UNIT - V

12 Kinematics of Rectilinear Motion

Important Points to Remember

- **Displacement :** It is defined as the change in position. Displacement is also a vector quantity.

  Displacement = Final position – Initial position.

- **Distance travelled :** It is positive scalar quantity which represents the total length of the path covered by the particle.

- **Velocity :** If a particle has displacement Δx in a time interval Δt, then the average velocity is given by

$$v_{av} = \frac{\Delta x}{\Delta t}$$

- The instantaneous velocity, which is generally referred to as velocity is given by

$$v = \frac{dx}{dt}$$

- **Acceleration :** If the velocity of particle changes by Δv in a time interval Δt, the average acceleration is given by

$$a_{av} = \frac{\Delta v}{\Delta t}$$

- The instantaneous acceleration, generally referred to as acceleration is given by

$$a = \frac{dv}{dt} = v\,\frac{dv}{dx}$$

- Problems of variable acceleration, where functions relating any two of the four variables is given, are solved by using the basic definitions and then either differentiating or integrating the functions.

- Problems of constant acceleration are solved using the equations

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

and $$v^2 = u^2 + 2as$$

- The displacement during n^{th} second is given by

$$s_n - s_{n-1} = u + \frac{1}{2}a(2n-1)$$

- Problems of motion under gravity can be solved by using equations of constant acceleration where, $a = -g = -9.81 \text{ m/s}^2$. In these problems, upward quantities are taken positive and downward negative.

Motion Diagrams

- Area under *a-t* graph gives change in velocity.

$$v_2 - v_1 = \text{Area under } a\text{-}t \text{ graph}$$

- The slope of tangent on *a-t* graph, which represents rate of change of acceleration is known as jerk.
- Area under *v-t* graph gives change in position which is displacement.

$$x_2 - x_1 = \text{Area under } v\text{-}t \text{ graph.}$$

- Slope of tangent on *v-t* graph is $\frac{dv}{dt}$ which is acceleration.

$$a = \text{Slope of tangent of } v\text{-}t \text{ graph.}$$

- Slope of tangent on *x-t* graph gives velocity.

$$v = \text{Slope of tangent of } x\text{-}t \text{ graph}$$

- Position at time t_1 can be obtained from *a-t* graph using moment-area method $x_1 = x_0 + v_0 t_1$ + Moment of area under *a-t* graph to the left of t_1 about t_1

Moment of area = Area × Distance of centroid from the point about which moment is taken

- To obtain distance travelled, add magnitudes of all areas on *v-t* graph.

Q.1 The motion of a particle starting from rest is defined by $a = 10t - t^2$, where a is in m/s^2 and t is in seconds. Find the displacement before it starts in reverse direction of motion and velocity when acceleration changes its direction. **[SPPU : May-08]**

Ans. :

$$a = 10t - t^2 = \frac{dv}{dt}$$

$$\int_0^v dv = \int_0^t (10t - t^2)\, dt$$

$$[v]_0^v = \left[5t^2 - \frac{t^3}{3}\right]_0^t$$

$$\therefore \quad v = 5t^2 - \frac{t^3}{3} \qquad \text{— (1)}$$

$$v = \frac{dx}{dt}$$

$$\therefore \quad \frac{dx}{dt} = 5t^2 - \frac{t^3}{3}$$

$$\int_0^x dx = \int_0^t \left(5t^2 - \frac{t^3}{3}\right) dt$$

$$\therefore \quad x = \frac{5t^3}{3} - \frac{t^4}{12} \qquad \text{— (2)}$$

When particle reverses direction, $v = 0$

$$\therefore \quad 5t^2 - \frac{t^3}{3} = 0$$

$$5 - \frac{t}{3} = 0$$

$$\therefore \quad t = 15 \text{ s}$$

At $t = 15$ s , displacement is,

$$x = \frac{5 \times 15^3}{3} - \frac{15^4}{12}$$

$$\therefore \quad \boxed{x = 1406.25 \text{ m}} \qquad \text{...Ans.}$$

Ans. :

$$x = t^2 - (t-3)^3$$

$$v = \frac{dx}{dt} = 2t - 3(t-3)^2$$

$$a = \frac{dv}{dt} = 2 - 6(t-3)$$

a) For maximum velocity, $a = 0$

$$\therefore \quad 2 - 6(t-3) = 0$$

$$2 - 6t + 18 = 0$$

$$6t = 16$$

$$\therefore \quad \boxed{t = 3.33 \text{ s}}$$ ...Ans.

b) $$x_{3.33} = 3.33^2 - (3.33-3)^3$$

$$\therefore \quad \boxed{x_{2.667} = 11.05 \text{ m}}$$ ...Ans.

$$v_{max} = 2 \times 3.33 - 3(3.33-3)^2$$

$$\therefore \quad \boxed{v_{max} = 6.33 \text{ m/s}}$$ ...Ans.

c) To find distance travelled, put $v = 0$

$$\therefore \quad 2t - 3(t-3)^2 = 0$$

$$2t - 3(t^2 - 6t + 9) = 0$$

$$3t^2 - 20t + 27 = 0$$

$$\therefore \quad t = 1.88 \text{ s}, 4.786 \text{ s}.$$

$$x_0 = 0 - (0-3)^3 = 27 \text{ m}$$

$$x_{1.88} = 1.88^2 - (1.88-3)^3 = 4.94 \text{ m}$$

$$x_{4.786} = 4.786^2 - (4.786-3)^3 = 17.21 \text{ m}$$

$$x_{12} = 12^2 - (12-3)^3 = -585 \text{ m}$$

Distance travelled $= |x_{1.88} - x_0| + |x_{4.786} - x_{1.88}| + |x_{12} - x_{4.786}|$

$$= |4.94 - 27| + |17.21 - 4.94| + |-585 - 17.21|$$

$$= 22.06 + 12.27 + 602.21$$

$$\therefore \quad \boxed{\text{Distance travelled} = 636.54 \text{ m}}$$ ...Ans.

Q.4 A sphere is fired into a medium with an initial speed of 27 m/s. If it experiences a deceleration a = (– 6t) m / s² where t is in seconds, determine distance traveled before it stops.

[SPPU : Dec.-13, Marks 4]

Ans. :

$$a = -6t$$

$$a = \frac{dv}{dt}$$

$$\therefore \quad \frac{dv}{dt} = -6t$$

$$\int_{27}^{v} dv = -6\int_{0}^{t} t\,dt$$

$$v - 27 = -3t^2$$

$$\therefore \quad v = 27 - 3t^2$$

The time at which particle stops is obtained by substituting $v = 0$

$$\therefore \quad 0 = 27 - 3t^2$$

$$\therefore \quad t = 3\,s$$

As $\quad v = \frac{dx}{dt}$,

$$\frac{dx}{dt} = 27 - 3t^2$$

$$\int_{0}^{x} dx = \int_{0}^{3} (27 - 3t^2)\,dt$$

$$x = [27t - t^3]_0^3 = 27 \times 3 - 3^3$$

$$\therefore \quad \boxed{x = 54\text{ m}}$$

...Ans.

Q.5 A particle moves along a straight line with an acceleration a = (4t² – 2), where a is in m/s² and t is in s. When t = 0, the particle is at 2 m to the left of origin and when t = 2s the particle is at 20 m to left of origin. Determine the position of particle at t = 4 s.

[SPPU : May-14, Marks 4]

Ans. : $a = \frac{dv}{dt} = 4t^2 - 2$

$\therefore \quad \int dv = \int (4t^2 - 2)\, dt$

$v = \frac{4t^3}{3} - 2t + C_1$

$v = \frac{dx}{dt}$

$\therefore \quad \frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$

$\int dx = \int \left(\frac{4t^3}{3} - 2t + C_1 \right) dt$

$\therefore \quad x = \frac{t^4}{3} - t^2 + C_1 t + C_2$

At $\quad t = 0 \ , \ x = -2 \text{ m}$

$\therefore \quad -2 = C_2$

$\therefore \quad x = \frac{t^4}{3} - t^2 + C_1 t - 2$

At $\quad t = 2 \, s \ , \ x = -20 \text{ m}$

$\therefore \quad -20 = \frac{2^4}{3} - 2^2 + C_1 \times 2 - 2$

$\therefore \quad C_1 = -19.33$

$\therefore \quad x = \frac{t^4}{3} - t^2 - 19.33\, t - 2$

at $\quad t = 4 \, s,$

$\therefore \quad x_4 = \frac{4^4}{3} - 4^2 - 19.33 \times 4 - 2$

$\therefore \quad \boxed{x_4 = -9.987 \text{ m}}$ ...Ans.

Q.6 A particle moves along a straight line with an acceleration $a = (4t^3 - 2t)$, where a is in m/s^2 and t is in s. When t = 0, the particle is at 2 m to the left of origin and when t = 2s the particle is at 20 m to left of origin. Determine the position of particle at t = 4s. **[SPPU : Dec.-16, Marks 4]**

Ans. : $a = \frac{dv}{dt} = 4t^3 - 2t$

$$\int dv = \int (4t^3 - 2t)\,dt + c_1$$

$$v = t^4 - t^2 + c_1$$

$$v = \frac{dx}{dy} = t^4 - t^2 + c_1$$

$$\int dx = \int (t^4 - t^2 + c_1)\,dt + c_2$$

$$x = \frac{t^5}{5} - \frac{t^3}{3} + c_1 t + c_2 \qquad \ldots (1)$$

At t = 0, x = – 2 m

$$\therefore \quad -2 = c_2$$

At $t = 2$ s, $x = 20$ m

$$\therefore \quad 20 = \frac{2^5}{5} - \frac{2^3}{3} + c_1 \times 2 - 2$$

$$\therefore \quad c_1 = 9.133$$

$$\therefore \quad x = \frac{t^5}{5} - \frac{t^3}{3} + 9.133\,t - 2$$

At $t = 4$ s, $x = \frac{4^5}{5} - \frac{4^3}{3} + 9.133 \times 4 - 2$

$$\therefore \quad \boxed{x = 218 \text{ m}} \qquad \text{...Ans.}$$

Q.7 The acceleration of a particle as it moves along a straight line is given by a = (2t – 1) m/s², where t is in seconds. If s = 1 m and v = 2 m/s when t = 0, determine the particle velocity and position when t = 6 s. **[SPPU : May-17, Marks 4]**

Ans. : $a = \frac{dv}{dt} = 2t - 1$

$$\int dv = \int_0^t (2t-1)\,dt$$

$$v - 2 = t^2 - t$$

$$\therefore \quad v = t^2 - t + 2$$

$$\therefore \quad \frac{ds}{dt} = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\, dt$$

$$s - 1 = \frac{t^3}{3} - \frac{t^2}{2} + 2$$

$$\therefore \quad s = \frac{t^3}{3} - \frac{t^2}{2} + 3$$

For $t = 6$

$$v = 6^2 - 6 + 2$$

$$\therefore \quad \boxed{v = 32 \text{ m/s}}$$

$$s = \frac{6^3}{3} - \frac{6^2}{2} + 3$$

$$\therefore \quad \boxed{s = 57 \text{ m}}$$

...Ans.

Q.8 The motion of particle is defined by, $x = t^3 - 6t^2 + 9t + 5$, where x expressed in meter and t in seconds. Determine the time at which velocity becomes zero. Also determine velocity and acceleration at $t = 5$ s. **[SPPU : May-18, Marks 6]**

Ans. :

$$x = t^3 - 6t^2 + 9t + 5$$

$$v = \frac{dx}{dt} = 3t^2 - 12t + 9$$

$$a = \frac{dv}{dt} = 6t - 12$$

$$v = 0 \Rightarrow 3t^2 - 12t + 9 = 0$$

$$\therefore \quad t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$$t(t-3) - 1(t-3) = 0$$

$$(t-1)(t-3) = 0$$

$$\therefore \quad \boxed{t = 1\text{ s} \quad \text{or} \quad t = 3\text{ s}}$$

...Ans.

At $t = 5s$,

$v = 3\times5^2 - 12\times5 + 9$

$\therefore$ $\boxed{v = 24\ m/s}$ ...Ans.

$a = 6\times5 - 12$

$\therefore$ $\boxed{a = 18\ m/s^2}$ ...Ans.

Q.9 Three telegraph poles A, B and C are spaced at 50 m intervals along a straight road. A car starting from rest accelerates uniformly, passes post A and then takes 8 s to reach post B and further 7 s to reach post C. Calculate a) The acceleration of the car b) The velocity of the car at A, B, and C, c) The distance of the post A from the starting point of the car.

Ans. : Let O be the starting point of the car as shown in Fig. Q.9.1.

Let v_A, v_B and v_C be the velocities of car at A, B and C respectively, x be the distance of post A from the starting point and a = acceleration of car.

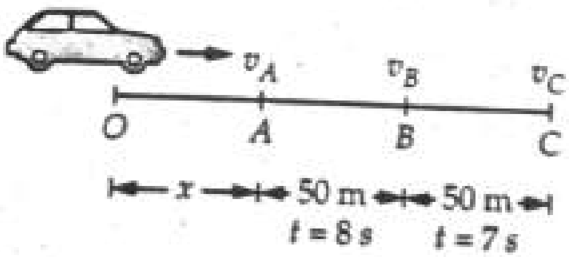

Fig. 13.4.1

For motion from A to B, using $s = ut + \frac{1}{2}at^2$,

$$50 = v_A \times 8 + \frac{1}{2}a \times 8^2 \quad \text{... (1)}$$

For motion from A to C,

$$100 = v_A \times 15 + \frac{1}{2}a \times 15^2 \quad \text{... (2)}$$

From equations (1) and (2),

$$\boxed{v_A = 5.774 \text{ m/s}}$$

$$\boxed{a = 0.119 \text{ m/s}^2}$$

For motion from A to B, using $v = u + at$,

$$v_B = 5.774 + 0.119 \times 8$$

$$\therefore \quad \boxed{v_B = 6.726 \text{ m/s}}$$

For motion from B to C,

$$v_C = 6.726 + 0.119 \times 7$$

$$\therefore \quad \boxed{v_C = 7.559 \text{ m/s}}$$

For motion from O to A, using $v^2 = u^2 + 2as$,

$$5.774^2 = 0^2 + 2(0.119)(x)$$

$$\therefore \quad \boxed{x = 140.08 \text{ m}}$$ ...Ans.

Q.10 A base ball is thrown downward from a 15 m tower with an initial speed of 5 m/s. Determine the speed at which it hits the ground and the time of travel. **[SPPU : May-11, 13, Marks 6]**

Ans. :

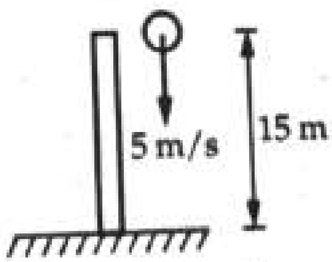

Fig. Q.10.1

$$u = 5 \text{ m/s}, \quad s = -15 \text{ m},$$

$$a = -9.81 \text{ m/s}^2$$

$$v^2 = u^2 + 2as$$

$$v^2 = (-5)^2 + 2(-9.81)(-15)$$

$$\therefore \quad \boxed{v = 17.87 \text{ m/s} \downarrow}$$

$$v = u + at$$

$$-17.87 = (-5) + (-9.81)t$$

$$\therefore \quad \boxed{t = 1.312 \text{ s}}$$

...Ans.

Q.11 A particle starts with an initial velocity of 2.5 m/s and uniformly accelerates at the rate 0.5 m/s^2. Determine the displacement in 2 s, time required to attain the velocity of 7.5 m/s and the distance travelled when it attain a velocity of 7.5 m/s. [SPPU : Dec.-12, Marks 6]

Ans. : $u = 2.5$ m/s, $a = 0.5$ m/s^2

$$s = ut + \frac{1}{2}at^2$$

At $t = 2$ s,

$$s = 2.5 \times 2 + \frac{1}{2} \times 0.5 \times 2^2$$

$$\therefore \quad \boxed{s = 6 \text{ m}}$$

$$v = u + at$$

For $v = 7.5$ m/s,

$$7.5 = 2.5 + 0.5 \times t$$

$$\therefore \quad \boxed{t = 10 \text{ s}}$$

$$v^2 = u^2 + 2as$$

$$7.5^2 = 2.5^2 + 2(0.5)(s)$$

$$\therefore \quad \boxed{s = 50 \text{ m}}$$

...Ans.

Q.12 A baseball is thrown downward from a 15 m tower with an initial speed of 5 m/s. Determine the speed at which it hits the ground and the time of travel. [SPPU : May-13, Marks 4]

Ans. : Taking downward direction positive as the complete motion is downward,

$u = 5$ m/s, $s = 15$ m, $a = 9.81$ m/s^2

$v^2 = u^2 + 2as$:

$$v^2 = 5^2 + 2 \times 9.81 \times 15$$

$$\therefore \quad \boxed{v = 17.87 \text{ m/s}}$$

$v = u + at$:

$$17.87 = 5 + 9.81\,t$$

$$\therefore \quad \boxed{t = 1.312 \text{ s}}$$

...Ans.

Q.13 A ball is projected vertically upward with a velocity of 9.81 m/s. Determine the maximum height travel by the ball, the velocity at which it strikes the ground and total time of journey.

[SPPU : Dec.-14, Marks 4]

Ans. : $u = 9.81$ m/s, $v = 0$, $a = -9.81$ m/s^2, $s = h$

$$v^2 = u^2 + 2as$$

$$0 = 9.81^2 + 2(-9.81)h$$

$$\therefore \quad \boxed{h = 4.905 \text{ m}}$$

... Ans.

When the ball strikes the ground, $s = 0$

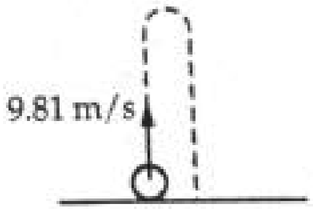

Fig. Q.13.1

$$s = ut + \frac{1}{2}at^2$$

$$0 = 9.81t + \frac{1}{2}(-9.81)t^2$$

$$\therefore \quad t = 2\ s$$

$$v^2 = u^2 + 2as = 9.81^2 + 0$$

$$\therefore \quad \boxed{v = 9.81 \text{ m/s} \downarrow}$$

...Ans.

Q.14 A car comes to complete stop from an initial speed of 50 kmph in a distance of 100 m. With the same constant acceleration, what would be the stopping distance s from an initial speed of 70 kmph. [SPPU : May-15, Marks 4]

Ans. :

$$v^2 = u^2 + 2as$$

$$v = 0, \; u = 50 \text{ km/h} = 50 \times \frac{5}{18} \text{ m/s}$$

$$s = 100 \text{ m}$$

$$0 = \left(50 \times \frac{5}{18}\right)^2 + 2a \times 100$$

$$\therefore \quad a = -0.9645 \text{ m/s}^2$$

For

$$u = 70 \text{ km/h} = 19.444 \text{ m/s}$$

$$0 = 19.444^2 + 2(-0.9645)\, s$$

$$\therefore \quad \boxed{s = 196 \text{ m}}$$ ...Ans.

Q.15 A stone is dropped from the top of a tower 50 m high. At the same time, another stone is thrown vertically upwards from the foot of tower with a velocity of 25 m/s. When and where the two stone cross each other ? [SPPU : Dec.-15, Marks 4]

Sol. : The conditions for the two stones A and B are shown in Fig. Q.15.1. Let t be the time when they meet.

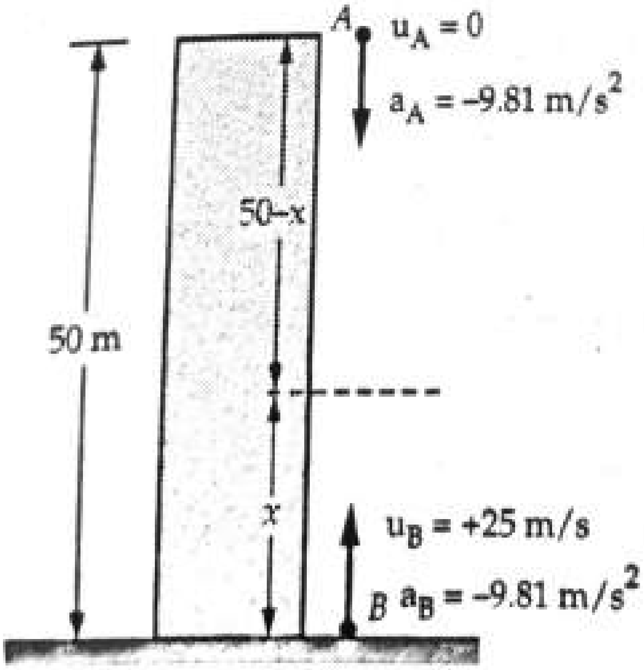

Fig. Q.15.1

For A, $s = ut + \frac{1}{2}at^2$

$$-(50-x) = 0 + \frac{1}{2}(-9.81)t^2$$

$$50 - x = 4.905\,t^2 \quad \text{... (1)}$$

For B, $s = ut + \frac{1}{2}at^2$

$$x = 25t + \frac{1}{2}(-9.81)t^2 = 25t - 4.905t^2 \quad \text{... (2)}$$

Add equations (1) and (2)

$$50 = 25\,t$$

$$\therefore \quad \boxed{t = 2\text{ s}}$$

Substitute in equation (2)

$$x = 25 \times 2 - 4.905 \times 2^2$$

$$\therefore \quad \boxed{x = 30.38\text{ m}}$$

Q.16 A stone thrown vertically upward from earth returns to the earth in 5 sec. How high does the stone reached. Also determine the velocity with which it is thrown. **[SPPU : Dec.-18, Marks 6]**

Ans. : A stone is thrown from earth and returns to earth hence displacement is zero i.e.

$$s = ut - \frac{1}{2}gt^2$$

$$0 = u \times 5 - \frac{1}{2} \times 9.81 \times 5^2$$

$$\therefore \quad \boxed{u = 24.525\text{ m/s}} \quad \text{...Ans.}$$

At the top most point v = 0

$$v^2 = u^2 - 2gs$$

$$\therefore \quad 0 = 24.525^2 - 2 \times 9.81 \times h$$

$$\therefore \quad \boxed{h = 30.656\text{ m}} \quad \text{...Ans.}$$

Q.17 A motorist travelling with 54 kmph when he observes that a traffic light, 240 m ahead of him, turns red. The traffic light is timed to stay red for 24 sec. If the motorist wishes to pass the light without stopping just as it turns green again, determine the required uniform deceleration of the car and also the speed with which he crosses the light signal. (Refer Fig. Q.17.1).

[SPPU : May-19, Marks 6]

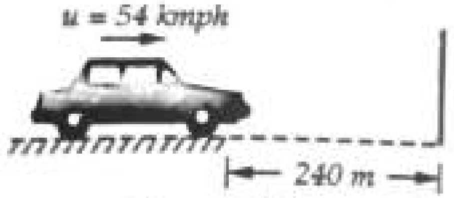

Fig. Q.17.1

Ans. :

$$V = 54 \text{ kmph} = 15 \text{ m/s}$$

$$s = ut + \frac{1}{2}at^2$$

$$240 = 15 \times 24 + \frac{1}{2} \times a \times 24^2$$

$$\therefore \quad \boxed{a = -0.417 \text{ m/s}^2} \quad \text{...Ans.}$$

Q.18 The Fig. Q.18.1 shows an a-t diagram for a car which starts from rest and comes to halt after time (60 + t) seconds. Find the value of 't' and 'f' shown in the diagram. Find also 1) Velocity at 40 seconds 2) Velocity at 60 seconds 3) Distance travelled during first 40 seconds.

[SPPU : May-05]

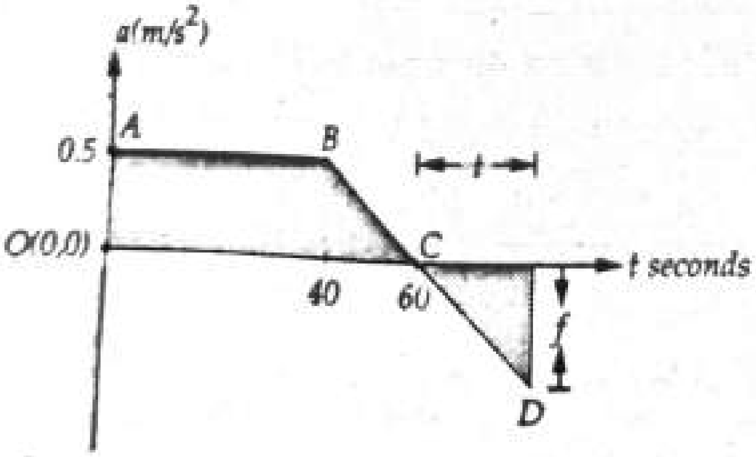

Fig. Q.18.1

Ans. : Change in velocity = Area under a-t graph.

Given $\quad v_0 = 0 ; \quad v_{60+t} = 0$

$$v_{60+t} - v_0 = (40)(0.5) + \frac{1}{2}(20)(0.5) - \frac{1}{2}(t)(f)$$

$$\therefore \quad t f = 50 \qquad \dots (1)$$

By similarity of triangles,

$$\frac{t}{f} = \frac{20}{0.5}$$

$$\therefore \quad t = 40 f \qquad \dots (2)$$

Substituting in equation (1),

$$40 f^2 = 50$$

$$\therefore \quad \boxed{f = 1.118 \text{ m/s}^2}$$

$$t = 40 \times 1.118$$

$$\therefore \quad \boxed{t = 44.72 \text{ s}}$$

$$v_{40} - v_0 = (40)(0.5)$$

$$\therefore \quad \boxed{v_{40} = 20 \text{ m/s}}$$

$$v_{60} - v_{40} = \frac{1}{2}(20)(0.5)$$

$$\therefore \quad v_{60} - 20 = 5$$

$$\therefore \quad \boxed{v_{60} = 25 \text{ m/s}}$$

Distance travelled during 40 s can be obtained using

$$x_1 = x_0 + v_0 t_1 + \text{moment of area about } t_1$$

$$x_{40} = 0 + 0 + (40)(0.5)(20)$$

$$\therefore \quad \boxed{x_{40} = 400 \text{ m}} \qquad \text{...Ans.}$$

Q.19 The v-t diagram for the motion of the train as it moves from station A to station B is shown in Fig. Q.19.1. Determine the average speed for the train and the distance between the stations. Also draw the a-t curve. **[SPPU : May-10]**

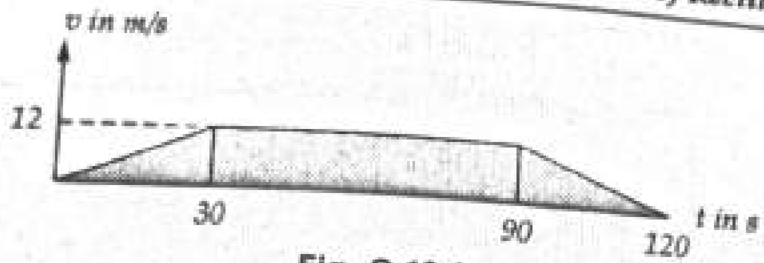

Fig. Q.19.1

Ans. : Distance between the stations is area under the v-t graph.

$$\therefore \quad s = \frac{1}{2} \times 30 \times 12 + 60 \times 12 + \frac{1}{2} \times 30 \times 12$$

$$\therefore \quad \boxed{s = 1080 \text{ m}}$$

$$\text{Average speed } v_{av} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{1080}{120}$$

$$\therefore \quad \boxed{v_{av} = 9 \text{ m/s}}$$

Acceleration is slope of v-t graph.

For $0 \leq t \leq 30$ s,

$$a = \frac{12-0}{30-0} = 0.4 \text{ m/s}^2$$

For $30 \text{ s} \leq t \leq 90$ s,

$$a = 0$$

For $90 \text{ s} \leq t \leq 120$ s

$$a = \frac{0-12}{120-90} = -0.4 \text{ m/s}^2$$

The a-t curve is shown in Fig. Q.19.1 (a).

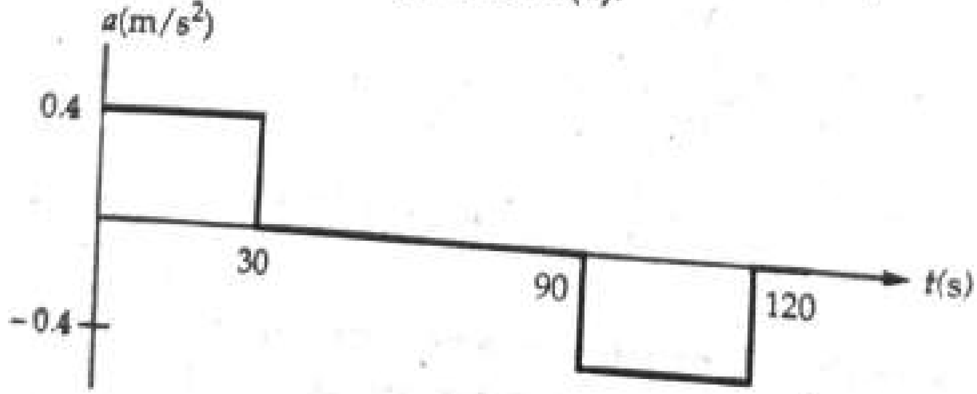

Fig. Q.19.1 (a)

UNIT - V

13 Kinematics of Curvilinear Motion

Important Points to Remember

- **Rectangular Components**
- The position vector $\vec{r}$ of a particle in rectangular components is written as

$$\vec{r} = x\hat{i} + y\hat{j}$$

- The velocity vector is given by

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

where $$v_x = \frac{dx}{dt} \text{ and } v_y = \frac{dy}{dt}$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \theta_v = \tan^{-1}\left(\frac{|v_y|}{|v_x|}\right)$$

- The acceleration vector is given by

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

where $$a_x = \frac{dv_x}{dt}; \quad a_y = \frac{dv_y}{dt}$$

$$a = \sqrt{a_x^2 + a_y^2}, \quad \theta_a = \tan^{-1}\left(\frac{|a_y|}{|a_x|}\right)$$

- If acceleration is variable in any direction (x or y), use basic definitions and either differentiate or integrate the functions.
- If acceleration is constant in any direction, use kinematical equations in that direction.

- To obtain equation of path, get x any y in terms of a parameter like 't' or any angle 'θ'. Then eliminate the parameter to obtain equation relating x and y which is equation of path.
- If an equation relating x and y is given and x , v_x, a_x are given, differentiate the given equation to get $\frac{dy}{dt}$ (i.e. v_y) in terms of $\frac{dx}{dt}$ (i.e. v_x). Again differentiate to get a_y.
- In projectile motion, $a_x = 0$ and $a_y = -g = -9.81 \text{ m/s}^2$. As the acceleration in x as well as y direction is constant, we can use kinematical equations in X and Y-directions.

Projectile Motion

- If a projectile is launched with velocity u at an angle α with the horizontal, the equations are

X-direction :

$$v_x = u\cos\alpha$$

$$x = (u\cos\alpha)\cdot t$$

Y-direction :

$$v_y = u\sin\alpha - gt$$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$v_y^2 = u^2\sin^2\alpha - 2gy$$

- Equation of path for projectile motion is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

- Range on horizontal plane is

$$R = \frac{u^2\sin 2\alpha}{g}$$

- Time of flight on horizontal plane is

$$T = \frac{2u\sin\alpha}{g}$$

- Maximum height above the point of projection is

$$H = \frac{u^2 \sin^2 \alpha}{2g}$$

- Range on inclined plane of inclination β with horizontal is

$$R = \frac{2u^2 \cos\alpha \sin(\alpha - \beta)}{g \cos^2 \beta}$$

Normal and Tangential Components (Path Variables)

- The normal component of velocity is always zero and the tangential component is the magnitude of velocity, i.e. speed.
- The acceleration is given by

$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$a_t = \frac{dv}{dt} = \text{Rate of change of speed}$$

- The normal component of acceleration is

$$a_n = \frac{v^2}{\rho}$$

- The resultant acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

- The radius of curvature for a particle moving along a curve $y = f(x)$ is given by

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right|$$

- When v_x, v_y, a_x and a_y are known or can be calculated, the radius of curvature can be calculated using

$$\rho = \left| \frac{\left(v_x^2 + v_y^2\right)^{3/2}}{v_x a_y - v_y a_x} \right|$$

- If object moves along the curve with constant acceleration, kinematical equations can be used in the tangential direction.
- In projectile motion, the radius of curvature is given by

$$\rho = \frac{v^2}{g \cos\theta}$$

where $v^2 = v_x^2 + v_y^2$

and $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Radial and Transverse Components (Polar Co-ordinates)

- The velocity of the particle is given by

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

- The radial component of velocity is

$$v_r = \dot{r}$$

- The transverse component of velocity is

$$v_\theta = r\dot{\theta}$$

- The magnitude of velocity is

$$v = \sqrt{v_r^2 + v_\theta^2}$$

- The direction of velocity is given by

$$\alpha_v = \tan^{-1}\left(\frac{v_\theta}{v_r}\right) \text{ with radial direction}$$

- The acceleration of the particle is given by

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\,\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{e}_\theta$$

- The radial component of acceleration is

$$a_r = \ddot{r} - r\dot{\theta}^2$$

- The transverse component of acceleration is

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$$

- The magnitude of acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2}$$

- The direction of acceleration is given by

$$\alpha_a = \tan^{-1}\left(\frac{a_\theta}{a_r}\right) \text{ with radial direction}$$

- If 'r' and 'θ' are given as functions of 't' then differentiate them twice w.r.t. 't' to obtain $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$.
- If equation relating r and θ is given, differentiate w.r.t. 't' to get $\dot{r}$ and $\ddot{r}$ in terms of θ, $\dot{\theta}$ and $\ddot{\theta}$.

Q.1 A particle moves along the path $y = 4x^2 + 8x + 10$ starting with an initial velocity $\vec{v}_0 = 5\hat{i} + 3\hat{j}$. If v_x is constant, determine v_y and a_y at $x = 3$ m.

Ans. : Given $\vec{v}_0 = 5\hat{i} + 3\hat{j}$

$\therefore$ $v_{x0} = 5$ m/s and $v_{y0} = 3$ m/s are the initial velocities in x and y directions respectively. But v_x is constant.

$$\therefore \quad v_x = 5 \text{ m/s}$$

We know that $v_x = \frac{dx}{dt}$, $a_x = \frac{dv_x}{dt}$, $v_y = \frac{dy}{dt}$,

$$a_y = \frac{dv_y}{dt}$$

$$y = 4x^2 + 8x + 10$$

Differentiate w.r.t. 't'

$$\frac{dy}{dt} = 8x\frac{dx}{dt} + 8\frac{dx}{dt}$$

$$\therefore \quad v_y = 8x\,v_x + 8\,v_x \quad \ldots (1)$$

Again differentiate w.r.t. 't'

$$\frac{dv_y}{dt} = 8\,v_x\frac{dx}{dt} + 0 \quad \text{as } v_x \text{ is constant}$$

$$\therefore \quad a_y = 8\, v_x \cdot v_x = 8\, v_x^2 \qquad \text{... (2)}$$

Substituting the values in equations (1) and (2),

$$v_y = 8 \times 3 \times 5 + 8 \times 5 = 120 + 40$$

$$\therefore \quad v_y = 160 \text{ m/s} \qquad \text{... Ans.}$$

and $$a_y = 8 \times 5^2 = 200 \text{ m/s}^2 \qquad \text{... Ans.}$$

Q.2 The Y-co-ordinate of a particle moving along a curve is $y = t^3 - 6t + 3$ where y is in metres and t in seconds. Its acceleration in x-direction is given by $a_x = 4t + 3$ m/s^2. If velocity of the particle in x direction is 2 m/s when t = 0, calculate the magnitude and direction of velocity and acceleration of the particle when $t = 1$ s.

Ans. :

$$y = t^3 - 6t + 3$$

$$v_y = \frac{dy}{dt} = 3t^2 - 6$$

$$a_y = \frac{d\,v_y}{dt} = 6t$$

$$a_x = 4t + 3$$

$$\therefore \quad \frac{dv_x}{dt} = 4t + 3$$

$$v_x = \frac{4t^2}{2} + 3t + C_1$$

$$\therefore \quad v_x = 2t^2 + 3t + C_1$$

At $t = 0$, $v_x = 2$ m/s

$$\therefore \quad 2 = C_1$$

$$\therefore \quad v_x = 2t^2 + 3t + 2$$

At $t = 1$ s,

$$v_x = 2 + 3 + 2 = 7 \text{ m/s}$$

$$v_y = 3 - 6 = -3 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + 3^2} = 7.62 \text{ m/s}$$

$$\theta_v = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{3}{7} = 23.2°$$

$$a_x = 4+3 = 7 \text{ m/s}^2 ;$$

$$a_y = 6 \times 1 = 6 \text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{7^2+6^2} = 9.22 \text{ m/s}^2$$

$$\theta_a = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{6}{7} = 40.6^\circ$$

Q.3 A projectile is fired with initial velocity 240 m/s at a target 'B' located 600 m above the gun and a horizontal distance of 3.6 km. Neglecting air resistance, determine the firing angles.

[SPPU : May-99, Dec.-09, Marks 8]

Ans. : Equation of path

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$600 = 3600\tan\alpha - \frac{9.81\times 3600^2}{2\times 240^2\cos^2\alpha}$$

$$600 = 3600\tan\alpha - 1103.625\sec^2\alpha$$

$$600 = 3600\tan\alpha - 1103.625(1+\tan^2\alpha)$$

$$1103.625\tan^2\alpha - 3600\tan\alpha + 1703.625 = 0$$

$$\tan\alpha = 0.574 \quad \text{or} \quad 2.688$$

$$\therefore \quad \boxed{\alpha = 29.86^\circ \text{ or } 69.6^\circ} \quad \text{... Ans.}$$

Q.4 The tennis player serves the ball from height 'H' with an initial velocity of 40 m/s at an angle of 4° with the horizontal as shown in Fig. Q.4.1. Knowing that the ball clears the 0.914 m net height by 152 mm, determine : a) The height 'H' b) The distance from the net 'd' where the ball will strike the floor.

[SPPU : Dec.-11, Marks 6]

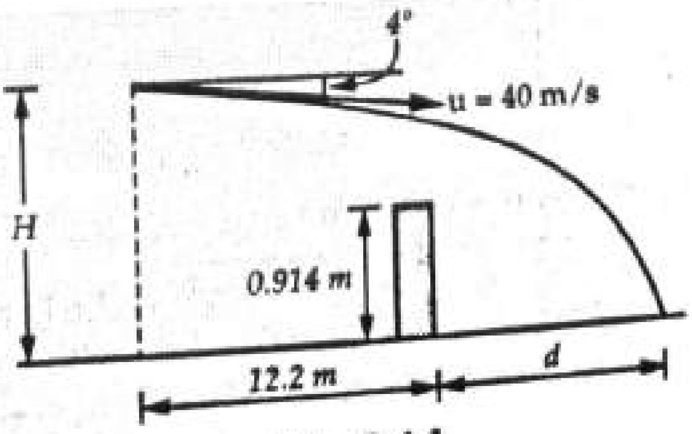

Fig. Q.4.1

Ans. : $u_x = 40 \cos 4,\ u_y = -40 \sin 4$

At the net,

$$x = 12.2 \text{ m},\ y = -(H - 0.152 - 0.914)$$

$$\therefore \quad y = -(H - 1.066)$$

$$a_x = 0, \quad a_y = -9.81 \text{ m/s}^2$$

$$x = u_x \cdot t$$

$$\therefore \quad 12.2 = 40 \cos 4 \times t$$

$$\therefore \quad t = 0.305745 \text{ s}$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\therefore \quad -(H - 1.066) = (-40 \sin 4)(0.305745) + \frac{1}{2}(-9.81)(0.305745)^2$$

$$\therefore \quad \boxed{H = 2.378 \text{ m}}$$ ... Ans.

At the floor,

$$x = 12.2 + d,\ y = -H = -2.378$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$-2.378 = (-40 \sin 4)\, t + \frac{1}{2}(-9.81) t^2$$

$$4.905 t^2 + (40 \sin 4) t - 2.378 = 0$$

$$\therefore \quad t = 0.4677 \text{ s}$$

$$x = u_x \cdot t$$

$$\therefore \quad 12.2 + d = (40 \cos 4)(0.4677)$$

$$\therefore \quad \boxed{d = 6.46 \text{ m}}$$

Q.6 During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal as shown in Fig. Q.5.1. If the point of landing is 6 m away, determine the approximate speed at which the bike was travelling just before it left the ground.

[SPPU : May-13, Marks 4]

Scanned by CamScanner

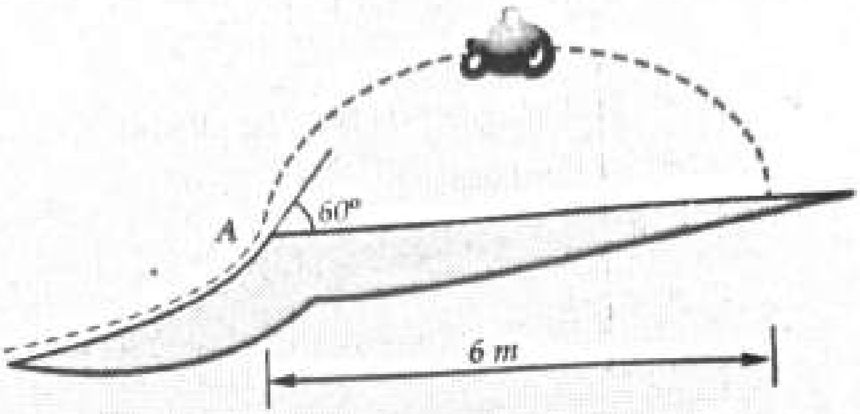

Fig. Q.6.1

Ans. : Let the initial speed of bike be u. As initial and final points are at the same level, the range is given by

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$R = 6$ m, $\alpha = 60°$

$$\therefore \quad 6 = \frac{u^2 \sin(2 \times 60)}{9.81}$$

$$\therefore \quad \boxed{u = 8.244 \text{ m/s} = 29.68 \text{ km/h}} \quad \text{...Ans.}$$

Q.6 A cricket ball shot by a batsman from a height of 1.8 m at an angle of 30° with the horizontal with a velocity of 18 m/s is caught by a fielder at a height of 0.6 m from the ground. Determine the distance between the batsman and fielder.

[SPPU : May-14, Marks 4]

Ans. : The trajectory of the ball is shown in Fig. Q.6.1.

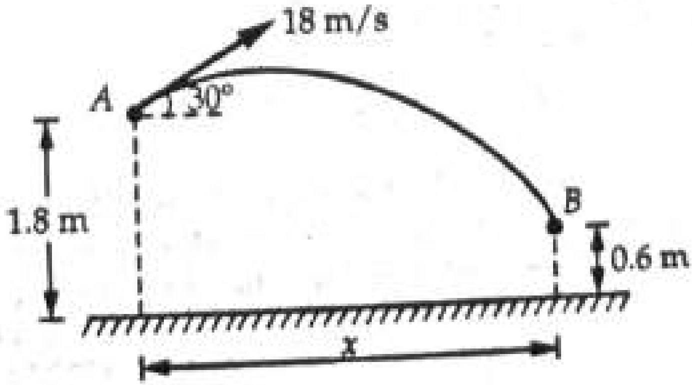

Fig. Q.6.1

The equation of path is

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2\alpha}$$

Here $y = -1.2$ m , $g = 9.81$ m/s^2 ,

$\alpha = 30°$, $u = 18$ m/s

$$\therefore \quad -1.2 = x \tan 30 - \frac{9.81 x^2}{2\times18^2 \cos^2 30}$$

$$\frac{9.81 x^2}{2\times18^2 \cos^2 30} - \tan 30 \times x - 1.2 = 0$$

$$\therefore \quad x = 30.55 \text{ m} \ , \ -1.95 \text{ m}$$

x cannot be negative.

$$\therefore \quad \boxed{x = 30.55 \text{ m}}$$

...Ans.

Q.7 A projectile is fired from the edge of a 150 m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find the horizontal distance from the gun to the point, where the projectile strikes the ground and the greatest elevation above the ground reached by the projectile. Refer Fig. Q.7.1. [SPPU : May-15, Marks 4]

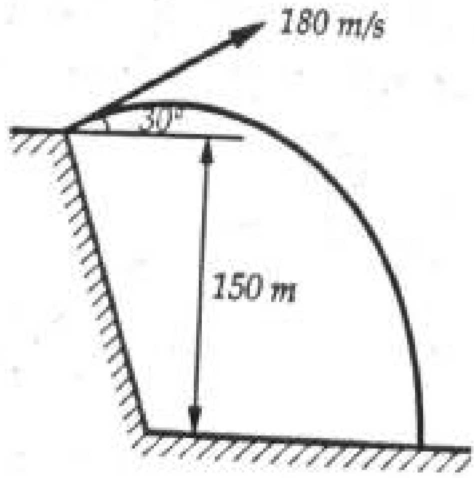

Fig. Q.7.1

Ans. : At the point where the projectile strikes the ground,

$$y = -150 \text{ m}$$

Equation of path is,

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

$$\alpha = 30°, g = 9.81 \text{ m/s}^2, u = 180 \text{ m/s}$$

$$\therefore \quad -150 = x \tan 30 - \frac{9.81 \times x^2}{2 \times 180^2 \cos^2 30}$$

$$\therefore 2.01852 \times 10^{-4} x^2 - (\tan 30)\, x - 150 = 0$$

$$\therefore \quad x = 3100 \text{ m} - 239.72 \text{ m}$$

$$\therefore \quad \boxed{x = 3100 \text{ m}}$$ ...Ans.

Maximum height above the point of projection is,

$$H = \frac{u^2 \sin^2 \alpha}{2g} = \frac{180^2 \sin^2 30}{2 \times 9.81} = 412.84 \text{ m}$$

Maximum height above the ground is

$$H_{max} = 150 + 412.84$$

$$\therefore \quad \boxed{H_{max} = 562.84 \text{ m}}$$ ...Ans.

Q.8 A cricket ball thrown by a fielder from a height of 2 m at an angle of 45° to the horizontal with an initial velocity of 25 m/s hit the wickets at the height of 0.6 m from the ground. Find distance of fielder from the wickets. ☞ [SPPU : Dec.-15, Marks 4]

Ans. : The equation of path for projectile motion is

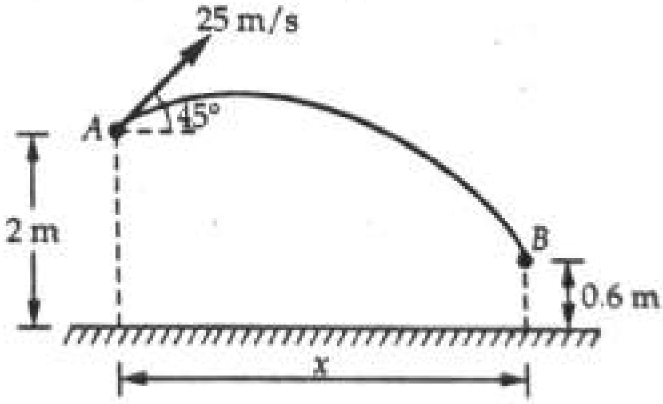

Fig. Q.8.1

$$y = x \tan\alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

$$y = -(2-0.6) = -1.4 \text{ m}$$

$$\alpha = 45°, \; g = 9.81 \text{ m/s}^2, \; u = 25 \text{ m/s}$$

$$\therefore \quad -1.4 = x \tan 45 - \frac{9.81 \times x^2}{2 \times 25^2 \cos^2 45}$$

$$0.015696x^2 - x - 1.4 = 0$$

$$\therefore \quad \boxed{x = 65.08 \text{ m}}$$

...Ans.

Q.9 Water flows from a drain spout with an initial velocity of 0.75 m/s at an angle of 75° with the vertical as shown in Fig. Q.9.1. Determine the range of values of the distance d for which the water will enter the trough BC. **[SPPU : May-16, Marks 4]**

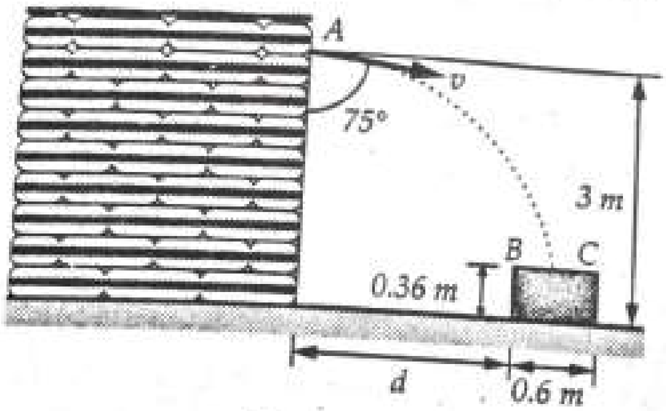

Fig. Q.9.1

Ans. : In y-direction,

$$y = u_y + \frac{1}{2} a_y t^2$$

$$y = -(3-0.36) = -2.64 \text{ m}$$

$$u_y = -0.75 \cos 75, \quad a_y = -9.81 \text{ m/s}^2$$

$$\therefore \quad -2.64 = -(0.75 \cos 75)t + \frac{1}{2}(-9.81)t^2$$

$$4.905 t^2 + 0.194 t - 2.64 = 0$$

$$\therefore \quad t = 0.714 \text{ s}, -0.754 \text{ s}$$

$$\therefore \quad t = 0.714 \text{ s}$$

For minimum d, water will enter the trough as C.

At C, $\quad x = d_{min} + 0.6$

In x - direction,

$$x = u_x t$$

$$u_x = 0.75 \sin 75$$

$$\therefore \quad d_{min} + 0.6 = 0.75 \sin 75 \times 0.714$$

$$\therefore \quad d_{min} = -0.083 \text{ m}$$

Negative values of d implies that point B will be to the left of the vertical at A. This is not possible due to pressure of wall.

$$\therefore \quad d_{min} = 0$$

For maximum d, water enters the trough at B. The y - coordinate of B is same as that of C.

$$\therefore \quad t = 0.714 \text{ s}$$

$$x = d_{max} = 0.75 \sin 75 \times 0.714$$

$$\therefore \quad d_{max} = 0.517 \text{ m}$$

$\therefore$ Range of d is

$$\boxed{0 \leq d \leq 0.517 \text{ m}}$$ ...Ans.

Q.10 A cricket ball shot by a batsman from a height of 2.0 m at an angle of 30° with the horizontal with a velocity of 20 m/s is caught by a fielder at a height of 0.8 m from the ground. Determine the distance between the batsman and fielder.

[SPPU : Dec.-16, Marks 4]

Ans. : The equation for projectile motion is

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2 \alpha}$$

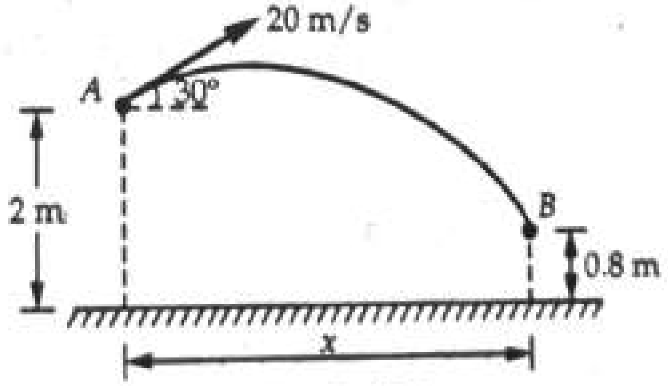

Fig. Q.10.1

$$y = -(2 - 0.8) = -1.2 \text{ m}$$

$$\alpha = 30°,\ g = 9.81 \text{ m/s}^2,\ u = 20 \text{ m/s}$$

$$\therefore \quad -1.2 = x\tan 30 - \frac{9.81x^2}{2\times 20^2 \cos^2 30}$$

$$0.01635x^2 - 0.57735x - 1.2 = 0$$

$$\therefore \quad \boxed{x = 37.28 \text{ m}}$$

...Ans.

Q.11 A projectile is launched with a speed of V_0 = 25 m/s at an angle of θ = 30° with horizontal as shown in Fig. Q.11.1. Determine the maximum distance travelled by projectile along horizontal and vertical direction. [SPPU : May-17, Marks 4]

Fig. Q.11.1

Ans. : Maximum horizontal distance is

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$v_0 = 25 \text{ m/s},\ \theta = 30°,\ g = 9.81 \text{ m/s}^2$$

$$\therefore \quad R = \frac{25^2 \sin 60}{9.81}$$

$$\therefore \quad \boxed{R = 55.17 \text{ m}}$$

Maximum height is

$$H = \frac{v_0^2 \sin^2\theta}{2g} = \frac{25^2 \sin^2 30}{2\times 9.81}$$

$$\therefore \quad \boxed{H = 7.96 \text{ m}}$$

...Ans.

Q.12 Measurements of shot recorded on a videotape during a basketball game are shown in Fig. Q.12.1. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude V_A of its initial velocity and height h of the ball when it passes player B. **[SPPU : Dec.-17, Marks 6]**

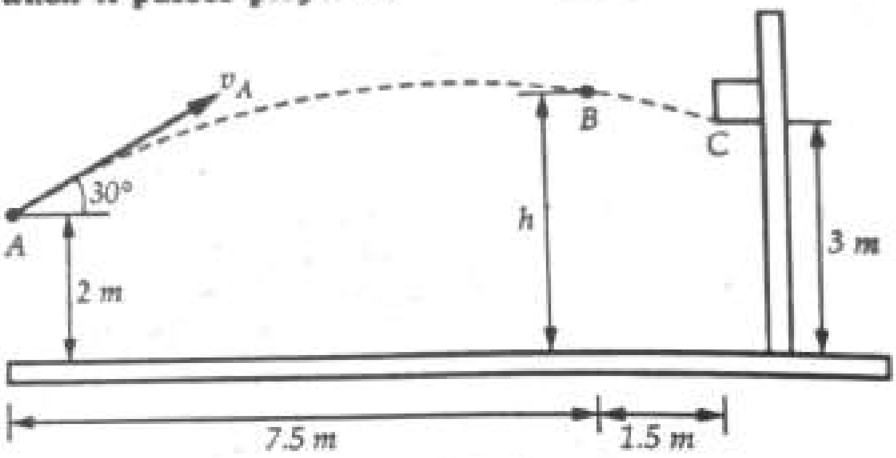

Fig. Q.12.1

Ans. : For motion from A to C,

$$x = 9 \text{ m}, \quad y = 1 \text{ m}$$

$$u = v_A, \quad \alpha = 30°$$

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2 \alpha}$$

$$1 = 9 \tan 30 - \frac{9.81 \times 9^2}{2 v_A^2 \cos^2 30}$$

$\therefore$ $\boxed{v_A = 11.236 \text{ m/s}}$ ... Ans.

For motion from A to B,

$$x = 7.5 \text{ m}, y = h - 2$$

$$u = 11.236 \text{ m/s}, \quad \alpha = 30°$$

$$\therefore \quad h - 2 = 7.5 \tan 30 - \frac{9.81 \times 7.5^2}{2 \times 11.236^2 \cos^2 30}$$

$\therefore$ $\boxed{h = 3.416 \text{ m}}$ ...Ans.

Q.13 A ball is thrown by a player from 5 m above ground level, clears the 25 m high wall placed 100 m from the player. If the angle of projection of the ball is 60 degrees, then determine the initial velocity of the ball. [SPPU : Dec.-18, Marks 6]

Ans. : The coordinates of point B (100, 20)

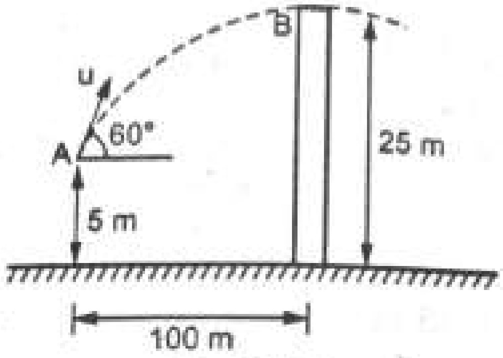

Fig. Q.13.1

The equation of path is,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$20 = 100\tan 60 - \frac{9.81\times 100^2}{2u^2\times(\cos 60)^2}$$

$$20 = 173.205 - \frac{196200}{u^2}$$

$$\therefore \quad \boxed{u = 35.786 \text{ m/sec}} \qquad \text{...Ans.}$$

Q.14 A handball player throws a ball from A horizontally with a velocity 'u' m/s. Knowing that d = 15 m, determine the range of the values of velocity for which the ball will strike the corner region BCD as shown in the Fig. Q.14.1. [SPPU : May-19, Marks 6]

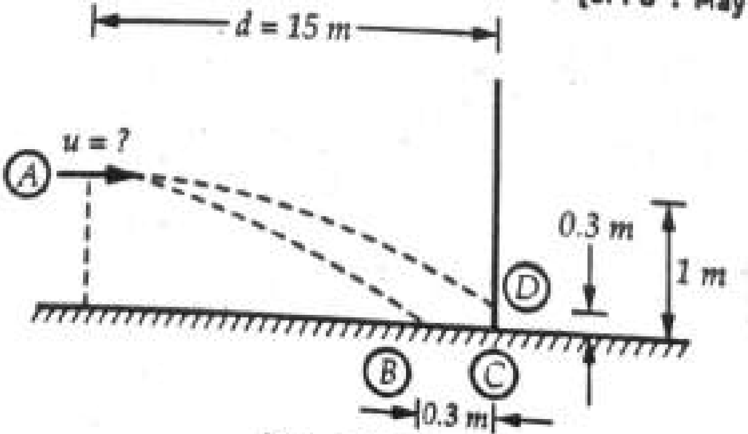

Fig. Q.14.1

Ans. : Here, $\alpha = 0°$

Motion A to B, $x = 15 - 0.3 = 14.7$ m

$$y = -1 \text{ m}$$

Equation of path is,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$-1 = 14.7\tan(0) - \frac{9.81 \times 14.7^2}{2 \times u^2 \times (\cos 0)^2}$$

$$\therefore \quad u = 32.556 \text{ m/s}$$

Motion A to D, $x = 15$ m

$$y = -(1 - 0.3) = -0.7 \text{ m}$$

$$\therefore \quad -0.7 = 15\tan(0) - \frac{9.81 \times 15^2}{2 \times u^2 \times (\cos 0)^2}$$

$$\therefore \quad u = 39.706 \text{ m/s}$$

Hence, range of velocity to enter region BCD is,

$$\boxed{32.556 \le u \le 39.706 \text{ m/s}}$$

... Ans.

Q.15 A rocket follows a curvilinear path such that its acceleration is given as $\vec{a} = 4\hat{i} + t\hat{j}\,(m/s^2)$. Also at t = 0, rocket started from rest. Find the speed of rocket and radius of curvature of its path at t = 10 s.

[SPPU : May-12]

Ans. :

$$\vec{a} = 4\hat{i} + t\hat{j}$$

$$\therefore \quad a_x = 4 \; ; \; a_y = t$$

$$\frac{dv_x}{dt} = 4 \; ; \; \frac{dv_y}{dt} = t$$

$$\int_0^{v_x} dv_x = 4\int_0^{10} dt \; ; \; \int_0^{v_y} dv_y = \int_0^{10} t\,dt$$

$$v_x = 40 \text{ m/s} ; v_y = \left[\frac{t^2}{2}\right]_0^{10} = 50 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 50^2}$$

$$\therefore \quad \boxed{v = 64.03 \text{ m/s}} \quad \text{...Ans.}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{50}{40}\right)$$

$$\theta = 51.04° \measuredangle$$

At $t = 10$ s, $a_x = 4 \text{ m/s}^2$, $a_y = 10 \text{ m/s}^2$

$$\rho = \left|\frac{(v_x^2 + v_y^2)^{3/2}}{v_x a_y - v_y a_x}\right| = \left|\frac{(40^2 + 50^2)^{3/2}}{(40)(10) - (50)(4)}\right|$$

$$\therefore \quad \boxed{\rho = 1312.64 \text{ m}} \quad \text{...Ans.}$$

Q.16 A particle is projected at an angle of 30° to the horizontal with a velocity of 100 *m/s*. Determine the range of radius of curvature of the path followed by the particle.

[SPPU : Dec.-12, Marks 6]

Ans. : The radius of curvature is maximum at the starting point.

$$R_{max} = \frac{v^2}{g \cos\theta}$$

$$v = 100 \text{ m/s}, \ g = 9.81 \text{ m/s}^2, \theta = 30°$$

$$\therefore \quad R_{max} = \frac{100^2}{9.81 \cos 30}$$

$$\therefore \quad \boxed{R_{max} = 1177.06 \text{ m}}$$

Radius of curvature is minimum at the top of the trajectory. It is given by,

$$R_{min} = \frac{v_x^2}{g}$$

$$v_x = 100 \cos 30$$

$$\therefore \quad R_{min} = \frac{(100\cos 30)^2}{9.81}$$

$$\therefore \quad \boxed{R_{min} = 764.53 \text{ m}} \quad \text{...Ans.}$$

Q.17 An outdoor track is 126 m in diameter. A runner increases her speed at a constant rate from 4.2 to 7.2 m/s over a distance of 28.5 m. Determine the total acceleration of the runner 2 s after she begins to increase her speed. **[SPPU : Dec.-13, Marks 4]**

Ans. : $u = 4.2$ m/s, $v = 7.2$ m/s, $s = 28.5$ m

$$v^2 = u^2 + 2a_t s$$

$$\therefore \quad 7.2^2 = 4.2^2 + 2a_t \times 28.5$$

$$\therefore \quad a_t = 0.6 \text{ m/s}^2$$

At $t = 2$ s,

$$v = u + a_t t = 4.2 + 0.6 \times 2$$

$$\therefore \quad v = 5.4 \text{ m/s}$$

$$\rho = \frac{126}{2} = 63 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{5.4^2}{63} = 0.463 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6^2 + 0.463^2}$$

$$\therefore \quad \boxed{a = 0.758 \text{ m/s}^2} \quad \text{...Ans.}$$

Q.18 A motorist is travelling on a curved section of highway of radius 762 m at the speed of 96 kmph. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after 8 s the speed has been reduced to 72 kmph, determine the acceleration of the automobile immediately after the brakes have been applied. **[SPPU : May-18, Marks 6]**

Ans. : For tangential direction,

$$u = 96 \text{ kmph} = 26.67 \text{ m/s}$$

$$v = 72 \text{ kmph} = 20 \text{ m/s}$$

$$t = 8 \text{ s}, \quad a = a_t = \text{constant}$$

$$v = u + a\,t$$

$$20 = 26.67 + a_t \times 8$$

$$\therefore \quad a_t = -0.834 \ m/s^2$$

At the begining, $v = u = 26.67 \ m/s$

$$a_n = \frac{v^2}{\rho} \ ; \ \rho = 762 \ m$$

$$\therefore \quad a_n = \frac{26.67^2}{762} = 0.933 \ m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.834)^2 + 0.933^2}$$

$$\therefore \quad \boxed{a = 1.25 \ \mathrm{m/s^2}}$$

...Ans.

Q.19 A particle moves in the x-y plane such that its position is defined by vector $\vec{r} = (2t\hat{i} + 4t^2\hat{j})$ m, where t is in seconds. Determine the radial and tangential components of the particle's velocity and acceleration when t = 0.25 s. [SPPU : Dec.-08]

Ans. : As $\vec{r} = 2t\,\hat{i} + 4t^2\,\hat{j}$

$$x = 2t \ , \ y = 4\,t^2$$

$$v_x = \frac{dx}{dt} = 2 \ , \ v_y = \frac{dy}{dt} = 8\,t$$

$$a_x = \frac{dv_x}{dt} = 0 \ ;$$

$$a_y = \frac{dv_y}{dt} = 8$$

At $t = 0.25$ s,

$$x = 2\,(0.25) = 0.5 \ \mathrm{m},$$

$$y = 4\,(0.25)^2 = 0.5 \ \mathrm{m}, \Rightarrow \mathrm{r} = 0.707 \ \mathrm{m}, \ \theta = 45^\circ$$

$$v_x = 2 \ \mathrm{m/s} \ ; \ v_y = 8\,(0.25) = 2 \ \mathrm{m/s}$$

$$a_x = 0 \ ; \ a_y = 8 \ \mathrm{m/s^2}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 2^2}$$

$$\therefore \quad v = 2.828 \text{ m/s}$$

$$\theta_v = \tan^{-1}\left(\frac{2}{2}\right) = 45°$$

As velocity is always tangential,

$$\therefore \quad \boxed{v_t = 2.828 \text{ m/s}}$$ ...Ans.

Also from Fig. Q.19.1 where the velocity and acceleration are shown along with the radial and transverse directions,

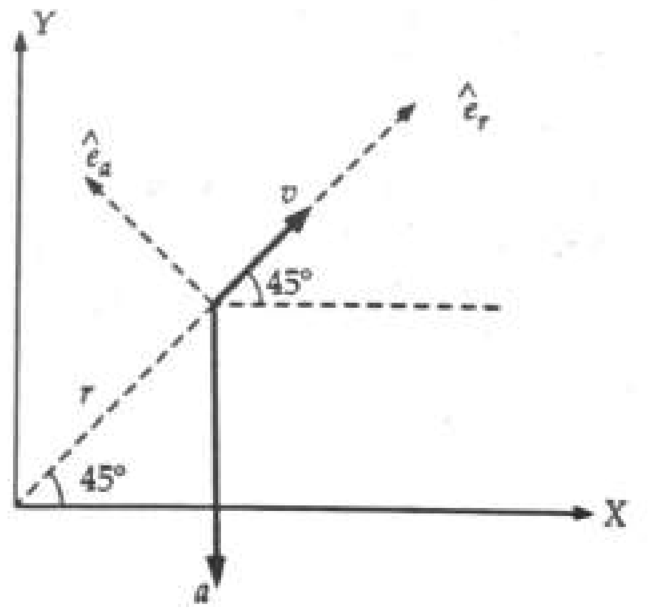

Fig. Q.19.1

$$\therefore \quad \boxed{v_r = v = 2.828 \text{ m/s}}$$

The radial and tangential components of acceleration will be the components parallel to 'v' (as velocity is in radial direction in this case)

$$\therefore \quad a_r = a_t = 8 \cos 135°$$

$$\therefore \quad \boxed{a_r = a_t = -5.657 \text{ m/s}^2}$$ ...Ans.

Q.20 A particle position is describe by the co-ordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds. Determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

[SPPU : May-11, Marks 6]

Ans. : $r = 2\sin 2\theta$, $\theta = 4t$

$\dot{\theta} = 4$, $\ddot{\theta} = 0$

At $t = 1$ s, $\theta = 4$ rad, $\dot{\theta} = 4$ rad/s, $\ddot{\theta} = 0$

$$\dot{r} = 4 \cos 2\theta \times \dot{\theta} = 4\dot{\theta} \cos 2\theta$$

$$\ddot{r} = -8\dot{\theta}^2 \sin 2\theta$$

At $t = 1$ s,

$$r = 2\sin(2 \times 4) = 1.9787 \text{ m}$$

$$\dot{r} = 4(4)\cos(2\times 4) = -2.328 \text{ m/s}$$

$$\ddot{r} = -8 \times 4^2 \sin(2\times 4) = -126.64 \text{ m/s}^2$$

(Note that θ is in radians. While calculating $\sin 2\theta$ and $\cos 2\theta$, either convert θ to degrees or use calculator in radian mode)

$$v_r = \dot{r}$$

$$\therefore \quad \boxed{v_r = -2.328 \text{ m/s}}$$

$$v_\theta = r\dot{\theta} = (1.9787)(4)$$

$$\therefore \quad \boxed{v_\theta = 7.9148 \text{ m/s}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -126.64 - (1.9787)(4^2)$$

$$\therefore \quad \boxed{a_r = -158.3 \text{ m/s}^2}$$

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 2(-2.328)(4) + 0$$

$$\therefore \quad \boxed{a_\theta = -18.624 \text{ m/s}^2}$$

...Ans.

UNIT - VI

14 Kinetics of Rectilinear Motion - Force and Acceleration

Important Points to Remember

- **Newton's Second Law of Motion :** The acceleration of a particle is directly proportional to magnitude of the resultant force acting on it and is in the direction of resultant force.

$$\sum \vec{F} = m\vec{a}$$

- The above equation in two dimensions can be written in scalar form as

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

- **D'Alembert's principle of dynamic equilibrium :**

$$\sum \vec{F} + (-m\vec{a}) = 0$$

- The vector $-m\vec{a}$, which has same magnitude as $m\vec{a}$ but opposite direction, is called the **inertia vector.**

General Procedure for Solving Problems

1) Draw free body diagram of the objects as in statics. The additional term now that has to be shown is the $m\vec{a}$ term, which is the effect. To differentiate the effect from the causes, we will show the effect with a double arrow ($\Rightarrow$).

2) Choose co-ordinate axes parallel and perpendicular to the direction in which motion can take place. For example, if an object is moving on an inclined plane, choose x-axis parallel to inclined plane and y-axis perpendicular to it.

3) Use $\sum F_x = ma_x$
and $\sum F_y = ma_y$
to find acceleration of the object.

4) Use methods of kinematics for further analysis.

5) If objects are connected through cables, relate their acceleration and tensile forces in connecting cables.

6) Take $m\vec{a}$ term in the direction of motion so that if 'a' comes out to be negative it will indicate deceleration.

7) If D'Alemberts principle is to be used, identify the direction of acceleration and show the $m\vec{a}$ term in opposite direction. Use

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

Q.1 The 50 kg crate is projected along the floor with an initial speed of 7 m/s at x = 0. The coefficient of kinetic friction μ_k = 0.4. Calculate the time required for the crate to come to rest and corresponding distance x traveled. Refer Fig. Q.1.1.

[SPPU : Dec.-06]

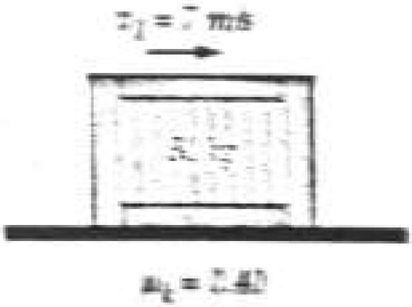

Fig. Q.1.1

Ans. : The F.B.D of crate is shown in Fig. Q.1.1 (a).

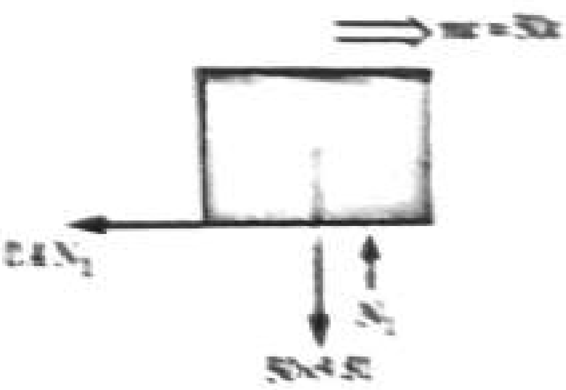

Fig. Q.1.1 (a)

$\sum F_y = 0$:

$$N_1 - 50 \times 9.81 = 0$$

$$\therefore \quad N_1 = 490.5 \text{ N}$$

$\sum F_x = ma_x$:

$$-0.4 N_1 = ma$$

$$\therefore \quad -0.4\,(490.5) = 50\,a$$

$$a = -3.924 \ m/s^2$$

$$u = 7 \ m/s, \ v = 0$$

$$v = u + at :$$

$$0 = 7 + (-3.924)t$$

$\therefore$ $\boxed{t = 1.784 \ s}$ ... Ans.

$$v^2 = u^2 + 2as$$

$$0 = 7^2 + 2(-3.924)s$$

$\therefore$ $\boxed{s = 6.244 \ m}$ ... Ans.

Q.2 A body of mass 'm' is projected up a 25° inclined plane with an initial velocity of 15 m/s. If the coefficient of friction μ_K = 0.25, determine how far the body will move up the plane and the time required to reach the highest point. [SPPU : May-99]

Ans. : From Fig. Q.2.1 (a),

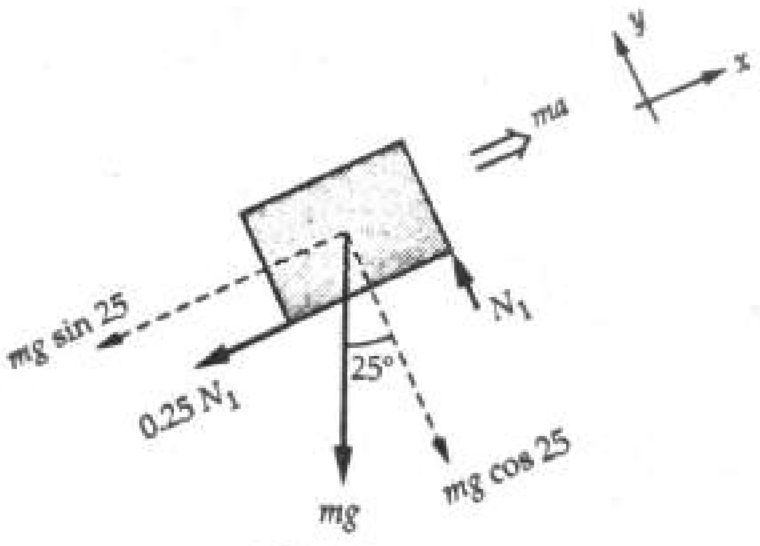

Fig. Q.2.1 (a)

$\sum F_y = 0$:

$N_1 - mg \cos 25 = 0$

$\therefore$ $N_1 = mg \cos 25$

$\sum F_x = ma_x$:

$-mg \sin 25 - 0.25 \ N_1 = ma$

$- mg \sin 25 - 0.25\, mg \cos 25 = ma$

$a = -9.81 (\sin 25 + 0.25 \cos 25)$

$\therefore \quad a = -6.369 \text{ m/s}^2$

$u = 15 \text{ m/s}, \quad v = 0$

$v = u + at$:

$0 = 15 + (-6.369)\, t$

$\therefore \quad \boxed{t = 2.355 \text{ s}}$ ... **Ans.**

$v^2 = u^2 + 2as$:

$0 = 15^2 + 2(-6.369)\, s$

$\therefore \quad \boxed{s = 17.66 \text{ m}}$ ... **Ans.**

Q.3 The conveyor belt as shown in Fig. Q.3.1 is designed to transport packages of various weights. Each 10 kg package has a coefficient of friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the required time and distance the package will slide on the conveyor before coming to rest. **[SPPU : Dec.-08, 17, May-11, Marks 6]**

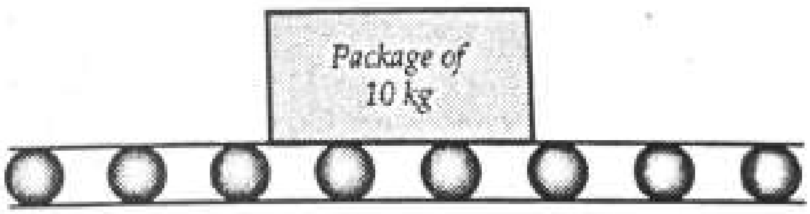

Fig. Q.3.1

Ans. : Consider that the package is moving towards right. When conveyor stops, package starts moving with velocity 5 m/s towards right due to inertia. Consider *F.B.D.* of package shown in Fig. Q.3.1 (a).

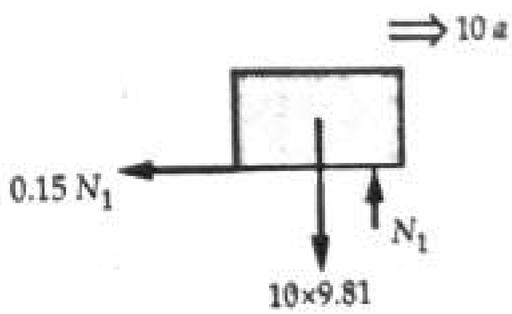

Fig. Q.3.1 (a)

$\sum F_y = 0$:

$N_1 - 10 \times 9.81 = 0$

$\therefore \quad N_1 = 98.1 \text{ N}$

$$\sum F_x = ma_x \quad :$$

$$-0.15\, N_1 = 10a$$

$$\therefore \quad a = -\frac{0.15 \times 98.1}{10} = -1.4715 \text{ m/s}^2$$

$$u = 5 \text{ m/s}, \quad v = 0$$

$$v = u + at \quad :$$

$$0 = 5 + (-1.4715)\, t$$

$$\therefore \quad \boxed{t = 3.4 \text{ s}} \quad \text{... Ans.}$$

$$v^2 = u^2 + 2as \quad :$$

$$0 = 5^2 + 2(-1.4715)s$$

$$\therefore \quad \boxed{s = 8.495 \text{ m}} \quad \text{... Ans.}$$

Q.4 A motorist travelling at a speed of 108 kmph suddenly applies the brake and comes to rest after skidding 75 m. Determine the time required for the car to stop and the coefficient of friction μ_k between the tyres and road. [SPPU : May-09 (Old), Marks 8]

Sol. : $u = 108$ km/h $= 30$ m/s, $\quad v = 0$

$$s = 75 \text{ m}$$

$$v^2 = u^2 + 2\,as$$

$$0 = 30^2 + 2a\,(75)$$

$$\therefore \quad a = -6 \text{ m/s}^2$$

Negative sign indicates deceleration.

$$v = u + a\,t$$

$$0 = 30 + (-6)\,t$$

$$\therefore \quad \boxed{t = 5 \text{ s}} \quad \text{... Ans.}$$

Consider the *FBD* of the motorist shown in Fig. Q.4.1 assuming motion towards right.

$$\sum F_y = 0$$

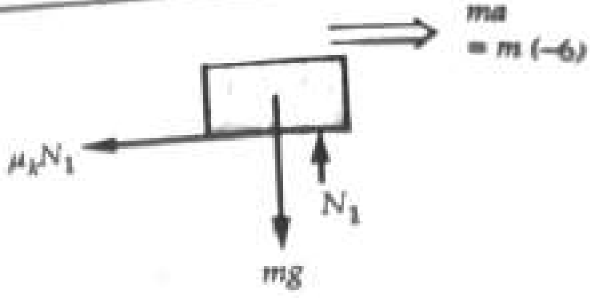

Fig. Q.4.1

$$N_1 = m\,g$$

$$\sum F_x = m\,a_x$$

$$-\mu_k N_1 = m\,(-6)$$

$$\mu_k\, mg = 6\text{ m}$$

$$\mu_k = \frac{6}{9.81}$$

$$\therefore \quad \boxed{\mu_k = 0.612} \quad \text{... Ans.}$$

Q.5 A light train made up of 2 cars 'A' and 'B' is travelling along a straight path at 54 kmph when brakes are applied to car 'A'. Braking force applied is 40 kN. Mass of car A is 25000 kg and that of car B is 20000 kg. Determine the distance traveled by the train before coming to a stop. Also find the force in the coupling between car A and car B. **[SPPU : May-09, Marks 9]**

Ans. : Assuming that A and B move towards right, their $FBDs$ are shown in Fig. Q.5.1 where F is the force in the coupling.

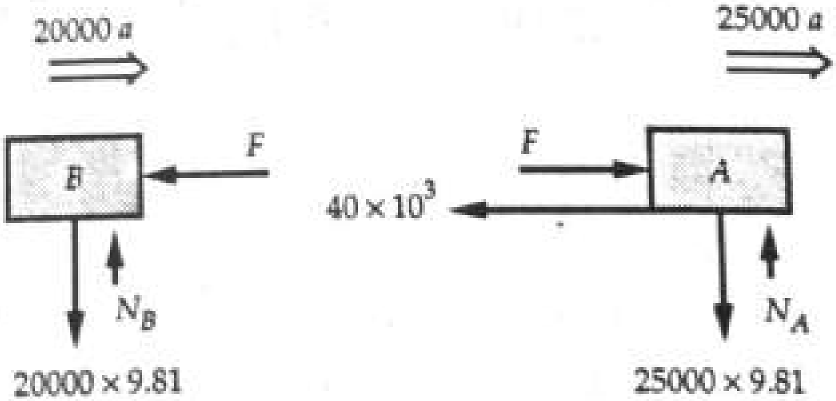

Fig. Q.5.1

$$\text{For} \quad B : -F = 20000\,a \quad \text{... (1)}$$

$$\text{For } A : F - 40000 = 25000\,a \quad \text{... (2)}$$

$$\therefore \quad -20000\,a - 40000 = 25000\,a$$

$$a = -\frac{8}{9}\text{ m/s}^2 = -0.889\text{ m/s}^2$$

Negative sign indicates that it is deceleration,

$$u = 54 \times \frac{5}{18} = 15\text{ m/s}, \; v = 0$$

$$v^2 = u^2 + 2as$$

$$0 = 15^2 + 2\left(-\frac{8}{9}\right)s$$

$$\boxed{s = 126.56 \text{ m}}$$

... Ans.

$$F = -20000\,a = 17777.78 \text{ N}$$

$$\boxed{F = 17.78 \text{ kN}}$$

... Ans.

Q.6 A particle of mass 10 kg is acted upon by a force $F = (20\,t^2 - 40)$ N, where 't' is time in s. At t = 0 s, velocity 'v' of the particle is 5 m/s and position x = 0. Find velocity and position of the particle at t = 2 sec. [SPPU : May-09, Marks 9]

Ans. :

$$F = ma$$

$$a = \frac{F}{m}$$

As $m = 10$ kg and $F = 20t^2 - 40$,

$$a = \frac{20\,t^2 - 40}{10}$$

$$\therefore \quad a = 2\,t^2 - 4$$

$$a = \frac{dv}{dt} = 2t^2 - 4$$

$$\int_5^v dv = \int_0^t (2t^2 - 4)\,dt$$

$$v - 5 = \frac{2t^3}{3} - 4t$$

$$\therefore \quad v = \frac{2t^3}{3} - 4t + 5$$

At $t = 2s$,

$$v_2 = 2 \times \frac{2^3}{3} - 4 \times 2 + 5$$

$$\therefore \quad \boxed{v_2 = 2.33 \text{ m/s}}$$

... Ans.

$$v = \frac{dx}{dt} = \frac{2t^3}{3} - 4t + 5$$

$$\int_0^{x_2} dx = \int_0^2 \left(2\frac{t^3}{3} - 4t + 5\right) dt$$

$$x_2 = \left[\frac{2t^4}{12} - 2t^2 + 5t\right]_0^2 = \frac{2 \times 2^4}{12} - 2 \times 2^2 + 5 \times 2$$

$\therefore$ $\boxed{x_2 = 4.667 \text{ m}}$ ... **Ans.**

Q.7 A 90 kg block rests on a horizontal plane. Find the magnitude of the force 'P' required to give the block an acceleration of 3 m/s^2 to the right. The coefficient of friction between the block and the plane is u_k = 0.25. [SPPU : Dec.-09, Marks 9, Dec.-18, May-19, Marks 6]

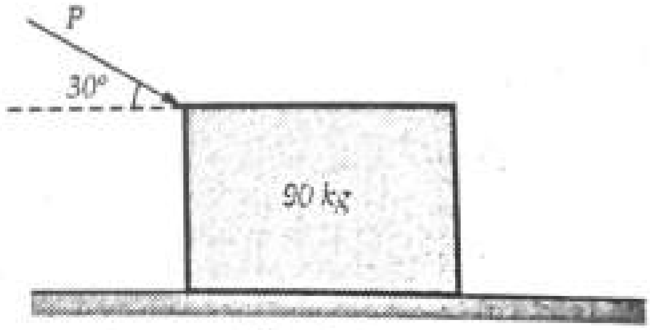

Fig. Q.7.1

Ans. : The *FBD* of block is shown in Fig. Q.7.1 (a)

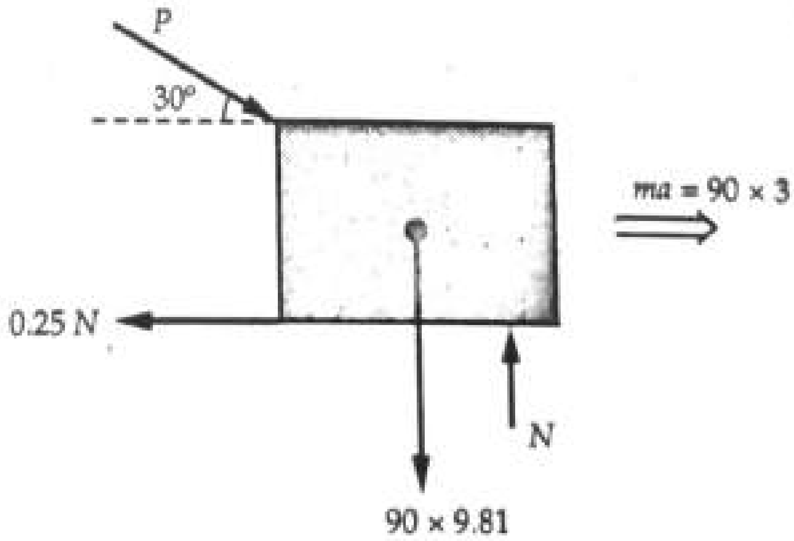

Fig. Q.7.1 (a)

$$\sum F_y = 0 :$$

$$N - 90 \times 9.81 - P \sin 30 = 0$$

$$\therefore \quad N = P \sin 30 + 90 \times 9.81$$

$$\sum F_x = m\, a_x :$$

$$P \cos 30 - 0.25\, N = 90 \times 3$$

$$\therefore P \cos 30 - 0.25\,(P \sin 30 + 90 \times 9.81) = 90 \times 3$$

$$P(\cos 30 - 0.25 \sin 30) = + 90 \times 3 + 90 \times 9.81 \times 0.25$$

$$\therefore \quad \boxed{P = 662.22 \text{ N}}$$

... Ans.

Q.8 The 50 kg crate shown in Fig. Q.8.1 rests on horizontal plane for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate does not tip over when it is subjected to a 400 N force, determine the velocity of the crate in 5 s starting from rest. [SPPU : May-10]

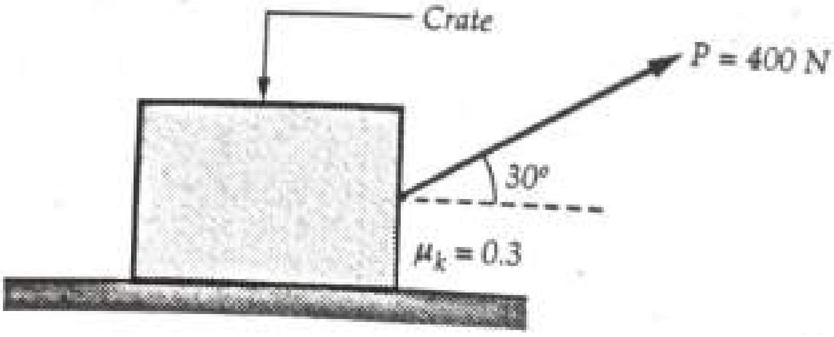

Fig. Q.8.1

Ans. : The F.B.D. of crate is shown in Fig. Q.8.1 (a) 'a' is acceleration of the crate.

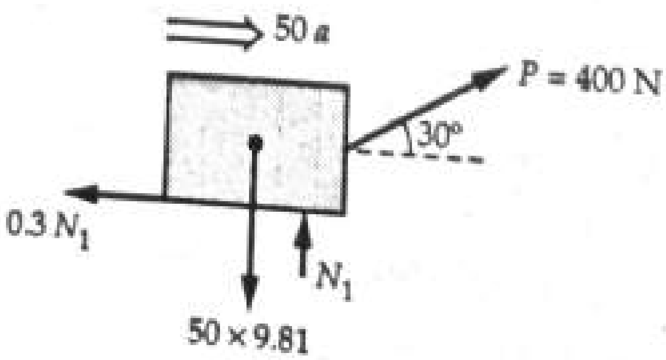

Fig. Q.8.1 (a)

$\sum F_y = 0$

$N_1 + 400 \sin 30 - 50 \times 9.81 = 0$

$\therefore \quad N_1 = 290.5$ N

$\sum F_x = ma_x$:

$400 \cos 30 - 0.3 \times N_1 = 50\,a$

$400 \cos 30 - 0.3 \times 29\,0.5 = 50\,a$

$\therefore \quad a = 5.185 \text{ m/s}^2$

$u = 0$

$t = 5\text{s}$

$v = u + at = 0 + 5.185 \times 5$

$\therefore \quad \boxed{v = 25.925 \text{ m/s}}$ ... Ans.

Q.9 Block A has a weight of 300 N and block B has a weight of 50 N. Determine distance block A must descend from rest before it attains a speed of 2.5 m/s. Neglect mass of pulleys and cord.

[SPPU : May-10]

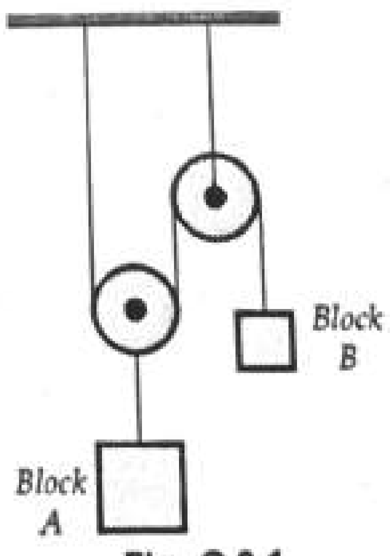

Fig. Q.9.1

Ans. : Consider distances x_A and x_B as shown in Fig. Q.9.1 (a).

$$2x_A + x_B = l$$

where l is the length of the cable which remains constant.

Differentiating w.r.t. 't' twice, we get

$$2\,a_A + a_B = 0$$

Let $a_B = 2a \uparrow$

them $a_A = -a$

$\therefore \quad a_A = a \downarrow$

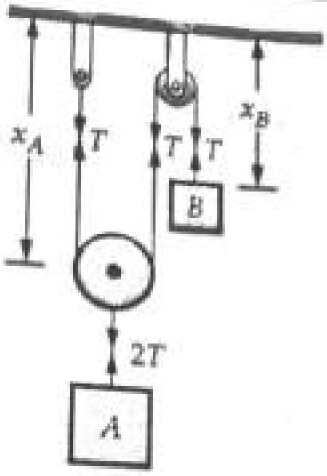

Fig. Q.9.1 (a)

The F.B.Ds of A and B are shown in Fig. Q.9.1 (b).

$\sum F_y = ma_y$ for A :

$$2T - 300 = -\frac{300\,a}{9.81} \quad \ldots(1)$$

For B $\quad T - 50 = \frac{50 \times 2a}{9.81} \quad \ldots(2)$

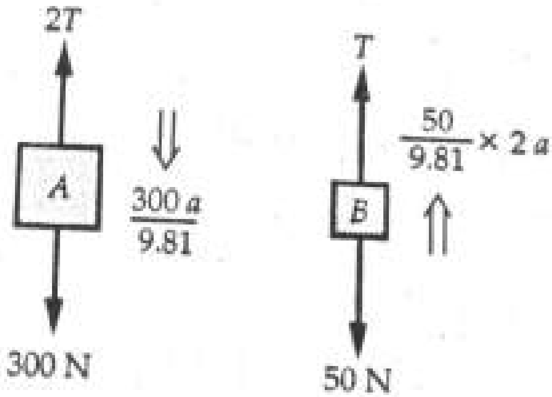

Fig. Q.9.1 (b)

From equations (1) and (2)

$$a = 3.924 \text{ m/s}^2$$

$$\therefore \quad a_A = 3.924 \text{ m/s}^2 \downarrow$$

$$u_A = 0,$$

$$v_A = 2.5 \text{ m/s} \downarrow$$

$$v^2 = u^2 + 2as$$

$$\therefore \quad 2.5^2 = 0^2 + 2\,(3.924)\,s_A$$

$$\therefore \quad \boxed{s_A = 0.796 \text{ m} \downarrow}$$

... Ans.

Q.10 A 50 kg body is initially at rest on a 20° inclined plane with coefficient of kinetic friction μ = 0.25. Find the distance and the time body travels before attaining the speed of 15 m/s. Refer Fig. Q.10.1.

[SPPU : Dec.-11, Marks 6]

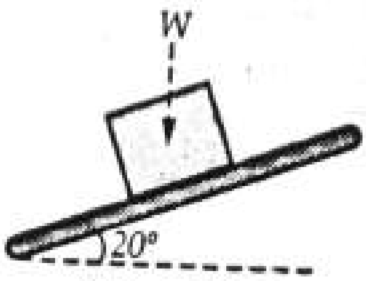

Fig. Q.10.1

Ans. : The F.B.D of the body is shown in Fig. Q.10.1 (a).

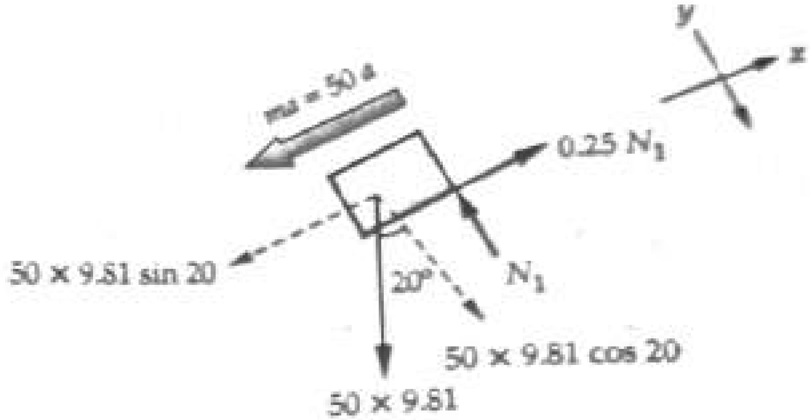

Fig. Q.10.1 (a)

$\sum F_y = 0:$

$$N_1 - 50 \times 9.81 \cos 20 = 0$$

$$N_1 = 50 \times 9.81 \cos 20$$

$\sum F_x = ma_x:$

$$0.25\, N_1 - 50 \times 9.81 \sin 20 = -50\, a$$

$$0.25 \times 50 \times 9.81 \cos 20 - 50 \times 9.81 \sin 20 = -50\, a$$

$\therefore \quad a = 1.05 \text{ m/s}^2, u = 0, v = 15 \text{ m/s}$

$$v = u + at:$$

$$15 = 0 + 1.05\, t$$

$\therefore \quad \boxed{t = 14.29 \text{ s}}$ ... Ans.

$$v^2 = u^2 + 2as$$

$$15^2 = 0^2 + 2 \times 1.05\, s$$

$\therefore \quad \boxed{s = 107.14 \text{ m}}$ ... Ans.

Q.11 Two weights 100 N and 30 N are connected by a string and move along a rough horizontal plane under the action of force 50 N applied to the first weight 100 N as shown in Fig. Q.11.1. The coefficient of friction between the sliding surfaces of the weights and plane is 0.25. Determine the acceleration of weights and the tension in the string using Newton's second law.

[SPPU : Dec.-12, Marks 6]

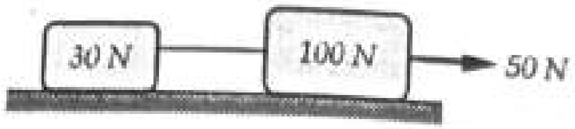

Fig. Q.11.1

Ans. : The free body diagrams of the two blocks are shown in Fig. Q.11.1 (a).

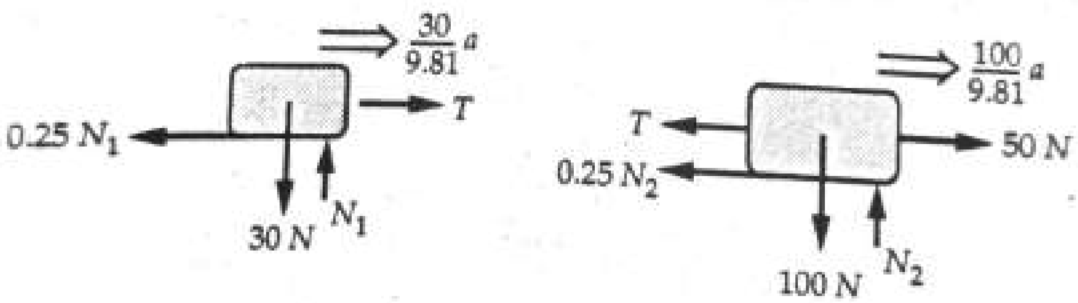

Fig. Q.11.1 (a)

For 30 N block, $\sum F_y = 0$;

$$N_1 - 30 = 0$$

$$\therefore \quad N_1 = 30 \text{ N}$$

$$\sum F_x = ma_x$$

$$T - 0.25\, N_1 = \frac{30}{9.81} a$$

$$\therefore \quad T - 0.25 \times 30 = \frac{30}{9.81} a \qquad \dots (1)$$

For 100 N block, $\sum F_y = 0$;

$$N_2 - 100 = 0$$

$$\therefore \quad N_2 = 100 \text{ N}$$

$$\sum F_x = ma_x :$$

$$\therefore \quad 50 - T - 0.25\,N_2 = \frac{100}{9.81}a$$

$$\therefore \quad 50 - T - 0.25 \times 100 = \frac{100}{9.81}a$$

$$\therefore \quad -T + 25 = \frac{100}{9.81}a \qquad \text{... (2)}$$

From equations (1) and (2),

$$\boxed{T = 11.54 \text{ N}} \qquad \boxed{a = 1.32 \text{ m/s}^2} \qquad \text{... Ans.}$$

Q.12 The 4.5×10^6 kg tanker is pulled with constant acceleration of 0.001 m/s^2 using cable that makes an angle of 15° with the horizontal as shown in Fig. Q.12.1. Determine the force in the cable using Newton's second law of motion.

[SPPU : May-13, Marks 4]

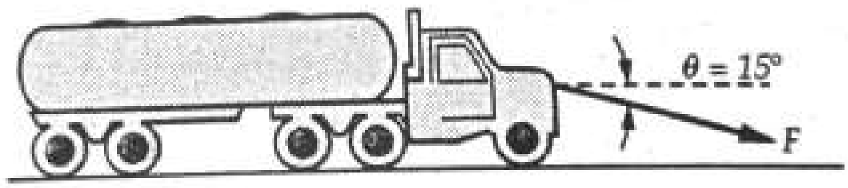

Fig. Q.12.1

Ans. : The horizontal component of force is $F \cos 15$.

Using $\sum F_x = m\,a_x$

$$F \cos 15 = 4.5 \times 10^6 \times 0.001$$

$$\therefore \quad F = 4.66 \times 10^3 \text{ N}$$

$$\therefore \quad \boxed{F = 4.66 \text{ kN}} \qquad \text{... Ans.}$$

Q.13 The tanker is pulled with constant acceleration of 0.001 m/s^2 using cable that makes an angle of 10° with the horizontal as shown in Fig. Q.13.1. If the force in the cable is 45.694 kN, determine the mass of tanker using Newton's 2^{nd} law of motion.

[SPPU : Dec.-13, Marks 4]

Fig. Q.13.1

Ans. :

$$\sum F_x = ma_x$$

$\therefore \quad 45.694 \times 10^3 \cos 10 = m \times 0.001$

$\therefore \quad \boxed{m = 45 \times 10^6 \text{ kg}}$ ... Ans.

Q.14 During a break test, the car of mass 1500 kg is stop from an initial speed of 100 kmph in a distance of 50 m. Determine the breaking force assuming uniform deceleration.

[SPPU : Dec.-14, Marks 4]

Ans. : $u = 100 \text{ km/h} = 100 \times \frac{5}{18} \text{ m/s}, v = 0, s = 50 \text{ m}$

$$v^2 = u^2 + 2as$$

$$0 = \left(100 \times \frac{5}{18}\right)^2 + 2a \times 50$$

$\therefore \quad a = -7.716 \text{ m/s}^2$

The minus sign indicates deceleration.

$$F = ma = 1500 \times 7.716$$

$\therefore \quad \boxed{F = 11574 \text{ N}}$ ... Ans.

Q.15 A 50 kg crate shown in Fig. Q.15.1 rests on a horizontal plane for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate does not tip over when it is subject to a 400 N towing force as shown, determine the velocity of the crate in 5 s starting from rest.

[SPPU : May-15, Marks 4]

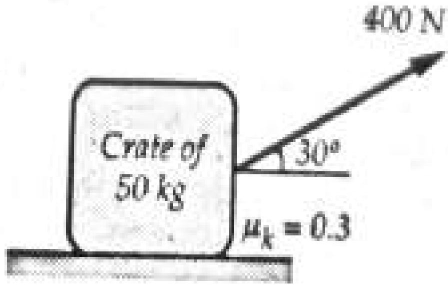

Fig. Q.15.1

Ans. : The FBD of crate is shown in Fig. Q.15.1 (a).

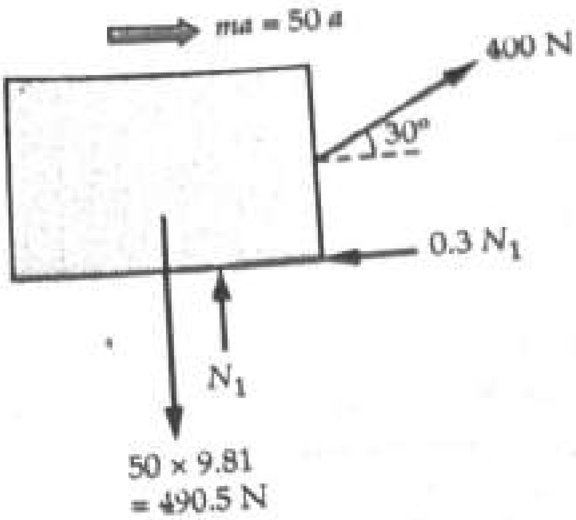

Fig. Q.15.1 (a)

$$\Sigma F_y = 0 :$$

$$N_1 - 490.5 + 400 \sin 30 = 0$$

$$\therefore \quad N_1 = 290.5 \text{ N}$$

$$\Sigma F_x = ma_x :$$

$$400 \cos 30 - 0.3 \text{ N}_1 = 50a$$

$$400 \cos 30 - 0.3 \times 290.5 = 50a$$

$$\therefore \quad a = 5.185 \text{ m/s}^2$$

$$t = 5 \text{ s}, u = 0$$

$$v = u + at$$

$$v = 0 + 5.185 \times 5$$

$$\boxed{v = 25.925 \text{ m/s}}$$ ... Ans.

Q.16 Block B rest on smooth surface. If the coefficient of static and kinetic friction between A and B are μ_s = 0.4 and μ_k = 0.3 respectively, determine the acceleration of each block if a block A is push with a force F : a) 30 N b) 250 N. Refer Fig. Q.16.1.

[SPPU : May-16, Marks 4]

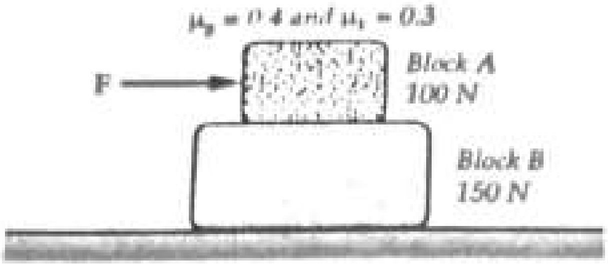

Fig. Q.16.1

Ans. : Force required for A to move on the surface of B is,

$$\mu_s \, mg = 0.4 \times 100 = 40 \text{ N}$$

For $F = 30$ N, A will not move with respect to B. Hence, frictional force between A and B will be 30 N and the two blocks will move together with same acceleration.

The FBD of A and B together is shown in Fig. Q.16.1 (a)

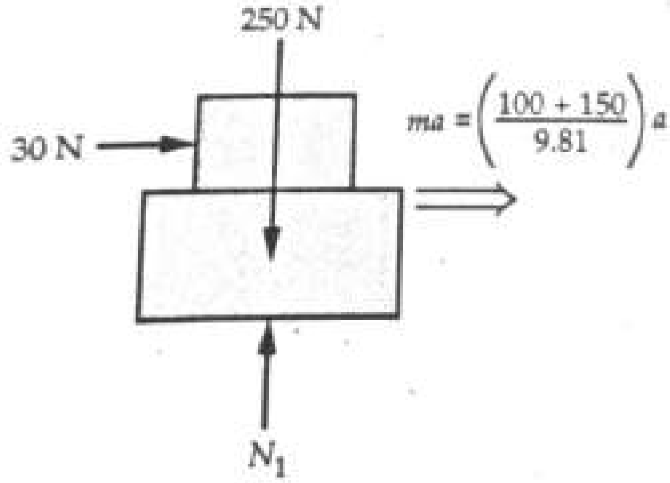

Fig. Q.16.1 (a)

$$\sum F_x = ma_x :$$

$$30 = \left(\frac{100+150}{9.81}\right) a$$

$$\therefore \quad \boxed{a = 1.18 \text{ m/s}^2} \quad \text{... Ans.}$$

For F = 250 N, A will accelerate on the surface of B.

The free body diagrams of A and B for this case are shown in Fig. Q.16.1 (b).

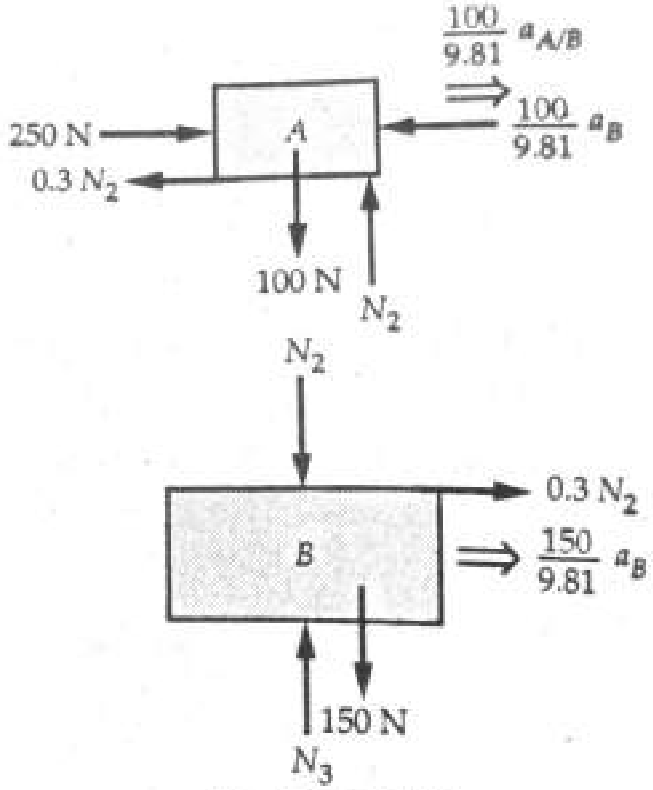

Fig. Q.16.1 (b)

From FBD of A,

$$\sum F_y = 0 : \quad N_2 = 100 \text{ N}$$

$$\sum F_x = ma_x$$

$$250 - 0.3 N_2 - \frac{100}{9.81} a_B = \frac{100}{9.81} a_{A/B}$$

$$250 - 0.3 \times 100 = \frac{100}{9.81} a_{A/B} + \frac{100}{9.81} a_B$$

$$a_{A/B} + a_B = 21.582 \quad \text{...(1)}$$

For B,

$$\sum F_x = ma_x$$

$$0.3 N_2 = \frac{150}{9.81} a_B$$

$$0.3 \times 100 = \frac{150}{9.81} a_B$$

$$\therefore \quad \boxed{a_B = 1.962 \text{ m/s}^2 \rightarrow} \quad \text{... Ans.}$$

From equation (1),

$$a_{A/B} = 19.62 \text{ m/s}^2$$

$$\therefore \quad a_A - a_B = 19.62$$

$$\therefore \quad \boxed{a_A = 21.582 \text{ m/s}^2 \rightarrow}$$

... Ans.

Q.17 The system shown in Fig. Q.17.1 is initially at rest. Neglecting axle friction and mass of pulley, determine the acceleration of block. **[SPPU : Dec.-16, May-17, Marks 4]**

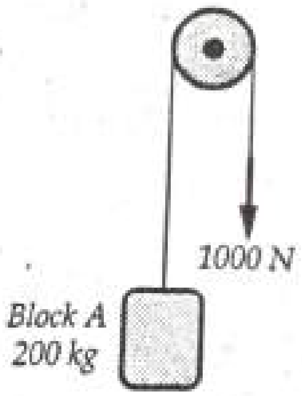

Fig. Q.17.1

Ans. : The tension in cable is same on both sides of the pulley. The FBD of block A is shown in Fig. Q.17.1 (a).

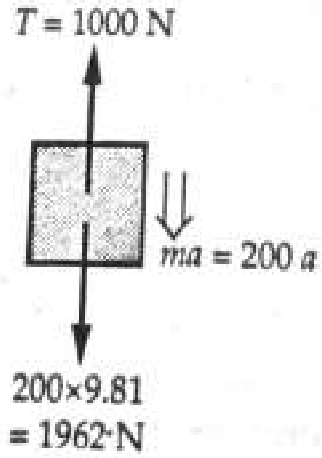

Fig. Q.17.1 (a)

$$\sum F_y = ma_y : \quad 1000 - 1962 = -200 \text{ a}$$

$$\therefore \quad \boxed{\text{a} = 4.81 \text{ m/s}^2}$$

... Ans.

END...

UNIT - VI

15 Kinetics of Curvilinear Motion

Important Points to Remember

- If normal and tangential components are used, the equations of motion would be

$$\sum F_t = ma_t \quad \text{and} \quad \sum F_n = ma_n$$

where $$a_n = \frac{v^2}{\rho}$$

- ma_n is called the centripetal force.
- a_t is directed along the tangent in the direction of velocity and a_n is directed towards the centre of curvature.
- If centrifugal force is used for normal direction, the equations are

$$\sum F_t = ma_t$$

and $$\sum F_n = 0$$

- If radial and transverse components are used , the equations of motion will be

$$\sum F_r = ma_r$$

and $$\sum F_\theta = ma_\theta$$

Q.1 A motor cyclist is moving in a spherical cage of 3.6 m radius in a circus show. The mass of motor cycle and the rider together is 240 kg. What shall be the minimum speed with which the motor cyclist can pass through the highest point without loosing the contact inside the cage ? If he is moving with 36 kmph, what force is transmitted to the cage ? ☞ [SPPU : Dec.-09, Marks 8]

Ans. : The forces acting on the bottle are shown in Fig. Q.2.1.

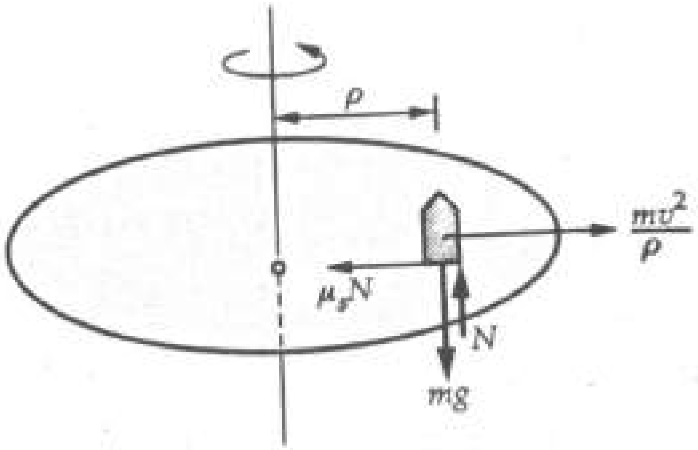

Fig. Q.2.1

$$\sum F_y = 0 :$$

$$N - mg = 0$$

$$\therefore \quad N = mg$$

$$\sum F_x = 0 :$$

$$\frac{mv^2}{\rho} - \mu_s N = 0$$

$$\therefore \quad \frac{mv^2}{\rho} - \mu_s mg = 0$$

$$\therefore \quad v = \sqrt{\mu_s \rho g}$$

$$\mu_s = 0.3, \rho = 0.9 \text{ m}, \ g = 9.81 \text{ m/s}^2 .$$

$$\therefore \quad v = \sqrt{0.4 \times 09 \times 981}$$

$$\boxed{v = 1.63 \text{ m/s}}$$ ...Ans.

Q.3 A small sphere of weight W = 10 N is held as shown by the wires AB and CD. If wire AB is cut, determine radius = 50 m the tension in the other wire a) Before AB is cut, b) Immediately after AB has been cut. Refer Fig. Q.3.1. **[SPPU : May-09 (Old), Marks 8]**

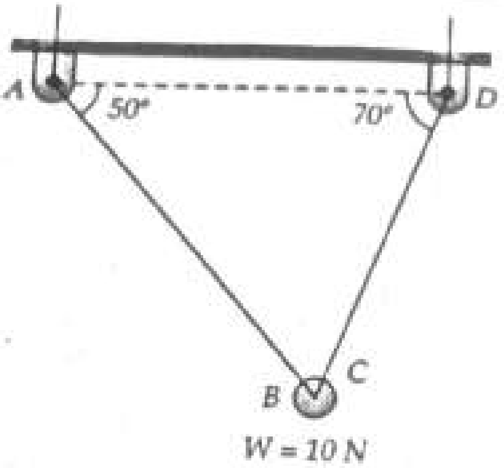

Fig. Q.3.1

Ans. : The *FBD* of sphere just before cutting *AB* is shown in Fig. Q.3.1 (a).

As the sphere is in equilibrium, using Lami's theorem.

$$\frac{T_{AB}}{\sin 160} = \frac{T_{CD}}{\sin 140} = \frac{10}{\sin 60}$$

$$\therefore \quad \boxed{T_{CD} = 7.42 \text{ N}} \quad \text{...Ans.}$$

The F.B.D. of sphere just after cutting *AB* is shown in Fig. Q.3.1 (b). As velocity at the instant *AB* is cut is zero,

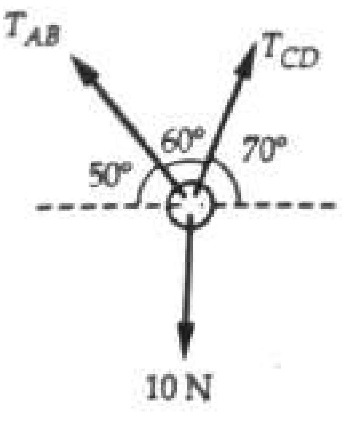

Fig. Q.3.1 (a)

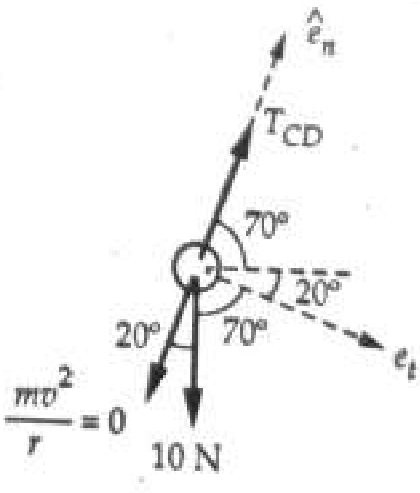

Fig. Q.3.1 (b)

$$\frac{mv^2}{r} = 0$$

$$\sum F_n = 0$$

$$T_{CD} - 10 \cos 20 = 0$$

$$\therefore \qquad \boxed{T_{CD} = 9.4 \text{ N}} \qquad \text{...Ans.}$$

Q.4 An automobile is moving on a hump with radius = 50 m, along a road, at a speed 40 kmph. Knowing that the co-efficient of friction between tyres and road surface is 0.4, determine the tangential acceleration produced if brakes are suddenly applied.

[SPPU : May-09, Marks 5]

Ans. : The FBD of automobile is shown in Fig. Q.4.1.

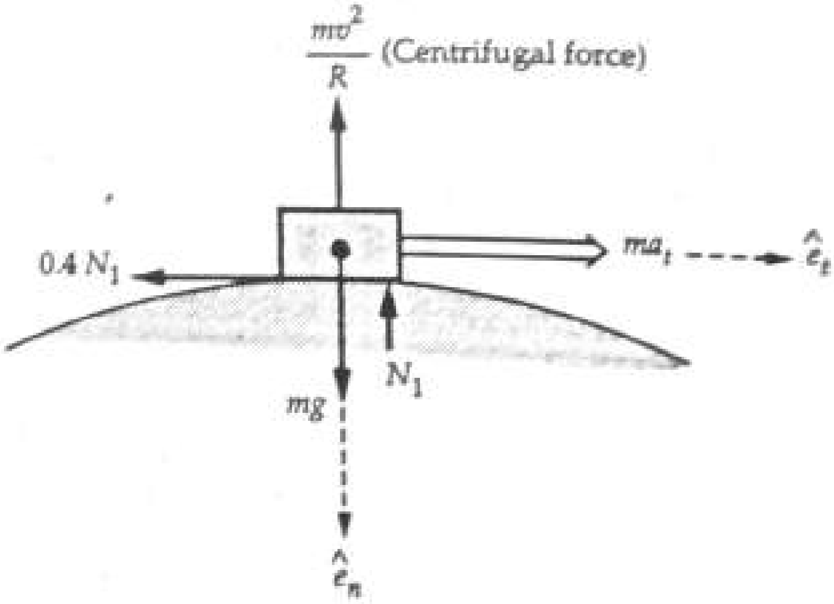

Fig. Q.4.1

$\sum F_n = 0$:

$$mg - \frac{mv^2}{R} - N_1 = 0$$

$$N_1 = mg - \frac{mv^2}{R}$$

$$= m \left[9.81 - \frac{\left(40 \times \frac{5}{18}\right)^2}{50} \right]$$

$$N_1 = 9.463\ m$$

$$\sum F_t = ma_t$$

$$-0.4\,N_1 = ma_t$$

$$-0.4 \times 9.463\ \text{m} = ma_t$$

$$\therefore \quad \boxed{a_t = -3.785\ \text{m/s}^2} \quad \text{...Ans.}$$

Q.5 A small block of mass 100 gm is kept on a rotating disc at radial distance 15 mm from centre of rotation. If co-efficient of friction between the block and surface of disc is 0.2, find out the rotational speed 'ω' at which the block will slide.

[SPPU : May-09, Marks 8]

Ans. : The block kept on the disc shown in Fig. Q.5.1 (a) and the *FBD* of block in Fig. Q.5.1 (b).

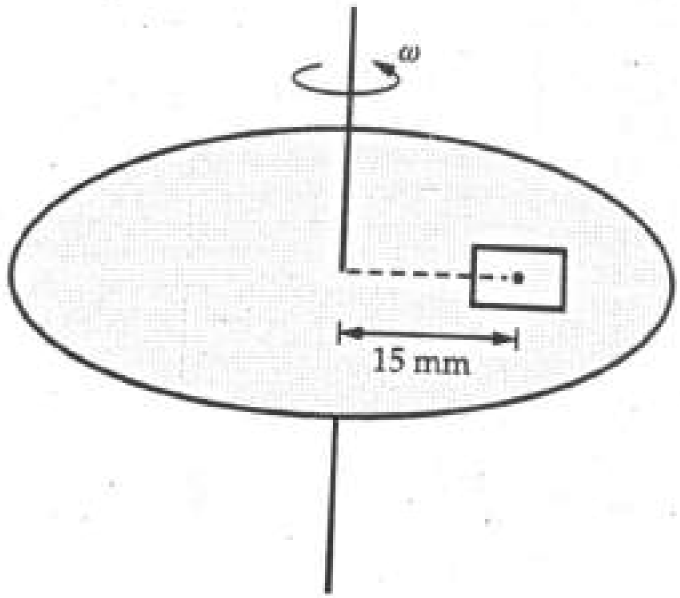

Fig. Q.5.1 (a)

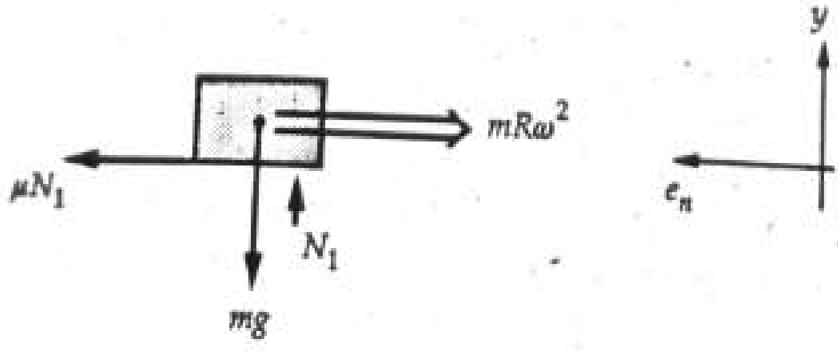

Fig. Q.5.1 (b)

$\sum F_y = 0$:

$$N_1 = mg$$

$\sum F_n = 0$:

$$\mu N_1 - m R\omega^2 = 0$$

$$\mu mg - m R\omega^2 = 0$$

$$\omega = \sqrt{\frac{\mu g}{R}} = \sqrt{\frac{0.2 \times 9.81}{15 \times 10^{-3}}}$$

$$\therefore \quad \boxed{\omega = 11.44 \text{ rad/s}} \quad \text{...Ans.}$$

Q.6 Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5$ m/s^2. If he has a mass of 70 kg, determine the normal force he can exert on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.

[SPPU : May-10, 11]

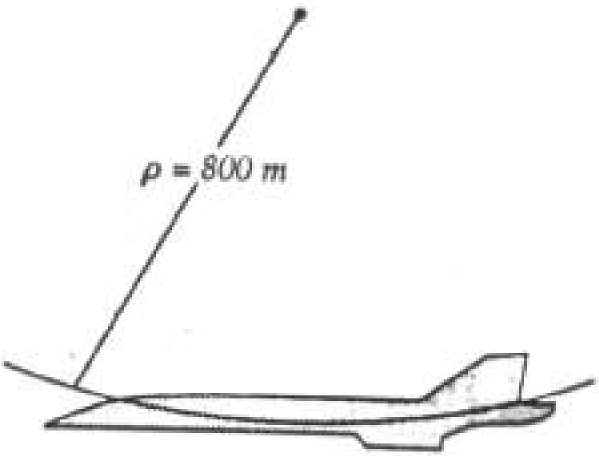

Fig. Q.6.1

Ans. :

$$a_n = \frac{v^2}{\rho}$$

$$a_n = 78.5 \text{ m/s}^2$$

$$\rho = 800 \text{ m}$$

$$\therefore \quad 78.5 = \frac{v^2}{800}$$

$$\therefore \quad \boxed{v = 250.6 \text{ m/s}} \quad \text{...Ans.}$$

The F.B.D. of pilot at the lowest point in the curve is shown in Fig. Q.6.1 (a). N_1 is the normal reaction exerted by the seat on the pilot which is same as the reaction that the pilot exerts on the seat.

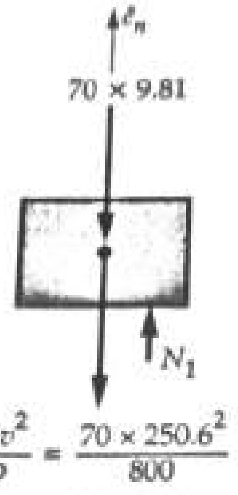

Fig. Q.6.1 (a)

$\sum F_n = 0$:

$$N_1 - 70 \times 9.81 - \frac{70 \times 250.6^2}{800} = 0$$

$\therefore$ $\boxed{N_1 = 6181.73 \text{ N}}$ ...Ans.

Q.7 The bob of a 2 m pendulum describes an arc of a circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown in Fig. Q.7.1 find velocity and acceleration of the bob in that position. [SPPU : May-10, Dec.-11]

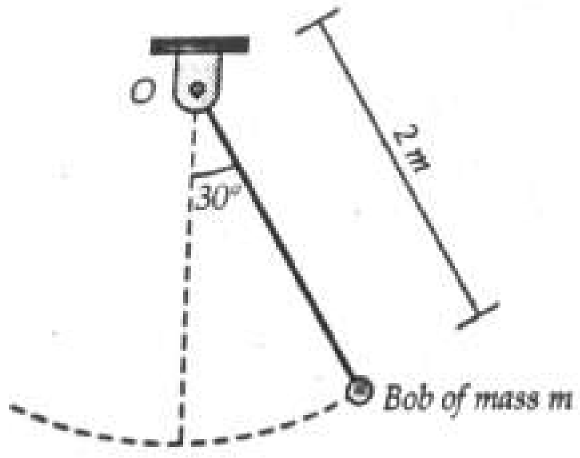

Fig. Q.7.1

Ans. : The F.B.D. of bob in the given position is shown in Fig. Q.7.1 (a).

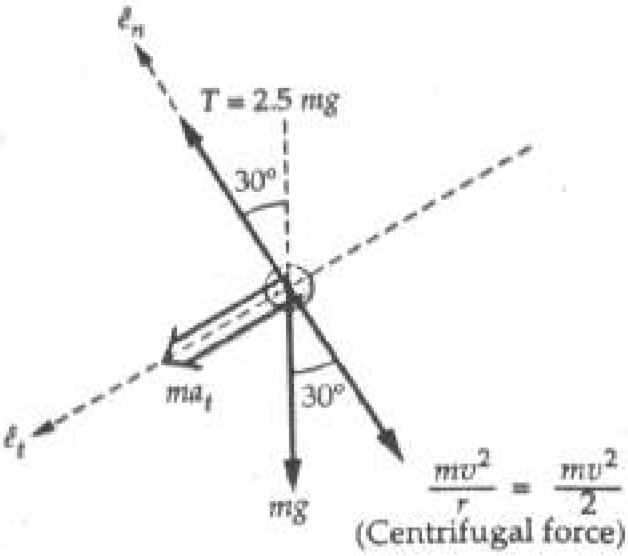

Fig. Q.7.1 (a)

$\sum F_t = ma_t$:

$$mg \sin 30 = ma_t$$

$$\therefore \quad a_t = g \sin 30 = 9.81 \sin 30$$

$$\therefore \quad a_t = 4.905 \text{ m/s}^2$$

$\sum F_n = 0$:

$$T - mg \cos 30 - \frac{mv^2}{r} = 0$$

$$2.5\ mg - mg \cos 30 = \frac{mv^2}{2}$$

$$\therefore \quad \boxed{v = 5.662 \text{ m/s at } 30^\circ}$$

$$a_n = \frac{v^2}{r} = \frac{5.662^2}{2} = 16.03 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.905^2 + 16.03^2}$$

$$\therefore \quad \boxed{a = 16.764 \text{ m/s}}$$

$$\theta_a = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{16.03}{4.905}\right)$$

$$\therefore \quad \theta_a = 73^\circ \text{ with tangential direction}$$

i.e., $$\boxed{\theta_a = 73 - 30° = 43°}$$ ...Ans.

Q.8 A 150 kg car enters a curved portion of the road of radius 200 m travelling at a constant speed of 36 km/h. Determine the normal and tangential component of force at curved portion.
[SPPU : Dec.-12, Marks 6]

Ans. : As the car is travelling at a constant speed, $a_t = 0$

$$\therefore \quad \boxed{F_t = 0}$$

$$a_n = \frac{v^2}{\rho} \; ; F_n = ma_n = \frac{mv^2}{\rho}$$

$v = 36$ km/h $= 10$ m/s, $\rho = 200$ m, $m = 150$ kg

$$\therefore \quad F_n = \frac{150 \times 10^2}{200}$$

$$\therefore \quad \boxed{F_n = 75 \text{ N}}$$ ...Ans.

Q.9 A girl having mass of 25 kg sits at the edge of the merry go-round so her centre of mass G is at a distance of 1.5 m from the centre of rotation as shown in Fig. Q.9.1. Neglecting tangential component of acceleration, determine the maximum speed which she can have before she begins to slip off the merry go-round. The coefficient of static friction is $\mu_s = 0.3$. Use Newton's second law of motion. **[SPPU : May-13, Marks 4]**

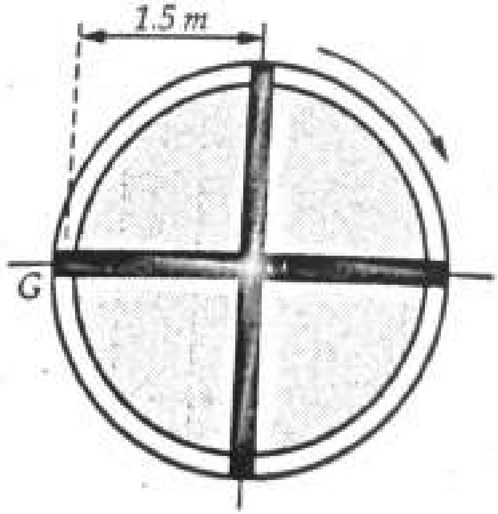

Fig. Q.9.1

Ans. : According to Newton's second law in normal direction,

$$\sum F_n = m\,a_n$$

$$\sum F_n = \mu_s\,m\,g$$

$$m\,a_n = \frac{mv^2}{\rho}$$

$$\therefore \quad \mu_s\,mg = \frac{mv^2}{\rho}$$

$$\therefore \quad v = \sqrt{\mu_s \rho g}$$

$$\mu_s = 0.3,\ \rho = 1.5\text{ m},\ g = 9.81\text{ m/s}^2$$

$$\therefore \quad v = \sqrt{0.3 \times 1.5 \times 9.81}$$

$$\therefore \quad \boxed{v = 2.1\text{ m/s}} \qquad \text{... Ans.}$$

Q.10 The small 0.6 kg block slides on a smooth circular path of radius 3 m in the vertical plane. If the speed of the block is 5 m/s as it passes point A and 4 m/s as it passes point B, determine the normal force exerted on the block by the surface at each of these location. Refer Fig. Q.10.1. **[SPPU : Dec.-13, Marks 4]**

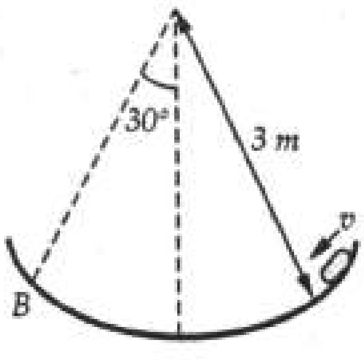

Fig. Q.10.1

Ans. : The F.B.D. of block at *A* with centrifugal is shown in Fig. Q.10.1 (a).

$$\sum F_n = 0 :$$

$$N_A - 0.6 \times 9.81 - \frac{0.6 \times 5^2}{3} = 0$$

$$\therefore \quad \boxed{N_A = 10.886\text{ N}} \qquad \text{...Ans.}$$

The F.B.D. of block at B is shown in Fig. Q.10.1 (b).

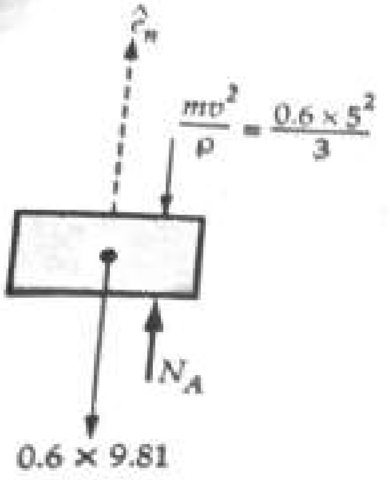

Fig. Q.10.1 (a)

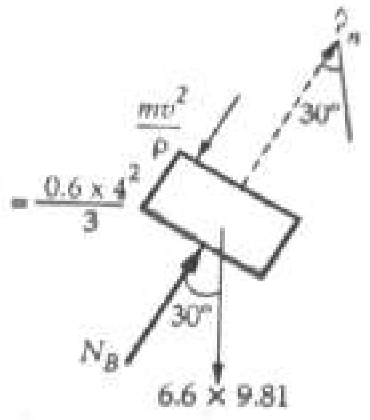

Fig. Q.10.1 (b)

$\sum F_n = 0:$

$$N_B - 0.6 \times 9.81 \cos 30 - \frac{0.6 \times 4^2}{3} = 0$$

$\therefore$ $\boxed{N_B = 8.3 \text{ N}}$ ...Ans.

Q.11 If the crest of the hill has a radius of curvature ρ = 60 m, determine the maximum constant speed at which the car of weight 17.5 kN can travel over it without leaving the surface of the road.

[SPPU : May-15, 17, Dec.-16, Marks 4]

Ans. : $v_{max} = \sqrt{\rho g}$

$$\rho = 60 \text{ m}, g = 9.81 \text{ m/s}^2$$

$\therefore$ $v_{max} = \sqrt{60 \times 9.81}$

$\therefore$ $\boxed{v_{max} = 24.26 \text{ m/s}}$ ...Ans.

Q.12 The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by 0.4 m/s^2. If the coefficient of static friction between his clothes and the platform is, μ_s = 0.3, determine the time required to cause him to slip. Refer Fig. Q.12.1.

[SPPU : May-16, Marks 4]

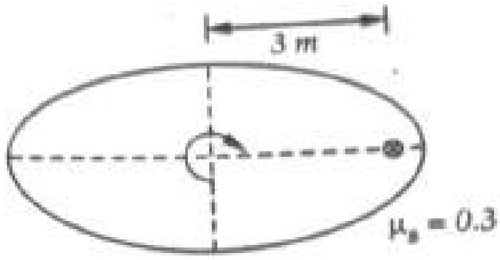

Fig. Q.12.1

Ans. :

$$a_t = 0.4 \text{ m/s}^2$$

$$a_n = \frac{V^2}{r}$$

$$V = \mu + a_t t = 0 + 0.4 \times t$$

$$\therefore \quad V = 0.4\, t$$

$$r = 3 \text{ m}$$

$$\therefore \quad a_n = \frac{(0.4t)^2}{3}$$

Net force on man = ma = $m\sqrt{a_t^2 + a_n^2}$

$$= 80\sqrt{0.4^2 + \frac{0.4^2\, t^2}{9}}$$

Just when the man is about to slip, the net force becomes equal to the maximum frictional force

$$\mu_s \text{ mg} = 0.3 \times 80 \times 9.81 = 235.44 \text{ N}$$

$$\therefore \quad 80\sqrt{0.4^2 + \frac{0.4^2\, t^2}{9}} = 235.44$$

$$0.4^2 + \frac{0.4^2\, t^2}{9} = \left(\frac{235.44}{80}\right)^2$$

$$\therefore \quad \boxed{t = 21.87 \text{ s}}$$ ...Ans.

Q.13 The small ball of mass m and its supporting wire becomes a simple pendulum when the horizontal cord is served. Determine the ratio of the tension T in the supporting wire immediately after the cord is cut to that in the wire before the wire is cut.

[SPPU : Dec.-17, Marks 6]

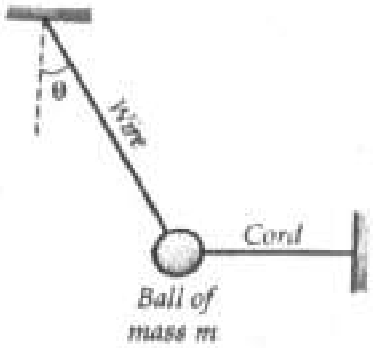

Fig. Q.13.1

Ans. : The ball will be in equilibrium before the wire is cut. The FBD of ball for this situation is shown in Fig. Q.13.1 (a)

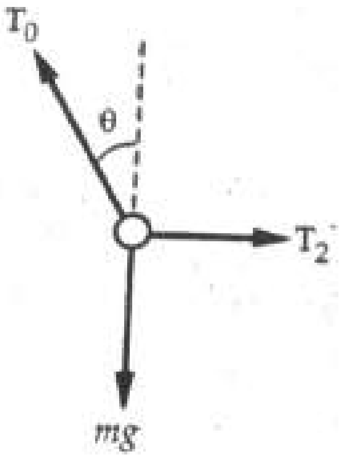

Fig. Q.13.1 (a)

Using $\sum F_y = 0$

$$T_0 \sin\theta - mg = 0$$

$$\therefore \quad T_0 = \frac{mg}{\sin\theta} \qquad \text{...(1)}$$

Just after the cord is cut the ball will begin to move in a circular path. The FBD for this situation is shown in Fig. Q.13.1 (b)

$$\sum F_n = 0 :$$

$$T_1 - mg\cos\theta = 0$$

$$\therefore \quad T_1 = mg\cos\theta \qquad \text{...(2)}$$

From equations (1) and (2).

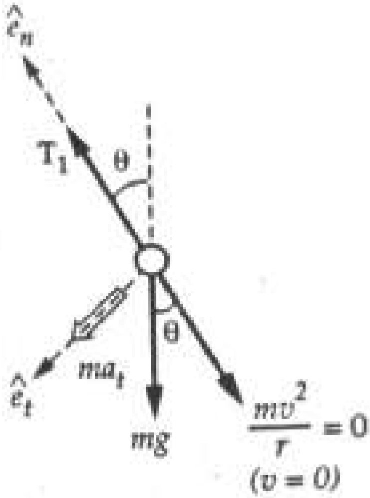

Fig. Q.13.1 (b)

$$\therefore \quad \boxed{\frac{T_1}{T_0} = \cos\theta \sin\theta = \frac{\sin 2\theta}{2}} \quad \text{...Ans.}$$

Q.14 If the 10 kg ball has a velocity of 3 m/s when it is at the position A as shown in Fig. Q.14.1 along the vertical path, determine the tension in the cord and the tangential component of acceleration of ball at this position. [SPPU : May-18, Marks 6]

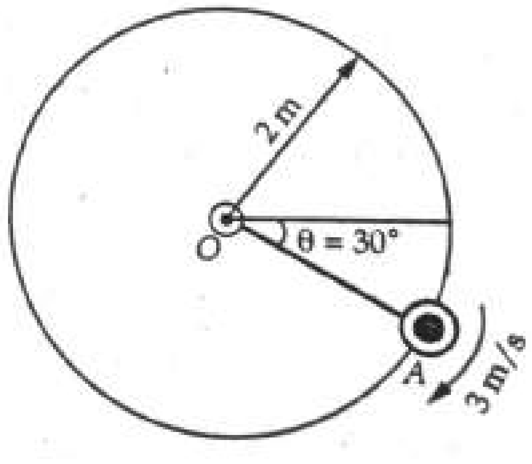

Fig. Q.14.1

Ans. : The forces acting on the ball are shown in Fig. Q.14.1 (a)

$$\sum F_t = m a_t :$$

$$mg \cos 30 = m a_t$$

$$\therefore \quad a_t = 9.81 \cos 30$$

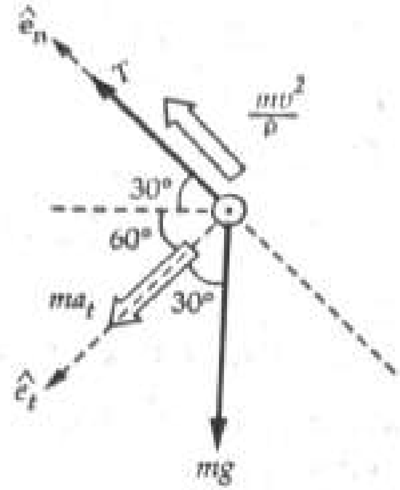

Fig. Q.14.1 (a)

$\therefore$ $$\boxed{a_t = 8.5 \text{ m/s}^2}$$ ...Ans.

$$\sum F_n = m a_n :$$

$$T - mg \sin 30 = \frac{mv^2}{\rho}$$

$$v = 3 \, m/s, \rho = 2 \, m, m = 10 \text{ kg}$$

$\therefore$ $$T = mg \sin 30 + \frac{mv^2}{\rho}$$

$$= 10 \times 9.81 \sin 30 + \frac{10 \times 3^2}{2}$$

$\therefore$ $$\boxed{T = 94.05 \text{ N}}$$ ...Ans.

END...

UNIT - VI

16 Kinetics of Particle : Energy Methods

Important Points to Remember

- If a force of magnitude F causes displacement ds at angle θ with it, the work done is

$$dU = F\,ds\cos\theta$$

- The work done by a force is positive when the component of force parallel to displacement (i.e. $F\cos\theta$) is in the direction of displacement. When they are in opposite directions, the work done is negative.
- **Principle of Work and Energy :** Initial kinetic energy plus the work done by all forces equals the final kinetic energy.

$$K.E_1 + U_{1\text{-}2} = K.E_2$$

- Power is defined as the rate at which work is done.

$$\text{Power} = \vec{F}\cdot\vec{v}$$

- **Conservation of Energy :** Principle of conservation of energy states that under the action of only conservative forces, the sum of kinetic energy and potential energy of a particle remains constant.
- If a particle under the action of only conservative forces is taken from position 1 to position 2,

$$K.E_1 + G.P.E_1 + S.P.E_1 = K.E_2 + G.P.E_2 + S.P.E_2$$

Gravitational Potential Energy (G.P.E) :

$$\text{G.P.E.} = mgh$$

Spring Potential Energy (S.P.E.) :

$$\text{S.P.E.} = \frac{1}{2}kx^2$$

where k = Spring constant and

x = Deformation of spring.

Q.1 A collar of mass 1.2 kg slides along a smooth path AB in vertical plane as shown in Fig. Q.1.1. The collar starts from rest 'A' under the action of constant horizontal force of 10 N. Calculate its velocity as it hits at 'B'.

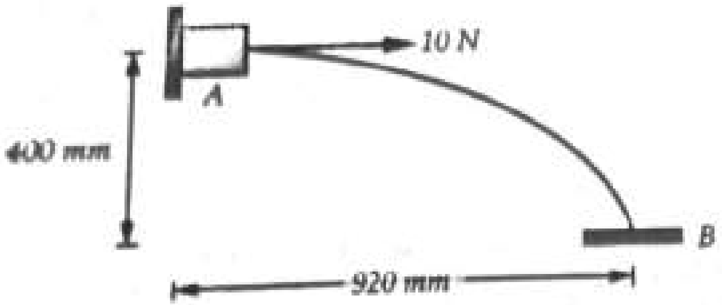

Fig. Q.1.1

[SPPU : Dec.-11, Marks 7]

Ans. : By work energy principle,

$K.E_A$ + Work done by all forces = $K.E_B$

$$K.E_A = 0, \qquad K.E_B = \frac{1}{2} \times 1.2\, v_B^2$$

Work done by gravity = $1.2 \times 9.81 \times 0.4$

Work done by 10 N force = 10×0.92

$$\therefore \quad 0 + (1.2 \times 9.81 \times 0.4 + 10 \times 0.92) = \frac{1}{2} \times 1.2\, v_B^2$$

$$\therefore \quad \boxed{v_{AB} = 4.815 \text{ m/s}}$$ ... Ans.

Q.2 A 2 kg stone is dropped from a height h and strikes the ground with a velocity of 24 m/s. Using work energy principle find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped.

[SPPU : May-12, Marks 6; Dec.-13, Marks 4]

Ans. : At the top

$$K.E_1 = 0$$

At the ground,

$$K.E_2 = \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times 24^2$$

$$\therefore \quad \boxed{K.E_2 = 576 \text{ J}}$$ ...Ans.

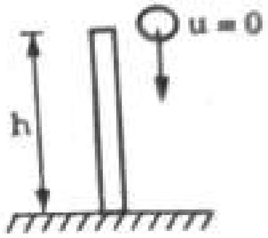

Fig. Q.2.1

By work-energy principle,

$K.E_1$ + Work done by all forces = $K.E_2$

Only gravitational force acts on the stone.

Work done by gravity = $mgh = 2 \times 981 \times h$

$$\therefore \quad 0 + 2 \times 981h = 576$$

$$\therefore \quad \boxed{h = 29.36 \text{ m}}$$ ...Ans.

Q.3 A car of mass 1500 kg is moving down a hill having a slope of 15° to the horizontal. At the time, when the car is moving at a speed of 10 m/s, the driver applies the brakes. Calculate the average force applied parallel to the hill slope that will stop the car in a distance of 30 m. Use work energy principle.

[SPPU : Dec.-12, Marks 6]

Ans. : When brakes are applied the frictional force stops the car. The F.B.D. of car is shown in Fig. Q.3.1. The average force that stops the car is F.

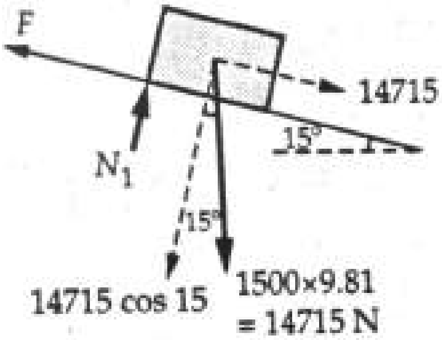

Fig. Q.3.1

$$\text{Initial K.E.} = \frac{1}{2} \times 1500 \times 10^2$$

$$\text{Final K.E.} = 0$$

Work done by all forces = 14715 sin 15 × 30 − F × 30

By work-energy principle,

Initial K.E. + Work done by all forces = Final K.E.

$$\therefore \quad \frac{1}{2} \times 1500 \times 10^2 + [14715 \sin 15 \times 30 - F \times 30] = 0$$

$$\therefore \quad \boxed{F = 6308.5 \text{ N}}$$ ...Ans.

Q.4 A woman having a mass of 70 kg stands in an elevator which has a downward acceleration of 4 m/s^2 starting from rest. Determine the work done by her weight and the normal force which the floor exerts on her when the elevator descends 6 m.

[SPPU : May-13, Dec-16, Marks 4, Dec.-17, Marks 6]

Ans. : As the weight acts in downward direction and the displacement is also downwards, the works done is positive.

$$W = m g \times h = 70 \times 9.81 \times 6$$

$$\boxed{W = 4120.2 \text{ J}} \qquad \text{...Ans.}$$

$$\text{Normal force } N_1 = mg - ma = 70 \times 9.81 - 70 \times 4$$

$$\therefore \quad N_1 = 406.7 \text{ N} \uparrow$$

$$\therefore \text{ Work done } W_1 = -406.7 \times 6$$

$$\therefore \quad \boxed{W_1 = -2440.2 \text{ J}} \qquad \text{...Ans.}$$

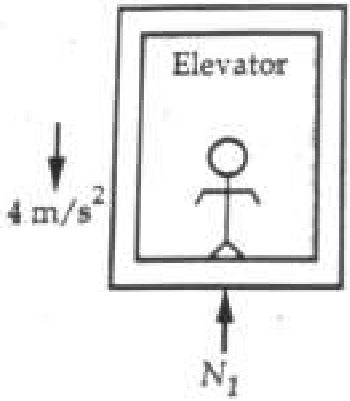

Fig. Q.4.1

Q.5 Two blocks shown in Fig. Q.5.1 have weight 90 N and 45 N. The coefficient of friction between the block A and the horizontal plane is 0.25. If the system is released from rest and the block B moves through a vertical distance of 0.6 m, determine the velocity of the block neglecting the friction in the pulley.

[SPPU : May-14, Marks 4]

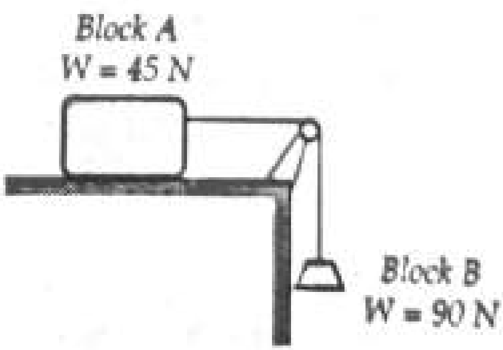

Fig. Q.5.1

Ans. : Using work-energy principle for the system,

Initial K.E. + Work done by all forces = Final K.E.

$$0 + [90 \times 0.6 - 0.25 \times 45 \times 0.6] = \frac{1}{2} \times \frac{90}{9.81} v^2 + \frac{1}{2} \times \frac{45}{9.81} v^2$$

$$\therefore \quad \boxed{v = 2.62 \text{ m/s}^2} \quad \text{...Ans.}$$

Q.6 Determine the work done by all forces acting on the block of 18 kg as shown in Fig. Q.6.1 as it moves 12 m upwards along the plane. Take coefficient of kinetic friction as 0.2.

[SPPU : May-14, Marks 4]

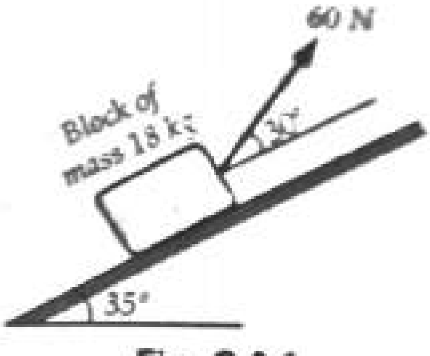

Fig. Q.6.1

Ans. : Work done by 60 N force is

$$W_1 = 60 \cos 30 \times 12$$

$$\therefore \quad \boxed{W_1 = 623.54 \text{ J}} \quad \text{...Ans.}$$

Work done by gravity is

$$W_2 = -18 \times 9.81 \times 12 \sin 35$$

$$\therefore \quad \boxed{W_2 = -1215.38 \text{ J}} \quad \text{...Ans.}$$

Work done by friction is

$$W_3 = -0.2 \times (18 \times 9.81 \cos 30 - 60 \sin 30) \times 12$$

$$\therefore \quad \boxed{W_3 = -295 \text{ J}} \quad \text{...Ans.}$$

Total work done by all forces

$$W = W_1 + W_2 + W_3 = 623.54 - 1215.38 - 295$$

$$\therefore \quad \boxed{W = -886.84 \text{ J}} \quad \text{...Ans.}$$

Q.7 The small collar of mass 0.5 kg is released from rest at A and strikes the base B with velocity 4.7 m/s as shown in Fig. Q.7.1. determine the work done by frictional force using work energy principle.

[SPPU : Dec.-14, Marks 4]

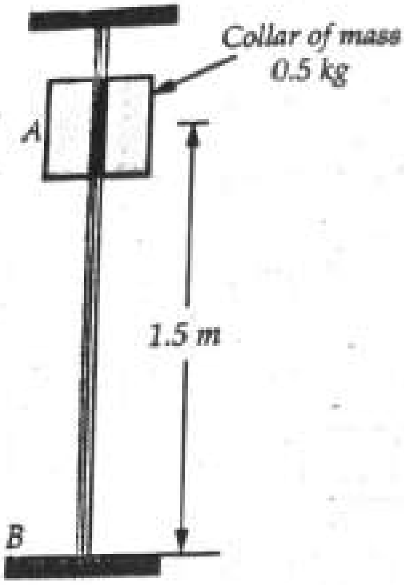

Fig. Q.7.1

Ans. : By work-energy principle,

$$K.E_A + \text{Work done by all forces} = K.E_B$$

$$K.E_A = 0$$

$$\text{Work done by gravity} = 0.5 \times 9.81 \times 1.5 = 7.3575 \text{ J}$$

$$\text{Work done by friction} = -W_f$$

$$K.E_B = \frac{1}{2} \times 0.5 \times 4.7^2 = 5.5225 \text{ J}$$

$$\therefore \quad 0 + [7.3575 - W_f] = 5.5225$$

$$\therefore \quad \boxed{W_f = 1.835 \text{ J}}$$ ...Ans.

Q.8 Two blocks are connected by an inextensible string as shown in Fig. Q.8.1. If the system is released from rest, determine the velocity of the block A after it has moved 2 m by work energy principle. The coefficient of friction between block A and the plane is μ_s = 0.25.

[SPPU : Dec.-15, Marks 4]

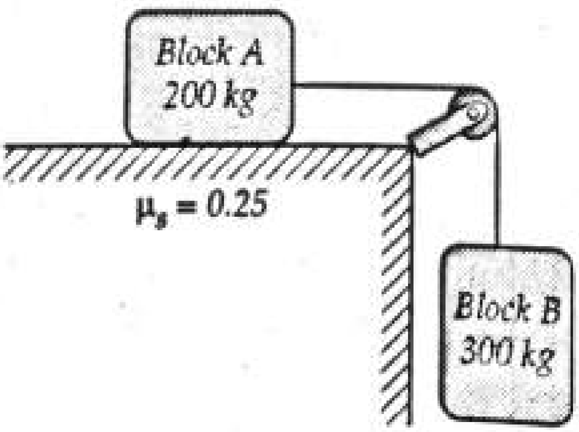

Fig. Q.8.1

Ans. : By work-energy principle,

$$\text{Work done by all forces} = K.E_f - K.E_i$$

$$K.E_i = 0 \quad \text{and} \quad v_A v_B = v$$

$$\therefore \quad m_B g \times s - \mu_K m_A g \times s = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2$$

$$300 \times 9.81 \times 2 - 0.25 \times 200 \times 9.81 \times 2 = \frac{1}{2} \times 200\, v^2 + \frac{1}{2} \times 300\, v^2$$

$$\therefore \quad \boxed{v = 4.429 \text{ m/s}} \quad \text{...Ans.}$$

Q.9 Block A has a weight of 300 N and block B has a weight of 50 N. Determine the speed of block A after it moves 1.5 m down the plane, starting from rest by work energy principle. Neglect the friction and mass of the pulleys.

Refer Fig. Q.9.1. **[SPPU : May-16, Marks 4]**

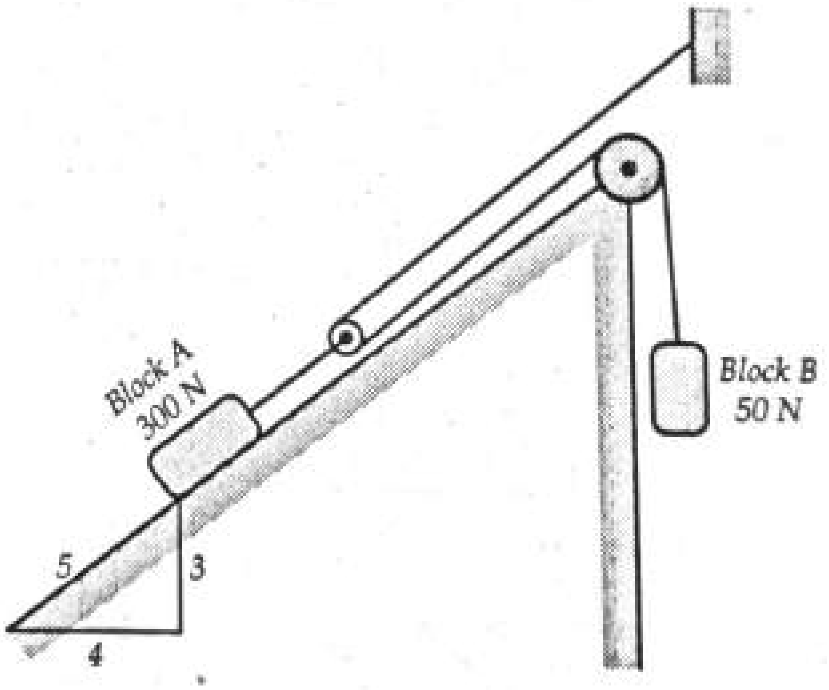

Fig. Q.9.1

Ans. : The external forces acting on the system are shown in Fig. Q.9.1 (a)

If A travels 1.5 m down the plane, B travels 3 m upward. Also, if v is speed of A, speed of B will be $2v$.

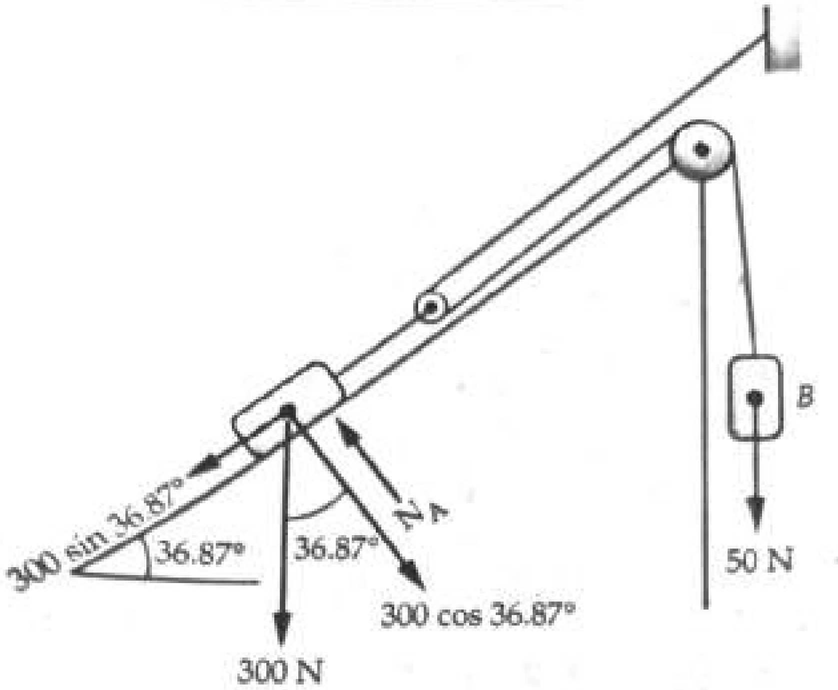

Fig. Q.9.1 (a)

R_y work - energy principle,

Initial K.E. + Work done by all forces = Final K.E

$$\text{Initial K.E} = 0$$

$$\text{Work done} = 300 \sin 36.87 \times 1.5 - 50 \times 3 = 120 \text{ J}$$

$$\text{Final K.E} = \frac{1}{2}\left(\frac{300}{9.81}\right)v^2 + \frac{1}{2}\left(\frac{50}{9.81}\right)(2v)^2 = 25.484\,v^2$$

$$\therefore \quad 0 + 120 = 25.484\,v^2$$

$$\therefore \quad \boxed{v = 2.17 \text{ m/s}}$$ ...Ans.

Q.10 The simple pendulum as shown in Fig. Q.10.1 is released from rest at A with the string horizontal and swings downward under the influence of gravity. Express the velocity v of the bob and the tension T in the string as a function of θ. [SPPU : May-08, Dec.-14, Marks 4]

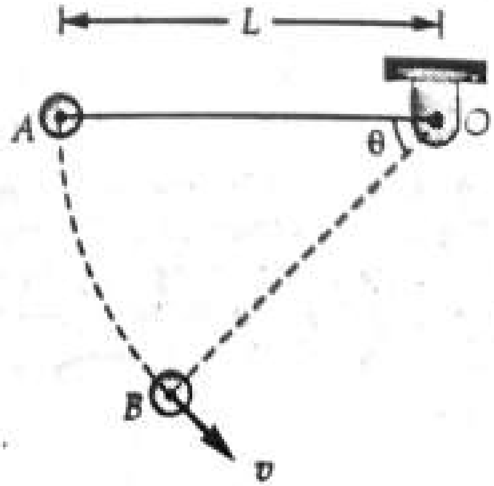

Fig. Q.10.1

Scanned by CamScanner

Ans. : From Fig. Q.10.1 (a),

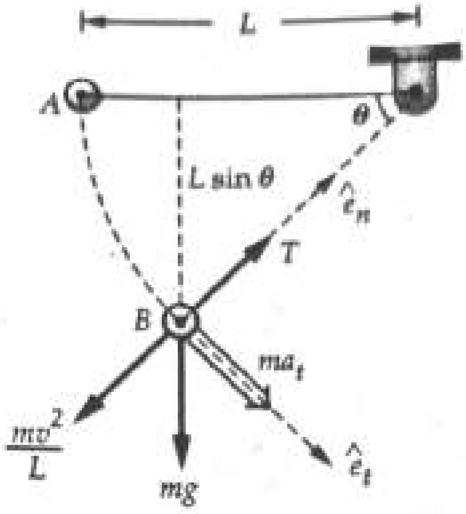

Fig. Q.10.1 (a)

$$\sum F_n = 0 \text{ at } B :$$

$$T - \frac{mv^2}{L} - mg \sin\theta = 0 \qquad \text{... (1)}$$

By conservation of energy,

$$E_A = E_B$$

$$mg\, L\sin\theta = \frac{1}{2} mv^2$$

$$\therefore \quad \boxed{v = \sqrt{2\, g\, L \sin\theta}} \qquad \text{...Ans.}$$

Substitute in equation (1)

$$T = \frac{m}{L}(2\, gL\sin\theta) + mg\, \sin\theta$$

$$\therefore \quad \boxed{T = 3\, mg\, \sin\theta} \qquad \text{...Ans.}$$

Q.11 The double spring bumper is used to stop the 7500N steel billet in a rolling mill. Determine the stiffness $k = k_1 = k_2$ of each spring so that no spring is compressed more than 0.06 m after it is struck by the billet travelling with a speed of 2.4 m/s. Neglect the mass of the springs, rollers and the plates A and B. Refer Fig. Q.11.1. **[SPPU : May-10]**

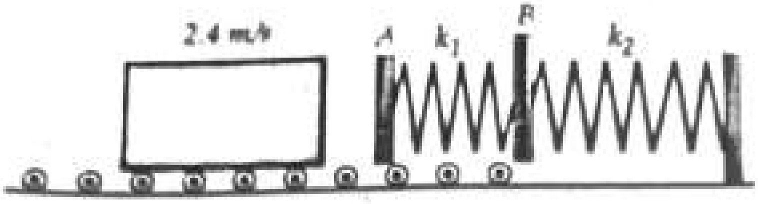

Fig. Q.11.1

Ans. : Both springs will be compressed by 0.06 m in the final position of the billet when it comes to rest. By conservation of energy,

$$\frac{1}{2} m v_1^2 = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

As $\quad k_1 = k_2 = k, \quad v_1 = 2.4$ m/s

and $\quad m = \frac{7500}{9.81}$ kg

$$\frac{1}{2}\left(\frac{7500}{9.81}\right)(2.4^2) = \frac{1}{2} k \times 0.06^2 + \frac{1}{2} k \times 0.06^2$$

$\therefore \quad \boxed{k = 6.116 \times 10^5 \text{ N/m}}$ ...Ans.

Q.12 A 2 kg stone is dropped from a height h and strikes the ground with a velocity of 24 m/s. Find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped. **[SPPU : May-15, Marks 4]**

Ans. :

$$\text{K.E.} = \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times 24^2$$

$\therefore \quad \boxed{\text{K.E.} = 576 \text{ J}}$ ...Ans.

By conservation of energy,

$$mgh = \frac{1}{2} m v^2$$

$$\therefore \quad gh = \frac{1}{2} V^2$$

$$9.81 \times h = \frac{1}{2} \times 24^2$$

$\therefore \quad \boxed{h = 29.36 \text{ m}}$ ...Ans.

Q.13 A 1.5 kg collar is attached to a spring and slides without friction along a circular rod in a horizontal plane. The spring has an undeformed length of 150 mm and a constant k = 400 N/m. Knowing that the collar is in equilibrium at A and is given a slight push to get moving, determine the velocity of the collar as it passes point B. Refer Fig. Q.13.1.

[SPPU : May-15, Marks 4]

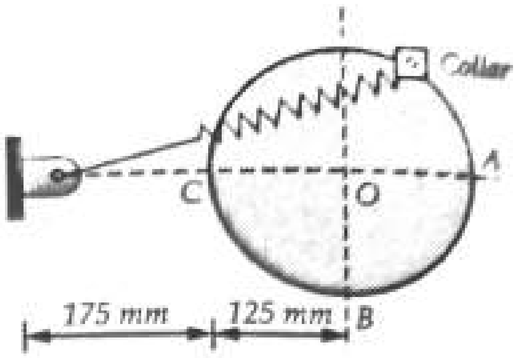

Fig. Q.13.1

Ans. : By conservation of energy,

$$K.E_A + S.P.E_A = K.E_B + S.P.E_B$$

$$K.E_A = 0$$

$$S.P.E_A = \frac{1}{2} kx_A^2 = \frac{1}{2} \times 400 \times (0.425 - 0.150)^2$$

$$= 15.125 \text{ J}$$

$$K.E_B = \frac{1}{2} \times 1.5 \times v_B^2$$

$$S.P.E_B = \frac{1}{2} kx_B^2$$

$$x_B = \sqrt{300^2 + 125^2} - 150 = 175 \text{ mm}$$

$$= 0.175 \text{ m}$$

$$\therefore \quad S.P.E_B = \frac{1}{2} \times 400 \times 0.175^2$$

$$= 6.125 \text{ J}$$

$$\therefore \quad 0 + 15.125 = \frac{1}{2} \times 1.5 v_B^2 + 6.125$$

$$\therefore \quad \boxed{v_B = 3.464 \text{ m/s}}$$ ...Ans.

Q.14 A pendulum bob has a mass of 10 kg and is relased from rest when θ = 0° as shown in Fig. Q.14.1. Determine the tension in the cord at θ = 30°. Neglect the size of bob.

[SPPU : Dec.-15, Marks 4]

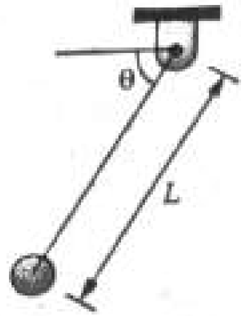

Fig. Q.14.1

Ans. : From Fig. Q.14.1 (a),

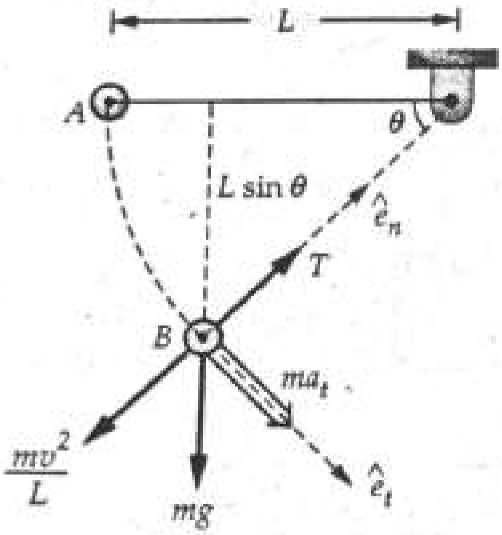

Fig. Q.14.1 (a)

$\sum F_n = 0$ at B :

$$T - \frac{mv^2}{L} - mg \sin\theta = 0 \qquad \text{... (1)}$$

By conservation of energy,

$$E_A = E_B$$

$$mg\, L\sin\theta = \frac{1}{2} mv^2$$

$$\therefore \qquad \boxed{v = \sqrt{2 g L \sin\theta}} \qquad \text{...Ans.}$$

Substitute in equation (1)

$$T = \frac{m}{L}(2\, gL\sin\theta) + mg \sin\theta$$

$$\therefore \quad T = 3\,mg \sin\theta$$

$$\theta = 30°, \; m = 10 \text{ kg}$$

$$\therefore \quad T = 3 \times 10 \times 9.81 \sin 30$$

$$\therefore \quad \boxed{T = 147.15 \text{ N}}$$ ...Ans.

Q.15 A bullet moving at a speed of 300 m/s has its speed reduce to 270 m/s when it passes through a board. Determine how many such boards the bullet will penetrate before it stops.

[SPPU : Dec.-15, Marks 4]

Ans. :

$$\text{Energy of bullet} = \frac{1}{2} m \times 300^2$$

Energy required to penetrate one board

$$= \frac{1}{2} m \times 300^2 - \frac{1}{2} m \times 270^2 = \frac{1}{2} m (300^2 - 270^2)$$

$\therefore$ Number of boards that the bullet will

$$\text{Penetrate} = \frac{\frac{1}{2} m \times 300^2}{\frac{1}{2} m (300^2 - 270^2)} = 5.26$$

$$\therefore \quad \boxed{\text{Number of boards} = 5}$$ ...Ans.

Q.16 The 2 kg pendulum bob is released from rest when it is at A as shown in Fig. Q.16.1. Determine the speed of the bob when it passes through its lowest position B.

[SPPU : May-18, Marks 6]

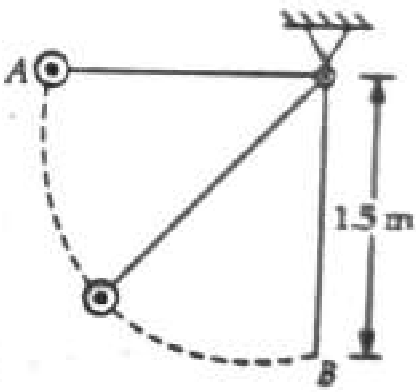

Fig. Q.16.1

Ans. : Using conservation of energy,

$$K.E_A + P.E_A = K.E_B + P.E_B$$

$$0 + m \times 9.81 \times 1.5 = \frac{1}{2} m v_B^2 + 0$$

$$\therefore \quad v_B = \sqrt{2 \times 9.81 \times 1.5}$$

$$\therefore \quad \boxed{v_B = 5.425 \text{ m/s}}$$ ...Ans.

Q.17 A 30 kg block dropped from a height of 2 m onto the 10 kg pan of spring scale as shown in the Fig. Q.17.1. Assuming the collision to be perfectly plastic. Determine the maximum deflection (Compression of the pan). The spring constant is k = 20 kN/m.

[SPPU : Dec.-18, Marks 6]

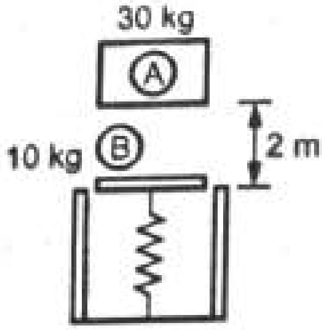

Fig. Q.17.1

Ans. : Applying principle of conservation of energy between 1 and 2 (on block A)

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$\therefore \quad 0 + 30 \times 9.81 \times 2 = \frac{1}{2} \times 30 \times v_{A2}^2 + 0$$

$$\therefore \quad v_{A2} = 6.26 \text{ m/s}$$

For perfectly plastic impact, velocity after impact is same

$$\therefore \quad m_A v_{A2} + m_B v_{B2} = (m_A + m_B) v_3$$

$$30 \times 6.26 + 0 = (30+10) v_3$$

$$\therefore \quad v_3 = 4.7 \text{ m/s}$$

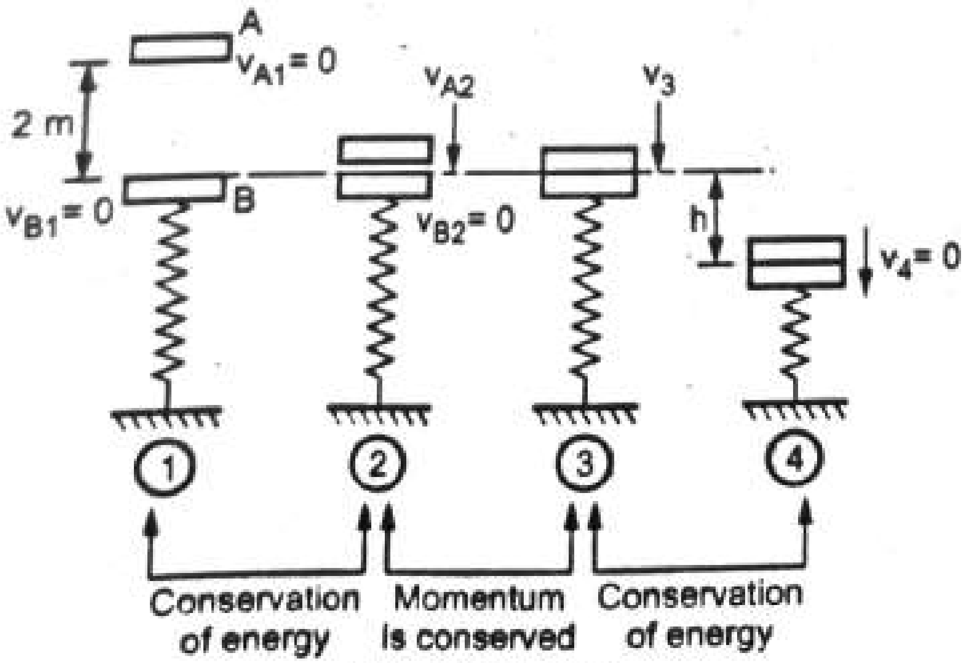

Fig. Q.17.1 (a)

Initial spring deflection due to pan weight is,

$$x_3 = \frac{W_B}{k} = \frac{10\times 9.81}{20\times 10^3} = 4.905\times 10^{-3}\,\text{m}$$

Again applying principle of conservation of energy between 3 and 4,

$$KE_3 + PE_3 = KE_4 + PE_4\ \frac{1}{2}(m_A + m_B)v_3^2 + \left(0 + \frac{1}{2}k\,x_3^2\right)$$

$$= \frac{1}{2}(m_A + m_B)v_4^2 + \left((m_A + m_B)g\,(-h) + \frac{1}{2}k\,x_4^2\right)$$

$$\therefore\ \frac{1}{2}(30+10)\times 4.7^2 + \frac{1}{2}\times 20\times 10^3 \times (4.905\times 10^{-3})^2$$

$$= \frac{1}{2}(30+10)\times 0 + \left[(30+10)\times 9.81\times(-1)(x_4 - x_3) + \frac{1}{2}\times 20\times 10^3 \times x_4^2\right]$$

$$\therefore\quad 442.0405 = -392.4\,(x_4 - 4.905\times 10^{-3}) + 10\times 10^3\,x_4^2$$

Solving the above equation, we get

$$x_4 = 0.23\text{ m}$$

Now, $\quad h = x_4 - x_3 = 0.23 - 4.905\times 10^{-3}$

$$\therefore\quad \boxed{h = 0.225\text{ m}}$$

...Ans.

END...

UNIT - VI

17 Kinetics of Particle : Momentum Methods

Important Points to Remember

Impulse - momentum principle

- According to impulse - momentum principle,

Initial momentum + Impulse due to all forces = Final momentum

$$m\vec{v}_1 + \sum imp_{1-2} = m\vec{v}_2$$

- If $\vec{F}$ is a function of t, i.e.,

$\vec{F} = \vec{F}(t)$, then impulse is obtained by integration

$$\text{imp}_{1-2} = \int_{t_1}^{t_2} \vec{F}(t)dt$$

- If $\vec{F}$ is constant,

$$\text{imp}_{1-2} = \vec{F}\Delta t$$

- If graph of F versus t is given,

$$\text{imp}_{1-2} = \text{Area under } F - t \text{ graph}$$

Conservation of Momentum for a System of Particles

- If a system of two objects A and B is not acted upon by any external force, the total momentum is conserved.

$$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B$$

Impact

- During all types of impacts, momentum is conserved.
- In perfectly elastic impacts, kinetic energy is also conserved.

- Coefficient of restitution is the negative ratio of relative velocity after collision to the relative velocity before collision.

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)$$

- If u is the speed of an object with a rigid surface before collision and v is the speed after collision,

$$v = e\ u$$

- If a ball released from height h_1 bounces from a rigid floor to a height h_2, the coefficient of restitution e is given by

$$e = \sqrt{\frac{h_2}{h_1}}$$

Q.1 The 400 kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200\ t^2)$ N, where t is in seconds. If the car has an initial velocity $v_0 = 2$ m/s when t = 0, determine the distance it moves up the plane when t = 2 s. Ref. Fig. Q.1.1. **[SPPU : May-12, Marks 6]**

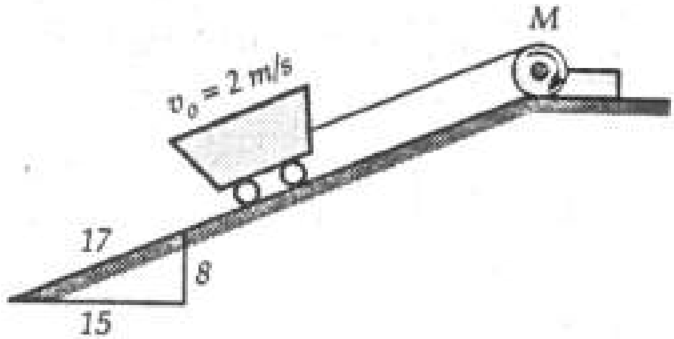

Fig. Q.1.1

Ans. :

- As force is given as a function of t and also the time for which it acts is given, we use impulse-momentum theorem. The F.B.D. of car is shown in Fig. Q.1.1 (a).

$\cos\theta = \frac{15}{17}$ and $\sin\theta = \frac{8}{17}$

- Impulse in positive x-direction is due to F which is variable.

$\therefore$ Impulse due to $F = \int_0^t F\,dt = \int_0^2 3200\,t^2\,dt$

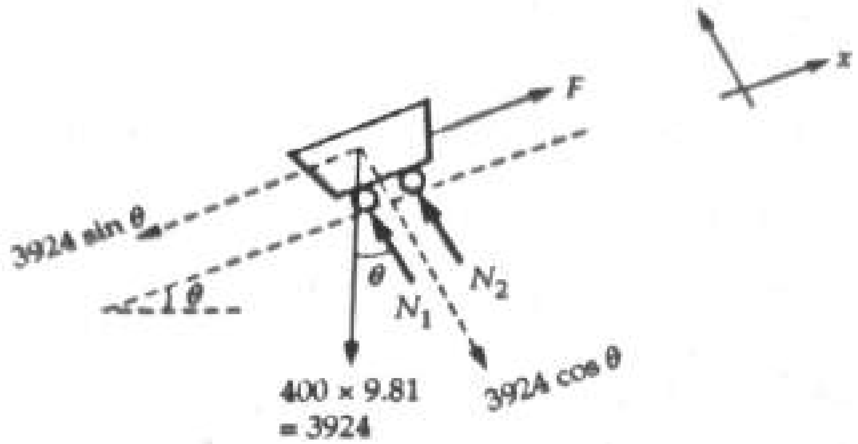

Fig. Q.1.1 (a)

$$= 3200\left[\frac{t^3}{3}\right]_0^2 = 3200 \times \frac{2^3}{3} = 8533.33 \text{ Ns}$$

- The component of weight parallel to plane is a constant force.

$\therefore$ Impulse due to component of weight = 3924 sin θ × Δ t

$$= 3924 \times \frac{8}{17} \times 2 = 3693.18 \text{ Ns}$$

- By impulse momentum theorem,

$$mv_1 + \text{Impulse due to all forces} = mv_2$$

$$v_1 = 2 \text{ m/s}, \quad m = 400 \text{ kg}$$

$$\therefore \quad 400 \times 2 + [8533.33 - 3693.18] = 400\, v_2$$

$$\therefore \quad \boxed{v_2 = 14.1 \text{ m/s}}$$ ... Ans.

Q.2 The force acting on the 250 N crate has a magnitude of F = (12t²) N, where t is in seconds. If the crate starts from rest, determine its speed when t = 5 s. The coefficient of static and kinetic friction between the floor and crate are 0.3 and 0.2 respectively. Refer Fig. Q.2.1.

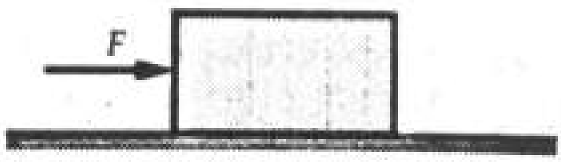

Fig. Q.2.1

[JNTU : SPPU : May-10]

Ans. : • The crate starts moving when

$$F = \mu_s mg$$

Ans. : Using impulse momentum principle,

$$mv_1 + \text{impulse} = mv_2 \quad \text{...(i)}$$

$m = 30$ kg, $v_1 = 15$ m/s, $v_2 = 0$ (Maximum height)

$$\text{Impulse} = mg \times t \downarrow = 30 \times 9.81\, t \downarrow$$

Put in (i)

$$\therefore \quad 30 \times 15 - 30 \times 9.81\, t = 0$$

$$\therefore \quad \boxed{t = 1.529 \text{ s}} \quad \text{... Ans.}$$

The maximum height can be obtained using conservation of energy.

$$\frac{1}{2} mv^2 = m\,g\,h$$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{15^2}{2 \times 9.81}$$

$$\therefore \quad \boxed{h = 11.47 \text{ m}} \quad \text{... Ans.}$$

Q.6 A block weighing 200 N is pulled up a 30° plane by force P producing a velocity of 5 m/s in 5 s. If the coefficient of friction is 0.2, determine the magnitude of force P using impulse momentum principle.

[SPPU : May-14, Marks 4]

Ans. : The forces acting on the block are shown in Fig. Q.6.1. The forces are resolved parallel and perpendicular to direction of motion.

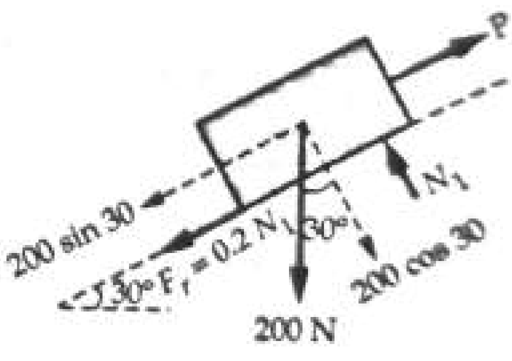

Fig. Q.6.1

$$\sum F_y = 0$$

$$N_1 - 200 \cos 30 = 0$$

$$\therefore \quad N_1 = 200 \cos 30$$

$$F_r = 0.2\, N_1$$

$$\therefore \quad F_r = 0.2 \times 200 \cos 30$$

By impulse - momentum principle,

$$mv_1 + \text{Impulse due to all forces} = mv_2$$

$$m = \frac{200}{9.81}\text{ kg} \ , v_1 = 0 \ , v_2 = 5 \text{ m/s}$$

$$\therefore \quad \frac{200}{9.81}\times 0 + [P\times 5 - 200\sin 30\times 5 - 02\times 200\cos 30\times 5]$$

$$= \frac{200}{9.81}\times 5$$

$$\boxed{P = 155.03 \text{ N}} \quad \text{... Ans.}$$

Q.7 Each of the cable can sustain a maximum tension of 25 kN. If the uniform beam has a weight of 25 kN, determine the shortest time possible to lift the beam with a speed of 3 m/s starting from rest by impulse momentum principle. Refer Fig. Q.7.1.

[SPPU : May-16, Marks 4]

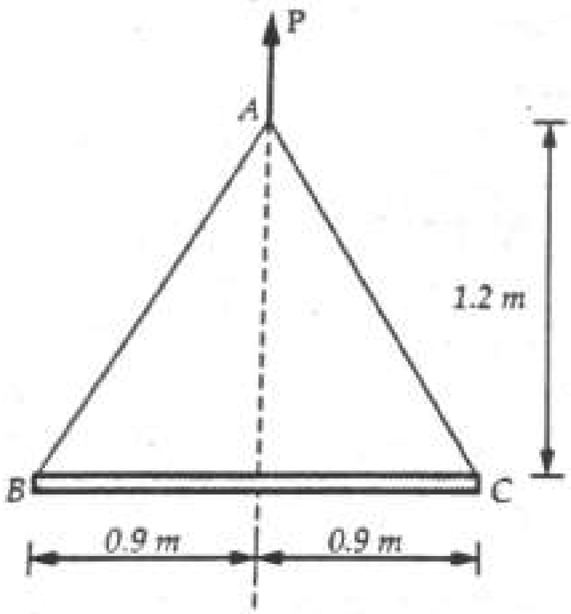

Fig. Q.7.1

Ans. : Force acting on the beam are shown in Fig. Q.7.1 (a).

By impulse momentum principles,

mv_1 + impulse due to all forces = mv_2

$$v_1 = 0$$

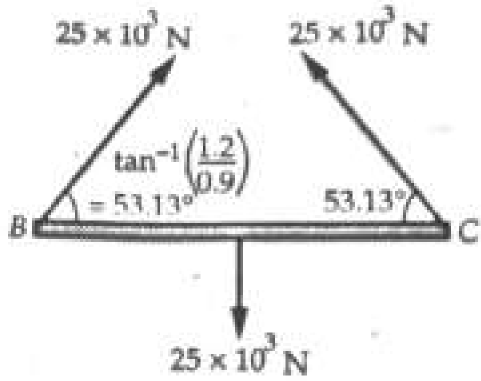

Fig. Q.7.1 (a)

Impulse due to all forces in vertical direction

$$= 25\times10^3 \sin 53.13\times t + 25\times10^3 \sin 53.13\times t - 25\times10^3\, t$$

$$= 15\times10^3\, t$$

$$v_2 = 3 \text{ m/s}$$

$$m = \frac{25\times10^3}{9.81}$$

$$\therefore 0+15\times10^3\, t = \left(\frac{25\times10^3}{9.81}\right)\times3$$

$$\therefore \quad \boxed{t = 0.51 \text{ s}}$$

... Ans.

Q.8 A 40 kg block A is connected to a 60 kg block B by a spring of constant k = 180 N/m. The blocks are placed on a smooth horizontal surface and are at rest when the spring is stretched 2 m. If they are released from rest, determine the speeds of blocks A and B, at the instant the spring becomes unstretched.

[SPPU : May-12, Marks 6]

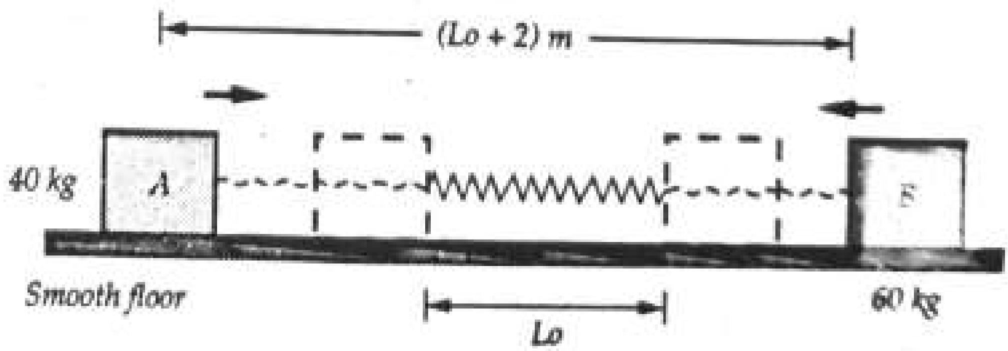

Fig. Q.8.1

Ans. : When spring is released, A moves towards right and B moves towards left. ($u_A = 0$ and $u_B = 0$)

By conservation of momentum,

$$0 = m_A v_A + m_B(-v_B)$$

$$40\,v_A - 60\,v_B = 0$$

$$v_A = 1.5\,v_B \qquad \dots (1)$$

By conservation of energy,

$$\frac{1}{2}kx^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$180 \times 2^2 = 40\,v_A^2 + 60\,v_B^2$$

$$180 \times 2^2 = 40(1.5\,v_B)^2 + 60\,v_B^2$$

As force is given as a function of t and also the time for which it acts is given, we use impulse-momentum theorem. The F.B.D. of cart is shown in Fig. 18.3.1 (a).

$$\therefore \quad \boxed{v_B = 2.19 \text{ m/s} \leftarrow} \qquad \dots \text{Ans.}$$

$$v_A = 1.5 \times 2.19$$

$$\therefore \quad \boxed{v_A = 3.285 \text{ m/s} \rightarrow} \qquad \dots \text{Ans.}$$

Q.9 A 900 kg car travelling at 48 km/h couples to a 680 kg car travelling at 24 km/h in the same direction. Determine their common velocity after coupling. What is the amount of energy lost.

[SPPU : Dec.-11]

Ans. : As collision is plastic, $v_1 = v_2 = v$

By conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(900)\left(48 \times \frac{5}{18}\right) + (680)\left(24 \times \frac{5}{18}\right) = 900\,v + 680\,v$$

$$\therefore \quad \boxed{v = 10.464 \text{ m/s} = 37.67 \text{ km/h}} \qquad \dots \text{Ans.}$$

$$\text{Energy lost} = \left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right) - \frac{1}{2}(m_1 + m_2)\,v^2$$

$$= \frac{1}{2}(900)\left(48 \times \frac{5}{18}\right)^2 + \frac{1}{2}(680)\left(24 \times \frac{5}{18}\right)^2$$

$$- \frac{1}{2}(900 + 680)(10.464^2) = 8609.8 \text{ J}$$

$$\therefore \quad \boxed{\text{Energy lost} = 8.61 \text{ kJ}} \quad \text{... Ans.}$$

Q.10 Two swimmers A and B of mass 75 kg and 50 kg respectively dive off the end of a 200 kg boat. Each swimmer has a relative horizontal velocity of 3 m/s when leaving the boat. If the boat is initially at rest, find its final velocity assuming that both the swimmers dive simultaneously. **[SPPU : May-09, Marks 8]**

Ans. : Let the masses of A, B and boat be

$$m_A = 75 \text{ kg} \quad m_B = 50 \text{ kg} \quad m_b = 200 \text{ kg}$$

By conservation of momentum,

$$m_A u_A + m_B u_B + m_b u_b = m_A v_A + m_B v_B + m_b v_b$$

$$0 + 0 + 0 = 75 \times 3 + 50 \times 3 + 200\, v_b$$

$$\therefore \quad v_b = -1.875 \text{ m/s}$$

i.e. $v_b = 1.875$ m/s opposite to the diretion of velocity of swimmers A and B.

Q.11 A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the car meet and couple together, determine the speed of both cars just after the coupling. **[SPPU : Dec.-13, Marks 4]**

Ans. :

$$m_1 = 15 \text{ Mg} = 15 \times 10^3 \text{ kg}$$

$$u_1 = +1.5 \text{ m/s}$$

$$m_2 = 12 \text{ Mg} = 12 \times 10^3 \text{ kg}$$

$$u_2 = -0.75 \text{ m/s}$$

$$v_1 = v_2 = v_C$$

By conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(15 \times 10^3)(1.5) + (12 \times 10^3)(-0.75) = 15 \times 10^3 v_C + 12 \times 10^3 v_C$$

$$\boxed{v_c = 0.5 \text{ m/s}}$$ ... Ans.

Q.12 Prove that the coefficient of restitution e = 1 for a perfectly elastic collision.

Ans. : In a perfectly elastic collision, momentum and kinetic energy are conserved.

By conservation of momentum for collision of two objects A and B,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$m_A(u_A - v_A) = m_B(v_B - u_B) \qquad \ldots (1)$$

By conservation of kinetic energy,

$$\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$m_A(u_A^2 - v_A^2) = m_B(v_B^2 - u_B^2)$$

$$m_A(u_A - v_A)(u_A + v_A) = m_B(v_B - u_B)(v_B + u_B) \qquad \ldots (2)$$

Dividing equation (1) by equation (2),

$$u_A + v_A = v_B + u_B$$

$$u_A - u_B = v_B - v_A$$

$$u_A - u_B = -(v_A - v_B)$$

$$\therefore \qquad -\left(\frac{v_A - v_B}{u_A - u_B}\right) = 1$$

$$\therefore \qquad \boxed{e = 1}$$ ... Ans.

Q.13 A ball 'A' of mass 0.25 kg, moving on smooth horizontal table with velocity of 10 m/s, strikes on identical stationary ball 'B' on the table. Find the velocity of ball 'B' just after the impact. Consider the impact as perfect plastic. [SPPU : Dec.-09, Marks 8]

Ans. : By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(0.25)(10) + 0 = 0.25 v_A + 0.25 v_B$$

As the impact is perfectly plastic, $v_A = v_B$

$$\therefore \qquad 2.5 = 0.25 v_B + 0.25 v_B$$

$\therefore \quad \boxed{v_B = 5 \text{ m/s}}$ ... Ans.

Q.14 Block A has a mass of 250 kg and is sliding on a smooth surface with an initial velocity of 2 m/s. It makes a direct impact with block B, which has a mass of 175 kg and is originally at rest. If both blocks are of the same size and the impact is perfectly elastic (e = 1), determine the velocity of each block just after impact. Show that the kinetic energy of the blocks before and after impact is the same. **[SPPU : May-10]**

Ans. : Let v_A and v_B be the velocities of the two blocks after collision and u_A and u_B before collision. By conservation of momentum.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$u_A = 2 \text{ m/s}, \quad u_B = 0$$

$$m_A = 250 \text{ g}, \quad m_B = 175 \text{ g}$$

$$\therefore \quad 250 \times 2 + 0 = 250\, v_A + 175\, v_B \quad \text{...(1)}$$

As collision is perfectly elastic, $e = 1$

$$\therefore \quad -\left(\frac{v_A - v_B}{u_A - u_B}\right) = 1$$

$$\therefore \quad -\left(\frac{v_A - v_B}{2 - 0}\right) = 1$$

$$v_A - v_B = -2 \quad \text{...(2)}$$

From equations (1) and (2)

$\therefore \quad \boxed{v_A = 0.353 \text{ m/s}}$ ... Ans.

$\therefore \quad \boxed{v_B = 2.353 \text{ m/s}}$ ... Ans.

Before impact, only A has kinetic energy

$$\therefore \quad K.E_1 = \frac{1}{2}(0.25)(2^2) = 0.5 \text{ J}$$

After impact, both A and B have kinetic energy

$$\therefore \quad K.E_2 = \frac{1}{2}(0.25)(0.353^2) + \frac{1}{2}(0.175)(2.353^2)$$

$$K.E_2 = 0.5 \text{ J}$$

$$\boxed{K.E_1 = K.E_2}$$ ... Ans.

Q.15 Determine the velocities of the two balls shown in Fig. Q.15.1 after impact. Take weight of ball A is 20 N, weight of ball B is 10 N and coefficient of restitution is 0.6.

[SPPU : Dec.-12, Marks 6]

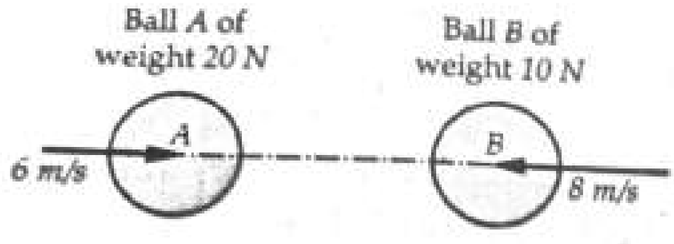

Fig. Q.15.1

Ans. : By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\therefore \quad \left(\frac{20}{981}\right)(6)+\frac{10}{981}(-8) = \left(\frac{20}{981}\right)v_A+\left(\frac{10}{981}\right)v_B$$

$$\therefore \quad 2v_A + v_B = 4 \quad \text{... (1)}$$

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)$$

$$\therefore \quad 0.6 = -\left[\frac{v_A - v_B}{6-(-8)}\right]$$

$$\therefore \quad v_A - v_B = -8.4 \quad \text{... (2)}$$

From equations (1) and (2),

$$v_A = -1.467 \text{ m/s} ; \quad v_B = 6.933 \text{ m/s}$$

$$\therefore \quad \boxed{v_A = 1.467 \text{ m/s}}$$ ... Ans.

$$\therefore \quad \boxed{v_B = 6.933 \text{ m/s}}$$ ... Ans.

Q.16 One of the requirement for tennis balls to be used in official competition is that, when dropped onto a rigid surface from a height of 2540 mm, the height of the first bounce of the ball must be in the range of 1346 mm ≤ h ≤ 1473 mm. Determine the range of the coefficient of restitution of the tennis balls satisfying this requirement. **[SPPU : Dec.-14, Marks 4]**

Ans. :
$$e = \sqrt{\frac{h_2}{h_1}}$$

$$\therefore \quad e_{max} = \sqrt{\frac{1473}{2540}} = 0.7615$$

$$e_{min} = \sqrt{\frac{1346}{2540}} = 0.728$$

$$\therefore \quad \boxed{0.728 < e < 0.7615} \quad \text{... Ans.}$$

Q.17 A ball is dropped from an unknown height on a horizontal floor from which it rebounds to height of 8 m. If e = 0.667 calculate the height from which the ball was dropped. **[SPPU : Dec.-15, Marks 4]**

Ans. :
$$e = \sqrt{\frac{h_2}{h_1}}$$

$$e = 0.667$$

$$h_2 = 8 \text{ m}$$

$$\therefore \quad 0.667 = \sqrt{\frac{8}{h_1}}$$

$$\therefore \quad \boxed{h_1 = 17.98 \text{ m}} \quad \text{... Ans.}$$

Q.18 A tennis ball is dropped from a height 1600 mm and it rebounds to a height 1100 mm. Determine the coefficient of restitution. **[SPPU : May-17, Marks 4]**

Ans. :
$$e = \sqrt{\frac{h_2}{h_1}}$$

$$h_2' = 1100 \text{ mm}, \ h_1 = 1600 \text{ mm}$$

$$\therefore \quad e = \sqrt{\frac{1100}{1600}}$$

$\therefore$ $\boxed{e = 0.83}$ ... Ans.

Q.19 Determine the coefficient of restitution e between two identical balls A and B. The velocities of balls A and B before and after impact are shown in Fig. Q.19.1. [SPPU : May-18, Marks 6]

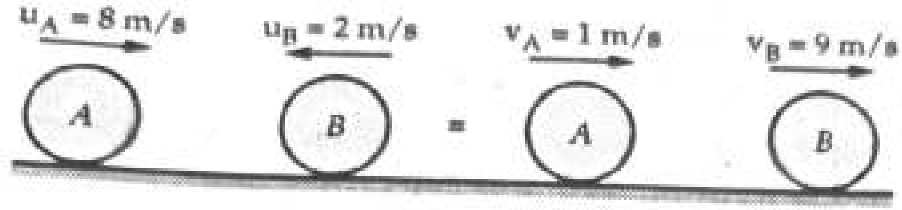

Fig. Q.19.1

Ans. :

$$u_A = +8\ m/s,\ u_B = -2\ m/s$$

$$u_A = +1\ m/s,\ v_B = +9\ m/s$$

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right) = -\left(\frac{1-9}{8+2}\right) = \frac{8}{10}$$

$\therefore$ $\boxed{e = 0.8}$ ... Ans.

Q.20 A 20 Mg railroad car moving with 0.5 m/s speed to the right collides with a 35 Mg car which is at rest. If after the collision the 35 Mg car is observed to move right with a speed of 0.3 m/s, determine the coefficient of restitution between the two cars.

[SPPU : Dec.-18, May-19, Marks 6]

Ans. : By law of conservation of momentum $\begin{bmatrix} \rightarrow + \\ \leftarrow - \end{bmatrix}$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$20 \times 0.5 \times 10^{+3} + 0 = 20 \times 10^{+3} \times v_1 + 35 \times 10^{+3} \times 0.3$$

$\therefore$

$$v_1 = -0.025 \text{ m/s} = 0.025 \text{ m/s} \leftarrow$$

Coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0.3-(-0.025)}{0.5-0}$$

$\therefore$ $\boxed{e = 0.65}$ ... Ans.

END