Algorithms for Numerical Computations

Bisection Method:

The method is also called the interval halving method, the binary search method or the dichotomy method. This method is used to find root of an equation in a given interval that is value of 'x' for which $f(x) = 0$

The method is based on **The Intermediate Value Theorem** which states that if $f(x)$ is a continuous function and there are two real numbers a and b such that $f(a)*f(b)$ 0 and $f(b) < 0$), then it is guaranteed that it has at least one root between them.

If $f(x)$ is a continuous function within the inteval [a,b] and if the root of $f(x)$ also lies in the given limit:

Algorithm:1**.** Find middle point **c**= $(a + b)/2$.

2. If $f(c) == 0$, then c is the root of the solution.

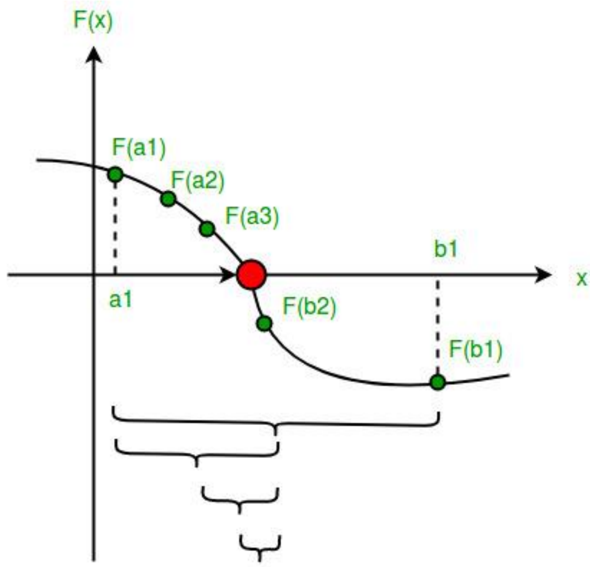
3. Else $f(c) != 0$

   A. If value $f(a)*f(c) < 0$ then root lies between a and c. So we recur for a and c

   B. Else If $f(b)*f(c) < 0$ then root lies between b and c. So we recur b and c.

   C. Else given function doesn't follow one of assumptions.

4. If A or B is satisfied then repeat from 2 till certain accuracy is achieved.

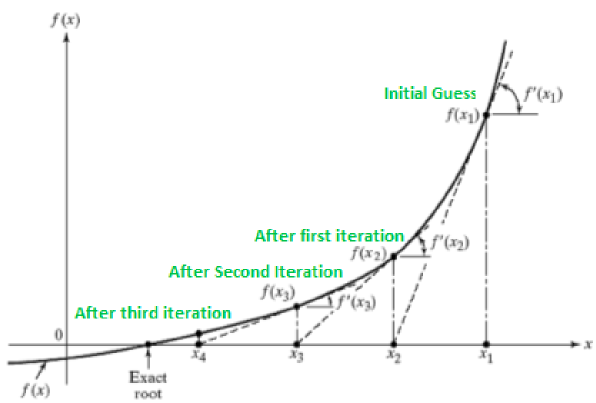


Newton Raphson Method:

Starting from initial guess x_1 the Newton Raphson method uses below formula to find next value of x, i.e., x_{n+1} from previous value x_n.

Algorithm:

1. Start the program.
2. Define the function $f(x)$, $f'(x)$
3. Enter the initial guess of the root , say x_0
4. Calculate $x_{n+1} = x_n - [f(x_n)/ f'(x_n)]$,
   where $n = 0, 1, 2, \ldots$
5. If $|x_{n+1} - x_n| < \epsilon$, ϵ being prescribed accuracy then go to step 7.
6. Set $x_n = x_{n+1}$ and go to step 4.
7. Print the value of x_n.
8. Stop the program.



$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \cdot \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

<u>Euler Method for Solving ODE (1st Order):</u>

In mathematics and computational science, the Euler method is a first-order numerical procedurefor solving ordinary differential equations (ODEs) with a given initial value. Consider a differential equation dy/dx = f(x, y) with initialc ondition y(x0)=y0
then succesive approximation of this equation can be given by:

$$y_{n+1} = y_n + hf(t_n, y_n).$$

Algorithm:

1. Define function f(x,y)
2. Get the values of x_0, y_0, h and n
   *Here x_0 and y_0 are the initial conditions
   h is the interval size
   n is the interval number
3. Start loop from i=1 to n
4. y = y_0 + h*f(x_0,y_0)
   x = x + h
5. Print values of y_0 and x_0
6. Check if i < n,
   If yes, assign x_0 = x and y_0 = y
   If no, goto 7
7. End loop i
8. Stop the program