Things I learned from Problems

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1 Bitwise XOR

1. If xor of an array is 0, there exists a prefix whose suffix == prefix
2. If suffix is x then xor of the suffixes are x

2 Lexicographically Smaller

1. $a_1 + a_2 + a_3$ is lexicographically smaller than $b_1 + b_2 + b_3$ if there's an index i for which $a_i < b_i$ and all previous indexes are equal.

3 Combinatorics

1.
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

means the number of k elements we can choose from set of n things

2.
$$\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$$

sum of numbers $1..n$

3. Sum of odd number of odd numbers is an odd number

4.
$$\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n} = 2^n$$

5.
$$\binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n} = 2^n - 1$$

6.
$$\binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n-1} = 2^n - 2$$

7.
$$\binom{n}{k} = \binom{n}{n-k}$$

8. Permutation of n things taken r at a time

$$p(n,r) = \frac{n!}{(n-r)!}$$

4 Mathematical Induction

You can prove a conjecture for all n using mathematical induction.

- **Steps** :

1. First show that the conjecture is true for some case n
2. Then find the value of the kth iteration of the problem
3. Then prove it for $(k+1)$

5 Number Theory

5.1 GCD

1. GCD of two number a and b is the largest integer that can divide a and b both.
2. For all $x \in \mathbb{N}$ and $x \geq 1$ then $gcd(x, x+1) = 1$