# (7) Fast-Fourier Transform. (FFT).

In direct computation of the DFT requires.

i.e for the given $x(n)$, Direct DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nK} \quad \text{requires}$$

$N^2$ Multiplications.

$N(N-1) \approx N^2$ Additions.

Collectively. Direct DFT requires $N^2$ MADS $\begin{bmatrix} \text{MADS} : \\ \text{Multiplication \&} \\ \text{Addition} \end{bmatrix}$

Eg: 8-point Direct DFT requires $(8)^2 = 64$ MADS.

i.e 64 additions & 64 Multiplications

Inorder to increase the Computation efficiency of DFT.

Fast fourier transform (FFT) algorithms are developed

## FFT algorithms

(1) Decimation-in-time FFT algorithm. (DIT)

(2) Decimation-in-frequency FFT algorithm (DIF).

The FFT algorithms listed above are Radix-2 algorithms i.e N is a power of 2.

DIT fft algorithm requires $2\left(\dfrac{N}{2}\right)^2 + N$ MADS.

DIF fft algorithm requires $2\left(\dfrac{N}{2}\right)^2 + N$ MADS

Eg: 8-point Direct DFT require 64 MADS.

8 point DIT FFT requires. 40 MADS.

8-point DIF FFT requires 40 MADS.

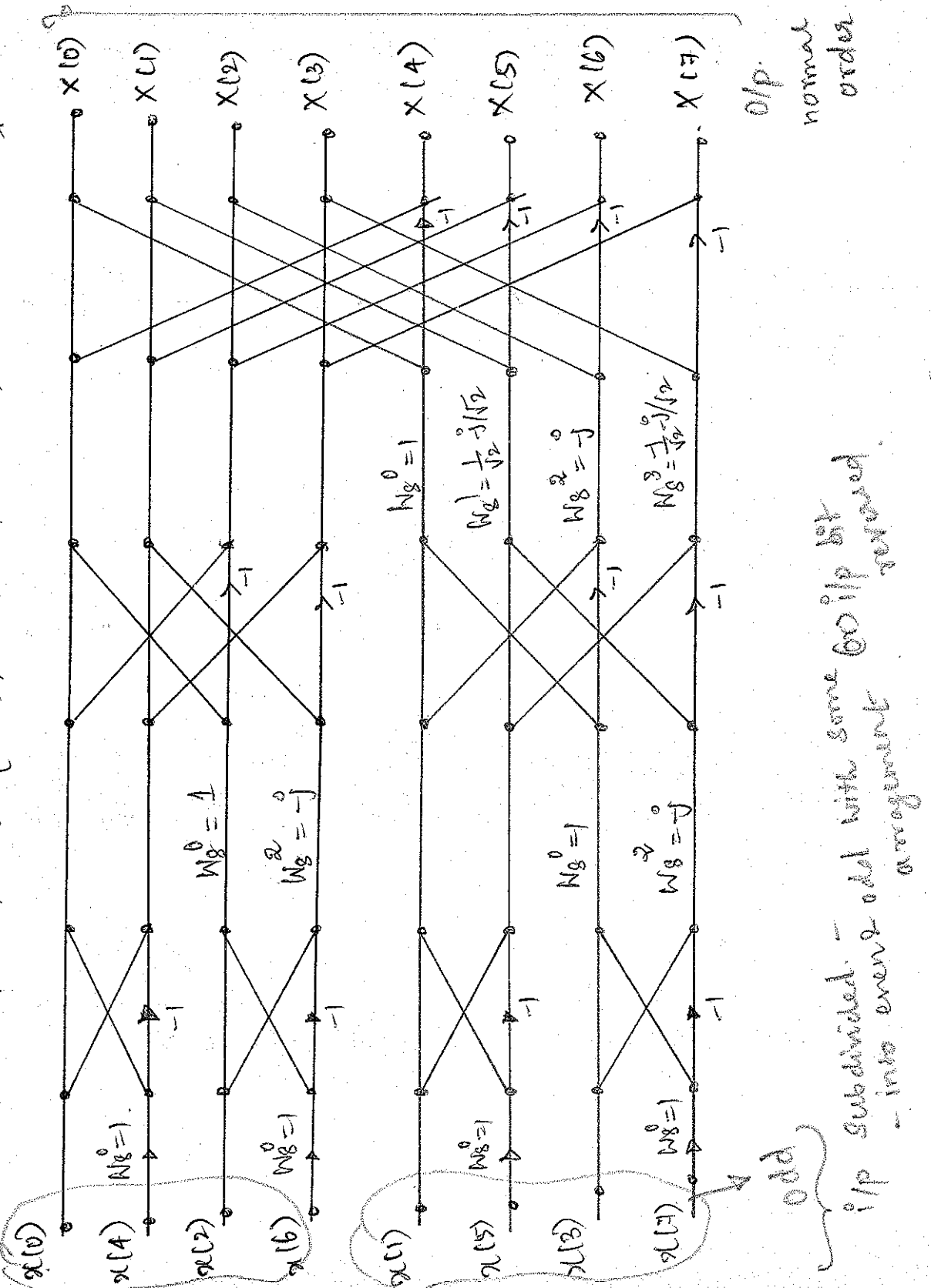Note: Radix-2 algorithms : N is a power of 2.

# DECIMATION - IN - TIME (DIT).

(1) Decimation-in-time (DIT) algorithms which are based on successive sub division of the input.

(2) DIT represents an <u>in-place computation</u> with the input in bit reversed order i.e. by dividing the input into its even and odd numbered points and the output in normal order.

**DIT-FFT FLOW GRAPH. for the given x(n).**

Eg:- $x(n) = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$.

To find $x(k) = \{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$

Inputs (bit-reversed order):
$x(0), x(4), x(2), x(6)$ (even), $x(1), x(5), x(3), x(7)$ (odd)

Twiddle factors:
$$W_8^0 = 1 \qquad W_8^0 = 1 \qquad W_8^0 = 1$$
$$W_8^2 = -j \qquad W_8^2 = -j$$
$$W_8^1 = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$
$$W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

Outputs (normal order): $X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)$ — o/p normal order.

i/p Subdivided - into even odd with some.
(i) i/p bit reversed.
(ii) i/p bit reversed arrangement.

# DECIMATION - IN - FREQUENCY. (DIF).

(1) DIF algorithm are class of algorithms developed on the basis of successive subdivisions of the output.

(2) DIF algorithm represents an in-place computation with the input in normal order and output in bit reversed order i.e the output divided into even and odd numbered points.
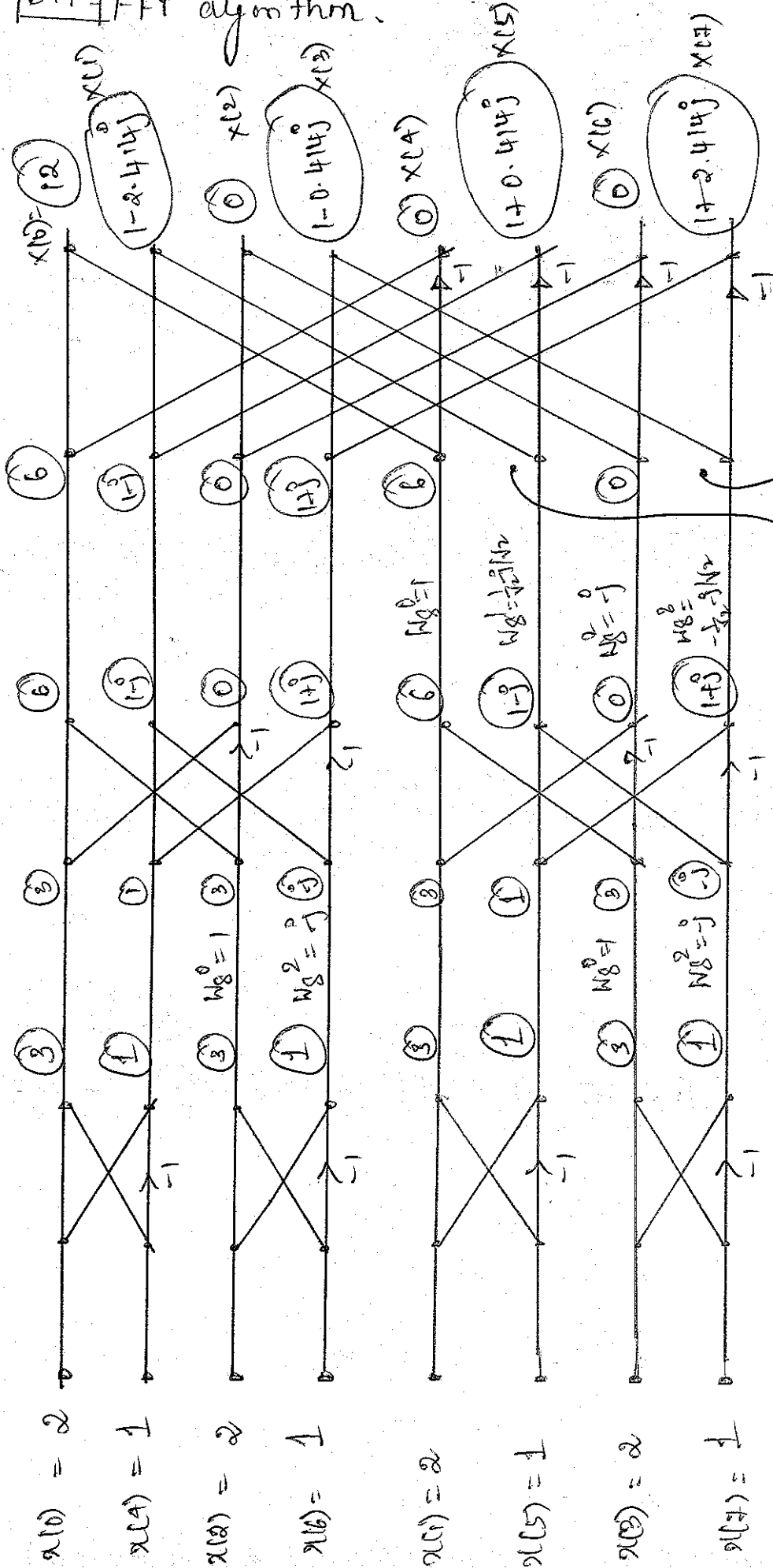
DECIMATION - IN - FREQUENCY (DIF)
FLOW GRAPH for given x(n).
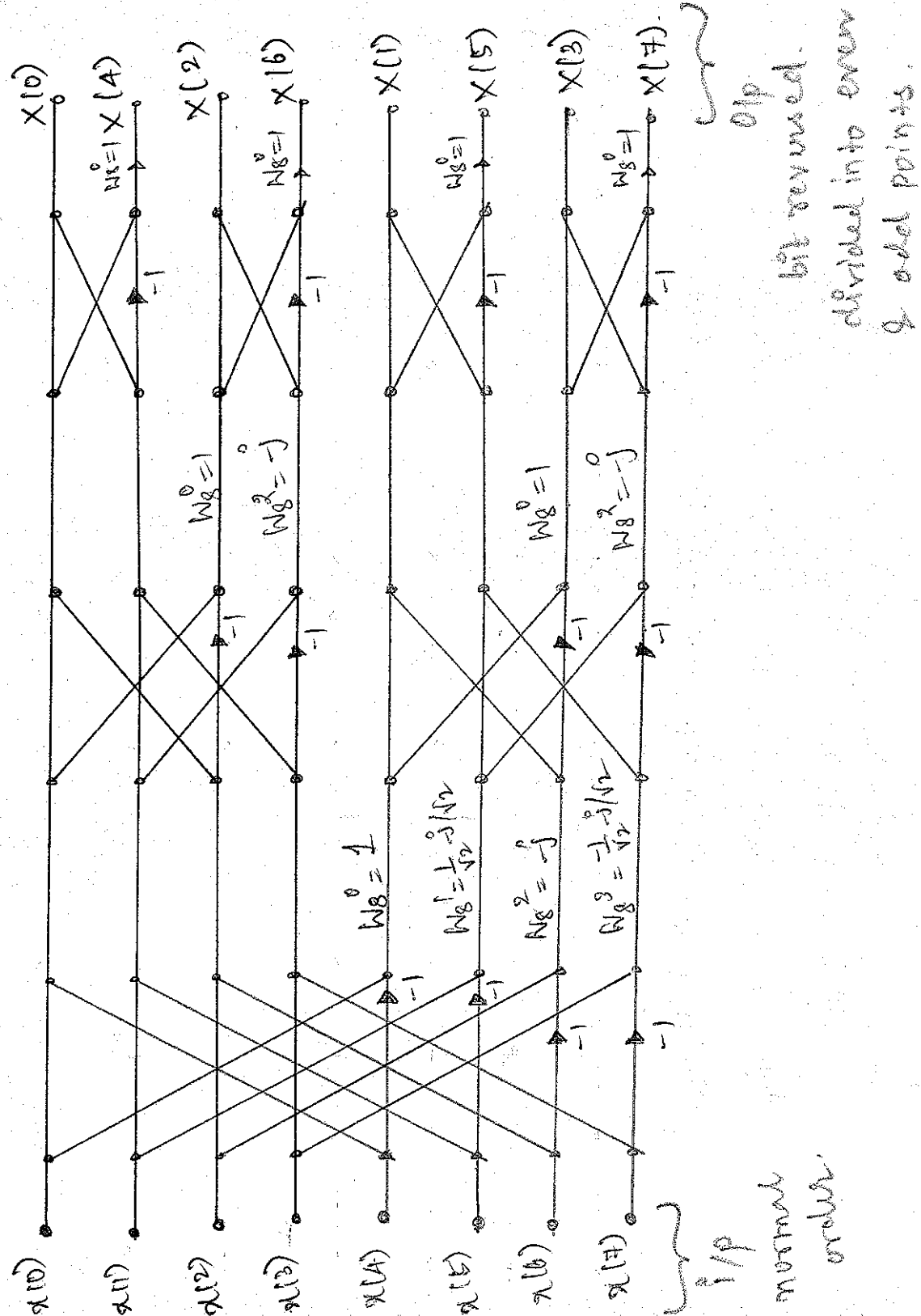
# Problems:-

**(1)** Compute the DFT for the sequence $\{2,2,2,1,1,1,1\}$ using radix-2 $\boxed{DIT}$ FFT algorithm.

Given $x(n) = \{\underset{\underset{N(0)}{\uparrow}}{2}, \underset{1}{2}, \underset{2}{2}, \underset{3}{2}, \underset{4}{1}, \underset{5}{1}, \underset{6}{1}, \underset{7}{1}\}$.

DIT-FFT Flow graph.



Input sequence (bit-reversed order):
$x(0) = 2$
$x(4) = 1$
$x(2) = 2$
$x(6) = 1$
$x(1) = 2$
$x(5) = 1$
$x(3) = 2$
$x(7) = 1$

Twiddle factors:
$W_8^0 = 1$
$W_8^2 = -j$
$W_8^0 = 1$
$W_8^2 = -j$
$W_8^0 = 1$
$W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$
$W_8^2 = -j$
$W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

Outputs:
$X(0) = 12$
$X(1) = 1 - 2.414j$
$X(2) = 0$
$X(3) = 1 - 0.414j$
$X(4) = 0$
$X(5) = 1 + 0.414j$
$X(6) = 0$
$X(7) = 1 + 2.414j$

$(1-j) \times \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$, $(1+j) \times \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$,

$1 + 2 + 414j$

$$X(K) = \left\{12, \; 1-2.414j, \; 0, \; 1-0.414j, \; 0, \; 1+0.414j, \; 0, \; 1+2.414j\right\}$$

**② Compute the DFT for the sequence {1,1,1,1,1,1,1,0} using radix-2 [DIT] FFT algorithm.**

(103)

Given $x(n) = \{1,1,1,1,1,1,1,0\}$
$\quad\quad\quad\quad\quad\quad 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7.$

DIT-FFT – Flow graph.



$x(0) = 1$
$x(4) = 1$
$x(2) = 1$
$x(6) = 1$
$x(1) = 1$
$x(5) = 1$
$x(3) = 1$
$x(7) = 0$

$X(0) = 4$
$X(1) = 0.707 - j0.707$
$X(2) = -j$
$X(3) = -0.707 - j0.707$
$X(4) = 1$
$X(5) = -0.707 + j0.707$
$X(6) = +j$
$X(7) = 0.707 + j0.707$
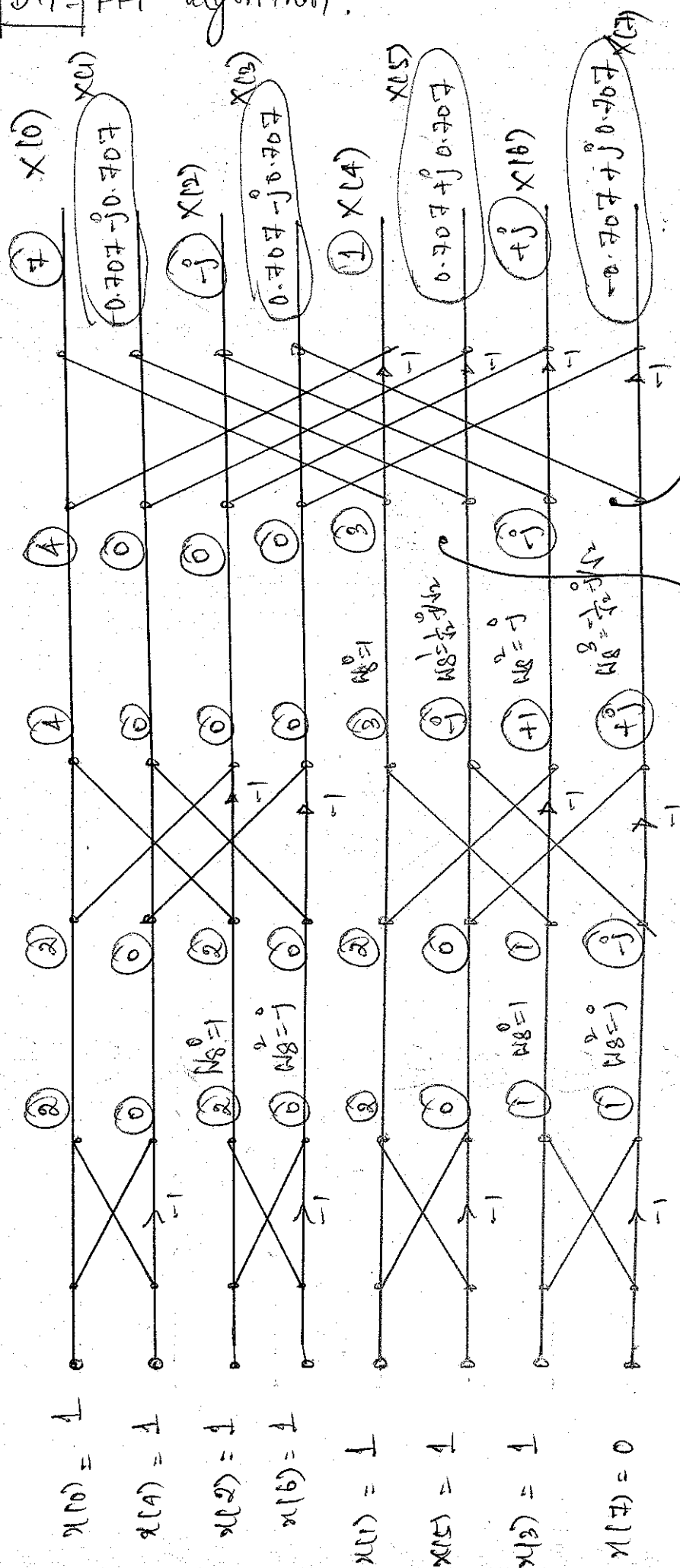
$X(k) = \left\{ 4,\ 0.707-j0.707,\ -j,\ -0.707-j0.707,\ 1,\ -0.707+j0.707,\ +j,\ 0.707+j0.707 \right\}$

③ Compute the DFT for the Sequence {1,2,3,4, 4,3,2,1} using radix-2 [DIF]-FFT algorithm.

Given x(n) = {1,2,3,4,4,3,2,1}

DIF-FFT-Flow graph.



x(0) = 1
x(1) = 2
x(2) = 3
x(3) = 4
x(4) = 4
x(5) = 3
x(6) = 2
x(7) = 1

Outputs: X(0), X(4), X(2), X(6), X(1), X(5), X(3), X(7)

Circled output values:
- 20 → X(0)
- 0 → X(4)
- 0 → X(2)
- 0 → X(6)
- -5.828 - j2.414 → X(1)
- -0.172 + j0.414 → X(5)
- -0.172 - j0.414 → X(3)
- -5.828 + j2.414 → X(7)

Twiddle factors:
$W_8^0 = 1$, $W_8^2 = -j$, $W_8^0 = 1$, $W_8^0 = 1$, $W_8^2 = -j$, $W_8^0 = 1$, $W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$, $W_8^2 = -j$, $W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$
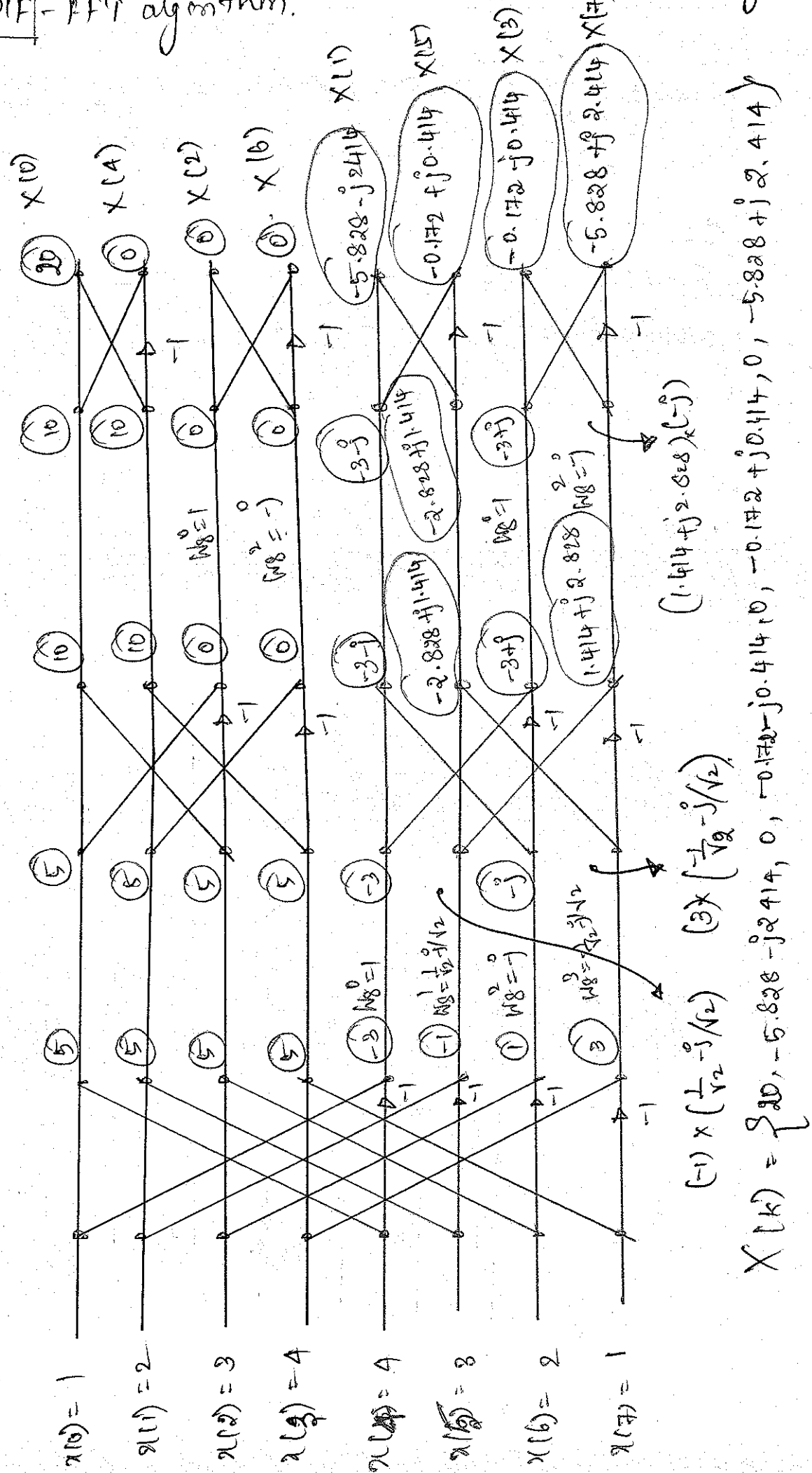
$(1) \; x\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$

$(3) \; x\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$

$(1.414 + j\,2.828)(-j)$

$X(k) = \{20, -5.828 - j2.414, 0, -0.172 + j0.414, 0, -0.172 - j0.414, 0, -5.828 + j2.414\}$

# Practice Problems :-

(1) Re-do / Repeat the Problem ① $\{2, 2, 2, 2, 1, 1, 1, 1\}$ using radix 2- [DIF] FFT algorithm. & Verify the results. that DIT & DIF answers are same.

(2) Repeat problem ② $\{1, 1, 1, 1, 1, 1, 1, 0\}$ using radix -2 [DIF] FFT & Verify the results that DIT & DIF answers are same.

(3) Repeat problem ③ $\{1, 2, 3, 4, 4, 3, 2, 1\}$ using radix -2 [DIT] FFT & Verify the results that DIF & DIT answers are same.