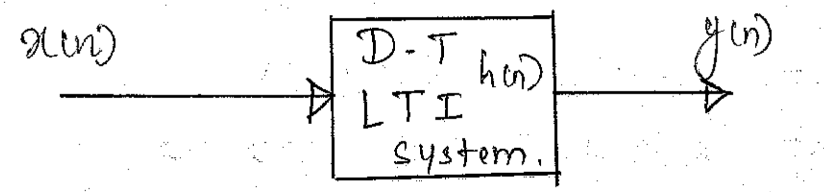
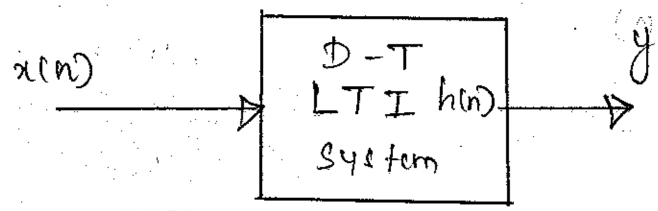
ANALYSIS OF DISCRETE-TIME LTI SYSTEM.



→ Linear Convolution
→ Correlation
↳ Auto correlation
↳ Cross-Correlation

$x(n) \to$ Input (D-T signal / sequence).
$y(n) \to$ Output (D-T signal / sequence).
$h(n) \to$ Impulse response of LTI system.
(or) Sample response of LTI system.

(1) Linear Convolution.



$y(n) = x(n) * h(n)$.

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k).\, h(n-k). = x(n) * h(n),}$$

Convolution is the process of obtaining the response from the given input and Impulse response. This is sometime known as System Analysis.

Properties of Discrete Convolution / Linear Convolution

(1) Commutative

$$x(n) * h(n) = h(n) * x(n)$$

(2) Distributive.

$$x(n) * \{h_1(n) + h_2(n)\} = x(n) * h_1(n) + x(n) * h_2(n).$$

(3) Associative.

$$x(n) * \{h_1(n) * h_2(n)\} = \{x(n) * h_1(n)\} * h_2(n).$$

(1) Determine the output $y(n)$ of a relaxed linear-time invariant system with impulse response

$$h(n) = a^n u(n) \; ; \quad 0 < a < 1$$

when the input is a unit step sequence, that is

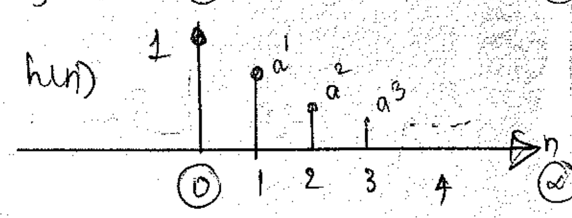
$$x(n) = u(n).$$

$x(n) = u(n)$

Given.

$x(n) = u(n)$

$h(n) = a^n u(n) \; ; \; 0 < a < 1$



$h(n)$ is exponentially decaying because $[0 < a < 1]$

WKT

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

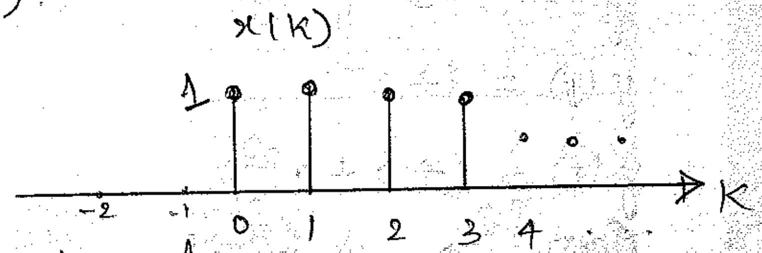
compute the limits of o/p $y(n)$.

Sum of lower limits of $x(n)$ & $h(n)$	$\leq$	$y(n)$ is defined for n	$\leq$	Sum of upper limits of $x(n)$ & $h(n)$.
$0 + 0$ $= 0$	$\leq$	$y(n)$ is defined. for n values	$\leq$	$\infty + \infty$ $= \infty$

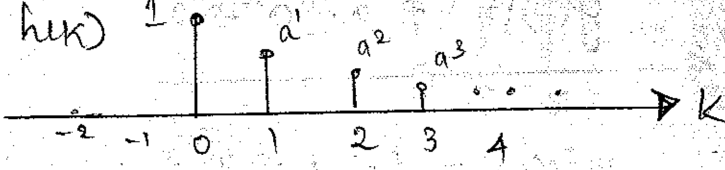
$0 \leq$ $y(n)$ is defined for n values $\leq \infty$

Simply $\quad 0 \leq n \leq \infty.$

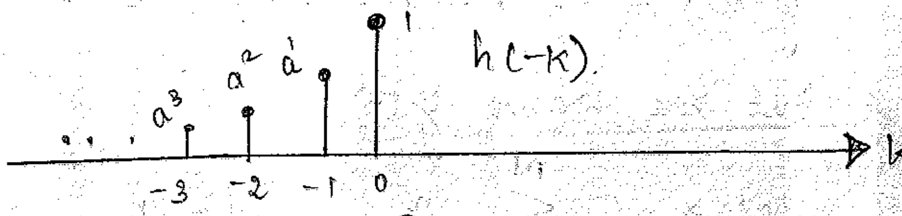
$$y(n) = \sum_{k=-\infty}^{\infty} x(k).\, h(n-k).$$



$x(k)$

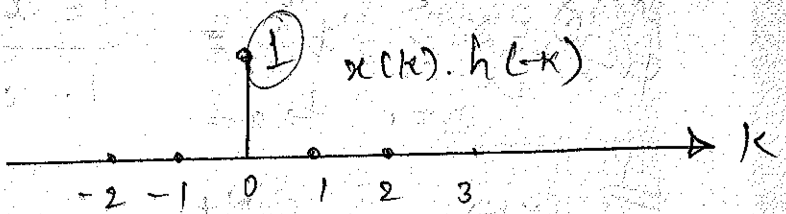


$h(k)$



$h(-k)$

put $n=0$

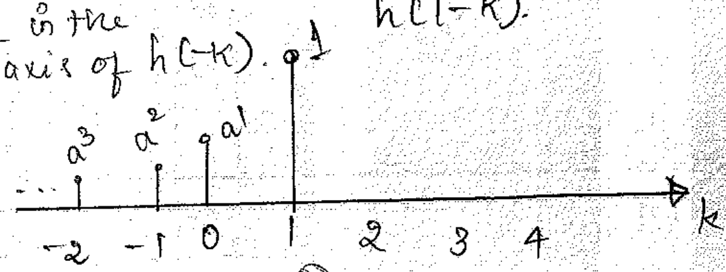


$$y(0) = \sum_{k=-\infty}^{\infty} x(k).\, h(-k).$$

Meaning multiply $x(k)$ & $h(-k)$ and add all the amplitudes.

$$y(0) = 1.$$

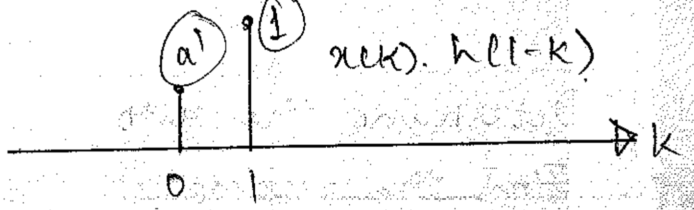
add 1 in the time axis of $h(-k)$.



put $n=1$

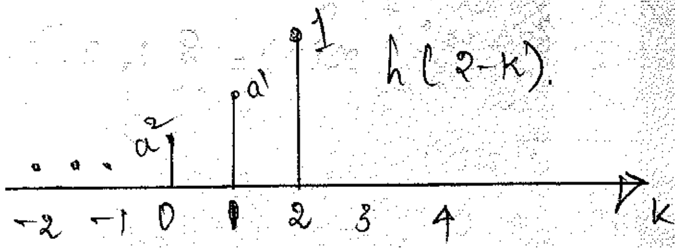
$$y(1) = \sum_{k=-\infty}^{\infty} x(k).\, h(1-k)$$

multiply $x(k).\, h(1-k)$ and add the amplitudes.



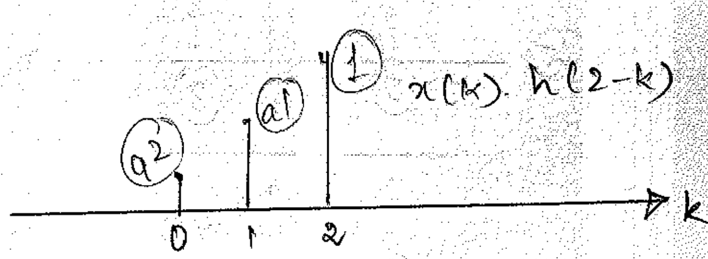
$$y(1) = 1 + a.$$

put $n=2$



$$y(2) = \sum_{k=-\infty}^{\infty} x(k).\, h(2-k).$$

$$y(2) = 1 + a + a^2.$$



$\vdots$

$$y(\infty) = 1 + a + a^2 + \cdots + a^{\infty}.$$

$y(0) = 1$

$y(1) = 1 + a$

$y(2) = 1 + a + a^2$

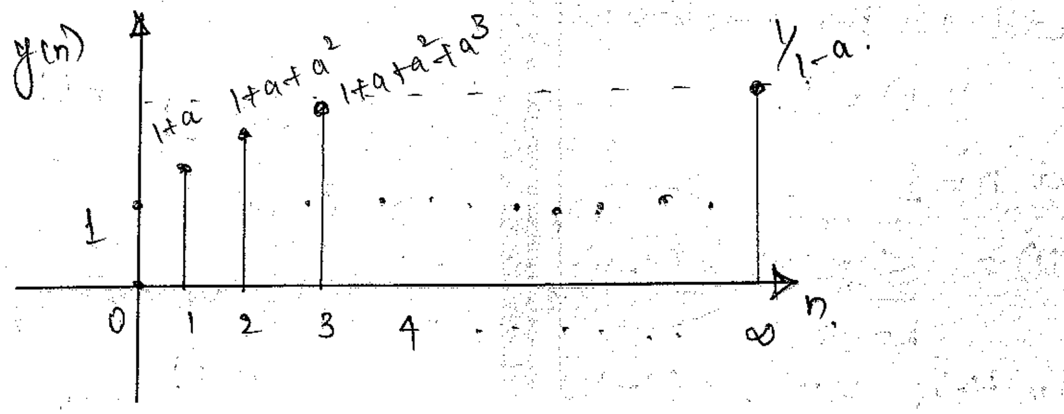
$y(2) = 1 + a + a^2 + a^3$

$\vdots$

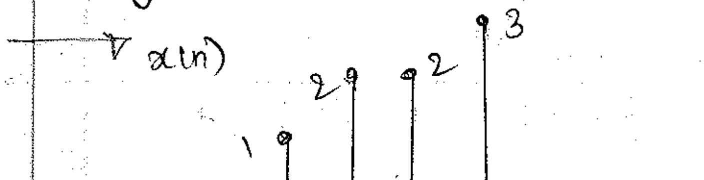
$$y(\infty) = 1 + a + a^2 + \cdots + a^{\infty} = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$y(n) = \left\{ 1, 1+a, 1+a+a^2, 1+a+a^2+a^3, \ldots, \frac{1}{1-a} \right\}$$

o/p



(2) Find the response of the system for the input signal $x(n) = \{1, 2, 2, 3\}$ and $h(n) = \{1, 0, 3, 2$



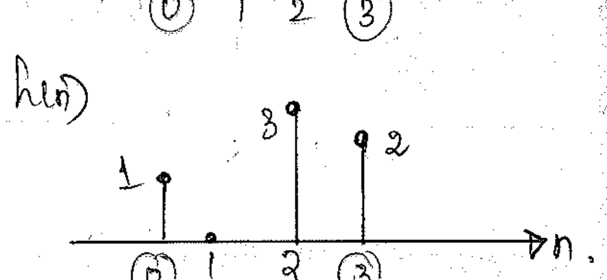
o/p.

$y(n)$

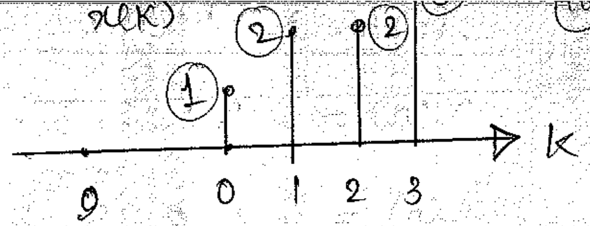
$0+0 \le n \le 3+3$

$=0 \qquad\qquad =6.$

$0 \le n \le 6.$

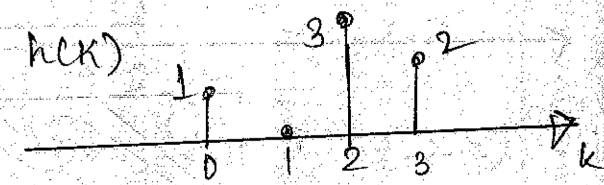


$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

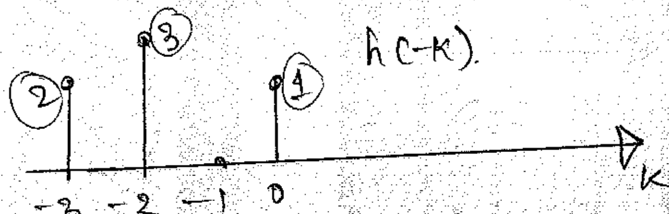


put $n = 0$.

$$y(0) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-k).$$

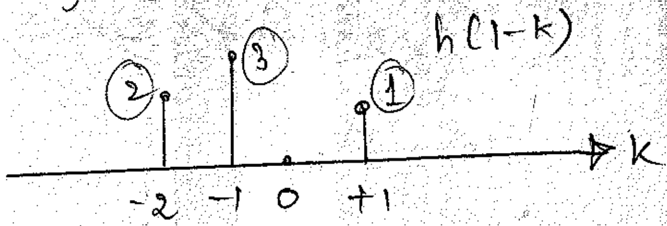


$$\boxed{y(0) = 1}$$



put $n = 1$

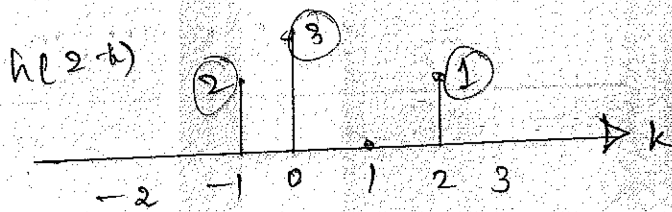
$$y(1) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(1-k)$$



$$\boxed{y(1) = 2}$$

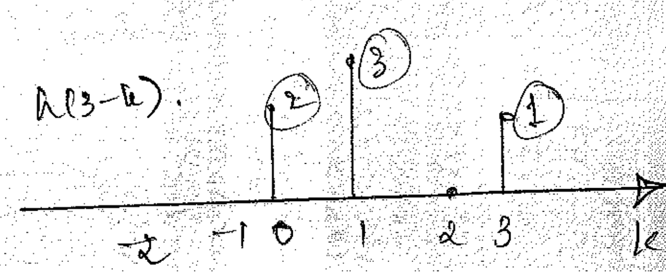
put $n = 2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(2-k)$$



$$= 3 + 2 = 5$$

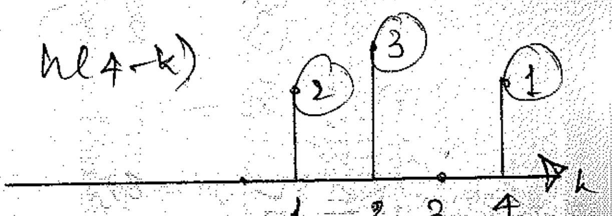
$$\boxed{y(2) = 5}$$



put $n = 3$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(3-k).$$

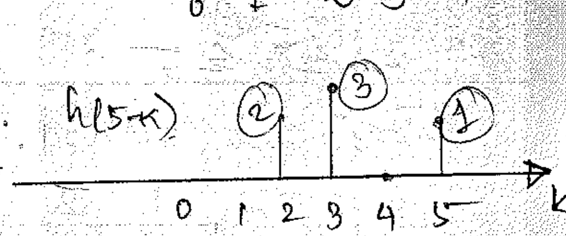
$$= 2 + 6 + 3 = 11$$



$$\boxed{y(3) = 11.}$$

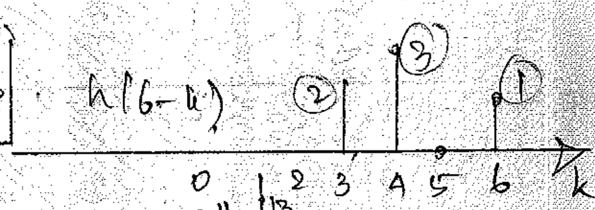
$$y(4) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(4-k).$$

$$= 4 + 6 + 0 = 10 \quad \boxed{y(4) = 10}$$

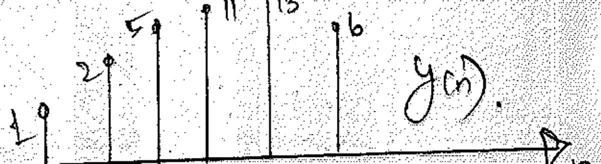


$$y(5) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(5-k)$$

$$= 4 + 9 + 0 = 13 \quad \boxed{y(5) = 13}$$



$$y(6) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(6-k).$$



$$\boxed{y(6) = 6}$$

Short-Cut Method

$x(n) = \{1, 2, 2, 3\}$

$h(n) = \{1, 0, 3, 2\}$

$\longrightarrow x(n)$

h(n) ↓	1	2	2	3
1	1	2	2	3
0	0	0	0	0
3	3	6	6	9
2	2	4	4	6

$y(n) = \{1,\ 0+2,\ 3+0+2,\ 2+6+0+3,\ 6+4+0,\ 9+4,\ 6\}$

$= \{1, 2, 5, 11, 10, 13, 6\}.$

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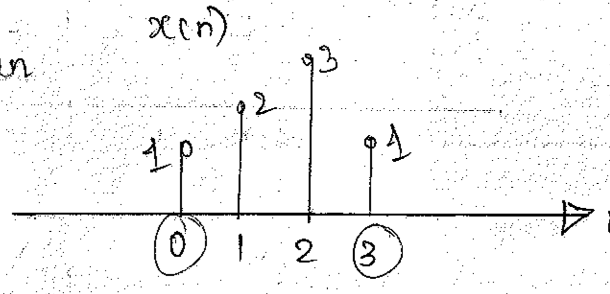
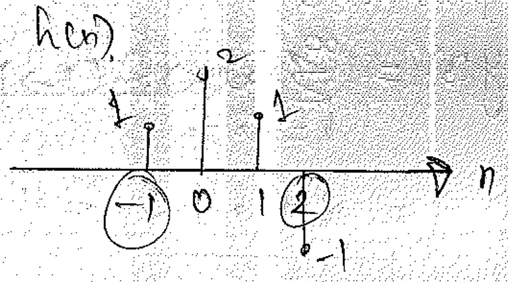
(3) Find the response of the system for the input signal $x(n) = \{1, 2, 3, 1\}$, $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$.

Given





$y(n)$; $n = ??$

$0 + (-1) \le n \le 3 + 2$

$-1 \le n \le 5$

WKT

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Short-cut Method. $\longrightarrow x(n)$

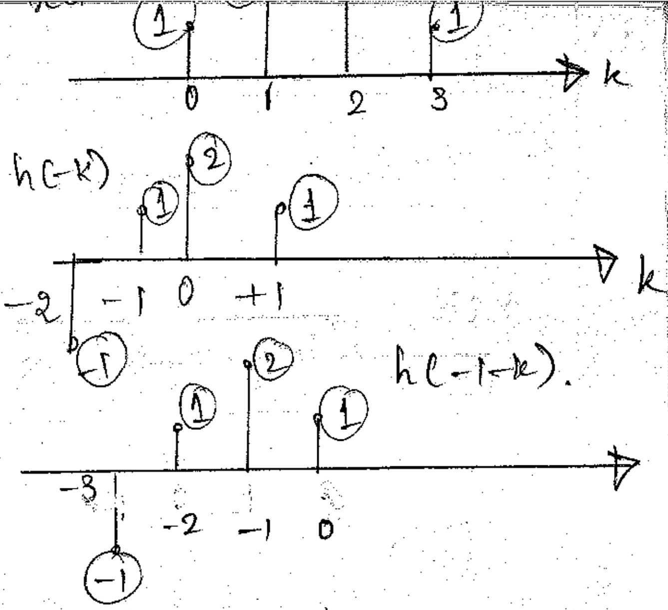
h(n) ↓	1	2	3	1
1	1	2	3	1
2	2	4	6	2
1	1	2	3	1
-1	-1	-2	-3	-1

$y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2,$

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k).h(-1-k)$$

$h(-1-k)$
Means add -1 in the time axis of $h(-k)$.

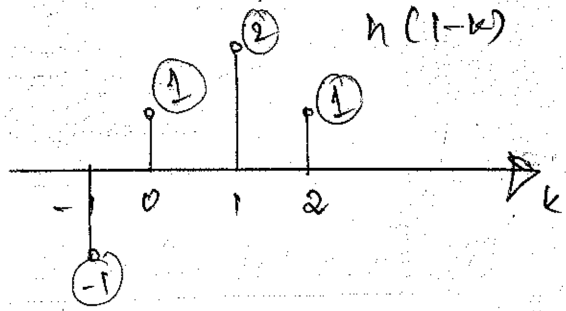
$$\boxed{y(-1) = 1}$$



put $n = 0$.

$$y(0) = \sum_{k=-\infty}^{\infty} x(k).h(-k).$$

$= 2 + 2 = 4$; $\boxed{y(0) = 4}$

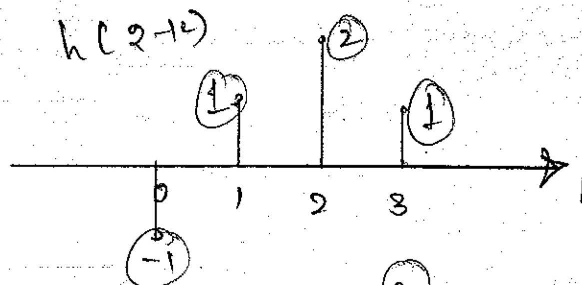


put $n=1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k).h(1-k).$$

$= 1 + 4 + 3 = 8$

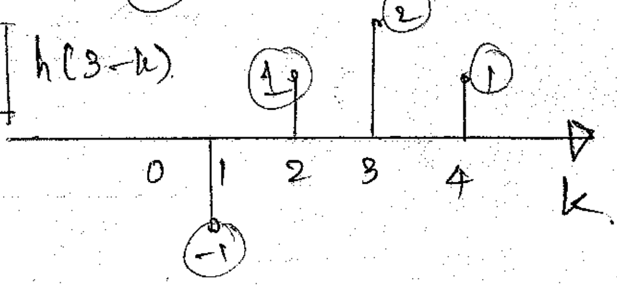
$\boxed{y(1) = 8}$



put $n=2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k).h(2-k).$$

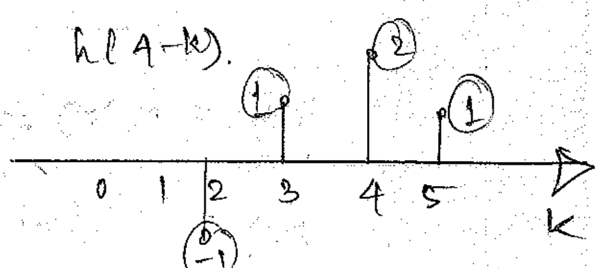
$y(2) = -1 + 2 + 6 + 1 = \boxed{8 = y(2)}$



put $n = 3$.

$$y(3) = \sum_{k=-\infty}^{\infty} x(k).h(3-k).$$

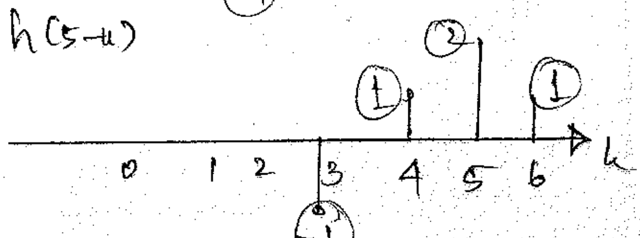
$y(3) = -2 + 3 + 2 = \boxed{3 = y(3)}$



put $n = 4$

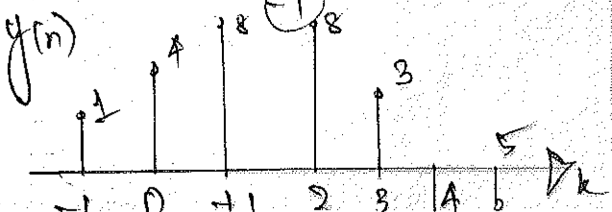
$$y(4) = \sum_{k=-\infty}^{\infty} x(k).h(4-k).$$

$= -3 + 1 = \boxed{-2 = y(4)}$



$$y(5) = \sum_{k=-\infty}^{\infty} x(k).h(5-k)$$

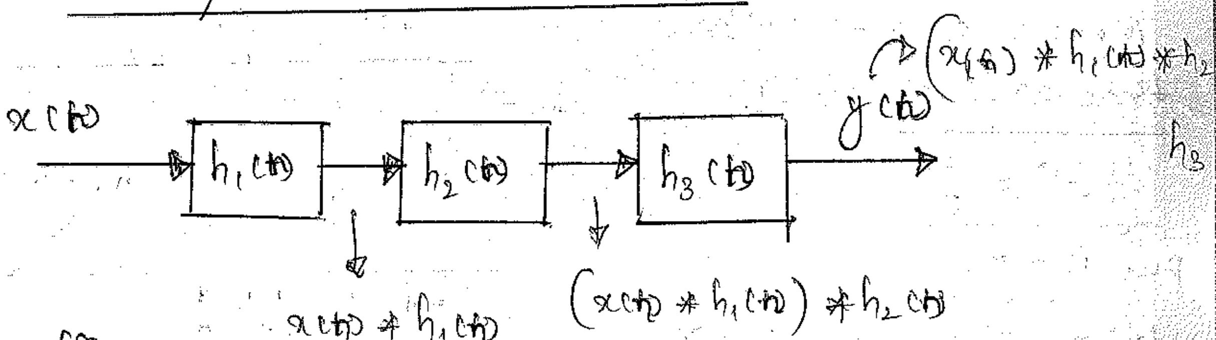
$\boxed{y(5) = -1}$



$y(n) = \{1, 4, 8, 8, 3, -2, -1\}$

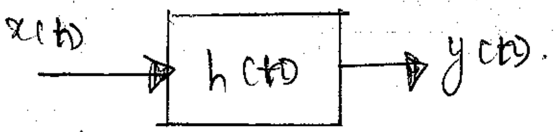
(4) Interconnection of Systems.

(1) Series / cascade Interconnection.

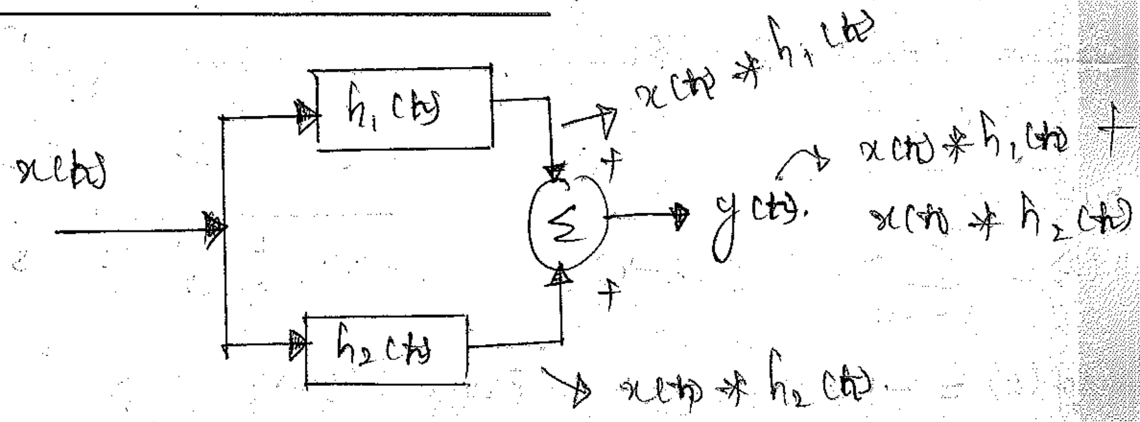


(Or)

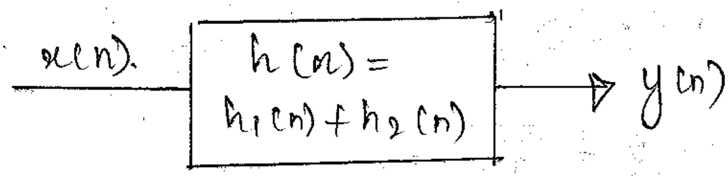
$$h(t) = h_1(t) * h_2(t) * h_3(t).$$



(2) Parallel Interconnection:



(Or)



$$y(n) = x(n) * \{h_1(n) + h_2(n)\}$$

$$y(n) = x(n) * h_1(n) + x(n) * h_2(n).$$