

Welcome 😊

Agenda : Knapsack.
2-4 questions.

Q Given a rod of length N and an array A of length N

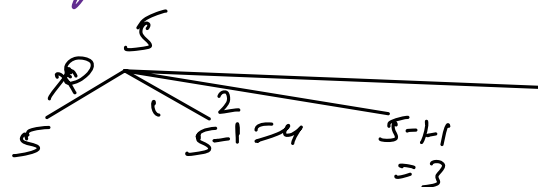
$A[i] \Rightarrow$ price of i length rod (index is 1 based)

Find max value that can be obtained by cutting rod into some pieces & selling them.

eg:

$N=5$

$A = [1 \quad 4 \quad 2 \quad 5 \quad 6]$
 1 2 3 4 5



sold length

prices.

5

6

1 + 4

6

3 + 2

6

3 + 1 + 1

4

2 + 2 + 1

9

2 + 1 + 1 + 1

7

1 + 1 + 1 + 1 + 1

5

Unbounded Knapsack.

state \Rightarrow length of rod.

$W \rightarrow$ length of rod.

$wt[i] \rightarrow$ length of piece of rod

$value[i] \rightarrow A[i]$

$dp[i] \Rightarrow$ max value if length is i

$\forall i, dp[i] = 0$

for ($i = 1 \rightarrow N$) // capacity / length.

{

for ($j \rightarrow 1$ to i)

{

$dp[i] = \max(dp[i], A[j] + dp[i-j])$

}

}

return $dp[N]$

Q In how many ways can we make N by using coins given in the arrays.

$A[i] \Rightarrow$ value of i^{th} coin

One coin can be used multiple times

A: Ordered selection

$(x, y) \neq (y, x)$

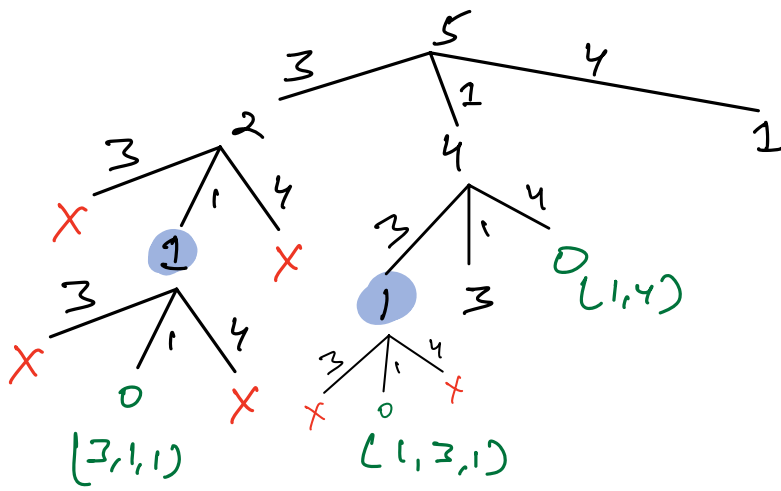
eg: $N=5$

A: [3 1 4]

$(1, 4) \{ 3, 1, 1 \} \{ 1, 1, 3 \}$

$(4, 1) \{ 1, 3, 1 \} \{ 1, 1, 1, 1, 1 \}$

ans = 6



$dp[i] \Rightarrow$ # ways to select coins s.t. sum = i

$\forall i \ dp[i] = 0$

$dp[0] = 1$

for ($i \rightarrow 1$ to N)

{

for ($j \rightarrow 0$ to $A.length() - 1$)

{

if ($A[j] \leq i$)

{

$dp[i] = dp[i] + dp[i - A[j]]$

}

}

}

return $dp[N]$

T.C = $O(N * A.length())$

S.C = $O(N)$

B. Un-ordered

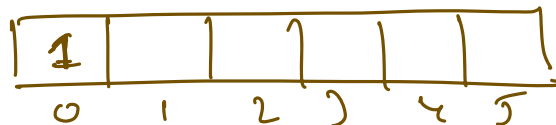
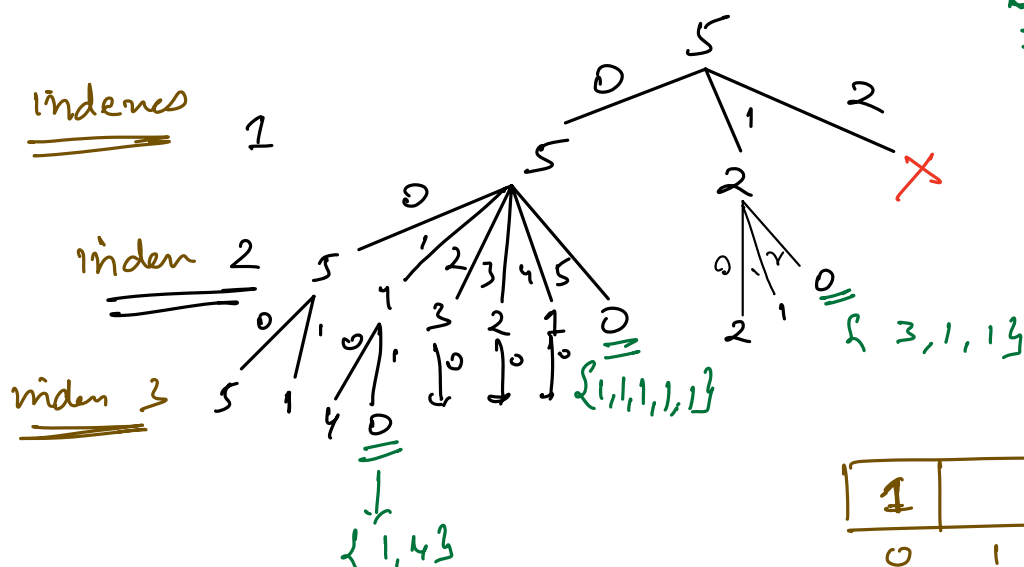
$$(x, y) = (y, x)$$

eg: $N=5$

A: $\begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix}$

$(1, 4)$ ~~$\{3, 1, 1\}$~~ ~~$\{1, 1, 3\}$~~
 ~~$(4, 1)$~~ $\{1, 3, 1\}$ $\{1, 1, 1, 1, 1\}$

~~ans = 6~~ 3



$dp[i] \Rightarrow$ # ways to select coins s.t. sum = i

$\forall i \ dp[i] = 0$

$dp[0] = 1$

for ($j \rightarrow 0$ to $A.length() - 1$) // inden

{

for ($i \rightarrow 1$ to N) // sum

{

if ($A[j] \leq i$)

{

$dp[i] = dp[i] + dp[i - A[j]]$

}

}

}

return $dp[N]$

$j = 0$
 $win = 3$

T.C = $O(N * A.length())$

S.C = $O(N)$

Q

We are given N bags with their happiness & weight. Find max. total happiness that can kept in a bag. with capacity W . 0-1

$$0 \leq N \leq 500$$

$$0 \leq h[i] \leq 50$$

$$0 \leq wt[i] \leq 10^9$$

$$0 \leq W \leq 10^9$$

change state

T.C $\Rightarrow O(N \times W)$

\swarrow

$500 \times 10^9 = 10^{11}$

TLE