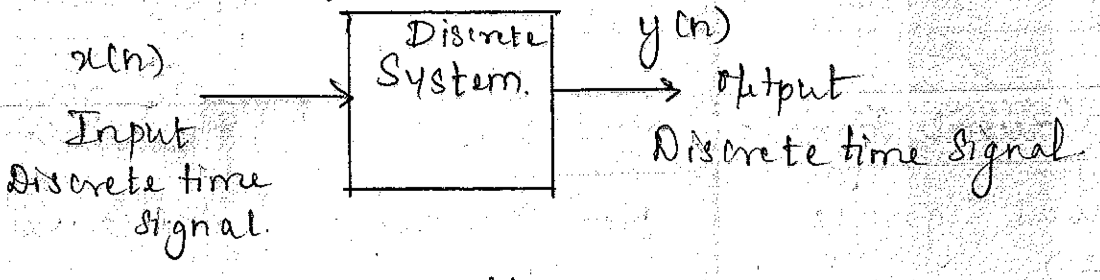
Analysis of Discrete-Time Systems.

(Types)
Classification of Discrete-time Systems.

(1) Linear System and Non-Linear System.
(2) Time-Invariant and Time Variant System.
(3) Causal System and Non-Causal System.
(4) Static (Memoryless) system and Dynamic (memory) - system
(5) Stable system and Unstable system.

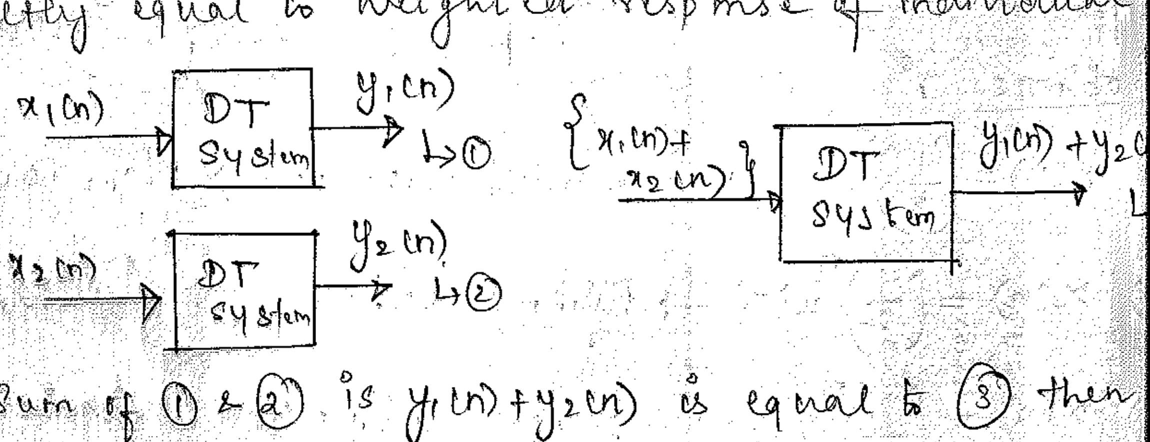
(1) Linear System and Non-Linear System.

A system is said to be linear it should satisfy two properties
(i) Superposition (or) Additivity
(ii) Homogenity (or) scaling.



Superposition or Additivity:

Weighted response of sum of the inputs is exactly equal to weighted response of individual



Sum of ① & ② is $y_1(n)+y_2(n)$ is equal to ③ then system is linear and satisfying the superposition property

Homogeneity (or) Scaling:

If $x(n) \rightarrow y(n)$ then $a x(n) \rightarrow a y(n)$

It is not valid for Complex Systems.

Problems.

① Determine whether the system described by the input-output relation is linear (or) non-linear.

(a) $y(n) = n\,x(n)$ (e) $y(n) = x^2(n)$

(b) $y(n) = 0.5\,x(n) + 9$ (f) $y(n) = \log x(n)$

(c) $y(n) = x(n) \cos \omega n$ (g) $y(n) = \sqrt{x(n)}$

(d) $y(n) = |x(n)|$ (h) $y(n) = x(n) \cdot x(n-1)$

(i) $y(n) + y(n-1) = x(n)$ (j) $= y(n) + y(n-2) = x(n-2) + x(n+1)$

(a) $y(n) = n\,x(n)$

$x(n) = x_1(n)$ then
o/p $y(n) = y_1(n) \rightarrow$ ①

$x(n) = x_2(n)$ then o/p
$y(n) = y_2(n) \rightarrow$ ②

$x(n) = [x_1(n) + x_2(n)]$ then
o/p $y(n) \rightarrow$ ③.

① + ② = ③ $\rightarrow$ then the system is Linear.

$y(n) = n\,x(n)$

$y_1(n) = n x_1(n) \rightarrow$ ①

$y_2(n) = n x_2(n) \rightarrow$ ②

①+② $n x_1(n) + n x_2(n)$.

$y(n) = n[x_1(n) + x_2(n)]$

$= n[x_1(n)] + n x_2(n)$.
$\hookrightarrow$ ③.

① + ② = ③ $\rightarrow y(n) = n x(n)$ is Linear

(b) $y(n) = 0.5\,x(n) + 9.$

$\rightarrow$ $y_1(n) = 0.5\,x_1(n) + 9 \rightarrow (1)$

$y_2(n) = 0.5\,x_2(n) + 9 \rightarrow (2)$

(1) + (2)

$0.5\,x_1(n) + 0.5\,x_2(n) + 18$

$y(n) = 0.5\left[x_1(n) + x_2(n)\right] + 9$

$= 0.5\left[x_1(n)\right] + 0.5\,x_2(n) + 9 \rightarrow (3)$

(1) + (2) $\neq$ (3)

$y(n) = 0.5\,x(n) + 9$ is Non-Linear System.

(c) $y(n) = x(n)\cos\omega n$

$\rightarrow$ $y_1(n) = x_1(n)\cos\omega n \rightarrow (1)$

$y_2(n) = x_2(n)\cos\omega n \rightarrow (2)$

(1) + (2) $= x_1(n)\cos\omega n + x_2(n)\cos\omega n$

$y(n) = \left[x_1(n) + x_2(n)\right]\cos\omega n$

$= x_1(n)\cos\omega n + x_2(n)\cos\omega n \rightarrow (3)$

(1) + (2) = (3)

$y(n) = x(n)\cos\omega n$ is Linear System

(d) $y(n) = |x(n)|.$

$\rightarrow$ $y_1(n) = |x_1(n)|$

$y_2(n) = |x_2(n)|.$

(1) + (2) $= |x_1(n)| + |x_2(n)|.$

$y(n) = |x_1(n) + x_2(n)| \rightarrow (3)$

WKT

$|a+b| \leq |a| + |b|.$

$|x_1(n) + x_2(n)| \leq |x_1(n)| + |x_2(n)|$

(3) $\leq$ (1) + (2)

$\rightarrow$ because of this inequality

$y(n) = |x(n)|$ is Non-Linear System.

(e) $y(n) = x^2(n).$

$\rightarrow$ $y_1(n) = x_1^2(n) \rightarrow (1)$

$y_2(n) = x_2^2(n) \rightarrow (2)$

(1) + (2) $= x_1^2(n) + x_2^2(n)$

$y(n) = \left[x_1(n) + x_2(n)\right]^2$

$= x_1^2(n) + x_2^2(n) + 2x_1(n)\,x_2(n) \rightarrow (3)$

(1) + (2) $\neq$ (3)

(f) $y(n) = \log x(n)$.

$\rightarrow y_1(n) = \log x_1(n)$ ⟶ ①

$y_2(n) = \log x_2(n)$ ⟶ ②

① + ② $= \log x_1(n) + \log x_2(n)$

$y(n) = \log [x_1(n) + x_2(n)]$

WKT =

$\log(a+b) = \log a . \log b$

$= \log x_1(n) . \log x_2(n)$ ⟶ ③

① + ② $\neq$ ③

$y(n) = \log x(n)$ is Non-Linear System

(g) $y(n) = \sqrt{x(n)}$.

$\rightarrow y_1(n) = \sqrt{x_1(n)}$ ⟶ ①

$y_2(n) = \sqrt{x_2(n)} \rightarrow$ ②

① + ② $= \sqrt{x_1(n)} + \sqrt{x_2(n)}$

$y(n) = \sqrt{x_1(n) + x_2(n)}$ ⟶ ③

WKT

$\sqrt{a+a} \neq \sqrt{a} + \sqrt{b}$.

① + ② $\neq$ ③

$y(n) = \sqrt{x(n)}$ is Non-linear System.

(h) $y(n) = x(n) . x(n-1)$.

$\rightarrow y_1(n) = x_1(n) . x_1(n-1) \rightarrow$ ①

$y_2(n) = x_2(n) . x_2(n-1) \rightarrow$ ②

① + ②

$x_1(n) . x_1(n-1) +$

$x_2(n) . x_2(n-1)$.

$y(n) = [x_1(n) + x_2(n)] . [x_1(n-1) + x_2(n-1)]$

$= x_1(n) . x_1(n-1) + x_1(n) x_2(n-1)$
$+ x_2(n) x_1(n-1) + x_2(n) x_2(n-1)$ ⟶ ③

① + ② $\neq$ ③

$y(n) = x(n) . x(n-1)$ is Non-linear System.

(i) $y(n) + y(n-1) = x(n)$.

$\rightarrow y_1(n) + y_1(n-1) = x_1(n). \rightarrow$ ①

$y_2(n) + y_2(n-1) = x_2(n) \rightarrow$ ②.

$y(n) + y(n-1) = [x_1(n) + x_2(n)]$ ⟶ ③

① + ② = ③

(j) $y(n) + y(n-2) = x(n-2) + x(n+1)$

$\rightarrow$

$y_1(n) + y_1(n-2) = x_1(n-2) + x_1(n+1)$ $\rightarrow$ (1)

$y_2(n) + y_2(n-2) = x_2(n-2) + x_2(n+1)$ $\rightarrow$ (2)

(1) + (2)

$x_1(n-2) + x_1(n+1) + x_2(n-2) + x_2(n+1)$

$y(n) = \Big[[x_1(n-2) + x_2(n-2)] + [x_1(n+1) + x_2(n+1)] \Big]$

$= [x_1(n-2) + x_2(n-2) + x_1(n+1) + x_2(n+1)]$ $\rightarrow$ (3)

(1) + (2) = (3)

$y(n) + y(n-2) = x(n-2) + x(n+1)$ is Linear System

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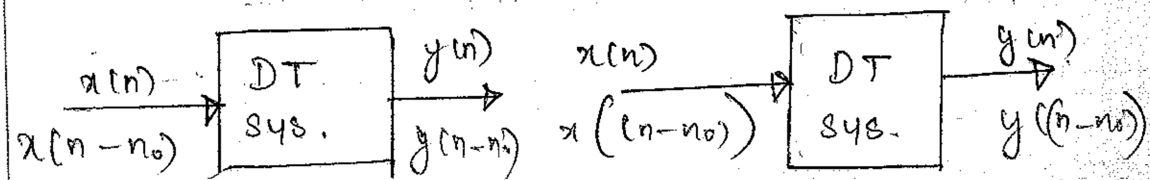
(2) Time Invariant and Time Variant System.



shift the input by n_0

$x(n-n_0)$ the o/p

$y(n-n_0) \rightarrow (1)$

put $n = n - n_0$ then o/p

$y(n-n_0) \rightarrow (2)$

If (1) = (2) $\rightarrow$ Time Invariant

(1) $\neq$ (2) $\rightarrow$ Time Variant.

(1) Determine whether the system described by the input output relation is Time Invariant (or) Time Variant

(a) $y(n) = 0.5\, x(n) + 9$

(b) $y(n) = x(n-1)$

(c) $y(n) = x(-n)$

(a) $y(n) = 0.5\,x(n) + 9$

→ Shift the i/p by n_0

$= 0.5\,x(n-n_0) + 9. \rightarrow (1)$

put $n = n - n_0$

$0.5\,x(n-n_0) + 9. \rightarrow (2)$

(1) = (2) $y(n) = 0.5\,x(n) + 9$ is Time Invariant System

(b) $y(n) = x(n-1)$

→ shif the i/p by n_0

$= x(n - n_0 - 1) \rightarrow (1)$

put $n = n - n_0$.

$= x((n - n_0) - 1)$

$= x(n - n_0 - 1) \rightarrow (2)$

(1) = (2) $y(n) = x(n-1)$ is Time Invariant system.

(c) $y(n) = x(-n)$

→ shift the i/p by n_0

$y(n) = x(-n - n_0) \rightarrow (1)$

put $n = n - n_0$

$= x(-(n-n_0))$

$= x(-n + n_0) \rightarrow (2)$

(1) ≠ (2) so

$y(n) = x(-n)$ is Time Variant systems.

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(3) Causal System & Non-Causal System.

The System is said to be causal system, the Present output of the system is depends on present input, past input & past outputs.

(1) Determine whether the system described by the input and output relation is Causal system (or) Non-causal system.

(a) $y(n) = x(-n)$

(b) $y(n) = 3x(n-2) + 3x(n+2)$

(a) $y(n) = x(-n)$.

→ put $n=1$

$y(1) = x(-1)$. pres. o/p, past i/p

present depends on past input. but put $n=-1$

$y(-1) = x(1)$ present o/p, future i/p.

present o/p depends on future input.

So. $y(n) = x(-n)$ is Non-causal System.

(b) $y(n) = 3x(n-2) + 3x(n+2)$

→ put $n=0$

$y(0) = 3x(-2) + 3x(2)$. past i/p, future i/p.

Present o/p. The system is Non-causal systems.

(c) $y(n) = x(n-1) + 4x(n-2)$.

→ put $n=0$

$y(0) = x(-1) + 4x(-2)$ past i/p

Present o/p, past i/p

The system is Causal System.

Note:-

(1) Reversal system } → Non-causal Systems
(2) ~~Advanced~~ Signal- system

(3) Signal Delaying system → Causal Systems.

(4) Static (Memoryless) System & Dynamic (Memory) System.

The System is said to be Static/Memoryless system, Present o/p depends on Present i/p. not on Past i/p (or) future i/p's.

(1) Test Whether the following systems are Static (or) Dynamic System.

(a) $y(n) = x(n) + 3u(n+1)$.

→ put $n=0$

$$\underbrace{y(0)}_{\text{Present o/p}} = \underbrace{x(0)}_{\text{present i/p}} + 3u(1).$$

$y(n) = x(n) + 3u(n+1)$ is Static System.

(b) $y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$; $n_0 \neq 0$

→ assume $n_0 = 2$

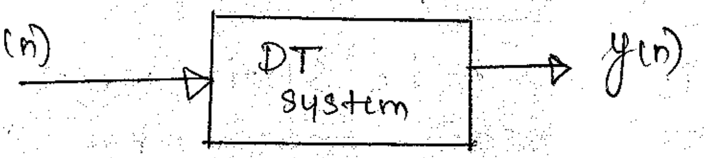
$$y(n) = \sum_{k=n-2}^{n+2} x(k) =$$

put $n = 0$.

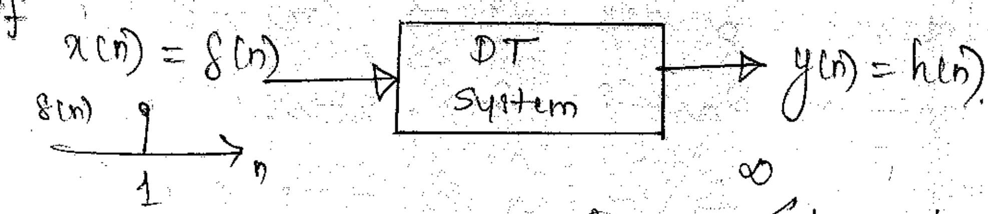
$$\underbrace{y(0)}_{\text{Present o/p}} = \sum_{k=-2}^{2} x(k) = \underbrace{x(-2) + x(-1)}_{\text{past i/p}} + \underbrace{x(0)}_{\text{Prest i/p}} + \underbrace{x(1) + x(2)}_{\text{future i/p}}$$

The system is Dynamic System/Memory system.

(5) Stable System & Unstable System.



if



The system is stable if $\sum_{n=-\infty}^{\infty} |h(n)| =$ finite i.e

$$0 < \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

(1) Test whether the following systems are stable (or) Unstable system.

(a) $y(n) = x(-n-2)$

(b) $y(n) = x(n) \cos \omega n$.

(a) $y(n) = x(-n-2)$.

→ $x(n) = \delta(n)$ then

$y(n) = h(n)$.

then $x(-n-2) = \delta(-n-2)$

$y(n) = h(n)$

$h(n) = \delta(-n-2)$.

$h(-1) = \delta(-1) = 0$

$h(0) = \delta(-2) = 0$

$h(1) = \delta(-3) = 0$

$h(2) = \delta(-4) = 0$

$h(-2) = \delta(0) = 1$.

$\vdots$

$\sum_{n=-\infty}^{\infty} |h(n)| = \cdots 0 + 0 + 1 + 0 + 0 \ldots$

$= 1$ finite

(b) $y(n) = x(n) \cos \omega n$.

→ $h(n) = \delta(n) \cos \omega n$

$h(-1) =$

$\delta(-1) \cdot \cos \omega(-1) = 0$

$h(0) = \delta(0) \cdot \cos \omega(0) = 1$

$h(1) = \delta(1) \cdot \cos \omega(1) = 0$

$\vdots$

$\sum_{n=-\infty}^{\infty} |h(n)| = 1$ finite

Stable System.

<u>Note:</u> $\delta(n) = \begin{cases} 1 & ; n = 0 \\ 0 & ; n \neq 0 \end{cases}$

Additional Problem

Classification of Discrete Time Systems.

(1) $y(n) = x(n) + x(n-1)$, determine whether the system is

(a) Linear (or) Non-Linear

(b) Time Invariant (or) Time Variant

(c) Static (or) Dynamic

(d) Causal (or) Non Causal

(e) Stable (or) Unstable.

$\longrightarrow$

(a) $y(n) = x_1(n) + x_1(n-1) \rightarrow (1)$

Step1: $y(n) = x_2(n) + x_2(n-1) \rightarrow (2)$

$x_1(n) + x_1(n-1) + x_2(n) + x_2(n-1) \rightarrow (I)$

Step2: $y(n) = \{x_1(n) + x_2(n)\} + \{x_2(n-1) + x_2(n-1)\} \rightarrow (II)$

$(I) = (II)$ Linear.

(b) $y(n) = x(n-n_0) + x(n-n_0-1) \rightarrow (1)$

Step1:

Step 2/ $y(n) = x(n-n_0) + x(n-n_0-1) \rightarrow (2)$

$(1) = (2)$

So the system is Time Invariant.

(c) $y(n) = x(n) + x(n-1)$

$\rightarrow$ $y(0) = x(0) + x(0-1)$ (0)

Case(i) $= x(0) + x(-1)$ $\rightarrow$ Present o/p depends on present i/p (0) & past i/p (-1)

Case(ii) $y(-1) = x(-1) + x(-2)$ $\rightarrow$ (-1) depends on (-1) & (-2)

So system is Dynamic (Memory)

(d) $y(n) = x(n) + x(n-1)$

$\rightarrow$ $y(0) = x(0) + x(-1)$ $\rightarrow$ Present o/p depends on Present i/p & past i/p

So system is Causal.

(e) $y(n) = x(n) + x(n-1)$

$\rightarrow$ step 1: $h(n) = \delta(n) + \delta(n-1)$

step 2: $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \delta(n) + \delta(n-1)$

$= \cancel{\delta(-\infty)}^{0} + \cdots + \underline{\delta(0)}^{1} + \cdots \cancel{\delta(\infty)}^{0} +$
$\cancel{\delta(-\infty-1)}^{0} + \cdots + \cancel{\delta(-1)}^{0} + \underline{\delta(0)}^{1} + \cdots + \cancel{\delta(\infty-1)}^{0}$

$= 2 \cdot (\text{finite}) < \infty$

So system is Stable.

Practise Question.

(2) Determine whether the given system is

$$y(n) = \sum_{k=-\infty}^{n} x(k) \text{ is}$$

(a) Linear (or) Non - Linear.
(b) Time Invariant (or) Variant
(c) Static (or) Dynamic
(d) Causal (or) Non Causal.
(e) Stable (or) Unstable.

→

(a) $y(n) = \sum_{k=-\infty}^{n} x(k)$

Step1: $y(n) = \sum_{k=-\infty}^{n} x_1(k) \rightarrow (1)$

$y(n) = \sum_{k=-\infty}^{n} x_2(k) \rightarrow (1)$

$(1) + (2) = \sum_{k=-\infty}^{n} x_1(k) + \sum_{k=-\infty}^{n} x_2(k) \rightarrow (I)$

Step2: $y(n) = \sum_{k=-\infty}^{n} [x_1(k) + x_2(k)]$

$= \sum_{k=-\infty}^{n} x_1(k) + \sum_{k=-\infty}^{n} x_2(k) \rightarrow (II)$

$(I) = (II)$ Linear.

(b) $y(n) = \sum_{K=-\infty}^{n} x(K)$

$\rightarrow$

Step1: $y(n) = \sum_{K=-\infty}^{n} x(K-n_0) \rightarrow (1)$

Step 2: $y(n) = \sum_{K=-\infty}^{n-n_0} x(K) \rightarrow (2)$

$(\mathrm{I}) \neq (\mathrm{II})$ Time Variant.

(c) $y(n) = \sum_{K=-\infty}^{n} x(K)$

$\rightarrow$

$y(0) = \sum_{K=-\infty}^{0} x(K)$

$= x(-\infty) + \dots x(0)$

Present o/p (0) depends on past i/p $(-\infty)$ & present i/p (0)

then system is Causal.

(d) $y(n) = \sum_{K=-\infty}^{n} x(K)$

$\rightarrow$

$y(0) = \sum_{K=-\infty}^{n} x(K) = x(-\infty) + \dots + x(0)$

Dynamic (memory).

(e) $y(n) = \sum_{K=-\infty}^{n} x(K)$

$\rightarrow$

Step1: $h(n) = \sum_{K=-\infty}^{n} \delta(K)$

Step2: $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \sum_{K=-\infty}^{n} \delta(K) = \sum_{n=-\infty}^{\infty} 1 = \infty$

(Ustable)