Math 113 Homework 1

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1 Problem 1

Solution Claim: $A \cap B = A \setminus B^c$.

Proof. Let $x \in A \cap B$. Then $x \in A$ and $x \in B$, so $x \notin B^c$. But then $x \in A$ and $x \notin B^c$, so $x \in A \setminus B^c$. Thus $A \cap B \subseteq A \setminus B^c$. On the other hand, let $x \in A \setminus B^c$. Then $x \in A$ and $x \notin B^c$, so $x \in A$ and $x \in B$, and thus $x \in A \cap B$. So $A \setminus B^c \subseteq A \cap B$. But then $A \cap B = A \setminus B^c$ $\square$

2 Problem 2

Solution Suppose $f_\rho = f_\sigma$. Then f_ρ and f_σ have the same domains, codomains, and assignments. In particular, for all $x_1, \ldots, x_n \in X_1, \ldots X_n$, $f_rho(x_1, \ldots, x_n) = f_\sigma(x_1, \ldots, x_n) = y$. By the definition of f_σ and f_ρ, this means that the relations $\sigma(x_1, \ldots, x_n, y)$ and $\rho(x_1, \ldots, x_n, y)$ both hold, so the relations ρ and σ are equipotent.
On the other hand, suppose σ and ρ are equipotent. Then $\sigma(x_1, \ldots, x_n, y)$ holds exactly when $\rho(x_1, \ldots, x_n, y)$ holds. But this means that for all $x_1, \ldots, x_n \in X_1, \ldots X_n$, $f_rho(x_1, \ldots, x_n) = f_\sigma(x_1, \ldots, x_n) = y$. Then f_ρ and f_σ have the same domains, codomains, and targets, and thus $f_\rho = f_\sigma$. Therefore, $f_\rho = f_\sigma$ if and only if ρ and σ are equipotent.

3 Problem 3

Solution Suppose $x = x'$ and $y = y'$. Then clearly $\{\{x\}, \{x, y\}\} = \{\{x'\}, \{x', y'\}\}$. On the other hand, if $\{\{x\}, \{x, y\}\} = \{\{x'\}, \{x', y'\}\}$, then by definition $x = x'$ and $y = y'$, since the ordered pairs $(x_1, \ldots, x_m)$ and $(y_1, \ldots, y_n)$ are equal if and only if $n = m$ and $x_i = y_i$ for $i = 1, \ldots, n$. In this case, our ordered pair consists of two elements, one of which is an ordered pair itself. Thus they are equal if $x = x'$ and $y = y'$, so $\{\{x\}, \{x, y\}\} = \{\{x'\}, \{x', y'\}\}$ if and only if $x = x'$ and $y = y'$.