

CS575 Homework 1

Instructor: KD Kang

Please remember to include the following paragraph at the beginning of your submitted assignment, write down your name, and SIGN it.

“I have done this assignment completely on my own. I have not copied it, nor have I given my solution to anyone else. I understand that if I am involved in plagiarism or cheating I will have to sign an official form that I have cheated and that this form will be stored in my official university record. I also understand that I will receive a grade of 0 for the involved assignment for my first offense and that I will receive a grade of “F” for the course for any additional offense.”

1. [30%] Prove the following using the original definitions of O , Ω , Θ , o , and ω .
 - (a) $10n^3 + 2n + 15 = O(n^3)$.
 - (b) $7n^2 = \Omega(n)$.
 - (c) $5n^2 = \omega(n)$.
 - (d) $7n^3 + 15n^2 + 5 = \Theta(n^3)$.
 - (e) $p(n) = \sum_{i=1}^k a_i n^i = \Theta(n^k)$ where $a_i > 0$.
2. [20%] Prove the following using limits.
 - (a) $n^k = o(3^n)$ where $k > 0$.
 - (b) $n = \omega(\lg n^5)$.
3. [10%] Prove that $3^n - 1$ is divisible by 2 for $n = 1, 2, 3, \dots$ by induction. Divide your proof into the three required parts: Induction Base, Induction Hypothesis, and Induction Steps.
4. [10%] Prove or disprove that $n^3 = O(n^2)$.
5. [10%] Just say True or False for the following.
 - (a) $1000000n^2 + 5000 = \Theta(n^2)$.

- (b) $2^{n+1} = O(2^n)$.
 - (c) $n^3 + n^2 + 100n = \Omega(n^3)$.
 - (d) $n^{1000} = \omega(2^n)$.
 - (e) $\log n^{100} = \Omega(\log n)$.
6. [10%] Analyze the worst case time complexity of recursive binary search using the iterative method. Assume the number of data $n = 2^k$.
 7. [10%] The following pseudo code computes a factorial for input parameter n that is expressed via s bits where $n, s \geq 1$. What is the time complexity of the pseudo code?

```

unsigned int fact(unsigned int n) {
    unsigned int p = 1;
    for (i=1; i ≤ n; i++) p = p * i;
    return p;
}

```