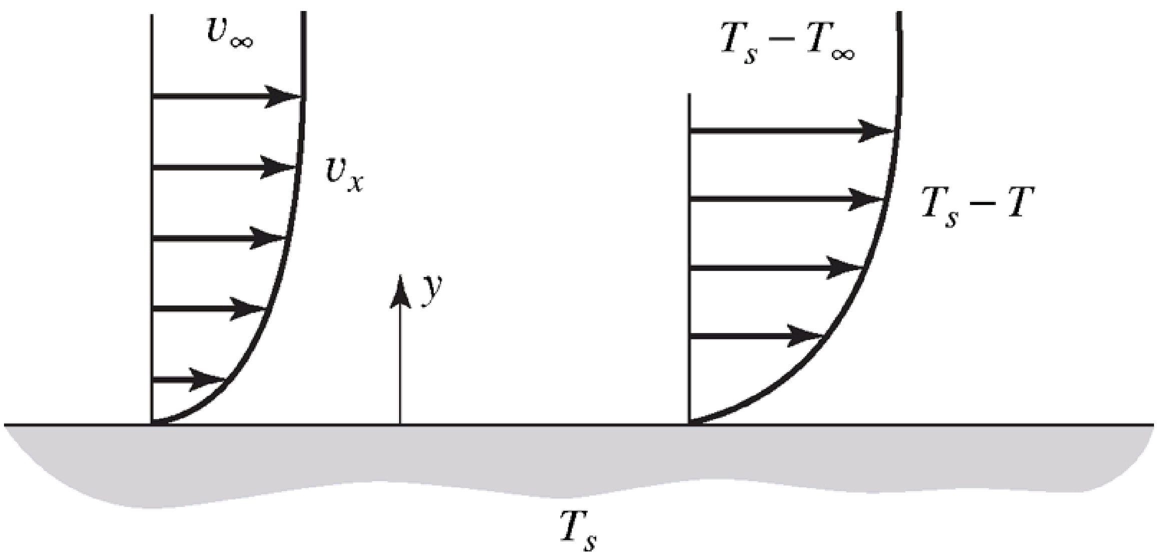
Lecture notes 2
Convective Heat Transfer

Convection

- Heat transfer by convection is associated with energy exchange between a surface and an adjacent fluid.

$$\frac{q}{A} = h\Delta T = h(T_s - T_\infty)$$

Newton's law of cooling



The problem of convection is to determine the coefficient *h*.

Recall the Momentum Transfer

- Momentum transfer in a fluid involves the study of the motion of fluids and the forces that produce these motions – Newton's second law of motion, this subject is often called Fluid mechanics.
  - Velocity profile
  - Laminar and turbulent flow – Reynolds number
  - Viscous drag – coefficient of skin friction

- **CHE 211 Fluid Mechanics**

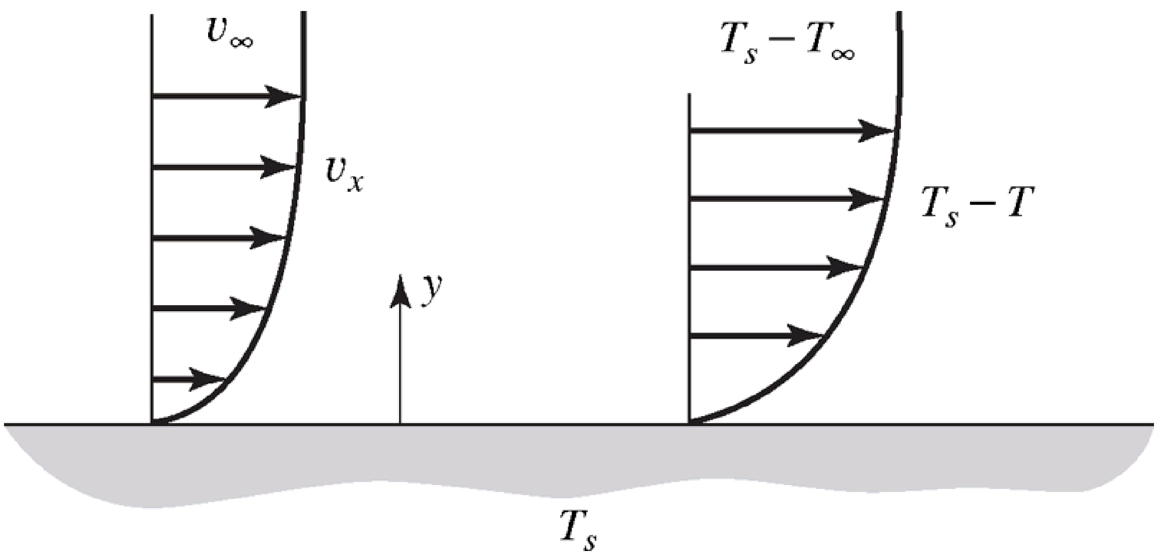
  Fundamentals of fluid flow. Conservation laws for mass, momentum and mechanical energy. Flow of fluids in conduits. Flow past immersed bodies. Flow through beds of solids, fluidization. Transportation and metering of fluids. Dimensional analysis.

Convection

- Heat transfer by convection is associated with energy exchange between a surface and an adjacent fluid.

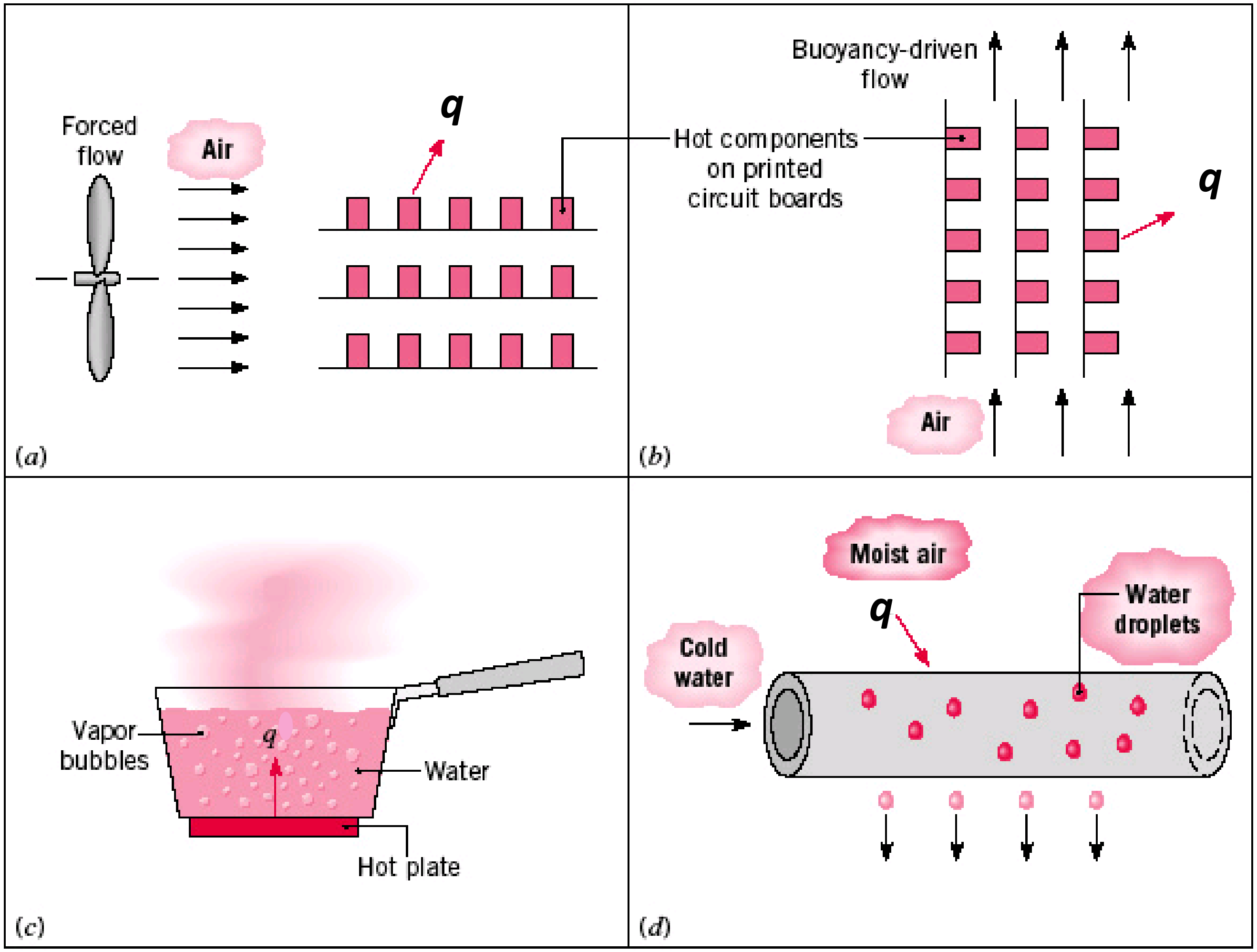
$$\frac{q}{A} = h\Delta T = h(T_s - T_\infty)$$

Newton's law of cooling



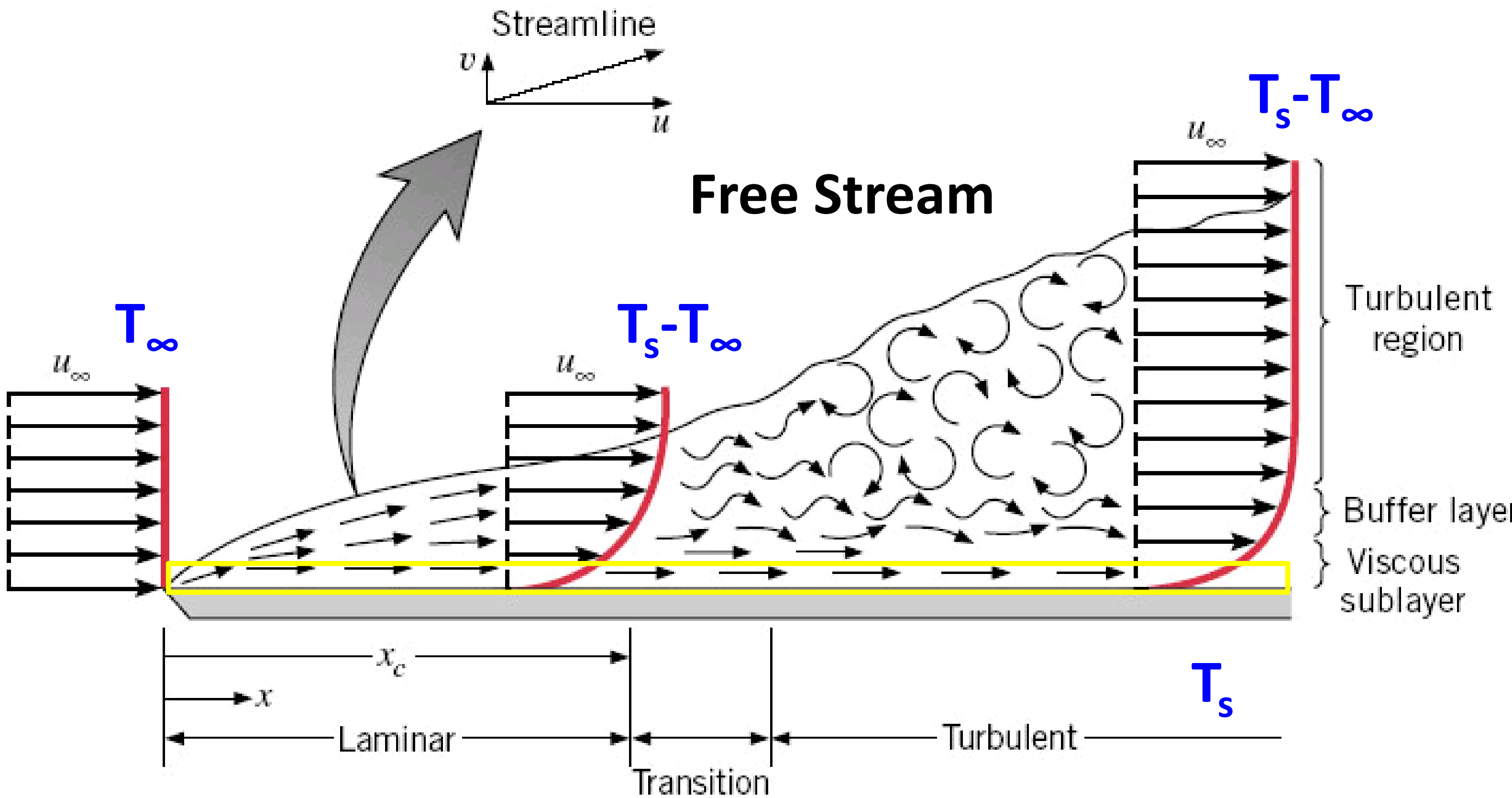
The problem of convection is to determine the coefficient *h*.

Convective Heat Transfer Processes



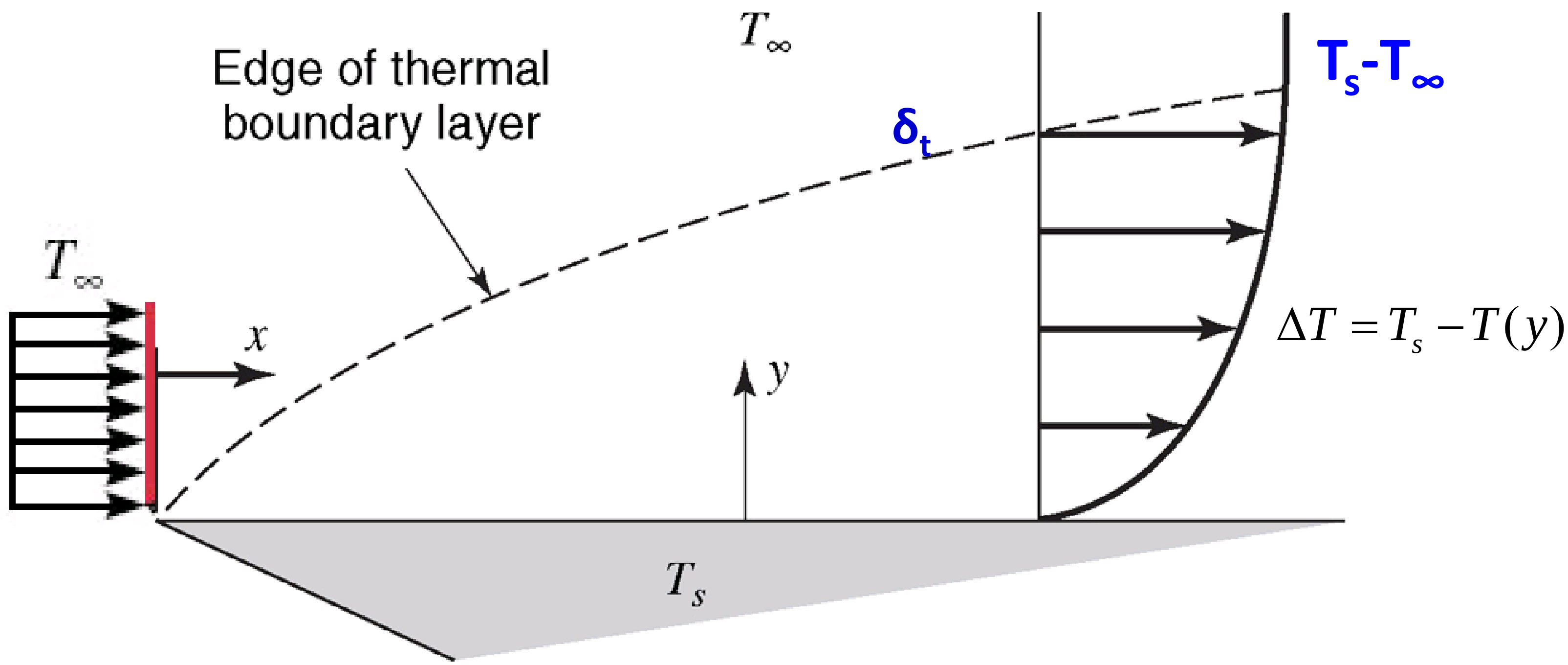
(a) Forced convection, (b) Natural convection, (c) Boiling, (d) Condensation

Fundamental Considerations



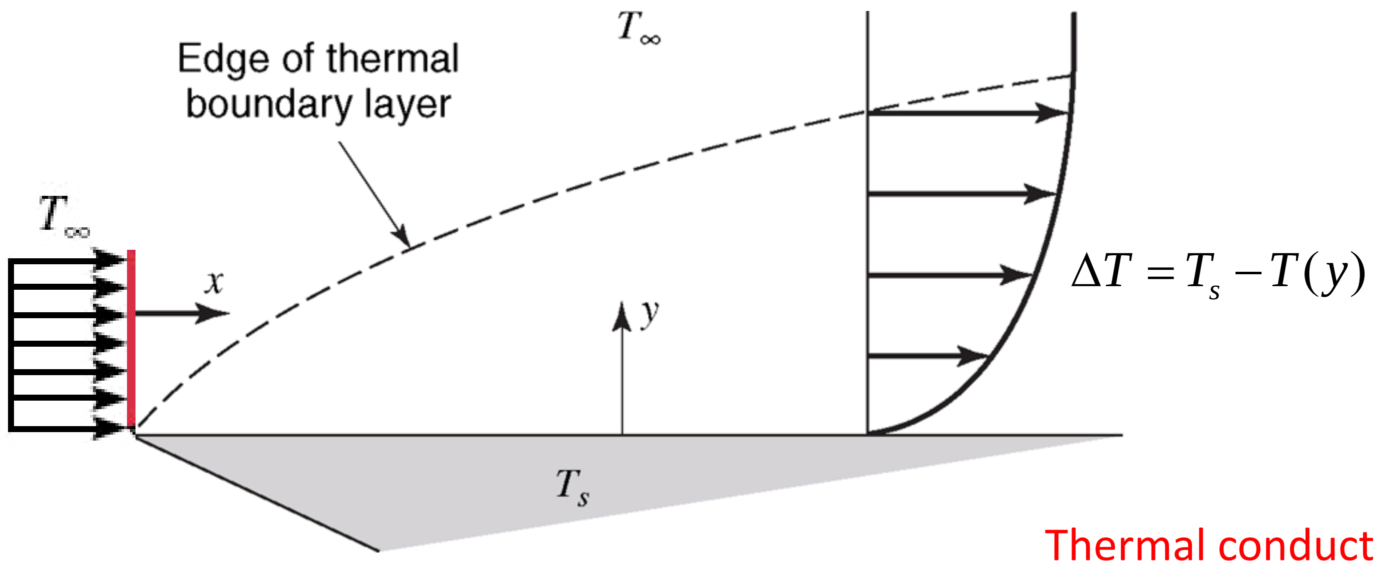
- **The molecular energy exchange or conduction effects are always be present in any convective situation.**
- The distinction between laminar and turbulent flow is a major consideration in any convective situation.

The Thermal Boundary Layer of Laminar Flow



- The TBL is the region of the fluid in which temperature gradients exist, and its thickness δ_t is typically defined as the value of y for which the ratio $[(T_s\text{-}T)/(T_s\text{-}T_\infty)]=0.99$.

The Thermal Boundary Layer



Thermal conductivity of the fluid

Heat transfer processes:

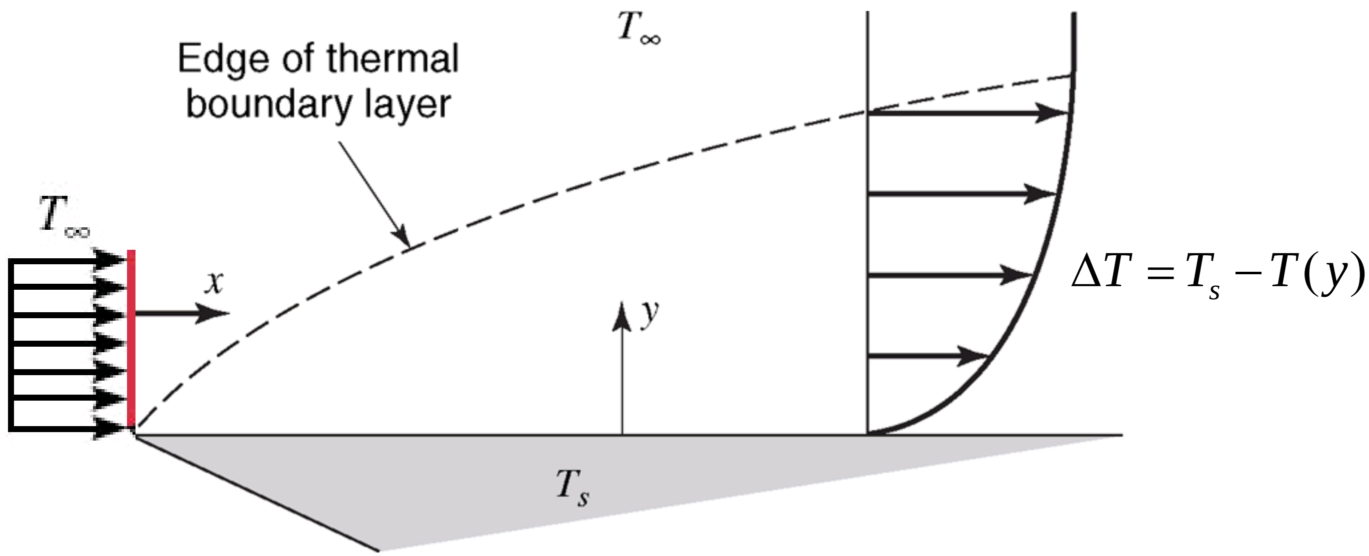
(a) Heat Transfer at the surface is by conduction of thin fluid layer

(b) Heat transfer between the surface and the fluid is by convection

$$q_y = -kA \left. \frac{\partial (T - T_s)}{\partial y} \right|_{y=0}$$

$$q_y = hA(T_s - T_\infty)$$

Local Convective Heat Transfer Coefficients

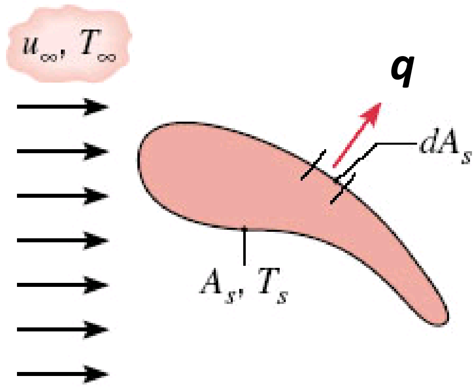


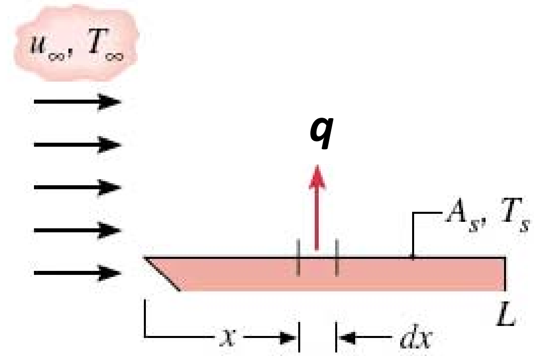
At any distance x from the leading edge, we have

$$q_y = hA(T_s - T_\infty) = -kA \left. \frac{\partial (T - T_s)}{\partial y} \right|_{y=0} \qquad h_x = -k \frac{\partial (T - T_s) / \partial y \big|_{y=0}}{T_s - T_\infty}$$

Note that T, h and q vary along the x direction.

Mean Convective Heat Transfer Coefficients





- Total heat transfer rate

$$q = (T_s - T_\infty) \int_{A_s} h_x \, dA_s$$

Mean convection coefficient

$$h = \frac{1}{A_s} \int_{A_s} h_x \, dA_s \,, \qquad h = \frac{1}{L} \int_0^L h_x \, dx$$

The determination of heat transfer coefficients (local h_x and mean h) is viewed as **the problem of convection**. Coefficients depend on fluid properties, surface geometry, flow conditions.

Example 1

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

Where $\boldsymbol{\alpha}$ is a coefficient ($W/m^{1.9}.K$) and $\boldsymbol{x}$ (m) is the distance from the leading edge of the plate.

(a) Develop an expression for the ratio of the mean heat transfer coefficient for a plate of length x to the local heat transfer coefficient at x.

(b) Show, in a qualitative manner, the variation of h_x and h as a function of x.

Dimensional Analysis of Convective Heat Transfer

- Apply the **Buckingham Method** to identify significant parameters in convective heat transfer
  - Reynolds number
  - Prandtl number
  - Nusselt number
  - Stanton number
  - Grashof number

- Identify the form of correlations of the convective heat transfer

Dimensional Analysis

- Dimensional analysis: to group the variables in a given situation into dimensionless parameters
  - An application of the requirement of dimensional homogeneity
  - The resultant dimensionless parameters are less numerous than the original variables
  - The relations between dimensionless parameters are useful in expressing the performance of the systems
- Dimensionless parameters

For example, stipulate the reference values for the length and velocity

L – reference length
υ_∞ – reference velocity

$$x^* = x / L$$

$$y^* = y / L$$

$$t^* = t\upsilon_\infty / L$$

Previous Application of Dimensionless Parameters

The Continuity Equation

$$\frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} = 0 \quad \Longrightarrow \quad \frac{\partial \upsilon_x^*}{\partial x^*} + \frac{\partial \upsilon_y^*}{\partial y^*} = 0 \qquad (11\text{-}1)$$

The Navier-Stokes Equation

$$\rho\left(\frac{\partial \upsilon}{\partial t} + \upsilon_x \frac{\partial \upsilon}{\partial x} + \upsilon_y \frac{\partial \upsilon}{\partial y}\right) = \rho g - \nabla P + \mu\left(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2}\right)$$

Stipulate the reference values for the length and velocity

L – reference length
υ_∞ – reference velocity

Dimensionless Parameters

$$x^* = x / L \qquad \upsilon_x^* = \upsilon_x / \upsilon_\infty$$

$$y^* = y / L \qquad \upsilon_y^* = \upsilon_y / \upsilon_\infty$$

$$t^* = t\upsilon_\infty / L \qquad \upsilon^* = \upsilon / \upsilon_\infty$$

$$\nabla^* = L\nabla$$

Dimensionless Navier-Stokes Differential Equation

$$\rho(\frac{\partial \upsilon}{\partial t}+\upsilon_x\frac{\partial \upsilon}{\partial x}+\upsilon_y\frac{\partial \upsilon}{\partial y})=\rho g-\nabla P+\mu(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2})$$

$$\frac{\rho\upsilon_\infty^2}{L}(\frac{\partial \upsilon^*}{\partial t^*}+\upsilon_x^*\frac{\partial \upsilon^*}{\partial x^*}+\upsilon_y^*\frac{\partial \upsilon^*}{\partial y^*})=\rho g-\frac{1}{L}\nabla^* P+\frac{\mu\upsilon_\infty}{L^2}(\frac{\partial^2 \upsilon^*}{\partial x^{*2}}+\frac{\partial^2 \upsilon^*}{\partial y^{*2}}) \qquad (11\text{-}2)$$

$\rho\upsilon_\infty^2/L$ Is an Inertial force [F/L^3]

ρg is a gravitational force

P/L Is a pressure force

$\mu\upsilon_\infty/L^2$ Is a viscous force

Significant dimensionless parameters

Reynolds Number: $\mathrm{Re}\equiv\frac{L\upsilon_\infty\rho}{\mu}=\frac{inetial}{viscous}$

Froude Number: $Fr\equiv\frac{\upsilon_\infty^2}{gL}=\frac{inertial}{gravitational}$

Euler Number: $Eu\equiv\frac{P}{\rho\upsilon_\infty^2}=\frac{pressure}{inertial}$

Buckingham Method – Review (Text p144-147)

- An empirical method of grouping the variables in a system to dimensionless groups in terms of the fundamental dimensions.

The number of dimensionless groups i used to describe a situation involving n variables is equal to n- r, where r is the rank of the dimensional matrix of the variables.

i = n- r

- First step is to construct a table of the variables and their dimensions. E.g. The flow of fluid external to a solid body.

Variable	Symbol	Dimensions
Force	F	ML/t^2
Velocity	υ	L/t
Density	ρ	M/L^3
Viscosity	μ	M/Lt
Length	L	L

Dimensional matrix

	F	*υ*	*ρ*	*μ*	*L*
M	1	0	1	1	0
L	1	1	-3	-1	1
t	-2	-1	0	-2	0

Dimensional Analysis of Convective Heat Transfer

- Fundamental dimensions in heat transfer

Dimensions	**Symbols**
Mass	M
Length	L
Time	t
Temperature	T
Heat	Q

- Dimensional Analysis of Forced Convection
- Dimensional Analysis of Natural Convection

Dimensional Analysis of Forced Convection

Variable	Symbol	Dimensions
Tube diameter	D	L
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/Lt
Fluid heat capacity	c_p	Q/MT
Fluid thermal conductivity	k	Q/tLT
Velocity	v	L/t
Heat-transfer coefficient	h	Q/tL^2T

Dimensional matrix

	D	ρ	μ	c_P	k	υ	h
L	1	-3	-1	0	-1	1	-2
M	0	1	1	-1	0	0	0
t	0	0	-1	0	-1	-1	-1
Q	0	0	0	1	1	0	1
T	0	0	0	-1	-1	0	-1

There are 7 variables and the rank of the dimensional matrix of the variables is 4; Thus, we need to have 3 dimensionless groups

i = n – r = 7 - 4 = 3

Dimensional Analysis of Forced Convection

Variable	Symbol	Dimensions
Tube diameter	D	L
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/Lt
Fluid heat capacity	c_p	Q/MT
Fluid thermal conductivity	k	Q/tLT
Velocity	v	L/t
Heat-transfer coefficient	h	Q/tL^2T

- Choosing D, k, μ, υ as core variables, we can find three groups

$$\pi 1 = \frac{D\upsilon\rho}{\mu} \equiv \mathrm{Re}; \qquad \pi 2 = \frac{\mu c_p}{k} \equiv \mathrm{Pr}; \qquad \pi 3 = \frac{hD}{k} \equiv Nu \qquad Nu = f_1(\mathrm{Re}, \mathrm{Pr})$$

- Choosing ρ, μ, c_p, υ as core variables, we can find three groups

$$\pi 1 = \frac{D\upsilon\rho}{\mu} \equiv \mathrm{Re}; \qquad \pi 2 = \frac{\mu c_p}{k} \equiv \mathrm{Pr}; \qquad \pi 3 = \frac{h}{\rho\upsilon c_p} \equiv St \qquad St = f_2(\mathrm{Re}, \mathrm{Pr})$$

Utilizing the Buckingham method of grouping the variables as presented in Chapter 11, the required number of dimensionless groups is found to be 3. Note that the rank of the dimensional matrix is 4, one more than the total number of fundamental dimensions.

Choosing D, k, μ, and v as the four variables comprising the core, we find that the three π groups to be formed are

$$\pi_1 = D^a k^b \mu^c v^d \rho$$
$$\pi_2 = D^e k^f \mu^g v^h c_p$$

and

$$\pi_3 = D^i k^j \mu^k v^l h$$

Writing π_1 in dimensional form,

$$1 = (L)^a \left(\frac{Q}{LtT}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{L}{t}\right)^d \frac{M}{L^3}$$

and equating the exponents of the fundamental dimensions on both sides of this equation, we have for

$$\begin{aligned} L: &\quad 0 = a - b - c + d - 3 \\ Q: &\quad 0 = b \\ t: &\quad 0 = -b - c - d \\ T: &\quad 0 = -b \end{aligned}$$

and

$$M: \quad 0 = c + 1$$

Solving these equations for the four unknowns yields

$$\begin{aligned} a &= 1 \qquad & c &= -1 \\ b &= 0 \qquad & d &= 1 \end{aligned}$$

and π_1 becomes

$$\pi_1 = \frac{Dv\rho}{\mu}$$

which is the Reynolds number. Solving for π_2 and π_3 in the same way will give

$$\pi_2 = \frac{\mu c_p}{k} = \mathrm{Pr} \quad \text{and} \quad \pi_3 = \frac{hD}{k} = \mathrm{Nu}$$

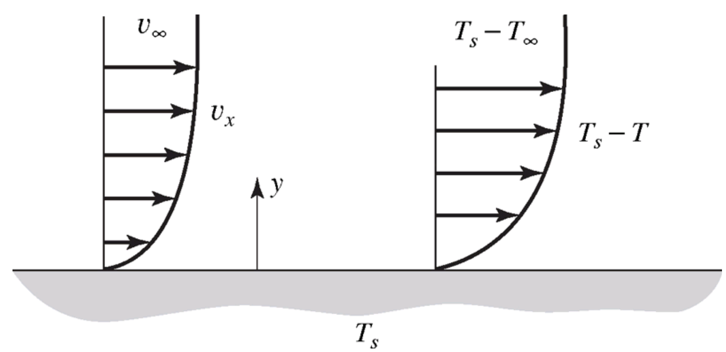
The Reynolds and Prandtl Numbers

- Reynolds number – a ratio of the inertial effect to viscous effect

$$\mathrm{Re} \equiv \frac{L\upsilon\rho}{\mu} \sim \frac{\text{inertial}}{\text{viscous}}$$

- Prandtl number – a measure of the relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers
  - For laminar boundary layers
  - For gases: $\delta \sim \delta_t$ and Pr ~ 1
  - For a liquid metal $\delta_t >> \delta$ and Pr <<1
  - For oils $\delta_t << \delta$ and Pr >>1

$$\mathrm{Pr} \equiv \frac{v}{a} = \frac{\mu c_p}{k} \square \frac{\text{momentum}}{\text{thermal}}$$



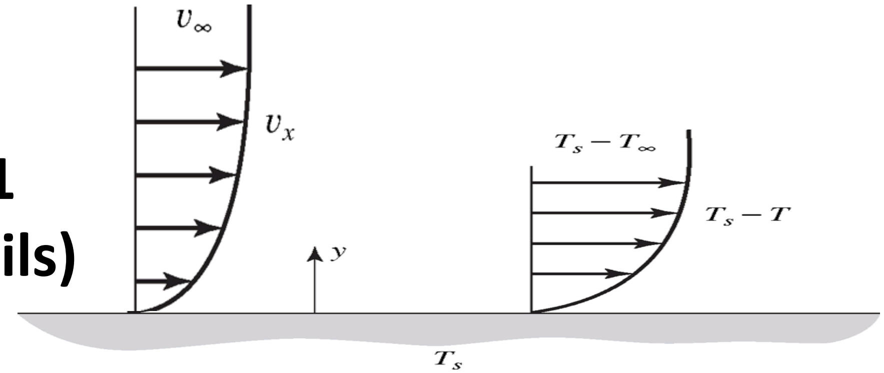
The value of Pr strongly influences the relative growth of the velocity and thermal boundary layers.

Momentum and Prandtl Number

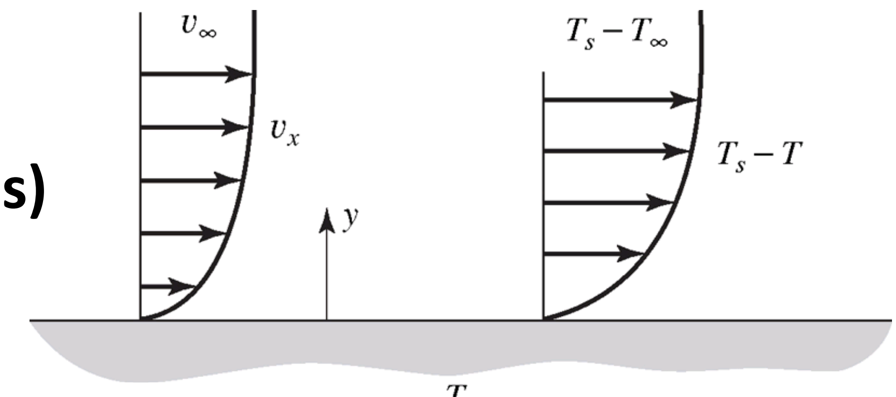
$$\mathrm{Pr} \equiv \frac{\nu}{a} = \frac{\mu c_p}{k} \sim \frac{momentum}{thermal}$$

$$\mathrm{Pr}^n = \frac{\delta}{\delta_t} \qquad [n > 0]$$

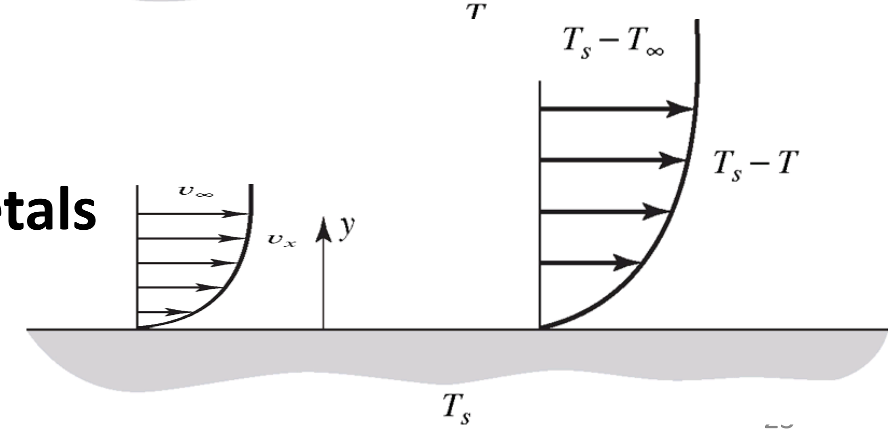
Pr >1
(e.g. oils)



Pr =1
(e.g. gasses)



Pr<1
e.g. liquid metals



Nusselt and Stanton Numbers

- Nusselt number – a ratio of convection to pure conduction heat transfer

$$Nu \equiv \frac{hL}{k} \sim \frac{convection}{conduction}$$

- Stanton number – a modified Nusselt number

$$St \equiv \frac{h}{\rho \upsilon c_p} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}}$$

Convective Heat Transfer Correlations:

$$Nu = f_1(\mathrm{Re}, \mathrm{Pr})$$

$$St = f_2(\mathrm{Re}, \mathrm{Pr})$$

Dimensional Analysis of Natural Convection

Variable	Symbol	Dimensions
Significant length	L	L
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/Lt
Fluid heat capacity	c_p	Q/MT
Fluid thermal conductivity	k	Q/LtT
Fluid coefficient of thermal expansion	β	$1/T$
Gravitational acceleration	g	L/t^2
Temperature difference	ΔT	T
Heat-transfer coefficient	h	Q/L^2tT

- Buoyant force is dominant $F_{buoyant} = \beta g \rho_0 \Delta T$
- Choosing L, μ, k, g, β as core variables, we can find four groups

$$\pi 1 = \frac{\mu c_p}{k} \equiv \mathrm{Pr}; \quad \pi 2 = \frac{L^3 g \rho^2}{\mu^2}; \quad \pi 3 = \beta \Delta T; \quad \pi 4 = \frac{hL}{k} \equiv Nu$$

Grashof Number

- The product of π2 and π3 must also be a dimensionless number. This parameter is the Grashof number

$$Gr \equiv \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} = \frac{\underline{\text{Thermal expansion}}}{\text{Viscous effect}}$$

- Grashof number replaces the Reynolds number in the heat transfer correlations

Natural convection

$$Nu = f_1(Gr, \mathrm{Pr})$$

Forced convection

$$Nu = f_1(\mathrm{Re}, \mathrm{Pr})$$

$$St = f_2(\mathrm{Re}, \mathrm{Pr})$$

Significant Parameters in Convective Heat Transfer

- Momentum diffusivity: $\nu \equiv \dfrac{\mu}{\rho}$
- Thermal diffusivity: $a \equiv \dfrac{k}{\rho c_p}$
- Reynolds number $\mathrm{Re} \equiv \dfrac{L \upsilon \rho}{\mu} \sim \dfrac{inertia}{viscous}$
- Prandtl number $\mathrm{Pr} \equiv \dfrac{\nu}{a} = \dfrac{\mu c_p}{k} \square \dfrac{momentum}{thermal}$
- Nusselt number $Nu \equiv \dfrac{hL}{k} \square \dfrac{convection}{conduction}$
- Stanton number $St \equiv \dfrac{h}{\rho \upsilon c_p}$
- Grashof number $Gr \equiv \dfrac{\beta g \rho^2 L^3 \Delta T}{\mu^2}$

All of them depends on the fluid temperature - See Appendix 1

Excerpt of Appendix I

686 Appendix I

T (K)	ρ (kg/m^3)	c_p (J/kg × K)	$\mu \times 10^6$ (Pa × s)	$\nu \times 10^6$ (m^2/s)	k (W/m × K)	$\alpha \times 10^6$ (m^2/s)	Pr	$g\beta\rho^2/\mu^2 \times 10^{-9}$ (1/K · m^3)
				Water				
273	999.3	4226	1794	1.795	0.558	0.132	13.6	
293	998.2	4182	993	0.995	0.597	0.143	6.96	2.035
313	992.2	4175	658	0.663	0.633	0.153	4.33	8.833
333	983.2	4181	472	0.480	0.658	0.160	3.00	22.75
353	971.8	4194	352	0.362	0.673	0.165	2.57	46.68
373	958.4	4211	278	0.290	0.682	0.169	1.72	85.09
473	862.8	4501	139	0.161	0.665	0.171	0.94	517.2
573	712.5	5694	92.2	0.129	0.564	0.139	0.93	1766.0

T (°F)	ρ (lb$_m$/ft^3)	c_p (Btu/lb$_m$ °F)	$\mu \times 10^5$ (lb$_m$/ft s)	$\nu \times 10^5$ (ft^2/s)	k (Btu/h ft °F)	$\alpha \times 10^3$ (ft^2/h)	Pr	$\beta \times 10^3$ (1/°F)	$g\beta\rho^2/\mu^2 \times 10^{-6}$ (1/°F · ft^3)
				Aniline					
60	64.0	0.480	305	4.77	0.101	3.29	52.3		
80	63.5	0.485	240	3.78	0.100	3.25	41.8		
100	63.0	0.490	180	2.86	0.100	3.24	31.8	0.45	17.7
150	61.6	0.503	100	1.62	0.0980	3.16	18.4		

Note that the numerical values of the Physical Properties are slightly different in different textbook.

Heat Transfer Correlations

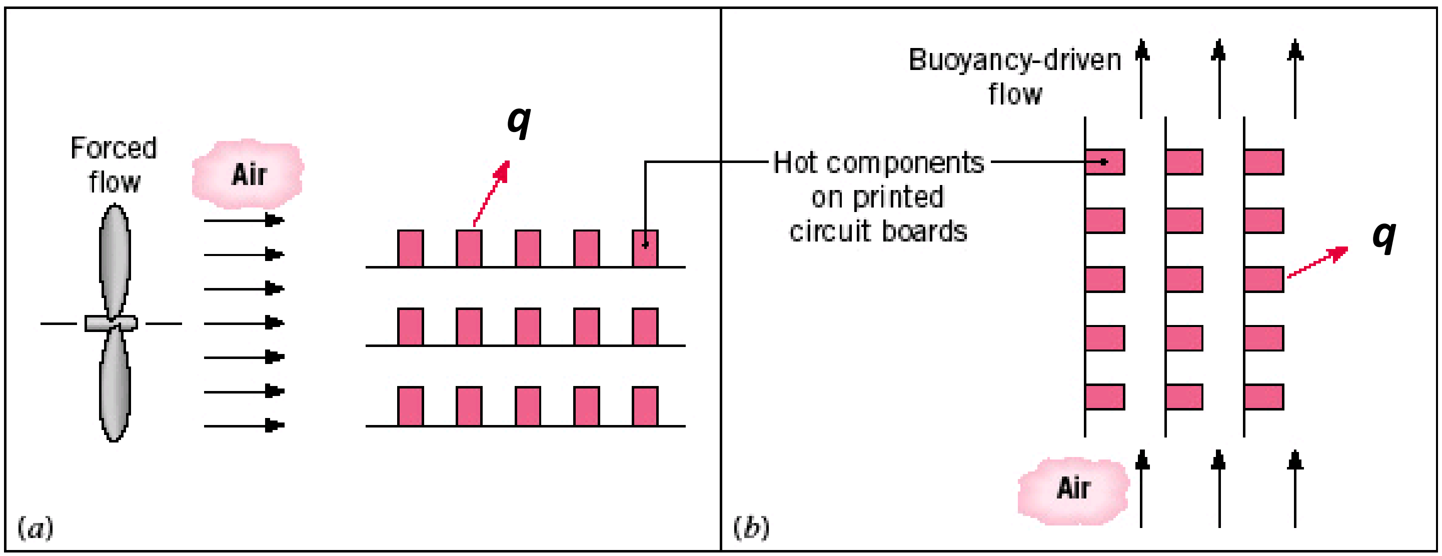
Forced convection

$$Nu = f_1(\mathrm{Re}, \mathrm{Pr})$$

$$St = f_2(\mathrm{Re}, \mathrm{Pr})$$

Natural convection

$$Nu = f_1(Gr, \mathrm{Pr})$$



- We will focus mainly on the forced convection correlations.

Example 2

- Experimental tests on a portion of the turbine blade shown below indicate a heat flux to the blade of $q'' = q/A = \ 95{,}000\ \mathrm{W/m^2}$. To maintain a steady-state surface temperature of $800°\mathrm{C}$, heat transferred to the blade is removed by circulating a coolant inside the blade.
  - Determine the heat flux to the blade if its temperature is reduced to $T_{s,1} = 700°\mathrm{C}$ by increasing the coolant flow.
  - Determine the heat flux to the blade for a similar turbine blade having a chord length of $L = 80\ \mathrm{mm}$, when the blade operates in an airflow at $T_\infty = 1150°\mathrm{C}$ and $\upsilon = 80\ \mathrm{m/s}$, with $T_s = 800°\mathrm{C}$.