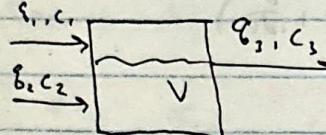


(1) 01/14/24

1.



## 1. Objective

- Derive a dynamic balance for outlet conc & vessel vol. as a  $f(g, c_1, c_2)$

2. Schematic  $\rightarrow$  above.

3. Assumptions:

- Well mixed vessel
- no heat added
- $C_p = \text{constant}$
- no rxn
- $P_1 = P_2 = P_3 = \text{constant}$
- no shaft work.

4. Spatial dependence

- no vars as a  $f(x, y, z)$ ,  $\therefore$  no spatial dependence

5. Dynamic balances:

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\because m_i = P_i V ; \dot{m}_i = P_i g_i ; P_1 = P_2 = P_3 \quad (\text{well mixed})$$

$$\frac{d(PV)}{dt} = g_1 P + g_2 P - g_3 P$$

$$\therefore P = \text{constant}$$

$$P \frac{dV}{dt} = g_1 P + g_2 P - g_3 P \quad \div P$$

$$\frac{dV}{dt} = g_1 + g_2 - g_3$$

Species

$$\frac{dN_i}{dt} = \sum \dot{n}_{i,in} - \sum \dot{n}_{i,out} + \cancel{\sum \dot{m}_i g_i} \quad \cancel{\text{no rxn}} \quad \cancel{V \neq constant}$$

$$\therefore N_i = C_i g_i ; \dot{n}_i = C_i \dot{g}_i$$

$$\frac{d(C_i V)}{dt} = C_1 \dot{g}_1 + C_2 \dot{g}_2 - C_3 \dot{g}_3$$

$$\cancel{C_1 \dot{g}_1} + C_1 \frac{dV}{dt}$$

$$V \frac{dC_1}{dt} + C_1 \frac{dV}{dt} = C_1 \dot{g}_1 + C_2 \dot{g}_2 - C_3 \dot{g}_3$$

$$V \frac{dC_1}{dt} = C_1 \dot{g}_1 + C_2 \dot{g}_2 - C_3 \dot{g}_3 - C_1 \frac{dV}{dt}$$

(2)

$$\frac{dC_3}{dt} = \frac{C_1 \dot{g}_1 + C_2 \dot{g}_2}{V} - \frac{C_3}{V} (\dot{g}_3 + \frac{dV}{dt})$$

6. Other relns:

$$m = PV \quad \dot{n} = Cg$$

~~assume~~

7. DOF:

7 vars ; 4 eqns  $\Rightarrow$  3 DOF

8. Inputs:

$$\dot{g}_1, C_1, g_2, C_2$$

9. ~~Outputs~~ V (state) ; C<sub>3</sub> (process var) ; g<sub>3</sub> (state)10. Simplify  $\rightarrow$  Done.

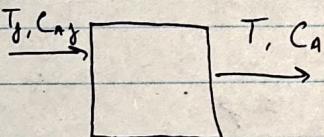
11. Steady State.

$$V = \text{constant}; \frac{dV}{dt} = 0$$

$$\dot{g}_3 = \dot{g}_1 + \dot{g}_2$$

$$V \frac{dC_3}{dt} = C_1 \dot{g}_1 + C_2 \dot{g}_2 - C_3 \dot{g}_3$$

2. Python script.

3. ~~to~~ ~~eq~~

1. objective.

- Develop the concentration response ; T response.

2. schematic  $\rightarrow$  Above  $\uparrow$ 

3. Assumptions

- Constant V ;  $\frac{dV}{dt} = 0$       -  $C_{P,A} = \text{constant}$   $\dot{V} = 100 \frac{m^3}{hr}$ - Well mixed      -  $P_A = \text{constant}$       - no rxn; no shaft work- 4. Spatial dependence  $\Rightarrow$  no var is  $f(x,y,z) \rightarrow \text{None}$

(3)

Species balance.

$$\frac{dn_A}{dt} = \sum \dot{n}_{A,in} - \sum \dot{n}_{A,out} + \sum \dot{n}_{A,gen} - \sum \dot{n}_{A,loss}$$

$$\frac{dn_A}{dt} = \dot{n}_{Af} - \dot{n}_A$$

$$\dot{n}_A = C_A \dot{V}$$

$$\frac{dC_{AV}}{dt} = \cancel{\rho_f} C_A \delta_f - C_A \delta_g$$

$$V, \text{ constant}, \therefore \delta_f = \delta$$

$$V \left( \frac{dC_A}{dt} \right) = \delta (C_{Af} - C_A)$$

$$\frac{dC_A}{dt} = \frac{\delta}{V} (C_{Af} - C_A)$$

### Energy Balance

$$m C_p \frac{dT}{dt} = \sum \dot{m}_{in} C_p (T_{in} - T_{ref}) - \sum \dot{m}_{out} C_p (T_{out} - T_{ref}) + Q + h_s$$

$$-Q=0$$

$$-h_s=0$$

$$m C_p \frac{dT}{dt} = \sum \dot{m}_{in} C_p (T_{in} - T_{ref}) - \sum \dot{m}_{out} C_p (T_{out} - T_{ref})$$

$$C_p = \text{constant; cancel out.}$$

$$m \frac{dT}{dt} = \sum \dot{m}_{in} (T_{in} - T_{ref}) - \sum \dot{m}_{out} (T_{out} - T_{ref})$$

$$m = PV; \dot{m} = PG$$

$$PV \frac{dT}{dt} = \cancel{P} \delta_f (T_{in} - T_{ref}) - PG (T_{out} - T_{ref})$$

$P$  is all same.

$$V \frac{dT}{dt} = \delta_f (T_{in} - T_{ref}) - PG (T_{out} - T_{ref})$$

$$\text{since } V = \text{constant} \rightarrow \delta_f = g$$

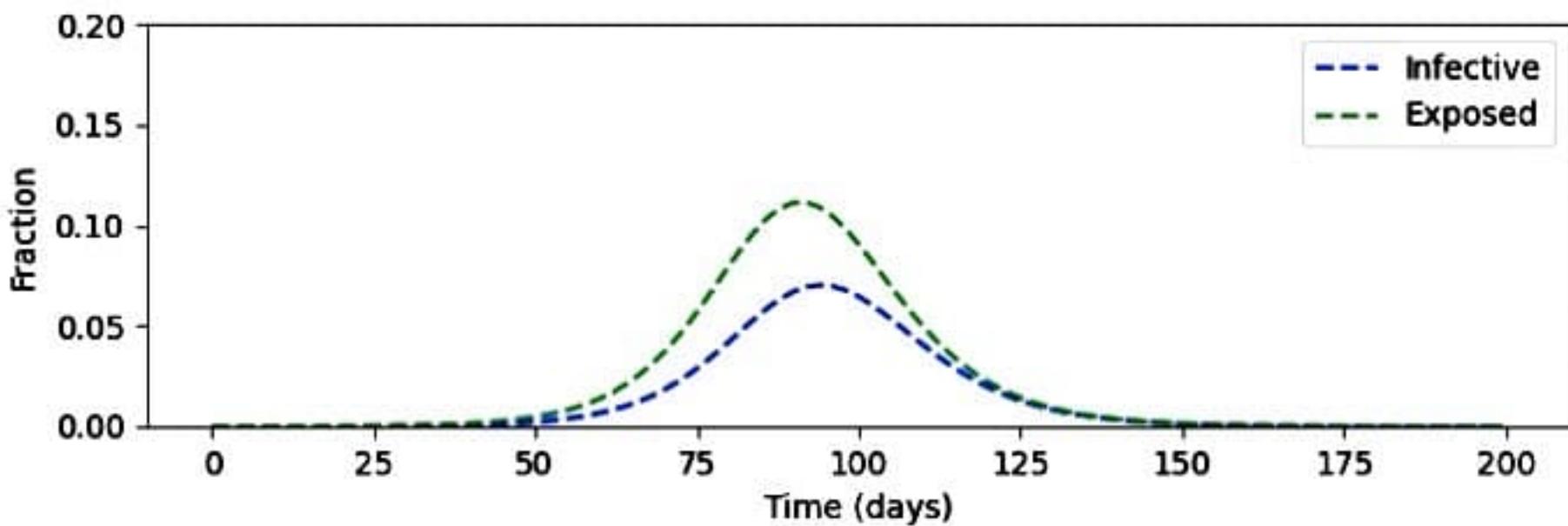
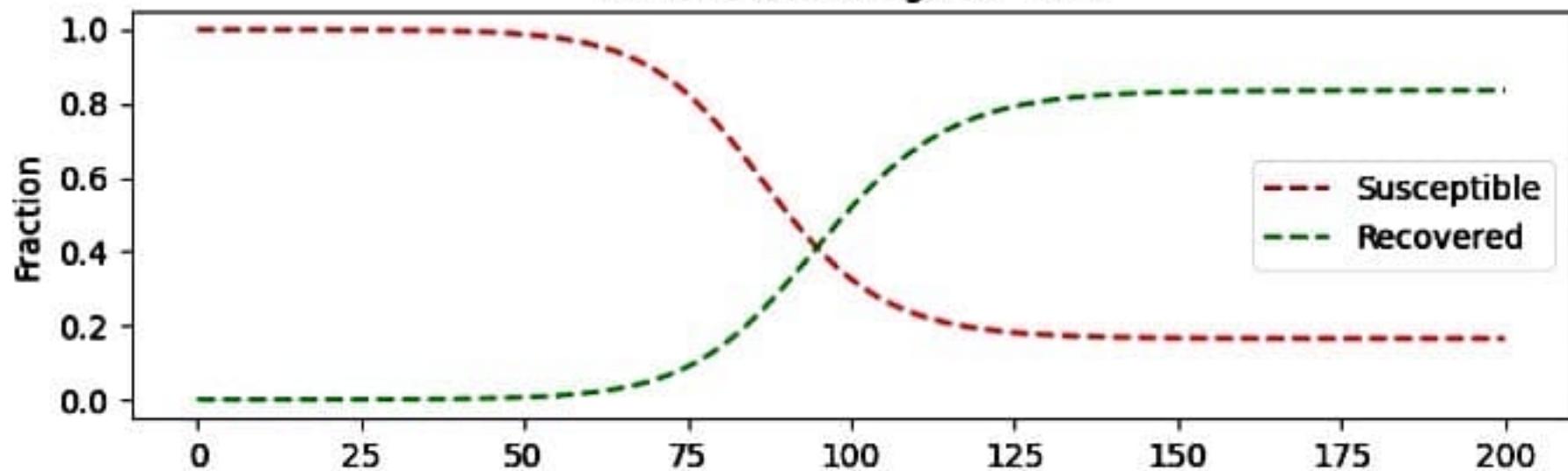
$$V \frac{dT}{dt} = g (T_{in} - T_{ref}) - (T_{out} - T_{ref})$$

$$V \frac{dT}{dt} = g [(T_{in} - T_{ref}) - (T_{out} - T_{ref})]$$

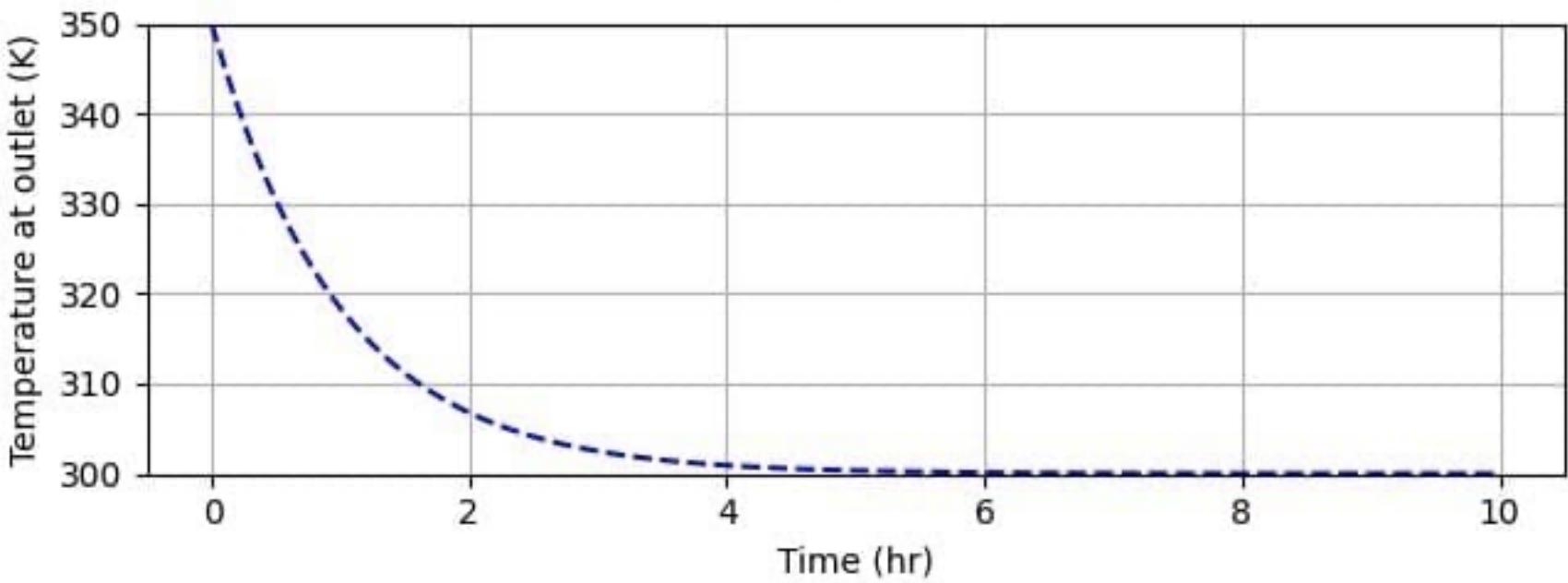
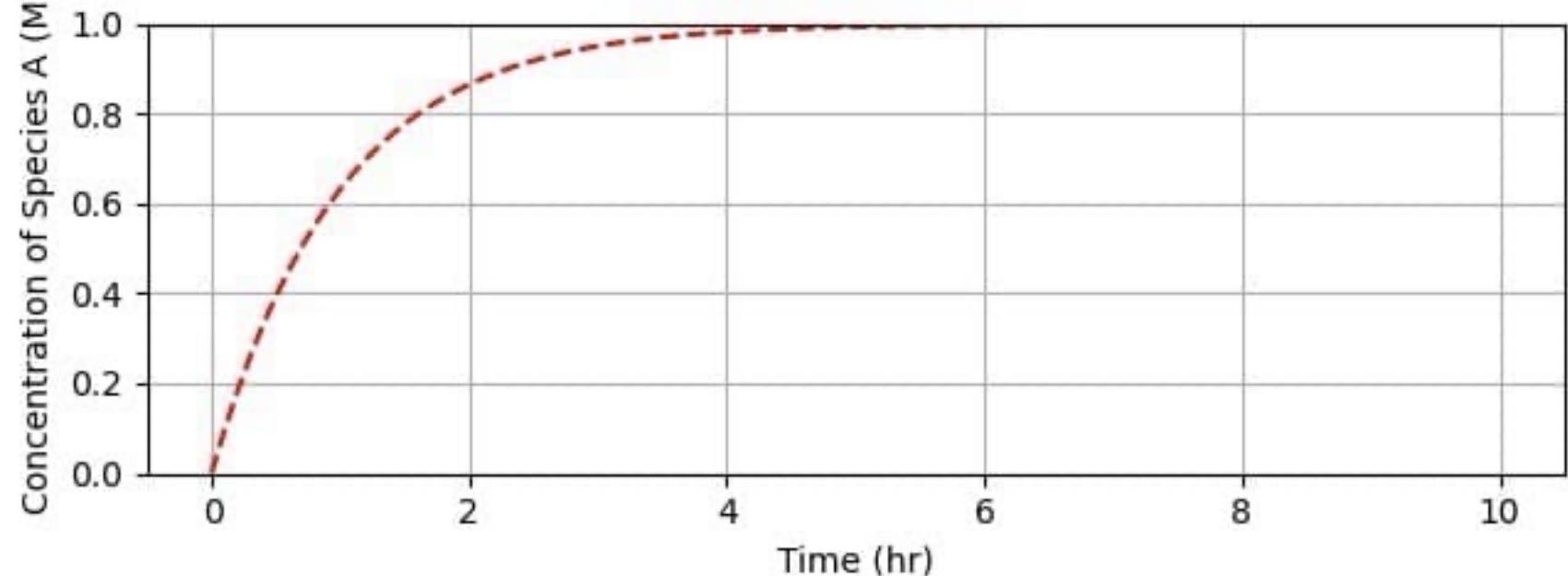
$$\frac{dT}{dt} = \frac{g}{V} [(T_{in} - T_{ref}) - (T_{out} - T_{ref})]$$

$$\frac{dT}{dt} = \frac{g}{V} [T_{in} - T_{ref} - T_{out} + T_{ref}] = \frac{g}{V} [T_{in} - T_{out}]$$

### Social Distancing, $u_r = 0.1$



```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.integrate import odeint
4
5 V = 100 # m3
6 q = 100 # m3/hr
7
8 # z = [Ca, T]
9 # dz = [dCa_dt, dT_dt]
10 def func(z, t):
11     q = 100 # volumetric flow rate -- 100 m^3/hr
12     V = 100 # volume of vessel, 100 m^3
13     Ca = z[0] # concentration
14     Ca_f = 1 # inlet concentration
15     T = z[1] # temperature at outlet
16     Tf = 300 # inlet temperature
17     dCa_dt = q/V * (Ca_f - Ca)
18     dT_dt = q/V * (Tf - T)
19
20     return [dCa_dt, dT_dt]
21
22 t = np.linspace(0, 10, 100)
23
24
25 z0 = [0, 350] # z[0] = Ca0, z[1] = T0
26
27
28 z = odeint(func, z0, t)
29 Ca = z[:, 0]
30 T = z[:, 1]
31
32 fig, axes = plt.subplots(2, 1, layout="constrained")
33 axes[0].plot(t, Ca, "r--")
34 axes[0].set_xlim(0, 10)
35 axes[0].set_xlabel("Time (hr)")
36 axes[0].set_ylabel("Concentration of Species A (M)")
37 axes[0].grid(True)
38
39 axes[1].plot(t, T, "b--")
40 axes[1].set_xlim(0, 10)
41 axes[1].set_xlabel("Time (hr)")
42 axes[1].set_ylabel("Temperature at outlet (K)")
43 axes[1].grid(True)
44
45 plt.show()
```



```
12 import csv
11 import time
10 import tclab
9 import numpy as np
8 import matplotlib.pyplot as plt
7 from scipy.integrate import odeint
6
5 n = 300 # number of second time points (5 min)
4 m = 0.004 # kg
3 tm = np.linspace(0, n, n+1)
2
1 # lab = tclab.setup(connected=False, speedup=5)
13 skip = True
1
2 # data
3 lab = tclab.TCLabModel()
4 if skip is False:
5     # simulate data if no data has been written out yet.
6     t1 = {"data" : [{"time (s)" : 0, "Temperature (degC)" : lab.T1}]}
7     lab.Q1(50) # set heater to 50%
8
9     for i in range(n):
10         data = {}
11         print(i)
12         time.sleep(1)
13         print(lab.T1)
14         data = {"time (s)" : i, "Temperature (degC)" : lab.T1}
15         t1["data"].append(data)
16     lab.close()
17
18
19 with open("tclab_temps.csv", "w", encoding = "utf-8") as outfile:
20     writer = csv.DictWriter(outfile, fieldnames = ["time (s)", "Temperature (degC)"])
21     writer.writerows(t1["data"])
22     outfile.close()
23
24 U = 5 # heat transfer coefficient, W/m2-K
25 m = 0.004 # mass, kg
26 A = 0.0012 # cross-sectional area, m2
27 Cp = 500 # heat capacity J/kg-K
28 T_inf = 23 + 273
29 T_a = 23 + 273
30 sigma = 5.67e-8
31 epsilon = 0.9
32 alpha = 0.01
33 def tclab_model(T, t):
34     T_k = T + 273
35
36     dT_dt = (U * A * (T_a - T_k) + epsilon * sigma * A * (T_inf**4 - T_k**4) + alpha * 50)/(m * Cp)
37
38     return dT_dt
39
40 T0 = lab.T1
41 print(f"T0 => [{T0}]")
42
```

```
32
31 T_model = odeint(tclab_model, T0, tm)
30 T_actual = []
29 with open("tclab_temps.csv", "r") as infile:
28 |   reader = csv.reader(infile)
27 |   for row in reader:
26 |     T_actual.append(row[1])
25
24 T_model_K = T_model + 273
23 convection = U * A * (T_a - T_model_K)
22 radiation = epsilon * sigma * A * (T_inf**4 - T_model_K**4)
21 loss = convection + radiation
20
19
18 plt.figure(1)
17 plt.title("Modelled and Measured Temperature vs Time")
16 plt.plot(tm, T_model, "r-", label="Model")
15 plt.plot(tm, T_actual, "b-", label="Measured")
14 plt.ylabel("Temperature (degC)")
13 plt.xlabel("Time (sec)")
12 plt.yticks(np.linspace(0, 100, 10))
11 plt.legend()
10 plt.show()
9
8 plt.title("Convective, Radiative, and Total Loss")
7 plt.plot(tm, convection, "r--", label="Convection")
6 plt.plot(tm, radiation, "g--", label="Radiation")
5 plt.plot(tm, loss, "g--", label="Loss")
4 plt.ylabel("Heat Loss (W)")
3 plt.xlabel("Time (s)")
2 plt.legend()
1 plt.show()
```

