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Capital Asset Pricing Model (CAPM)

Capital Asset Pricing Model (CAPM) aims to describe the relationship between systematic risk and the expected returns of assets. The model was introduced by Jack Treynor (1961, 1962), William F. Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) independently.

The formula for calculating the expected return of an asset given its risk is given by:

$$ER_i = R_f + \beta_i \cdot (ER_m - R_f)$$

where ER_i is the expected return of the investment.
R_f is the risk free rate (usually the rate on the US Treasury Bill).
β_i is the beta of the investment. Beta is a measure of the volatility, or systematic risk, of a security or portfolio in comparison to the market as a whole.
and $(ER_m - R_f)$ is the market risk premium, that is the return expected from the market above the risk-free rate.

Assumptions	Limitations
Investors are rational and are risk-averse	Difficult to estimate beta. The lookback period to estimate beta is not well established because stock returns are not normally distributed.
Only Systematic Risk is present.	If portfolio is not fully well-diversified, the portfolio will be open to risks other than the systematic risk.
Markets are efficient and all information is available to all participants.	Risk free rate may not remain constant.
There are no transaction costs and taxes involved	The beta of an asset may not be constant.
Investors can borrow and lend at risk-free rate	It does not include the possibility of investors who are willing to take higher risk for lower expected returns.

Arbitrage Pricing Theory (APT)

Arbitrage Pricing Theory (APT) is a multi-factor asset pricing model. It describes assets expected returns using a linear relationship between the expected return and several macroeconomic variables which capture systematic risk. The sensitivity to each of these factors is represented by a factor-specific beta coefficient. It was introduced by Stephen Ross (1976).

If the Asset Returns for asset j is following a factor intensity structure given by

$$r_j = a_j + \lambda_{j1} \cdot f_1 + \lambda_{j2} \cdot f_2 + \lambda_{j3} \cdot f_3 + \cdots + \lambda_{jn} \cdot f_n$$

where a_j is constant for asset j
f_i is systematic factor
λ_{ji} is sensitivity of asset j to factor i and ϵ_j is risky asset's idiosyncratic random shock with mean zero, and it is assumed to be uncorrelated across assets and factors.

Then APT states that the expected returns for this asset j will have the following formula:

$$ER_j = r_f + \lambda_{j1} \cdot RP_1 + \lambda_{j2} \cdot RP_2 + \cdots + \lambda_{jn} \cdot RP_n$$

where, ER_j is the expected return of asset j, RP_i is the risk premium of the factor and r_f is the risk-free rate.

Assumptions	Limitations
Investors are rational and are risk-averse	APT does not specify the factors.
Markets are efficient and all information is available to all participants.	It is difficult to accurately calculate the beta of each factor and their stability.
There are no transaction costs and taxes involved	Risk free rate may not remain constant.
Factors that are sensitive are uncorrelated.	It is difficult to construct a pure factor portfolio.

Literature Review

In Devinaga, J. and Peongkwee, M. (2011). "The Effectiveness of Arbitrage Pricing Model in Modern Financial Theory,", the authors emphasize on the potential of **APT to overcome CAPM's** limitations since it requires **less assumptions which also are more realistic**. Its **explanatory power is also higher** since it is a multi-factor model as opposed to CAPM which only focusses on market risk premium. In the same paper, we also see**, CAPM theory is more intuitive and easier to implement as compared to APT which is complex and does not outline the factors.**

Richard, C.G. and Ronald, N.K. (2011). "Active Portfolio Management: A Quantitative Approach for Providing Superior Returns and Controlling Risk,", investors usually employ a common index like the S&P 500 as a comparison between the asset's sensitivity to the market. This provides an easy way to model an assets risk level and find the expected rate of return in an intuitive and easier way.

In Levy, I. and Solomon, H. (2000). "Arbitrage Pricing Theory: Evidence from an Emerging Stock Market,", we find **that CAPM is based on over simplifying assumptions which are unrealistic**. Assumptions like no transaction costs and taxes are impossible to achieve, assumptions of homogenous assumptions have also been criticised as market participants usually have divergent expectations.

One of the major critiques to CAPM have been in found in the paper Roll, R. (1977). "A Critique of the Asset Pricing Theory's Tests, Part I: On Past and Potential Testability of the Theory,". **CAPM assumptions that systematic risk can be captured by one beta factor is unreliable**. Also, the assumption that asset returns are normally distributed, or that firm-specific risks are usually eliminated are incorrect, and hence a multi-factor model like APT would be more reliable.

We find the most obvious criticism of APT in Morel, L. (2001). "An Empirical Investigation of Asset-Pricing Models in Australia,", where the author highlights that APT does not identify the factors and are not supported by the theoretical foundations of CAPM. It identifies APT as a too general asset pricing model.

We find that the **multi-collinearity of economic variables is a major drawback to APT** in Paavola, B. (2006). "The Effects of Macroeconomics Variables on Stock Returns: Evidence from Turkey,".

Fama-French Model

Nobel Laureate Eugene Fama and researcher Kenneth French, former professors at the University of Chicago Booth School of Business, attempted to better measure market returns and, through research, found that value stocks outperform growth stocks. Similarly, small-cap stocks tend to outperform large-cap stocks.

The Fama and French model has three factors: size of firms, book-to-market values, and excess return on the market. In other words, the three factors used are SMB (small minus big), HML (high minus low) and the portfolio's return less the risk-free rate of return. SMB accounts for publicly traded companies with small market caps that generate higher returns, while HML accounts for value stocks with high book-to-market ratios that generate higher returns in comparison to the market.

The underlying concept behind the model is that the returns generated by portfolio managers are due in part to factors that are beyond the managers' control. Specifically, value stocks have historically outperformed growth stocks on average, while smaller companies have outperformed larger ones.

The first of these factors (the outperformance of value stocks) is referred to by the term High Minus Low (HML), whereas the second factor (the outperformance of smaller companies) is referred to by the term Small Minus Big (SMB).

$$r - R_f = \beta * (K_m - R_f) + b_3 * SMB + b_\gamma * HML + \alpha$$

where r is the portfolio's return rate
R_f is the Risk-free return rate
K_m is the return of the whole stock market
and α is the outperformance.

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