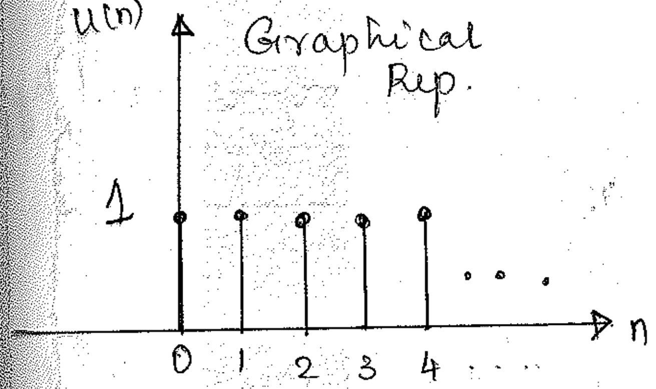
# Discrete-Time Signals (DTs).
(or)
# Discrete-Time Sequences.

## Basic/standard discrete-time signals

### (1) Unit step sequence : $u(n)$.

Graphical Rep.

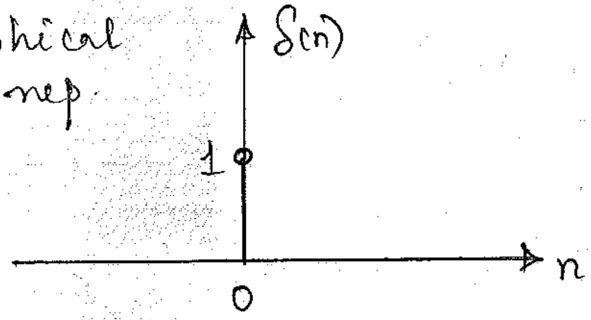



Mathematical rep.

$$u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$

### (2) Kroneckers delta (or) Dirac delta (or) Unit sample function (or) Unit Impulse Sequence: $\delta(n)$

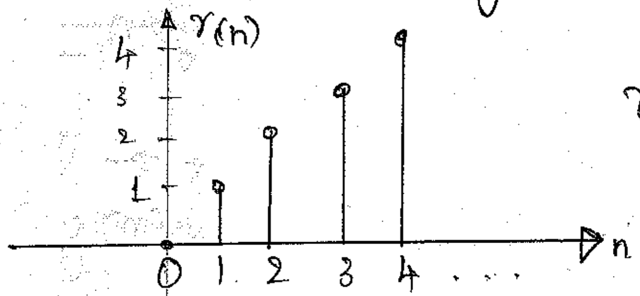
Graphical rep.




Mathematical rep.

$$\delta(n) = \begin{cases} 1 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

### (3) ~~Unit~~ Ramp sequence/signal : $r(n)$.




$$r(n) = \begin{cases} n ; & n \geq 0 \\ 0 ; & n < 0. \end{cases}$$

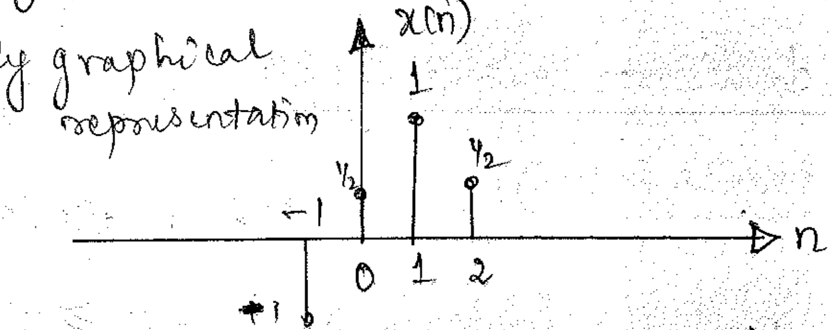
## Relationship between $u(n)$ & $\delta(n)$.

(i) $\delta(n) = u(n) - u(n-1)$

(ii) $u(n) = \sum_{k=0}^{\infty} \delta(n-k)$

(4) Random signal (or) Random discrete-time signal/sequence (or) arbitrary discrete-time signal/sequence.

only graphical representation



Mathematically the above signal be represented by

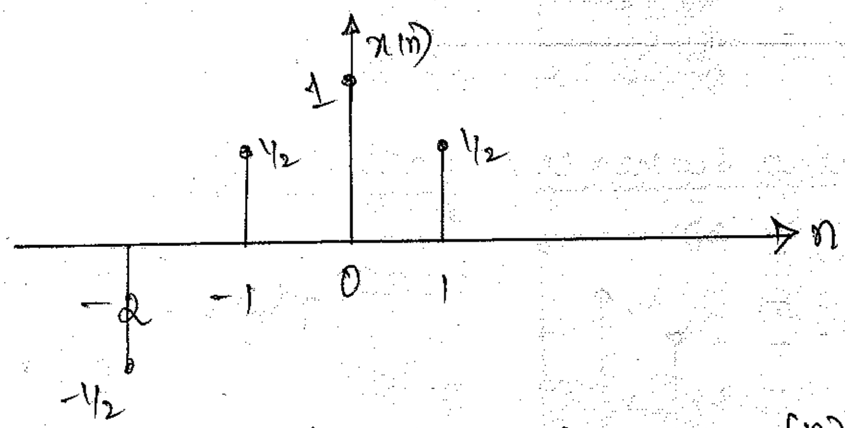
$$x(n) = \{\underset{n=-1}{-1},\ \underset{\underset{n=0}{\uparrow}}{1/2},\ \underset{n=1}{1},\ \underset{n=2}{1/2}\}$$

## Basic operations on Discrete-time Signals.

Consider DTS $x(n) = \{-1/2, 1/2, \underset{\uparrow}{1}, 1/2\}$.

→

$$x(n) = \{\underset{n=-2}{-1/2},\ \underset{n=-1}{1/2},\ \underset{\underset{n=0}{\uparrow}}{1},\ \underset{n=1}{1/2}\}$$



(a) $x(n-1)$

(b) $x(n+1)$

(c) $x(-n)$

(d) $x(-n-1)$

(e) $x(-n+1)$

(f) $x(2n)$

(h) $x(2n+1)$

(i) $x(-2n-1)$

(j) $x(-2n+1)$

(k) $y(n) = x(n) + x(-n)$

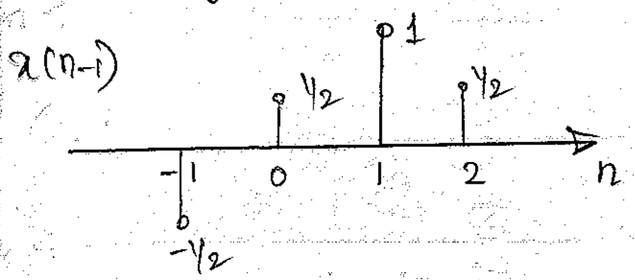
(l) $y(n) = x(n) - x(-n)$

(m) $y(n) = x(n) \times x(-n)$

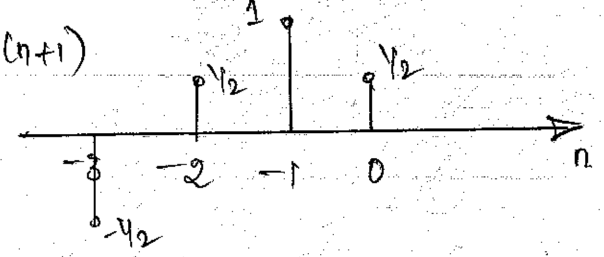
(n) $y(n) = 2x(n) - 2x(-n$

(o) $x(-2n)$

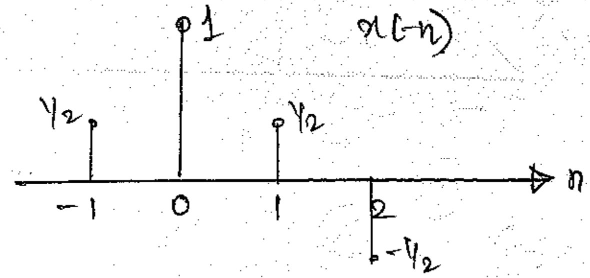
(a) $x(n-1)$ → delay [right shift $x(n)$ by 1 unit].



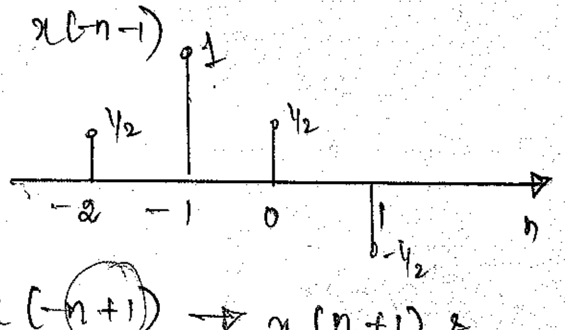

(b) $x(n+1)$ → advance
[left shift $x(n)$ by 1 unit]



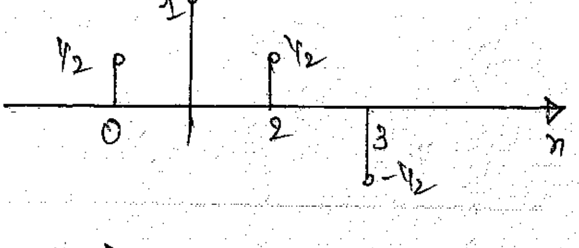

(c) $x(-n)$ → Reversal - time



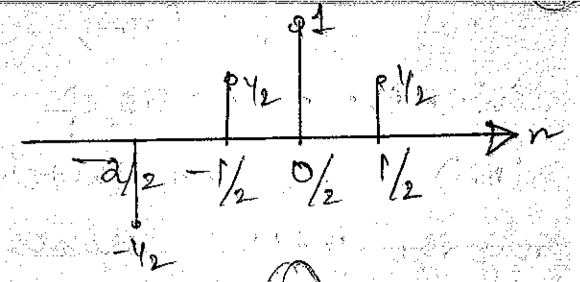

(d) $x(-n-1)$ → $x(n-1)$ & reverse $x(n-1)$.

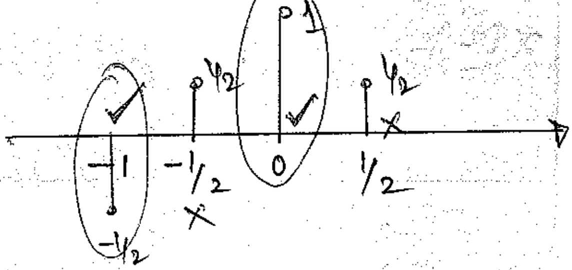


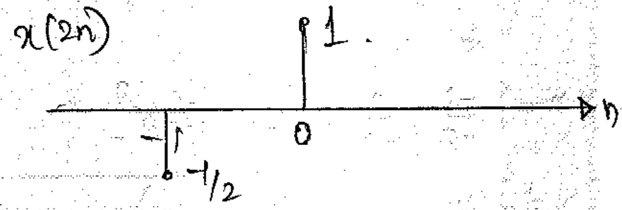

(e) $x(-n+1)$ → $x(n+1)$ & reverse $x(n+1)$



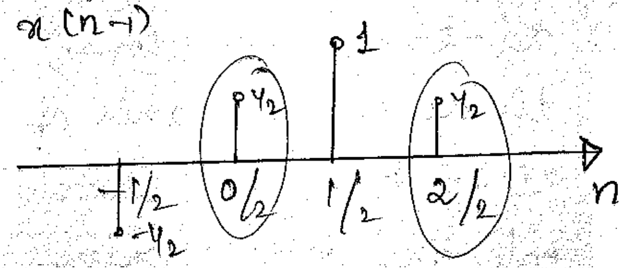

(f) $x(2n)$ → divide time axis of $x(n)$ by 2 and Sketch only the integer values

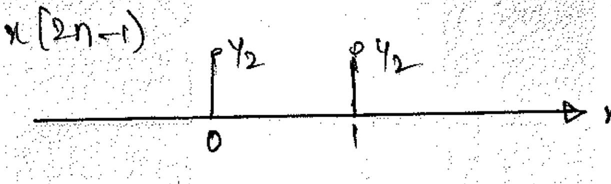


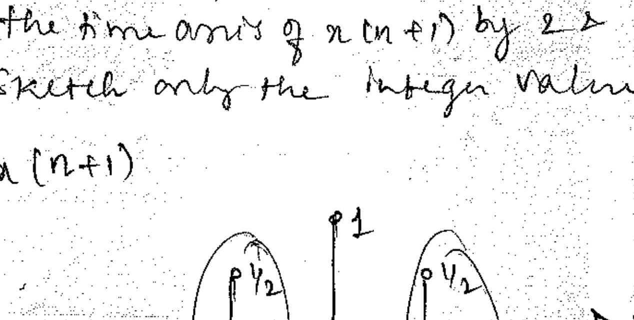



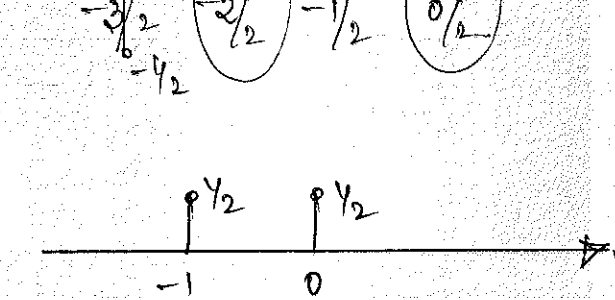

(g) $x(2n-1)$ → $x(n-1)$ & divide the time axis of $x(n-1)$ by 2 & Sketch only the integer values.




(h) $x(2n+1)$ → $x(n+1)$ & divide the time axis of $x(n+1)$ by 2 & Sketch only the integer values

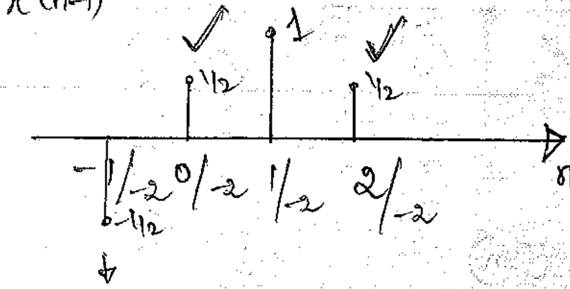


(i) $x(-2n-1) \rightarrow x(n-1)$ &

(x) divide the time axis of $x(n-1)$ by $-2$ and sketch only the integer values

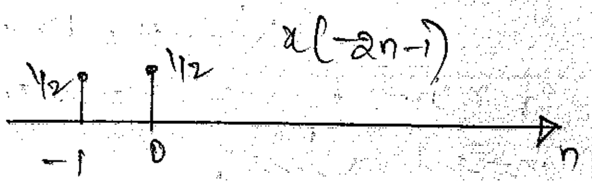
$x(n-1)$




$n = \left\{ \frac{-1}{-2}, \frac{0}{-2}, 1/_{-2}, 2/_{-2} \right\}$

$n = \left\{ 1/2, 0, -1/2, -1 \right\}$.

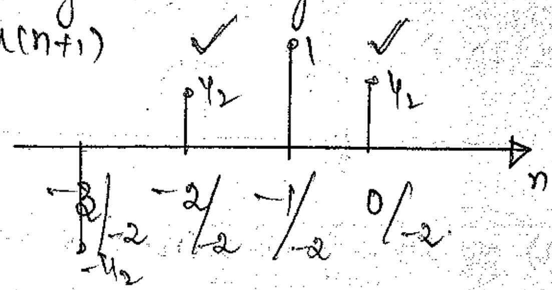
$x(-2n-1)$




(j) $x(-2n+1) \rightarrow x(n+1)$

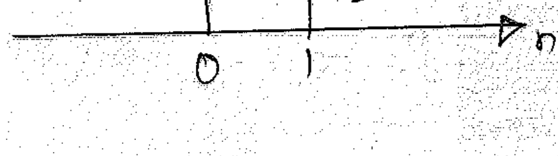
divide the time axis of $x(n+1)$ by $-2$ and sketch only the integer value.

$x(n+1)$



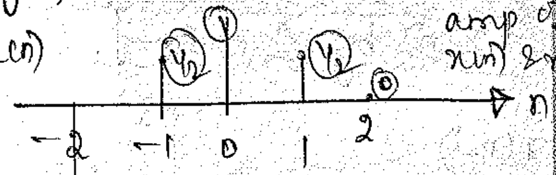

$n = \left\{ \frac{-3}{-2}, \frac{-2}{-2}, \frac{-1}{-2}, 0/2 \right\}$

$= \left\{ 3/2, 1, +1/2, 0 \right\}$




(k) $y(n) = x(n) + x(-n)$ add the amp of $x(n)$ & $x(-n)$

$x(n)$

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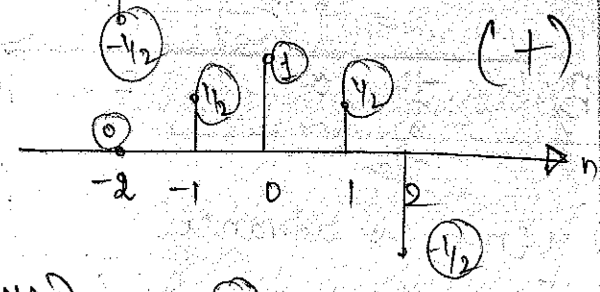
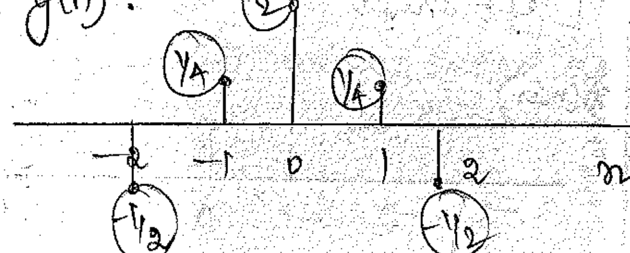
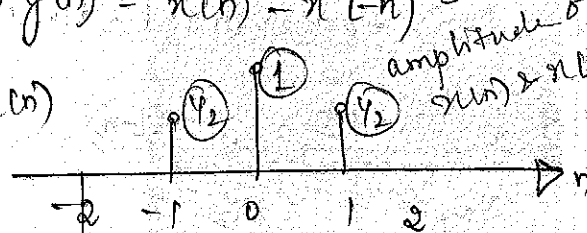



$y(n)$.



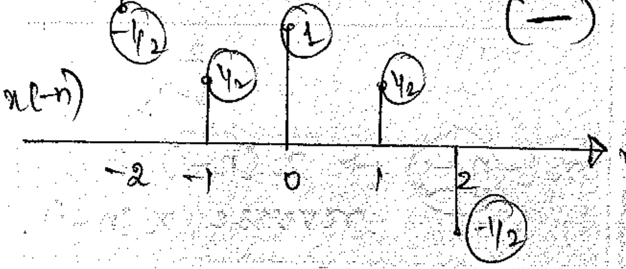

(l) $y(n) = x(n) - x(-n)$ Subtract amplitude of $x(n)$ & $x(-n)$

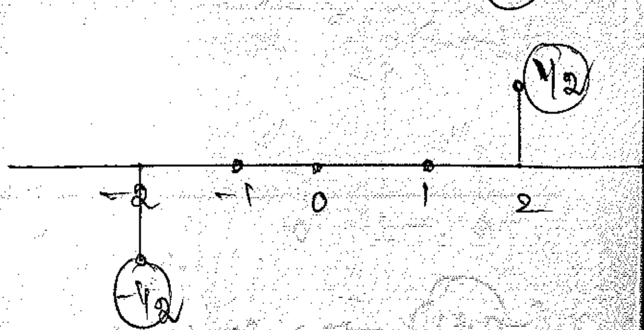
$x(n)$




(−)

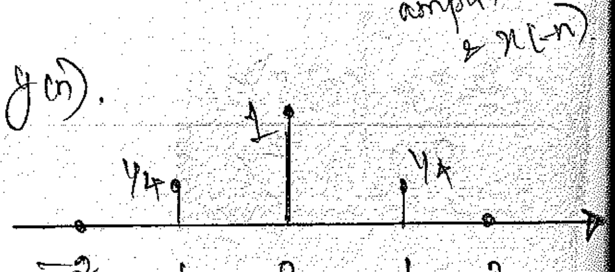
$x(-n)$



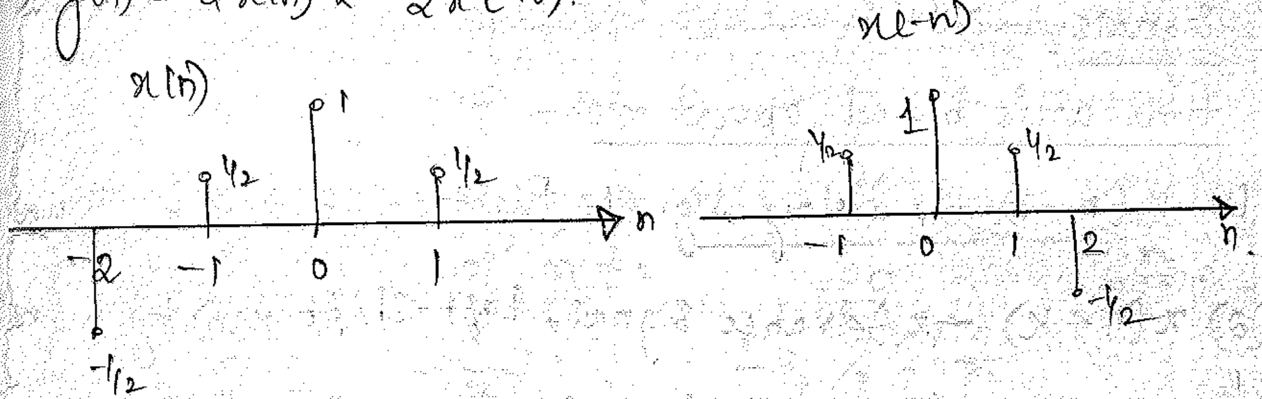




(m) $y(n) = x(n) \times x(-n)$ Multiply amplitude of $x(n)$ & $x(-n)$.

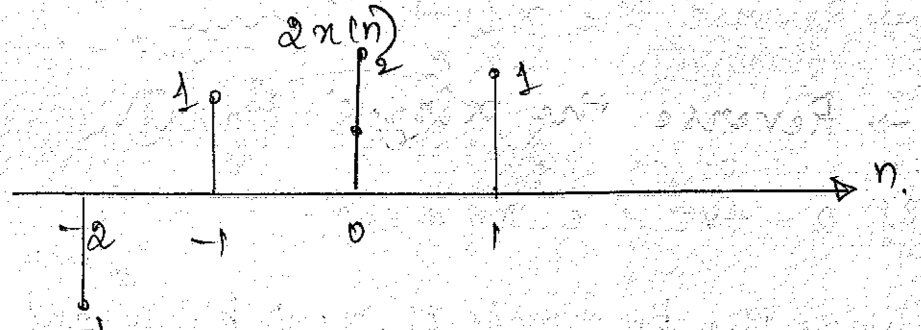
$y(n)$.



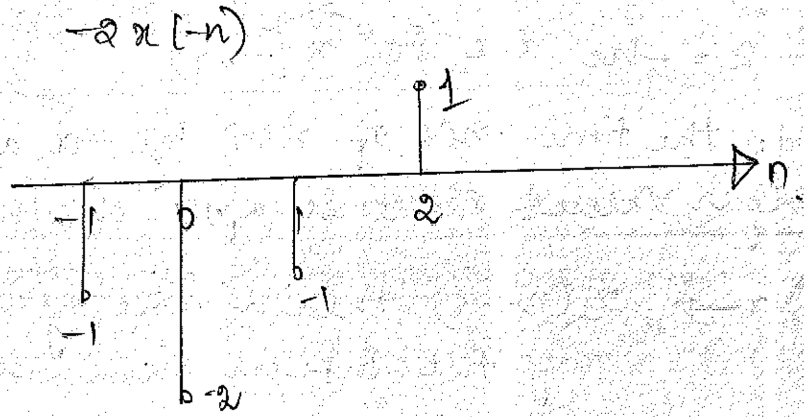
(n) $y(n) = 2x(n) \times -2x(-n)$.




$2x(n) \rightarrow$ Multiply the amplitude of $x(n)$ by 2.

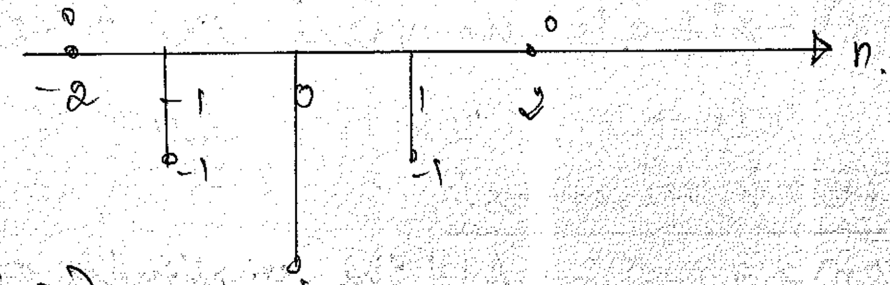



$-2x(-n) \rightarrow$ Multiply the amplitude of $x(-n)$ by $-2$.




$y(n) = 2x(n) \times -2x(-n)$. Multiply $2x(n)$ & $-2x(-n)$.

$y(n) = 2x(n) - 2x(-n)$.




(o) $x(-2n)$

divide $x(n)$ by $-2$ and sketch only the integer value

$n = \left\{\frac{-2}{-2}, \frac{-1}{-2}, \frac{0}{-2}, \frac{1}{-2}\right\}$; $n = \{+1, 1/2, 0, -1/2\}$

$x(n)$ $x(-2n)$ Note: Reversal

## Summary of Basic Operation on Discrete time Signals.

### time axis based operations.

(1) $x(n-k)$ → delay signal, Right shift $x(n)$ by K units

(2) $x(n+k)$ → Advance Signal, Left shift $x(n)$ by K units

(3) $x(-n)$ → Reversal Signal; Reverse $x(n)$ in time axis

(4) $x(-n-k)$ → Reverse the $x(n-k)$ signal.

(5) $x(-n+k)$ → Reverse the $x(n+K)$ signal.

(6) $x(an)$ if $a$ = +ve i.e Eg: $x(+2n)$.

↳ Divide the time axis of $x(n)$ by $a$ → +ve and keep only the integer valued sequence after division.

(7) $x(an)$ if $a$ = -ve i.e Eg: $x(-2n)$

↳ Divide the time axis of $x(n)$ by $+a$ → -ve value and keep only the integer valued sequence after division.

(8) $x(an+k)$ → $x(n+k)$ then divide by $a$ & keep only the integer values sequences

(9) $x(an-k)$ → $x(n-k)$ " "

(10) $x(an+k)$ → if $a$ is -ve value like $x(-2n+1)$
$x(n+k)$, divide by $+a$ 2 -ve value.

(11) $x(+an-k)$ → if $a$ is -ve value like $x(-2n+-1)$
$x(n-k)$ " "

### Amplitude based operations.

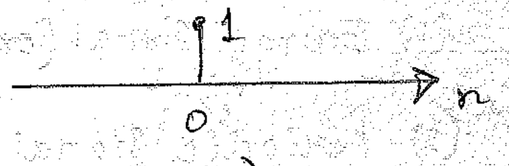
(1) $y(n) = x(n) + x(-n)$ → Add the amplitudes of $x(n)$ & $x(-n)$

(2) $y(n) = x(n) - x(-n)$ → Subtract the amp. of two signals

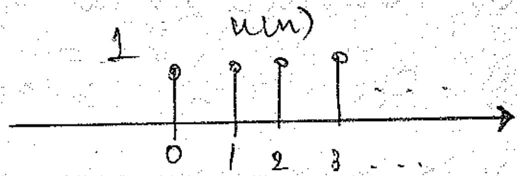
# Problems [based on operation on DTs].

(1) Prove that $\delta(n) = u(n) - u(n-1)$.

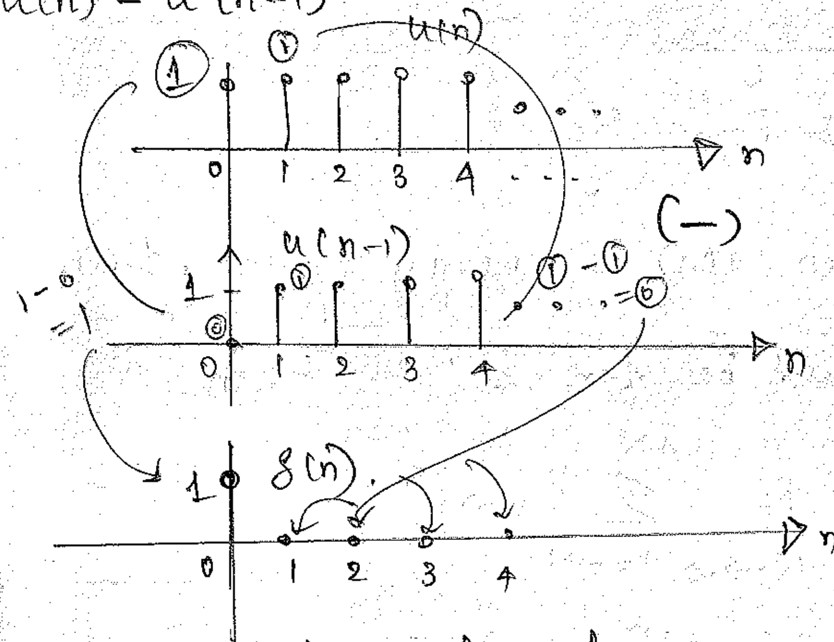
$\rightarrow$ WKT $\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$



$u(n) = \begin{cases} 1, & n \geq 0 \\ 0; & n < 0. \end{cases}$



$u(n) - u(n-1)$



WKT $u(n-1)$ delay Rs by 1 unit

Hence proved.

## Summation Series formulas.

1. $\sum_{n=0}^{N-1} 1 = N$

2. $\sum_{n=-N}^{N} 1 = 2N+1$

3. $\sum_{n=0}^{N} 1 = N+1$,

4. $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$; $|a|<1$.

5. $\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2}$; $|a|<1$

6. $\sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a} & ; a \neq 1 \\ N & ; a=1 \end{cases}$

7. $\sum_{n=0}^{N} a^n = \begin{cases} \frac{1-a^{N+1}}{1-a} & ; a \neq 1 \\ N+1 & ; a=1. \end{cases}$

8. $\sum_{n=1}^{N} n = \frac{N(N+1)}{2}$

9. $\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$

10. $\sum_{n=1}^{N} n^3 = \frac{N^2(N+1)^2}{4}$.