ANALYSIS OF DISCRETE-TIME SIGNALS.

Classification of Discrete-time Signals:

(1) Energy Signal (..) & power signal.

(2) Periodic Signal & Aperiodic Signal

(3) Even Signal & odd signal.

(4) Causal Signal, Non-Causal signal & anticausal signal.

(1) Energy Signal & Power Signal.

If the given $x(n)$ is energy signal, then $E_{x(n)}$ should be finite i.e $0 < E_{x(n)} < \infty$

$$E_{x(n)} = \lim_{N \to \infty} \sum_{n=-N}^{+N} |x(n)|^2$$

If the given $x(n)$ is power signal, then $P_{av}\,x(n)$ should be finite i.e $0 < P_{av}\,x(n) < \infty$

$$P_{av}\,x(n) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x(n)|^2$$

Problems.

(1) Determine the energy and power of the following signals.

(a) $x(n) = u(n)$

(b) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

(c) $x(n) = \sin \frac{\pi}{3} n$

(e) $x(n) = r(n)$.

(a) $x(n) = u(n)$.

$$\rightarrow E_{x(n)} = \underset{N\to\infty}{lt} \sum_{n=-N}^{N} |x(n)|^2$$

WKT $u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$

$$E_{x(n)} = \underset{N\to\infty}{lt} \sum_{n=0}^{N} |1|^2 = \underset{N\to\infty}{lt} \sum_{n=0}^{N} 1 \rightarrow N+1$$

WKT

$$\underset{N\to\infty}{lt} N+1 = \infty \qquad \boxed{E_{x(n)} = \infty}$$

$$P_{av} = \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=0}^{N} |1|^2 = \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=0}^{N} 1 \rightarrow N+1$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \times N+1 \qquad \left[\because \underset{N\to\infty}{lt} \frac{N+1}{aN+1} = \frac{1}{a} \right]$$

$$\boxed{P_{av} = \frac{1}{2}}$$

$x(n) = u(n)$ Power Signal

(b) $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

$$\rightarrow E_{x(n)} = \underset{N\to\infty}{lt} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{lt} \sum_{n=0}^{N} \left|\left(\frac{1}{2}\right)^n\right|^2$$

$$= \underset{N\to\infty}{lt} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

WKT $\displaystyle\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$

$$P_{av} = \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left|\left(\frac{1}{2}\right)^n\right|^2$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$\underset{N\to\infty}{lt} \frac{1}{2N+1} \left[\frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-\frac{1}{4}} \right]$$

$$= \frac{1-\left(\frac{1}{4}\right)^{\infty+1} \to 0}{1-\frac{1}{4}}$$

$$= \frac{1-0}{\frac{3}{4}} = \frac{4}{3}.$$

$$E_{x(n)} = \frac{4}{3}.$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1}\left[\frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-\frac{1}{4}}\right]$$

$$= \frac{1}{\infty+1} \times \frac{1}{3/4} \quad \left[\because \frac{1}{\infty} = \right.$$

$$P_{av} = 0$$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$ is Energy signal

(c) $x(n) = \sin \frac{\pi}{3} n.$

$$E_{x(n)} = \underset{N\to\infty}{lt} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{lt} \sum_{n=-N}^{N} \left|\sin \frac{\pi}{3} n\right|^2$$

$$= \underset{N\to\infty}{lt} \sum_{n=-N}^{N} \sin^2 \frac{\pi}{3} n$$

$$\left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= \underset{N\to\infty}{lt} \sum_{n=-N}^{N} \left[\frac{1-\cos \frac{2\pi}{3} n}{2}\right]$$

$$= \frac{1}{2} \underset{N\to\infty}{lt} \left[\sum_{n=-N}^{N} 1 - \cancel{\sum_{n=-N}^{N} \frac{\cos 2\pi n}{3}}^{\,0}\right]$$

$$= \frac{1}{2} \underset{N\to\infty}{lt} \left[2N+1\right]$$

$$= \frac{1}{2}\left(2(\infty)+1\right)$$

$$E_{x(n)} = \infty$$

$$P_{av}\, x(n) = \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin \frac{\pi}{3} n\right|$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2 \frac{\pi}{3} n$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos \frac{2\pi}{3} n}{2}$$

$$= \underset{N\to\infty}{lt} \frac{1}{2N+1} \frac{1}{2}\left[\sum_{n=-N}^{N} 1 - \cancel{\sum_{n=-N}^{N} \frac{\cos 2\pi n}{3}}^{\,0}\right]$$

$$= \frac{1}{2} \underset{N\to\infty}{lt} \frac{1}{\cancel{2N+1}} \times \cancel{2N+1}$$

$$P_{av} = \frac{1}{2}$$

$x(n) = \sin \frac{\pi}{3} n$ is Power Signal.

Note:

$A \sin \omega n \longrightarrow$ Power signal $= \frac{Amp^2}{2} = \frac{A^2}{2}$

$A \cos \omega n \longrightarrow$ Power signal $= \frac{Amp^2}{2} = \frac{A^2}{2}$

Energy is always ∞

(d) $x(n) = 2e^{j\left(\frac{\pi}{2}n + \pi/4\right)}$

$$E_{x(n)} = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|2e^{j\underbrace{\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}_{\theta}}\right|^2$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} |2|^2 \cdot \left|e^{j\underbrace{\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}_{\theta}}\right|^2$$

WKT $|e^{j\theta}| = 1$.

$$= 4 \lim_{N\to\infty} \sum_{n=-N}^{N} 1^2$$

$$= 4 \lim_{N\to\infty} \sum_{n=-N}^{N} 1 \rightarrow 2N+1$$

$$E_{x(n)} = \infty.$$

$$P_{av}\, x(n) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|2e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} 4(2N+1)$$

$$= 4 \lim_{N\to\infty} \frac{\cancel{2N+1}}{\cancel{2N+1}} = 4.$$

$P_{av}\, x(n) = 4.$

$x(n) = 2e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$ is Power Signal

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(e) $x(n) = r(n)$.

$\rightarrow$ WKT

$$r(n) = \begin{cases} n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

$$E_{x(n)} = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} n^2$$

$$= \lim_{N\to\infty} \left[0 + \sum_{n=1}^{N} n^2\right]$$

$$= \frac{N(N+1)(2N+1)}{6}$$

$E_{x(n)} = \infty$

$$P_{av}\, x(n) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

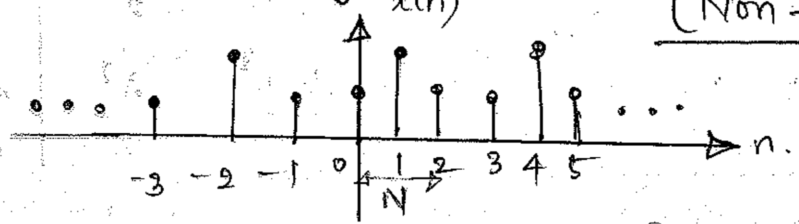
$$= \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cdot \frac{N(N+1)\cdot\cancel{(2N+1)}}{6}$$

$$= \lim_{N\to\infty} \frac{N(N+1)}{6}.$$

$P_{av} = \infty$

$x(n) = r(n)$ is not a Energy & power signal.

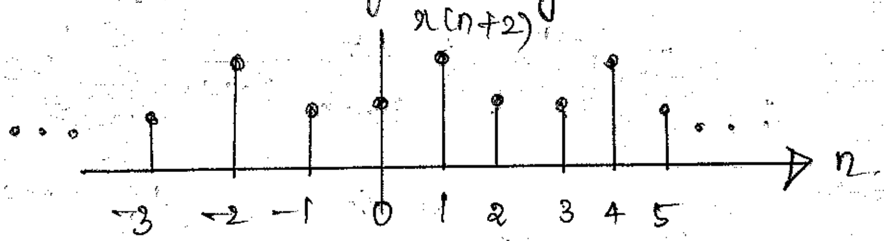
(2) Periodic Signal and Aperiodic Signal.
(Non-periodic)



$x(n)$ is said to be periodic if $x(n) = x(n+N)$.

for the above $x(n)$ the period is $N=2$.

then $x(n+2)$ is given by.



So the given $x(n)$ is periodic.

Note:-

(1) Sum of two Continuous-time periodic signal may not be periodic.

(2) Sum of two Discrete-time periodic signal is periodic signal.

Problems.

(1) Determine the following Signals are periodic (or) Aperiodic Signals

(a) $x(n) = \sin\left(\frac{6\pi}{31}n + \frac{\pi}{6}\right)$

(b) $x(n) = e^{j\frac{7\pi n}{4}}$

(c) $x(n) = \cos\left(\frac{\pi}{4}\right)n + \cos\frac{\pi}{6}n - \sin\left(\frac{\pi}{8}n - \frac{\pi}{3}\right)$.

(d) $x(n) = \cos\left(\frac{n}{4}\right) \cdot \cos\left(\frac{\pi}{4}n\right)$.

(a) $x(n) = \sin\left(\frac{6\pi}{31}n + \frac{\pi}{6}\right)$

$x(n) = x(n+N)$.

WKT $\sin \omega n$.

$\omega = 2\pi f = \frac{\cancel{2}\pi}{N} = \frac{\cancel{6}^{3}\pi}{31}$

$N = \frac{31}{3}$ not an integer

So $mN = \frac{31}{3} \times m$

$m = 3$ then $N = \frac{31}{3} \times 3$

$N = 31$.

$x(n+31) = \sin\left(\frac{6\pi}{31}(n+31) + \frac{\pi}{6}\right)$

$= \sin\left(\frac{6\pi n}{31} + 6\pi + \frac{\pi}{6}\right)$.

$= \sin\left(2\pi(3) + \frac{6\pi n}{31} + \frac{\pi}{6}\right)$

WKT $\sin(2\pi + \theta) = \sin\theta$

$= \sin\left(\frac{6\pi n}{31} + \frac{\pi}{6}\right)$

$x(n) = x(n+31)$

Periodic signal.

(b) $x(n) = e^{j\frac{7\pi n}{4}}$

WKT $e^{j\omega n}$

$\omega = \frac{2\pi}{N} = \frac{7\pi}{4}$

$N = \frac{8}{7}$ not an Integer

$mN = \frac{8}{7} \times m$; $m = 7$

$N = 8$

$x(n+8) = e^{j\frac{7\pi(n+8)}{4}}$

$= e^{j\frac{7\pi}{4}(n+8)}$. $\left[e^{a+b} = e^{a} \cdot e^{b}\right]$

$= e^{j\frac{7\pi}{4}n} \cdot e^{j\frac{7\pi}{\cancel{4}} \times \cancel{8}2}$

$= e^{j\frac{7\pi n}{4}} \cdot e^{j14\pi}$

$= e^{j\frac{7\pi n}{4}} \cdot e^{j2\pi(7)}$.

WKT $e^{j2\pi n} = 1$

$= e^{j\frac{7\pi n}{4}}$

$x(n) = x(n+8)$

Periodic signal.

(c) $x(n) = \cos\frac{\pi}{4}n + \cos\frac{\pi}{6}n - \sin\left(\frac{\pi}{8}n - \pi/3\right)$.

$\omega_1 = \frac{2\pi}{N_1} = \frac{\pi}{4}$; $\omega_2 = \frac{2\pi}{N_2} = \frac{\pi}{6}$

$\omega_3 = \frac{2\pi}{N_3} = \frac{\pi}{8}$

$N_1 = 8$; $N_2 = 12$; $N_3 = 16$.

$\frac{N_1}{N_2} = \frac{\cancel{8}\,2}{\cancel{12}\,3}$, $\frac{N_1}{N_3} = \frac{\cancel{8}\,1}{\cancel{16}\,2}$

Ratio's are rational so it is Periodic signal.

Period N = LCM (denominators of ratio's) $\times N_1$

LCM denominators of ratio's

LCM $(3, 2) = 6$.

$N = 6 \times N_1 = 6 \times 8 = 48$.

(d) $x(n) = \cos\frac{n}{4} \cdot \cos\frac{\pi}{4}n$

WKT

$\rightarrow x(n) = \cos\underbrace{\frac{n}{4}}_{A} \cdot \cos\underbrace{\frac{\pi}{4}}_{B}n$; $\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$

$$= \frac{\cos\left(\frac{n}{4} + \frac{\pi}{4}n\right) + \cos\left(\frac{n}{4} - \frac{\pi}{4}n\right)}{2} = \frac{1}{2}\left[\cos\left(\frac{1}{4}+\frac{\pi}{4}\right)n + \cos\left(\frac{n}{4} - \frac{\pi}{4}\right)n\right]$$

$$\omega_1 = \frac{2\pi}{N_1} = \frac{1}{4} + \frac{\pi}{4} = \frac{\pi+1}{4}$$

$$N_1 = \frac{8\pi}{\pi+1}$$

$$\omega_2 = \frac{2\pi}{N_2} = \frac{1}{4} - \frac{\pi}{4} = \frac{1-\pi}{4} \; ; \; N_2 = \frac{8\pi}{1-\pi}$$

$$\frac{N_1}{N_2} = \frac{\frac{8\pi}{\pi+1}}{\frac{8\pi}{1-\pi}} = \frac{\cancel{8\pi}}{\pi+1} \times \frac{1-\pi}{\cancel{8\pi}} = \frac{1-\pi}{\pi+1} \text{ [irrational]}$$

So $x(n)$ is Non-periodic (or) Aperiodic Signal.

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(3) Even signal and Odd signal.

① Given $x(n)$ is said to be even signal when $x(n) = x(-n) \rightarrow$ Even signal.

$x(n) = -x(-n) \rightarrow$ Odd signal.

② Given $x(n)$

even part of $x(n)$ is given by

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

odd part of $x(n)$ is given by

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Even Signals $\rightarrow$ Symmetric Signals

Odd Signals $\rightarrow$ Asymmetric Signals

Problems

(1) Determine the following signals are even (or) odd signals

(a) $x(n) = \sin n$

(b) $x(n) = \cos n$

(a) $x(n) = \sin n$

$\rightarrow$ $x(n) = \sin n$

$x(-n) = \sin(-n) = -\sin n$

$x(n) = -x(-n)$

So $x(n) = \sin n \rightarrow$ odd signal

(b) $x(n) = \cos n$

$\rightarrow$ $x(n) = \cos n$

$x(-n) = \cos(-n) = \cos n$

$x(n) = x(-n)$

$x(n) = \cos n$ is Even Signal.

(2) Determine the even and odd part of the given signal.

$x(n) = \cos n + \sin n$.

$\rightarrow$ $x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

$= \frac{1}{2}\left[\cos n + \cancel{\sin n} + \cos n - \cancel{\sin n}\right]$

$x_e(n) = \frac{\cancel{2}\cos n}{\cancel{2}} = \cos n$.

$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$

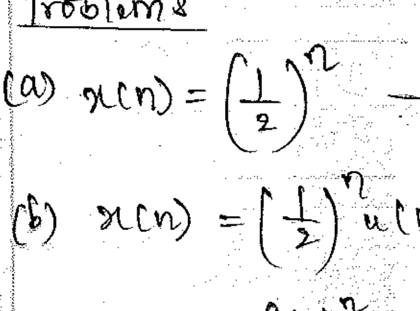
$= \frac{1}{2}\left[\cancel{\cos n} + \sin n - \cancel{\cos n} + \sin n\right]$

$= \frac{\cancel{2}\sin n}{\cancel{2}} = \sin n$

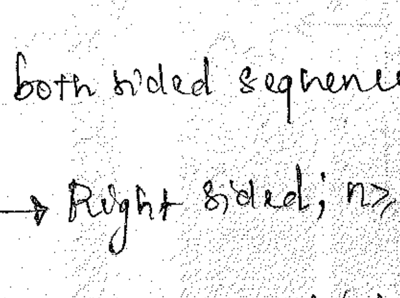
$x_o(n) = \sin n$.

(4) Causal Signals, Non-Causal & Anticausal Signals

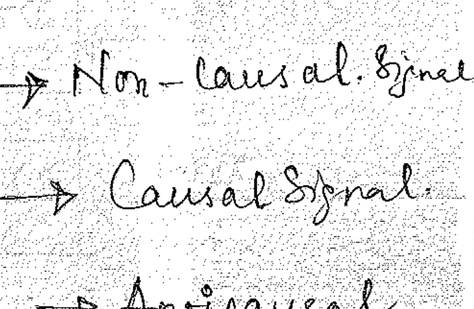
Causal signals



Non-Causal.



Anticausal signals



Problems

(a) $x(n) = \left(\frac{1}{2}\right)^n \rightarrow$ both sided sequence $\rightarrow$ Non-Causal. Signal

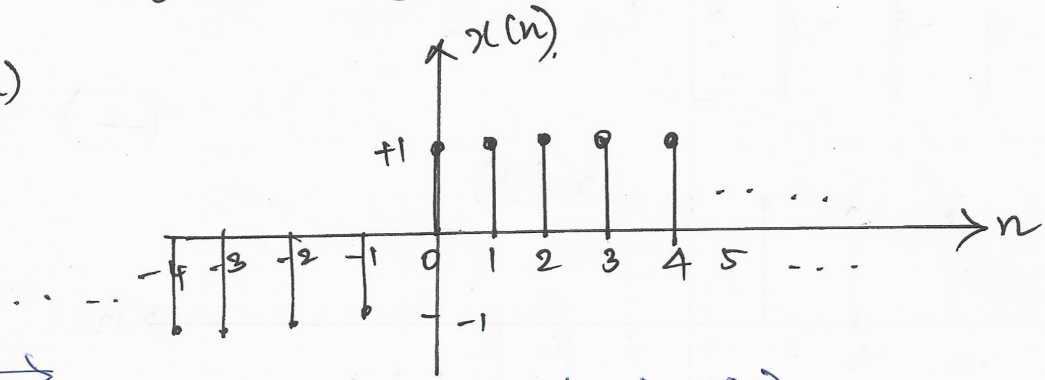
(b) $x(n) = \left(\frac{1}{2}\right)^n u(n) \rightarrow$ Right sided; $n \geq 0 \rightarrow$ Causal Signal.

(c) $x(n) = \left(\frac{1}{2}\right)^n u(-n) \rightarrow$ left sided; $n \leq 0 \rightarrow$ Anticausal signals

Even & odd Signal: DT signals.

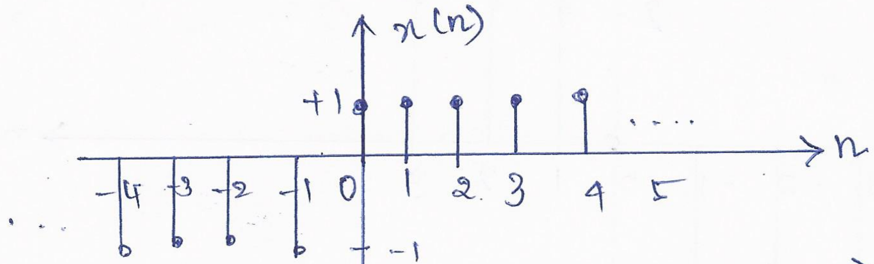
(1) Determine and sketch the even & odd parts of the given signal $x(n)$

(a)

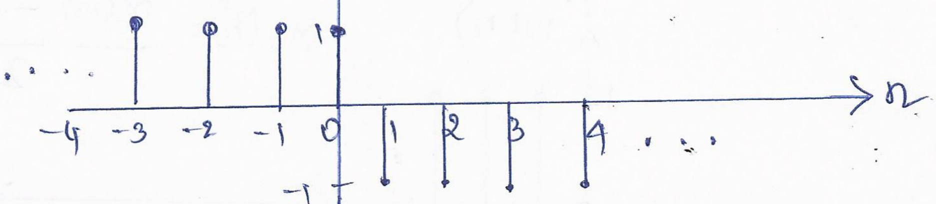


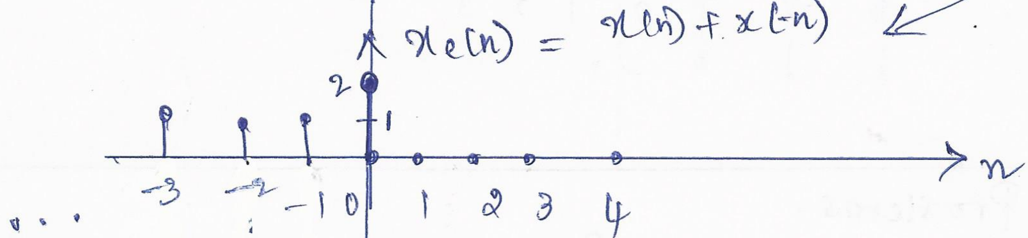
To find even part of $x(n)$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

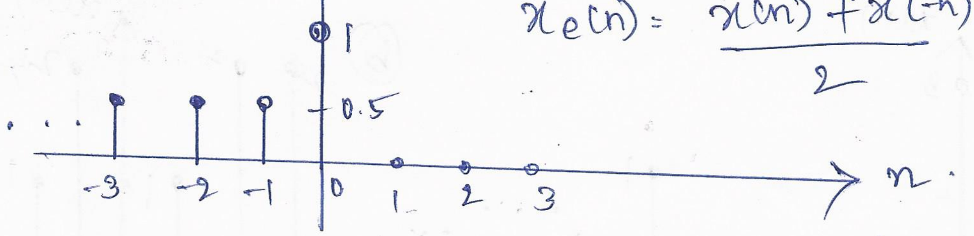


$x(-n)$ +



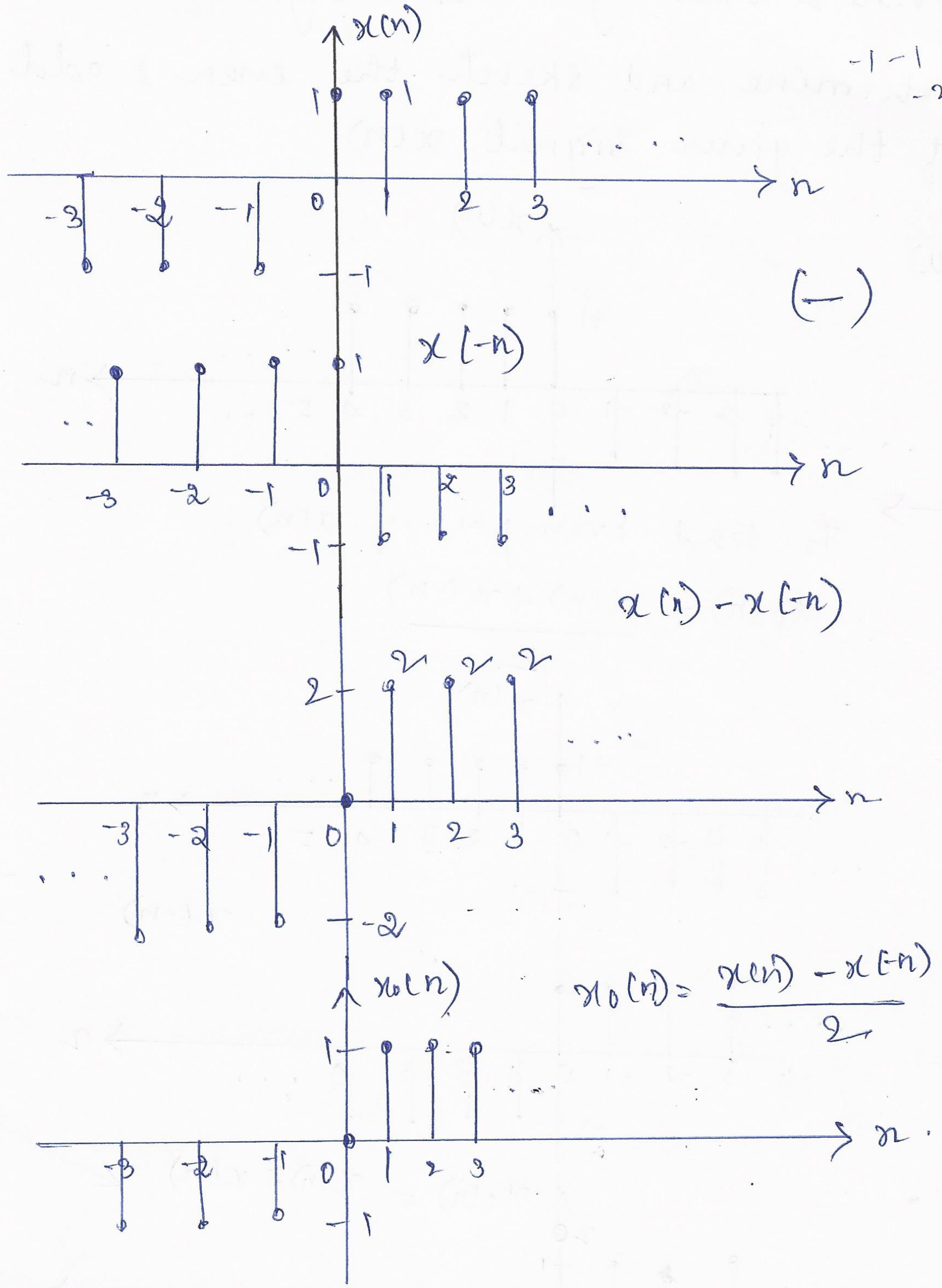


$$x_e(n) = \frac{x(n) + x(-n)}{2}$$



To find odd part of $x(n)$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$



Practice Problems.

(1) Find $x_e(n)$ & $x_o(n)$ for the signal $x(n)$