Practice Questions

University Theory Questions Unit-IV – Application of Partial Differential Equation

| Sr. No | Jacobian | Year |
|--------|---|--------------------------|
| 1 | If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ | (Dec 2013) |
| 2 | If $u = x(1 - y) \& v = xy$ find $\frac{\partial(x,y)}{\partial(u,v)}$ | (Dec 2012) |
| 3 | If $x = uv$ and $y = \frac{u+v}{u-v}$ find $\frac{\partial(u,v)}{\partial(x,y)}$ | (May 2013) (May 2019) |
| 4 | If $x = e^{v}$ secu, $y = e^{v}$ tanu find $\frac{\partial(u,v)}{\partial(x,y)}$ | (May 2014) |
| 5 | If $x = rsin\theta cos\emptyset$, $y = rsin\theta sin\emptyset$, $z = rcos\theta$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$ | (Dec 2014) |
| 6 | If $u = \frac{y-x}{1+xy}$, $v = \tan^{-1} y - \tan^{-1} x$ find $\frac{\partial(u,v)}{\partial(x,y)}$ | (Dec 2015) |
| 7 | If $x = uv$ $y = \frac{u+v}{u-v}$ find $\frac{\partial(u,v)}{\partial(x,y)}$ | (May 2016) |
| 8 | If $x = v^2 + w^2$, $y = w^2 + u^2$, $z = u^2 + v^2$ Prove that JJ'=1 | (May 2017) |
| 9 11 | $fux = yz, vy = zx, wz = xy \text{ find } \frac{\partial(u,v,w)}{\partial(x,y,z)}$ | (Dec 2018) |

| Sr. No | Jacobian- Functional Dependency/Independency | Year |
|--------|--|--------------------------|
| 1 | Examine for functional dependence for $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$ | (Dec 2012) |
| 2 | Examine the functional dependence of $u = \sin^{-1} x + \sin^{-1} y$ $v = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}$ | (May 2013, Dec 2014) |
| 3 | Examine for functional dependence for $u=x+y+z$, $v=x^2+y^2+z^2$, $w=x^3+y^3+z^3-3xyz$ | (Dec 2013) |
| 4 | Examine for functional dependence for $u = \frac{x}{y-z} \ , v = \frac{y}{z-x} \ , w = \frac{z}{x-y}$ | (May 2014) |
| 5 | Determine whether the following functions are functionally dependent. If functionally dependent, find the relation between them: $u = \sin x + \sin y$, $v = \sin(x + y)$ | (May 2015) |
| 6 | Prove that the functions $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$ are functionally dependent and find the relation between them. | (Dec 2015) (Dec 2018) |
| 7 | Examine whether; $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ are functionally dependent; if so find the relation between them. | (May 16,Dec |
| 8 | Examine for functional dependence : $u = \frac{x-y}{x+z}$, $v = \frac{x+z}{y+z}$. | (May 2017) |
| | If $u=x+y+z$, $v=x^2+y^2+z^2$, $w=xy+yz+zx$. Examine whether u, v, w are functionally dependent. If so find the relation between them. | (May 2018) (May 2019) |

Partial Derivative

| Sr. No | Jacobian of Implicit Function | Year |
|--------|---|--------------------------|
| 1 | If $u^2 + xv^2 = x + y$, $v^2 + yu^2 = x - y$ Find $\frac{\partial v}{\partial y}$ | (Dec 2013) |
| 2 | If $x = u^2 - v^2$, $y = uv$ find $\frac{\partial u}{\partial x}$ | (Dec 2012) |
| 3 | If $u^2 + xv^2 - uxy = 0$ and $v^2 - xy^2 + 2uv + u^2 = 0$ find $\frac{\partial u}{\partial x}$ | (May 2013) |
| 4 | If $x = \cos\theta - r\sin\theta$, $y = \sin\theta + r\cos\theta$ find $\frac{\partial r}{\partial x}$ | (May 2014) |
| 5 | If $u^2 + xv^2 - uxy = 0$, $v^2 - xy^2 + 2uv + u^2 = 0$ find $\frac{\partial u}{\partial x}$ | (Dec 2014) |
| 6 | If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$, find $\frac{\partial(u,v)}{\partial(x,y)}$ | (May 2015) |
| 7 | If $u + v^2 = x$, $v + w^2 = y$, $w + y^2 = z$ find $\frac{\partial u}{\partial x}$. | (Dec 2015) |
| 8 | If $x = u + v$; $y = v^2 + w^2$; $z = w^3 + u^3$ then show that $\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}$ | (May 2015) |
| 9 | If $u+v=x^2+y^2, u-v=x+2y$ find $\frac{\partial u}{\partial x}$ treating y constant. | (May 2017) |
| 10 | If $u = x + y^2$; $v = y + z^2$; $w = z + x^2$ find $\frac{\partial x}{\partial u}$ | (May 2018) (Dec 2018) |
| 11 | If $u = x + y + z$; $v = x^2 + y^2 + z^2$; $w = x^3 + y^3 + z^3$ find $\frac{\partial x}{\partial u}$ | (May 2019) |

| Sr. No | Errors & Approximation | Year |
|--------|---|--------------------------|
| 1 | Find the percentage error in computing the parallel resistance 'r' of 2 resistances $r_1 \& r_2$ from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$, where both $r_1 \& r_2$ have percentage error of 2% each respectively. | (Dec 2012) |
| 2 | The resistance R of a circuit was calculated using the formula I = E/R. If there is an error of 0.1 Ampere in reading I and 0.5 Volts in reading E, Find the corresponding % error in R when I=15Ampere and E = 100Volts. | (Dec 2013) |
| 3 | If the kinetic energy is given by $T = \frac{mv^2}{2}$ and m changes from 49 to 49.5 and v changes from 1600 to 1590, find approximate change in T. | (May 2013) |
| 4 | The resonant frequency in series electrical circuit is given by $f = \frac{1}{2\pi\sqrt{LC}}.$ If the measurement in L and C are in error by 2% and $-$ 1% respectively. Find the percentage error in f. | (May 2014) |
| 5 | The area of triangle ABC is calculated from the formula $\Delta = \frac{1}{2}bcsinA$ errors of 1%,2%, and 3% respectively are made in measuring b,c, A if the corrected value of A is 30^{0} find the percentage error in the calculated value of area of triangle. | (Dec 2014) (Dec 2018) |
| 6 | If $e^z = \sec x \cos y$ and errors of magnitude h and - h are made in estimating x and y, where x and y are found to be $\frac{\pi}{3}$. | (May 2015) |
| 7 | In estimating the cost of a pile of bricks measured $2m \times 15m \times 1.2m$ the top of the pile is stretched 1% beyond the standard length. If the count is 450 bricks pile cubic meter and bricks cost Rs 450 per thousand, find the approximate error in cost. | (Dec 2015) |
| 8 | Find the percentage error in the area of an ellipse when an error of 1% is made in measuring its major and minor axes | (May 2016) (May 2019) |
| 9 | A balloon is in the form of right circular cylinder of radius 1.5 cm and length 4m and is surrounded by hemispherical ends of the radius is increased by 0.01m and the length by 0.05m, find the % change in the volume of balloon | (May 2016) |
| 10 | Find the percentage error in computing the parallel resistance r of three resistances r_1 , r_2 , r_3 from the formula : $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1 , r_2 , r_3 are in error by 2% each | (May 2017) |
| 11 | In calculating the volume of a right circular core, errors of 2% and 1% are made in measuring the height and base respectively. Find the error in calculating volume. | (May 2018) |

| Sr. No | Maxima & Minima | Year |
|--------|--|--------------------------|
| 1 | Discuss the maxima and minima of $f(x,y) = xy + a^3 \left(\frac{1}{x} + \frac{1}{y}\right)$, $a > 0$ | (Dec 2012) |
| 2 | Find the extreme values of $f(x,y) = xy(a-x-y)$ | (May 2013, 2016) |
| 3 | Find all stationary values of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. Find maximum values of $f(x,y)$ at suitable point. | (Dec 2014) (May 2018) |
| 4 | Examine maxima and minima of the following function and find their extreme values: $x^2 + y^2 + 6x + 12$ | (May 2015) |
| 5 | Find the minimum value of $x^2 + y^2$ subject to the condition $ax + by = c$ | (Dec 2015) |
| 6 | Find stationary points of : $f(x,y) = x^3y^2(1-x-y)$ and find f_{\max} where it exists. | (May 2017) |
| 7 | Find the extreme values of $f(x,y) = 3x^2 - y^2 + x^3$. | (Dec 2018) |
| 8 | Find the extreme values of $f(x,y) = x^3 + y^3 - 3axy$, a>0 | (May 2019) |

| Sr. No | Lagrange Method of Undetermined Multipliers | Year |
|--------|---|--------------------|
| 1 | Find the points on the surface $z^2 = xy + 1$ nearest to the origin by using Lagrange's Method. | (Dec 2012) |
| 2 | Use the Lagrange's method to find the minimum distance from origin to the plane $3x + 2y + z = 12$ | (May13, 15 |
| 3 | Divide 24 into three parts such that, the continued product of the first, square of the second and cube of the third may be maximum. Use Lagrange's method. | (Dec 2013) |
| 4 | Use Lagrange's method to find stationary value of $u = \frac{x^2}{a^3} + \frac{y^2}{b^3} + \frac{z^2}{c^3} \text{where } x + y + z = 1$ | (May 2014) |
| 5 | As dimension of triangle ABC are varied ,show that the maximum value of cosA cosB cosC is obtained when the triangle is equilateral | (Dec 15,May 16) |
| 6 | Find the stationary points of : $T(x, y, z) = 8x^2 + 4yz - 16z + 600$ if the condition $4x^2 + y^2 + 4z^2 = 16$ | (May 2017) |
| 7 | Find the stationary value of $u = x^m y^n z^p$ under the condition $x + y + z = a$. | (May 2018) |
| 8 | Find the stationary value of $a^3x^2 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ | (Dec 2018) |
| 9 | Divide the number 120 into three parts so that the sum of their products taken two at a time shall be Maximum. | (May 2019) |