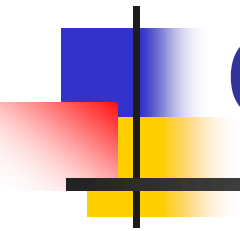
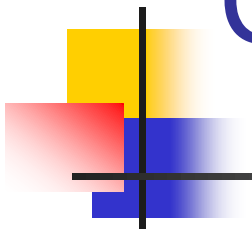


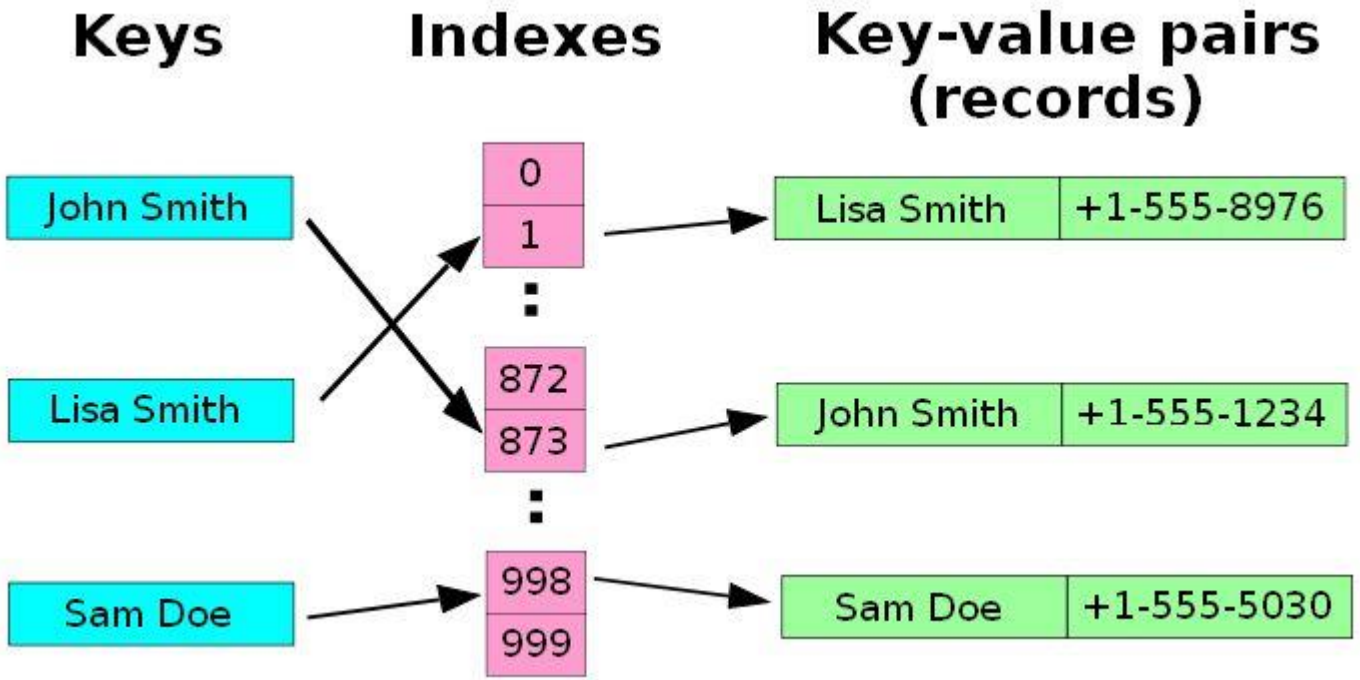
Chapter 14 Hashing

Concept of Hashing



- A **hash table** is a data structure that associates keys (names) with values (attributes).
  - Look-Up Table
  - Dictionary
  - Cache
  - Memcached: a.k.a. distributed hash table

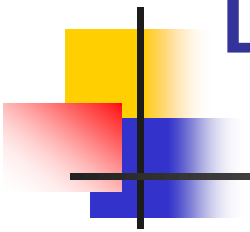
Example



A small phone book as a hash table.

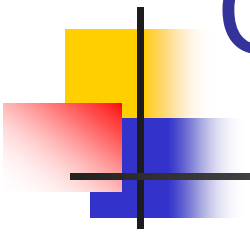
(Figure from Wikipedia)

Dictionaries



- Collection of pairs
  - (key, value)
  - Each pair has a unique key
- Operations.
  - Get(key)
  - Delete(key)
  - Insert(key, value)

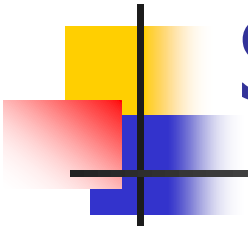
Overall Idea



- Hash table :
  - Collection of pairs,
  - Lookup function (Hash function)
- Hash tables are often used to implement associative arrays,
  - Worst-case time for Get, Insert, and Delete is O(size).
  - Expected time is O(1).

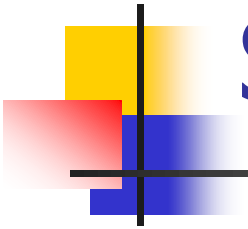
Origins of the Term

- The term "hash" comes by way of analogy with its standard meaning in the physical world, to "chop and mix." **D. Knuth** notes that **Hans Peter Luhn** of IBM appears to have been the first to use the concept, in a memo dated January 1953; the term hash came into use some ten years later.



Search vs. Hashing

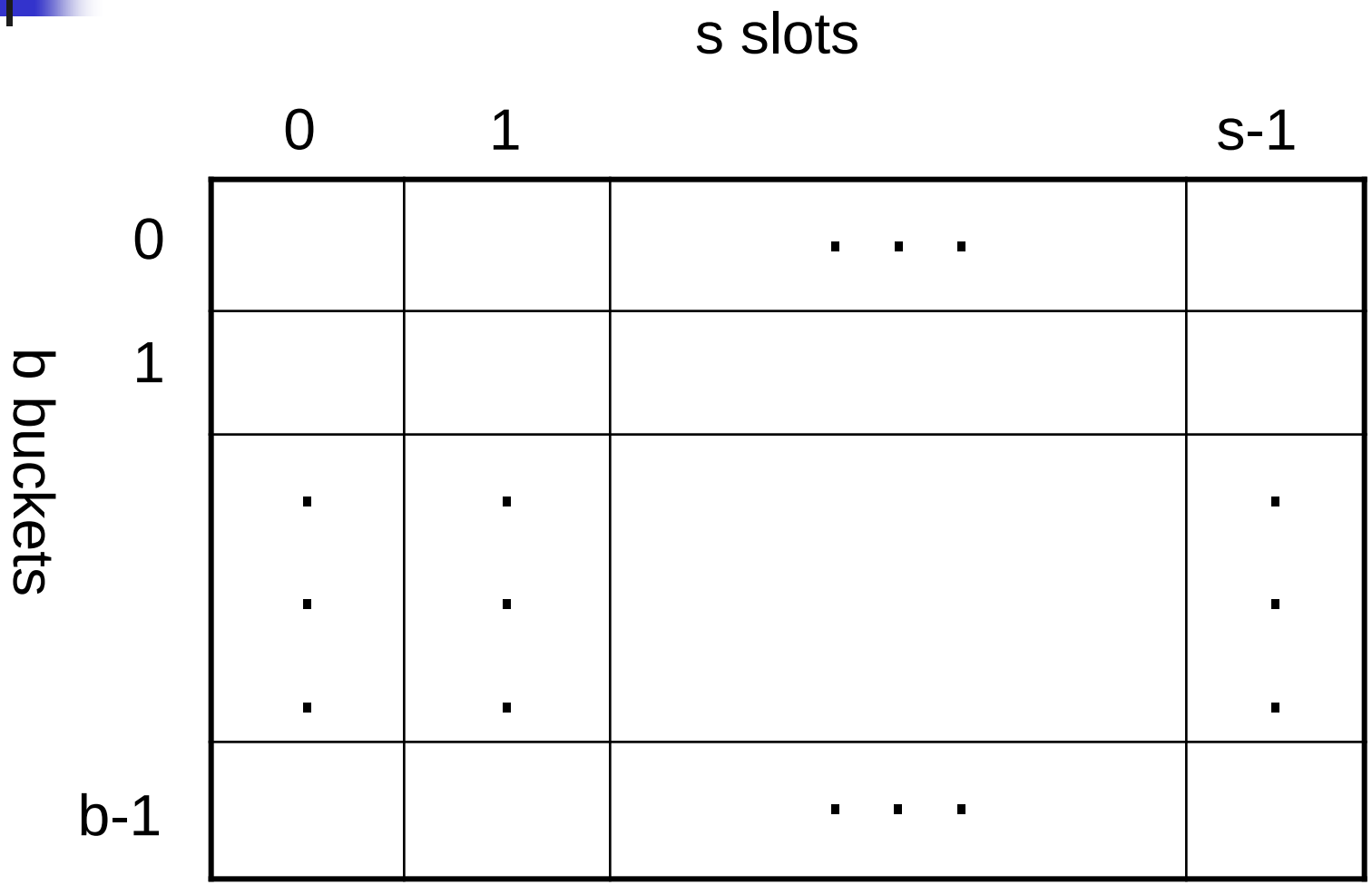
- Search tree methods: key comparisons
  - Time complexity: O(size) or O(log n)
- Hashing methods: hash functions
  - Expected time: O(1)
- Types
  - Static hashing
  - Dynamic hashing



Static Hashing

- Key-value pairs are stored in a fixed size table called a *hash table*.
  - A hash table is partitioned into many ***buckets***.
  - Each bucket has many ***slots***.
  - Each slot holds one record.
  - A hash function f(x) transforms the identifier (key) into an address in the hash table

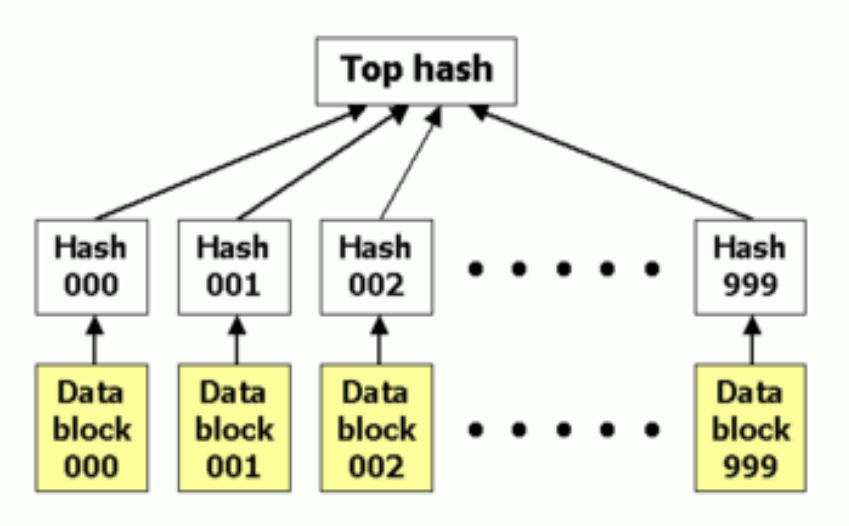
Hash table

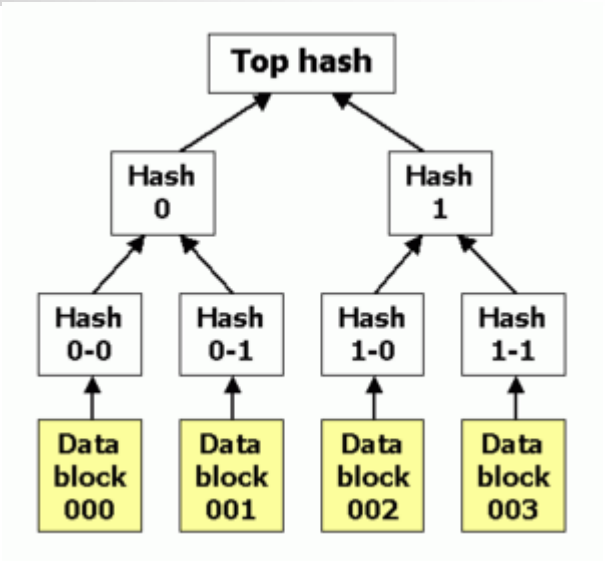


Data Structure for Hash Table

```
#define MAX_CHAR  10
#define TABLE_SIZE  17
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
element hash_table[TABLE_SIZE];
```

Other Extensions

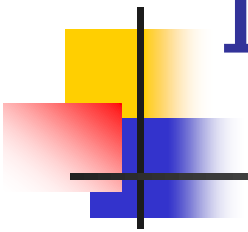




Hash List and Hash Tree

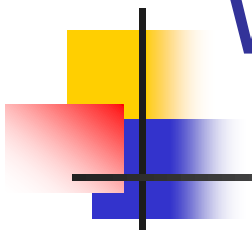
(Figure is from Wikipedia)

Ideal Hashing



- Uses an array table[0:b-1].
  - Each position of this array is a bucket.
  - A bucket can normally hold only one dictionary pair.
- Uses a hash function f that converts each key k into an index in the range [0, b-1].
- Every dictionary pair (key, element) is stored in its home bucket table[f[key]].

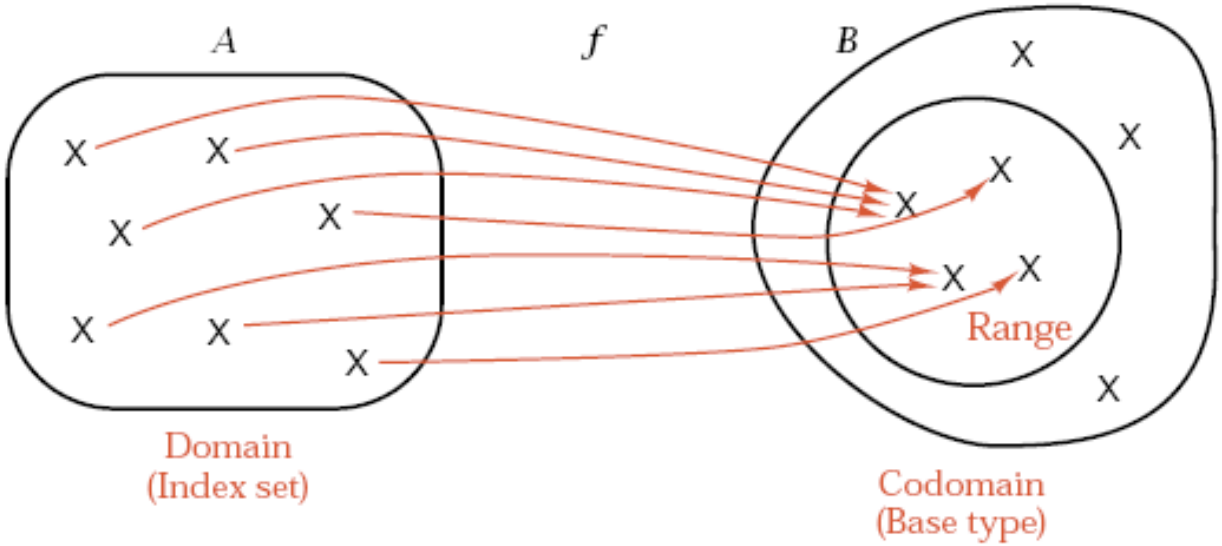
What Can Go Wrong?



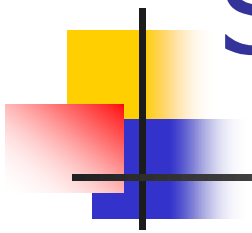
- More than one keys can be hashed to the same bucket
- Keys that have the same home bucket are synonyms
- How to deal with this?
  - First, choose a good hash function to minimize collisions
  - Second, handle overflow efficiently

Example

- More than one keys can be hashed to the same bucket



Some Issues



- **Choice of hash function**
  - ***Really tricky!***
  - To avoid collision where two different pairs are in the same bucket
  - Size (number of buckets) of hash table is desired to be small
- **Overflow handling method**
  - Overflow: there is no space in the bucket for the new pair.

Example

synonyms:
char, ceil,
clock, ctime

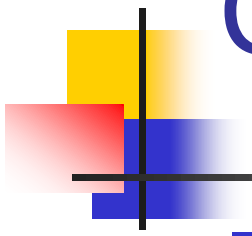
overflow

	Slot 0	Slot 1
0	acos	atan
1		
2	char	ceil
3	define	
4	exp	
5	float	floor
6		
. . .		
25		

synonyms

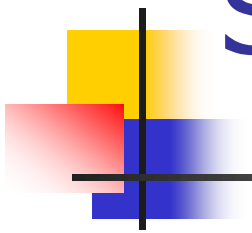
synonyms

Choice of Hash Function

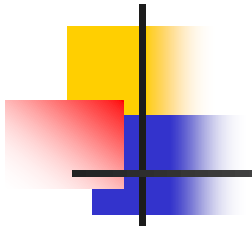


- Requirements
  - easy to compute
  - minimal number of collisions
- If a hashing function groups key values together, this is called clustering of the keys
- A good hashing function distributes key values uniformly throughout the range
- Ideally, simple uniform hashing

Some hash functions



- Middle of square
  - H(x):= return middle digits of x^2
- Division
  - H(x):= return x % m
  - Choosing good m is tricky
  - Bad example: m is power of 2
  - Generally, choose a prime that is not so close to power of 2 as m

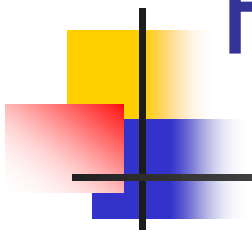


- Multiplicative:
  1. Choose A where $0 < A < 1$
  2. Multiply key k by A
  3. Extract the fractional part of k*A
  4. Multiply the fractional part by the number of slots m
  5. Take the floor of the result
- Pro: Value of m is not critical
- Con: slower than division
  - **advocated by D. Knuth in TAOCP vol. III.**

Some hash functions II

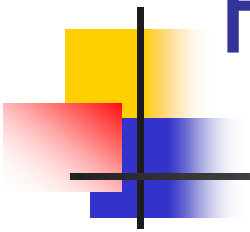
- Folding:
  - Partition the identifier x into several parts, and add the parts together to obtain the hash address
  - e.g. x=12320324111220; partition x into 123,203,241,112,20; then return the address 123+203+241+112+20=699
- Digit analysis:
  - If all the keys have been known in advance, then we could delete the digits of keys having the most skewed distributions, and use the remaining digits as hash address.
  - Distribution of keys may not be known in advance

Hashing By Division



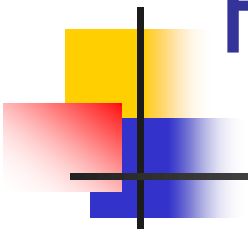
- Domain is all integers.
- For a hash table of size *b*, the number of integers that get hashed into bucket *i* is approximately $\mathbf{2^{32}/b}$.
- The division method results in a uniform hash function that maps approximately the same number of keys into each bucket.

Hashing By Division II



- In practice, keys tend to be correlated.
  - If divisor is an even number, odd integers hash into odd home buckets and even integers into even home buckets.
    - 20%14 = 6, 30%14 = 2, 8%14 = 8
    - 15%14 = 1, 3%14 = 3, 23%14 = 9
  - divisor is an odd number, odd (even) integers may hash into any home.
    - 20%15 = 5, 30%15 = 0, 8%15 = 8
    - 15%15 = 0, 3%15 = 3, 23%15 = 8

Hashing By Division III



- Similar biased distribution of home buckets is seen in practice, when the divisor is a multiple of prime numbers such as 3, 5, 7, …
- Ideally, choose large prime number b.
- Alternatively, choose b so that it has no prime factors smaller than 20.

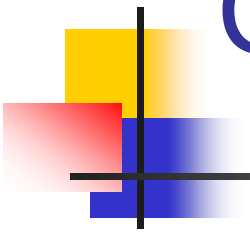
Hash Algorithm via Division

```
void init_table(element ht[])
{
  int i;
  for (i=0; i<TABLE_SIZE; i++)
    ht[i].key[0]=NULL;
}

int transform(char *key)
{
  int number=0;
  while (*key) number += *key++;
  return number;
}
```

```
int hash(char *key)
{
  return (transform(key)
          % TABLE_SIZE);
}
```

Criterion of Hash Table



- The key density (or identifier density) of a hash table is the ratio n/T
  - n is the number of keys in the table
  - T is the number of distinct possible keys
- The loading density or loading factor of a hash table is α = n/(sb)
  - s is the number of slots
  - b is the number of buckets

Example

synonyms:
char, ceil,
clock, ctime

overflow

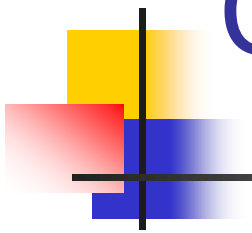
	Slot 0	Slot 1
0	acos	atan
1		
2	char	ceil
3	define	
4	exp	
5	float	floor
6		
…		
25		

synonyms

synonyms

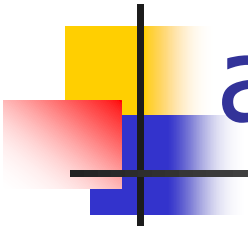
b=26, s=2, n=10, α=10/52=0.19, f(x)=the first char of x

Overflow Handling



- An overflow occurs when the home bucket for a new pair (key, element) is full.
- We may handle overflows by:
  - Search the hash table in some systematic fashion for a bucket that is not full.
    - Linear probing (linear open addressing).
    - Quadratic probing.
    - Random probing.
  - Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
    - Array linear list.
    - Chain.

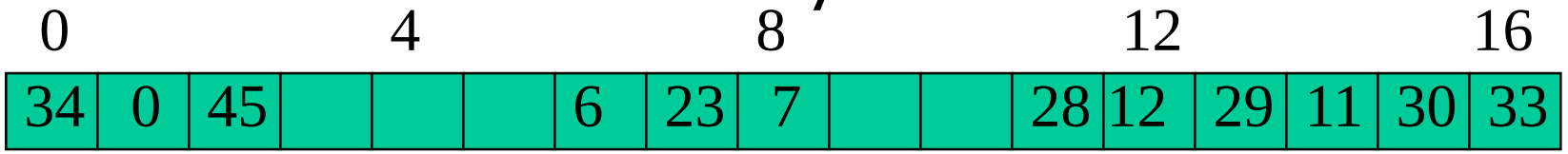
Linear probing (linear open addressing)



- **Open addressing** ensures that all elements are stored directly into the hash table. It attempts to resolve collisions using various methods.
  - **Linear Probing** resolves collisions by placing the data into the next open slot in the table.

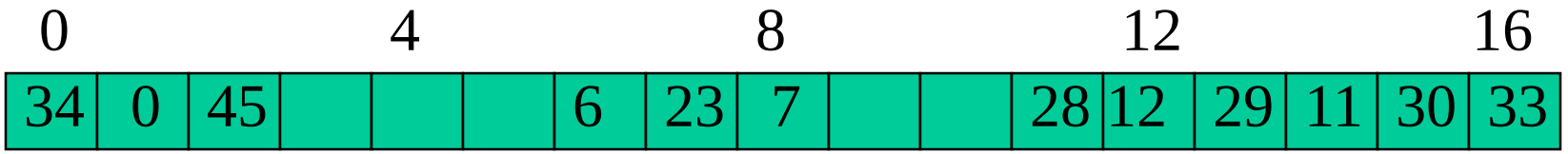
Linear Probing – Get And Insert

- Compute hash function H(k)
- If occupied, probe H(k) + 1, H(k)+ 2, …
- divisor = b (number of buckets) = 17.
- Home bucket = key % 17.



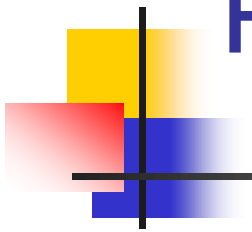
- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Performance Of Linear Probing



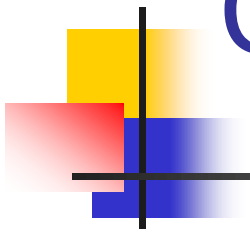
- Pro: Easy to implement
- Con: Primary clustering
  - Long runs of occupied slots build up. An empty slot preceded by i full slots gets filled next with probability (i+1)/m
- Worst case time is $\Theta(n)$. This happens when all pairs are in the same cluster

Problem of Linear Probing



- Identifiers tend to cluster together
- Adjacent cluster tend to coalesce
- Increase the search time

Quadratic Probing

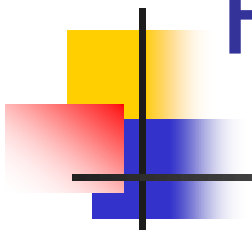


- Compute H(k)
- If occupied, probe H(k) + 1^2, H(k) + 2^2, H(k) + 3^2, …
- Secondary clustering
  - If the initial hash value is the same for two different keys, their probe sequences are the same
  - Could be better than primary clustering though

Random Probing

- Random Probing works incorporating with random numbers.
  - Buckets are examined in the order H(x), (H(x) + R[i]) % b where $1 \leq i < b$.
  - R[i] is a table with size b-1
  - R[i] is a (pseudo) random permutation of integers 1,2, …, b-1.

Rehashing



- **Rehashing**: Try H_1, H_2, …, H_m in sequence if collision occurs. (H_i is a hash function.)

Rehashing (cont'd)

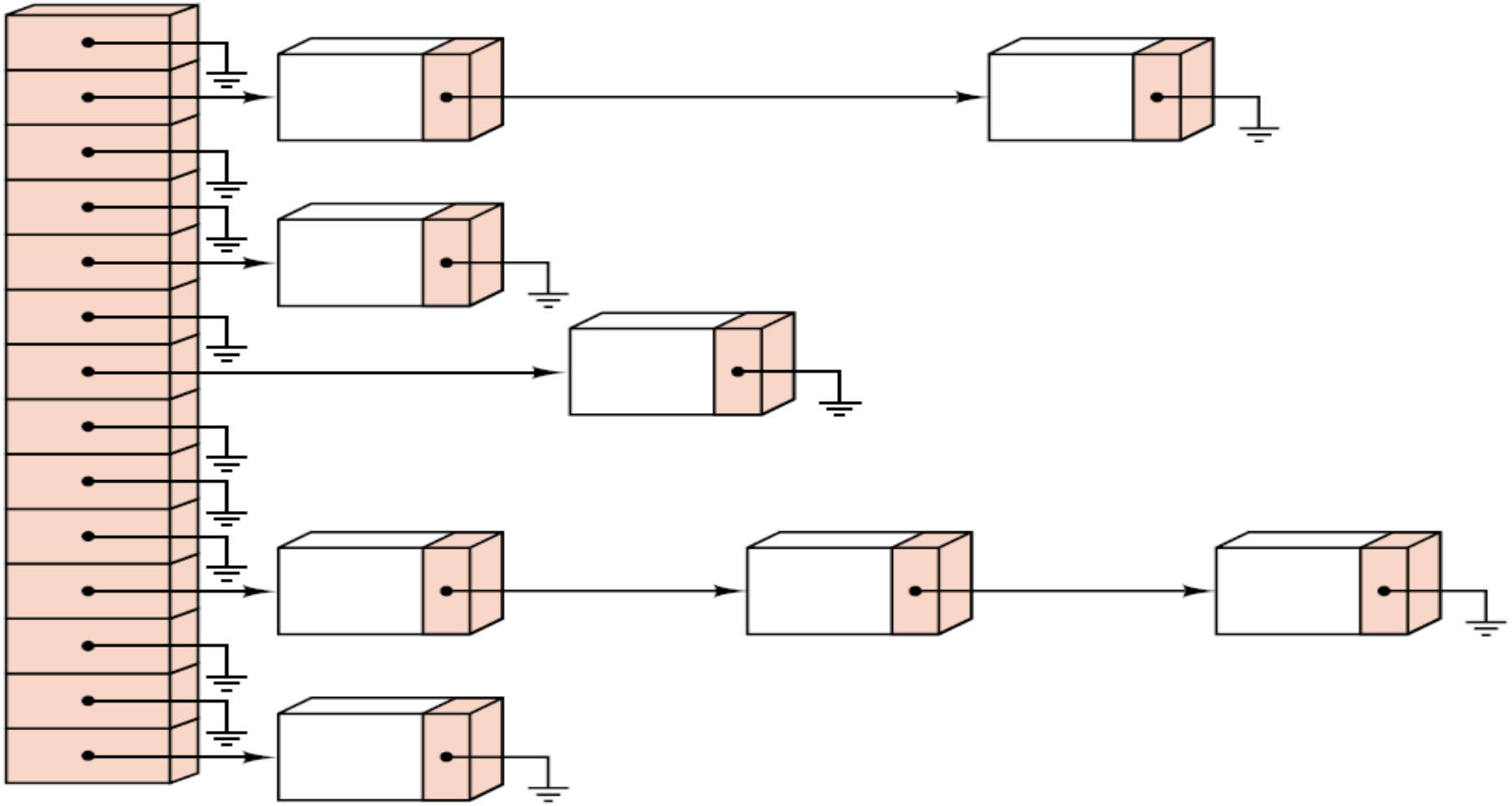
- **Double hashing** is one of the best methods for dealing with collisions.
  - If the slot is full, then a second hash function is calculated and combined with the first hash function.
  - $H(k, i) = (H_1(k) + i\, H_2(k)) \% b$
  - Probe sequence is:
    - $H_1(k) \% b$
    - $(H_1(k) + 1H_2(k)) \% b$
    - $(H_1(k) + 2H_2(k)) \% b$
    - …

Data Structure for Chaining

```
#define MAX_CHAR 10
#define TABLE_SIZE 13
#define IS_FULL(ptr) (!(ptr))
typedef struct {
    char key[MAX_CHAR];
    /* other fields */
} element;
typedef struct list *list_pointer;
typedef struct list {
    element item;
    list_pointer link;
};
list_pointer hash_table[TABLE_SIZE];
```

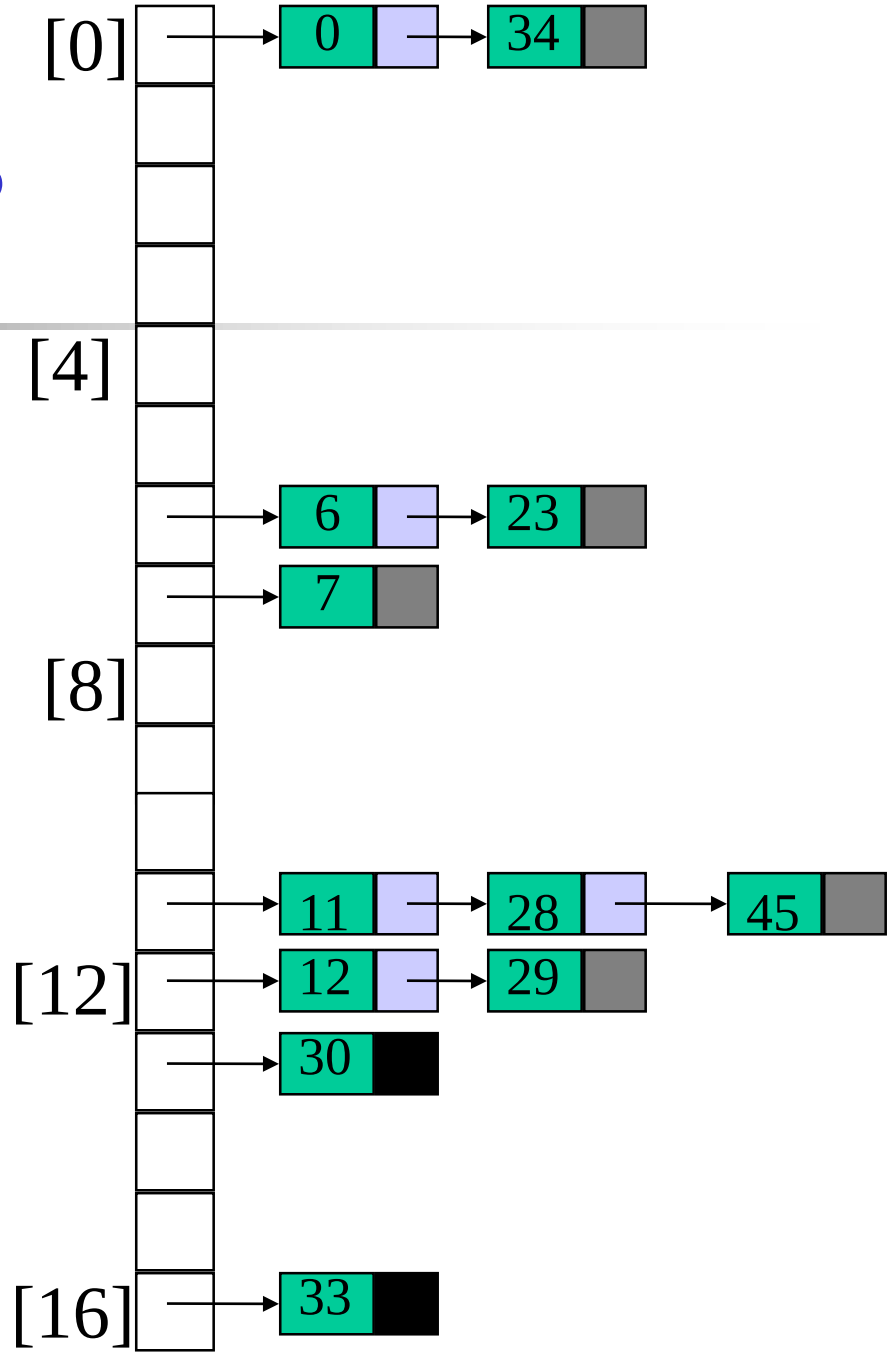
The idea of **Chaining** is to combine the linked list and hash table to solve the overflow problem.

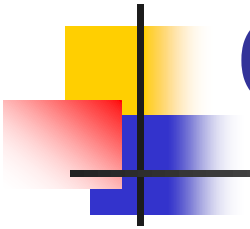
Figure of Chaining



Sorted Chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Bucket = key % 17.





Conclusion

- **Linear probing** has the best cache performance but is most sensitive to clustering
- **Double hashing** has poorer cache performance but exhibits virtually no clustering
- **Quadratic probing** falls in between the two methods above