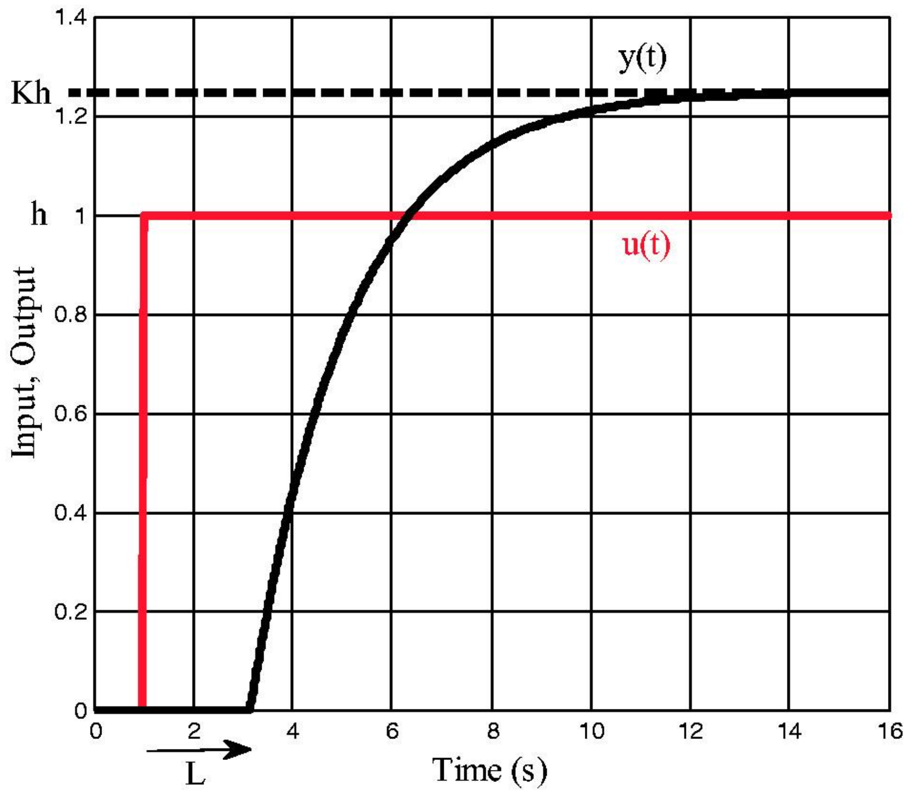
FIRST ORDER PLUS DEAD TIME MODELS

Data driven linear ODEs for process control



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LEARNING OUTCOMES

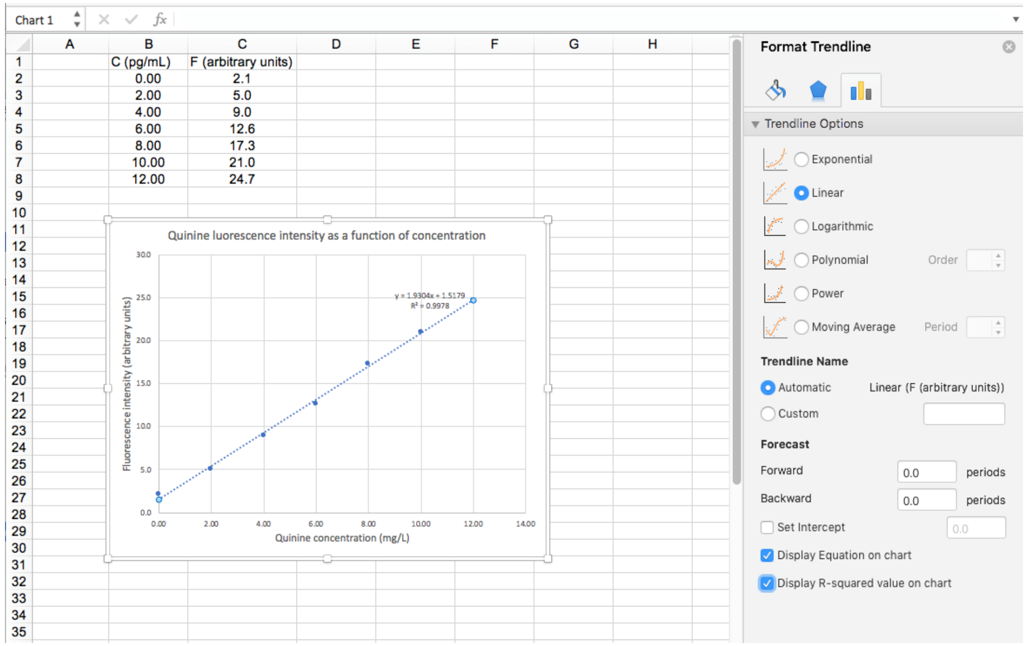
- Explain and predict how FOPDT parameters affect the response
- Graphically identify FOPDT parameters using step test data

RESOURCES

- APMonitor course
  - https://apmonitor.com/pdc/index.php/Main/FirstOrderSystems
  - https://apmonitor.com/pdc/index.php/Main/FirstOrderPlusDeadTime
  - https://apmonitor.com/pdc/index.php/Main/FirstOrderGraphical
- Seborg 3rd Edition
  - Section 7.2

DATA DRIVEN MODELS

- Mathematical models with adjustable parameters to fit data
- Parameters are chosen such that the model “best” fits empirical data
- The process of choosing the best parameters based on data is called **regression**
- *Example:* Trend lines in Excel



FOPDT MODEL

- Data driven linear SISO model for process control
  - Uses deviation variables $y' = y - y_{ss}$ $u' = u - u_{ss}$

Note: The reading simply writes y and u for convenience, but really should be y' and u'

$$\tau_p \frac{dy'(t)}{dt} = -y'(t) + K_p u'\left(t - \theta_p\right)$$

  - *Process gain K_p:* The total change in y' caused by a unit change in u'
  - *Process time constant τ_p*: Describes how fast y' changes
  - *Process time delay (i.e., dead time) θ_p*: How long it takes y' to start responding after a change to u'
- Let's try it out!

1ST PRINCIPLES BASED PARAMETER SELECTION

- Start with nonlinear SISO ODE model

$$\frac{dy}{dt} = f(y, u)$$

- Linearize

$$\frac{dy'}{dt} = \alpha y' + \beta u' \qquad \alpha = \left.\frac{\partial f}{\partial y}\right|_{y_{ss}, u_{ss}} \qquad \beta = \left.\frac{\partial f}{\partial u}\right|_{y_{ss}, u_{ss}}$$

- Reformulate in FOPDT form (time delay excluded)

$$\tau_p \frac{dy'}{dt} = -y' + K_p u' \qquad \frac{-1}{\alpha}\frac{dy'}{dt} = -y' + \frac{-\beta}{\alpha} u'$$

- Relate linearized ODE parameters to FOPDT parameters

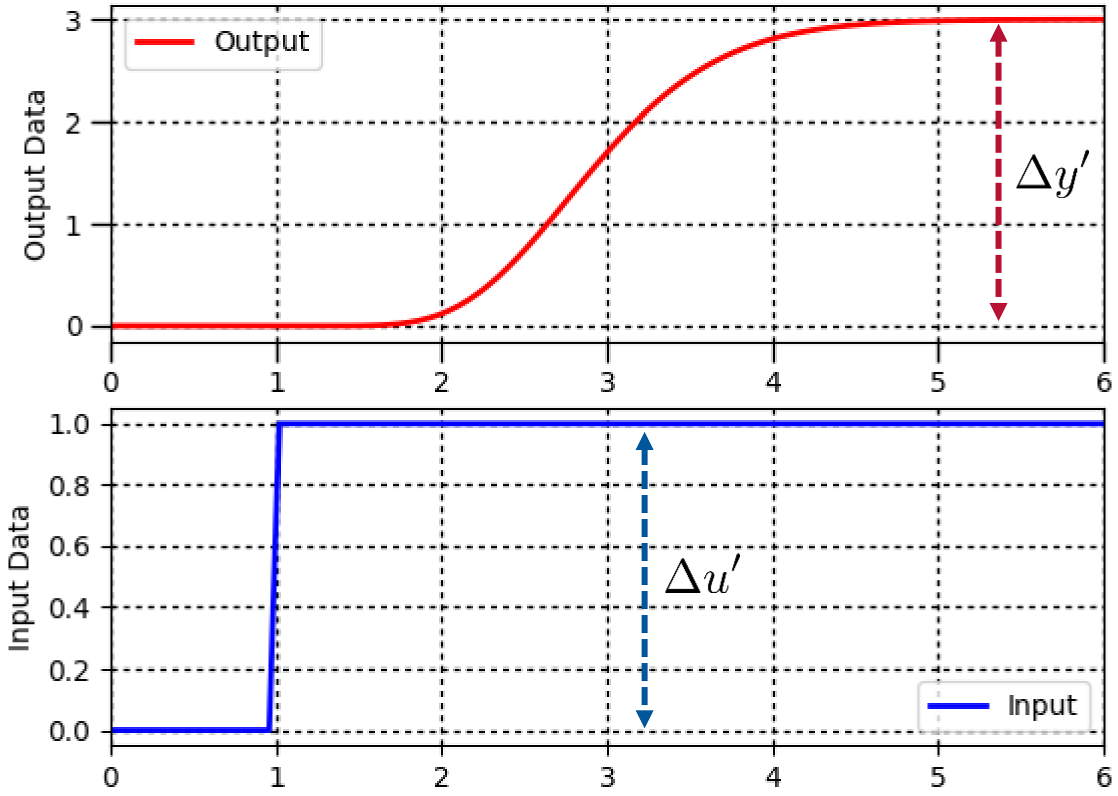
$$\boxed{\tau_p = \frac{-1}{\alpha} \qquad K_p = \frac{-\beta}{\alpha}}$$

PROCESS GAIN (K_p)

- Captures the ratio relating how a step change in u' changes the steady-state value of y'

$$K_p = \frac{\Delta y'}{\Delta u'} = \frac{y'_{ss_2} - y'_{ss_1}}{u'_{ss_2} - u'_{ss_1}}$$

- Can be positive or negative
- Only valid if there is a new steady-state



PROCESS TIME CONSTANT (τ_p)

- The time for y' to reach 63.2% of the way to the new steady state after step change

$$y'(\tau_p) = 0.632\Delta y'$$

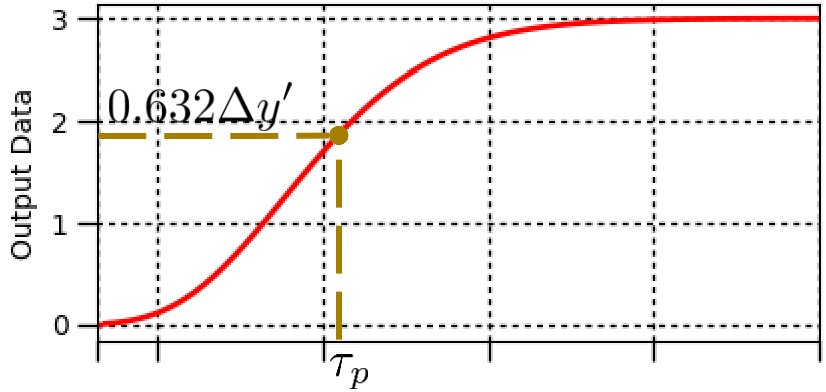
- Derivation

FOPDT solution w/ step change $\Delta u'$ & $\theta_p = 0$

$$y'(t) = \left(e^{-t/\tau_p}\right) y'(0) + \left(1 - e^{-t/\tau_p}\right) K_p \Delta u'$$

Let $y'(0) = 0$ and consider $t = \tau_p$

$$\begin{aligned} y'(\tau_p) &= \left(1 - e^{-\tau_p/\tau_p}\right) K_p \Delta u' \\ &= \left(1 - e^{-1}\right) \frac{\Delta y'}{\Delta u'} \Delta u' \\ &= 0.632\Delta y' \end{aligned}$$

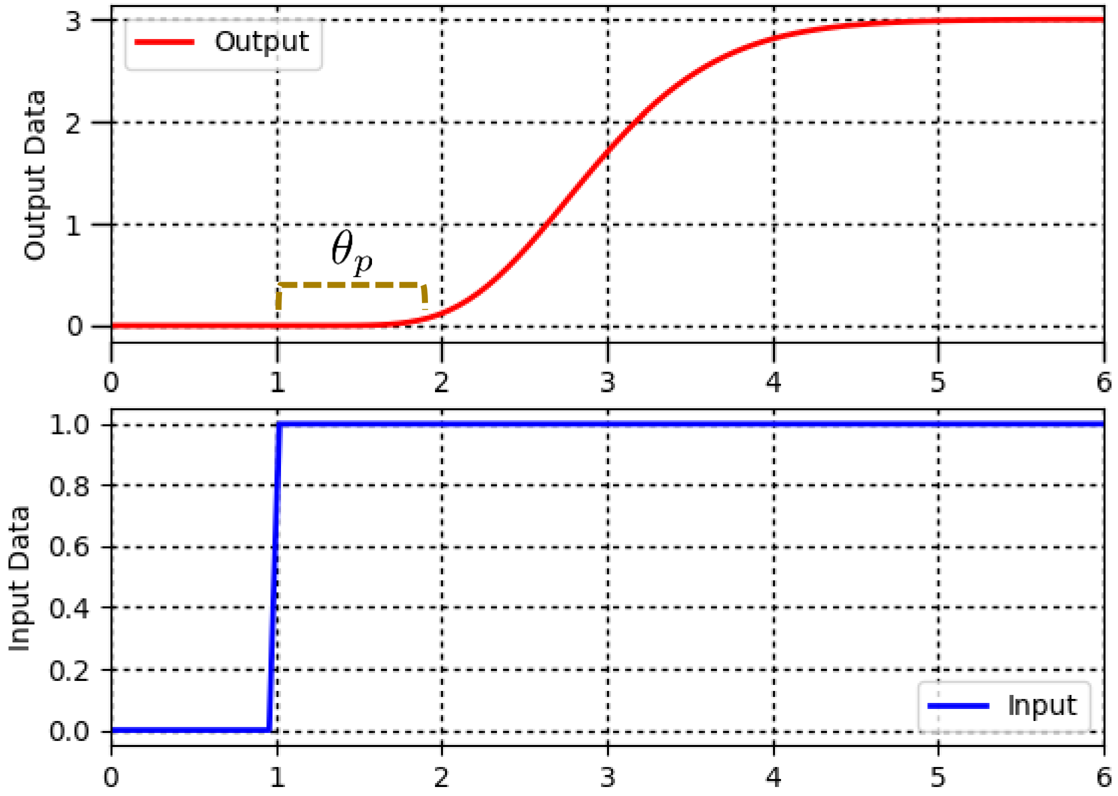


PROCESS TIME DELAY (θ_p)

- The time duration until y' starts to change

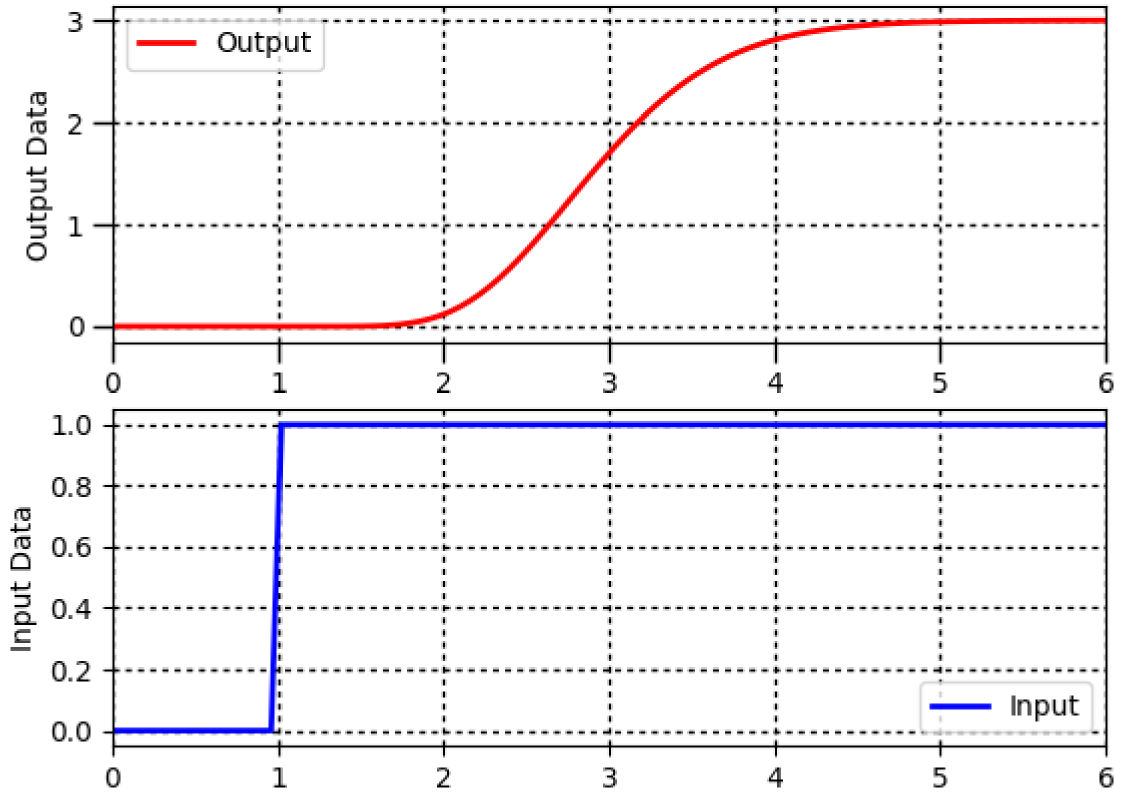
$$y'(t < \theta_p) = y'(0)$$

$$y'(t \geq \theta_p) = \left(e^{-(t-\theta_p)/\tau_p}\right) y'(0) + \left(1 - e^{-(t-\theta_p)/\tau_p}\right) K_p \Delta u'$$



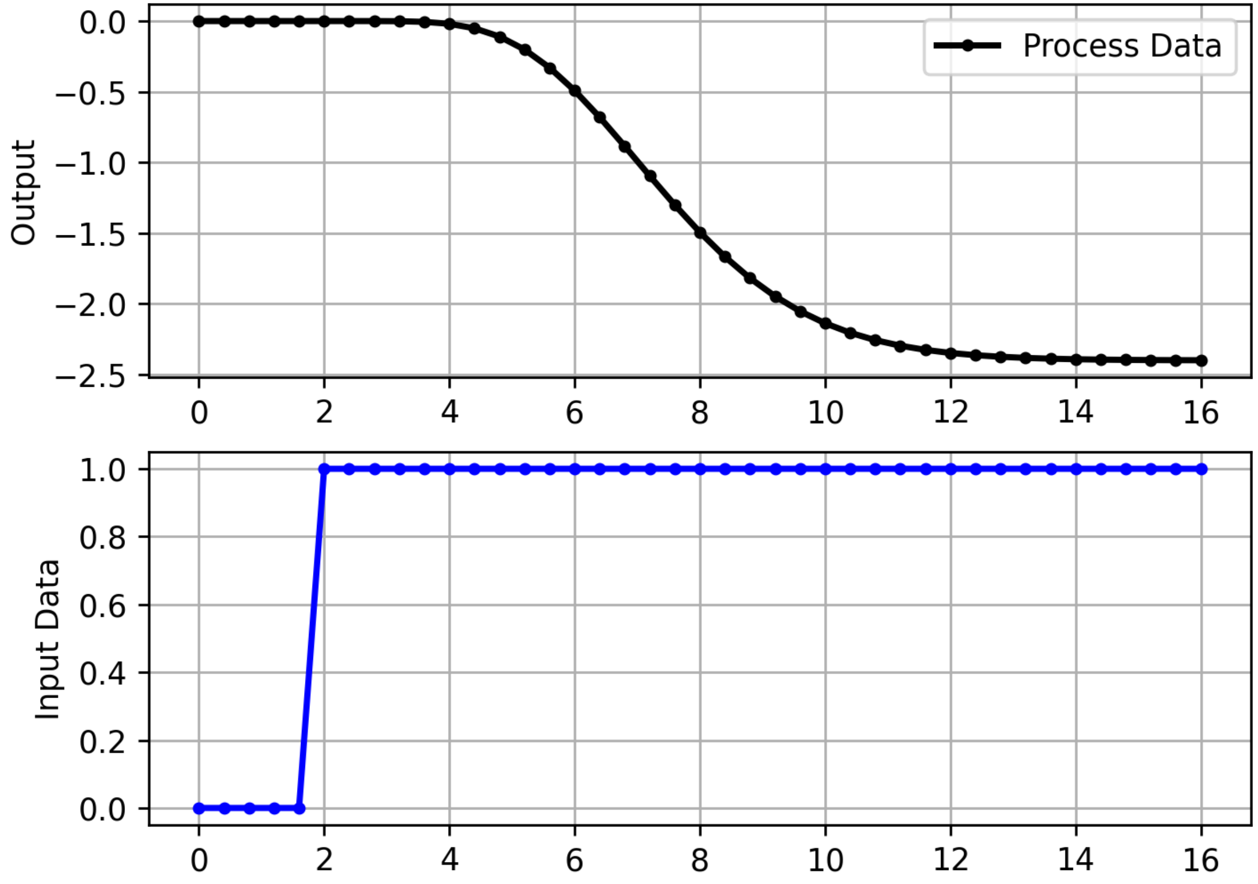
GRAPHICAL FIT FROM STEP TEST DATA

1. Determine $\Delta y'$
2. Determine $\Delta u'$
3. Compute $K_p = \frac{\Delta y\prime}{\Delta u\prime}$
4. Determine θ_p (using inflection point)
5. Determine $0.632\Delta y'$
6. Determine $t_{0.632}$ for $y'(t_{0.632})$
7. Compute $\tau_p = t_{0.632} - \theta_p - \text{time before step}$



EXERCISE

- Compute FOPDT parameters w/ the step test data



BEFORE NEXT TIME

- Quiz 3: Due at 11:59pm
- Reading
  - https://apmonitor.com/pdc/index.php/Main/FirstOrderOptimization
  - https://apmonitor.com/pdc/index.php/Main/ExamModeling
- Assignment 2: Due Jan. 22 (same time as Test 1)
  - Get started now, you now can complete all but one question