WORLDQUANT UNIVERSITY

Group Work Project

MScFE 620
Discrete-time Stochastic Processes

Revision Date: 08/03/2020

Overview

This document describes the requirements for the three Group Work Project assignments which must be submitted at the end of week 3, 5, and 7 respectively. Within a week of each submission, your group will receive feedback from the WQU Instructional Team, enabling you to use the feedback to revise your assignment ahead of the second and third submissions. You will use the Group Work Forum to communicate with your peers throughout the course.

Please make use of the **LIRN Library** located on the left pane of your screen as the primary resource for your research.

Your research should favor authoritative, scholarly sources, and you must cite all sources where relevant. The task is not to reproduce the research of others, but instead to develop your own systematic narrative that addresses the research topic and is informed by the research of others. Not only are you required to cite accurate and relevant facts, but you must also present your own clear logic when linking and contextualizing these facts.

Visit the **Student Resource Center (SRC)** where you can find resources on **how to conduct research**, how to use different sources of information, how to **cite references to** a**void plagiarism**, and how to use the **MLA citation style.**

Note: All Group Work Project assignments must be submitted via **Turnitin**, the anti-plagiarism software.

Group Work Project Objectives

Submission 1: Understand probability and discrete stochastic processes terminology by exploring the history of measure-theoretic probability and martingales

Submission 2: Develop trading strategies in discrete time based on binomial and trinomial tree models

Submission 3: Develop trading strategies in discrete time based on binomial and trinomial tree models

Submission 1: The History of Measure-theoretic Probability and Martingales

Submit a **report** illustrating the early history of probability theory from a measure- theoretic perspective. Your discussion must address the following:

1. Kolmogorov and his axioms of probability
2. Markov and Markov processes
3. J. L. Doob and the development of martingales

Submission Requirements

Submit a PDF report including a response of about 500 words for each part, for a total of 1,500 words.

To develop your assignment, use LaTeX editor (https://www.overleaf.com), which significantly simplifies the writing of complex mathematical formulae. You can replicate in LaTeX the "**Submission Template**" provided in the course room along with this Group Work Project Requirements. Then, convert the assignment to PDF and submit.

Note: If you submit multiple files, the PDF file with your report must be uploaded **separately** from the zipped folder that includes your other files. This allows Turnitin to generate the similarity report.

Submission 2: Trinomial Trees

A trinomial tree is a discrete-time model for stock prices that assumes that the evolution of the risky stock price X is:

$$X_{n+1} = X_n Z_n + 1, \qquad X_0 = \text{const},$$

where $(Zn)_{n=1}^{T}$ is a sequence of independent random variables each taking three distinct values.

For your submission, complete the following:

a) Consider a market with $T = 2$ and $d = 1$ risky stock. Construct a concrete trinomial tree model for X by specifying a $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ filtered probability space and defining the random variables Zn. Show explicit filtration – F0, F1 and F2.

b) Use the model above to place conditions under which it is arbitrage-free. Find the set of all EMMs for X under this condition. After finding EMM, discuss the possibility of making the market complete via additional assumptions.

Submission Requirements

Submit a PDF report including a response of about 1,000 words **for each part**, for a total of 2,000 words.

To develop your assignment, use LaTeX editor (https://www.overleaf.com), which significantly simplifies the writing of complex mathematical formulae. You can replicate in LaTeX the "**Submission Template**" provided in the course room along with this Group Work Project Requirements. Then, convert the assignment to PDF and submit.

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Submission 3: American Options

Consider a discrete-time market $((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}), S)_{n=0}^{T}$ with one stock S and a fixed (continuously compounded) interest rate of $r > 0$ per period.

a) Provide a theoretical proof that for an American call option on S, there is no benefit for early exercise and the price of this option is the same as the price of its European counterpart.

b) Show that the same does not hold generally for an American put option. First, show where the theoretical proof from part a) breaks down in this case. Then, give a numerical example with Snell envelope, optimal exercise strategy, Doob's decomposition, and hedging portfolio in the Binomial tree model with T = 2 and r = 10%. (Keep only two decimal places in all calculations.)

c) Describe and price the so-called perpetual American put option within the Binomial tree model. Begin with short theory, then proceed with pricing the example from part b) with the same r and K.

Submission Requirements

Submit a PDF report including a response of about 1,000 words for **each part**, for a total of 3,000 words.

To develop your assignment, use LaTeX editor (https://www.overleaf.com), which significantly simplifies the writing of complex mathematical formulae. You can replicate in LaTeX the "**Submission Template**" provided in the course room along with this Group Work Project Requirements. Then, convert the assignment to PDF and submit.

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