Design of State Feedback Controller and Luenberger Observer for Inverted Pendulum systems

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Contents

Introduction

- Inherently a nonlinear system

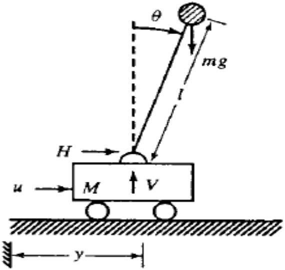


Figure 1: Inverted Pendulum model

- Unstable system
- x,$\dot{x}$,ϕ and $\dot{\phi}$ needs to be controlled

Modelling

The state space model about $\phi = 0$ can be given by;

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)g}{Ml} & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix}$$

C=$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$ $\quad \mathbf{D} = 0$

Substituting the following values:
$m = 5kg, M = 10kg, l = 20m, g = 9.8m/s^2$

Modelling

Final matrices are:

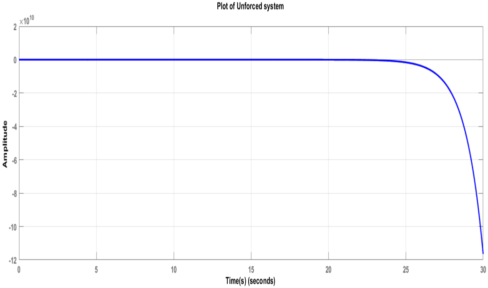
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -4.905 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.73575 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ -0.005 \end{bmatrix}$$

C=$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$ $\quad \mathbf{D} = 0$

Objective

- To make the system stable
- To place the poles at desired locations in the left half plane of s-plane
- To observe the states even when initial conditions are known or unknown
- System response of the unforced system with poles=

$$[0, 0, 0.8578, -0.8578]$$



State Feedback Controller

The control signal is defined as the weighted sum of the state variables, i.e.

$$u = -\mathbf{kx} = -k_1 x_1 - k_2 x_2 - \cdots - k_N x_N$$

The closed loop system can be re-written as:

$$\dot{x} = Ax + Bu$$
$$\dot{x} = (A - BK)x$$

where $A_{cl} = (A - BK)$

- Using the state feedback controller

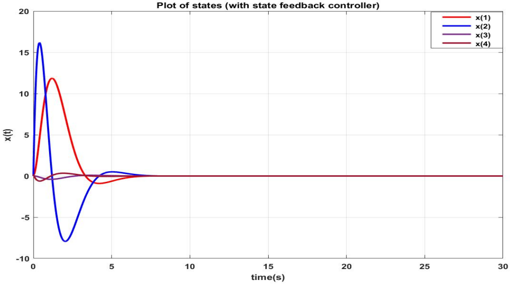


Figure 3: System states

Observer Design

Luenberger Observer (Closed-loop estimator)

- Used to estimate the unknown states
- Initial conditions may or may not be known

The estimated state space system can be written as:

$$\dot{\hat{x}} = A\hat{x} + Bu$$

$$\dot{\hat{x}} = A\hat{x} + B(-Kx) + L(y - \hat{y})$$

$$\dot{\hat{x}} = (A - LC)\hat{x} + B(-Kx) + Ly$$

Observer Results

- Using the controller and observer together

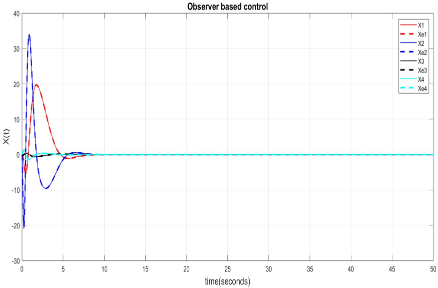


Figure 4: Estimated States along with real states

Thank You