In[•]:=
```
Rz = RotationMatrix[2 π / 3, {0, 0, 1}]
```
Out[•]=
$$\left\{\left\{-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right\}, \left\{\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right\}, \{0, 0, 1\}\right\}$$

In[•]:=
```
data = {sp → 76 * 10^(-3), L → 524 * 10^(-3), l → 1244 * 10^(-3), wb → 164 * 10^(-3),
  wp → 22 * 10^(-3), up → 44 * 10^(-3), sb → 567 * (10^(-3)), ub → 327 * 10^(-3)}
```
Out[•]=
$$\left\{\text{sp} \to \frac{19}{250}, \text{L} \to \frac{131}{250}, \text{l} \to \frac{311}{250}, \text{wb} \to \frac{41}{250}, \text{wp} \to \frac{11}{500}, \text{up} \to \frac{11}{250}, \text{sb} \to \frac{567}{1000}, \text{ub} \to \frac{327}{1000}\right\}$$

In[•]:=
```
b1 = {0, -wb, 0}
```
Out[•]=
$$\{0, -\text{wb}, 0\}$$

In[•]:=
```
h = {x, y, z}
```
Out[•]=
$$\{x, y, z\}$$

In[•]:=
```
b2 = Rz.b1
```
Out[•]=
$$\left\{\frac{\sqrt{3}\ \text{wb}}{2}, \frac{\text{wb}}{2}, 0\right\}$$

In[•]:=
```
b3 = Rz.b2
```
Out[•]=
$$\left\{-\frac{\sqrt{3}\ \text{wb}}{2}, \frac{\text{wb}}{2}, 0\right\}$$

In[•]:=
```
p1 = {0, -up, 0}
```
Out[•]=
$$\{0, -\text{up}, 0\}$$

In[•]:=
```
p2 = {sp / 2, wp, 0}
```
Out[•]=
$$\left\{\frac{\text{sp}}{2}, \text{wp}, 0\right\}$$

In[•]:=
```
p3 = {-sp / 2, wp, 0}
```
Out[•]=
$$\left\{-\frac{\text{sp}}{2}, \text{wp}, 0\right\}$$

In[•]:=
```
Rx1 = RotationMatrix[(ϴ1), {1, 0, 0}]
```
Out[•]=
$$\{\{1, 0, 0\}, \{0, \text{Cos}[\text{ϴ1}], -\text{Sin}[\text{ϴ1}]\}, \{0, \text{Sin}[\text{ϴ1}], \text{Cos}[\text{ϴ1}]\}\}$$

In[•]:=
```
Rx2 = RotationMatrix[π / 2, {1, 0, 0}]
```
Out[•]=
$$\{\{1, 0, 0\}, \{0, 0, -1\}, \{0, 1, 0\}\}$$

In[•]:=
```
L1 = Rx2.Rx1.{0, 0, L}
```
Out[•]=
$$\{0, -\text{L}\,\text{Cos}[\text{ϴ1}], -\text{L}\,\text{Sin}[\text{ϴ1}]\}$$

In[•]:=
```
Rx3 = RotationMatrix[θ2, {1, 0, 0}]
```
Out[•]=
$$\{\{1, 0, 0\}, \{0, \text{Cos}[\theta 2], -\text{Sin}[\theta 2]\}, \{0, \text{Sin}[\theta 2], \text{Cos}[\theta 2]\}\}$$

In[•]:=
```
L2 = Rz.Rx2.Rx3.{0, 0, L}
```
Out[•]=
$$\left\{\frac{1}{2}\sqrt{3}\ \text{L Cos}[\theta 2],\ \frac{1}{2}\text{L Cos}[\theta 2],\ -\text{L Sin}[\theta 2]\right\}$$

In[•]:=
```
Rx4 = RotationMatrix[θ3, {1, 0, 0}]
```
Out[•]=
$$\{\{1, 0, 0\}, \{0, \text{Cos}[\theta 3], -\text{Sin}[\theta 3]\}, \{0, \text{Sin}[\theta 3], \text{Cos}[\theta 3]\}\}$$

In[•]:=
```
L3 = Rz.Rz.Rx2.Rx4.{0, 0, L}
```
Out[•]=
$$\left\{-\frac{1}{2}\sqrt{3}\ \text{L Cos}[\theta 3],\ \frac{1}{2}\text{L Cos}[\theta 3],\ -\text{L Sin}[\theta 3]\right\}$$

In[•]:=
```
v1a = b1 + L1
```
Out[•]=
$$\{0, -\text{wb} - \text{L Cos}[\theta 1], -\text{L Sin}[\theta 1]\}$$

In[•]:=
```
v1b  = p1 + h
```
Out[•]=
$$\{x, -\text{up} + y, z\}$$

In[•]:=
```
eq1 = Simplify[Expand[(v1b - v1a).(v1b - v1a)]] - l^2
```
Out[•]=
$$-\text{l}^2 + \text{L}^2 + \text{up}^2 - 2\,\text{up wb} + \text{wb}^2 + x^2 - 2\,\text{up}\,y + 2\,\text{wb}\,y + y^2 + z^2 - 2\,\text{L}\,(\text{up} - \text{wb} - y)\,\text{Cos}[\theta 1] + 2\,\text{L}\,z\,\text{Sin}[\theta 1]$$

In[•]:=
```
v2a = b2 + L2
```
Out[•]=
$$\left\{\frac{\sqrt{3}\ \text{wb}}{2} + \frac{1}{2}\sqrt{3}\ \text{L Cos}[\theta 2],\ \frac{\text{wb}}{2} + \frac{1}{2}\text{L Cos}[\theta 2],\ -\text{L Sin}[\theta 2]\right\}$$

In[•]:=
```
v2b  = p2 + h
```
Out[•]=
$$\left\{\frac{\text{sp}}{2} + x,\ \text{wp} + y,\ z\right\}$$

In[•]:=
```
eq2 = Simplify[Expand[(v2b - v2a).(v2b - v2a)]] - l^2
```
Out[•]=
$$-\text{l}^2 + \text{L}^2 + \frac{\text{sp}^2}{4} - \frac{1}{2}\sqrt{3}\ \text{sp wb} + \text{wb}^2 - \text{wb wp} + \text{wp}^2 + \text{sp}\,x - \sqrt{3}\ \text{wb}\,x + x^2 - \text{wb}\,y +$$
$$2\,\text{wp}\,y + y^2 + z^2 - \frac{1}{2}\text{L}\left(\sqrt{3}\ \text{sp} + 2\left(-2\,\text{wb} + \text{wp} + \sqrt{3}\ x + y\right)\right)\text{Cos}[\theta 2] + 2\,\text{L}\,z\,\text{Sin}[\theta 2]$$

In[•]:=
```
v3a  = b3 + L3
```
Out[•]=
$$\left\{-\frac{\sqrt{3}\ \text{wb}}{2} - \frac{1}{2}\sqrt{3}\ \text{L Cos}[\theta 3],\ \frac{\text{wb}}{2} + \frac{1}{2}\text{L Cos}[\theta 3],\ -\text{L Sin}[\theta 3]\right\}$$

In[•]:=
```
v3b = p3 + h
```
Out[•]=
$$\left\{-\frac{sp}{2}+x,\ wp+y,\ z\right\}$$

In[•]:=
```
eq3 = Simplify[Expand[(v3b - v3a).(v3b - v3a)]] - l^2
```
Out[•]=
$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\ sp\ wb+wb^2-wb\ wp+wp^2-sp\ x+\sqrt{3}\ wb\ x+x^2-wb\ y+$$
$$2\ wp\ y+y^2+z^2-\frac{1}{2}L\left(\sqrt{3}\ sp+2\left(-2\ wb+wp-\sqrt{3}\ x+y\right)\right)\text{Cos}[\theta 3]+2\ L\ z\ \text{Sin}[\theta 3]$$

In[•]:=
```
sub3 = {Cos[θ3] → (1 - t3^2) / (1 + t3^2), Sin[θ3] → 2 * t3 / (1 + t3^2)}
```
Out[•]=
$$\left\{\text{Cos}[\theta 3]\to\frac{1-t3^2}{1+t3^2},\ \text{Sin}[\theta 3]\to\frac{2\ t3}{1+t3^2}\right\}$$

In[•]:=
```
eq3a = eq3 /. sub3
```
Out[•]=
$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\ sp\ wb+wb^2-wb\ wp+wp^2-sp\ x+\sqrt{3}\ wb\ x+x^2-wb\ y+$$
$$2\ wp\ y+y^2-\frac{L\left(1-t3^2\right)\left(\sqrt{3}\ sp+2\left(-2\ wb+wp-\sqrt{3}\ x+y\right)\right)}{2\left(1+t3^2\right)}+\frac{4\ L\ t3\ z}{1+t3^2}+z^2$$

In[•]:=
```
sub2 = {Cos[θ2] → (1 - t2^2) / (1 + t2^2), Sin[θ2] → 2 * t2 / (1 + t2^2)}
```
Out[•]=
$$\left\{\text{Cos}[\theta 2]\to\frac{1-t2^2}{1+t2^2},\ \text{Sin}[\theta 2]\to\frac{2\ t2}{1+t2^2}\right\}$$

In[•]:=
```
eq2a = eq2 /. sub2
```
Out[•]=
$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\ sp\ wb+wb^2-wb\ wp+wp^2+sp\ x-\sqrt{3}\ wb\ x+x^2-wb\ y+$$
$$2\ wp\ y+y^2-\frac{L\left(1-t2^2\right)\left(\sqrt{3}\ sp+2\left(-2\ wb+wp+\sqrt{3}\ x+y\right)\right)}{2\left(1+t2^2\right)}+\frac{4\ L\ t2\ z}{1+t2^2}+z^2$$

In[•]:=
```
sub1 = {Cos[θ1] → (1 - t1^2) / (1 + t1^2), Sin[θ1] → 2 * t1 / (1 + t1^2)}
```
Out[•]=
$$\left\{\text{Cos}[\theta 1]\to\frac{1-t1^2}{1+t1^2},\ \text{Sin}[\theta 1]\to\frac{2\ t1}{1+t1^2}\right\}$$

In[•]:=
```
eq1a = eq1 /. sub1
```
Out[•]=
$$-l^2+L^2+up^2-2\ up\ wb+wb^2+x^2-\frac{2\ L\left(1-t1^2\right)(up-wb-y)}{1+t1^2}-2\ up\ y+2\ wb\ y+y^2+\frac{4\ L\ t1\ z}{1+t1^2}+z^2$$

In[•]:=
```
eq1b = Collect[Numerator[Together[eq1a]], t1]
```
Out[•]=
$$-l^2+L^2-2\ L\ up+up^2+2\ L\ wb-2\ up\ wb+wb^2+x^2+2\ L\ y-2\ up\ y+2\ wb\ y+y^2+4\ L\ t1\ z+z^2+$$
$$t1^2\left(-l^2+L^2+2\ L\ up+up^2-2\ L\ wb-2\ up\ wb+wb^2+x^2-2\ L\ y-2\ up\ y+2\ wb\ y+y^2+z^2\right)$$

In[•]:=
```
Sol1 = Solve[eq1b == 0, t1]
```

Out[•]=

$$\left\{\left\{t1 \to \left(-4\,L\,z - \sqrt{\left(16\,L^2\,z^2 - 4\left(-l^2 + L^2 + 2\,L\,up + up^2 - 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 - 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\left(-l^2 + L^2 - 2\,L\,up + up^2 + 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 + 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\right)}\right) \Big/ \left(2\left(-l^2 + L^2 + 2\,L\,up + up^2 - 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 - 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\right)\right\},\right.$$
$$\left.\left\{t1 \to \left(-4\,L\,z + \sqrt{\left(16\,L^2\,z^2 - 4\left(-l^2 + L^2 + 2\,L\,up + up^2 - 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 - 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\left(-l^2 + L^2 - 2\,L\,up + up^2 + 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 + 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\right)}\right) \Big/ \left(2\left(-l^2 + L^2 + 2\,L\,up + up^2 - 2\,L\,wb - 2\,up\,wb + wb^2 + x^2 - 2\,L\,y - 2\,up\,y + 2\,wb\,y + y^2 + z^2\right)\right)\right\}\right\}$$

In[•]:=
```
eq2a = eq2 /. sub2
```

Out[•]=

$$-l^2 + L^2 + \frac{sp^2}{4} - \frac{1}{2}\sqrt{3}\,sp\,wb + wb^2 - wb\,wp + wp^2 + sp\,x - \sqrt{3}\,wb\,x + x^2 - wb\,y + 2\,wp\,y + y^2 - \frac{L\left(1 - t2^2\right)\left(\sqrt{3}\,sp + 2\left(-2\,wb + wp + \sqrt{3}\,x + y\right)\right)}{2\left(1 + t2^2\right)} + \frac{4\,L\,t2\,z}{1 + t2^2} + z^2$$

In[•]:=
```
eq2b = Collect[Numerator[Together[eq2a]], t2]
```

Out[•]=

$$-4\,l^2 + 4\,L^2 - 2\sqrt{3}\,L\,sp + sp^2 + 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 - 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 - 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 - 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 16\,L\,t2\,z + 4\,z^2 + t2^2\left(-4\,l^2 + 4\,L^2 + 2\sqrt{3}\,L\,sp + sp^2 - 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 + 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 + 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 + 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)$$

In[•]:=
```
Sol2 = Solve[eq2b == 0, t2]
```

Out[•]=

$$\left\{\left\{t2 \to \left(-16\,L\,z - \sqrt{\left(256\,L^2\,z^2 - 4\left(-4\,l^2 + 4\,L^2 - 2\sqrt{3}\,L\,sp + sp^2 + 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 - 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 - 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 - 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\left(-4\,l^2 + 4\,L^2 + 2\sqrt{3}\,L\,sp + sp^2 - 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 + 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 + 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 + 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\right)}\right) \Big/ \left(2\left(-4\,l^2 + 4\,L^2 + 2\sqrt{3}\,L\,sp + sp^2 - 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 + 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 + 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 + 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\right)\right\},\right.$$
$$\left.\left\{t2 \to \left(-16\,L\,z + \sqrt{\left(256\,L^2\,z^2 - 4\left(-4\,l^2 + 4\,L^2 - 2\sqrt{3}\,L\,sp + sp^2 + 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 - 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 - 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 - 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\left(-4\,l^2 + 4\,L^2 + 2\sqrt{3}\,L\,sp + sp^2 - 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 + 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 + 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 + 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\right)}\right) \Big/ \left(2\left(-4\,l^2 + 4\,L^2 + 2\sqrt{3}\,L\,sp + sp^2 - 8\,L\,wb - 2\sqrt{3}\,sp\,wb + 4\,wb^2 + 4\,L\,wp - 4\,wb\,wp + 4\,wp^2 + 4\sqrt{3}\,L\,x + 4\,sp\,x - 4\sqrt{3}\,wb\,x + 4\,x^2 + 4\,L\,y - 4\,wb\,y + 8\,wp\,y + 4\,y^2 + 4\,z^2\right)\right)\right\}\right\}$$

In[•]:= **eq3a = eq3 /. sub3**

Out[•]=

$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\ sp\ wb+wb^2-wb\ wp+wp^2-sp\ x+\sqrt{3}\ wb\ x+x^2-wb\ y+$$
$$2\ wp\ y+y^2-\frac{L\left(1-t3^2\right)\left(\sqrt{3}\ sp+2\left(-2\ wb+wp-\sqrt{3}\ x+y\right)\right)}{2\left(1+t3^2\right)}+\frac{4\ L\ t3\ z}{1+t3^2}+z^2$$

In[•]:= **eq3b = Collect[Numerator[Together[eq3a]], t3]**

Out[•]=

$$-4\ l^2+4\ L^2-2\sqrt{3}\ L\ sp+sp^2+8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2-4\ L\ wp-4\ wb\ wp+4\ wp^2+$$
$$4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2-4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+16\ L\ t3\ z+4\ z^2+$$
$$t3^2\left(-4\ l^2+4\ L^2+2\sqrt{3}\ L\ sp+sp^2-8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2+4\ L\ wp-4\ wb\ wp+\right.$$
$$\left.4\ wp^2-4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2+4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+4\ z^2\right)$$

In[•]:= **Sol3 = Solve[eq3b == 0, t3]**

Out[•]=

$$\left\{\left\{t3\to\left(-16\ L\ z-\sqrt{\left(256\ L^2\ z^2-4\left(-4\ l^2+4\ L^2-2\sqrt{3}\ L\ sp+sp^2+8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2-4\ L\ wp-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ wb\ wp+4\ wp^2+4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2-4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ z^2\right)\left(-4\ l^2+4\ L^2+2\sqrt{3}\ L\ sp+sp^2-8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2+4\ L\ wp-4\ wb\ wp+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ wp^2-4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2+4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+4\ z^2\right)\right)\right)\Big/$$
$$\left(2\left(-4\ l^2+4\ L^2+2\sqrt{3}\ L\ sp+sp^2-8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2+4\ L\ wp-4\ wb\ wp+\right.\right.$$
$$\left.\left.4\ wp^2-4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2+4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+4\ z^2\right)\right)\Big\},\ \Big\{t3\to$$
$$\left(-16\ L\ z+\sqrt{\left(256\ L^2\ z^2-4\left(-4\ l^2+4\ L^2-2\sqrt{3}\ L\ sp+sp^2+8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2-4\ L\ wp-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ wb\ wp+4\ wp^2+4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2-4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ z^2\right)\left(-4\ l^2+4\ L^2+2\sqrt{3}\ L\ sp+sp^2-8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2+4\ L\ wp-4\ wb\ wp+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\ wp^2-4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2+4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+4\ z^2\right)\right)\right)\Big/$$
$$\left(2\left(-4\ l^2+4\ L^2+2\sqrt{3}\ L\ sp+sp^2-8\ L\ wb-2\sqrt{3}\ sp\ wb+4\ wb^2+4\ L\ wp-4\ wb\ wp+\right.\right.$$
$$\left.\left.\left.4\ wp^2-4\sqrt{3}\ L\ x-4\ sp\ x+4\sqrt{3}\ wb\ x+4\ x^2+4\ L\ y-4\ wb\ y+8\ wp\ y+4\ y^2+4\ z^2\right)\right)\right\}\Big\}$$

In[•]:= **solu3 = Flatten[Sol3 /. data /. {x → 0} /. {y → 0} /. {z → -0.9}]**

Out[•]=

$$\{t3\to -0.181037,\ t3\to -3.10312\}$$

In[•]:= **solu2 = Flatten[Sol2 /. data /. {x → 0} /. {y → 0} /. {z → -0.9}]**

Out[•]=

$$\{t2\to -0.181037,\ t2\to -3.10312\}$$

In[•]:= **solu1 = Flatten[Sol1 /. data /. {x → 0} /. {y → 0} /. {z → -0.9}]**

Out[•]=

$$\{t1\to -0.181105,\ t1\to -3.10347\}$$

In[•]:= **valθ3 = {θ3 → 2 * ArcTan[t3 /. solu3[[1]]], θ3 → 2 * ArcTan[t3 /. solu3[[2]]]}**

Out[•]=

$$\{\theta 3\to -0.358194,\ \theta 3\to -2.5181\}$$

In[•]:=
```
valθ2 = {θ2 → 2 * ArcTan[t2 /. solu2[[1]]], θ2 → 2 * ArcTan[t2 /. solu2[[2]]]}
```
Out[•]=
$$\{\theta 2 \to -0.358194, \theta 2 \to -2.5181\}$$

In[•]:=
```
valθ1 = {θ1 → 2 * ArcTan[t1 /. solu1[[1]]], θ1 → 2 * ArcTan[t1 /. solu1[[2]]]}
```
Out[•]=
$$\{\theta 1 \to -0.358327, \theta 1 \to -2.51816\}$$

In[•]:=
```
i1 = {0, 0}
```
Out[•]=
$$\{0, 0\}$$

In[•]:=
```
i2 = p1 + {x, y, z} /. data /. {x → 0} /. {y → 0} /. {z → -0.9}
```
Out[•]=
$$\left\{0, -\frac{11}{250}, -0.9\right\}$$

In[•]:=
```
i3 = p2 + {x, y, z} /. data /. {x → 0} /. {y → 0} /. {z → -0.9}
```
Out[•]=
$$\left\{\frac{19}{500}, \frac{11}{500}, -0.9\right\}$$

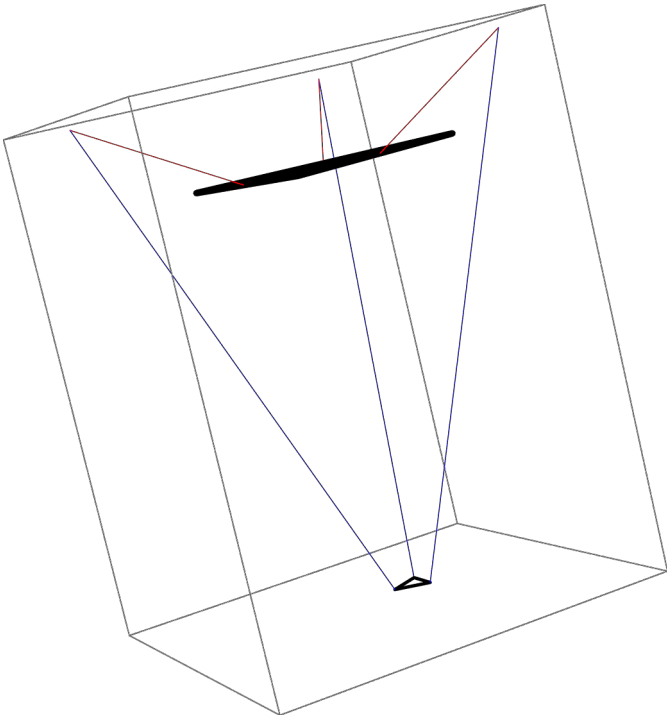
In[•]:=
```
i4 = p3 + {x, y, z} /. data /. {x → 0} /. {y → 0} /. {z → -0.9}
```
Out[•]=
$$\left\{-\frac{19}{500}, \frac{11}{500}, -0.9\right\}$$

In[•]:=
```
B1 = {sb / 2, -wb, 0} /. data;
B2 = {0, ub, 0} /. data;
B3 = {-sb / 2, -wb, 0} /. data;

bb1 = b1 /. data;
bb2 = b2 /. data;
bb3 = b3 /. data;

ll1 = L1 /. data /. valθ1[[1]];
ll2 = L2 /. data /. valθ2[[1]];
ll3 = L3 /. data /. valθ3[[1]];
```

In[•]:=
```
line1 = Graphics3D[{Thickness[0.005], Line[{i2, i3}]}];
line2 = Graphics3D[{Thickness[0.005], Line[{i3, i4}]}] ;
line3 = Graphics3D[{Thickness[0.005], Line[{i4, i2}]}];

line4 = Graphics3D[{Thickness[0.01], Line[{B1, B2}]}];
line5 = Graphics3D[{Thickness[0.01], Line[{B2, B3}]}];
line6 = Graphics3D[{Thickness[0.01], Line[{B3, B1}]}];

line7 = Graphics3D[{Red, Line[{bb1, ll1}]}];
line8 = Graphics3D[{Red, Line[{bb2, ll2}]}];
line9 = Graphics3D[{Red, Line[{bb3, ll3}]}];

line10 = Graphics3D[{Blue, Line[{ll1, i2}]}];
line11 = Graphics3D[{Blue, Line[{ll2, i3}]}];
line12 = Graphics3D[{Blue, Line[{ll3, i4}]}];
```

In[•]:=
```
Show[{line1, line2, line3, line4, line5,
  line6, line7, line8, line9, line10, line11, line12}]
```

Out[•]=



In[•]:=
```
eq1
eq2
eq3
```

Out[•]=
$$-l^2+L^2+up^2-2\,up\,wb+wb^2+x^2-2\,up\,y+2\,wb\,y+y^2+z^2-2\,L\,(up-wb-y)\,\text{Cos}[\theta 1]+2\,L\,z\,\text{Sin}[\theta 1]$$

Out[•]=
$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\,sp\,wb+wb^2-wb\,wp+wp^2+sp\,x-\sqrt{3}\,wb\,x+x^2-wb\,y+2\,wp\,y+y^2+z^2-\frac{1}{2}L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp+\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 2]+2\,L\,z\,\text{Sin}[\theta 2]$$

Out[•]=
$$-l^2+L^2+\frac{sp^2}{4}-\frac{1}{2}\sqrt{3}\,sp\,wb+wb^2-wb\,wp+wp^2-sp\,x+\sqrt{3}\,wb\,x+x^2-wb\,y+2\,wp\,y+y^2+z^2-\frac{1}{2}L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp-\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 3]+2\,L\,z\,\text{Sin}[\theta 3]$$

In[•]:=
```
eqa = Simplify[eq1 - eq2]
```

Out[•]=
$$-\frac{sp^2}{4}+up^2+\frac{1}{2}\sqrt{3}\,sp\,wb-2\,up\,wb+wb\,wp-wp^2-sp\,x+\sqrt{3}\,wb\,x-2\,up\,y+3\,wb\,y-2\,wp\,y-2\,L\,(up-wb-y)\,\text{Cos}[\theta 1]+\frac{1}{2}L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp+\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 2]+2\,L\,z\,\text{Sin}[\theta 1]-2\,L\,z\,\text{Sin}[\theta 2]$$

In[•]:=
```
eqb = Simplify[eq2 - eq3]
```

Out[•]=
$$\frac{1}{2}\left(4\,sp\,x-4\sqrt{3}\,wb\,x-L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp+\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 2]+L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp-\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 3]+4\,L\,z\,\text{Sin}[\theta 2]-4\,L\,z\,\text{Sin}[\theta 3]\right)$$

In[•]:=
```
eqc = Simplify[eq1 - eq3]
```

Out[•]=
$$-\frac{sp^2}{4}+up^2+\frac{1}{2}\sqrt{3}\,sp\,wb-2\,up\,wb+wb\,wp-wp^2+sp\,x-\sqrt{3}\,wb\,x-2\,up\,y+3\,wb\,y-2\,wp\,y-2\,L\,(up-wb-y)\,\text{Cos}[\theta 1]+\frac{1}{2}L\left(\sqrt{3}\,sp+2\left(-2\,wb+wp-\sqrt{3}\,x+y\right)\right)\text{Cos}[\theta 3]+2\,L\,z\,\text{Sin}[\theta 1]-2\,L\,z\,\text{Sin}[\theta 3]$$

In[•]:=
```
sol = Numerator[Solve[{eqb, eqc} == 0, {x, y}]][[1]]
```

Out[•]=

$$\left\{x \to -\left(\left((-L\cos[\theta 2]+L\cos[\theta 3])\left(-\frac{sp^2}{4}+up^2+\frac{1}{2}\sqrt{3}\,sp\,wb-2\,up\,wb+wb\,wp-wp^2-2\,L\,up\cos[\theta 1]+2\,L\,wb\cos[\theta 1]+\frac{1}{2}\sqrt{3}\,L\,sp\cos[\theta 3]-2\,L\,wb\cos[\theta 3]+L\,wp\cos[\theta 3]+2\,L\,z\sin[\theta 1]-2\,L\,z\sin[\theta 3]\right)-(-2\,up+3\,wb-2\,wp+2\,L\cos[\theta 1]+L\cos[\theta 3])\left(-\frac{1}{2}\sqrt{3}\,L\,sp\cos[\theta 2]+2\,L\,wb\cos[\theta 2]-L\,wp\cos[\theta 2]+\frac{1}{2}\sqrt{3}\,L\,sp\cos[\theta 3]-2\,L\,wb\cos[\theta 3]+L\,wp\cos[\theta 3]+2\,L\,z\sin[\theta 2]-2\,L\,z\sin[\theta 3]\right)\right)\Big/\left((-L\cos[\theta 2]+L\cos[\theta 3])\left(sp-\sqrt{3}\,wb-\sqrt{3}\,L\cos[\theta 3]\right)-(-2\,up+3\,wb-2\,wp+2\,L\cos[\theta 1]+L\cos[\theta 3])\left(2\,sp-2\sqrt{3}\,wb-\sqrt{3}\,L\cos[\theta 2]-\sqrt{3}\,L\cos[\theta 3]\right)\right)\right),$$

$$y \to \left(2\,sp^3-8\,sp\,up^2-6\sqrt{3}\,sp^2\,wb+16\,sp\,up\,wb+8\sqrt{3}\,up^2\,wb+12\,sp\,wb^2-16\sqrt{3}\,up\,wb^2-8\,sp\,wb\,wp+8\sqrt{3}\,wb^2\,wp+8\,sp\,wp^2-8\sqrt{3}\,wb\,wp^2+16\,L\,sp\,up\cos[\theta 1]-16\,L\,sp\,wb\cos[\theta 1]-16\sqrt{3}\,L\,up\,wb\cos[\theta 1]+16\sqrt{3}\,L\,wb^2\cos[\theta 1]-3\sqrt{3}\,L\,sp^2\cos[\theta 2]+4\sqrt{3}\,L\,up^2\cos[\theta 2]+20\,L\,sp\,wb\cos[\theta 2]-8\sqrt{3}\,L\,up\,wb\cos[\theta 2]-8\sqrt{3}\,L\,wb^2\cos[\theta 2]-4\,L\,sp\,wp\cos[\theta 2]+8\sqrt{3}\,L\,wb\,wp\cos[\theta 2]-4\sqrt{3}\,L\,wp^2\cos[\theta 2]-8\sqrt{3}\,L^2\,up\cos[\theta 1]\cos[\theta 2]+8\sqrt{3}\,L^2\,wb\cos[\theta 1]\cos[\theta 2]-3\sqrt{3}\,L\,sp^2\cos[\theta 3]+4\sqrt{3}\,L\,up^2\cos[\theta 3]+20\,L\,sp\,wb\cos[\theta 3]-8\sqrt{3}\,L\,up\,wb\cos[\theta 3]-8\sqrt{3}\,L\,wb^2\cos[\theta 3]-4\,L\,sp\,wp\cos[\theta 3]+8\sqrt{3}\,L\,wb\,wp\cos[\theta 3]-4\sqrt{3}\,L\,wp^2\cos[\theta 3]-8\sqrt{3}\,L^2\,up\cos[\theta 1]\cos[\theta 3]+8\sqrt{3}\,L^2\,wb\cos[\theta 1]\cos[\theta 3]+12\,L^2\,sp\cos[\theta 2]\cos[\theta 3]-16\sqrt{3}\,L^2\,wb\cos[\theta 2]\cos[\theta 3]+8\sqrt{3}\,L^2\,wp\cos[\theta 2]\cos[\theta 3]-16\,L\,sp\,z\sin[\theta 1]+16\sqrt{3}\,L\,wb\,z\sin[\theta 1]+8\sqrt{3}\,L^2\,z\cos[\theta 2]\sin[\theta 1]+8\sqrt{3}\,L^2\,z\cos[\theta 3]\sin[\theta 1]+8\,L\,sp\,z\sin[\theta 2]-8\sqrt{3}\,L\,wb\,z\sin[\theta 2]-8\sqrt{3}\,L^2\,z\cos[\theta 3]\sin[\theta 2]+8\,L\,sp\,z\sin[\theta 3]-8\sqrt{3}\,L\,wb\,z\sin[\theta 3]-8\sqrt{3}\,L^2\,z\cos[\theta 2]\sin[\theta 3]\right)\Big/\left(4\left(-4\,sp\,up+6\,sp\,wb+4\sqrt{3}\,up\,wb-6\sqrt{3}\,wb^2-4\,sp\,wp+4\sqrt{3}\,wb\,wp+4\,L\,sp\cos[\theta 1]-4\sqrt{3}\,L\,wb\cos[\theta 1]+L\,sp\cos[\theta 2]+2\sqrt{3}\,L\,up\cos[\theta 2]-4\sqrt{3}\,L\,wb\cos[\theta 2]+2\sqrt{3}\,L\,wp\cos[\theta 2]-2\sqrt{3}\,L^2\cos[\theta 1]\cos[\theta 2]+L\,sp\cos[\theta 3]+2\sqrt{3}\,L\,up\cos[\theta 3]-4\sqrt{3}\,L\,wb\cos[\theta 3]+2\sqrt{3}\,L\,wp\cos[\theta 3]-2\sqrt{3}\,L^2\cos[\theta 1]\cos[\theta 3]-2\sqrt{3}\,L^2\cos[\theta 2]\cos[\theta 3]\right)\right)\right\}$$

In[•]:=
```
fkin = {val𝜃1[[1]], val𝜃2[[1]], val𝜃3[[1]]}
```

Out[•]=

$$\{\theta 1 \to -0.358327,\ \theta 2 \to -0.358194,\ \theta 3 \to -0.358194\}$$

In[•]:=
```
fineq = Numerator[Together[eq1 /. sol]]
```

Out[•]=

$$sp^6 - 64\,l^2\,sp^2\,up^2 + 64\,L^2\,sp^2\,up^2 + 8\,sp^4\,up^2 + 16\,sp^2\,up^4 - 6\,\sqrt{3}\,sp^5\,wb + 192\,l^2\,sp^2\,up\,wb - 192\,L^2\,sp^2\,up\,wb - 24\,sp^4\,up\,wb + 128\,\sqrt{3}\,l^2\,sp\,up^2\,wb - 128\,\sqrt{3}\,L^2\,sp\,up^2\,wb - 32\,\sqrt{3}\,sp^3\,up^2\,wb - 96\,sp^2\,up^3\,wb - 32\,\sqrt{3}\,sp\,up^4\,wb - 144\,l^2\,sp^2\,wb^2 + 144\,L^2\,sp^2\,wb^2 + 63\,sp^4\,wb^2 - 384\,\sqrt{3}\,l^2\,sp\,up\,wb^2 + 384\,\sqrt{3}\,L^2\,sp\,up\,wb^2 + \langle\langle 2124 \rangle\rangle + 64\,L^4\,z^2\,\text{Cos}[\theta 2]\,\text{Cos}[\theta 3]\,\text{Sin}[\theta 2]\,\text{Sin}[\theta 3] + 16\,L^2\,sp^2\,z^2\,\text{Sin}[\theta 3]^2 + 64\,L^2\,up^2\,z^2\,\text{Sin}[\theta 3]^2 - 32\,\sqrt{3}\,L^2\,sp\,wb\,z^2\,\text{Sin}[\theta 3]^2 - 192\,L^2\,up\,wb\,z^2\,\text{Sin}[\theta 3]^2 + 192\,L^2\,wb^2\,z^2\,\text{Sin}[\theta 3]^2 + 128\,L^2\,up\,wp\,z^2\,\text{Sin}[\theta 3]^2 - 192\,L^2\,wb\,wp\,z^2\,\text{Sin}[\theta 3]^2 + 64\,L^2\,wp^2\,z^2\,\text{Sin}[\theta 3]^2 - 128\,L^3\,up\,z^2\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 3]^2 + 192\,L^3\,wb\,z^2\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 3]^2 - 128\,L^3\,wp\,z^2\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,z^2\,\text{Cos}[\theta 1]^2\,\text{Sin}[\theta 3]^2 - 32\,\sqrt{3}\,L^3\,sp\,z^2\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 - 64\,L^3\,up\,z^2\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 192\,L^3\,wb\,z^2\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 - 64\,L^3\,wp\,z^2\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,z^2\,\text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,z^2\,\text{Cos}[\theta 2]^2\,\text{Sin}[\theta 3]^2$$

Full expression not available (original memory size: 0.6 MB)

In[•]:=
```
solz = Solve[fineq == 0, z]
```

Out[•]=

$$\Big\{\Big\{z \to \langle\langle 1 \rangle\rangle \Big/ \Big(2\,\big(64\,sp^2\,up^2 - 192\,sp^2\,up\,wb - 128\,\sqrt{3}\,sp\,up^2\,wb + 144\,sp^2\,wb^2 + 384\,\sqrt{3}\,sp\,up\,wb^2 + 192\,up^2\,wb^2 - 288\,\sqrt{3}\,sp\,wb^3 - 576\,up\,wb^3 + 432\,wb^4 + \langle\langle 311 \rangle\rangle + 192\,L^3\,wb\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 3]^2 - 128\,L^3\,wp\,\text{Cos}[\theta 1]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,\text{Cos}[\theta 1]^2\,\text{Sin}[\theta 3]^2 - 32\,\sqrt{3}\,L^3\,sp\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 - 64\,L^3\,up\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 192\,L^3\,wb\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 - 64\,L^3\,wp\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,\text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,\text{Cos}[\theta 2]^2\,\text{Sin}[\theta 3]^2\big)\Big)\Big\},$$
$$\Big\{z \to \Big(\langle\langle 585 \rangle\rangle + \sqrt{\big(-16\,L\,sp^4\,\text{Sin}[\theta 1] + \langle\langle 567 \rangle\rangle + 96\,L^5\,\text{Cos}[\theta 1]\,\langle\langle 1 \rangle\rangle^2\,\text{Cos}[\theta 3]\,\text{Sin}[\theta 3]\big)^2 - 4\,\big(\langle\langle 1 \rangle\rangle\big)\,\big(\langle\langle 335 \rangle\rangle + 64\,\langle\langle 3 \rangle\rangle\,\langle\langle 1 \rangle\rangle^2 + 64\,L^4\,\text{Cos}[\theta 2]^2\,\text{Sin}[\theta 3]^2\big)}\Big) \Big/ \Big(2\,\big(64\,sp^2\,up^2 - 192\,sp^2\,up\,wb - 128\,\sqrt{3}\,sp\,up^2\,wb + 144\,sp^2\,wb^2 + 384\,\sqrt{3}\,sp\,up\,wb^2 + \langle\langle 325 \rangle\rangle + 192\,L^3\,wb\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 - 64\,L^3\,wp\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,\text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 3]^2 + 64\,L^4\,\text{Cos}[\theta 2]^2\,\text{Sin}[\theta 3]^2\big)\Big)\Big\}\Big\}$$

Full expression not available (original memory size: 1.6 MB)

In[•]:=
```
solz /. fkin /. data
```

Out[•]=

$$\{\{z \to -0.9\}, \{z \to 1.26746\}\}$$

In[•]:=
```
ceq = fineq /. fkin /. data
```

Out[•]=

$$-68.5868 - 22.0938\,z + 60.1265\,z^2$$

In[•]:=
```
invz = NSolve[ceq == 0, z]
```

Out[•]=

$$\{\{z \to -0.9\}, \{z \to 1.26746\}\}$$

In[•]:=
```
{invx, invy} = sol /. fkin /. data /. invz[[1]]
```

Out[•]=

$$\{x \to 0., y \to -1.02182 \times 10^{-16}\}$$

In[•]:=
```
num1 = N[Numerator[t1 /. Sol1[[1]] /. data]]
```

Out[•]=

$$-2.096\,z - 1.\,\sqrt{\left(4.39322\,z^2 - 4.\,\left(-1.38432 + x^2 - 0.808\,y + y^2 + z^2\right)\,\left(-1.1328 + x^2 + 1.288\,y + y^2 + z^2\right)\right)}$$

In[•]:=
```
num2 = Numerator[t2 /. Sol2[[1]] /. data ]
```

Out[•]=

$$-\frac{1048\,z}{125}-\sqrt{\left(\frac{1\,098\,304\,z^2}{15\,625}-4\left(-\frac{5437}{1250}-\frac{1634\,\sqrt{3}}{15\,625}+\frac{38\,x}{125}-\frac{344\,\sqrt{3}\,x}{125}+4\,x^2-\frac{322\,y}{125}+4\,y^2+4\,z^2\right)\left(-\frac{176\,011}{31\,250}+\frac{171\,\sqrt{3}}{3125}+\frac{38\,x}{125}+\frac{36\,\sqrt{3}\,x}{25}+4\,x^2+\frac{202\,y}{125}+4\,y^2+4\,z^2\right)\right)}$$

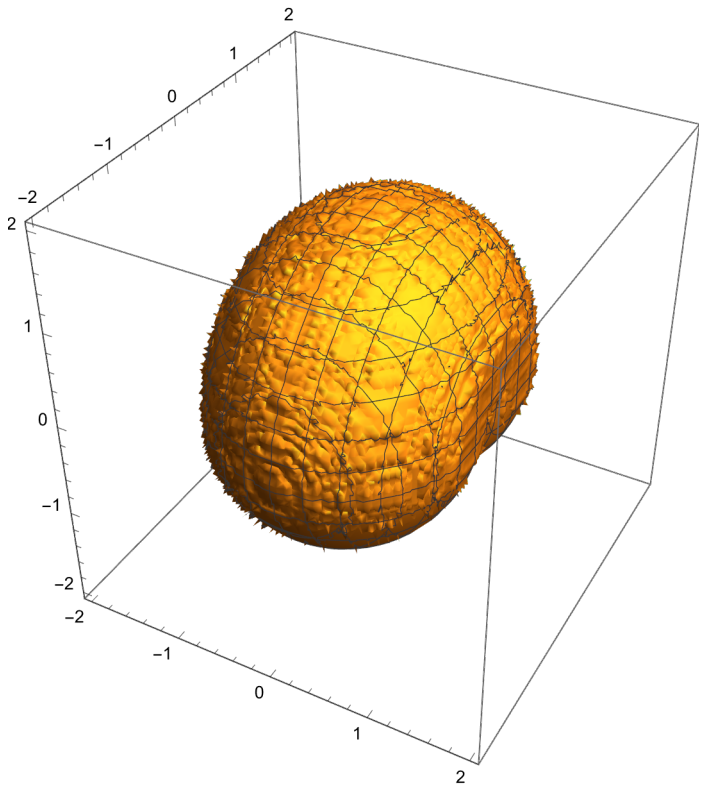
In[•]:=
```
num3 = Numerator[t3 /. Sol3[[1]] /. data ]
```

Out[•]=

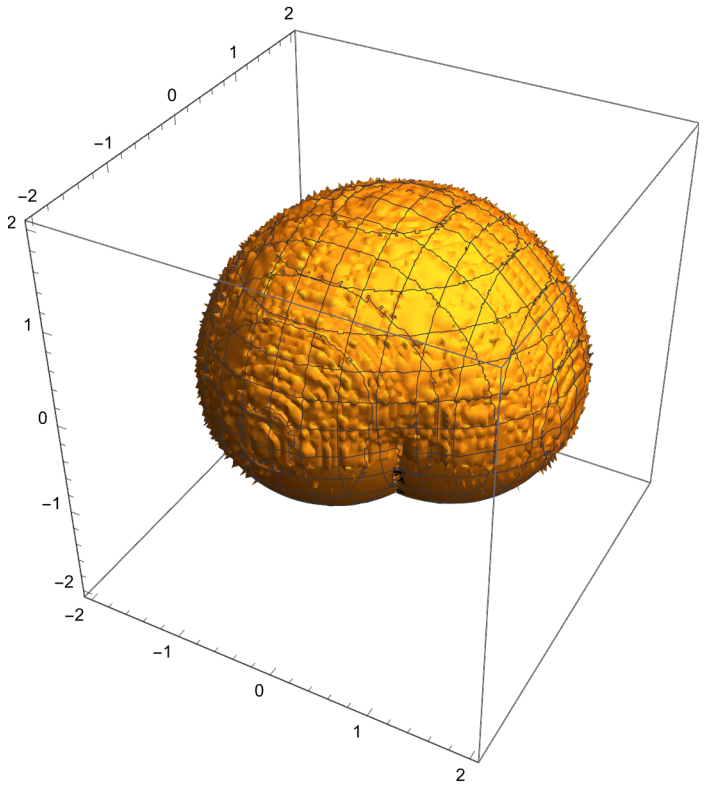
$$-\frac{1048\,z}{125}-\sqrt{\left(\frac{1\,098\,304\,z^2}{15\,625}-4\left(-\frac{5437}{1250}-\frac{1634\,\sqrt{3}}{15\,625}-\frac{38\,x}{125}+\frac{344\,\sqrt{3}\,x}{125}+4\,x^2-\frac{322\,y}{125}+4\,y^2+4\,z^2\right)\left(-\frac{176\,011}{31\,250}+\frac{171\,\sqrt{3}}{3125}-\frac{38\,x}{125}-\frac{36\,\sqrt{3}\,x}{25}+4\,x^2+\frac{202\,y}{125}+4\,y^2+4\,z^2\right)\right)}$$

In[•]:=
```
lim = 2;
ContourPlot3D[num1 == 0, {x, -lim, lim}, {y, -lim, lim}, {z, -lim, lim}, Mesh → Full]
ContourPlot3D[num2 == 0, {x, -lim, lim}, {y, -lim, lim}, {z, -lim, lim}, Mesh → Full]
ContourPlot3D[num3 == 0, {x, -lim, lim}, {y, -lim, lim}, {z, -lim, lim}, Mesh → Full]
```

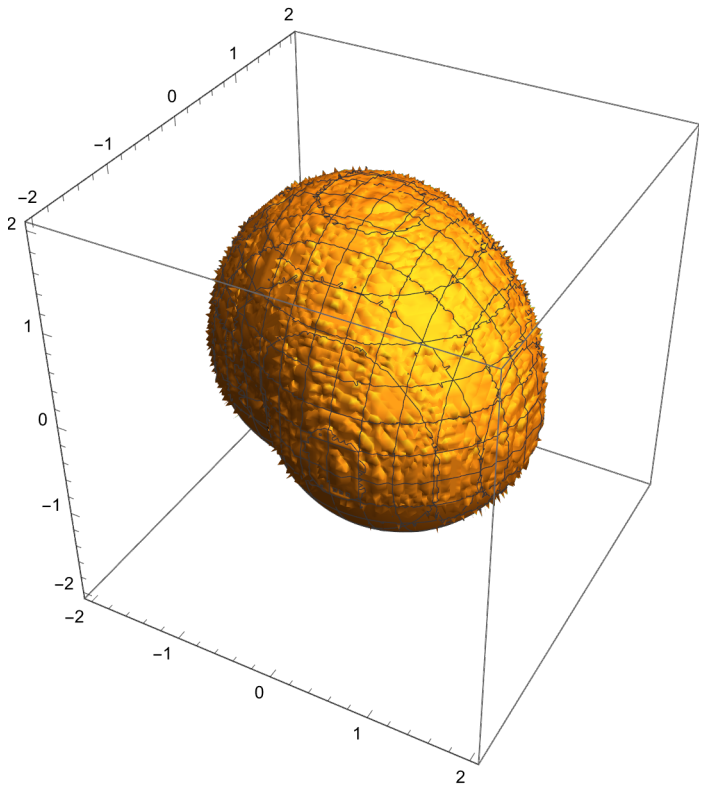
Out[•]=



Out[•]=



Out[•]=



In[•]:=
```
ContourPlot3D[{num1 == 0, num2 == 0, num3 == 0},
 {x, -lim, lim}, {y, -lim, lim}, {z, -lim, lim},
 ContourStyle → {Directive[Red, Opacity[0.5]], Directive[Green, Opacity[0.5]],
   Directive[Blue, Opacity[0.5]]}, Mesh → Full, AxesLabel → {"x", "y", "z"},
 PlotLabel → "Intersection of Three Surfaces", PlotPoints → 30]
```

Out[•]=