

# MUSA 174 Homework 1

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## 1 Problem 1.1.i

**Solution** Let  $C$  be a category, and let  $f : x \rightarrow y$ ,  $g : y \rightarrow x$  and  $h : x \rightarrow y$  be morphisms such that  $gf = 1_x$  and  $fh = 1_y$ . Then,  $g = h$ , since

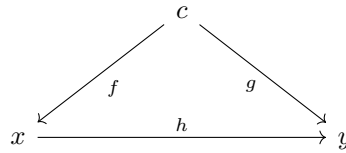
$$g = g1_y = g(fh) = (gf)h = 1_xh = h$$

Then  $f$  is an isomorphism as well, since there is a morphism  $f^{-1} = g = h$  such that  $f^{-1}f = 1_x$  and  $ff^{-1} = 1_y$ .

## 2 Problem 1.1.iii.i

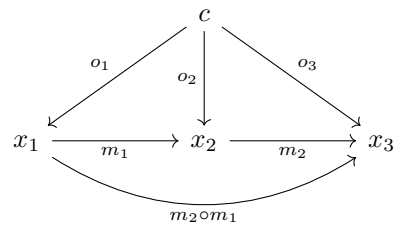
**Solution** Fixing a category  $C$  and an object  $c \in C$ , define a new category  $c/C$  such that:

- Objects in  $c/C$  are morphisms in  $C$  whose domain is  $c$ , or morphisms in  $C$  of the form  $f : c \rightarrow x$
- A morphism in  $c/C$  with domain  $f : c \rightarrow x$  and codomain  $g : c \rightarrow y$  is another morphism  $h : x \rightarrow y$  in  $C$  such that  $g = hf$ , or so that the following diagram commutes:



- Identity elements for each object  $f : c \rightarrow x$  will be the corresponding identity morphism  $1_x : x \rightarrow x$  in  $C$
- Given three objects  $o_1 : c \rightarrow x_1$ ,  $o_2 : c \rightarrow x_2$ , and  $o_3 : c \rightarrow x_3$ , with morphisms  $m_1 : x_1 \rightarrow x_2$  and  $m_2 : x_2 \rightarrow x_3$ , define the composition

$m_2 \circ m_1$  so that the following diagram commutes.



By the first diagram,  $o_2 = m_1 o_1$  and  $o_3 = m_2 o_2$ . Combining these gives  $o_3 = m_2 m_1 o_1$  or  $o_3 = (m_2 \circ m_1) o_1$

- Since all morphisms are some type of mapping, they are associative as well

Thus  $c/C$  meets all the requirements to be a category.