Data Analytics - Assignment-I (Second Part)

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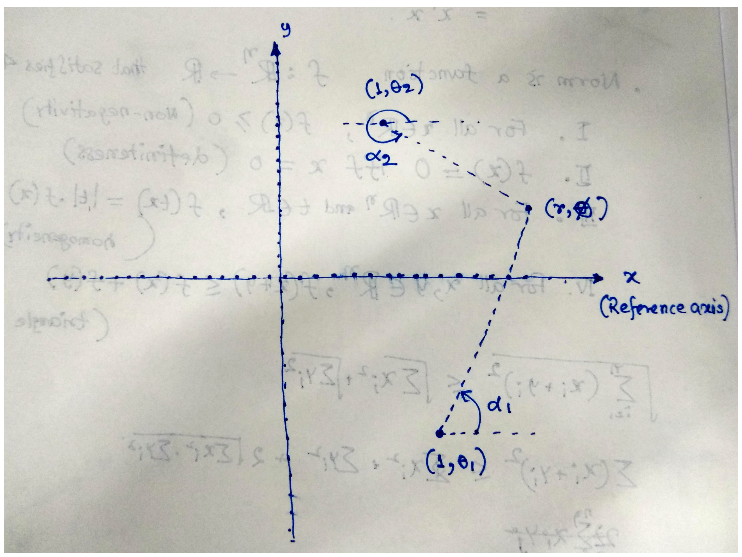
Problem 2(i)



In the above diagram, let us consider the x axis to be the reference axis (line of aries) i.e. all the angles are measured w.r.t x axis. In this problem, the task is to triangulate and find the projections of mars' location on the ecliptic plane.

In the diagram, let's take one pair of points: $(1, \theta_1)$ and $(1, \theta_2)$ be the two location of earth and (r, ϕ) be the position of mars. These are radial positions. We can easily convert these radial co-ordinates to cartesian coordinates.

Let (y, x) be the cartesian coordinates of mars. We can get the values of y and x by solving the following two equations:

$$y - rsin\theta_1 = tan\alpha_1(x - rcos\theta_1)$$

$$y - rsin\theta_2 = tan\alpha_2(x - rcos\theta_2)$$

The above two equations can also be written in a more convenient matrix form as follows:

$$\begin{bmatrix} 1 & -tan\alpha_1 \\ 1 & -tan\alpha_2 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} sin\theta_1 - tan\alpha_1 * cos\theta_1 \\ sin\theta_2 - tan\alpha_2 * cos\theta_2 \end{bmatrix} \tag{1}$$

Once we obtain the solution of the above matrix equation, we'll have y and x. Then we can easily obtain the radial coordinates (r, ϕ) for mars location on ecliptic plane.

$$r = \sqrt{y^2 + x^2}$$

$$\phi = tan^{-1} \frac{y}{x}$$

The five different position of mars on ecliptic plane are:

Radius	Projection(in degree)
1.687	148.9
1.382	329.5
1.517	43.7
1.645	175.4
1.665	157.5

Problem 2(ii)

The solution of previous problem gave us five different locations $(r_i, \phi_i), i \in \{1, 2, 3, 4, 5\}$ of mars position on ecliptic plane. We can then use these locations to find out a best fit circle of mars orbit using squared euclidean distance as loss function.

The given loss function for this problem is -

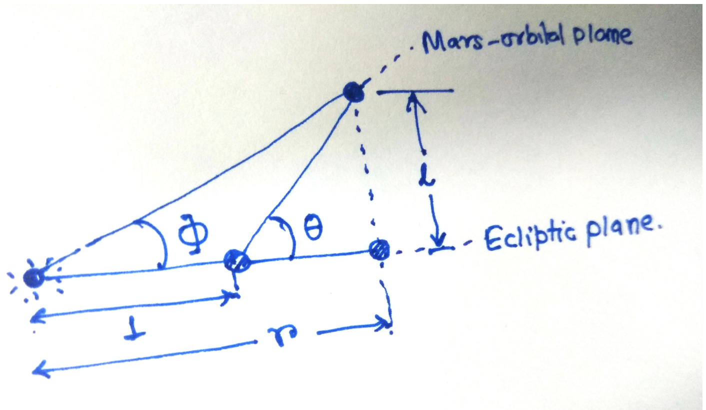
$$L_r = \frac{1}{5} \sum_{i=1}^{5} (r_i - r)^2$$

I have used *scipy* package's *minimize* function for the above mentioned function optimization. The parameters of the function is r. The minimize function returns the parameter values and the values are :

$$\boxed{r = 1.579}$$

Problem 3(i)

The task is to find the heliocentric latitudes of mars using opposition and geocentric latitudes of mars.



In the above figure, r AU is the distance of mars projection from sun in ecliptic plane. Earth is at 1 AU distance away from sun. Also, we know θ is the geocentric latutude of mars. We wan tot find ϕ - the heliocentric latitude of mars.

Using simple trigonometry we can derive from the figure that,

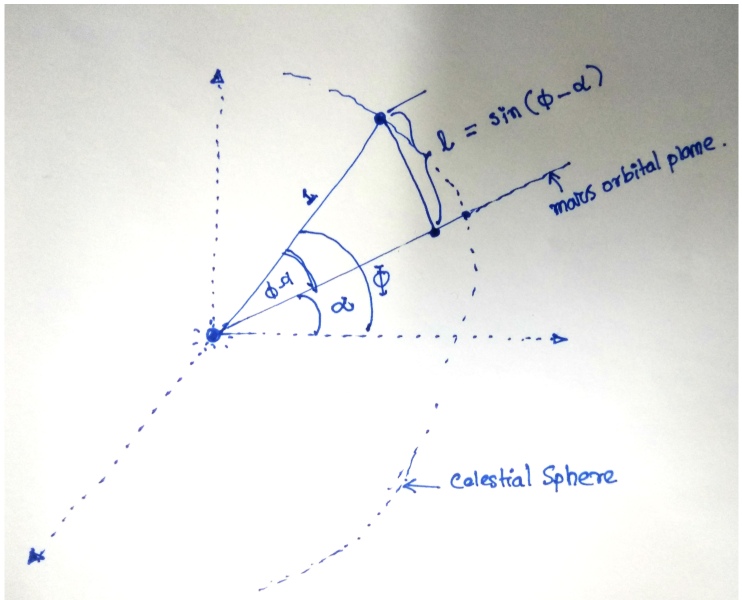
$$l = (r-1)\tan\theta = r\tan\phi$$

$$\phi = \tan^{-1}(\frac{r-1}{r}\tan\theta)$$

Using the twelve opposition geocentric latitudes of mars, we obtain the following twelve heliocentric latitudes of mars:

Heliocentric Latitude(in degree)
0.367
1.470
1.470
1.101
0.367
-1.470
-2.580
0.0
1.101
1.470
1.470
0.734

Problem 3(ii)



According to the above figure, mars orbital plane makes an angle α with ecliptic plane. Each of the twelve position makes an angle ϕ_i with ecliptic plane.

Hence, the distance between the mars position on celestial sphere and its projection on orbital plane can be written as:

$$l = sin(\phi_i - \alpha), i \in \{1, 2, 3, 4, 5\}$$

The loss function (euclidean loss) can be written as

$$L_\alpha = \frac{1}{12} \sum_{i=1}^{12} sin(\phi_i - \alpha)^2$$

Problem 3(iii)

I have used *scipy* package's *minimize* function for the above mentioned function optimization. The parameters of the function is α and it denotes the inclination of mars orbital plane with ecliptic plane. The minimize function returns the parameter value and the value is :

$$\boxed{\alpha = 0.60886977°}$$

Problem 4(i)

All the five locations are on the nodes, hence the latitudes will be zero. The longitudes are same as what we obtained in first part of second problem. Only difference will be the radial distances.

Say, the radial distance for i^{th} location be $r_i, i \in \{1,2,3,4,5\}$ and the angle between mars orbital plane and ecliptic plane is α. The radial distance of projections of these points on ecliptic plane be $r_i cos\alpha$. Using this formulation, we can find the radial distances. The five different 3-d position of mars on orbital plane are:

Radius	Longitude(in degree)	Latitude(in degree)
1.69	148.9	0.6
1.38	329.5	0.6
1.52	43.7	0.6
1.65	175.4	0.6
1.67	157.5	0.6

Problem 4(ii)

This problem is about finding best fit circle for mars orbital plane. The loss function is as follows:

$$L_r = \frac{1}{5}\sum_{i=1}^{5}(\frac{r_i}{cos\alpha} - r)^2$$

I have used *scipy* package's *minimize* function for the above mentioned function optimization. The parameters of the function is r. The minimize function returns the parameter value as :

$$\boxed{r = 1.58013542}$$

The loss for this circular fit is : **0.071218**