UNIT-I : SIGNALS AND SYSTEMS.

Signal :-

A signal is defined as any physical quantity that varies with time, space or any other independent variable (or) variables.

Mathematically, we describe a signal as a function of one (or) more independent variables.

Types of signals:

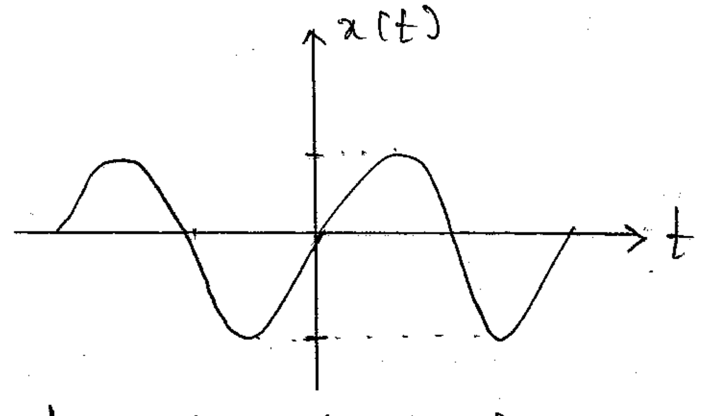
(i) Continuous time signals (CT) (or) Analog signal.

(ii) Discrete - time signal (DT).

Continuous time signals (CT) (or) Analog signal.

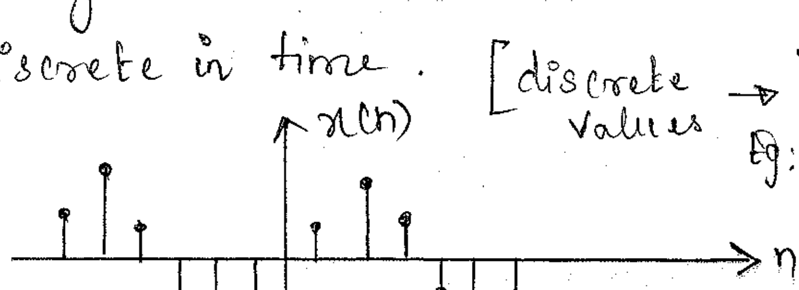
A signal which is continuous both in time and amplitude is continuous time signal.

Eg:-



Discrete time signals (DT).

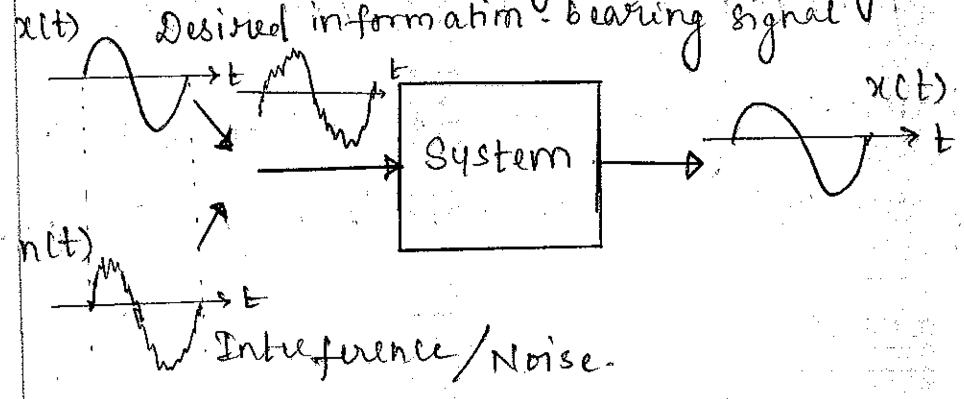
A signal which is continuous in amplitude but discrete in time. [discrete values → Integers Eg: -1, 0, 1, 2.]

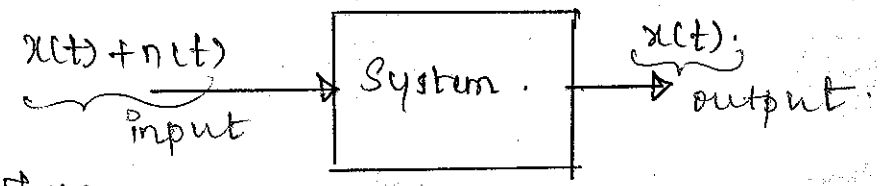


System:

A system may also be defined as a physical device that performs an operation on a signal.

Usually, the operation performed on signals are referred as Signal processing.





Where

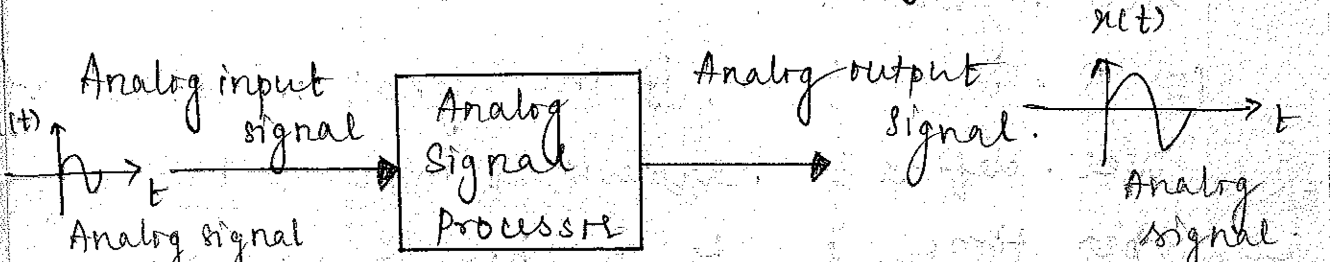
$x(t) \rightarrow$ desired Information-bearing signal.

$n(t) \rightarrow$ Interference/Noise.

Basic Elements of a Digital Signal Processing (DSP) System.

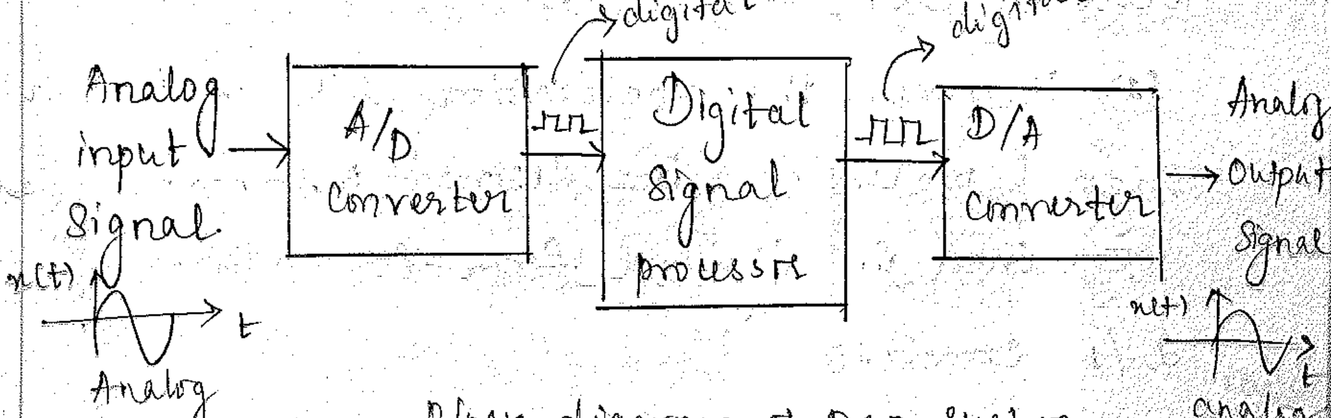
(1) Most of the signals encountered in science and engineering are along is nature.

(2) Such signals may be processed directly by appropriate analog systems. for the purpose of extracting some desired information. This is called as Analog signal processing.



Digital Signal processing (DSP).

(1) DSP provides an alternative method for processing the analog signal. To perform the processing digitally, there is a need for an interface between the analog signal and digital processor. This interface is called an analog-to-digital (A/D) converter.



Analog

Block diagram of DSP system.

analog

(2) Digital signal processor (DSP) may be a large programmable digital computer (or) a small microprocessor programmed to perform the desired operations on the input signal.

(3) DSP may be harwired digital processor (or) Programmable machines.

(4) Generally programmable signal processors are very common in use because of its flexibility and reconfigureable.

(5) But in contrast, if the signal operations are well-defined, hardwired implementation of the operations results in cheaper signal processor and it runs faster than the programmable processors.

D/A Converter:

(1) In speech signal processing applications, the processed signal is to given in analog form to the user. So an interfacing called Digital to analog converter is required between DSP and user.

(2) However, the applications like in Radar signal processing, the desired informations are required in digital from so no need of D/A converter.

Advantages of Digital Signal processing (DSP) over Analog Signal processing (ASP).

(1) Reconfigurability:-

Digital programmable system allows flexibility in reconfiguring the digital signal processing operations simply by changing the program.

But in Analog systems, reconfiguration means redesign of hardware followed by testing & verification to see that it operates properly.

(2) Accuracy:-

Digital systems provides much better control of accuracy requirements. But it by specifying the accuracy requirements of A/D & Dsp's.

Tolerances in analog circuits components make it extremely difficult for the system designer to control the accuracy of Analog signal processing systems.

(3) Memory: Digital signals are easily stored on magnetic tapes or disk and it is transportable.

(4) Precise mathematical operations on signals in Dsp's are implemented using software. But it is difficult to perform the precise mathematical operation on signals which is in analog form.

(5) Dsp's are cheaper over the Analog signal

Applications of DSP's

(1) Speech processing

(2) Signal transmission on telephone channels.

(3) Image processing

(4) Seismology

(5) Geophysics in oil exploration.

(6) Detection of nuclear explosions.

(7) Outer space signal processing.

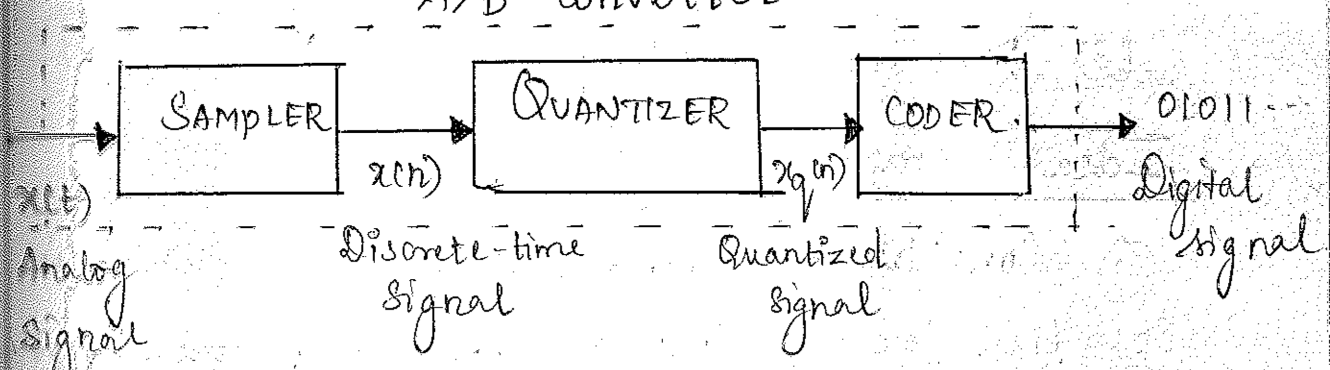
Limitations of DSP's.

(1) One of the practical limitation is the Speed of operation of A/D converters and DSP's.

(2) Signals having extremely wide bandwidths require fast-sampling rate A/D converters and fast DSP's.

SAMPLING THEOREM.

A/D Converter.



Sampling:- This is the conversion of a continuous-time signal (Analog signal) into a Discrete-time signal obtained by taking "Samples" of the continuous-time signal at discrete time instants. $x(t) \longrightarrow x(n)$

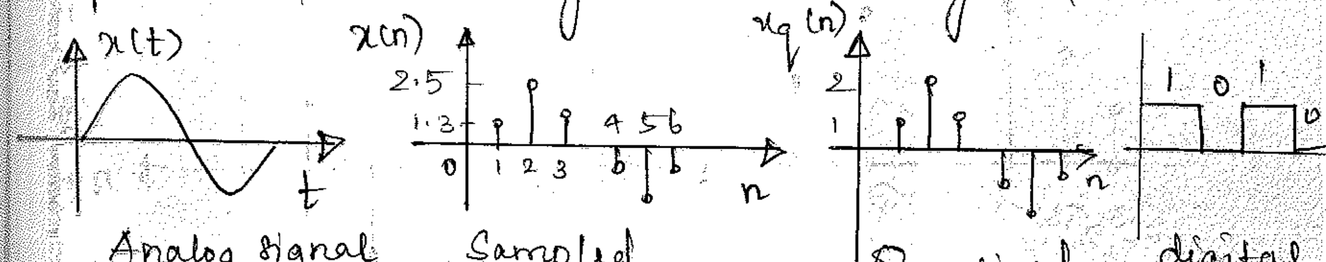
Quantization:- This is conversion of a discrete-time continuous valued signal into a discrete-time discrete valued signal. $x(n) \longrightarrow x_q(n)$.

Quantization Error:

Quantization error is the difference between the unquantized sample $x(n)$ and the quantized o/p $x_q(n)$ is called as quantization error.

$$\boxed{\text{Quantization error} = x(n) - x_q(n)}$$

Coding: In the coding process, each discrete value $x_q(n)$ is represented by a b-bit binary sequence.



Analog signal | Sampled signal (DTS) | Quantized signal | digital signal

Quantization can be performed by using either truncation (or) Rounding of the ...

Generally sampling can be performed in 3 Ways

(1) Ideal Sampling

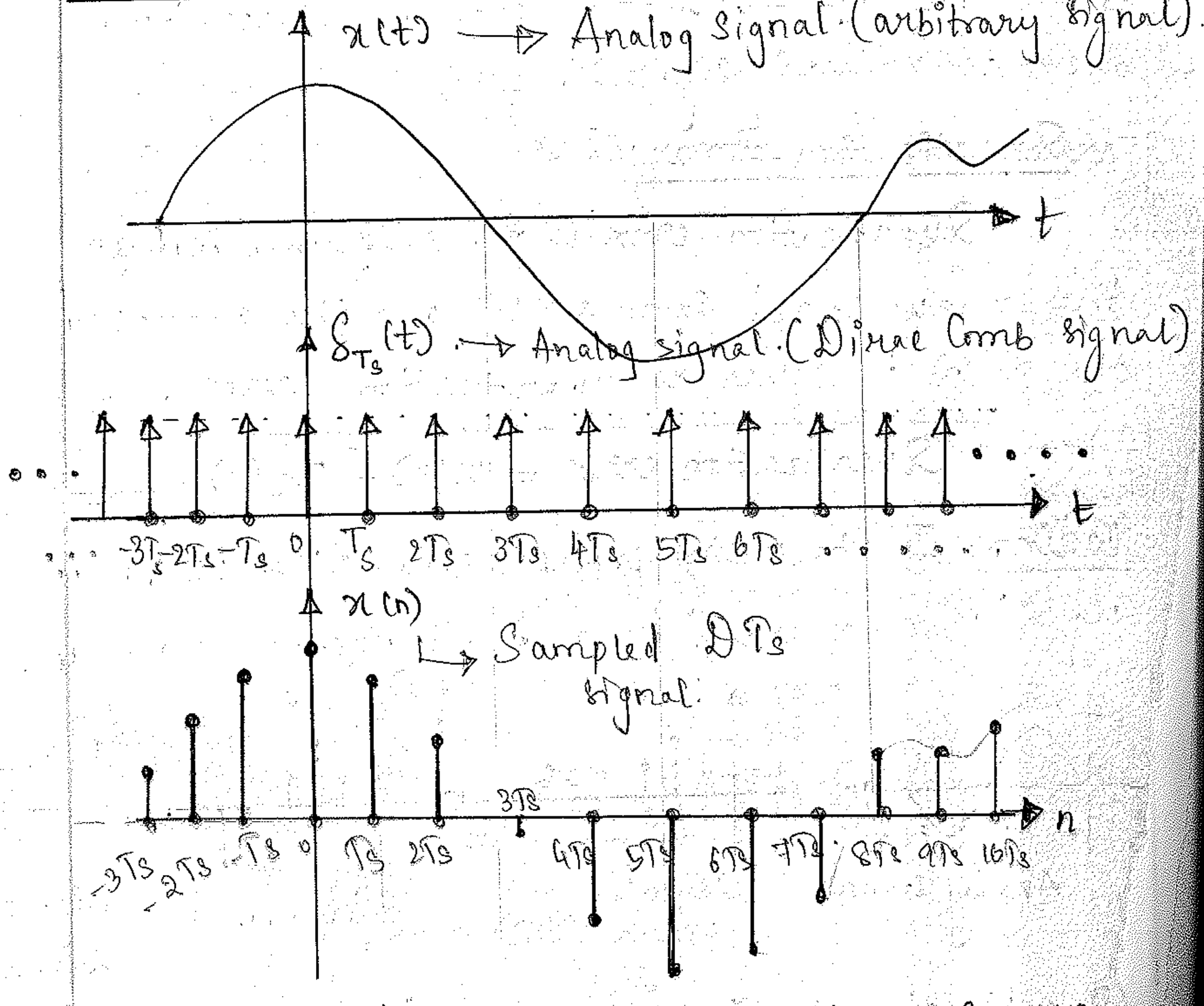
(2) Natural (or) practical Sampling

(3) Flat top Sampling.

Ideal Sampling:-

Consider an arbitrary signal $x(t)$ which is specified for all time as shown in figure. If we multiply this signal with an ideal sampling function (Dirac Comb), the resultant signal occurs for every T_s seconds which is known as ideal version of the sampled signal denoted by $x(n)$.

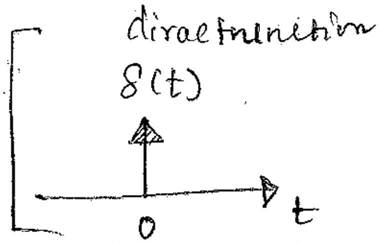
Graphical description:



Note:- T_s → Sampling period (or) Sampling

Mathematical description.

$$x(n) \triangleq x_s(t) = x(t) \times \delta_{T_s}(t)$$



WKT $\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$$x(n) \triangleq x_s(t) = x(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \longrightarrow (1)$$

Apply Fourier Transform on both sides of (1)

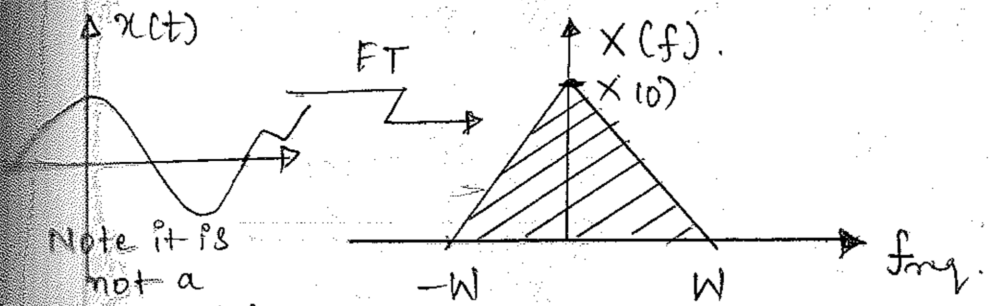
$$X_s(f) = X(f) * \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Spectrum of ideally sampled signal $X_s(f)$ now

$$X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s) \longrightarrow (2)$$

Assume the spectrum of $X(f)$ which is the fourier transform of the signal $x(t)$.

$$X(f) = FT(x(t)).$$



Note it is not a sinusoidal signal.

Some arbitrary signal.

$$\boxed{\text{Band Width of } x(t) = W.}$$

Note-

$x(t) = A \sin \omega t$

Case (i) WKT $T_s \rightarrow$ Sampling period (or) Sampling Interv (s)

then $f_s = \frac{1}{T_s} \rightarrow$ Sampling rate (or) Sampling frequency (samples/sec) (Hz).

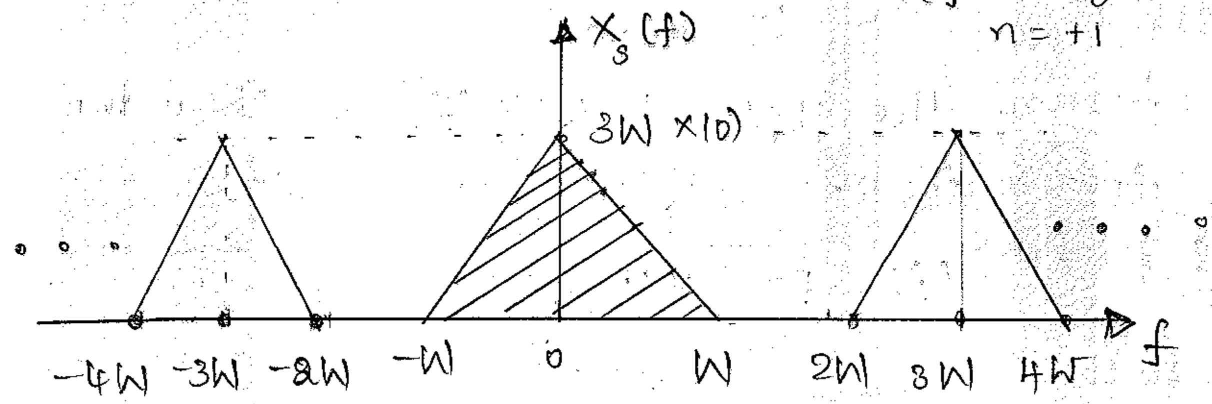
(i) $f_s > 2W$. i.e $f_s = 3W$.

↳ No loss of Message

(2) ↳ $X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - n f_s)$ — then (2) becom

$$X_s(f) = 3W \sum_{n=-\infty}^{\infty} X(f - n\,3W).$$

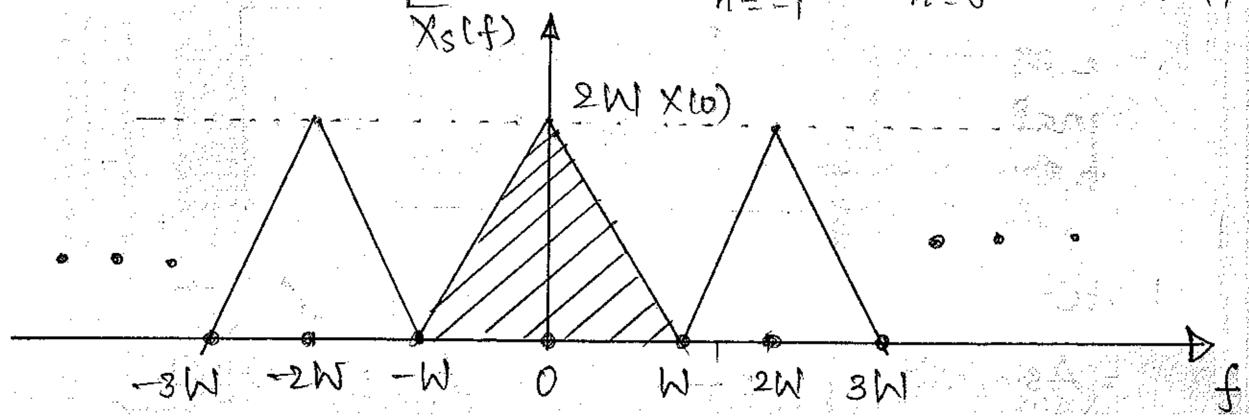
$$= 3W \Big[\cdots + \underbrace{X(f+3W)}_{n=-1} + \underbrace{X(f)}_{n=0} + \underbrace{X(f-3W)}_{n=+1} + \cdots$$



Case (ii) $f_s = 2W$. → No loss of Message.

then (2) becomes

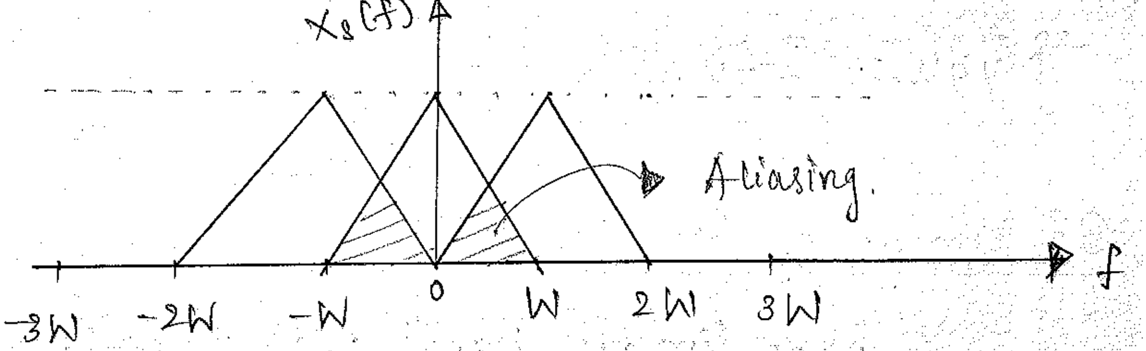
$$X_s(f) = 2W \Big[\cdots + \underbrace{X(f+2W)}_{n=-1} + \underbrace{X(f)}_{n=0} + \underbrace{X(f-2W)}_{n=+1} \cdots$$

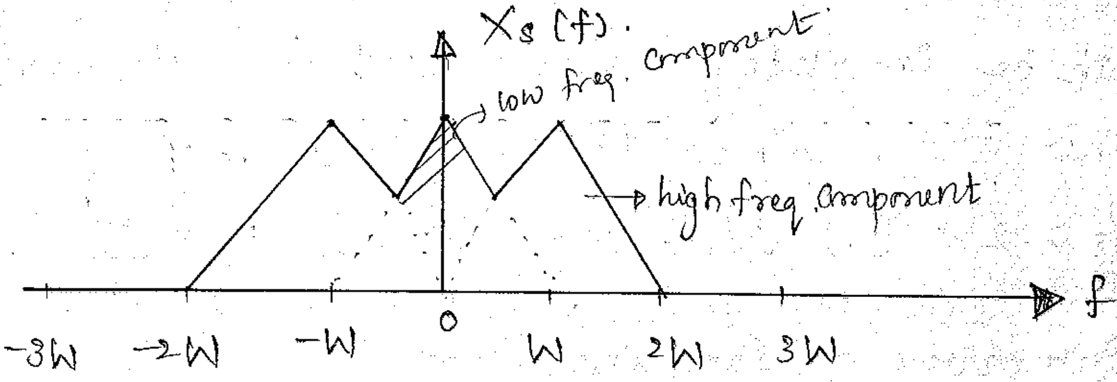


case (iii) $f_s < 2W$. i.e $f_s = W$.

then (2) becomes $\rightarrow$ loss of Message.

$$X_s(f) = W\left[\cdots + \underset{n=-1}{X(f+W)} + \underset{n=0}{X(f)} + \underset{n=+1}{X(f-W)} + \cdots\right]$$





Aliasing:- The phenomenon of high frequency component taken on the identity of low frequency component is called aliasing.

Aliasing : Sampling rate $\left(\frac{\text{samples}}{\text{sec}}\right)$ (or) Sampling frequency (Hz) < 2 Msg Bandwidth (Hz)

$\boxed{\text{Aliasing: } f_s < 2W}$

How to avoid aliasing:-

Sampling frequency should be greater than or equal to twice of the message bandwidth.

$$f_s \geq 2W \;;\; T_s \leq \frac{1}{2W}$$

Nyquist rate = Minimum sampling frequency to avoid aliasing.

Nyquist Interval = Minimum Sampling period (or) Sampling Interval to avoid aliasing.

Nyquist Interval $= \frac{1}{2W}$.

Statement of Sampling theorem:-

A band limited signal $x(t)$ of finite energy can be completely reconstructed from its samples when the samples taken at the rate of $f_s \geq 2W$ (or) Sampling interval $T_s \leq \frac{1}{2W}$.

Summary of Sampling theorem:-

$W \rightarrow$ Message Bandwidth (Hz) (or) Max msg freq.

(1) $T_s \rightarrow$ Sampling period (sec) (or) Sampling Interval (Sec).

(2) $f_s \rightarrow$ Sampling frequency (Hz) (or) Sampling Rate $\left(\frac{\text{Samples}}{\text{Sec}}\right)$.

(3) Aliasing: $f_s < 2W$.

(4) Avoid aliasing: $f_s \geq 2W$

(5) Nyquist rate = Minimum Sampling frequency to avoid aliasing
$= 2W$.

(6) Nyquist Interval $= \frac{1}{}$

Consider signals $x_1(t) = \cos 2\pi(10)t$

$x_2(t) = \cos 2\pi(50)t$ Which are

sampled at a rate of 40 Hz. Find the discrete time signals or sequences of $x_1(t)$ & $x_2(t)$.

Given $x_1(t) = \cos 2\pi(10)t$; $f_s = 40\,Hz$.

$x_2(t) = \cos 2\pi\, 50t$

WKT $x(t) = A\cos\omega t = A\cos 2\pi f t$.

then $x_1(t) = \cos 2\pi(10)t \longrightarrow f_1 = 10\,Hz$

$x_2(t) = \cos 2\pi(50)t \longrightarrow f_2 = 50\,Hz$.

To get or to obtain DTs of $x_1(t)$ & $x_2(t)$

To get $x(n)$ put $t = nT_s$ in $x(t)$.

then $x_1(n) = \cos 2\pi(10)nT_s$ WKT $f_s = 40\,Hz$

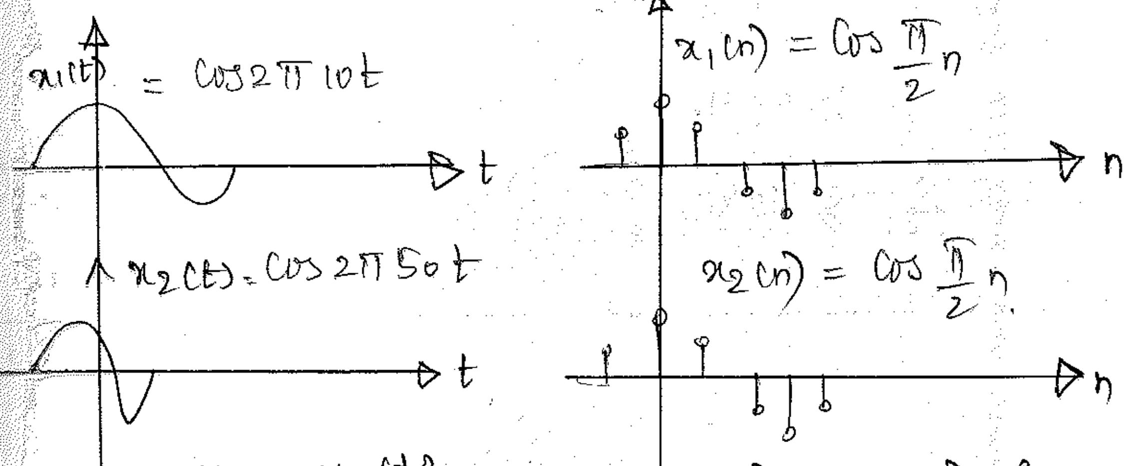
$x_2(n) = \cos 2\pi(50)nT_s$ $T_s = \frac{1}{40}$ sec.

$$x_1(n) = \cos \cancel{2}\pi\left(\frac{10}{\cancel{40}_{2}}\right)n = \cos\frac{\pi}{2}n$$

$$x_2(n) = \cos\cancel{2}\pi\left(\frac{50}{\cancel{40}_{2}}\right)n = \cos\frac{5\pi}{2}n. = \cos\left(2\pi n + \frac{\pi n}{2}\right)$$

$$= \cos\frac{\pi}{2}n.$$

$$\left[\cos(2\pi n + \theta) = \cos\theta\right]$$



$x_1(t)$ & $x_2(t)$ ... $x_1(n) = x_2(n)$ equal

From the problem the given signals are different in continuous-time domain but its corresponding discrete-time representations are same.

WKT Inorder to do sampling i.e. $x(t) \longrightarrow x(n)$

the $f_s \geq 2W$.

for $x_1(t) \rightarrow W = 10\,Hz$ then $f_{s_1} \geq 20\,Hz$

$x_2(t) \rightarrow W = 50\,Hz$ then $f_{s_2} \geq 100\,Hz$

But in question/problem $f_s = 40\,Hz$ (given).

for $x_1(t)$ f_s is greater than f_{s_1} $[f_s > f_{s_1}]$

for $x_2(t)$ f_s is smaller than f_{s_2} $[f_s < f_{s_2}]$

thats the reason why both sampled version of $x_1(t)$ & $x_2(t)$ are same.

* What is the choice of sampling frequency so that $x_1(t)$ & $x_2(t)$ samples will be different ??

By considering $f_s \geq 100\,Hz$

(or)

By considering individual sampling frequency of $x_1(t)$ & $x_2(t)$

$f_{s_1} \geq 20\,Hz$

$f_{s_2} \geq 100\,Hz$.

Consider the analog signal $x_a(t) = 3\cos 100\pi t$

(a) Determine the minimum Sampling rate required to avoid aliasing

(b) Suppose that the signal is sampled at the rate $F_s = 200\,Hz$. What is the discrete-time signal obtained after sampling?

(c) Suppose that the signal is sampled at the rate $F_s = 75\,Hz$. What is the discrete-time signal obtained after sampling?

(d) Given $x_a(t) = 3\cos 100\pi t$; $\cancel{2\pi} f = \cancel{100\pi}\,50$

$f = 50\,Hz$.

(a) $f_s = 2W = 2 \times 50 = 100\,Hz$. ; $f_s = 100\,Hz$.

(b) $f_s = 200\,Hz$

check the given f_s is Correct (or) not means it leads to aliasing (or) not.

Nyquist rate $= 100\,Hz$

$f_s = 200\,Hz$ is greater than Nyquist rate

So $x_a(t) = 3\cos 100\pi t$ is sampled by $f_s = 200\,Hz$ won't lead to aliasing.

$x_a(n) = 3\cos \frac{1\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}}\pi n = 3\cos\frac{\pi}{2}n$. [$x_a(n)$ by put nT_s in $x_a(t)$]

(c) $f_s = 75\,Hz$ < Nyquist sampling rate

So it leads to aliasing.

but

$x_a(n) = 3\cos\frac{\cancel{100}\,4}{\cancel{75}\,3}\pi n = 3\cos\frac{4\pi n}{3}$

$= 3\cos\left(2\pi n - \frac{2\pi}{3}n\right)$

WKT $\cos(2\pi n + \theta) = \cos\theta$

$$3\cos\left(2\pi n - \frac{2\pi}{3}n\right) = 3\cos\left(2\pi n + \left(\frac{-2\pi}{3}n\right)\right)$$

$$= 3\cos\left(\frac{-2\pi}{3}n\right) = 3\cos\left(\frac{2\pi}{3}\right)n$$

WKT

$\cos(-\theta) = \cos\theta$.

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(3) Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

Find Nyquist rate for this signal.)

→ Given

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

$2\cancel{\pi}f_1 = 50\cancel{\pi}$ $\quad$ $2\cancel{\pi}f_2 = 300\cancel{\pi}$ $\quad$ $2\cancel{\pi}f_3 = 100\cancel{\pi}$

$f_1 = 25\,Hz$ $\quad$ $f_2 = 150\,Hz$ $\quad$ $f_3 = 50\,Hz$

Nyquist rate $= 2 \times$ Max. Msg. frequency.

$= 2 \times 150\,Hz$

$= 300\,Hz$.

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(4) Consider the analog signal

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12{,}000\pi t$$

(a) What is the Nyquist rate for this signal.

(b) Assume now that we sample this signal using a sampling rate $F_s = 5000$ Samples/sec. What is the discrete-time signal obtained after sampling

(c) What is the analog signal $y_a(t)$ that we can reconstruct from the samples if we use ideal interpolation.

Given

$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12{,}000\pi t$

$2\pi f_1 = 2000\pi \qquad 2\pi f_2 = 6000\pi \qquad 2\pi f_3 = 12{,}000\pi$

$f_1 = 1000\,Hz \qquad f_2 = 3000\,Hz \qquad f_3 = 6{,}000\,Hz$

(a) Nyquist rate $= 2 \times$ Max. Msg frequency.

$= 2 \times 6000$

$= 12{,}000\,Hz$

(b) $f_s = 5000\,Hz$

Given f_s is less than Nyquist rate of $x_a(t)$

So if we do sampling with $f_s = 5000\,Hz$ it leads to aliasing.

$x_a(n) = 3\cos \frac{2000}{5000}\pi n + 5\sin \frac{6000}{5000}\pi n + 10\cos \frac{12{,}000}{5000}\pi n.$

$= 3\cos \frac{2\pi}{5}n + 5\sin \frac{6\pi}{5}n + 10\cos \frac{12\pi}{5}n.$

$= 3\cos \frac{2\pi}{5}n + 5\sin\left(2\pi n - \frac{4\pi n}{5}\right) + 10\cos\left(2\pi n + \frac{2\pi n}{5}\right)$

$= 3\cos \frac{2\pi}{5}n + 5\sin\left(-\frac{4\pi}{5}n\right) + 10\cos \frac{2\pi n}{5}$

$= 3\cos \frac{2\pi}{5}n - 5\sin \frac{4\pi}{5}n + 10\cos \frac{2\pi}{5}n$ $\quad [\because \sin(-\theta) = -\sin\theta]$

$\boxed{x_a(n) = 13\cos \frac{2\pi}{5}n - 5\sin \frac{4\pi}{5}n}.$

(c) Reconstruct $x_a(t)$ from $x_a(n)$ by using $f_s = 5000\,Hz$.

$\cancel{x(t) = 13\cos \frac{2\pi}{5}(}$

1. $x(t) \rightarrow x(n)$; put $t = nT_s$ in $x(t)$

$x_a(n) = 13 \cos \frac{2\pi}{5} n - 5 \sin \frac{4\pi}{5} n.$

to get $x(t)$ put $n = \frac{t}{T_s} = t f_s$ in $x_a(n)$

So

$$x_a(t) = 13 \cos \frac{2\pi}{\cancel{5}} \times \overset{1000}{\cancel{5000}} t - 5 \sin \frac{4\pi}{\cancel{5}} \times \overset{1000}{\cancel{5000}} t$$

$$x_a(t) = 13 \cos 2000\pi t - 5 \sin 4000\pi t$$

Note: The reconstructed $x_a(t)$ from $x_a(n)$ by using $f_s = 5000\,Hz$, is not the exact signal $x_a(t)$ given.

Reason, the $f_s = 5000\,Hz$ is less than the Nyquist rate so it leads to aliasing.

$$\underset{\text{reconstructed}}{x_a(t)} \neq x_a(t) = 2\cos 2000\pi t + 5\sin 6000\pi t + 10 \cos 12{,}000\pi t$$

$13 \cos 2000\pi t - 5 \sin 4000\pi t$

\# What is the right choice of f_s so that $x_a(t)$ can be reconstructed from $x_a(n)$.

$\rightarrow$ (1) firstly given $x_a(t)$ has to be sampled by using $f_s \geq 12000\,Hz$

(2) The obtained $x_a(n)$ after sampling, can be reconstructed to $x_a(t)$ by using $f_s \geq 12000$

then $\underset{\text{(reconstructed)}}{x_a(t)} = \underset{\text{(given)}}{x_a(t)}$.

Formulae:

(1) $\cos(2\pi n + \theta) = \cos\theta$

(4) $\sin(2\pi n + \theta) = \sin\theta$

(5) $\sin(2\pi n - \theta) = -\sin\theta$