

Assignment 1

Part 2

1

$$\sum_{i=0}^{n+1} 1 = 1 + 1 + 1 + \dots + n+1 = \boxed{O(n)}$$

2

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1 = \sum_{j=0}^{n-1} n = n^2. \quad \boxed{O(n^2)}$$

3.

$$\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1 = \sum_{i=0}^{n-1} i = n \times i = n \times (n-1) \because i = n-1$$

$$\therefore n^2 - n \quad \boxed{O(n^2)}$$

4

$$\sum_{i=0}^{n^2-1} \sum_{j=0}^{i-1} 1 = n^2 \times i \quad \because i = n^2-1$$

$$\therefore n^2 \times (n^2-1) = n^4 - n^2$$

$$\boxed{O(n^4)}$$

5

$$\sum_{i=0}^{n^2-1} \sum_{j=0}^{i-1} \sum_{k=0}^{j-1} 1 = \sum_{i=0}^{n^2-1} \sum_{j=0}^{i-1} j = \sum_{i=0}^{n^2-1} i j = n^2 i j$$

$$= n^2 i (i-1)$$

$$= n^2 (i^2 - i) \quad i = n^2 - 1$$

$$= n^2 ((n^2-1)^2 - n^2 + 1)$$

$$= n^2 (n^4 - 2n^2 + 1 - n^2 + 1)$$

$$= n^2 (n^4 - 3n^2 + 2)$$

$$= n^6 - 3n^4 + 2n^2$$

$$\boxed{O(n^6)}$$

6.

$$\sum_{i=0}^{n-1} \sum_{j=0}^{i^2-1} \sum_{k=0}^{j-1}$$

$$= n \cdot i^2 \cdot j$$

$$j = i^2 - 1$$

$$\frac{n(i^2)(i^2-1)}{n(i^4-i^2)}$$

$$i = n-1$$

$$n(i^2-i)(i^2+i)$$

$$n\{(n-1)^2-(n-1)\}\{(n-1)^2+(n-1)\}$$

$$n(n^2-2n+1-n+1)(n^2-2n+1+n-1)$$

$$n(n^2-3n+2)(n^2-n)$$

$$n(n^4-n^3-3n^3+3n^2+2n^2-2n)$$

$$n^5-4n^4+5n^3-2n^2$$

$$O(n^5)$$