Practice Problems

University Theory Questions

Unit III – Partial Differentiation

Sr. No.	Partial Differentiation	Year
1	If $u = \log\left(\sqrt{x^2+y^2+z^2}\right)$ show that $(x^2+y^2+z^2)\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] = 1$	(Dec 2012)
2	Verify $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$ for $u = \tan^{-1}\left[\frac{y}{x}\right]$	(Dec 2013)
3	If $u = (x^2-y^2)f(xy)$ then show that $u_{xx}+u_{yy} = (x^4-y^4)f''(xy)$	(Dec 2013)
4	If $u = r^m$ where $r = \sqrt{x^2+y^2+z^2}$ then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$	(May 2013)
5	Find the value of n such that $u = x^n(3\cos^2 y - 1)$ satisfies the partial differential equation $\frac{\partial}{\partial x}\left(x^2\frac{\partial u}{\partial x}\right)+\frac{1}{\sin y}\frac{\partial}{\partial y}\left(\sin y\frac{\partial u}{\partial y}\right) = 0$	(May 2014)
6	$u = \tan(y+ax)+(y-ax)^{\frac{3}{2}},$ where a is constant, then show that, $\frac{\partial^2 u}{\partial x^2} = a^2\frac{\partial^2 u}{\partial y^2}$.	(Dec 2015) (May 2019)
7	Find the value of n for which z= $Ae^{-gx}\sin(nt-gx)$ satisfies the partial differential equation $\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2}$	(May 2015)
8	If $u = \log(x^3+y^3+z^3-3xyz)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$	(May 2016)
9	If $u = \log(x^3+y^3-x^2y-xy^2)$ prove that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy} = -3$	(Dec 2014)
10	If $u = (x^2-y^2)f(xy)$ then show that $u_{xx}+u_{yy} = (x^4-y^4)f''(xy)$	(Dec 2014)
11	If $T = \sin\left(\frac{xy}{x^2+y^2}\right)+\sqrt{x^2+y^2}+\frac{x^2y}{x+y}$, find the value of $x\frac{\partial T}{\partial x}+y\frac{\partial T}{\partial y}$.	(May 2017)
12	Find the value of n for which $z = t^n e^{-r^2/4t}$ satisfies the partial differential equation: $\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial z}{\partial r}\right)\right] = \frac{\partial z}{dt}$.	(May 2017)
13	If $u = \log(x^3+y^3-x^2y-xy^2)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$	(Dec 2018)

Sr. No.	Euler's Theorem on Homogeneous Function	Year
1	If $u = \frac{x^3+y^3}{y\sqrt{x}} + \frac{1}{x^7}\sin^{-1}\left(\frac{x^2+y^2}{2xy}\right)$ then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ at point (1,1)	(Dec 2013)
2	Verify Euler's theorem for homogeneous functions $f(x,y,z) = 3x^2yz + 5xy^2z + 4z^4$	(Dec 2013)
3	If $u = \sin(\sqrt{x}+\sqrt{y})$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x}+\sqrt{y})\cos(\sqrt{x}+\sqrt{y})$	(May 2013)
4	If $u = \sin^{-1}\sqrt{x^2+y^2}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$	(May 2013)
5	If $u = x^8 f\left(\frac{y}{x}\right) + \frac{1}{y^8}\emptyset\left(\frac{x}{y}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 64u$	(May 2014)
6	If $u = \cos^{-1}\left[\frac{x^3y^2+4y^3x^2}{\sqrt{x^4+6y^4}}\right]$, find the value of (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$	(May 2014)
7	If $u = x^3 f\left(\frac{y}{x}\right) + \frac{1}{y^3}\emptyset\left(\frac{x}{y}\right)$ Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 9u$	(Dec 2015)
8	If $u = tan^{-1}\left[\frac{x^3+y^3}{x+y}\right], then\ prove\ that$ $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u[1 - 4\sin^2 u]$.	(Dec 2015)
9	If $u = \frac{x^4+y^4}{x^2y^2} + x^6\tan^{-1}\frac{x^2+y^2}{x^2+2xy}$ find value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at x=1,y=2.	(May 2015)
10	If $u = \sin^{-1}(\frac{x+y)}{\sqrt{x}+\sqrt{y}})$ Prove that $x^2(\frac{\partial^2 u}{\partial x^2}) + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u)$	May 2015)

11	If $z = x^8 f\left(\frac{y}{x}\right) + y^{-8}\emptyset\left(\frac{x}{y}\right)$ prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\,\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 64z + 8y^{-8}\emptyset\left(\frac{x}{y}\right) - 8x^8 f\left(\frac{y}{x}\right)$	**(May 2016)**
12	If $u = \sec^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-1}{4}\cot u[3 + \cot^2 u]$	**(May 2016)**
13	If $u = \frac{x^2y^2z^2}{x^2+y^2+z^2} + \cos\left(\frac{xy+yz}{x^2+y^2+z^2}\right)$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$	**(Dec 2012)**
14	If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$	**(Dec 2012)** **(May 2018)**
15	If $u = cos\left[\frac{xy}{x^2+y^2}\right] + \sqrt{x^2+y^2} + \frac{xy^2}{x+y}$ then find the value of $xu_x + yu_y$ at point (3, 4)	**(Dec 2014)**
16	If $u = \tan^{-1}\left(\frac{\sqrt{x^3+y^3}}{\sqrt{x}+\sqrt{y}}\right)$, then show that : $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u\cos u.$	**(May 2017)**
17	If $u = \frac{x^3+y^3}{x+y} + \frac{1}{x^5}sin^{-1}\left[\frac{x^2+y^2}{x^2+2xy}\right]$ Find the value $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ at the point (1,2).	**(May 2018)**
18	If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u\cos 2u}{4\cos^3 u}$	**(Dec 2018)**
19	If $x = e^u tanv$ and $y = e^u secv$, find the value of $\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right].\left[x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right]$	**(Dec 2018)**
20	If $u = \sin^{-1}(x^2+y^2)^{1/5}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{5}\tan u\left[\frac{2}{5}\tan^2 u - \frac{3}{5}\right]$	**(May 2019)**
21	If $f(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{logx-logy}{x^2+y^2}$ than prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$	**(May 2019)**

Sr. No.	Variables to be treated as Constant	Year
1	If $x = u\tan v, y = u\sec v$ prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$	(Dec 2013)
2	If $x = \frac{\cos\theta}{u}$ $y = \frac{\sin\theta}{u}$ then Evaluate $\left(\frac{\partial x}{\partial u}\right)_\theta \left(\frac{\partial u}{\partial x}\right)_y + \left(\frac{\partial y}{\partial u}\right)_\theta \left(\frac{\partial u}{\partial y}\right)_x$	(May 2013)
3	If $x = r\cos\theta$, $y = r\sin\theta$ then prove that (i) $\left(\frac{\partial y}{\partial r}\right)_x \left(\frac{\partial y}{\partial r}\right)_\theta = 1$ (ii) $\left(\frac{\partial x}{\partial \theta}\right)_r = r^2 \left(\frac{\partial \theta}{\partial x}\right)_y$	(May 2014)
4	$u = mx + ny,\ v = nx - my$, where m, n are constants, then find the value of $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial x}{\partial u}\right)_v \cdot \left(\frac{\partial v}{\partial y}\right)_u$	(Dec 2015)
5	If $x = \frac{\cos\theta}{r},\ y = \frac{\sin\theta}{r}$ find the value of $\left(\frac{\partial x}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial x}\right)_y + \left(\frac{\partial y}{\partial r}\right)_\theta \left(\frac{\partial y}{\partial r}\right)_\theta \left(\frac{\partial r}{\partial y}\right)_x$	(May 2015)
6	If $ux + vy = 0, \frac{u}{x} + \frac{v}{y} = 1$ prove that $\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{y^2+x^2}{y^2-x^2}$	(May 16,18) (Dec 12,14)
7	If $x^2 = a\sqrt{u} + b\sqrt{v}$ and $y^2 = a\sqrt{u} - b\sqrt{v}$ where a and b are constants, prove that : $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$.	(May 2017)
8	If $z = f(x, y)$, where $x = u\cos\alpha - v\sin\alpha,\ y = u\sin\alpha - v\cos\alpha$, where α is the constant, show that: $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$.	(May 2017)
9	If $u = 2x + 3y,\ v = 3x - 2y$ find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$.	(May 2018)
10	If $x^2 = au + bv$ and $y^2 = au - bv$ where a and b are constants, prove that : $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$.	(Dec 2018) (May 2019)

Sr. No.	Composite Functions	Year
1	$x = u + v + w, y = uv + uw + vw, z = uvw$. and F is function of x, y, z then prove that $x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z} = u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w}$	(Dec 2013)
2	If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$ then prove that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$	(May 2013) (May 2019)
3	If $z = f(x, y)$ and $x = r\cosh\theta$ and $y = r\sinh\theta$ then show that $(x - y)(z_x - z_y) = rz_r - z_\theta$	(May 2013)
4	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$	(May 2014)
5	If $u = x^2 - y^2, v = 2xy$ and $z = f(u, v)$, then show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2}\frac{\partial z}{\partial u}$	(Dec 2012)
6	If u = f (x-y, y-z, z-x), then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.	(Dec 2015)
7	$Z = f(x, y),\ x = \frac{\cos u}{v},\ y = \frac{\sin u}{v}$. Then P.T $v\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}$.	(Dec 2015)
8	If $z = f(u, v)$ and $u = \log(x^2 + y^2)$ $v = \frac{y}{x}$ Show that $x\frac{\partial z}{\partial y} - y\frac{\partial z}{\partial x} = (1 + v^2)\frac{\partial z}{\partial v}$	(May 2015)
9	If z= $f(u,v)$ and $u = x\cos t - y\sin t, v = x\sin t + y\cos t$ where t is constant prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}$	(May 2015)
10	If $x = r\cos\theta, y = r\sin\theta$, where r and θ are functions of t, Then prove that : $x\frac{dy}{dt} - y\frac{dx}{dt} = r^2\frac{d\theta}{dt}$	(Dec 2012)
11	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ find the value of $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$	(May 2016)
12	If $\emptyset = f(x, y, z)$ $x = \sqrt{vw}$ $y = \sqrt{uw}$ $z = \sqrt{uv}$ prove that $x\frac{\partial\emptyset}{\partial x} + y\frac{\partial\emptyset}{\partial y} + z\frac{\partial\emptyset}{\partial z} = u\frac{\partial\emptyset}{\partial u} + v\frac{\partial\emptyset}{\partial v} + w\frac{\partial\emptyset}{\partial w}$	(May 2016)

13	If $x = u + v + w$, $y = uv + vw + uw$, $z = uvw$ and $\emptyset$ is function of x, y, z. Then prove that $u\frac{\partial \emptyset}{\partial u} + v\frac{\partial \emptyset}{\partial v} + w\frac{\partial \emptyset}{\partial w} = x\frac{\partial \emptyset}{\partial x} + 2y\frac{\partial \emptyset}{\partial y} + 3z\frac{\partial \emptyset}{\partial z}$	**(Dec 2014)**
14	If $x = \frac{\cos\theta}{u}$, $y = \frac{\sin\theta}{u}$ then prove that $u\frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}$	**(Dec 2014)**
15	If $u=x^2-y^2$, $v=2xy$ and $z=f(u, v)$, then show that : $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2+v^2}\frac{\partial z}{\partial u}$.	**(May 2017)** **(May 2018)**
16	If z= $f(r)$ where $r = \sqrt{x^2+y^2}$ Then Prove that $u_{xx} + u_{yy} = f''(r) + \frac{1}{r}f'(r)$	**(May 2019)**
17	If $z = f(x, y)$ where $x = u + v, y = uv$, then prove that $u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y}$	**(May 2018)**
18	If $v = f(e^{x-y}, e^{y-z}, e^{z-x})$ then show that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$	**(Dec 2018)**