DIFFERENTIAL EQUATIONS (UNIT – 3)

Practice Sheet – 01

(Topic: Introduction to Differential Equations**)**

<table>
<tr><th>SL No.</th><th>Question</th></tr>
<tr><td colspan="2">SECTION – A
(This section tests your basic conceptual understanding and ability to identify key features of a DE.)</td></tr>
<tr><td>1.</td><td>Determine the order and degree (if defined) of the following differential equations:
(a) $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = k\left(\frac{d^2y}{dx^2}\right)$
(b) $\frac{\delta^2 u}{\delta t^2} = c^2\left(\frac{\delta^2 u}{\delta x^2}\right)$
(c) y'' + sin (y') = 0</td></tr>
<tr><td>2.</td><td>Classify the following as Ordinary (ODE) or Partial (PDE) Differential Equations:
(a) $\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$
(b) $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2}$</td></tr>
<tr><td>3.</td><td>For the following linear ODEs, state whether they are Homogeneous or Non-homogeneous:
(a) $\frac{dy}{dx} + Py = Q$, where P and Q are constants and $Q \neq 0$.</td></tr>
</table>

	(b) $(1 + x^2)\dfrac{dy}{dx} + 2xy = 4x^2$ (c) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 0$

SECTION – B

(This section tests your ability to apply definitions and perform basic derivations.)

4.	In each of the following, verify that the given function is a solution of the corresponding differential equation: (a) $y = e^{2x}(A + Bx)$, DE: $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$ (b) $v = \dfrac{1}{A} e^{-kt}$, DE: $\dfrac{dv}{dt} + kv = 0$
5.	Identify the dependent and independent variables in the following differential equations: (a) Newton's Law of Cooling: $\dfrac{dT}{dt} = -k(T - T_{env})$ (b) The equation of simple harmonic motion: $m\dfrac{d^2x}{dt^2} + kx = 0$

SECTION – C

(This section tests your ability to form differential equations by eliminating arbitrary constants)

6.	Form the differential equation representing the following families of curves, where a, b, c are arbitrary constants: (a) All circles touching the x – axis at the origin. (Hint: The equation of such a family is $x^2 + y^2 - 2ky = 0$) (b) $y = e^x(a \cos x + b \sin x)$

	(c) $y = ax^2 + bx + c$ (d) All straight lines in a plane which are at a constant distance p from the origin. (Hint: The equation of such a line is $x \cos \alpha + y \sin \alpha = p$)

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DIFFERENTIAL EQUATIONS (UNIT – 3)

Practice Sheet – 02

(Topic: Exact Differential Equations - Concepts, Conditions, and Solution Methods**)**

<table>
<tr><th>SL No.</th><th>Question</th></tr>
<tr><td colspan="2">SECTION – A
(This section tests your understanding of the fundamental concepts and conditions)</td></tr>
<tr><td>1.</td><td>State whether the following statements are True or False. Justify your answer.
(a) The differential equation $(x^2 + y)dx + (y^2 + x)dy = 0$ is exact.
(b) If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then the equation $Mdx + Ndy = 0$ is always exact.
(c) The function h(y) in the solution process of an exact DE is a function of y only.
(d) The general solution of an exact DE contains only one arbitrary constant.</td></tr>
<tr><td>2.</td><td>Define an Exact Differential Equation. State the necessary and sufficient condition for the equation $Mdx + Ndy = 0$ to be exact.</td></tr>
<tr><td>3.</td><td>In each of the following, verify the exactness condition $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$:
(a) $(2x + 3y)dx + (3x + 2y)dy = 0$
(b) $(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 2y)dy = 0$
(c) $(4x^3y + y^2)dx + (x^4 + 2xy)dy = 0$
(d) $\frac{y}{x}dx + (y^3 + \ln x)dy = 0$</td></tr>
</table>

SECTION – B

(This section tests your ability to solve exact differential equations using the standard algorithm)

4. Solve the following exact differential equations:

(a) $(2x + y)dx + (x + 2y)dy = 0$

(b) $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$

(c) $(\cos y + y\cos x)\,dx + (\sin x - x\sin y)\,dy = 0$

(d) $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$

(e) $\dfrac{xdy - ydx}{x^2 + y^2} = 0$ (Hint: Recognize the differential form)

SECTION – C

(This section combines exact ODE solving with initial conditions to find particular solutions)

5. Solve the following initial value problems:

(a) $(2xy + y^2 - 2x)dx + (x^2 + 2xy - 2y)dy = 0,\ y(1) = 2$

(b) $\left(ye^x + 2e^x + y^2\right)dx + (e^x + 2xy)dy = 0,\ y(0) = 6$

(c) $\left(\frac{1}{y}\sin x - \frac{y}{x^2}\right)dx + \left(\frac{1}{x}\cos y + \frac{1}{y} - \frac{\cos x}{y^2}\right)dy = 0,\ y(\pi) = \pi$

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