Lecture notes 3

Convection Heat-Transfer Correlations

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Correlation functions:

(a) Forced convection

$$Nu = f_1(\mathrm{Re}, \mathrm{Pr})$$

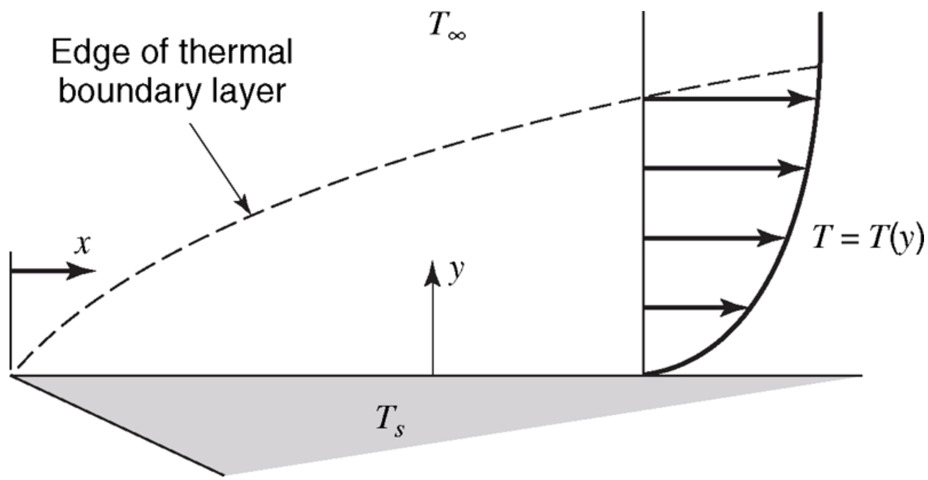
$$St = f_2(\mathrm{Re}, \mathrm{Pr})$$

(b) Natural convection

$$Nu = f_3(Gr, \mathrm{Pr})$$

- ❑**Theoretical /analytical approach** – solving boundary layer equation for a particular geometry
- ❑**Energy and Momentum Analogy**
- ❑**Experimental or empirical approach** – performing heat transfer measurements under controlled laboratory conditions

Exact Analysis of the Laminar Boundary Layer



Considering an incompressible fluid without energy sources and with constant k, and laminar flow condition.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \upsilon_x \frac{\partial}{\partial x} + \upsilon_y \frac{\partial}{\partial y} + \upsilon_z \frac{\partial}{\partial z}$$

General energy (balance) equation

In two-dimensional form

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T \quad \Rightarrow \quad \frac{\partial T}{\partial t} + \upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (19\text{-}14)$$

(16-14)

$$\Rightarrow \quad \upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \qquad (19\text{-}15)$$

In a steady state

Analytical Approach

For steady, incompressible, two-dimensional, isobaric flow, the energy equation can be solved in conjunction with the continuity equation, giving **the temperature gradient at the surface** .

$$\begin{cases} \upsilon_x \dfrac{\partial T}{\partial x} + \upsilon_y \dfrac{\partial T}{\partial y} = a \dfrac{\partial^2 T}{\partial y^2} \\[2ex] \dfrac{\partial \upsilon_x}{\partial x} + \dfrac{\partial \upsilon_y}{\partial y} = 0 \end{cases}$$

$$\frac{\upsilon_x}{\upsilon_\infty} = \frac{\upsilon_y}{\upsilon_\infty} = 0 \qquad y = 0;$$

$$\frac{\upsilon_x}{\upsilon_\infty} = 1 \qquad y = \infty$$

Boundary conditions

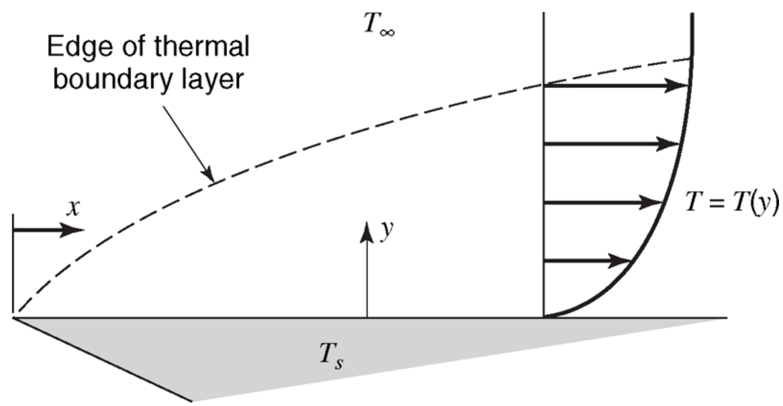
$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)[\frac{0.332}{x}(\frac{x\upsilon\rho}{\mu})_x^{1/2}(\frac{\mu c_p}{k})^{1/3}] = (T_\infty - T_s)[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}]$$

(19-23)

Balsius solution

Local Nusselt Number

Application of Newton and Fourier rate equations yields the local Nusselt number.

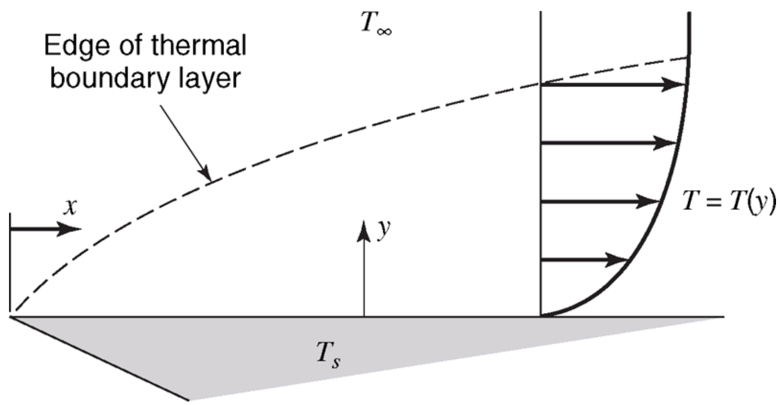


$$\frac{q_y}{A} = h_x (T_s - T_\infty) = -k \frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$h_x = -\frac{k}{(T_s - T_\infty)} \frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{0.332k}{x} \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3}$$

The local Nusselt number is $$Nu_x = \frac{h_x x}{k} = 0.332\, \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3}$$

Mean Nusselt number



The mean heat-transfer coefficient applying over a plate of width *w* and length *L* may be obtained by integration.

$$q_y = hA(T_s - T_\infty) = \int_{A_s} h_x (T_s - T_\infty)\, dA$$

$$h(wL)(T_s - T_\infty) = 0.332 kw \,\mathrm{Pr}^{1/3} (T_s - T_\infty) \int_0^L \frac{\mathrm{Re}_x^{1/2}}{x} dx$$

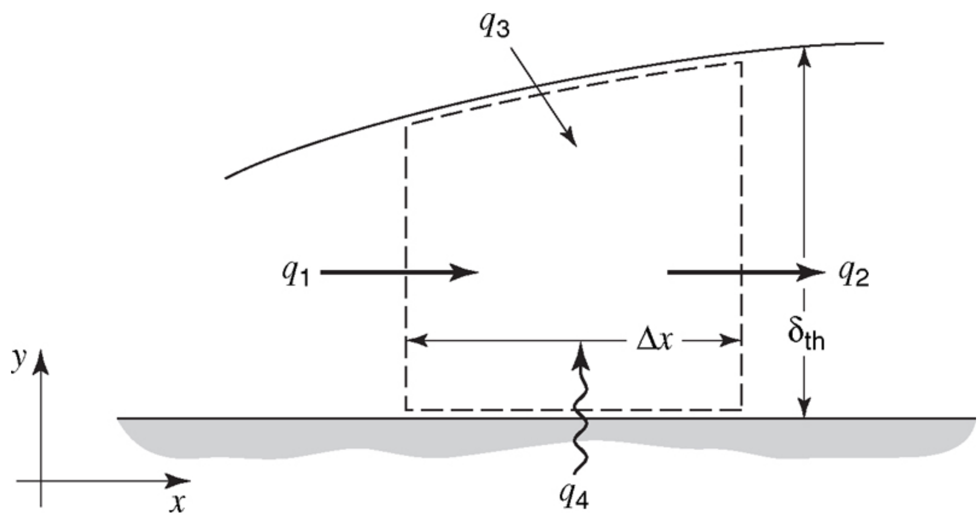
$$hL = 0.664 k \,\mathrm{Pr}^{1/3}\, \mathrm{Re}^{1/2}$$

The mean Nusselt number becomes $Nu = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$

(19-26)

Approximate Integral Analysis of the Thermal Boundary layer – von Karman Analysis

- Useful for flow other than laminar or for a configuration other than a flat surface.



Control volume for integral energy analysis under steady-state conditions

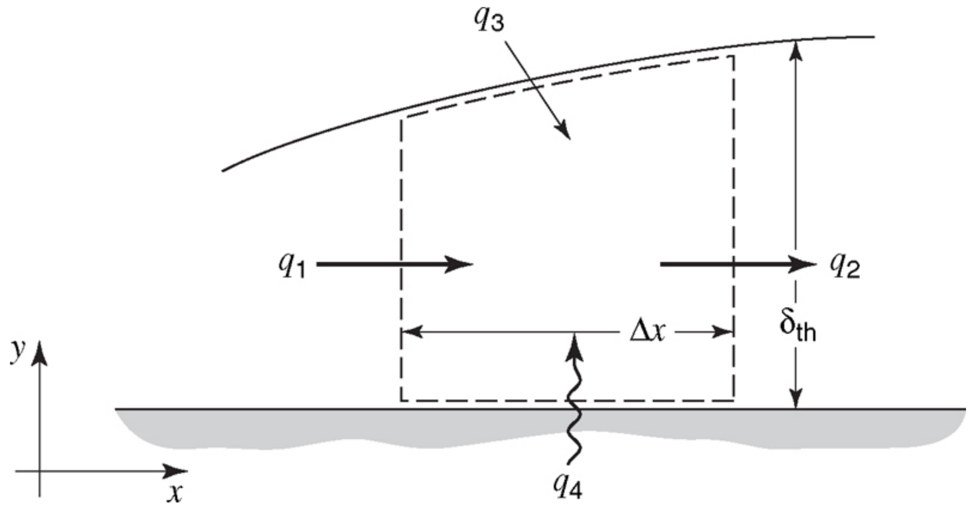
Energy Balance: $$q_4 = q_2 - q_1 - q_3$$

Convection:

Molecular energy exchange or conduction effects	Energy transport due to bulk fluid motion, also called advection

General Thermal Considerations

- Application of the first law of thermodynamics in integral form under steady-state conditions



$$\frac{\partial Q}{\partial t} = \iint_{c.s} (e + P/\rho)\rho(V \cdot n)dA \quad \text{(6-10)}$$

$$-k\Delta x \left.\frac{\partial T}{\partial y}\right|_{y=0} = \left.\int_0^{\delta_t} \rho \upsilon_x c_p T dy\right|_{x+\Delta x} - \left.\int_0^{\delta_t} \rho \upsilon_x c_p T dy\right|_{x} - \Delta x \frac{d}{dx}\int_0^{\delta_t} \rho \upsilon_x c_p T_\infty dy$$

$$q_4 = q_2 - q_1 - q_3$$

Approximate Integral Analysis of the Thermal Boundary layer – von Karman Analysis

- If the flow is incompressible and an average value of c_p is used

$$-k\Delta x \frac{\partial T}{\partial y}\bigg|_{y=0} = \int_0^{\delta_t} \rho \upsilon_x c_p T dy\bigg|_{x+\Delta x} - \int_0^{\delta_t} \rho \upsilon_x c_p T dy\bigg|_{x} - \rho c_p \Delta x \frac{d}{dx}\int_0^{\delta_t} \upsilon_x T_\infty dy$$

Differential Energy Balance Equation

$$\frac{k}{\rho c_p}\frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^{\delta_t} \upsilon_x (T_\infty - T) dy$$

To be solvable, both velocity and temperature profile need to be known.

von Karman Analysis

To solve the differential equation, we need to know both velocity and temperature profiles which must satisfy the boundary conditions

Assuming a power-series expressions

(1) At y=0 $\quad T - T_s = 0$

(2) At y = δ_t $\quad T - T_s = T_\infty - T_s$

(3) At y = δ_t $\quad \dfrac{\partial}{\partial y}(T - T_s) = 0$

(4) At y = 0 $\quad \dfrac{\partial^2}{\partial y^2}(T - T_s) = 0$

$$T - T_s = a + by + cy^2 + dy^3$$

$$\upsilon_x = a' + b'y + c'y^2 + d'y^3$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

$$\frac{\upsilon}{\upsilon_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\frac{k}{\rho c_p}\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{d}{dx}\int_0^{\delta_t} \upsilon_x (T_\infty - T)\,dy$$

$$Nu_x = \frac{h_x x}{k} = 0.36\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

Convection Heat-Transfer Correlations

Correlation functions:

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- ❑**Theoretical /analytical approach** – solving boundary layer equation for a particular geometry
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Energy and Momentum Transfer Analogies

Comparison to The Equation of Motion

- 2-D equation of energy

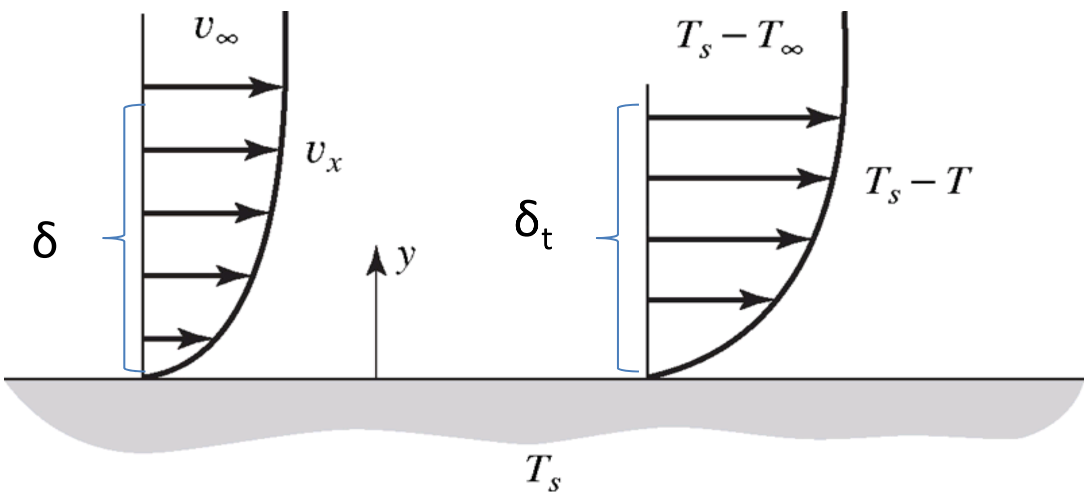
$$\upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}$$

- 2-D equation of motion

$$\upsilon_x \frac{\partial \upsilon_x}{\partial x} + \upsilon_y \frac{\partial \upsilon_y}{\partial y} = \nu \frac{\partial^2 \upsilon_x}{\partial y^2}$$

$$\Pr \equiv \frac{\nu}{a} = \frac{\mu c_p}{k} \sim \frac{momentum}{thermal}$$

$$\Pr^n \approx \frac{\delta}{\delta_t}, \quad [n > 0]$$



Reynolds Analogy for Pr =1 and no form drag.

Reynolds Analogy

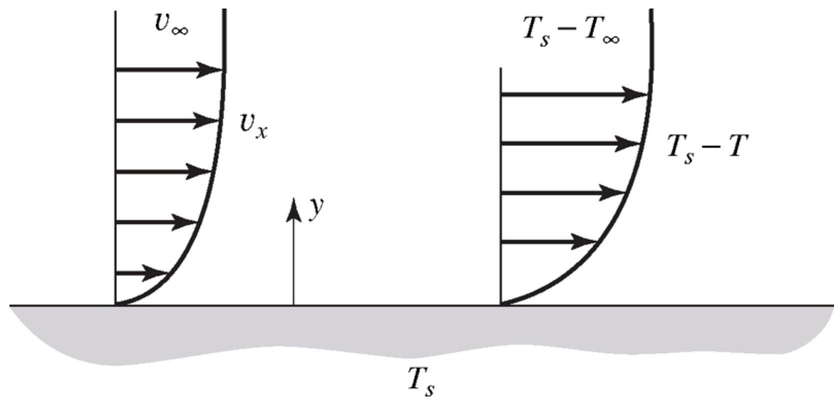
The boundary conditions for temperature and velocity are compatible, i.e., Pr =1 and no form drag.

$$\frac{d}{dy}\frac{\upsilon_x}{\upsilon_\infty}\bigg|_{y=0} = \frac{d}{dy}\left(\frac{T-T_s}{T_\infty - T_s}\right)\bigg|_{y=0}$$

$$\mathrm{Pr} \equiv \frac{v}{a} = \frac{\mu c_p}{k} \sim \frac{momentum}{thermal} = 1$$

$$\mu c_p \frac{d}{dy}\frac{\upsilon_x}{\upsilon_\infty}\bigg|_{y=0} = k\frac{d}{dy}\left(\frac{T-T_s}{T_\infty - T_s}\right)\bigg|_{y=0}$$

$$\frac{\mu c_p}{\upsilon_\infty}\frac{d\upsilon_x}{dy}\bigg|_{y=0} = -\frac{k}{T_s - T_\infty}\frac{d(T-T_s)}{dy}\bigg|_{y=0}$$



$$h_x = \frac{\mu c_p}{\upsilon_\infty}\frac{d\upsilon_x}{dy}\bigg|_{y=0}$$

Reynolds Analogy

The Coefficient of Skin Friction – a key dimensionless parameter in Momentum Transfer.

Surface shear stress

$$C_{f,x} \cong \frac{\tau_0}{\rho \upsilon_\infty^2 / 2} = \frac{2\mu c_p}{\rho \upsilon_\infty^2} \frac{d\upsilon_x}{dy}\bigg|_{y=0} \Longrightarrow h_x = \frac{\mu c_p}{\upsilon_\infty} \frac{d\upsilon_x}{dy}\bigg|_{y=0}$$

Dynamic pressure

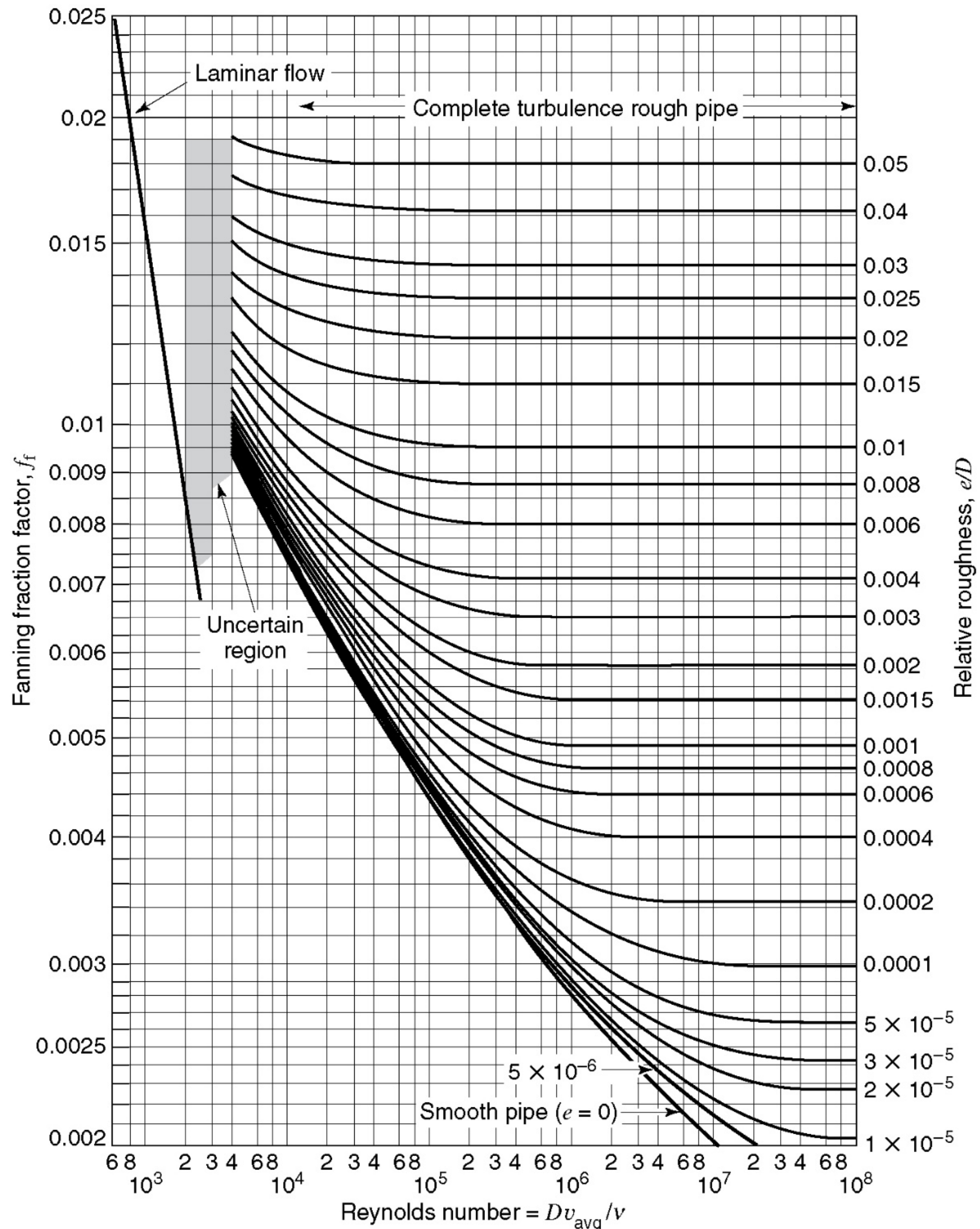
$$h_x = \frac{C_{f\,x}}{2}(\rho \upsilon_\infty c_p) \qquad St_x \equiv \frac{h_x}{\rho \upsilon_\infty c_p} = \frac{C_{fx}}{2} \qquad St \equiv \frac{h}{\rho \upsilon_\infty c_p} = \frac{C_f}{2}$$

A knowledge of the coefficient of frictional drag will enable the convective heat-transfer coefficient to be readily evaluated.

Restrictions: (1) Pr =1 (2) no form drag

Fig 13.1. The Fanning friction factor as a function of Re and D/e.

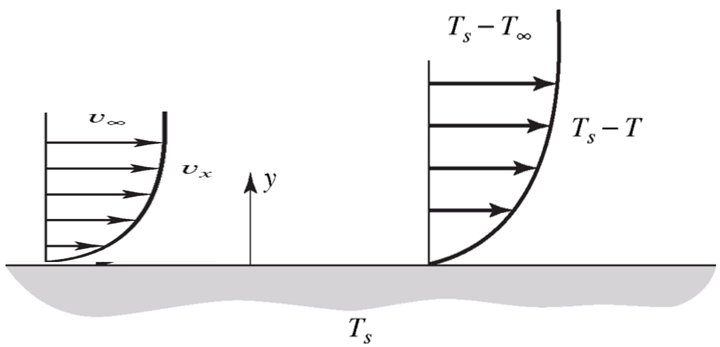
For Pipe flow



Colburn Analogy

The boundary conditions for temperature and velocity are not compatible, i.e., Pr ≠ 1; there is no form drag

$$St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2} \qquad [0.5 < \mathrm{Pr} < 50]$$



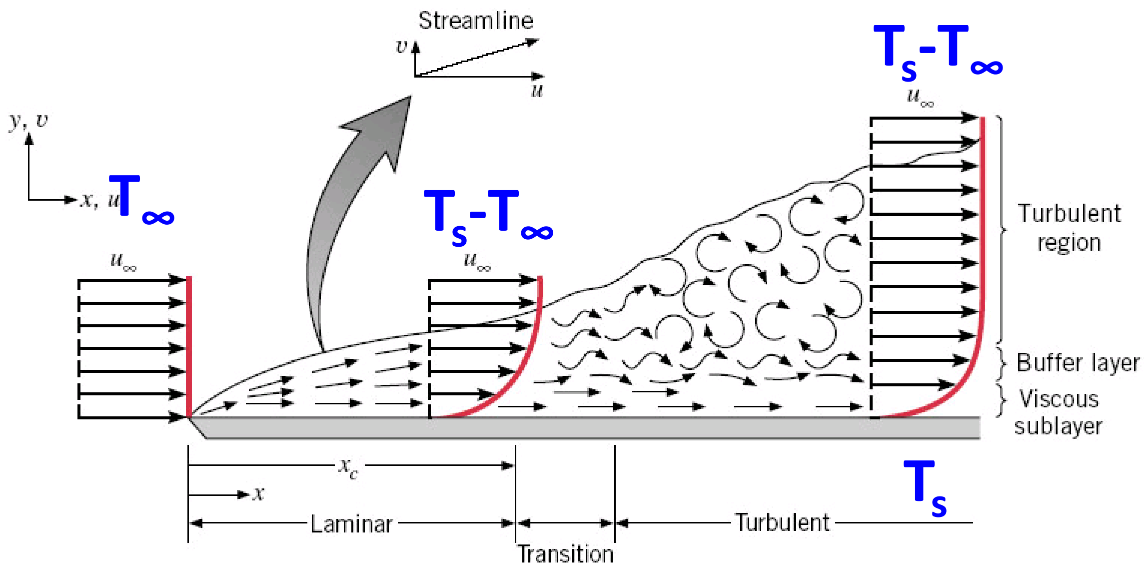
Colburn j factor

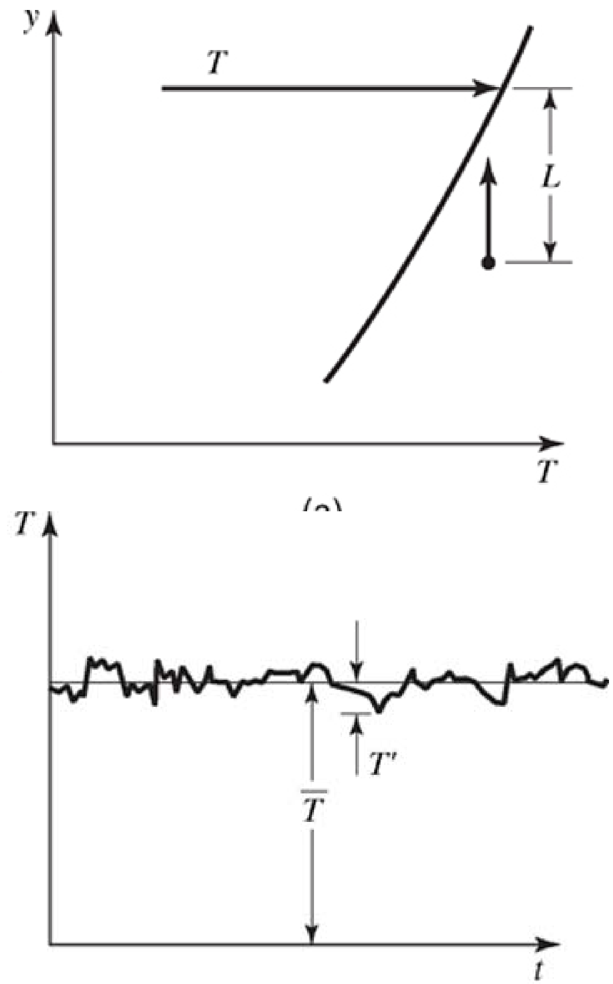
$$j_H = St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2}$$

Note that for Pr =1, the Colburn and Reynolds analogies are the same. High and low Prandtl number fluids falling outside this range would be heavy oils at one extreme and liquid metals at the other.

The Colburn analogy is particularly helpful for evaluating heat transfer internal forced flows, also quite accurate for turbulent flows.

Turbulent Flow Considerations – Eddy Diffusion





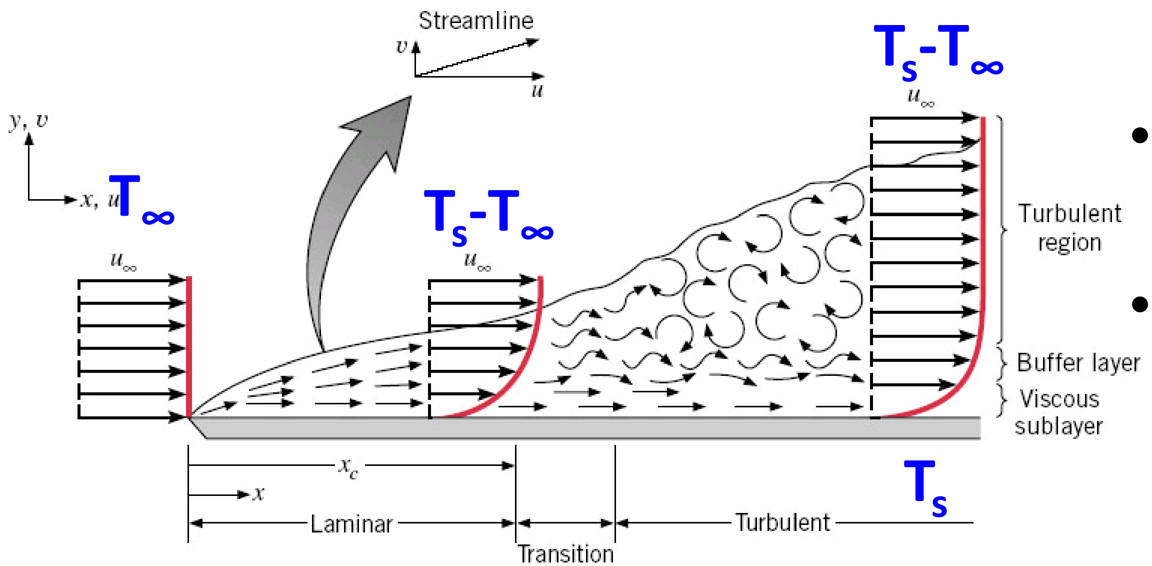
(b)

Prandtl mixing length, L - the distance moved by a fluid "packet" in the y direction through which the fluid "packet" retain the mean temperature from its point of origin.

Fluctuating temperature T'
Fluctuating velocity, υ'_y

Eddy diffusion of heat

Total Heat Flux



- Molecular diffusivity of heat – a
- Eddy diffusivity of heat – ε_H

- Total heat flux are due to both microscopic molecular and macroscopic turbulent contributions

$$\frac{q_y}{A} = -\rho c_p [a + \overline{|\upsilon_y' L|}] \frac{d\overline{T}}{dy} = -\rho c_p [a + \varepsilon_H] \frac{d\overline{T}}{dy}$$

- Applies to both the laminar and turbulent regions.

Prandtl and von Karman Analogy

$$\frac{q_y}{A} = -\rho c_p [a + \overline{|\upsilon_y' L|}] \frac{d\overline{T}}{dy} = -\rho c_p [a + \varepsilon_H] \frac{d\overline{T}}{dy}$$

Prandtl analogy

$$St = \frac{C_f / 2}{1 + 5\sqrt{C_f / 2}(\mathrm{Pr} - 1)}$$

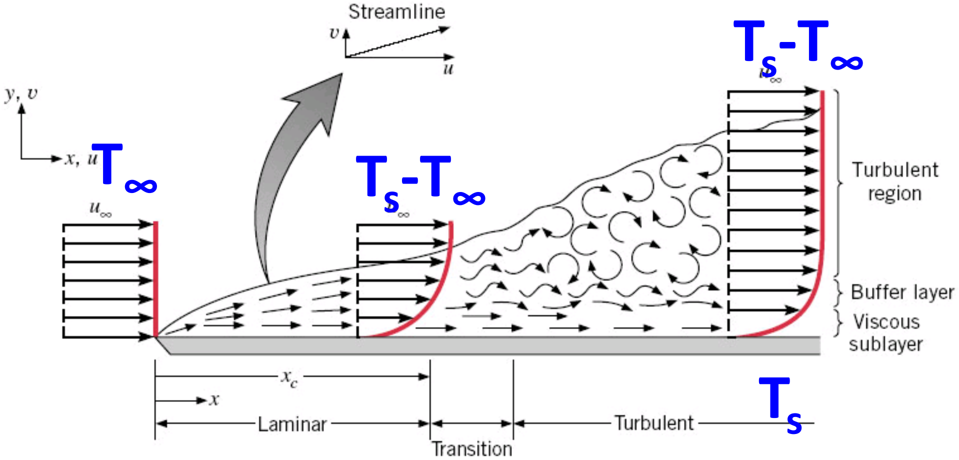
Von Karman Analogy

$$St = \frac{C_f/2}{1 + 5\sqrt{C_f/2}\left\{Pr - a + \ln[1 + \frac{5}{6}(Pr - 1)]\right\}}$$

These equations yield the most accurate results for Pr > 1

Convective Correlations

Critical Reynolds Number



Laminar flow

Exact Analysis

$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Reynolds Analogy

$$St \equiv \frac{h}{\rho \upsilon_\infty c_p} = \frac{C_f}{2} \qquad [\mathrm{Pr}=1]$$

Laminar and turbulent flows

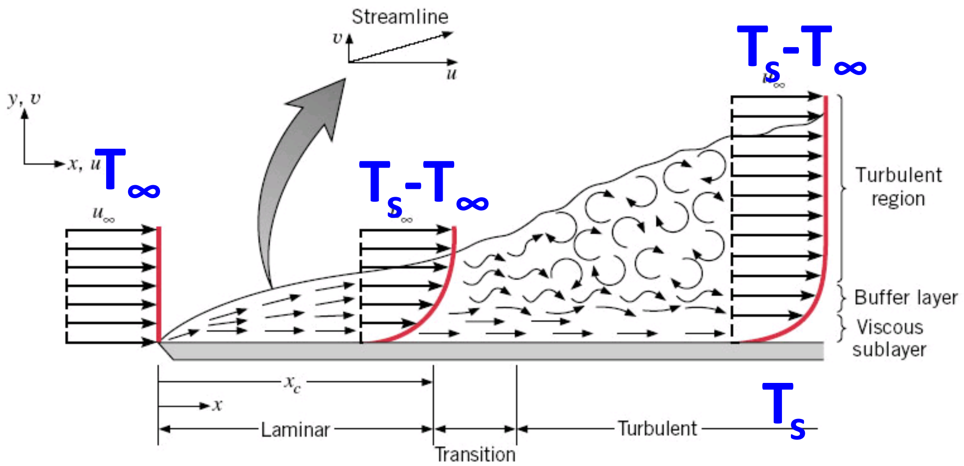
Colburn Analogy

$$St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2} \qquad [0.5 < \mathrm{Pr} < 50]$$

Prandtl analogy

$$St = \frac{C_f/2}{1+5\sqrt{C_f/2}(\mathrm{Pr}-1)}$$

Film Temperature



$$T_f \equiv \frac{T_s + T_\infty}{2}$$

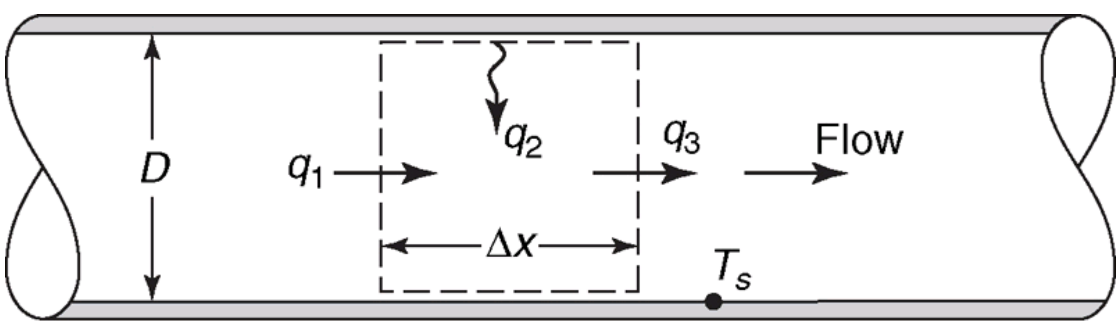
- Fluid properties vary with temperature across the boundary layer and this variation influence the Reynolds, Prandtl, Nusselt, Stanton and other numbers, and the heat transfer rate.

- Note it is customary to evaluate all fluid properties at a mean boundary layer temperature T_f, termed the film temperature for the external flows if the fluid properties are not specified.

Example 3 (Text pp291-293)

Water at 50°F enters a heat-exchange tube having an inside diameter of 1 in. and a length of 10ft. The water flows at 20 gal/min. For a constant wall temperature of 210°F, estimate the exit temperature of the water.

Entrance effects are to be neglected, and the free stream temperature may be estimated as the arithmetic –mean bulk temperature and properties of water may be evaluated at the film temperature.



Note for internal flow, the mean bulk temperature is typically used to evaluate fluid properties for internal flows

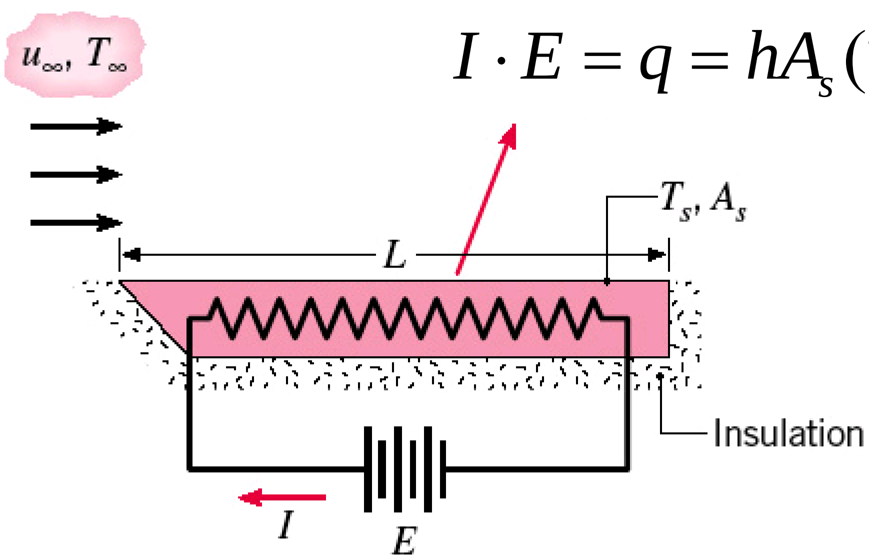
The Empirical Approach

- Experiment for measuring the mean convection heat transfer coefficient.
  - Repeat for a variety of test conditions
  - Varying velocity, plate length , nature of the fluid

$$\mathrm{Re} = \frac{L\upsilon\rho}{\mu}$$

$$\mathrm{Pr} = \frac{\mu c_p}{k}$$

$$I \cdot E = q = hA_s(T_s - T_\infty)$$



$$Nu = \frac{hL}{k}$$

Algebraic correlation function

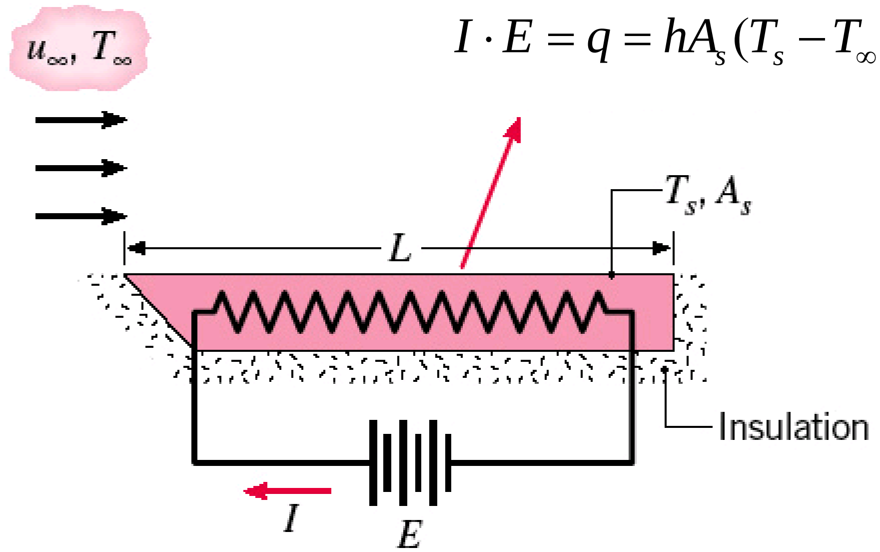
$$Nu = f_1(\mathrm{Re}, \mathrm{Pr}) = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

The Empirical Approach

- Experiment for measuring the mean convection heat transfer coefficient.
  - q, T_s , T_∞ and A_s can be easily measured.
  - h can be computed from the Newton's law of cooling.
  - From the knowledge of the characteristic length and the fluid properties
  - Repeat for a variety of test conditions by

    varying velocity, plate length

    Varying nature of the fluid



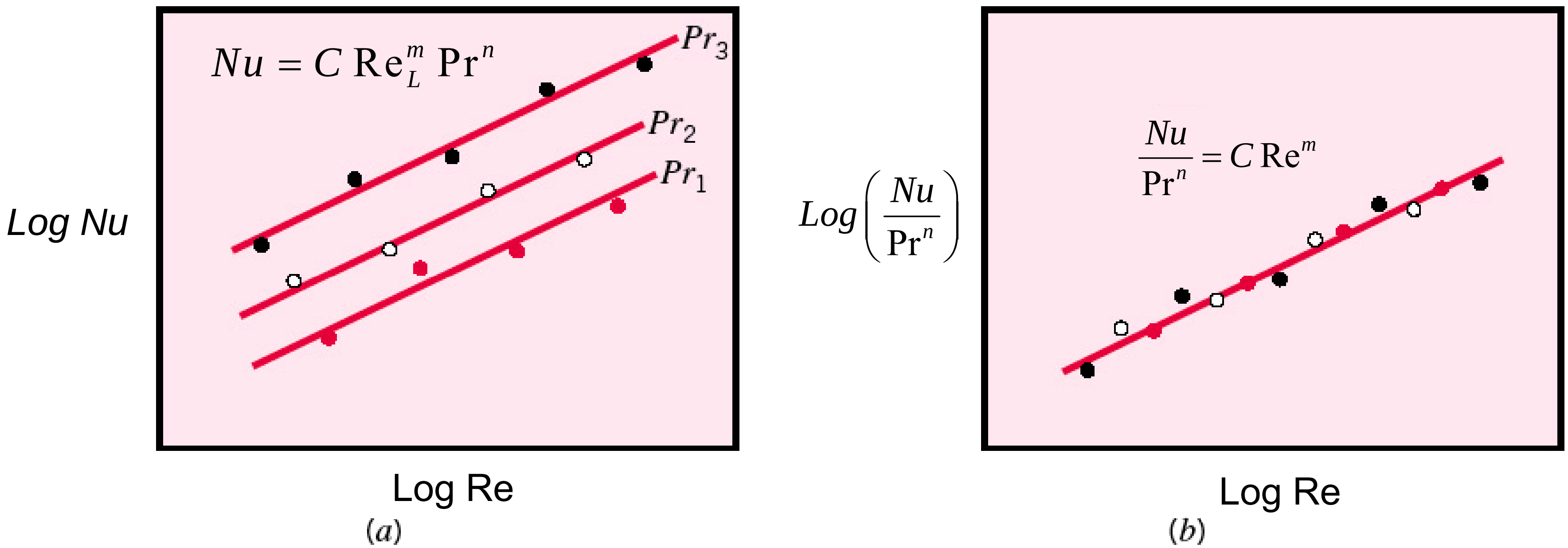
$$I \cdot E = q = hA_s(T_s - T_\infty)$$

$$Nu = \frac{hL}{k} \qquad \mathrm{Re} = \frac{L\upsilon\rho}{\mu} \qquad \mathrm{Pr} = \frac{\mu c_p}{k}$$

$$Nu = f_1(\mathrm{Re}, \mathrm{Pr}) = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

Commonly evaluating all fluid properties at the film temperature $T_f \equiv \dfrac{T_s + T_\infty}{2}$

Dimensionless representation of convection heat transfer measurements by plotting in a log-log scale



Values of C, m, n are often independent of the nature of the fluid, the family of straight lines corresponding to different Prandtl numbers can be collapsed to a single line.

Common Convection Correlations

- Forced Convection for External Flow
- Forced Convection for Internal Flow
- Natural or free convection
- Boiling and condensation