Belur University
Mid-Semestral Examination: (2021)
Introduction to Econometrics

Date: November 16 2021 Maximum Marks 50 Duration 1.30 hours

All notations are self-explanatory. This question paper carries 55 marks. You can answer any part of any question. However, the maximum that you can score is 50. Marks allotted to each question are given within parentheses.

1. Consider the linear regression model $y_n = \mu + \beta x_n + \varepsilon_n, n = 1,2,\ldots\ldots\ldots N$. Assume that $\{\varepsilon_n\}$ *is a sequence of independent random variables with the following distribution*:
   $P\{\varepsilon_n = 5\} = \frac{1}{2}, P\{\varepsilon_n = -5\} = \frac{1}{2}$.
   a. Check if CLRM conditions hold for ε_n.
   b. Show that OLS estimator of β is consistent. [7+8=15]

2. Consider the linear regression model $y_n = \mu + \varepsilon_n, n = 1,2,\ldots\ldots\ldots N$. Assume that $\{\varepsilon_n\}$ *is a sequence of dependent random variables with the following distribution*:
   $P\{\varepsilon_n = 1\} = \frac{1}{2}, P\{\varepsilon_n = -1\} = \frac{1}{2}, cov(\varepsilon_i, \varepsilon_j) = \begin{cases} \rho \; if \; |i-j| \le 2 \\ 0 \; otherwise \end{cases} \forall i \ne j = 1,2\ldots N.$

Examine if the least square estimator of μ is consistent or not.

[15]

3. Consider the multiple linear regression model as Y¿$X_1\beta_1 + X_2\beta_2 + \epsilon$, where Y is $n \times 1$, X_1 is $n \times k_1$, and X_2 is $n \times k_2$ matrices, respectively. Let $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$, and $\hat{\beta}_{ols} = \begin{bmatrix} \hat{\beta}_{1ols} \\ \hat{\beta}_{2ols} \end{bmatrix}$ be the OLS estimator of β.
   a. Without estimating the full parameter vector β, derive an estimator of β_1 *which is identical* to $\hat{\beta}_{1ols}$.
   b. Derive the variance of your estimator. [15+10=25]