(2) Correlation.

- Auto-Correlation
- Cross-Correlation.

Auto-Correlation. ($\gamma_{xx}(l)$).

Given $x(n)$ the autocorrelation is defined as

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l).$$

Where

$l \rightarrow$ lag parameter.

Properties of Auto-Correlation.

(1) $\gamma_{xx}(0) = E_{x(n)} \rightarrow$ Energy of Signal $x(n)$.

(2) $\gamma_{xx}(l) = \gamma_{xx}(-l) \rightarrow$ Auto correlation is Even function (or) Symmetric.

(3) Normalized $\gamma_{xx}(l) = \dfrac{\gamma_{xx}(l)}{\gamma_{xx}(0)}.$

Problems.

Find the auto-correlation of the signal $x(n) = a^n u(n)$; $0<a<1$.

$\rightarrow$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l).$$

$x(n) = a^n u(n)$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} a^n u(n) \cdot a^{n-l} \cancel{u}u(n-l)$$

$$= \sum_{n=0}^{\infty} a^n \cdot a^{n-l} = \sum_{n=0}^{\infty} a^n \cdot a^n \cdot a^{-l}.$$

$$= a^{-l} \sum_{n=0}^{\infty} a^{2n} = a^{-l} \left(\sum_{n=0}^{\infty} (a^2)^n \right) \rightarrow \frac{1}{1-a^2}$$

$$\gamma_{xx}(l) = \frac{a^{-l} \times 1}{1-a^2} = \frac{a^{-l}}{1-a^2}$$

W.K.T $\gamma_{xx}(l)$ is a even function so $\gamma_{xx}(l) = \gamma(-l)$

So $l = 0 \ldots \infty$ values will be exactly available in $l = 0 \ldots -\infty$

$$\gamma_{xx}(0) = \frac{a^0}{1-a^2} = \frac{1}{1-a^2} \;; \quad \gamma_{xx}(-1) = \frac{a^1}{1-a^2}$$

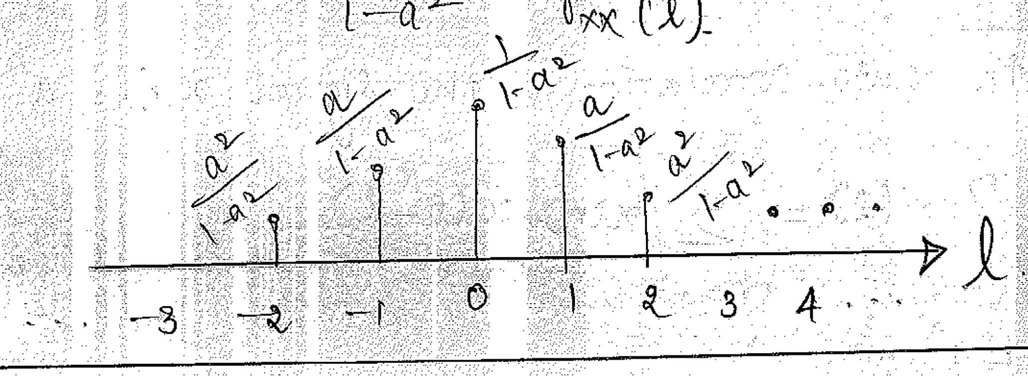
$$\gamma_{xx}(1) = \frac{a^{-1}}{1-a^2} \;; \quad \gamma_{xx}(-2) = \frac{a^2}{1-a^2}$$

$$\gamma_{xx}(2) = \frac{a^{-2}}{1-a^2}$$

But $\gamma_{xx}(1) = \frac{a^{-1}}{1-a^2}$; $\gamma_{xx}(-1) = \frac{a^1}{1-a^2}$ both are diffe values so

What approximation makes both same/equal

$$\gamma_{xx}(l) = \frac{a^{|l|}}{1-a^2}$$



(2) Find the Auto-correlation of the signal

$x(n) = \{1, 2, 3, 4\}$.

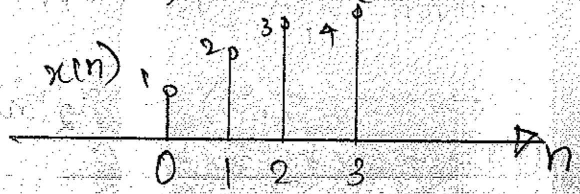
$\rightarrow$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

l values ranges from if $x(n) = \{\overset{n=0}{1}, \overset{n=1}{2}, \overset{n=2}{3}, \overset{n=3}{4}$

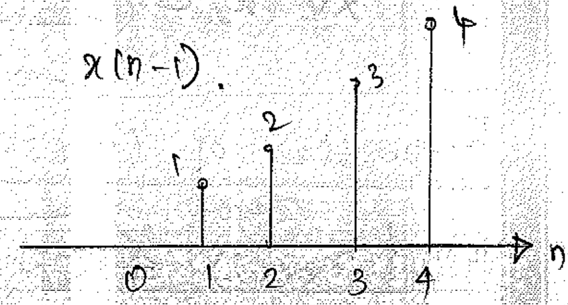
Calculate for the values of l so that we can get the -ve values of l by using $r_{xx}(l) = r_{xx}(-l)$.

put $l = 0$.



$$r_{xx}(0) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n).$$

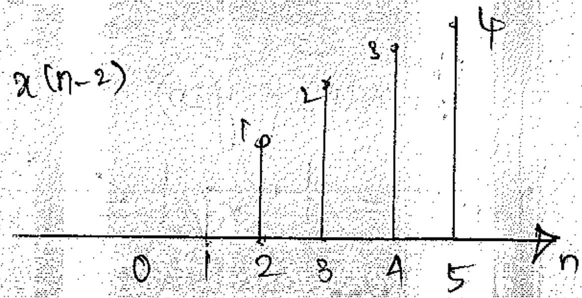
$$r_{xx}(0) = 1 + 4 + 9 + 16 = 30.$$



put $l = 1$

$$r_{xx}(1) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-1)$$

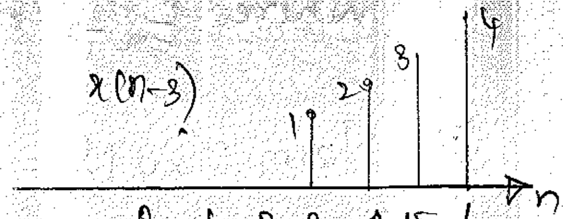
$$r_{xx}(1) = 2 + 6 + 12 = 20.$$



put $l = 2$

$$r_{xx}(2) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-2)$$

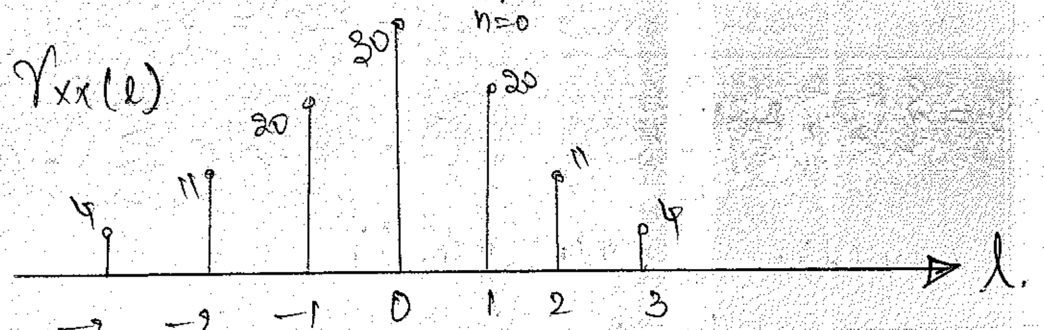
$$= 3 + 8 = 11.$$



$$r_{xx}(3) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-3)$$

$$= 4.$$

$$r_{xx}(l) = \{\underset{n=-3}{4}, \underset{n=-2}{11}, \underset{n=-1}{20}, \underset{\substack{\uparrow \\ n=0}}{30}, \underset{n=1}{20}, \underset{n=2}{11}, \underset{n=3}{4}\}.$$



2. Cross - Correlation.

Given $x(n)$ & $y(n)$ is a Energy signal, then Cross-Correlation between $x(n)$ & $y(n)$ is given by,

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-l)$$

Properties of Cross- Correlation.

(1) $|\gamma_{xy}(l)| \leq \sqrt{\gamma_{xx}(0) \cdot \gamma_{yy}(0)} = \sqrt{E_x E_y}$.

if $x(n) = y(n)$

$|\gamma_{xx}| \triangleq |\gamma_{xy}| \leq \sqrt{\gamma_{xx}(0) \cdot \gamma_{xx}(0)} = \sqrt{E_x E_x} = \sqrt{E_x^2}$
$= E_x$

Where $E_x \rightarrow$ Energy of signal $x(n)$

(2) Norm $\gamma_{xy}(l) = \dfrac{\gamma_{xy}(l)}{\sqrt{\gamma_{xx}(0) \gamma_{yy}(0)}}$.

(3) $\gamma_{xy}(l) = \gamma_{yx}(-l)$.

Problems

(1) Find the cross-correlation of $x(n) = \{1, 2, 1, 1\}$ $y(n) = \{1, 1, 2, 1\}$.

→ $$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-l)$$

To find the limit for l is the sum of lower limit & upper limit of $x(n)$ & $y(-n)$

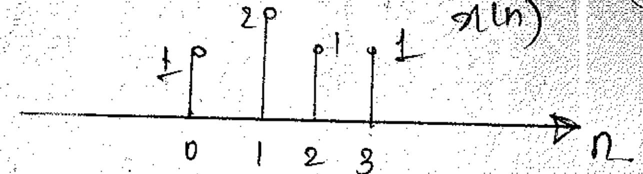
$x(n)$; $0 \leq n \leq 3$.

$y(n)$; $0 \leq n \leq 3$.

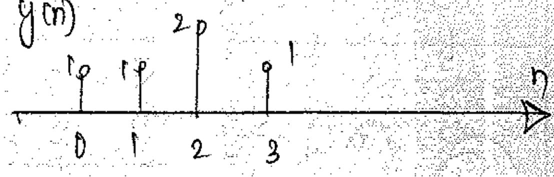
$y(-n)$; $-3 \leq n \leq 0$

Put $l = -3$.

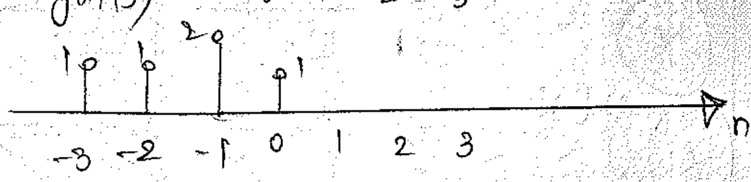
$$r_{xy}(-3) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+3)$$



$y(n+3)$ is advanced so left shift by 3 units



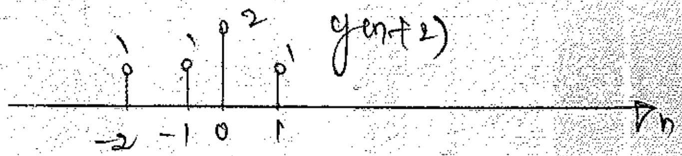
$r_{xy}(-3) = 1$



put $l = -2$.

$$r_{xy}(-2) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+2)$$

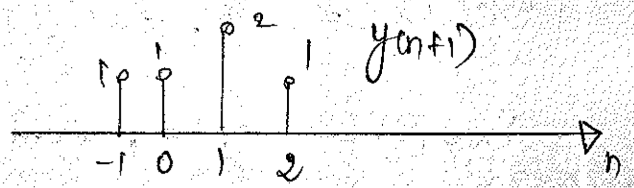
$$= 2+2 = 4.$$



Put $l = -1$

$$r_{xy}(-1) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+1)$$

$$= 1 + 4 + 1 = 6.$$



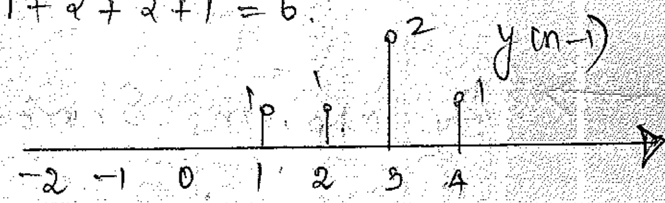
Put $l = 0$.

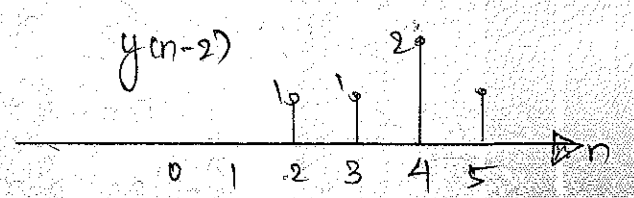
$$r_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n) = 1+2+2+1 = 6.$$

put $l = 1$

$$r_{xy}(1) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-1)$$

$$= 2+1+2 = 5$$

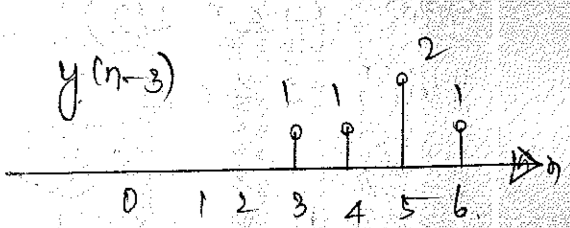




put $l = 2$

$$r_{xy}(2) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-2)$$

$$= 1+1 = 2$$

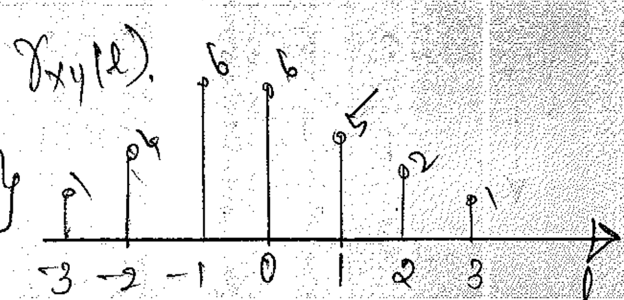


Put $l = 3$

$$r_{xy}(3) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-3)$$

$$= 1$$

$$r_{xy}(l) = \{\underset{n=-3}{1}, \underset{n=-2}{4}, \underset{n=-1}{6}, \underset{\uparrow}{6}, \underset{n=+1}{5}, \underset{n=2}{2}, \underset{n=3}{1}\}$$



Short-Cut Method:-

$x(n) = \{\underset{n=0}{1}, 2, 1, \underset{n=3}{1}\}$

$y(n) = \{1, 1, 2, 1\}$; $y(-n) = \{\underset{\uparrow\, n=-3}{1}, 2, 1, \underset{\uparrow\, n=0}{1}\}$

$y(-n)\downarrow$ \ $x(n)\rightarrow$	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1
1	1	2	1	1

$0-3 \le l \le 3+0$

$-3 \le l \le 3$

$r_{xy}(l) = \{1, 2+2, 1+4+1, 1+2+2+1, 2+1+2, 1+1,$

$r_{xy}(l) = \{\underset{n=-3}{1}, \underset{n=-2}{4}, \underset{n=-1}{6}, \underset{\uparrow\, n=0}{6}, \underset{n=1}{5}, \underset{n=2}{2}, \underset{n=3}{1}\}$

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(2) Find the Cross-Correlation of $x(n) = \{1, \underset{\uparrow}{3}, 2, 1\}$ &

$y(n) = \{4, 3, \underset{\uparrow}{1}, 2\}$.

→ By applying Short-cut method.

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-l)$$

• $l \rightarrow$ lag parameter limit.

$x(n)$; $-1 \le n \le 2$

$y(n)$; $-2 \le n \le 1$.

$y(-n)$; $-1 \le n \le 2$

$-1-1 \le l \le 2+2$

$-2 \le l \le 4$,

$y(-n) = \{2, 1, \underset{\uparrow}{3}$

$y(-n)\downarrow$ \ $x(n)\rightarrow$	1	3	2	1
2	2	6	4	2
1	1	3	2	1
3	3	9	6	3
	4	12	8	4

$r_{xy}(l) = \{\underset{n=-2}{2}, \underset{n=-1}{7}, \underset{\uparrow}{10}, 17, \underset{n=1}{19}, \underset{n=2}{11}, \ldots$

$r_{xy}(l)$.