

Homework Assignment #6
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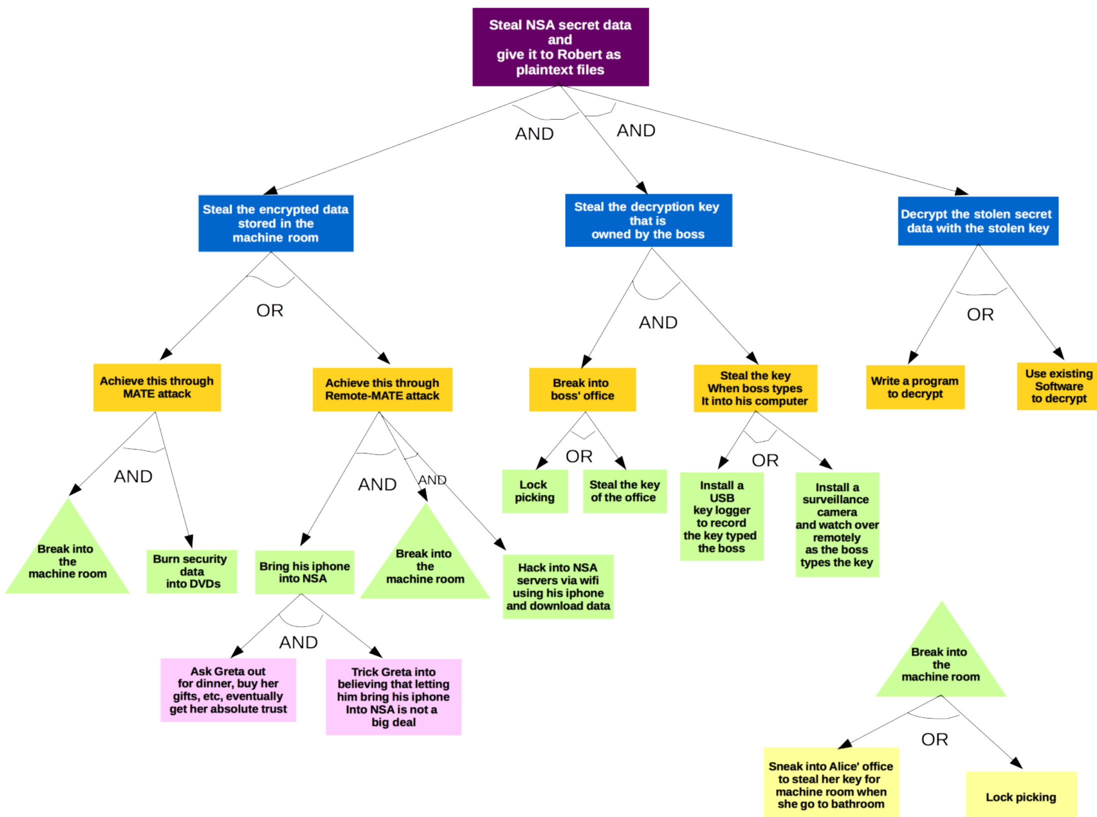
Problem 1



Problem 2

1. $\phi(210)$

   First, identify the prime factors of 210:
   $210 = 2 * 3 * 5 * 7$

   Then, according to Euler's Totient Function:

$$\phi(210) = 210 * (1 - \frac{1}{2}) * (1 - \frac{1}{3}) * (1 - \frac{1}{5}) * (1 - \frac{1}{7}) \tag{1}$$

$$= 210 * \frac{1}{2} * \frac{2}{3} * \frac{4}{5} * \frac{6}{7} \tag{2}$$

$$= 48 \tag{3}$$

2. $\phi(187)$

   First, identify the prime factors of 187:
   $187 = 11 * 17$

   Then, according to Euler's Totient Function:

$$\phi(187) = 187 * (1 - \frac{1}{11}) * (1 - \frac{1}{17}) \tag{4}$$
$$= 187 * \frac{10}{11} * \frac{16}{17} \tag{5}$$
$$= 160 \tag{6}$$

3. $(47^{78}) \mod 79$

   Since $GCD(47, 79) = 1$, 47 is relatively prime to 79. Since 79 is a prime number, we know that $\phi(79) = 79 - 1 = 78$.

   Thus according to Euler's Theorem, we have:

$$(47^{78}) \mod 79 = (47^{\phi(79)}) \mod 79 \tag{7}$$
$$= 1 \tag{8}$$

4. $(77777777^{2028}) \mod 2029$

   Since 2029 is a prime number (I verified this with a simple Python script), $\phi(2029) = 2028$. And since 2029 is not a prime factor of 77777777, we know that 77777777 is relatively prime to 2029.

   According Euler's Theorem:

$$(77777777^{2028}) \mod 2029 = (77777777^{\phi(2029)} \mod 2029 \tag{9}$$
$$= 1 \tag{10}$$

5. $(223213128736127386218^{7906}) \mod 7907$

   Since 7907 is a prime number (I verified this with a simple Python script), $\phi(7907) = 7906$. And since 7907 is not a prime factor of 223213128736127386218, we know that 223213128736127386218 is relatively prime to 7907.

   According Euler's Theorem:

$$(223213128736127386218^{7906}) \mod 7907 = (223213128736127386218^{\phi(7907)} \mod 7907 \tag{11}$$
$$= 1 \tag{12}$$