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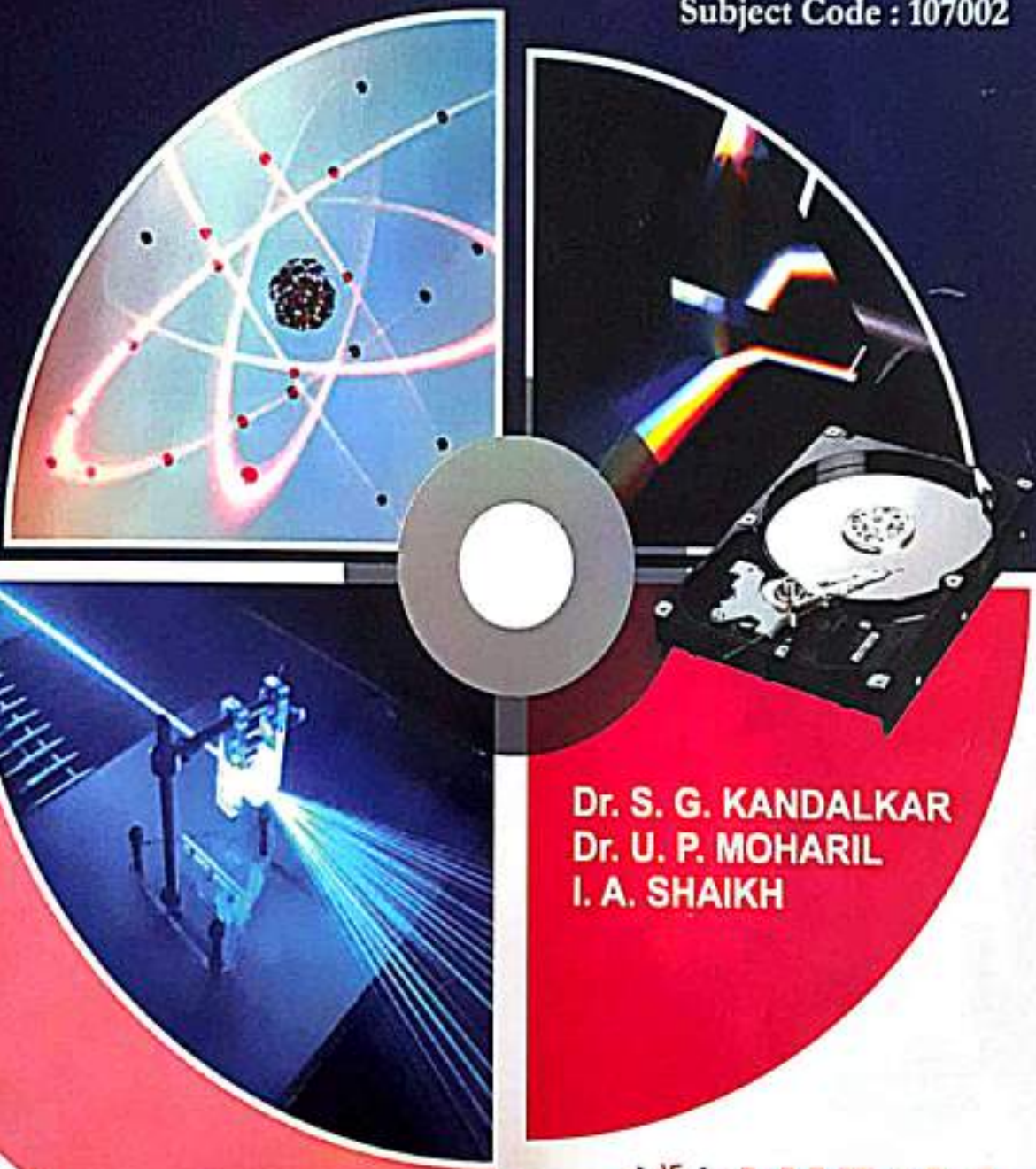
Subject Code : 107002



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ENGINEERING PHYSICS

(Course Code : 107002)

FOR

SEMESTER – I & II

FIRST YEAR DEGREE COURSES IN ENGINEERING

Strictly According to Choice Based Credit System Syllabus 2019 of

Savitribai Phule Pune University, Pune.

(Effective from Academic Year – 2019-2020)

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SYLLABUS

Unit I : Wave Optics (08 Hrs)

Interference

- Introduction to electromagnetic waves and electromagnetic spectrum
- Interference in thin film of uniform thickness (with derivation)
- Interference in thin film wedge shape (qualitative)
- Applications of interference: testing optical flatness, anti-reflection coating

Diffraction

- Diffraction of light
- Diffraction at a single slit, conditions for principal maxima and minima, diffraction pattern
- Diffraction grating, conditions for principal maxima and minima starting from resultant amplitude equations, diffraction pattern
- Rayleigh's criterion for resolution, resolving power of telescope and grating

Polarization

- Polarization of light, Malus law
- Double refraction, Huygen's theory of double refraction
- Applications of polarization: LCD

Unit II : Laser and Optic Fibre (08 Hrs)

Laser

- Basics of laser and its mechanism, characteristics of laser
- Semiconductor laser: Single Hetro-junction laser
- Gas laser: CO_2 laser
- Applications of lasers: Holography, IT, industrial, medical

Optic Fiber

- Introduction, parameters: Acceptance Angle, Acceptance Cone, Numerical Aperture
- Types of optical fiber- step index and graded index
- Attenuation and reasons for losses in optic fibers (qualitative)
- Communication system: basic building blocks
- Advantages of optical fiber communication over conventional methods.

Unit III : Quantum Mechanics (08 Hrs)

- De-Broglie hypothesis
- Concept of phase velocity and group velocity (qualitative)
- Heisenberg Uncertainty Principle
- Wave-function and its physical significance
- Schrodinger's equations: time independent and time dependent
- Application of Schrodinger's time independent wave equation - Particle enclosed in infinitely deep potential well (Particle in RigidBox)
- Particle in Finite potential well (Particle in Non Rigid box) (qualitative)
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UNIT I - WAVE OPTICS

INTERFERENCE

1.1 INTRODUCTION TO ELECTROMAGNETIC WAVES AND ELECTROMAGNETIC SPECTRUM

- The most common type of radiation which we come across in day to day life is electromagnetic wave or photon (the light quanta). Some of the electromagnetic waves can stimulate retina and some cannot. The part of the electromagnetic wave which can stimulate the retina is called ***'light'***.
- The branch of physics which deals with light is called **'optics'**. Further, optics can be broadly classified as (a) ***'geometrical optics'***, (b) ***'physical or wave optics'*** and (c) ***'quantum optics'*** depending upon the basic behaviour of light assumed for explaining the optical phenomena. In the current course our main focus will be on physical optics, where we assume the wave nature of light.
- From basic optical phenomena such as interference and diffraction, we can conclude that the light has a wave nature. In this unit, we will be studying these two basic properties of light.
- But these properties fail to explain the type of oscillations involved i.e. polarisation. The polarisation of light will be studied in later part of the text.
- The wave theory of light was proposed by Chritian Huygen in 1679 but interference was demonstrated by Thomas Young only in 1802. There are several examples of interference that can be observed in everyday life. Basically, oil is colourless but a film of oil floating on water shows bright colours and also keeps on changing colour. Similarly a soap bubble, a compact disc, a thin sheet of mica or cellophane appear coloured. All this is due to interference of light.
- In engineering too, interference has wide applications such as measurement of thickness and stress, testing flatness of a surface, anti-reflecting coating etc.
- Electromagnetic radiation is created when a charges particle (electron or proton), is accelerated by an electric field. This movement produces oscillating electric and magnetic fields, which travel at right angles to each other. The electromagnetic waves travel with the speed of 3×10^8 m/s.

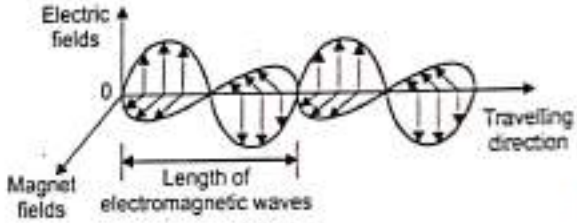


Fig. 1.1

- The electromagnetic spectrum consists of span of electromagnetic waves of range of wavelengths and frequencies. The EM spectrum is generally divided into seven regions, in order of decreasing wavelength and increasing energy and frequency. The common designations are cosmic rays, gamma rays, X-rays, ultraviolet (UV), visible light, infrared (IR), microwaves and radio waves. (See Table 1.1).

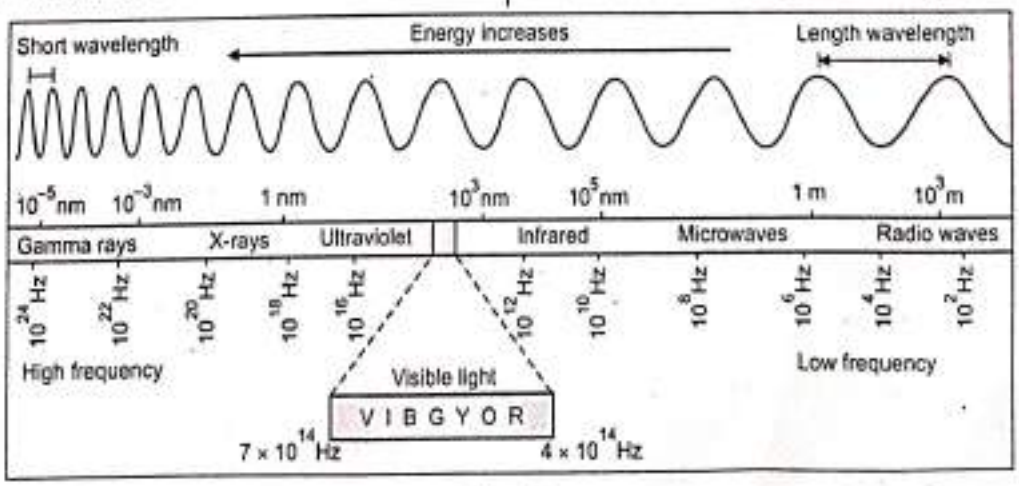


Fig. 1.2

Table 1.1

Color	Wavelength
Violet	380 – 450 nm
Blue	450 – 495 nm
Green	495 – 570 nm
Yellow	570 – 590 nm
Orange	590 – 620 nm
Red	620 – 750 nm

Table 1.2

Type	Frequency (Hz)	Wavelength (m)	Origin	Sources
Cosmic Rays	Upto 10^{22}	less than 3×10^{-15}	Supernova	Cosmos
Y-Rays	6×10^{22}–10^{18}	3×10^{-11}–5×10^{-13}	Energy changes in nucleus of an atom	Radioactive substances
X-Rays	6×10^{18}–6×10^{15}	5×10^{-8}–5×10^{-12}	1. Very high energy changes in electrons 2. Decele-rating electrons	X-Rays tubes
Ultraviolet Rays	2×10^{16}–7×10^{14}	4.3×10^{-7}–1.5×10^{-8}	Fairly high energy changes in electrons	Very hot bodies
Visible Rays	7×10^{14}–3×10^{14}	1×10^{-6}–4.3×10^{-7}	Energy changes in electrons	Lamps, flames, hot bodies
Infra-red Rays	4×10^{14}–3×10^{11}	1×10^{-3}–7.5×10^{-7}	Low energy changes in electrons	Wide range of temperature from 0 K
Radio waves	10^{12}–10^{3}	3×10^{-4}–3×10^{-5}	Oscillatory circuits, very low energy changes in electrons	Trans-mission circuits

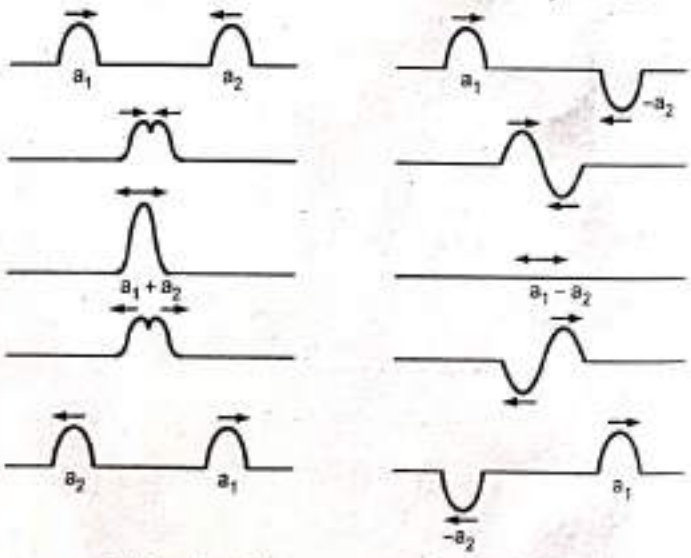
1.1.1 Interference of Waves

- If two waves of same frequency travel in same direction with a constant phase difference with time, they combine so that their energy is not uniformly distributed in space, but is maximum at certain points and minimum (or zero) at other points. This phenomenon is called ***interference***.
- In interference, energy is neither created (at maxima) nor destroyed (at minima) but is redistributed so that there is more energy at certain points (maxima) and less energy at other points (minima). Even after interference the total energy of the system remains constant.
- Thus, interference is the redistribution of energy due to superposition of two or more waves.

Principle of Superposition

'The principle of superposition states that when two or more waves are superposed in space or a medium, the waves travel independently, through each other and the resultant displacement of each position is the algebraic/vector sum of the displacements due to each wave'. Fig. 1.3 shows superposition of two waves.

- In Fig. 1.3 (a), two crests, with amplitude a_1 and a_2, are approaching each other and the point where they meet the resultant amplitude $(a_1 + a_2)$ is more than the individual amplitudes. After this, they pass through each other as though they have not interfered at all. Similarly, in Fig. 1.3 (b), one crest and one trough, with amplitude a_1 and $-a_2$, are approaching each other. At the point where they meet the resultant amplitude $(a_1 - a_2)$ is less than the individual amplitudes.
- The first case is called ***'constructive Interference'*** and the second case is called ***'Destructive Interference'***.



(a) Constructive interference **(b) Destructive interference**

Fig. 1.3 : Superposition

1. Constructive Interference

- When the crest of one wave overlaps the crest of the other or the trough of one overlaps with the trough of the other, the displacement is maximum. This is called as '***Constructive Interference***'.

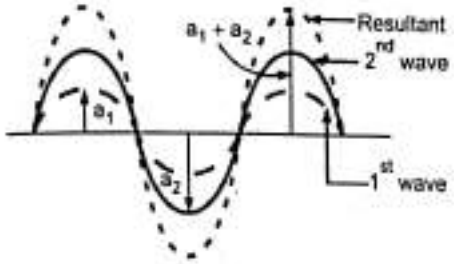


Fig. 1.4 : Constructive interference

- In this case, the waves are in phase and the resultant amplitude equals the sum of the two component amplitudes.

i.e. $A = a_1 + a_2$ ... (1)

If $a_1 = a_2 = a$

then $A = 2a$ ... (2)

The resultant intensity will be

$$I = A^2 = 4a^2 \quad \text{... (3)}$$

- Here the path difference between two waves is 0. Constructive interference will also take place when the path difference is λ or 2λ. In general, condition for constructive interference is,

Path difference,

$$\Delta = n\lambda \quad \text{... (4)}$$

where $n = 0, 1, 2, \ldots n.$

In terms of phase difference,

Phase difference,

$$\delta = k\Delta, \text{ where } k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \delta = \frac{2\pi}{\lambda} \cdot n\lambda$$

$$\delta = 2n\pi \quad \text{... (5)}$$

2. Destructive Interference

- In the other case, when the crest of one wave overlaps the trough of other or vice-versa, the displacement is minimum. This is called as '***Destructive Interference***'.

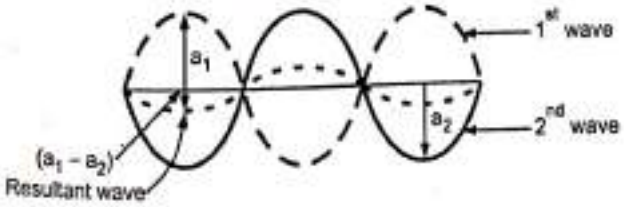


Fig. 1.5 : Destructive interference

- In this case, the waves are 180° out of phase and the resultant amplitude is the difference of the two component amplitudes.

i.e. $A = a_1 - a_2$ ... (6)

If $a_1 = a_2 = a$

then $A = 0$ ... (7)

The resultant intensity,

$$I = A^2 = 0 \quad \text{... (8)}$$

- Here the path difference between two waves is $\lambda/2$. The same thing will happen when the path difference is $3\lambda/2$ or $5\lambda/2$. In general, destructive interference will occur when

Path difference,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda \quad \text{... (9)}$$

where $n = 0, 1, 2, \ldots, n.$

or

Phase difference,

$$\delta = k \cdot \Delta \text{ where } k = \frac{2\pi}{\lambda}$$

$$\delta = \frac{2\pi}{\lambda}\left(n + \frac{1}{2}\right)\lambda$$

$$\delta = (2n + 1)\pi \quad \text{... (10)}$$

3. Conditions Necessary for Stable Interference

- The two interfering waves should emit light of ***same wavelength or frequency***.
- The two interfering waves must be ***coherent***.
- The interfering waves must have ***equal amplitudes***.
- The two interfering waves must be propagated along ***the same line***.
- The **separation between the two sources must be as small as possible** so that the path difference between the two interfering waves, reaching a particular point, is not very large.
- The source of two interfering wave must be ***narrow***, as a broad source will be equivalent to many fine sources.

4. Intensity Distribution in the Fringe System

- If the amplitude of interfering waves are a_1 and a_2, and the phase difference is δ, then the vector sum of these two amplitude vectors will be A, as shown in Fig. 1.6.

- A plane monochromatic light ray, which can be considered as a parallel beam, is incident on the upper surface of the film.

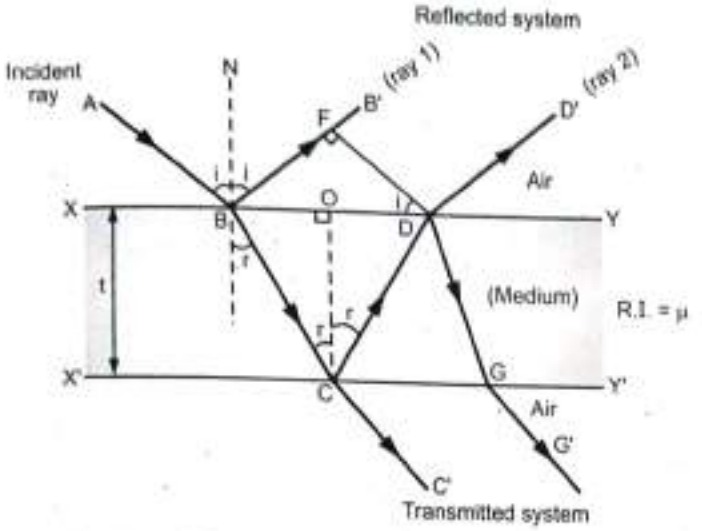


Fig. 1.9 : Reflection of light from a parallel thin film

- Let AB represent one of the incident rays. The light ray travelling along AB is incident at an angle 'i' on the upper surface of the film. A part of the incident light is reflected at the upper face along BB' and a part is refracted at an angle 'r' along BC.
- At C, it is partly reflected back into the film along CD, while a major portion is transmitted along CC'. The ray along CD emerges along DD' parallel to BB'.
- Both rays BB' (ray 1) and DD' (ray 2) are obtained from the same incident ray. They are, therefore, coherent and can produce interference. The condition of interference depends on the optical path difference between rays 1 and 2.
- To compute the optical path difference between the reflected ray BB' (ray 1) and the refracted ray DD' (ray 2), draw a normal DF from ray DD' on line BB'. Beyond points F and D, both rays travel equal distances.
- While ray BB' has covered a distance BF in air, the ray DD' has covered a distance BCD in the film of refractive index μ.
- The geometric '***path difference***' between ray 1 and ray 2 is, BC + CD – BF.

∴ Optical path difference is,

$$\Delta = \mu (BC + CD) - BF \quad \text{... (1)}$$

(Since, distances BC and CD are travelled in medium of R.I. = μ)

From Δ BCO and Δ DCO, $\cos r = \frac{CO}{BC}$ and $\cos r = \frac{CO}{CD}$

$$\therefore \quad \cos r = \frac{t}{BC} \text{ and } \cos r = \frac{t}{CD} \quad (\because CO = t)$$

$$\therefore \quad BC = CD = \frac{t}{\cos r} \quad \text{... (2)}$$

and $BF = BD \sin i$ (from Δ BFD)

But $BO = OD = BC \sin r$ (from Δ BCO and Δ DCO)

$$\therefore \quad BF = 2\,BC \sin r \sin i \quad \text{as } BD = BO + OD$$

From equation (2),

$$BF = \frac{2t}{\cos r} \sin r \sin i$$

Dividing and multiplying by sin r,

$$BF = \frac{2t}{\cos r} \sin^2 r \frac{\sin i}{\sin r}$$

$$BF = \frac{2\mu t}{\cos r} \sin^2 r \quad \left(\because \frac{\sin i}{\sin r} = \mu\right) \quad \text{... (3)}$$

Substituting equations (2) and (3) in equation (1),

$$\Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - \frac{2\mu t}{\cos r} \sin^2 r$$

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \sin^2 r$$

$$\Delta = \left(1 - \sin^2 r\right) \frac{2\mu t}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r} \cos^2 r \quad (\because \sin^2 r + \cos^2 r = 1)$$

$$\Delta = 2\mu t \cos r \quad \text{... (4)}$$

In Reflected System **(May 15)**

- The path difference given by (4) is not the true optical path difference between rays 1 and 2. The phase change due to reflection is to be taken into account.
- At B, the reflection is in a rarer medium. So, a path change of λ/2 occurs in the reflected ray BB'. At C, reflection is in a denser medium, therefore no path change occurs in ray 2.

∴ Total path difference between rays 1 and 2 is given by,

Total path difference = Path difference due to thin film + Path difference due to reflections

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2} \quad \text{... (5)}$$

(i) Condition for Constructive Interference

- If the total path difference is equal to an integral multiple of λ then rays 1 and 2 meet in phase and undergo constructive interference.

i.e., $\Delta = n\lambda$

$\therefore \quad 2\mu t \cos r \pm \lambda/2 = n\lambda$

$2\mu t \cos r = (2n \pm 1)\lambda/2$

where $n = 0, 1, 2, 3$ ... (6)

(ii) Condition for Destructive Interference

- If the optical path difference is equal to an odd integral multiple of $\lambda/2$, then rays 1 and 2 meet in opposite phase and undergo destructive interference.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$\therefore \quad 2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$2\mu t \cos r = n\lambda$ where $n = 0, 1, 2$ ... and is called **order of interference** ... (7)

- The rays incident on the film at the same angle are divided into two rays which become parallel on reflection from the surfaces of the film. Parallel rays do not intersect at finite distances, hence fringes are not observed at finite distances.
- The rays are to be condensed by a lens and interference is observed in its focal plane. Else, it can be observed by the unaided eye focused at infinity. Therefore, these interference fringes are said to be ***localized at infinity***.

Important Cases

- If the film is extremely thin i.e. $t << \lambda$ or $t \to 0$ then the path difference, $\Delta = \lambda/2$. The film will appear dark in reflected light.
- When monochromatic light is incident normal to the film then $\cos r = 1$.

$2\mu t = (2n + 1)\frac{\lambda}{2}$ for brightness

and $2\mu t = n\lambda$ for darkness.

This implies that the film will appear bright in reflected light if the film has thickness of $t = \frac{\lambda}{4\mu}, \frac{3\lambda}{4\mu}, \ldots\ldots$

and it will appear dark for a thickness of $t = \frac{\lambda}{2\mu}, \frac{2\lambda}{2\mu}, \frac{3\lambda}{2\mu} \ldots\ldots$

- If the incident monochromatic light is parallel, the whole film will be uniformly bright or dark as film thickness 't' and angle of refraction 'r' are constant. For a given incident wavelength (say green) the condition of constructive interference causes the incident colour to intensify (intense green).
- A change in the angle of incidence of the rays causes a change in the path difference. The optical path difference decreases with increase in angle of incidence. Hence, as inclination of the film is changed, it will appear alternately dark and bright for incident monochromatic light.
- If white light is incident on the film, the optical path difference will vary from one colour to the other as λ is different. Hence, the film will appear coloured, the colour being that of the rays which interfered constructively. Further, as the inclination of the film is changed, the film will appear coloured.
- If the incident white light is not parallel, the optical path difference will change due to change in the incident angle. Hence, the film will show different colours when viewed from different directions.

In Transmitted System

- When the film is observed in transmitted light, it can be shown that the path difference between rays CC' and GG' (Fig. 1.9) is equal to $2\mu t \cos r$. Reflections at C and D are in a denser medium. So, no additional path change will occur due to reflection.

Total path difference = Path difference due to thin film + Path difference due to reflections

$\therefore \quad \Delta = 2\mu t \cos r + 0$ ... (8)

(i) Condition for Constructive Interference

- For constructive interference the total phase difference should be an integral multiple of λ.

i.e. $\Delta = n\lambda$

$\therefore \quad 2\mu t \cos r = n\lambda$ ... (9)

(ii) Condition for Destructive Interference

- For destructive interference the total phase difference should be an odd integral multiple of $\lambda/2$.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$\therefore \quad 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ ... (10)

As is evident, the condition for brightness on reflection becomes the condition for darkness on transmission and vice versa.

1.2.2 Interference in Films of Non-Uniform Thickness (Wedge Shaped Film)

- A ***wedge*** is a plate or film of varying thickness, having zero thickness at one end and progressively increasing to a particular thickness at the other end.
- Consider two plane surfaces XY and X'Y' inclined at an angle α. The thickness of the film increases linearly from X to Y.
- When the wedge is illuminated by a parallel beam of monochromatic light, the rays reflected from its two surfaces will not be parallel. They appear to diverge from a point S near the film.
- When the film is viewed with reflected monochromatic light, ***equidistant interference fringes*** are observed which are parallel to the line of intersection of the two surfaces. The fringes are alternately bright and dark and are localised at the surface of the film.
- On illuminating the film with monochromatic light, one system of rays is reflected from the front surface XY and the other system of rays is obtained by transmission at the back surface X'Y' (not shown in Fig. 1.10) and consequent reflections at the front surface. As both rays are obtained from a single source, they are coherent and produce interference.

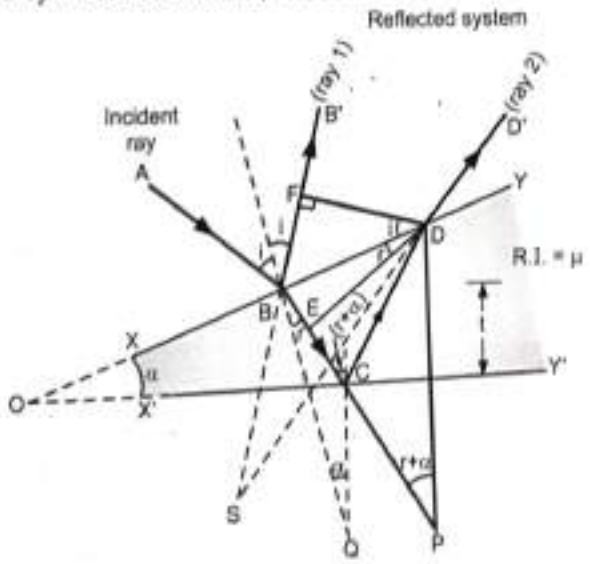


Fig. 1.10: Wedge-shaped film

- The optical path difference between them is given by,

$$\Delta = 2\mu t \cos (r + \alpha)$$

[**Note:** Derivation not expected.]

(i) In Reflected System

- Due to reflection, an additional path change is introduced in the reflected system at point B.

∴ Total path difference = Path difference due to thin film + Path difference due to reflections

$$\Delta = 2\mu t \cos (r + \alpha) \pm \frac{\lambda}{2} \quad \text{... (1)}$$

Condition for Constructive Interference

For constructive interference the total phase difference should be an integral multiple of λ.

$$\therefore \quad \Delta = n\lambda$$

$$\text{i.e. } 2\mu t \cos (r + \alpha) \pm \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2\mu t \cos (r + \alpha) = (2n \pm 1)\frac{\lambda}{2} \quad \text{... (2)}$$

Condition for Destructive Interference

For destructive interference the total phase difference should be an odd integral multiple of λ/2.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\text{i.e. } 2\mu t \cos (r + \alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \quad 2\mu t \cos (r + \alpha) = n\lambda \quad \text{... (3)}$$

(ii) Nature of Interference Pattern

- If the film is illuminated by parallel light, then 'i' is constant everywhere and so is 'r', the angle of refraction. In addition, if monochromatic light is used, the path change will occur only due to 't'. In this case, the fringes will be of ***'equal thickness'***.
- For a wedge shaped film, 't' remains constant only in a direction parallel to the thin edge of the wedge. So, straight fringes parallel to the edge of the wedge are obtained. The fringes are alternately bright or dark for monochromatic light.
- For white light, coloured fringes are obtained.

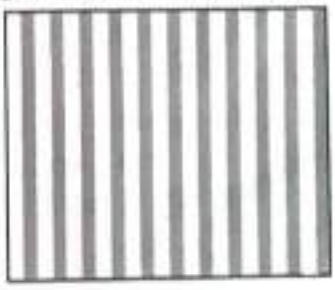


Fig. 1.11: Interference pattern in wedge-shaped film alternately bright and dark bands are parallel

[**Note:** In transmitted system we will get exactly opposite of reflected system]

- The fringe width will be given by formula

$$\beta = \frac{\lambda}{2\mu \sin \alpha} \quad \ldots (4)$$

which is the same as that of a bright fringe.

- The width of a dark or bright fringe is however equal to half the fringe width.
- In equation (4), as all quantities on the right side are constant, β is constant for a given wedge angle α. It means that the ***interference fringes*** are ***equidistant*** from each other.
- According to relation (4), as angle α increases, the fringes move closer. At α = 1°, the interference pattern vanishes. If α is gradually decreased, the fringe separation increases and ultimately the fringes disappear as the faces of the film become parallel (α = 0°).

1.2.3 Formation of Colours in Thin Films

- The above discussions makes it clear that the conditions of interference are complementary i.e. the conditions of interference in the case of reflected and transmitted systems are opposite. For a given path difference if a bright fringe is observed in reflected system, a dark fringe will be observed in the transmitted system.
- The condition for maxima in reflected system is $2\mu t \cos r = (n \pm 1/2)\lambda$. If white light is incident on the film the wavelength satisfying above relation will be present in reflected system and corresponding colour will be seen.
- The colour appearing on the film will change if μ, t and r are changed. So different colours in reflected (or transmitted) rays are seen if, one of the three parameters μ, t and r is changed.
- When oil is dropped on water surface, it spreads slowly and a thin film is formed. The film itself is although ***colourless*** but ***appears colourful*** and keeps on changing colour, ***due to interference***.
- The thin film keeps on changing colour as the oil film changes the thickness. The thickness changes as the oil spreads on the water surface and due to waves present in the water surface.
- A soap bubble is water film which is basically colourless but appears colourful due to interference and the colour also keeps on changing. The colour changes due to change in the thickness of the bubble which is caused by the gravitational force.
- When the soap bubble is about to burst the thickness is negligible and total path difference is just λ/2. Due to which the waves interfere destructively and bubble becomes black.
- A soap bubble floating in air also changes the colour as the angle of incidence and hence angle of refraction changes.

SOLVED PROBLEMS

Problem 1.1: *In a certain region of interference we get 490th order maximum for sodium 5890 A° line. What will be the order of interference maximum at the same plane for sodium 5896 A° ?*

Data: n = 490, λ = 5890 A°

Formula: Δ = nλ

Solution: Path difference for n^{th} order maximum, Δ = n λ

∴ Path difference for 490th order maximum when λ is 5890 A°

$$\Delta = 490 \times 5890 \times 10^{-8}$$

$$\Delta = 2.89 \times 10^{-2} \text{ cm}$$

For sodium light of wavelength λ_1 = 5896 A°, the order of interference is n_1. Then the

Path difference= $n_1 \lambda_1 = 2.89 \times 10^{-2}$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{\lambda_1}$$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{5896 \times 10^{-8}}$$

$$= 489.5$$

∴ The order of interference maximum = **489**

Problem 1.2: *Fringes are produced with monochromatic light of λ = 5450 A°. A thin glass plate of μ = 1.5 is then placed normally in the path of one of the interferring beams and the central band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. Calculate the thickness of the glass plate.*

Data: λ = 5450 A°, μ = 1.5, n = 3

Formula: t (μ − 1) = n λ

Solution:

$$t = n \frac{\lambda}{(\mu - 1)}$$

Substituting,

$$t = \frac{3 \times 5450 \times 10^{-8}}{1.5 - 1}$$

$$t = 0.000327 \text{ cm}$$

Problem 1.3: *When light falls normally on a soap film, whose thickness is 5×10^{-5} cm and whose refractive index is 1.33; which wavelength in the visible region will be reflected most strongly ?*

Data: $t = 5 \times 10^{-5}$ cm, $\mu = 1.33$

Formula: $2 \mu t \cos r = (2n + 1) \lambda/2$

where $n = 0, 1, 2, 3, \ldots$ etc.

Solution: For normal incidence,

$$\cos r = 1$$

$$\therefore \quad 2 \mu t = (2n + 1) \frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2 \times 2 \mu t}{2n + 1}$$

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n + 1)}$$

For $n = 0$, $\lambda = \dfrac{4 \times 1.33 \times 5 \times 10^{-5}}{1} = 2.66 \times 10^{-4}$ cm

$$\boxed{\lambda = 26{,}600 \text{ A}^\circ}$$

For $n = 1$,

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{3} = 8.866 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 8866 \text{ A}^\circ}$$

For $n = 2$,

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{5} = 5.32 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 5320 \text{ A}^\circ}$$

For $n = 3$,

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{7} = 3.8 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 3800 \text{ A}^\circ}$$

The wavelength 5320 A° will be most strongly reflected in the visible region.

Problem 1.4: *A parallel beam of sodium light $\lambda = 5890$ A° strikes a film of oil floating on water. When viewed at an angle of 30° from the normal, 8th dark band is seen. Determine the thickness of the film if refractive index of oil = 1.5*

Data: $\lambda = 5890$ A°, $\angle i = 30°$, $\mu = 1.5$, $n = 8$

Formulae: (i) $2\mu t \cos r = n\lambda$ or $t = \dfrac{n\lambda}{2\mu \cos r}$ ... (1)

(ii) $\mu = \dfrac{\sin i}{\sin r}$

Solution:

$$\sin r = \frac{\sin i}{\mu}$$

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$\therefore \quad \cos r = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$$

$$\therefore \quad \cos r = \sqrt{1 - \frac{\sin^2 30}{(1.5)^2}}$$

$$\cos r = 0.943$$

Substituting in (1),

$$\therefore \quad t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.943}$$

$$= \boxed{1.6302 \times 10^{-4} \text{ cm}}$$

Problem 1.5: *Two glass plates enclose a wedge-shaped air film, touching at one edge and are separated by a wire of 0.03 mm diameter at a distance of 15 cm from the edge. Monochromatic light of $\lambda = 6000$ A° from a broad source falls normally on the film. Calculate the fringe width of the fringes thus formed.*

Data: $\lambda = 6000 \times 10^{-8}$ cm; For air film, $\mu = 1$

Diameter = 0.03 mm = 0.003 cm

Distance of fringe from the edge = 15 cm

Formula: Fringe width,

$$\beta = \frac{\lambda}{2\mu \sin \alpha} \approx \frac{\lambda}{2\mu \tan \alpha}$$

Solution:

$$\beta = \frac{6000 \times 10^{-8}}{2 \times 1 \times \frac{0.003}{15}} = \boxed{0.15 \text{ cm}}$$

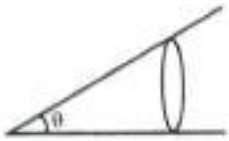


Fig. 1.12

Problem 1.6: *Interference fringes are produced by monochromatic light falling normally on a wedge-shaped film of cellophane whose refractive index is 1.4. The angle of the wedge is 20 sec of an arc and the distance between the successive fringes is 0.25 cm. Calculate the wavelength of light.*

Data: $\beta = 0.25$ cm, $\mu = 1.4$, $\theta = 20$ sec

$$= \frac{20}{60 \times 60} \times \frac{\pi}{180} = \frac{1}{180} \times \frac{\pi}{180} \text{ radians}$$

Formula: $\beta = \dfrac{\lambda}{2\mu\alpha}$

$\therefore \quad \lambda = 2\mu\,\alpha\cdot\beta$

Solution: $\lambda = 2 \times 1.4 \times \dfrac{\pi}{180 \times 180} \times 0.25$

$$\boxed{\lambda = 6.79 \times 10^{-5}\ \text{cm}}$$

Problem 1.7: *Two plane rectangular pieces of glass are in contact at one edge and separated by a hair at opposite edge, so that a wedge is formed. When light of wavelength 6000 A° falls normally on the wedge, nine interference fringes are observed. What is the thickness of the hair ?*

Data: $\lambda = 6000 \times 10^{-8}$ cm, $n = 9$, $r = 0$ for normal incidence

Formula: $2\mu t \cos(r + \alpha) = n\lambda$

Solution: If the fringes are seen normally and the angle of wedge is very small, then $r = 0$, so that

$$\cos(r + \alpha) = \cos\alpha = 1$$

For air film, $\mu = 1$

$$\therefore \quad 2\mu t = n\lambda$$

$$2 \times 1 \times t = 9 \times 6000 \times 10^{-8}$$

$$\therefore \quad t = \frac{9 \times 6000 \times 10^{-8}}{2} = \boxed{27 \times 10^{-5}\ \text{cm}}$$

Problem 1.8: *A square piece of cellophane film with index of refraction 1.5 has a wedge-shaped section, so that its thickness at two opposite sides is t_1 and t_2. If with light of λ = 6000 A°, the number of fringes appearing on the film is 10, calculate the difference $t_2 - t_1$.*

Data: $\lambda = 6000 \times 10^{-8}$ cm, $\mu = 1.5$

Formula:

$$2\mu t_1 \cos(r + \alpha) = n\lambda \quad \ldots (1)$$

Solution: For $(n + 10)^{th}$ dark fringe,

$$2\mu t_2 \cos(r + \alpha) = (n + 10)\lambda \quad \ldots (2)$$

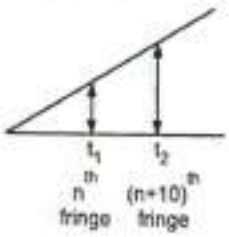


Fig. 1.13

For normal incidence, $r = 0$ and if the angle of wedge is small,

$$\cos(r + \alpha) = \cos\alpha = 1$$

$\therefore$ Equations (1) and (2) become

$$2\mu t_1 = n\lambda \quad \ldots (3)$$

$$2\mu t_2 = (n + 10)\lambda \quad \ldots (4)$$

$\therefore$ Subtracting equation (3) from equation (4), we get

$$2\mu(t_2 - t_1) = 10\lambda$$

$$\therefore \quad t_2 - t_1 = \frac{10\lambda}{2\mu} = \frac{10 \times 6000 \times 10^{-8}}{2 \times 1.5}$$

i.e. $\boxed{t_2 - t_1 = 2 \times 10^{-4}\ \text{cm}}$

Problem 1.9: *A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.*

Data: $\lambda = 5890$ A°, $r = 60°$, $\mu = 1.5$

Formula: For darkness,

$$2\mu t \cos r = n\lambda. \quad \text{Let } n = 1$$

Solution: $t = \dfrac{\lambda}{2\mu\cos r} = \dfrac{5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$

$$\boxed{t = 3.926 \times 10^{-5}\ \text{cm}}$$

Problem 1.10: *The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used.*

Data: $\Delta_1 = 199.5\lambda$, $\Delta_2 = 0.12$ cm

Formula: For 199.5 λ it is odd path difference, therefore point is a dark fringe and for darkness,

$$\Delta = (2n - 1)\frac{\lambda}{2} \qquad \left(\because \left(\frac{2n-1}{2}\right) = 199.5\right)$$

Solution: $0.012 = 199.5\lambda$

$$\lambda = \frac{0.012}{199.5}$$

$$\lambda = 6.015 \times 10^{-8}\ \text{cm}$$

$$\boxed{\lambda = 6015\ \text{A}^\circ}$$

Problem 1.11: *Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds of the wedge shaped air film between the plate, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm.*

Solution: 18 bands per cm

$\therefore$ Band width,

$$\beta = \frac{1}{18}$$

$$\beta = 0.0556 \text{ cm}$$

We know, $\beta = \dfrac{\lambda}{2\alpha}$

$$\alpha = \frac{\lambda}{2\beta} = \frac{4800 \times 10^{-8}}{2 \times 0.0556}$$

$$\alpha = 4.3165 \times 10^{-4} \text{ rad}$$

Note conversion of radians into seconds.

$$\alpha = 4.3165 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60$$

$$\boxed{\alpha = 89.02 \text{ seconds}}$$

Problem 1.12: *Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes.*

Solution: $\tan\alpha = \dfrac{t}{x}$

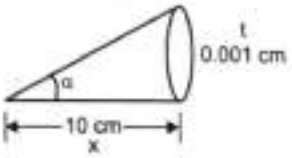


Fig. 1.14

As α is very small, tan α = α

$$\alpha = \frac{t}{x} = \frac{0.001}{10}$$

$$\alpha = 0.0001 \text{ rad}$$

$$\beta = \frac{\lambda}{2\alpha} = \frac{5.9 \times 10^{-5}}{2 \times 0.0001}$$

$$\boxed{\beta = 0.295 \text{ cm}}$$

Note while using α in calculation it must be all the time in radians. If it is given in seconds then convert it in rad.

Problem 1.13: *Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°.*

Data: n = 4, λ = 5893 A°

Formula: For bright band and wedge-shaped film,

$$2\mu t \cos(r + \alpha) = (2n - 1)\frac{\lambda}{2}$$

Let normal incidence, r = 0 and α is very small

$$\therefore \cos(r + \alpha) = 1, \mu = 1$$

$$\therefore \quad 2t = (2n - 1)\frac{\lambda}{2}$$

Solution: $2t = \dfrac{(2 \times 4 - 1)\lambda}{2}$

$$t = \frac{7}{4} \times 5893 \times 10^{-8}$$

$$\boxed{t = 1.031275 \times 10^{-4} \text{ cm}}$$

Problem 1.14: *Monochromatic light emitted by a broad source of light of wavelength 6×10^{-5} cm falls normally on two glass plates which enclose a thin wedge-shaped film of air. The plates touch at one end and are separated at a point 15 cm from the end by a wire 0.5 mm in diameter. Find the width between any two consecutive bright fringes.*

Solution:

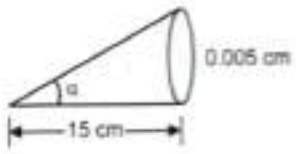


Fig. 1.15

$$\alpha = \frac{t}{x} = \frac{0.005}{15} = 3.333 \times 10^{-4} \text{ rad}$$

Bandwidth, $\beta = \dfrac{\lambda}{2\alpha}$

$$\beta = \frac{6 \times 10^{-5}}{2 \times 3.333 \times 10^{-4}}$$

$$\boxed{\beta = 0.09 \text{ cm}}$$

Problem 1.15: *Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 A°. Calculate the angle of wedge in seconds of an arc.*

(04) (Dec. 03, May 07)

Data: μ = 1.52

λ = 5893 A°

β = 1 mm

Formula: Fringe width is,

$$\beta = \frac{\lambda}{2\alpha}$$

Solution: $1 \times 10^{-1} = \dfrac{5893 \times 10^{-8}}{2\alpha}$

$$\therefore \quad \alpha = \frac{5893 \times 10^{-8}}{2 \times 1 \times 10^{-1}}$$

$$\alpha = 2.9 \times 10^{-4} \text{ radian}$$

or $\alpha = \dfrac{2.9 \times 10^{-4} \times 180 \times 60 \times 60}{3.14}$ of an arc

$$= \boxed{59.8 \text{ sec.}}$$

Problem 1.16: *A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, in the reflected light, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 A°.* **(04) (May 04)**

Data:

$i = 30°$

$\mu = 1.46$

$\lambda = 5890\ A°$

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\frac{\sin 30}{\sin r} = 1.46$

$$\sin r = \frac{0.5}{1.46} = 0.34247$$

$\therefore$ $\boxed{r = 20°}$

(ii) The thickness is given by relation (condition for minima)

$$2\mu t \cos r = n\lambda$$

$$t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times \cos 20°}$$

$$\boxed{t = 1.7 \times 10^{-4}\ cm}$$

Problem 1.17: *A soap film of refractive index 4/3 and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark and corresponding to wavelength of 5×10^{-5} cm. Calculate the order of interference band.* **(04) (Nov/Dec. 05)**

Data:

$\mu = 4/3 = 1.33$

$t = 1.5 \times 10^{-4}$ cm

$i = 45°$

$\lambda = 5 \times 10^{-5}$ cm

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\frac{\sin 45°}{\sin r} = 1.33$

$$\sin r = \frac{0.707}{1.33}$$

$$\sin r = 0.53038$$

or $\boxed{r = 32°}$

(ii) The order of interference will be given by (condition for dark band)

$$2\mu t \cos r = n\lambda$$

$$n = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times \cos 32°}{5 \times 10^{-5}}$$

$$\boxed{n = 6.78}$$

The order of interference,

$$n = 6$$

Problem 1.18: *A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes in reflected system are observed through a microscope. The distance between the consecutive bright fringes was measured as 0.12 cm. Calculate the wavelength of light used.* **(04) (May 06)**

Data:

$\alpha = 40$ sec.

$\beta = 0.12$ cm

$\alpha = 40$ sec

$$\alpha = \frac{40 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

Formula: $\alpha = 1.9 \times 10^{-4}$ rad.

The fringe width is given by,

$$\beta = \frac{\lambda}{2\alpha}$$

$$\lambda = 2\beta\alpha$$

Solution: $\lambda = 2 \times 0.12 \times 1.9 \times 10^{-4}$

$$\lambda = 5 \times 10^{-5}\ cm$$

$$\boxed{\lambda = 5000\ A°}$$

Problem 1.19: *A parallel beam of monochromatic light of wavelength λ = 5890 A° is incident on a thin film of μ = 1.5 such that the angle of refraction is 60°. Find the maximum thickness of the film so that it appears dark for normal incidence, what is the thickness required ?* **(04) (May 08)**

Data:

$\lambda = 5890\ A°$

$r = 60°$

$\mu = 1.5$

Formula: For dark band,

$$2\mu t \cos r = n\lambda$$

$$t = \frac{n\lambda}{2\mu t \cos r}$$

Solution: For maximum thickness, n = 1

$$t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$

$$t = 4 \times 10^{-5} \text{ cm}$$

For normal incidence, r = 0 and hence cos r = 1.

$$\therefore \quad t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 0}$$

$$\boxed{t = 2 \times 10^{-5} \text{ cm}}$$

Problem 1.20: *An oil drop of volume 0.2 cc is dropped on the surface of a water tank of area 1 sq. m. The thin film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain a first dark band whose centre has a wavelength of 5.5 × 10⁻⁵ cm. Find the refractive index of oil.* **(04) (Dec. 08)**

Data: $V = 0.2$ cc

$A = 1$ sq. m.

$n = 1$

$\lambda = 5.5 \times 10^{-5}$ cm

Formulae: (i) Volume = Area × thickness (ii) $2\mu t \cos r = n\lambda$

Solution: (i) $0.2 = 1 \times 10^{4} \times t$

$t = 2 \times 10^{-5}$ cm

(ii) For minima,

$$2\mu t \cos r = n\lambda$$

Let, $r = 0$

$$\mu = \frac{n\lambda}{2t}$$

$$\mu = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5}}$$

$$\boxed{\mu = 1.375}$$

Problem 1.21: *A beam of monochromatic light of wavelength 5.82 × 10⁻⁷ m falls normally on a glass wedge of wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length.* **(04) (Dec. 08)**

Data: $\lambda = 5.82 \times 10^{-7}$ m

$\theta = 20$ seconds

$\mu = 1.5$

The angle in degrees,

$$\theta = \frac{20}{60 \times 60} \times \frac{\pi}{180}$$

$$\theta = 9.69 \times 10^{-5}$$

Formula:

The fringe width, $\beta = \dfrac{\lambda}{2\mu\theta}$

Solution: $\beta = \dfrac{5.82 \times 10^{-7}}{2 \times 1.5 \times 9.69 \times 10^{-5}}$

$\therefore \quad \beta = 0.2 \times 10^{-2}$ m $= 0.2$ cm

$\therefore$ Number of dark fringes/cm

$$= \frac{1}{\beta} = \frac{1}{0.2} = \boxed{5}$$

Problem 1.22: *A parallel beam of sodium light of wavelength 5890 × 10⁻⁸ cm is incident on a thin glass plate of refractive index 1.5, such that the angle of refration into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.* **(04) (May 09)**

Data: $\lambda = 5890 \times 10^{-8}$ cm

$\mu = 1.5$

$r = 60°$

Formula: The condition for dark fringe in reflected system is

$$2\mu t \cos r = n\lambda$$

Solution: Taking n = 1

$$2 \times 1.5 \times t \times \cos 60 = 5890 \times 10^{-8}$$

$$\therefore \quad t = 3.926 \times 10^{-3} \text{ cm} = \boxed{3.926 \times 10^{-5} \text{cm}}$$

Problem 1.23: *Interference fringes are produced with monochromatic light falling normally on a wedge shaped film of refractive index 1.4. The angle of wedge is 10 sec of an arc and the distance between successive fringes is 0.5 cm. What is the wavelength of light used ?* **(3) (Nov. 13)**

Data: $\mu = 1.4$

$\alpha = 10$ sec

$$= \frac{10}{3600} \times \frac{\pi}{180} \text{ radian},$$

$\beta = 0.5$ cm

Formula: $\beta = \dfrac{\lambda}{2\mu\alpha}$

Solution : $\lambda = \dfrac{2 \times 1.4 \times 10 \times 3.14 \times 0.5 \times 10^{-2}}{3600 \times 180}$

$\therefore \quad \lambda = 0.6783 \times 10^{-6}$ cm

or $\quad \lambda = 6783$ A°

Problem 1.24: *A soap film having refractive index 1.33 and thickness 5×10^{-5} cm is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light.* **[3]**

Solution: Data: $\mu = 1.33$

$t = 5 \times 10^{-5}$ cm

$Q = 35°$

Formula: (i) By Snell's Law $\frac{\sin i}{\sin r} = \mu$

(ii) $2\mu t \cos r = n\lambda$

Solution:

(i) $\frac{\sin 35}{\sin r} = 1.33$

$\therefore \sin r = 0.431$

$\therefore r = 25.53°$

(ii) $\lambda = \frac{2 \times 1.33 \times 5 \times 10^{-5} \times \cos 25.53}{n}$

$\lambda = \frac{12000}{n}$ A°

for $n = 1$

$\lambda = 12000$ A° (not is visible region)

for $n = 2$

$\lambda = \frac{12000}{2} = 6000$A° (In visible region)

for $n = 3$

$\lambda = \frac{12000}{3} = 4000$A° (In visible region)

for $n = 4$

$\lambda = \frac{12000}{4} = 3000$A°

(not in visible region)

$\therefore$ Wavelengths absent from visible region are 6000A° and 4000A°.

Problem 1.25: *A parallel beam of light 622 nm incident on a glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.*

(3) (May 16)

Data : $\lambda = 622$ nm, $\mu = 1.5$, $r = 60$, $n = 1$

Formula : $2\mu t \cos r = n\lambda$,

Solution: $2 \times 1.5 \times t \times \cos 60 = 622 \times 10^{-9}$

$$t = \left[\frac{6.22 \times 10^{-9}}{2 \times 1.5 \times \cos 60°}\right]$$

$$t = \frac{622 \times 10^{-9}\text{ m}}{3 \times \frac{1}{2}}$$

$$t = \frac{622 \times 2 \times 10^{-9}}{3}$$

$$t = \frac{1244}{3} = 414.3 \times 10^{-9}$$

1.3 APPLICATIONS OF INTERFERENCE

1.3.1 Testing of Optical Flatness of Surfaces

- Interference is now widely used for testing the quality of a surface finish. Even after machining, machine components retain certain surface irregularities. These act as sources of stress, leading to cracks thereby limiting the suitability of the components to certain applications. Therefore, the surfaces of components which will be subjected to high stresses, are required to have a high surface finish. There are two ways in which the optical flatness or smoothness of a surface may be tested.
- The specimen surface is placed over the optically plane surface. If the surface is optically plane, no interference fringes will be observed. Because there will be no air film enclosed between the two planes and the path difference between the reflected pairs will be $\lambda/2$ which is the condition for destructive interference. If the given surface is not optically flat but has some imperfections, fringe pattern will be observed.

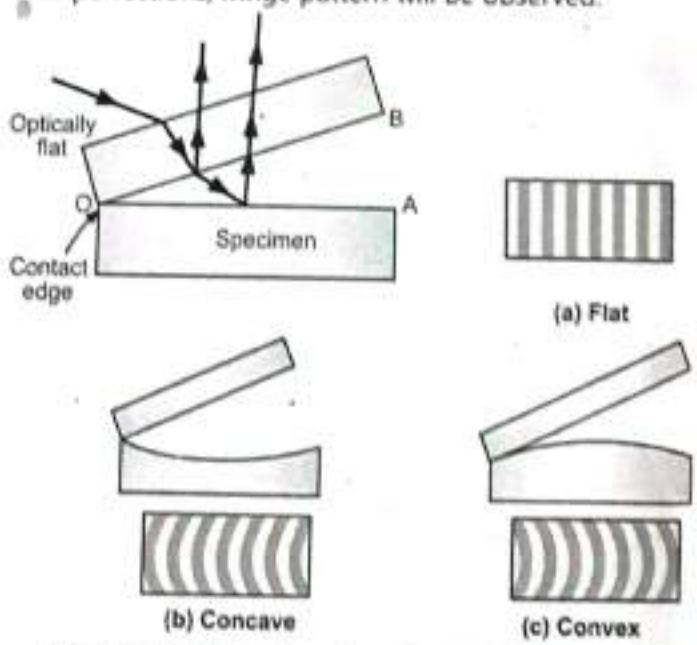


Fig. 1.16: Testing of surfaces for optical flatness

- If the specimen to be tested is placed inclined to an optically flat surface, a ***wedge shaped air film*** will be enclosed between them. The air wedge produces ***straight*** and ***equidistant fringes*** if the surface of the component is ***smooth***. These fringes are parallel to the line of intersection of two surfaces. The fringes or bands are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a

constant value. If the fringes are curved towards the contact edge, the surface is concave and if the fringes curve away, it is convex.

1.3.2 Antireflection or Non-Reflecting Coatings

(May 13, 16)

- Optical instruments like telescopes, cameras, etc. use multi component glass lenses. When light is incident on these lenses, part of the light is reflected. Hence, the transmitted component has reduced intensity. When the number of reflections are large, the quality of the image produced will be poor.
- It was in 1935 that Alexander Smakula discovered that the phenomenon of interference could be used to reduce reflections from a surface by coating it with a thin transparent film. Such a film coated to reduce reflections is called ***an antireflection film (AR coating) or non-reflecting film***.
- A thin film can act as an AR coating if the waves reflected from its top and bottom surfaces are exactly 180° out of phase and the waves have equal amplitudes. To achieve this, surfaces of lenses, prisms, etc. are coated with a thin layer or film of hard transparent material with refractive index intermediate between that of air and glass (less than that of glass).
- Light is reflected, from both surfaces of the layer, from a medium of greater refractive index than that in which it is travelling. So, the same phase change occurs in both reflections.
- If the film thickness is a quarter of the wavelength in the film, the total path difference is a half wavelength. Light reflected from the first surface is out of phase with light reflected from the second surface leading to destructive interference.
- The path difference due to non-reflecting film,

$$\Delta = \text{path difference due to film} + \text{Path difference due to reflections.}$$

$$\therefore \quad \Delta = 2\mu t \cos r + 0 \qquad \ldots (1)$$

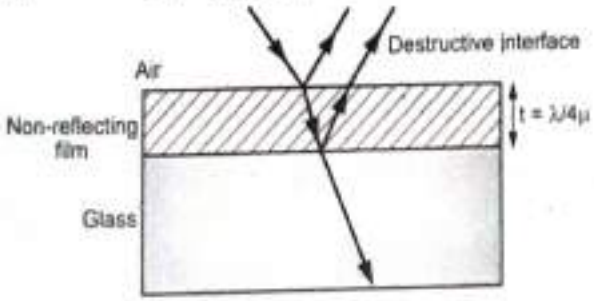


Fig. 1.17: Anti-reflection film

Considering normal incidence, the condition for destructive interference is,

$$2\mu t = (2n \pm 1)\frac{\lambda}{2}$$

For $n = 0, 2\mu t = \frac{\lambda}{2}$

$$\text{or} \qquad t = \frac{\lambda}{4\mu} \qquad \ldots (2)$$

- This means that the optical thickness, μt, of the material that has to be coated for reducing reflections is a quarter of the wavelength.
- For glass ($\mu = 1.5$), the materials having refractive index nearer to this value are magnesium fluoride ($\mu = 1.38$) and cryolite ($\mu = 1.36$) are usually used. In addition, the materials should adhere well, should be durable, scratch proof and insoluble in ordinary solvents. The coating is obtained by evaporating magnesium fluoride on the surface in vacuum.
- It is seen that condition (2) is satisfied only at a particular wavelength. This is usually chosen in the central yellow-green portion of the spectrum ($\lambda = 5500$ A°), where the eye is most sensitive. This leads to more reflection at both longer (red) and shorter (blue) wavelengths. The reflected light will therefore have a ***'purple hue'***.
- The overall reflection from a lens or a prism surface can be reduced, in this manner, from about 5% to 1%. This helps in increasing transmitted light. The same principle is used to minimize reflections from silicon photovoltaic cells (solar cells) by using a thin layer of silicon monoxide. This helps in increasing the amount of light actually reaching the solar cell.

Problem 1.26: *MgF_2 of refractive index 1.38 is coated on a glass plate in order to reduce the reflection from the glass surface using interference. How thick is the coating needed to produce a minimum reflection at the centre of visible spectrum (5500 A°) ?*

Data: $\mu = 1.38$, $\lambda = 5500 \times 10^{-8}$ cm

Considering normal incidence, $\cos r = 1$.

Formula: $2\mu t = \frac{\lambda}{2}$

$\therefore$ Thickness of coating,

$$t = \frac{\lambda}{4\mu}$$

Solution: $$t = \frac{5500 \times 10^{-8}}{4 \times 1.38}$$

$$\boxed{t = 996.37 \times 10^{-8} \text{ cm}}$$

SUMMARY

- **Interference** : The superposition of two waves of equal amplitude, frequency and a constant phase difference is called as interference. The result is alternate dark and bright fringes.

  Conditions on path difference (x):

  (a) Constructive Interference or Bright Fringe : Whole number multiple of λ.

  (b) Destructive Interference or Dark Fringe : Odd integer multiple of $\frac{\lambda}{2}$.

- **Thin Film :** A film is said to be thin if its thickness is of the order of a few wavelengths.
- In a **Thin Film of Uniform Thickness** in reflected system, condition for

  (a) constructive interference:

  $$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$$

  (b) destructive interference:

  $$2\mu t \cos r = n\lambda$$

  (c) in transmitted system, the conditions reverse.
- In a **Wedge-Shaped Film**, the condition for

  (a) constructive interference:

  $$2\mu t \cos (r + \alpha) = (2n + 1)\frac{\lambda}{2}$$

  (b) destructive interference:

  $$2\mu t \cos (r + \alpha) = n\lambda$$

  (c) fringe width:

  $$\beta = \frac{\lambda}{2\mu\alpha}$$

  (d) fringes obtained are: equal in thickness, straight, parallel and equidistant.
- Interference is used to test **Optical Flatness** of a surface.
- **Antireflection Coatings** are thin transparent coatings of quarter wave thickness

  $$\mu t = \frac{\lambda}{4}$$

IMPORTANT FORMULAE

- Constructive interference, $x = n\lambda$, $n = 0, 1, 2 \ldots$
- Destructive interference, $x = (2n + 1)\frac{\lambda}{2}$, $n = 0, 1, 2 \ldots$
- $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$, $n = 0, 1, 2 \ldots$
- $2\mu t \cos r = n\lambda$, $n = 0, 1, 2 \ldots$
- $2\mu t \cos (r + \alpha) = (2n \pm 1)\frac{\lambda}{2}$, $n = 0, 1, 2 \ldots$
- $2\mu t \cos (r + \alpha) = n\lambda$, $n = 0, 1, 2 \ldots$
- $\beta = \frac{\lambda}{2\mu \sin \alpha}$
- $t = \frac{\lambda}{4\mu}$

EXERCISE

1. Explain the phenomena of interference.
2. What is constructive and destructive interference ?
3. Derive the conditions for constructive and destructive interference.
4. Explain the phenomenon of interference in thin films in reflected light.
5. Explain the formation of colours in thin films.
6. Explain the phenomenon of interference in thin film in transmitted light.
7. What have you understood by non-reflecting films ? Explain.
8. Explain how the phenomenon of interference is utilized in testing the planeness of a surface.
9. When seen by reflected light, why does an excessively thin film appear to be perfectly black when illuminated by a white light ?
10. Explain, why colours are not observed in the case of a thick film when illuminated by a white light.
11. Explain how the principle of thin film interference can be used for measurement of thickness of thin film.

UNSOLVED PROBLEMS

1. A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.

   (**Ans.** 3.926×10^{-5} cm)
2. Two pin holes separated by a distance of 0.5 mm are illuminated by a monochromatic light of wavelength 6000 A°. An interference pattern is obtained on a screen placed at a distance of 100 cm from the pin holes. Find the distance on the screen between the fifth and tenth dark fringes. (**Ans.** 0.6 cm)
3. An oil drop of volume 0.2 cc is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain first

dark band whose centre has wavelength of 5.5×10^{-5} cm. Find the refractive index of oil. (**Ans.** 1.375)

4. A soap film of refractive index $\frac{4}{3}$ and of thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference of the dark band. (**Ans.** n = 6)
5. The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used. (**Ans.** Dark, 6015 A°)
6. Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds, of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm. (**Ans.** 89.1 seconds)
7. Two rectangular pieces of a plane glass are laid one upon the other and a thin wire is placed between them, so that a thin wedge shaped air film is formed between them. The plates are illuminated with sodium light of λ = 5893 A° at normal incidence. Bright and dark bands are formed, there being 10 of each per cm length of the wedge measured normal to the edge in contact. Find the angle of the wedge. (**Ans.** 2.94×10^{-4} radians)
8. Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes. (**Ans.** 0.295 cm)
9. Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°. (**Ans.** 1.03×10^{-4} cm)

UNIVERSITY QUESTIONS

December 2012

1. Define fringe width for wedge shaped film, obtain an expression for it. **[3]**

May 2013

1. Explain with diagram how interference Principle is used to design anti reflection coating. **[3]**

May 2014

1. Derive the equation of path difference between reflected rays when monochromatic light of wavelength 'λ' falls with angle of incidence 'I' on the uniform thickness film of refractive index 'μ'. Write the conditions of maxima and minima. **[6]**

December 2014

1. A monochromatic beam of light of wavelength 5893 A° is incident normally on the top of a glass which is coated by transparent material MgF_2 having R.I. 1.38. Calculate smallest thickness of the MgF_2 layer which will act as a non reflecting surface. **[3]**

May 2015

1. Derive an equation for path difference in reflected light when monochromatic light falls on the uniform thickness film and hence state the conditions for maxima and minima. **[6]**

May 2016

1. Explain with suitable diagram how interference is used to design anti-reflection coating. **[3]**

Dec. 2016

1. Derive expression for path difference in reflected light and derive the conditions for constructive and destructive interference for a film of uniform thickness. **[6]**
2. Explain an application of interference Antireflection coating. **[3]**

May 2017

1. A thin film of uniform thickness is illuminated by a monochromatic light. Derive an expression for path difference for the reflected rays system. Hence obtain the conditions for constructive and destructive interference. **[6]**

May 2018

1. Explain with a suitable diagram how the principle of interference is used in an anti-reflection coating. Derive and expression for its thickness. [3]

November 2018

1. Explain with a suitable diagram how the principle of interference is used in an anti-reflection coating. Derive an expression for its thickness. **[3]**

DIFFRACTION

1.4 INTRODUCTION (Nov. 13, May 14)

- Along with interference, diffraction is another phenomenon which supports the wave theory of light. Diffraction is responsible for the appearance of brilliant colours in a wide variety of natural phenomena. Diffraction with white light results into a beautiful rainbow like pattern.
- The flamboyant colours in the feathers of a peacock, the iridescent colours on the neck of pigeons, on the skins of snakes and on the back of beetles are due to diffraction of light.
- In practical applications, the diffraction affects the resolution of optical instruments such as microscopes and telescopes. Thus diffraction decides the usefulness of these devices.
- Diffraction of X-rays from the crystals is used in understanding the crystal structure. In engineering, diffraction is used for measurement of dimensions, stress, pressure etc.

1.5 DIFFRACTION OF LIGHT

- Diffraction is ***bending of light*** due to presence of an obstacle in the path of light. A diffraction pattern results from the interference of waves, diffracted by an obstacle, coming from the same source of light.
- For visible diffraction pattern,
  - The size of obstacle should be ***comparable to the wavelength*** of light.
  - The source of light must be a ***point source***.
- Diffraction is also defined as the encroachment of light in the region of geometrical shadow.
- According to Huygen's wave theory, each progressive wave produces secondary waves, the envelope of which produces the secondary wavefront.

OR

- Every point of a wavefront may be considered as a source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of waves.

Huygen's Theory

- In Fig. 1.18 (a), 'S' is a monochromatic source of light, MN is a small aperture, M'N' is the screen and AB is the illuminated portion on the screen in the absence of the aperture.
- Above A and below B, it is supposed to be geometrically shadow region. But practically, the shadows formed are not sharp and light encroaches the geometrical shadow region.

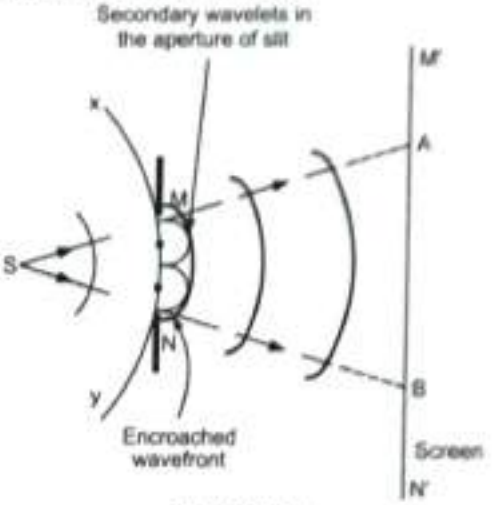


Fig. 1.18 (a)

- This is because when the primary wavefront XY is incident on the aperture MN, every point within the aperture can be viewed as creating secondary waves which propagate outward from the aperture. The envelope of all these secondary waves gives a new circular wavefront, thus slit works as a new source of light.
- The centre of the wave has more intensity and it fades out at the edges in the geometrical shadow region. Thus light through the aperture does not create a perfect image of the aperture and the diffraction observed can be explained.
- Similarly, if an opaque obstacle MN is placed in the path of light [See Fig. 1.18 (b)], the geometrical shadow region 'AB' is not sharp.

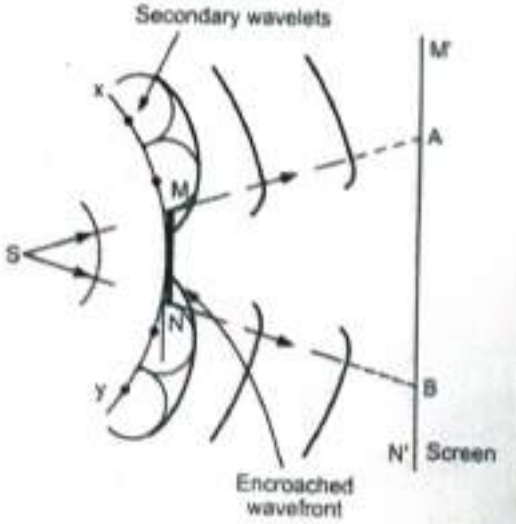


Fig. 1.18 (b)

- This is because when the primary wavefront XY is incident on the aperture MN, it developes a secondary wavefront. The envelope of this secondary wavefront encroaches in the geometrical shadow region.

1.6 DIFFRACTION AT SINGLE SLIT (GEOMETRICAL METHOD)

- For obtaining a Fraunhofer's diffraction pattern, the incident wavefront must be plane. **Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.**

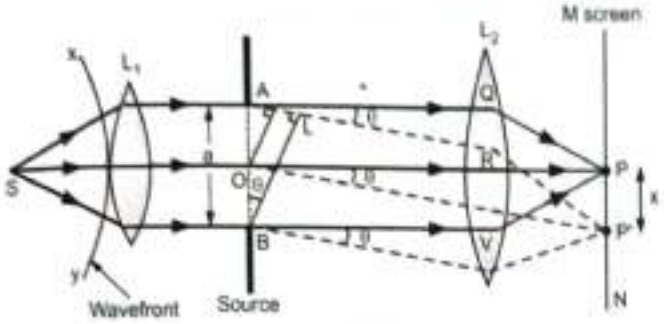
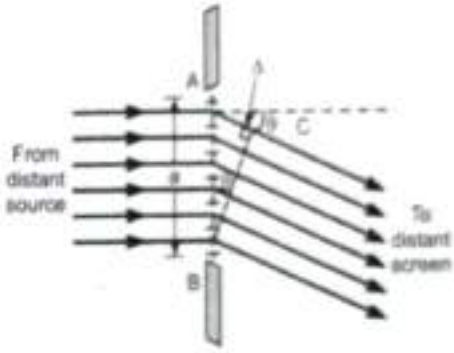


Fig. 1.19

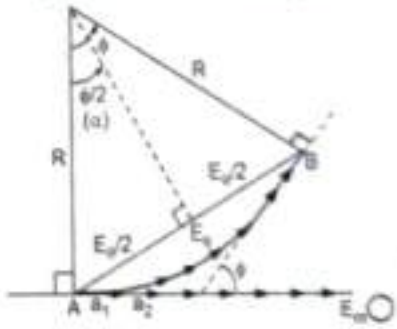
- In Fig. 1.19, 'S' is a narrow slit perpendicular to the plane of the paper and illuminated by a monochromatic light to act as a source of light. The wave coming from source S is made parallel by the collimating lens L_1.
- A plane wavefront is incident on the slit AB and each point on this wavefront is a source of the secondary wavefront. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the wavefront, travel the same distance to reach P and hence the path difference is zero. The secondary waves interfere with one another and P will be a point of ***maximum intensity***.
- Now, consider that the secondary waves are travelling in the direction AR, inclined at an angle θ to the direction OP. All the secondary waves travelling in this direction reach the point P' on the screen.
- The point P' will be of ***maximum*** or ***minimum intensity*** depending on the path difference between the secondary waves originating from the corresponding points of the wavefront.

Phasor Method

- The diffraction pattern for a single slit can be found out by the phasor method. Our approach is as follows.
- We divide the slit into a large number of narrow ***pseudoslits N*** of equal width, $\Delta x = \frac{a}{N}$, where 'a' is the width of the slit. The contribution of each pseudoslit will be represented by a phasor. To find the resultant wave amplitude, we add the N phasors.
- For convenience in drawing, the slit of width 'a' in Fig. 1.20 (a) has been split into six pseudoslits. If we call the phase difference between the first and the last phasor as φ, it will be the angle labelled as φ in Fig. 1.20 (b).



(a) Diffraction from singe slit



(b) Phasor diagram

Fig. 1.20

- The path difference between first and last wave is

$$\Delta = AC$$

$$\Delta = a \sin\theta \quad \text{[From Fig. 1.20 (a)]} \qquad \ldots (1)$$

∴ Phase difference,

$$\phi = k \cdot \Delta$$

where $k = \frac{2\pi}{\lambda}$

$$\therefore \quad \phi = \frac{2\pi}{\lambda} \cdot \Delta = \frac{2\pi}{\lambda}(a \sin\theta) \qquad \ldots (2)$$

For simplicity we will define angle α, such that,

$$\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(a \sin\theta) \qquad \ldots (3)$$

The phase difference between the waves reaching at the point from adjacent slit is

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x \sin\theta) \quad \text{... (4)}$$

- The amplitude a_n contributed at a point on the screen by any one of the pseudoslit will be the same, since they are of equal width. But the phases of these amplitudes will be different at different points.
- Let δ be the phase difference between two adjacent amplitudes which is constant. So each amplitude is inclined at an angle δ with the preceding one and their vector sum E_θ is the resultant amplitude.
- If the wavefront is divided into large or infinite number of equal elements, the vector a_n will become shorter and δ will decrease by the same proportion. In this way, the vector diagram will approach ***an arc of a circle***. The resultant amplitude E_θ is still the same and equal to ***the length of the chord of arc***.
- The length of arc is just the amplitude E_m obtained when all of the amplitudes are in phase i.e. slit is not there and diffraction is absent. The radius of arc is R and a perpendicular has been dropped from the centre on the chord E_m. This will divide the apex angle and chord into two equal halves ($E_\theta/2$ and $E_\theta/2$).
- In Fig. 1.20 (b), from the right triangle with apex angle $\phi/2$, we see that

$$\frac{E_\theta}{2} = R\sin\phi/2 \quad \text{... (5)}$$

$$\therefore \quad E_\theta = 2R\sin\alpha \quad \left(\because \alpha = \frac{\phi}{2}\right)$$

Also, $$\phi = \frac{\text{length of arc}}{\text{radius}} = \frac{E_m}{R}$$

This gives $E_m = R\phi$ or $E_m = 2R\alpha$ ... (6)

Dividing (5) by (6)

$$\frac{E_\theta}{E_m} = \frac{2R\sin\alpha}{2R\alpha}$$

$$\therefore \quad E_\theta = E_m\frac{\sin\alpha}{\alpha} \quad \text{... (7)}$$

Equation (7) gives the amplitude for the single slit diffraction pattern at any angle θ.

The intensity I_θ is proportional to the square of the amplitude.

$$\therefore \quad I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{... (8)}$$

where $I_m = E_m^2$ is the maximum amplitude.

1.7 CONDITIONS FOR MAXIMA AND MINIMA

(i) Principal Maximum

- The resultant amplitude in diffraction pattern of a single slit is given by,

$$E_\theta = E_m\frac{\sin\alpha}{\alpha} = \frac{E_m}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\right]$$

when $\sin\alpha$ is written in ascending powers of α.

where $\alpha = \frac{\pi}{\lambda}a\sin\theta$.

$$\therefore \quad E_\theta = E_m\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\right]$$

For E_θ to be maximum, the negative terms in the bracket must vanish. This is possible only when $\alpha = 0$

i.e. $\alpha = \frac{\pi}{\lambda}a\sin\theta = 0$ or $\sin\theta = 0$ or $\theta = 0$.

- Thus, the maximum value of E_θ is E_m and the principal maximum is formed at $\theta = 0$. The condition $\theta = 0$ simply means that this maximum is formed by parts of the secondary wavelets which travel normally to the slit. The position of principal maximum is directly opposite to the slit and it is bordered symmetrically by dark and bright bands.

(ii) Minimum Intensity Positions (Minima)

- The intensity $I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$ will be zero in the diffraction pattern if,

$$\sin\alpha = 0 \text{ and } \alpha \neq 0.$$

The values of α which satisfy this condition are

$$\alpha = m\pi \quad \text{where } m = \pm 1, \pm 2, \pm 3, \ldots$$

$$\therefore \quad \alpha = \frac{\pi}{\lambda}a\sin\theta = m\pi$$

Thus, the condition for minima is

$$a\sin\theta = m\lambda \quad \text{... (1)}$$

where $m = 0$ is not possible, because then θ becomes zero which corresponds to the principal maximum.

Equation (1) gives the positions of minima on either side of the principal maximum in the diffraction pattern of a single slit.

(iii) Secondary Maxima

- Analysis shows that the secondary maxima lie approximately half way between the two minima. They are found from

$$\alpha = \pm\left(m + \frac{1}{2}\right)\pi, \qquad m = 1, 2, 3, ...$$

or $\quad a \sin\theta = (2m + 1)\cdot\lambda/2$

Substituting this value of α in

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \text{, we get}$$

$$\frac{I_\theta}{I_m} = \left\{\frac{\sin\left(m + \frac{1}{2}\right)\pi}{\left(m + \frac{1}{2}\right)\pi}\right\}^2 = \frac{1}{\left(m + \frac{1}{2}\right)^2 \cdot \pi^2}$$

For m = 1, 2, 3, ...

$$\frac{I_\theta}{I_m} = 0.045, 0.016, 0.0083 .....$$

1.8 INTENSITY PATTERN DUE TO A SINGLE SLIT (May 14)

- Consider monochromatic plane waves incident on a single slit of width 'a'. When aperture is very small, only one secondary wavelet comes through and the wavefront is spherical. Suppose the slit width is such that several secondary wavelets pass through the slit. At the distance screen, these secondary wavelets superpose giving a rippled intensity distribution which is called ***the single-slit diffraction pattern*** as shown in Fig. 1.21.

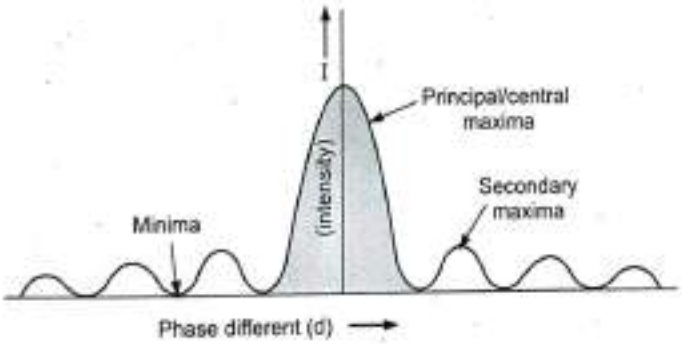


Fig. 1.21: Intensity distribution

- The intensity in the pattern has a central maximum and then falls to zero (the first minimum), past the first minimum the intensity rises to a relatively small secondary maximum before again dropping to zero (the second minimum). The rippling continues with each secondary maximum having less magnitude than the previous secondary maximum.

Dependence of Spectrum on Width and Wavelength

- The position of the minimum is given by, $a \sin\theta = m\lambda$. Therefore, the angular width of the spectrum will depend upon the slit width a, wavelength λ and order of interference m.
- The width of spectrum inversely depends upon the slit width, hence ***smaller the slit width, wider will be the spectrum***. Fig. 1.22 shows spectrum width for different slit width.

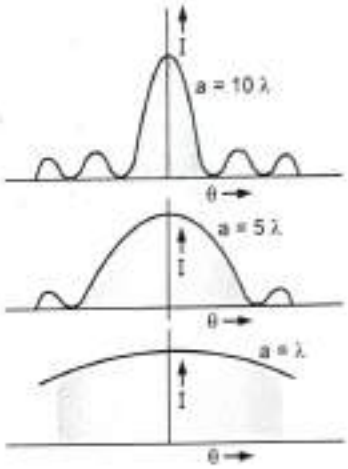


Fig. 1.22

- Fig. 1.22 shows the variation of single slit diffraction pattern as the slit width varies. Decreasing the aperture size from 10λ to 5λ causes the diffraction pattern to spread out about twice as far.
- The wavelength directly decides the spectrum width. If the slit is illuminated by white light, the light of different wavelengths will form a spectrum of different angular width. All this will overlap giving a colourful spectrum.
- The spectrum width directly depends on the order 'm'. But for higher order, the intensity decreases.

1.9 DIFFRACTION GRATING

- **Grating:** An arrangement consisting of a ***large number of parallel*** slits of the same width and separated by equal opaque spaces is known as ***diffraction grating***.
- A grating is prepared by ruling equidistant parallel lines on a plane glass plate with the help of a diamond point. The lines act as opaque spaces and the incident light cannot pass through them. The space between any two lines acts as a slit and is transparent to light.
- A plane transmission grating generally contains 15,000 to 20,000 lines per inch. It was first constructed by Fraunhofer.
- If the lines are drawn on a silvered surface, it acts as a ***reflection grating***.

Theory of Plane Diffraction Grating

- Consider the diffraction pattern of N parallel slits each of width 'a' and separated by equal opaque spaces 'b'. The distance between the centres of adjacent slits is d = (a + b) and it is called the ***grating element***.
- The grating element is defined as the reciprocal of number of lines per cm i.e.

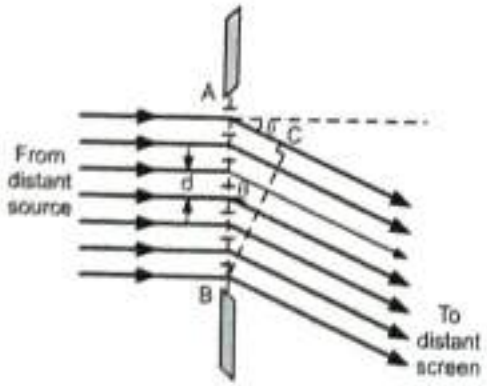
$$(a + b) = \frac{1}{N}, \text{ where N is lines per cm}$$

or $(a + b) = 2.54/N$ where N is lines per inch.

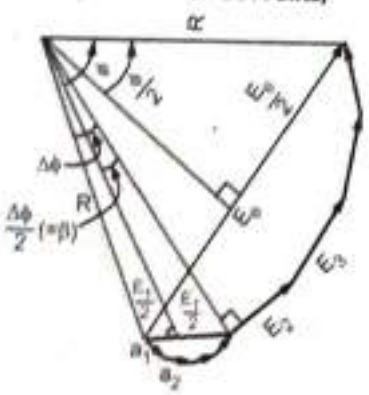
- Let a plane wavefront of wavelength λ be incident normally on the slits. According to Huygen's principle every point in the slit is regarded as the origin of secondary wavelets which spread out in all directions. Therefore, rays are diffracted from each slit in all directions.
- The resultant amplitude of light from a single slit of width 'a' in a direction making angle θ with the normal is given by

$$E_N = E_m \left(\frac{\sin \alpha}{\alpha}\right) \quad \text{... (1)}$$

where $$\alpha = \frac{\pi}{\lambda} a \sin \theta \quad \text{... (2)}$$



(a) Diffraction at N slits,



(b) Phasor diagram

Fig. 1.23

- We can replace all the secondary wavelets in each slit by a single wave of amplitude $E_m \frac{\sin \alpha}{\alpha}$, starting from its mid-point and travelling at an angle θ with the normal.
- We need, therefore, find only the N-slit interference pattern and multiply it by $E_m \frac{\sin \alpha}{\alpha}$ to obtain the complete pattern.
- To find the interference pattern for N slits, we make use of phasors. Assuming each phasor to have amplitude E_N and at an angle $\Delta\phi$ the phasor diagram is as shown in Fig. 1.23 (b).
- The phase angle $\Delta\phi$ is the phase difference between the waves coming from adjacent slits and is given by

$$\Delta\phi = k \cdot \text{(path difference between consecutive slits)}$$

$$\Delta\phi = \frac{2\pi}{\lambda}(a + b) \sin \theta$$

$$\Delta\phi = \frac{2\pi}{\lambda} d \sin \theta \quad \text{... (3)}$$

- We will define angle β such that

$$\beta = \frac{\Delta\phi}{2} = \frac{\pi}{\lambda}(a + b) \sin \theta \quad \text{... (4)}$$

- The total phase difference between first and last wave will simply be sum of the phase differences added by each slit individually.

$$\therefore \quad \phi = N\Delta\phi$$

$$\phi = N\frac{2\pi}{\lambda}(a + b) \sin \theta \quad \text{... (5)}$$

or $$\phi = 2N\beta \quad \text{[from equation (4)] ... (6)}$$

- As the slits of a grating are of equal width, the amplitude of light diffracted from each slit will be same. The amplitude of diffracted wave will be,

$$E_1 = E_2 = \ldots\ldots = E_N = E_m\left(\frac{\sin \alpha}{\alpha}\right) \quad \text{... (7)}$$

- Mathematically, it can be proved that the resultant amplitude is,

$$E_\phi = E_m \frac{\sin \alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin \beta} \quad \text{... (8)}$$

Intensity is square of the amplitude.

$$\therefore \quad I_\phi = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2 \cdot \frac{\sin^2 N\beta}{\sin^2 \beta} \quad \text{... (9)}$$

- The first factor $\left(\frac{\sin\alpha}{\alpha}\right)^2$ in equation (9) gives the intensity distribution in the ***diffraction pattern due to a single slit***. The second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ may be said to give the interference pattern for N slits.
- Thus, we can say that each of the N slits gives rise to a diffracted beam in which the intensity distribution depends on the width of the slit. These diffracted beams then interfere with one another to produce the final diffraction pattern.

1.10 CONDITIONS FOR MAXIMA AND MINIMA (May 16)

(i) Principal Maxima

- The condition for principal maxima is that, the path difference between the waves from adjacent slits must be an integer multiple of λ. Therefore, the condition for principal maxima is

$$(a + b)\sin\theta = m\lambda, \text{ where, } m = 0, 1, 2, 3, \ldots m \quad \ldots (1)$$

Here m is called the ***order of interference***.

This is equivalent to saying that,

$$\beta = \frac{\pi}{\lambda}(a + b)\sin\theta$$

$$\beta = \frac{\pi}{\lambda}(m\lambda) = m\pi \quad \ldots (2)$$

For these values of β, $\frac{\sin N\beta}{\sin\beta}$ becomes indeterminate.

But by L'Hospital's rule,

$$\lim_{\beta \to m\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to m\pi} \frac{N\cos N\beta}{\cos\beta} = \frac{N\cos Nm\pi}{\cos m\pi}$$

$$= \pm N$$

∴ The intensity of the principal maxima is given as

$$I_\theta = N^2\left(I_m \frac{\sin^2\alpha}{\alpha^2}\right) \quad \ldots (3)$$

Thus, the intensity of principal maxima increases with increasing N. The intensity of the principal maximum is greatest while on either side of it, the intensities of other maxima go on decreasing.

(ii) Minima

- The intensity is given by,

$$I_\theta = \left(I_m \frac{\sin\alpha}{\alpha}\right)^2 \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$$

For minima, $\sin N\beta = 0$ but $\sin\beta \neq 0$ i.e. $N\beta = m\pi$, where m has any integral value except N, 2N, 3N ... etc., because for these values of m, $\beta = 0, \pi, 2\pi$ ... etc. and these correspond to principal maxima.

Thus, for minima,

$$N_\beta = m\pi$$

$$\text{or } \beta = \frac{m\pi}{N} \qquad \text{But } \beta = \frac{\pi}{\lambda}(a + b)\sin\theta$$

∴ The condition for minima becomes

$$\frac{\pi}{\lambda}(a + b)\sin\theta = \frac{m\pi}{N}$$

$$\text{or } (a + b)\sin\theta = \frac{m}{N}\lambda, \text{ but } m \neq nN \quad \ldots (4)$$

where n = 0, 1, 2, 3, ...

Hence, the positions of minima are given by

$$(a + b)\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N} \ldots \quad \ldots (5)$$

There are (N – 1) minima between any two consecutive principal maxima.

(iii) Secondary Maxima

- As there are (N – 1) minima between two consecutive principal maxima, there must be (N – 2) other maxima coming alternatively with the minima between two consecutive principal maxima. These maxima are called the secondary maxima. The positions of secondary maxima are obtained by differentiating the expression for intensity with respect to β and equating to zero.

1.11 INTENSITY PATTERN

- A diffraction pattern due to diffraction grating consists of m principal maximum, one each for integer value of m. But the intensity of the principal maxima goes on decreasing with order.
- In between any two principal maxima there are minima and secondary. The intensity of secondary maxima is negligible in comparison with the intensity of principal maxima. The intensity from maxima to minima or minima to maxima changes gradually as a function of sine.
- The number of minima and secondary maxima in between principal maxima is not fixed but depends upon the number of slits in grating.
- If the number of slits in grating are N, then the number of minima will be (N – 1) and secondary maxima will be (N – 2).

- Fig. 1.24 shows diffraction pattern for N = 5, which has (N – 1 = 4) minima and (N – 2 = 3) secondary maxima.

Diffraction pattern for N = 5

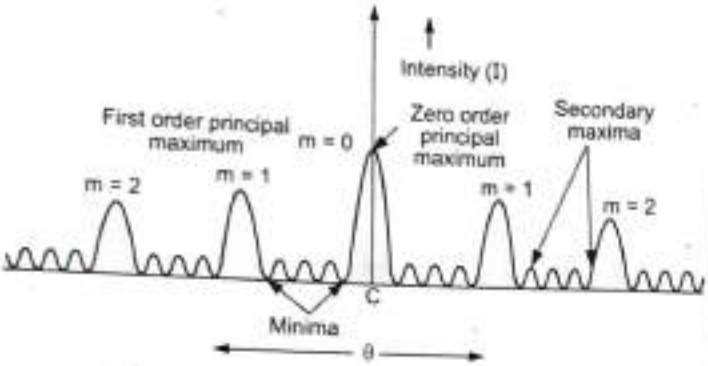


Fig. 1.24: Diffraction pattern for a grating

SOLVED PROBLEMS

Problem 1.27: *A slit of width 0.16 mm is illuminated by a light of wavelength 5600 A°. Find the half angular width of the central maximum.*

Data: a = 0.016 cm, $\lambda = 5600 \times 10^{-8}$ cm, n = 1.

Formula: $a \sin\theta = n\lambda$.

Solution: $a \sin\theta = n\lambda$

but $n = 1$

$$\therefore \quad \sin\theta = \frac{\lambda}{a}$$

$$\theta = \sin^{-1}\frac{\lambda}{a}$$

Substituting, $\theta = \sin^{-1}\left(\frac{5600 \times 10^{-8}}{0.016}\right)$

$$\therefore \quad \boxed{\theta = 12'.}$$

Problem 1.28: *A slit of variable width is illuminated by red light of λ = 6500 A°. At what width of the slit, the first minimum will fall at θ = 30° ?* **(03) (May 06)**

Data: $\lambda = 6500 \times 10^{-8}$ cm, $\theta = 30°$, n = 1

Formula: $a \sin\theta = n\lambda$.

Solution: $a \sin\theta = n\lambda$

but $n = 1$

$$a = \frac{\lambda}{\sin\theta}$$

Substituting, $a = \frac{6500 \times 10^{-8}}{\sin 30}$

$$\therefore \quad \boxed{a = 0.13 \times 10^{-3} \text{ cm}}$$

Problem 1.29: *A light of λ = 6000 A° falls on a screen at a distance of 200 cm from a narrow slit. Find the width of the slit, if the first minimum lies 5 mm on either side of the central maximum.*

Data: $\lambda = 6000 \times 10^{-8}$ cm, f = 200 cm, x = 0.5 cm, n = 1.

Formula: $a \sin\theta = n\lambda$, $\tan\theta = \frac{x}{f} \approx \sin\theta$

Solution: $\sin\theta = \frac{\lambda}{a} = \frac{x}{f}$ $\quad \because n = 1$

$$\frac{\lambda}{a} = \frac{x}{f}$$

$$a = \frac{\lambda f}{x}$$

Substituting, $a = \frac{6000 \times 10^{-8} \times 200}{0.5}$

$$\boxed{a = 0.024 \text{ cm}}$$

Problem 1.30: *A lens whose focal length is 40 cm forms a Fraunhofer diffraction pattern of a slit 0.3 mm wide. Calculate the distance of the first dark band and of the next bright band from the axis. Given λ = 5890 A°.*

Data: $\lambda = 5890 \times 10^{-8}$ cm, f = 40 cm, a = 0.03 cm

Formula: $a \sin\theta = n\lambda$

$$\sin\theta = \frac{x}{f}$$

$$a \sin\theta = (2n + 1)\frac{\lambda}{2}$$

Solution:

For $n = 1$, $\sin\theta = \frac{\lambda}{a}$

For first dark band,

$$\sin\theta = \frac{x}{f}$$

$$\therefore \quad \frac{\lambda}{a} = \frac{x}{f}$$

$$\therefore \quad x = \frac{\lambda f}{a}$$

Substituting,

$$x = \frac{5890 \times 10^{-8} \times 40}{0.03}$$

$$\therefore \quad \boxed{x = 0.0785 \text{ cm}}$$

For bright band, n = 1

$$a \sin\theta = (2n+1)\frac{\lambda}{2} \quad \text{... (1)}$$

$$\sin\theta = \frac{x_1}{f} \quad \text{... (2)}$$

From (1) and (2),

$$\frac{x_1}{f} = (2n+1)\frac{\lambda}{2a}$$

$$x_1 = \frac{3}{2}\frac{\lambda}{a} f$$

Substituting, $x_1 = \frac{3}{2}\frac{5890 \times 10^{-8}}{0.03} \times 40$

$$\boxed{x_1 = 0.11775 \text{ cm}}$$

Problem 1.31: *A lens of focal length 100 cm forms Fraunhofer's diffraction pattern of a single slit of width 0.04 cm in its focal plane. The incident light contains two wavelengths λ_1 and λ_2. It is found that, fourth minimum corresponding to λ_1 and the fifth minimum corresponding to λ_2 occur at the same point 0.5 cm from the central maximum. Calculate λ_1 and λ_2.*

Data: f = 100 cm, a = 0.04 cm, x = 0.5 cm

Formula: $a \sin\theta = n\lambda$, $\sin\theta = \frac{x}{f}$.

Solution: Condition for fourth minimum

$$a \sin\theta_1 = 4\lambda_1$$

$$\therefore \quad \sin\theta_1 = \frac{4\lambda_1}{a} \quad \text{... (1)}$$

Also, $\sin\theta_1 = \frac{x}{f}$ ... (2)

From (1) and (2),

$$\frac{x}{f} = 4\frac{\lambda_1}{a}$$

$$\therefore \quad \lambda_1 = \frac{x}{f} \cdot \frac{a}{4}$$

$$\lambda_1 = \frac{0.5}{100} \times \frac{0.04}{4}$$

$$\boxed{\lambda_1 = 5 \times 10^{-5} \text{ cm}}$$

It is given that fourth minimum of λ_1 coincides with fifth minimum of λ_2. In that case, $\theta_1 = \theta_2 = \theta$, where θ_1 and θ_2 are the angular deviations for the fourth and fifth minimum respectively, then

$$a \sin\theta_1 = 4\lambda_1 = 5\lambda_2$$

or $5\lambda_2 = 4\lambda_1$

$$\lambda_2 = \frac{4}{5}\lambda_1$$

$$\therefore \quad \lambda_2 = \frac{4}{5} \times 5 \times 10^{-5}$$

$$\boxed{\lambda_2 = 4 \times 10^{-5} \text{ cm}}$$

Problem 1.32: *In Fraunhofer's diffraction pattern due to a single slit, the screen is at a distance of 100 cm from the slit and the slit is illuminated by a monochromatic light of wavelength 5893 A°. The width of the slit is 0.1 mm. Calculate the separation between the central maximum and the first minimum.*

Data: f = 100 cm, $\lambda = 5893 \times 10^{-8}$ cm, a = 0.1 mm = 0.01 cm

Formula: $a \sin\theta = n\lambda$, $\tan\theta = \frac{x}{f} = \sin\theta$

Solution:

$$x = f\frac{\lambda}{a}$$

$$x = \frac{100 \times 5893 \times 10^{-8}}{0.01}$$

$$\boxed{x = 0.5893 \text{ cm}}$$

Problem 1.33: *Calculate the angular position of the first minimum in Fraunhofer's diffraction pattern of a slit 10^{-4} cm wide, if it is illuminated by light of wavelength 5000 A°.*

Data: $\lambda = 5000$ A°, $a = 10^{-4}$ cm, n = 1

Formula: $a \sin\theta = n\lambda$.

Solution: $a \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{a}$$

$$\theta = \sin^{-1}\left(\frac{n\lambda}{a}\right)$$

$$\theta = \sin^{-1}\left(\frac{1 \times 5000 \times 10^{-8}}{10^{-4}}\right)$$

$$\boxed{\theta = 30^\circ}$$

Problem 1.34: *A light of wavelength 5×10^{-5} cm is incident normally on the plane transmission grating of width 3 cm and having 15000 lines. Find the angle of diffraction in the first order.*

Data: $\lambda = 5 \times 10^{-5}$ cm, $(a+b) = \frac{3}{15000}$, n = 1

Formula: $(a+b)\sin\theta = n\lambda$.

Solution: $(a+b)\sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{a+b}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{a+b}\right)$$

Substituting, $\theta = \sin^{-1}\left(\frac{5\times10^{-5}}{3}\times15000\right)$

$$\boxed{\theta = 14^\circ\ 29'}$$

Problem 1.35: *The limits of a visible spectrum are approximately 400×10^{-7} cm to 700×10^{-7} cm. Find the angular width of the first order visible spectrum produced by a plane grating having 15000 lines/inch when light is incident normally on the grating.*

Data: $\lambda_1 = 400\times10^{-7}$ cm, $\lambda_2 = 700\times10^{-7}$ cm.

$$a+b = \frac{1}{\frac{15000}{2.54}} = \frac{2.54}{15000}\text{ cm}$$

Formula: $(a+b)\sin\theta = n\lambda$.

Solution: Let θ_1 and θ_2 be the angles corresponding to λ_1 and λ_2.

Here, $n = 1$

$$\therefore \quad \sin\theta_1 = \frac{\lambda}{a+b}$$

or $$\theta_1 = \sin^{-1}\frac{\lambda_1}{(a+b)}$$

$$\sin\theta_2 = \frac{\lambda_2}{a+b}$$

$$\theta_2 = \sin^{-1}\left(\frac{\lambda_2}{(a+b)}\right)$$

$\therefore$ Angular width of first order visible spectrum will be $\theta_2 - \theta_1$.

Substituting, $\theta_1 = \sin^{-1}\left(\frac{400\times10^{-7}\times15000}{2.54}\right)$

$$\theta_1 = 13^\circ\ 40'$$

$$\theta_2 = \sin^{-1}\frac{700\times10^{-7}\times15000}{2.54}$$

$$\theta_2 = 24^\circ\ 30'$$

$$\therefore \quad \theta_2 - \theta_1 = 24^\circ\ 30' - 13^\circ\ 40'$$

$$= \boxed{10^\circ\ 50'}$$

Problem 1.36: *A parallel beam of sodium light is allowed to be incident normally on a plane grating having 4250 lines per cm and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.*

Data: $\theta = 30^\circ$, $a+b = \frac{1}{4250}$, $n = 2$. , $\lambda = ?$

5880 Å

Formula: $(a+b)\sin\theta = n\lambda$.

Solution: $(a+b)\sin\theta = n\lambda$

but $n = 2$

$$\therefore \quad \lambda = \frac{(a+b)}{2}\sin\theta$$

Substituting, $\lambda = \frac{1}{4250}\times\frac{1}{2\times2}$

$$\lambda = 5.880\times10^{-5}\text{cm} = \boxed{5880\ \text{A}^\circ}$$

Problem 1.37: *How many orders will be visible, if the wavelength of the incident radiation is 5000 A° and the number of lines on the grating is 2620 in one inch ?*

Data: $\lambda = 5000\times10^{-8}$ cm, $(a+b) = \frac{2.54}{2620}$ cm

Formula: $(a+b)\sin\theta = n\lambda$. $\sin\theta = 1$

Solution: $(a+b)\sin\theta = n\lambda$

To have maximum order, the maximum possible value of $\sin\theta = 1$

$$\therefore \quad n = \frac{(a+b)}{\lambda}$$

Substituting, $n = \frac{2.54}{2620}\times\frac{1}{5\times10^{-5}}$

$$\therefore \quad \boxed{n = 19.38}$$

Hence, the highest order of the spectrum which can be seen is 19.

Problem 1.38: *In a plane transmission grating, the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines in one cm of the grating surface.*

Data: $\lambda = 5\times10^{-5}$ cm, $\theta = 30^\circ$, $n = 2$.

Formula: $(a+b)\sin\theta = n\lambda$.

Solution:

$$(a+b)\sin\theta = n\lambda$$

but, $n = 2$

$$\therefore \quad (a+b) = \frac{n\lambda}{\sin\theta}$$

Substituting,

$$(a+b) = \frac{2\times5\times10^{-5}}{1/2}$$

$$\therefore \quad (a+b) = 20\times10^{-5}\text{ cm}$$

But $$(a+b) = \frac{1}{\text{no. of lines/cm}}$$

$$\therefore \text{No. of lines/cm} = \frac{1}{a+b}$$

$$= \frac{1}{20 \times 10^{-5}}$$

$$= \boxed{5000 \text{ lines/cm}}$$

Problem 1.39: *Monochromatic light of wavelength 6×10^{-5} cm falls normally on a slit of width 0.001 cm. Calculate the angular width of the central bright maximum.*

Data: $\lambda = 6 \times 10^{-5}$ cm, $a = 0.001$ cm

Formula: $a \sin\theta = n\lambda$.

Solution: $a \sin\theta = n\lambda$

but, $n = 1$

$$\therefore \quad \sin\theta = \frac{\lambda}{a}$$

$$\theta = \sin^{-1}\frac{\lambda}{a}$$

$$\theta = \sin^{-1}\frac{6 \times 10^{-5}}{0.001} = 3^\circ\, 26'$$

$\therefore$ The angular width of the central bright maximum $= 2\theta$

$$= \boxed{6^\circ\, 52'}$$

Problem 1.40: *Monochromatic light of wavelength 6.56×10^{-5} cm falls normally on a grating 2 cm wide. The first order spectrum is produced at an angle of $18^\circ\, 14'$ from the normal. What is the total number of lines on the grating ?* **(04) (Dec. 06)**

Data: $\lambda = 6.56 \times 10^{-5}$ cm, $\theta = 18^\circ\, 14'$, $n = 1$.

Total width of the grating $= 2$ cm

Formula: $(a + b) \sin\theta = n\lambda$.

Solution: $(a + b) \sin\theta = n\lambda$

As $n = 1$

$$\therefore \quad (a + b) = \frac{\lambda}{\sin\theta}$$

Substituting,

$$(a + b) = \frac{6.56 \times 10^{-5}}{\sin 18^\circ\, 14'} = \frac{6.56 \times 10^{-5}}{0.3123}$$

$$(a + b) = 21.005 \times 10^{-5} \text{ cm}$$

But $$(a + b) = \frac{1}{\text{No. of lines/cm}}$$

$$21.005 \times 10^{-5} = \frac{1}{\text{No. of lines/cm}}$$

$\therefore$

$$\text{No. of lines/cm} = \frac{1}{21.005 \times 10^{-5}}$$

$$= \boxed{4761 \text{ lines/cm}}$$

Since, the grating has 2 cm width, total number of lines on the grating is $4761 \times 2 = \boxed{9{,}522}$

Problem 1.41: *A grating has 6000 lines/cm. Find the angular separation of two yellow lines of mercury of wavelengths 5770 A° and 5791 A° in the second order.* **(May 99)**

Data: $a + b = \frac{1}{6000}$ cm.

$\lambda_1 = 5770 \times 10^{-8}$ cm

$\lambda_2 = 5791 \times 10^{-8}$ cm

$n = 2$

Formula: $(a + b) \sin\theta = n\lambda$.

Solution:

$(a + b) \sin\theta_n = n\lambda$

For λ_1,

$(a + b) \sin\theta_2 = n\lambda_1$

$$\sin\theta_2 = \frac{n\lambda_1}{(a + b)} = 2 \times 5770 \times 10^{-8} \times 6000$$

$$= 0.6924$$

$$\theta_2 = 43.82^\circ$$

For λ_2,

$(a + b) \sin\theta_2 = n\lambda_2$

$$\sin\theta_2 = \frac{n\lambda_2}{(a + b)} = 2 \times 5791 \times 10^{-8} \times 6000$$

$$= 0.6949$$

$$\theta_2 = 44.02^\circ$$

Angular separation between the two yellow lines $= \theta_2' - \theta_2$

$$= 44.02 - 43.82 = \boxed{0.2^\circ \text{ or } 12'}$$

Problem 1.42: *Calculate the angles at which the first dark band and the next bright band are formed in Fraunhofer's diffraction pattern of a slit of 0.2 mm wide.*
Given $\lambda = 5890$ A°.

Data: $a = 0.2$ mm, $\lambda = 5890$ A°.

Formulae: (i) $a \sin\theta = n\lambda$ (ii) $a \sin\theta' = \left(n + \frac{1}{2}\right)\lambda$

Solution:

(i) For $n = 1$ $\Rightarrow \sin\theta$

$$= \frac{n\lambda}{a} = \frac{5890 \times 10^{-10}}{0.2 \times 10^{-3}}$$

$$= 2.945 \times 10^{-3}$$

$$\theta = \sin^{-1}(2.945 \times 10^{-3}) = 0.1687°$$

$$= \boxed{10.12°}$$

(ii) For n = 2 $\Rightarrow a \sin\theta'$

$$= \left(n + \frac{1}{2}\right)\lambda ; \sin\theta' = \left(n + \frac{1}{2}\right)\frac{\lambda}{a} = \frac{3\lambda}{2a}$$

$$\sin\theta' = 4.4175 \times 10^{-3}; \theta' = 0.2531° = \boxed{15.18°}$$

Problem 1.43: *Examine if two spectral lines of wavelengths 5890 A° and 5896 A° can be clearly resolved in the (i) first order and (ii) second order by a diffraction grating 2 cm wide and having 425 lines per cm.*

Data: λ_1 = 5890 A°, λ_2 = 5896 A°, w = 2 cm, N = 425 lines/cm

Formula: $(a + b) \sin\theta = m\lambda$

Solution: $(a + b) = \frac{2}{425} = 4.7 \times 10^{-3}$ cm, n = 1

In first order:

$$\sin\theta_1 = \frac{n\lambda_1}{a + b} = \frac{1 \times 5890 \times 10^{-8}}{4.7 \times 10^{-3}}$$

$$\sin\theta_1 = 0.0125$$

$$\theta_1 = 0.71°$$

$$\sin\theta_1 = \frac{n\lambda_2}{a + b} = \frac{1 \times 5896 \times 10^{-8}}{4.7 \times 10^{-3}}$$

$$\sin\theta_1 = 0.01254$$

$$\boxed{\theta_1 = 0.71°}$$

In second order:

$$\sin\theta_2 = \frac{n\lambda_1}{a + b} = \frac{2 \times 5890 \times 10^{-8}}{4.7 \times 10^{-3}}$$

$$\sin\theta_2 = 0.02506$$

$$\sin\theta_2 = 1.44°$$

$$\sin\theta_2 = 0.02508$$

$$\boxed{\theta_2 = 1.4376}$$

θ_2 both values slightly differ means slightly resolved.

Problem 1.44: *Calculate the wavelength of light whose diffraction maximum in the diffraction pattern due to a single slit falls at θ = 30° and coincides with the first minimum for red light of wavelength 6500 A°.*

Data: θ = 30°, λ = 6500 A°

Formulae: (i) $a \sin\theta = n\lambda_1$,

(ii) $a \sin\theta = \left(n + \frac{1}{2}\right)\lambda_2$ for bright band

'a' is constant, θ is constant order.

Solution: (i) $n\lambda_1 = \left(n + \frac{1}{2}\right)\lambda_2$ here n = 1

(ii) $\lambda_2 = \frac{n\lambda_1}{\left(n + \frac{1}{2}\right)} = \frac{\lambda_1}{3/2} = \boxed{4333.33 \text{ A°}}$

Problem 1.45: *Monochromatic light of wavelength λ = 6560 A° falls normally on a grating. The spectral line is diffracted at an angle of 19° 9' from the normal in the first order. Find the grating element.*

Data: λ = 6560 A°, θ = 19° 9', n = 1

Formula: $(a + b) \sin\theta = m\lambda$

Solution: $(a + b) = \frac{m\lambda}{\sin\theta} = \frac{1 \times 6560 \times 10^{-8}}{\sin(19° 9')}$

$$= \frac{6560 \times 10^{-8}}{0.340}$$

$$= 1.9 \times 10^{-4} = \boxed{2 \times 10^{-4} \text{ cm}}$$

Problem 1.46: *Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angles of diffraction for the principal maxima of the two sodium lines in the first order spectrum λ_1 = 5890 A° and λ_2 = 5896 A°.*

Data: w = 0.5 cm, Total lines = 2500 lines, λ_1 = 5890 A°, λ_2 = 5896 A°.

Solution: In 1 cm = 2500 × 2 = 5000 lines $\therefore (a + b) = \frac{1}{5000} = 2 \times 10^{-4}$ cm, order n = 1. For wavelength λ_1 = 5890,

$$\sin\theta_1 = \frac{n\lambda_1}{a + b} = \frac{1 \times 5890 \times 10^{-8}}{2 \times 10^{-4}}$$

$$\sin\theta_1 = 0.2945$$

$$\boxed{\theta_1 = 17.12°}$$

$$\sin\theta_2 = \frac{n\lambda_2}{a + b}$$

$$= \frac{1 \times 5896 \times 10^{-8}}{2 \times 10^{-4}}$$

$$\sin\theta_2 = 0.2948$$

$$\boxed{\theta_2 = 17.2°}$$

Problem 1.47: *What is the highest order spectrum that is visible with light of wavelength 6000 A° by means of a grating having 5000 lines per cm ?*

Data: $\lambda = 6000$ A°

$N = 5000$ lines per cm, $a + b = \frac{1}{5000}$ cm

Formula: $(a + b) \sin\theta = n\lambda$

Solution: Take, $\sin\theta = 1$

$\therefore \quad a + b = n\lambda$

$$\frac{1}{5000} = n \times 6000 \times 10^{-8}$$

$$\boxed{n = 3.3}$$

The highest order is n = 3.

Problem 1.48: *Light of wavelength 5460 A° falls on a diffraction grating normal to its surface. The grating is ruled with 7500 lines per cm. What is the angle corresponding to the first bright fringe produced by the grating ?*

(04) (Jan. 05)

Data: $\lambda = 5460$ A°

$N = 7500$ lines per cm

$n = 1$

$a + b = \frac{1}{7500}$ cm

Formula:

$(a + b) \sin\theta = n\lambda$

Solution:

$$\frac{1}{7500} \sin\theta = 1 \times 5460 \times 10^{-8}$$

$$\sin\theta = 0.4095$$

$$\boxed{\theta = 24.17°}$$

Problem 1.49: *Calculate the angular separation between the first order minima on either side of the central bright maxima when slit is 6×10^{-4} cm wide and $\lambda = 6000$ A°.*

(04) (Jan. 05)

Data: $a = 6 \times 10^{-4}$ cm

$\lambda = 6000$ A°

Formula: $a \sin\alpha = n\lambda$

Solution: Take $n = 1$

$$\sin\theta = \frac{1 \times 6000 \times 10^{-8}}{6 \times 10^{-4}}$$

$$\sin\theta = 0.1$$

$$\theta = 5.739° = \boxed{5° 44'}$$

Problem 1.50: *How many orders will be visible if the wavelength of the incident light is 6000 A° and the number of lines on the grating is 5.0×10^3 lines per cm ?*

(03) (Nov./Dec. 05)

Data: $\lambda = 6000$ A°

$N = 5.0 \times 10^3$ lines per cm

Solution:

$(a + b) \sin\theta = n\lambda$

take $\sin\theta = 1$

$$\frac{1}{N} = n\lambda$$

$$n = \frac{1}{5 \times 10^3 \times 6000 \times 10^{-8}}$$

$$\boxed{n = 3.33}$$

The number of order, n = 3

Problem 1.51: *What is the longest wavelength that can be observed in the third order for a transmission grating having 7000 lines per cm ? Assume normal incidence.* **(03) (May 07)**

Data: $n = 3$

$N = 7000$ lines per cm, $(a + b) = \frac{1}{7000}$ cm

Formula: $(a + b) \sin\theta = n\lambda$

Solution: Take $\sin\theta = 1$

$$\frac{1}{N} = n\lambda$$

$$\lambda = \frac{1}{7000 \times 3}$$

$$\lambda = 4.761 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 4761 \text{ A°}}$$

Problem 1.52: *What is the highest order of spectrum which may be seen with the light of wavlength 6328 A° by means of a grating with 3000 lines km ?* **(03) (May 08)**

Data: $N = 3000$ lines /cm

$\lambda = 6328$ A°, $(a + b) = \frac{1}{3000}$ cm

Formula:

$$(a + b) \sin\theta = n\lambda$$

Solution: Take $\sin\theta = 1$

$$\frac{1}{N} = n\lambda$$

$$n = \frac{1}{3000 \times 6328 \times 10^{-8}}$$

$$\boxed{n = 5.26}$$

The highest order, n = 5

Problem 1.53: *A grating has 6000 lines per cm. How many orders of light of wavelength 4500 A° can be seen ?*

(03) (Dec. 08)

Data: N = 6000 lines per cm

$\lambda = 4500$ A°, $(a + b) = \frac{1}{6000}$ cm

Formulae:

(i) $(a + b) = \frac{1}{N}$ cm (ii) $(a + b)\sin\theta = m\lambda$

Solution:

(i) $(a + b) = \frac{1}{6000} = 1.666 \times 10^{-4}$ cm

(ii) $(a + b)\sin\theta = m\lambda$

Take $\sin\theta = 1$

$$m = \frac{1.666 \times 10^{-4}}{4500 \times 10^{-8}}$$

$$\therefore \quad \boxed{m = 3.7}$$

Hence, highest order visible = 3

Problem 1.54: *A single slit diffraction pattern is formed using white light. For what wavelength of light does the second minimum coincide with the third minimum for the wavelength 4000 A° ?*

(04) (Dec. 08)

Data: $\lambda_1 = 4000$ A°

$n_1 = 3$

$n_2 = 2$

Formula: For single slit minima,

$$a \sin\theta = n\lambda$$

Solution: $a \sin\theta = 2 \times \lambda_2$

and $a \sin\theta = 3\lambda_1$

$\therefore \quad 2\lambda_2 = 3\lambda_1$

$\therefore \quad 2 \times \lambda_2 = 3 \times 4000$ A°

$\therefore \quad \boxed{\lambda_2 = 6000 \text{ A°}}$

Problem 1.55: *A light of wavelength 5.8×10^{-7} m is incident on a slit having a width of 0.3×10^{-3} m. The viewing screen is 2.00 m from the slit. Find the position of the first dark fringes and the width of the central bright fringe. What happens to the diffraction pattern if the slit width is increased ?*

(05) (May 09)

Data: $\lambda = 5.8 \times 10^{-7}$ m

$a = 0.3 \times 10^{-3}$ m

D = 2.00 m

n = 1

Formulae: **(i)** For single slit,

$$a \sin\theta = n\lambda$$

(ii) From Fig. 1.28,

$$\sin\theta = \frac{d}{D} \text{ (for large D)}$$

Solution: From (1) and (2),

$$\therefore \quad a\frac{d}{D} = \lambda$$

$$\therefore \quad d = \frac{D\lambda}{a}$$

$$d = 3.87 \times 10^{-3} \text{ m}$$

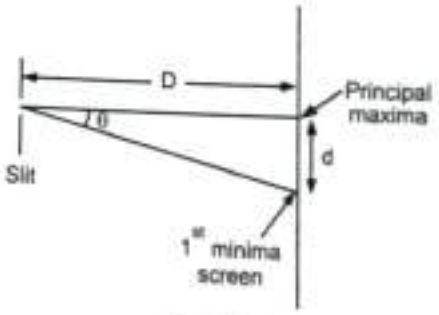


Fig. 1.25

The width of central bright fringe

$$= 2d$$

$$= \boxed{7.74 \times 10^{-3} \text{ m}}$$

If the slit width is increased, width of principal maxima decreases.

Problem 1.56: *A monochromatic light from a helium-neon laser (λ = 623.8 nm) is incident normally on a diffraction grating containing 6000 lines/cm. Find the angles at which the first and second order maxima are obtained.*

(04) (May 09)

Data: $\lambda = 623.8$ nm $= 623.8 \times 10^{-7}$ cm

N = 6000 lines/cm, $(a + b) = \frac{1}{6000}$ cm

Formulae: (i) $(a + b) = \frac{1}{N}$ cm. (ii) $(a + b) \sin\theta = n\lambda$

Solution: (i) $(a + b) = \frac{1}{6000} = 1.66 \times 10^{-4}$ cm

(ii) $(a + b) \sin\theta = n\lambda$

For $n = 1$, $\theta_1 = \sin^{-1}\left(\frac{1 \times 623.8 \times 10^{-5}}{1.66 \times 10^{-4}}\right) = \boxed{22.31°}$

For $n = 2$, $\theta_2 = \sin^{-1}\left(\frac{2 \times 623.8 \times 10^{-5}}{1.66 \times 10^{-4}}\right) = \boxed{49.39°}$

Problem 1.57: *Find the half angular width of the central maxima in the fraunhofer diffraction pattern of slit having width 10×10^{-5} cm. When illuminated by light having wave length 5000 A°.* **(03) (Dec. 12)**

Data: $a = 10 \times 10^{-5}$ cm, $\lambda = 5000$ A° $= 5000 \times 10^{-8}$ cm, $n = 1$

Formula: $a \sin\theta = n\lambda$

$\therefore$ $a \sin\theta = n\lambda$

But $n = 1$

$\therefore$ $\sin\theta = \frac{\lambda}{a}$

$\therefore$ $\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$

$\therefore$ Substituting,

$$\theta = \sin^{-1}\left(\frac{5000 \times 10^{-8}}{10 \times 10^{-5}}\right)$$

$\therefore$ $\theta = 30°$

Problem 1.58: *Monochromatic light from He-Ne laser source (λ=6328A0) is incident normally on a diffraction grating having 6000 lines/cm. find the angle at which one would observe second order maximum.* **(03) (May 13)**

Data: $\lambda = 6328$ A°

$N = 6000$ Lines/cm

$n = 2$

Formula: (i) $(a + b) = \frac{1}{N}$

(ii) $(a + b) \sin\theta = n\lambda$

Solution: (i) $(a + b) = \frac{1}{6000}$

$(a + b) = 1.6667 \times 10^{-4}$ cm

(ii) $\theta = \sin^{-1}\left(\frac{2 \times 6328 \times 10^{-8}}{1.6667 \times 10^{-4}}\right)$

$\theta = 49.4°$

Problem 1.59: *In a grating, the angle of diffraction for the second order principal maximum for the light of wavelength 5×10^{-5} cm is 30°. Calculate the number of lines per centimetre of the grating surface.* **(03) (May 14)**

Data: $\lambda = 5 \times 10^{-5}$ cm

$a = 30°$

$n = 2$

Formula: (i) $(a + b) \sin\theta = 2\lambda$

(ii) $N = \frac{1}{(a + b)}$

Solution: $\frac{1}{N} \sin 30° = 2 \times 5 \times 10^{-5}$

$$N = \frac{1}{2 \times 2 \times 5 \times 10^{-5}}$$

$N = 5000$ Lines per cm

Problem 1.60: *Interference fringes are produced with monochromatic light falling normally on a wedge shaped film of refractive index 1.4. The angle of wedge is 10 sec of an arc and the distance between successive fringes is 0.5 cm. What is the wavelength of light used ?* **(03) (Nov. 13)**

Data: $\mu = 1.4$

$\alpha = 10 \text{ sec} = \frac{10}{3600} \times \frac{\pi}{180}$ radian,

$\beta = 0.5$ cm

Formula: $\beta = \frac{\lambda}{2\mu\alpha}$

$$\lambda = \frac{2 \times 1.4 \times 10 \times 3.14 \times 0.5 \times 10^{-5}}{3600 \times 180}$$

$\therefore$ $\lambda = 0.6783 \times 10^{-4}$ cm

or $\lambda = 6783$ A°

Problem 1.61: *Find the half angular width of the central maxima in the fraunhofer diffraction pattern of slit having width 10×10^{-5} cm. When illuminated by light having wave length 5000 A°.* **(3) (Dec. 12)**

Data: $a = 10 \times 10^{-5}$ cm, $\lambda = 5000$ A° $= 5000 \times 10^{-8}$ cm, $n = 1$

Formula : $a \sin\theta = n\lambda$

Solution :

$\therefore$ $a \sin\theta = n\lambda$

But $n = 1$

$\therefore$ $\sin\theta = \frac{\lambda}{a}$

$\therefore \quad \theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$

$\therefore$ Substituting,

$$\theta = \sin^{-1}\left(\frac{5000 \times 10^{-8}}{10 \times 10^{-5}}\right) = 30^\circ$$

Problem 1.62: *Monochromatic light from He-Ne laser source (λ=6328A°) is incident normally on a diffraction grating having 6000 lines/cm. find the angle at which one would observe second order maximum.* **(3) (May 13)**

Data: $\lambda = 6328A^0$

$N = 6000$ Lines/cm

$\eta = 2$

Formula: (i) $(a+b) = \frac{1}{N}$

(ii) $(a+b)\sin\theta = \eta\lambda$

Solution: (i) $(a+b) = \frac{1}{6000}$

$(a+b) = 1.6667 \times 10^{-4}$ cm

(ii) $\theta = \sin^{-1}\left(\frac{2 \times 6328 \times 10^{-8}}{1.6667 \times 10^{-4}}\right) = 49.4^0$

Problem 1.63 : *In a grating, the angle of diffraction for the second order principal maximum for the light of wavelength 5×10^{-5} cm is 30°. Calculate the number of lines per centimetre of the grating surface.* **(3) (May 19)**

Data: $\lambda = 5 \times 10^{-5}$ cm

$a = 30^\circ$

$n = 2$

Formula: (i) $(a + b)\sin\theta = 2\lambda$

(ii) $N = \frac{1}{(a + b)}$

Solution:

$$\frac{1}{N}\sin 30^\circ = 2 \times 5 \times 10^{-5}$$

$$N = \frac{1}{2 \times 2 \times 5 \times 10^{-5}} = 5000 \text{ Lines per cm}$$

Problem 1.64: *A laser light of wavelength 6328 A.U. falls normally on a grating which is 2 cm long. The first order spectrum is observed at an angle of 20°. Find the total number of slits on grating.* **(3) (Nov. 17)**

Data: $\lambda = 6328$ A°

Length of grating = 2 cm

$n = 1$

$Q = 20^\circ$

Formula: (i) $(a + b)\sin\theta = n\lambda$

(ii) $N = \frac{1}{a + b}$

Solution: $(a + b) = \frac{1 \times 6328 \times 10^{-8}}{\sin 20}$

$= 1.850 \times 10^{-4}$ cm

$\therefore \quad N = \frac{1}{1.850 \times 10^{-4}} = 5404$ lines/cm

Therefore, total lines on grating of length 2 cm = 5404 × 2 = 10808 lines.

1.12 RAYLEIGH'S CRITERION FOR RESOLUTION

- According to Rayleigh's criterion, two point sources or two spectral lines of equal intensity are just resolved by an optical instrument if the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.
- Let us consider the intensity distribution pattern of two wavelengths λ and $\lambda + d\lambda$.
- The separation between their central maxima will depend upon $d\lambda$. If $d\lambda$ is considerably large, two wavelengths will be quite apart [See Fig. 1.26 (a)] and the two spectral lines will be well resolved.

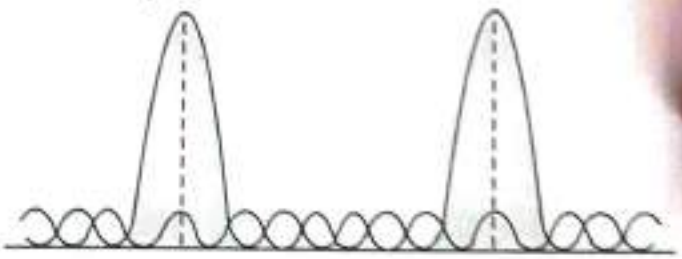


Fig. 1.26 (a)

- But $d\lambda$ will have a limiting value in which case the angular separation between their maxima is such that the central maxima of one (λ) coincides with the first minimum of the other (λ and $d\lambda$) and vice versa [See Fig. 1.26 (b)].

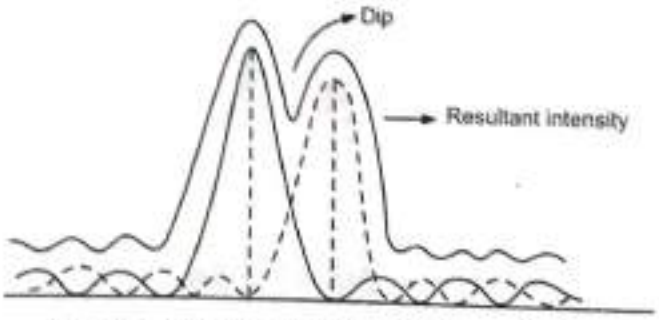


Fig. 1.26 (b) : Rayleigh's criterion for resolution

- In Fig. 1.26 (b), two spectral lines of wavelengths λ and $\lambda + d\lambda$ are shown. The central maximum in the diffraction pattern for λ is shown to occur at angle θ and the first minimum at $\theta + d\theta$. If the central maximum in the diffraction pattern for $(\lambda + d\lambda)$ is also found to occur at $(\theta + d\theta)$, where the first minimum for λ occurs, then only the two spectral lines will be been, just as separate or resolved.
- If $d\lambda$ is lesser then the limiting value satisfying the condition of Rayleigh's criterion, the two spectral lines will overlap and the resultant intensity pattern is as shown in
  Fig. 1.26 (c). Hence, in this case, the spectral lines are not resolved.

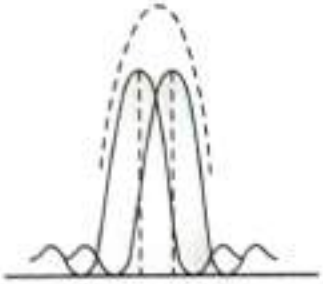


Fig. 1.26 (c)

1.12.1 Resolving Power of a Telescope

- A telescope is used to see distant objects with clarity. The details it reveals, depend on the angle the two point objects make at the objective rather than on the linear separation between them.
- The resolving power of a telescope is defined as the reciprocal of the least angle subtended at the objective by two distant point objects which can be distinguished just as separate in the focal plane of the telescope.
- The ring holding the objective lens acts as the circular aperture. The diffraction pattern due to a circular aperture consists of a central bright disc surrounded by concentric dark and bright rings.

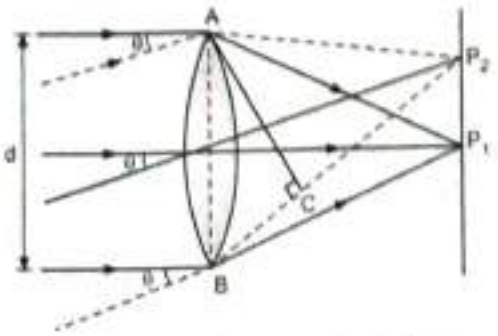


Fig. 1.27 : Resolving power of a telescope

- Extending Rayleigh's criterion for the resolution of diffraction patterns to the circular aperture, we say that, two patterns are resolved when the central maximum of one falls on the first dark ring of the other.
- Let 'd' be the diameter of the objective AB of the telescope.
- Consider the incident rays of light from two neighbouring points of a distant object. The full lines show rays of light from one point object and the dotted lines show the rays of light from the neighbouring point object. The image of each point object is a Fraunhofer diffraction pattern. The diffraction pattern of the first object consists of a central bright disc surrounded by concentric dark and bright rings with centre at P_1. The diffraction pattern of the second object is also a similar diffraction pattern with centre at P_2.
- According to Rayleigh, these two images are said to be resolved, if the position of the central maximum of the second image coincides with the first dark ring of the first image and vice versa.
- The path difference between the secondary waves travelling in the directions AP_1 and BP_1 is zero and hence, they reinforce with one another at P_1. Similarly, all the secondary waves, from the corresponding points between A and B will have zero path difference. Thus, P_1 corresponds to the central maximum of the first image.
- The secondary waves travelling in the directions AP_2 and BP_2 will meet at P_2. The path difference between the secondary waves travelling in the directions BP_2 and AP_2 is equal to BC (See Fig. 1.26).

  From the

  $$\Delta ABC,\ BC = AB \sin\theta = AB \cdot \theta = d \cdot \theta$$

  (for small angles)
- If this path difference $d \cdot \theta = \lambda$, the position of P_2 corresponds to the first minimum of the first image. But P_2 is also the position of the central maximum of the second image. Thus, Rayleigh's criterion of resolution is satisfied, if

  $$d \cdot \theta = \lambda$$

  i.e. $$\theta = \frac{\lambda}{d} \quad \text{... (1)}$$

  Equation (1) holds good for rectangular aperture. For circular aperture, according to Airy, this equation becomes

$$\theta = 1.22 \frac{\lambda}{d} \quad \text{... (2)}$$

Equation (2) gives the least angle the two distant point objects should subtend at the objective for their resolution. Hence, resolving power of the telescope is given by

$$R = \frac{1}{\theta} = \frac{d}{1.22\,\lambda} \quad \text{... (3)}$$

Equation (3) shows that resolving power of a telescope depends on wavelength λ and diameter d of the aperture.

1.12.2 Resolving Power of a Grating

Method - I :

- The angular separation between two spectral lines may be very small indeed. However, a grating is capable of resolving the images of the slit formed by the two spectral lines of wavelengths λ and $(\lambda + d\lambda)$.
- The resolving power of a grating is defined as, the smallest wavelength difference $d\lambda$ for which the spectral lines can be just resolved at the wavelength λ, and is mathematically expressed as

$$R = \frac{\lambda}{d\lambda} \quad \text{... (1)}$$

- Let (a + b) be the grating element and N the number of slits in the grating. If a plane wavefront of wavelength λ is incident on the grating and if the mth order maximum in the diffraction pattern is formed at angle θ_m then, the condition for this maximum to be formed is that, the path difference between the waves from adjacent slits must be

$$(a + b) \sin \theta_m = m\lambda \quad \text{... (2)}$$

- If the mth order maximum for $(\lambda + d\lambda)$ is formed at $(\theta_m + d\theta_m)$, the angular separation $d\theta_m$ between the mth order maxima for wavelengths λ and $(\lambda + d\lambda)$ is obtained by differentiating equation (2). We have

$$(a + b) \cos \theta_m \, d\theta_m = m \cdot d\lambda$$

i.e. $$d\theta_m = \frac{m \cdot d\lambda}{(a + b) \cos \theta_m} \quad \text{... (3)}$$

- By Rayleigh's criterion of resolution, these two spectral lines can be just resolved, if the first minimum adjacent to the mth order maximum for λ in the diffraction pattern is also formed at $(\theta_m + d\theta_m)$. Then, the condition for this first minimum adjacent to m^{th} order maximum for λ to occur at $(\theta_m + d\theta_m)$ is,

$$(a + b) \sin (\theta_m + d\theta_m) = m\lambda + \frac{\lambda}{N} \quad \text{... (4)}$$

because the path difference between a maximum and its adjacent minimum is $\frac{\lambda}{N}$.

Now,

$$\sin (\theta_m + d\theta_m) = \sin \theta_m \cos d\theta_m + \sin d\theta_m \cos \theta_m$$

i.e.

$$\sin (\theta_m + d\theta_m) = \sin \theta_m \cdot 1 + d\theta_m \cos \theta_m \quad \text{... (5)}$$

since, $\cos d\theta_m = 1$ and $\sin d\theta_m = d\theta_m$ if $d\theta_m$ is small.

$\therefore$ From (4) and (5),

$$(a + b) (\sin \theta_m + d\theta_m \cos \theta_m) = m\lambda + \frac{\lambda}{N}$$

By equation (1), this gives

$$(a + b) \cos \theta_m \cdot d\theta_m = \frac{\lambda}{N}$$

or $$d\theta_m = \frac{1}{(a + b) \cos \theta_m} \cdot \frac{\lambda}{N} \quad \text{... (6)}$$

From equations (3) and (6), we have

$$\frac{m \cdot d\lambda}{(a + b) \cos \theta_m} = \frac{\lambda}{(a + b) \cos \theta_m N}$$

or $$\frac{\lambda}{d\lambda} = m \cdot N$$

$\therefore$ $$R = \frac{\lambda}{d\lambda} = m \cdot N \quad \text{... (7)}$$

Equation (7) gives the resolving power of a grating.

- It is seen that, the resolving power is independent of the grating element (a + b).
- It increases with the order of the spectrum m, and also with the number of lines on the grating surface.

Method - II :

- Resolving power of a grating can also be found as follows :

By definition, $R = \frac{\lambda}{d\lambda}$ ... (1)

- In Fig. 1.27, **XY** is the grating surface and MN is the field of view of the telescope. P_1 is the mth primary maximum of a spectral line of wavelength λ at angle of diffraction θ_m. P_2 is the mth primary maximum of a second spectral line of wavelength $\lambda + d\lambda$ at a diffracting angle $\theta_m + d\theta_m$.

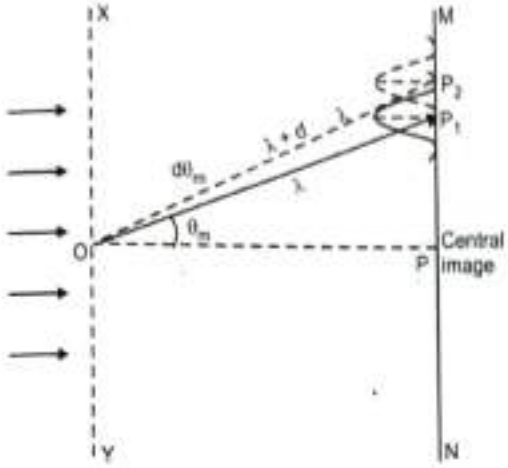


Fig. 1.28

- P_1 and P_2 are the spectral lines in the mth order. These two spectral lines according to Rayleigh's criterion, will appear resolved, if the position of P_2 also corresponds to the first minimum of P_1.
- The direction of the mth primary maximum for a wavelength λ is given by

$$(a + b) \sin \theta_m = m \lambda \quad \text{... (2)}$$

- The direction of the mth primary maximum for a wavelength $(\lambda + d\lambda)$ is given by

$$(a + b) \sin (\theta_m + d\theta_m) = m (\lambda + d\lambda) \quad \text{... (3)}$$

- The two lines will appear just resolved if the angle of diffraction $(\theta_m + d\theta_m)$ also corresponds to the direction of first minimum after the mth primary maximum at P_1 (corresponding to λ). This is possible, if the extra path difference introduced is λ/N, where N is the total number of lines on the grating surface.

$$\therefore \quad (a + b) \sin (\theta_m + d\theta_m) = m\lambda + \lambda/N \quad \text{... (4)}$$

From equations (3) and (4),

$$m (\lambda + d\lambda) = m \lambda + \frac{\lambda}{N}$$

or $$m\, d\lambda = \frac{\lambda}{N}$$

$$\therefore \quad R = \frac{\lambda}{d\lambda} = mN \quad \text{... (5)}$$

Equation (5) gives the resolving power of a grating.

- The resolving power of a grating is independent of the grating element. The resolving power is directly proportional to (i) the order of the spectrum, and (ii) the total number of lines on the grating surface.

SOLVED PROBLEMS

Problem 1.65 : *Calculate the least width that a grating must have to resolve the components of sodium D lines in the second order, the grating having 1000 lines/cm. D_1 = 5896 A°, and D_2 = 5890 A°.*

Data : λ_1 = 5896 A°, λ_2 = 5890 A°, n = 2, No. of lines/cm = 1000.

Formula : $$\frac{\lambda}{d\lambda} = nN$$

Solution : $$N = \frac{\lambda}{n \cdot d\lambda}$$

Substituting, $$N = \frac{5896 \times 10^{-8}}{6 \times 10^{-8} \times 2}$$

$$= 491 \text{ lines}$$

$$\therefore \text{ Width of the grating} = \frac{491}{1000}$$

$$= 0.491 \text{ cm.}$$

Problem 1.66 : *A plane grating just resolves two lines in the second order. Calculate the grating element if $d\lambda$ = 6 A°, $\lambda = 6 \times 10^{-5}$ cm and width of the ruled surface is 2 cm.*

Data : $\lambda = 6 \times 10^{-5}$ cm, $d\lambda = 6 \times 10^{-8}$ cm, width of the grating = 2 cm, n = 2.

Formula : $$\frac{\lambda}{d\lambda} = nN.$$

Solution : $$\frac{\lambda}{d\lambda} = n N$$

Substituting, $$N = \frac{6 \times 10^{-5}}{6 \times 10^{-8} \times 2}$$

$$N = 500$$

$$\therefore \text{ No. of lines/cm} = \frac{500}{2}$$

$$= 250 \text{ lines/cm}$$

$$\therefore \text{ Grating element } (a + b) = \frac{1}{250 \text{ lines/cm}}$$

$$= 0.004.$$

Problem 1.67 : *A grating has 100 lines ruled on it. What is the difference between two wavelengths that just appear separated in the second order spectrum in the region, given $\lambda = 6 \times 10^{-5}$ cm ?*

Data : $\lambda = 6 \times 10^{-5}$ cm, N = 100 lines, n = 2.

Formula : $\frac{\lambda}{d\lambda} = nN$.

Solution : $\frac{\lambda}{d\lambda} = nN$

$$d\lambda = \frac{\lambda}{n \cdot N}$$

Substituting, $d\lambda = \frac{6 \times 10^{-5}}{2 \times 100} = 3 \times 10^{-7}$ cm $= 30$ A°

Problem 1.68 : *A telescope objective has a focal length of 3 metres and diameter of 10 cm. Find the distance between the centres of the images of two stars just resolved by it, given $\lambda = 5500$ A°.*

Data : $\lambda = 5500 \times 10^{-8}$ cm, d = 10 cm, D = 300 cm.

Formula : $d\theta = 1.22 \frac{\lambda}{d}$, $d\theta = \frac{x}{D}$.

Solution : Since, the telescope has just resolved the two stars, the limit of resolution ($d\theta$) of the telescope will be equal to the angle subtended by the two stars $\left(\frac{x}{D}\right)$ at the objective of the telescope. i.e. $d\theta = \frac{x}{D}$.

But $d\theta = 1.22 \frac{\lambda}{d}$

$$\therefore \quad 1.22 \frac{\lambda}{d} = \frac{x}{D}$$

where x is the separation between the centres of the images of the two stars, D is the focal length of the objective.

$$\therefore \quad x = 1.22 \frac{\lambda}{d} \times D$$

Substituting, $x = \frac{1.22 \times 5500 \times 10^{-8}}{10} \times 300$

$$= 2.013 \times 10^{-3} \text{ cm.}$$

Problem 1.69 : *The angular separation of two stars is 1.5 seconds. Find the minimum aperture of a telescope objective, if the two stars are to be distinguished as separate, given $\lambda = 5700$ A°.*

Data : $\lambda = 5700 \times 10^{-8}$ cm, $d\theta = 1.5$ seconds,

$$1.5 \text{ seconds} = \frac{1.5}{3600} \times \frac{\pi}{180}$$

$$= 0.72 \times 10^{-5} \text{ radians.}$$

Formula : $d\theta = 1.22 \frac{\lambda}{d}$.

Solution : $d\theta = 1.22 \frac{\lambda}{d}$

$$\therefore \quad \text{aperture, } d = 1.22 \frac{\lambda}{d\theta}$$

Substituting, $d = \frac{1.22 \times 5700 \times 10^{-8}}{0.72 \times 10^{-5}}$

$$d = 9.65 \text{ cm.}$$

Problem 1.70 : *The objective of a telescope has a diameter of 40 inches. Calculate the smallest angular separation of two stars that may be resolved by it. Mean wavelength of light, $\lambda = 5600$ A°.*

Data : x = 40 inches = 40 × 2.54 cm = 101.6 cm = 1.016 m, $\lambda = 5600 \times 10^{-10}$ m.

Formula : $d\theta = 1.22 \frac{\lambda}{d}$.

Solution : $d\theta = 1.22 \frac{\lambda}{d}$

$$= 1.22 \times \frac{5600 \times 10^{-10}}{1.016}$$

$$\therefore \quad d\theta = 6.724 \times 10^{-7} \text{ radians}$$

$d\theta$ is the smallest angular separation between the stars that can be resolved by the telescope.

Problem 1.71: *A monochromatic light of wavelength 5500 Å incident normally on a slit of width 2×10^{-4} cm. Calculate the angular position of first and second minimum.*

(3) (May 17)

Solution : Data: $a = 2 \times 10^{-4}$ cm, $\lambda = 5500$ A° $= 5500 \times 10^{-8}$ cm, n = 1

Formula: $a \sin\theta = n\lambda$

(i) For first order,

$$n = 1$$

$$\therefore \quad \sin\theta = \frac{\lambda}{a}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$\therefore$ Substituting, $\theta = \sin^{-1}(5500 \times 10^{-8}/2 \times 10^{-4})$

$$\therefore \quad \theta = \mathbf{15.96°}$$

(ii) For second order,

$$n = 2$$

$$\therefore \quad \sin\theta = 2\lambda/a$$

$$\therefore \quad \theta = \sin^{-1} 2\lambda/a$$

$$\therefore \text{ Substituting, } \theta = \sin^{-1} \sin^{-1}(2 \times 5500 \times 10^{-8}/2 \times 10^{-4})$$

$$\therefore \quad \theta = \mathbf{33.36°}$$

Problem 1.72: *How many lines per cm are there on the surface of a plane transmission grating which gives 1st order of light of wavelength 6000 AU at an angle of diffraction 30°.* **(3) (Nov. 17)**

Given Data : $\lambda = 6000$ A.U.

$n = 1$

$Q = 30°$

Formula : (i) $(a + h) \sin Q = n\lambda$

(ii) $N = \dfrac{1}{(a + h)}$

Solution :

$$\frac{1}{N} \sin 30 = 1 \times 6000 \times 10^{-8}$$

$$\therefore \quad N = \frac{1}{2 \times 1 \times 6000 \times 10^{-8}}$$

$$N = 8333 \text{ line per cm} \quad \text{... Ans.}$$

Problem 1.73: *Find the half angular width of the central maxima in the fraunhofer diffraction pattern due to a single slit having a width of 7.07×10^{-5} cm, when illuminated by light having wavelength 5000 AU.* **(3) (May 18)**

Given : $a = 7.07 \times 10^{-5}$ cm, $\lambda = 5000$ A°

$= 5000 \times 10^{-8}$ cm, $n = 1$

Solution :

$$\therefore \quad a \sin\theta = n\lambda$$

$$n = 1$$

$$\sin\theta = \frac{\lambda}{a}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$\theta = \sin^{-1}\left(\frac{5000 \times 10^{-8}}{7.07 \times 10^{-5}}\right) = 45°$$

Problem 1.74 : *In a plane transmission grating, the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines/cm of the grating surface.* **(3) (May 19)**

Solution : $\theta = 30'$, $n = 2$, $k = 5 \times 10^{-5}$ cm,

Formula : (i) $d\sin\theta = n\lambda$

(ii) $N = \dfrac{\sin\theta}{n\lambda}$

$$= \frac{\sin 30°}{2 \times 85 \times 10^{-5}}$$

$$= 500 \text{ lines/cm}$$

SUMMARY

- **Diffraction :** The bending of waves around the corners of the obstacle and their encroachment into the region of geometrical shadow is called "diffraction of waves".
- **Three Elements of Diffraction :** Source, obstacle or aperture and the screen.
- **Diffraction Phenomenon :** Can be observed only when the size of the obstacle or aperture is comparable to the wavelength of incident waves.
- **The Intensities :** Of the secondary maxima lying on either side of central maxima go on decreasing away from the central maxima.
- **The Intensity For :** A single slit diffraction pattern at any angle θ is $I_\theta = I_m \dfrac{\sin^2\alpha}{\alpha^2}$
  - ➢ Condition for maxima in the single slit diffraction pattern is

    $$\alpha = 0, \quad \theta = 0.$$

  - ➢ Condition for minima in the single slit diffraction pattern is

    $$\alpha = \pm m\pi$$

    $$a \sin\theta = m\lambda$$

  - ➢ Condition for secondary maxima in the single slit diffraction pattern is

    $$\alpha = \pm\left(m + \frac{1}{2}\right)\pi$$

    $$a \sin\theta = (2m + 1)\frac{\lambda}{2}$$

- **A Plane Diffraction :** Grating is a device having N narrow slits side by side separated by opaque spaces.
- **The Intensity :** For diffraction grating is

$$I_\theta = I_m \frac{\sin^2 \alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2 \beta}$$

➢ Condition for principal maxima in the intensity pattern of a grating,

$$(a + b) \sin \theta = m\lambda$$

➢ Condition for minima,

$$(a + b) \sin \theta = \frac{m}{N} \lambda$$

- **Rayleigh's Criterion of Resolution :** When principal maxima of one coincides with first minima other the images are said to be resolved.

IMPORTANT FORMULAE

- $a \sin \theta = n\lambda$
- $\sin \theta = \frac{\lambda}{a}$ if $n = 1$
- $\sin \theta = \frac{x}{f}$
- $a \sin \theta = (2n + 1) \frac{\lambda}{2}$
- $(a + b) \sin \theta = n\lambda$
- $(a + b) \sin \theta = \frac{m}{N} \lambda$
- $a + b = \frac{1}{N \text{ lines/cm}}$
- Resolving power of telescope, $R = \frac{1}{Q} = \frac{d}{1.22\,\lambda}$
- Resolving power of grating, $R = \frac{\lambda}{d\lambda} = mN$

EXERCISE

1. Explain the phenomenon of diffraction of light. State its types and distinguish between them.
2. Explain: (i) Diffraction of light (ii) Diffraction grating.
3. Explain the difference between interference and diffraction.
4. Describe the phenomenon observed when monochromatic light falls on a single slit.
5. Derive an expression for the intensity at a point the Fraunhofer's type of diffraction produced by single slit.
6. Explain the formation of spectra by a plane transmission grating.
7. Obtain the conditions for maxima and minima Fraunhofer diffraction due to a single slit.
8. What is plane transmission grating ? Explain how can be used for determining the wavelength of given monochromatic light.
9. Give the theory of plane transmission grating.
10. Discuss Fraunhofer's diffraction at a single slit and derive the condition for obtaining secondary minimum intensity positions.
11. State Rayleight's criterion of resolution. Give an example of it.

UNSOLVED PROBLEMS

1. Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angles of diffraction for the principal maxima of the two sodium lines in the first order spectrum $\lambda_1 = 5890$ A° and $\lambda_2 = 5896$ A°.

   (**Ans.** $\theta_1 = 17.1°$, $\theta_2 = 17.2°$)

2. What is the highest order spectrum which can be seen with monochromatic light of wavelength 6000 A° by means of a diffraction grating with 5000 lines/cm ?

   (**Ans.** n = 3)

3. A plane grating has 15000 lines per inch. Find the angle of separation of 5408 A° and 5016 A° lines of helium in the second order spectrum.

   (**Ans.** $(\theta_2 - \theta_1) = 16'$)

4. A diffraction grating used at normal incidence gives a line $\lambda_1 = 6000$ A° in a certain order superimposed on another line, $\lambda_2 = 4500$ A° of the next highest order. If the angle of diffraction is 30°, how many lines are there in a cm in the grating ?

   (**Ans.** 2778 lines/cm)

5. What is the highest order spectrum which may be seen with light of wavelength 5000 A° by means of a grating with 3000 lines/cm ?

(**Ans.** n = 6)

6. A plane diffraction grating has the value of grating constant equal to 15×10^{-4} cm. Calculate the position of third order maximum for $\lambda = 2.4 \times 10^{-4}$ cm.

(**Ans.** $\theta_3 = 28.7°$)

7. Monochromatic light of wavelength $\lambda = 6560$ A° falls normally on a grating. The spectral line is diffracted at an angle of 19° 9' from the normal in the first order. Find the grating element.

(**Ans.** 2×10^{-4} cm)

8. A single slit Fraunhofer's diffraction pattern is formed using white light. For what wavelength of light does the second minimum coincide with the third minimum for the wavelength 4000 A° ?

(**Ans.** 6000 A°)

9. In a plane transmission grating with 5000 lines/cm and for wavelength 6000 A°, if the opaque spaces are exactly 2.0 lines the transparent spaces, which order of spectra will be absent ?

(**Ans.** 3rd, 6th, 9th etc. orders)

10. Light of wavelength 6×10^{-5} cm falls on a screen at a distance of 100 cm from a narrow slit. Find the width of the slit if the first minimum has 1 mm on either side of the central maximum.

(**Ans.** 0.06 cm)

11. Calculate the angles at which the first dark band and the next bright band are formed in Fraunhofer diffraction pattern of a slit of 0.3 mm wide, given $\lambda = 5890$ A°.

(**Ans.** 6.7', 10')

12. Calculate the angular separation between the first order minima on either side of central bright maximum, when slit is 6×10^{-4} cm wide and $\lambda = 6000$ A°.

(**Ans.** 73° 4')

13. Light of wavelength 5500 A° falls normally on slit of width 22×10^{-1} cm. Calculate the angular position of the first two minima from the central bright maximum.

(**Ans.** 14° 29', 30°)

14. Plane waves of $\lambda = 6000$ A° fall normally on a single slit of width 0.2 mm. Calculate the total angular width of the central bright maximum and also the linear width as observed on a screen placed 2 m away.

(**Ans.** 20', 1.2 cm)

UNIVERSITY QUESTIONS

May 2014

1. Define diffraction of light. Draw intensity distribution pattern obtained because of diffraction of light at a single slit and label the significant points in the same. **[3]**

December 2014

1. What is diffraction? What are the types of diffraction? Distinguish between them (any two point). **[3]**

November 2015

1. The resultant amplitude of a wave when monochromatic light is diffracted from a single slit is $E_\theta = E_m \frac{\sin \alpha}{\alpha}$. Then derive the condition of minima. **[3]**

May 2016

1. For a plane diffraction grating, starting from the equations of resultant amplitude and intensity, derive conditions for maxima and minima of the diffraction pattern. **[6]**

November 2018

1. What is diffraction grating? For a plane transmission grating, starting from equation of resultant amplitude and intensity, specify the terms involved. Derive conditions of maxima and minima of diffraction pattern. **[6]**

POLARIZATION

1.13 INTRODUCTION

- The phenomenon like interference or diffraction prove the wave nature of light. But it does not tell us whether the light waves are longitudinal or transverse. Because even longitudinal waves, like sound waves, show the phenomena of interference and diffraction.
- The phenomenon of polarization can be explained only by considering the transverse nature of light. And it has been proved by electromagnetic theory that the light is transverse wave.
- In ***longitudinal waves***, the particles of the medium vibrate to and fro in the direction of propagation of the wave.
- But in the ***transverse waves***, the particles of ***the medium vibrate up and down*** at right angles to the direction of propagation of the wave.
- The important difference between longitudinal and transverse wave is that the transverse waves can be polarized. Hence the following experiments are considered to explain what polarization is.

1.14 POLARIZATION OF WAVES

- The transverse nature of waves leads to the characteristic phenomenon called ***polarization***. The characteristic, polarization is not exhibited by longitudinal waves. Thus only transverse waves could be polarized.
- In a transverse wave, if the directions of all the vibrations at all the points are restricted to one particular plane, then the wave is called ***polarized***, more specific plane polarized. A plane polarized wave is the simplest of a transverse wave, which is also termed as linearly polarized wave.

1.15 POLARIZATION OF LIGHT

1.15.1 Mechanical Experiment

- Consider a string AB passing through slits S_1 and S_2. The end B of the string is fixed while the end A is free. The free end A is given up and down motion rapidly such that transverse waves are set up in the string. These transverse waves travel towards the end B. In this case the particles of string vibrate along the direction parallel to S_1 therefore, passes through S_1.
- Since S_2 is parallel to S_1, these transverse vibrations pass through S_2 also, to reach the end B. If the end A is given motion in all possible directions instead of the up and down motion, the particle of the string will vibrate in all directions. When these vibrations reach vertical slit S_1, they are restricted to the vertical plane only. These vertical vibrations will reach the end B if S_2 is parallel to S_1.
- If on the other hand, S_2 is perpendicular to S_1, the vibrations of the string are completely stopped by S_2. This is because the displacement of the particles are now at right angles to the slit S_2. Thus, the string does not vibrate between S_2 and B.

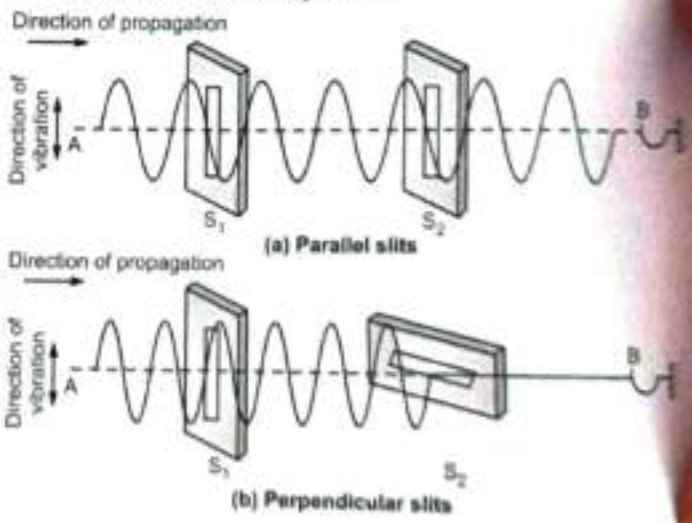


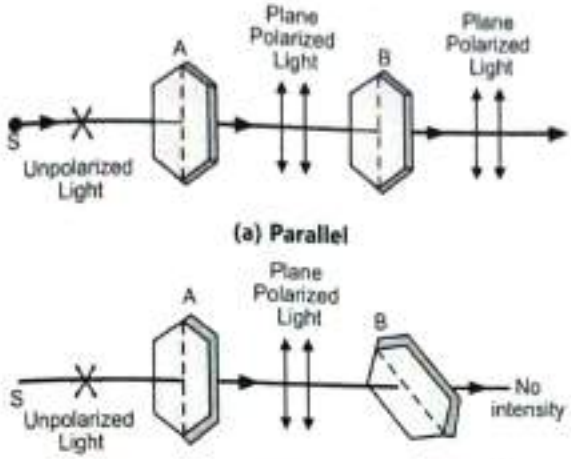
Fig. 1.29

- If longitudinal waves are produced by moving the string forward and backward along it's length, then the waves will freely pass through S_1 and S_2 irrespective of their positions.

1.15.2 Optical Experiment

- Light from a source is allowed to fall normally on the flat surface of a thin plate of tourmaline crystal, cut parallel to it's axis.
- The crystal A is rotated and the intensity of light transmitted through A is noted. No change is observed. Now, another crystal B is placed with it's axis parallel to A. On observing the intensity of the light transmitted through B, no change is observed.
- If now the crystal A is kept fixed and B is gradually rotated, it is seen that the intensity of light emerging out of B decreases. It becomes zero when the axis of B

is perpendicular to that of A. If B is further rotated, the intensity increases and becomes maximum when the axes of A and B are again parallel.



(a) Parallel

(b) Perpendicular

Fig. 1.30

- Thus the intensity of transmitted light is maximum when axes of A and B are parallel, and zero when they are perpendicular.
- From these experiments, it is clear that light waves are transverse. If they were longitudinal, then the rotation of crystal B would not have produced any change in the intensity of light.

1.15.3 Polarization

- Thus, light from the source S consists of transverse waves having vibrations in all directions in a plane perpendicular to the direction of propagation of light. When such a beam reaches the crystal A, vibrations parallel to it's axis are only allowed to pass. Thus, the light coming out of crystal A is not symmetrical about the direction of propagation of light.
- It's vibrations are confined only to a single plane, in a plane perpendicular to the direction of propagation.
- Such a light is called a ***plane polarized light***, the crystal A is called ***polarizer*** and crystal B which detects these vibrations is known as ***analyzer***. The phenomenon is called as ***polarization***.

1.15.4 Representation of Plane Polarized Light (PPL), Unpolarized Light (UPL) and Partially Polarized Light

- According to the electromagnetic theory, light consists of electric and magnetic vectors vibrating continuously with time in a plane, transverse to the direction of propagation of light and to each other. However, ***in explaining polarization only the vibrations of the electric vector are considered***.
- It does not mean that magnetic field vectors are absent, they are present. But for drawing simplicity they are not shown in the diagram.

Unpolarized Light

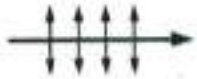
- The light having vibrations along all possible directions perpendicular to the direction of propagation of light, is called an ***unpolarized light***. It is symmetrical about it's direction of propagation.

Fig. 1.31: Unpolarized light

- It can be considered to consist an infinite number of waves each having its own vibration. Therefore, for any position of the crystal there will be one vibration parallel to its axes, so when such a light is passed through a single tourmaline crystal and is rotated no change in the intensity of the emergent light is observed.
- Since unpolarized light has vibrations along all possible directions, at right angles to the directions of propagation of light, it is represented by a star.

Polarized Light

- The light having vibration only along a single plane perpendicular to the direction of propagation of light is called a ***polarized light***. It's vibrations are one sided, therefore it is disymmetrical about the direction of propagation of light.
- When polarized light is passed through a single rotating crystal, a change in the intensity of emergent light is observed.
- The polarized beam of light has vibrations along a single plane. If they are parallel to the plane of the paper, they are represented by arrows [See Fig. 1.32 (a)]. If they are perpendicular to the plane of the paper, they are represented by dots on a ray of light. [See Fig. 1.32 (b)].



(a) Parallel to plane of paper

(b) Perpendicular to plane of paper

Fig. 1.32 : Plane polarized light

Partially Polarized Light

- A partially polarized light is a mixture of plane polarized and unpolarized light. It is represented as shown in Fig. 1.33.
- In partially polarized light the vibrations in the plane of plane polarized light dominate over the vibrations in other directions.

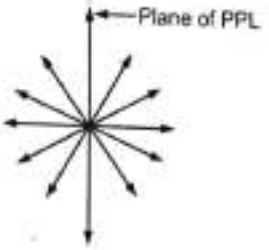


Fig. 1.33: Partially polarized light

1.16 LAW OF MALUS (Dec. 14, May, Nov. 15)

Statement

- The law states that, the intensity of polarized light emerging from the analyzer is proportional to the **square of the cosine** of the angle between the plane of the transmission for the analyzer and the plane of the polarizer.

Proof

- Let A_o be the amplitude of the incident plane polarized light. Let θ be the angle between the planes of transmissions of the analyzer and the polarizer. The amplitude A_o may be resolved into two components $A_o \cos\theta$ and $A_o \sin\theta$.
- The vibrations of the former are parallel to the plane of the transmission of the analyzer, while the vibrations of latter are perpendicular to this i.e. $A_o \cos\theta$ is ***transmitted*** by the analyzer while $A_o \sin\theta$ is ***eliminated***.

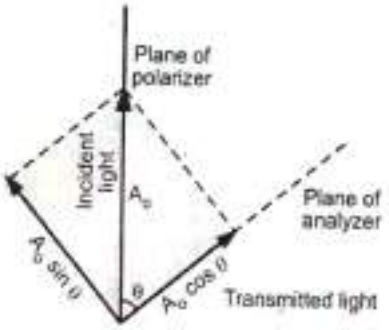


Fig. 1.34

- The amplitude of the transmitted beam,

$$A = A_o \cos\theta$$

- Therefore, the intensity of the transmitted beam, $I = A^2$ but $A = A_o \cos^2\theta$

$$\therefore \quad I = A_o^2 \cos^2\theta$$

$$I = I_o \cos^2\theta$$

- As the intensity of incident beam I_o is constant, therefore,

$$I \propto \cos^2\theta$$

This is called as ***law of Malus***.

Note

- In the case of incident unpolarized light, the law of Malus is $I = \frac{1}{2} I_0$. Because unpolarized light vibrates in all possible directions in a plane perpendicular to the direction of propagation. Therefore, the average value of $\cos^2\theta$ will have to be considered.

  Then the intensity of the transmitted light is

$$I = I_0 \cos^2\theta = \frac{1}{2} I_0$$

  Since the average value of $\cos^2\theta$ over all possible values of θ is $\frac{1}{2}\left(\text{i.e. } \cos^2\theta = \frac{1}{2}\right)$ i.e. an ideal polarizer will be one which will transmit 50 % of the incident unpolarized light as plane polarized light.

1.17 DOUBLE REFRACTION

(Dec. 12, May 13, 14, 16)

Calcite Crystal

- Calcite or Iceland spar is crystallised calcium carbonate ($CaCO_3$). It is found in large quantities in Iceland as a large transparent crystal. It crystallises in many forms and readily breaks into simple rhombohedron.

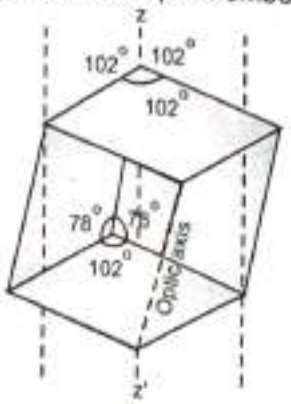


Fig. 1.35: Calcite crystal form

The direction of the optic axis is indicated by broken lines

- For a point in calcite, there are ***three principal sections***, one for each pair of opposite faces.

- The principal section does not always suffice in describing the direction of vibrations. Here, we make use of the two other planes, ***the principal plane of the ordinary ray***, a plane containing the ordinary ray and the optic axis, and ***the principal plane of the extraordinary ray*** or a plane containing the E-ray and the optic axis.
- The O-ray always lies in the plane of incidence. This is not generally true for the E-ray. The principal planes of the two refracted rays do not coincide except in special cases. The special cases are those for which the plane of incidence is a principal section as shown in Fig. 1.35. Under these conditions, the plane of incidence, the principal section and the principal planes of the O and E rays all coincide.

Optic Axis

- The ***optic axis*** is the direction of symmetry of unisotropic media along which double refraction does not take place.
- A line drawn through any of the blunt corners making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore, optic axis is not a line but ***it is a direction*** (as shown in Fig. 1.35).

Principal Section

- A plane containing ***the optic axis*** and ***perpendicular to the opposite faces*** of the crystal is called the ***principal section of the crystal***. The principal section cuts the surfaces of a calcite crystal in a parallelogram with angles 109° and 71°.

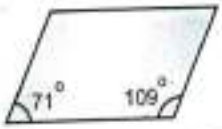


Fig. 1.36: Principal section of calcite crystal

Principal Plane

- The plane containing the optic axis and the ordinary ray is called principal plane of the ordinary ray. Similarly the plane containing the optic axis and the extraordinary ray is called the principal plane of the extraordinary ray.
- Experiments revealed that the vibrations of the ordinary rays are perpendicular to the principal section of the crystal while the vibrations of the extraordinary rays are parallel to the principal section of the crystal. Thus, the two rays are plane polarized, their vibrations being at right angles to each other.

Double Refraction

- The phenomenon of double refraction was discovered by Erasmus Bartholinus in 1669 during his studies on calcite. When light is incident on a calcite crystal, it is found to produce two refracted rays which are different in properties. The phenomenon of causing ***two refracted rays*** by a crystal is called ***birefringence*** or ***double refraction***. The crystals are said to be ***birefringent***.

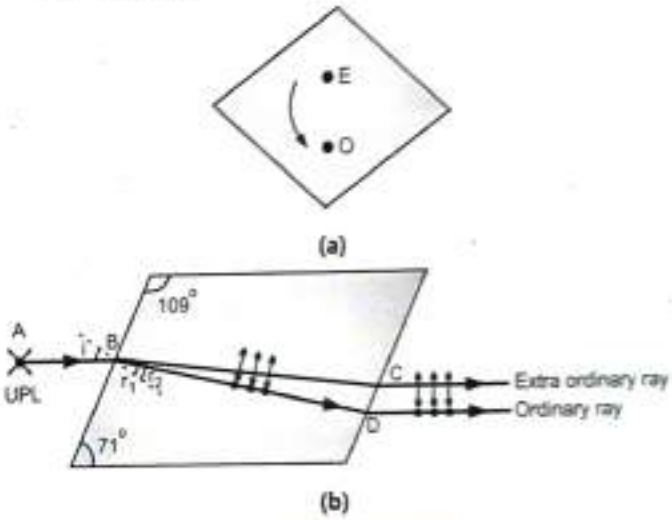


Fig. 1.37: Double reflection

- All anisotropic materials exhibit double refraction. The two rays formed in double refraction are linearly polarized in mutually perpendicular directions.
- One of the rays ***obeys Snell's law*** of refraction and hence is called ***an ordinary ray or O-ray***. The other ray ***does not obey Snell's law*** and is called an ***extraordinary ray or E-ray.*** If one of the rays is eliminated, the light transmitted by the crystal will be a linearly polarized light.
- When a ray of light AB is incident on the calcite crystal making an angle of incidence i, it is refracted along two paths inside the crystal: (i) along BC making an angle of refraction r_2, (ii) along BD making an angle of refraction r_1. These two rays emerge out along DO and CE which are parallel as the crystal faces are parallel.

1.18 HUYGEN'S THEORY OF DOUBLE REFRACTION

Huygen explained the phenomenon of double refraction on the basis of the principle of secondary wavelets.

He assumed:

- When a beam of ordinary unpolarized light strikes a doubly refracting crystal, each point on the surface

sends out ***two wavefronts***, one for ordinary ray and the other for extraordinary ray.

- The ***ordinary-ray*** travels with the ***same speed*** v_o in all directions and the crystal has a single refractive index $\mu_o = \frac{c}{v_o}$ for this wave. Thus, the O-ray has a ***spherical wavefront***.
- The ***speed of extra-ordinary ray v_e varies with direction***. So, the refractive index, $\mu_e = \frac{c}{v_e}$ also varies with direction for the E-ray. Therefore, the extra-ordinary ray develops a wavefront which is ***ellipsoidal***.

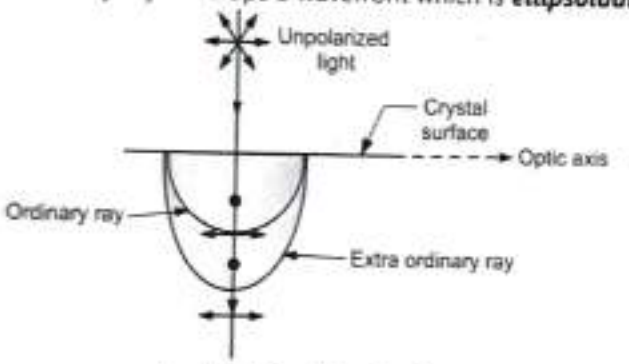


Fig. 1.38: Double refraction

- The velocity v_e measured is perpendicular to the optic axis.
- The velocities of the O-ray and E-ray are the same along the optic axis.
- When rays are incident along the optic axis, the spherical and ellipsoidal wavefronts touch each other at points of intersection with the optic axis and double refraction does not take place.
- If $v_o > v_e$ or $\mu_o < \mu_e$, the spherical wavefront lies outside the elliptical wavefront. Such crystals are called ***positive crystals***. The examples of positive crystal are quartz, ice etc.
- If $v_e > v_o$ or $\mu_e < \mu_o$, the elliptical wavefront lies outside the spherical wavefront. Such crystals are called ***negative crystals***. The examples of negative crystals are calcite, tourmaline, etc.

Positive and Negative Crystals

Positive Crystals	Negative Crystals
1. For positive crystals, $v_o > v_e$ and $\mu_o < \mu_e$.	1. For negative crystals, $v_o < v_e$ and $\mu_o > \mu_e$.
2. The velocity of O-ray is same in all directions.	2. The velocity of O-ray is same in all directions.
3. The wavefront of O-ray lies outside the wavefront of E-ray.	3. The wavefront of O-ray lies inside the wavefront of E-ray.
4. **Examples:** Quartz, Ice.	4. **Examples:** Calcite, Tourmaline.

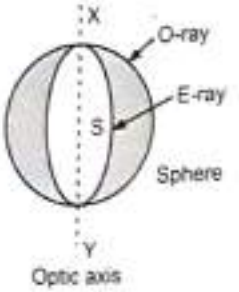


Fig. 1.39 (a)

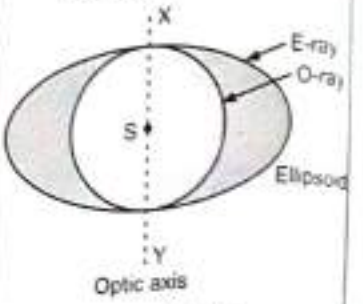


Fig. 1.39 (b)

1.18.1 Cases of Double Refraction of Crystal Cut with Optic Axis Lying in the Plane of Incidence (Nov. 1

Parallel to the Surface (Dec. 14

- Fig. 1.40 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. The optic axis lies along XY and is in the plane of incidence.
- At the points A and B, it develops two wavefronts, one spherical for O-ray and one elliptical for E-ray. The envelope of O-ray and E-ray gives the corresponding wavefront which is plane polarized.
- It should be noted that both O-ray and E-ray are plane polarized light. Here both O-ray and E-ray travel along the same direction with different velocities. As O-ray and E-ray travel along the same direction with different velocities, a path difference is introduced between them.
- This principle is used in the construction of quarter and half-wave plates.

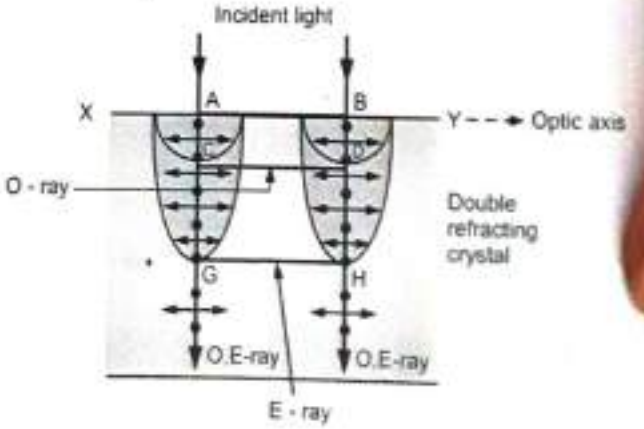


Fig. 1.40

Perpendicular to the Surface

- Fig. 1.41 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. Optic axis

lies in the plane of incidence and perpendicular to the crystal surface.

- As the light is incident in the direction of optic axis, O-ray and E-ray travel with the same speed along the optic axis. As a result O-ray and E-ray travel along the same directions with same velocity. Hence the phenomenon of ***double refraction is absent*** in this case. Ordinary and extraordinary wavefronts CD and GH coincide at all instants.

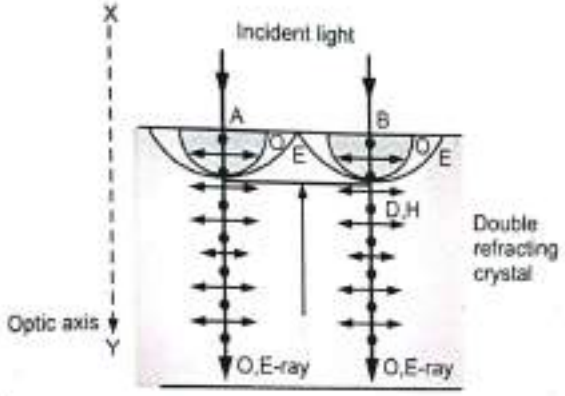


Fig. 1.41

Inclined to the Surface

- Fig. 1.42 shows an unpolarized plane wavefront incident normally on the crystal surface so that the optic axis makes an angle with the crystal surface.
- O-ray and E-ray travel with different velocities in different direction in the crystal. Hence double refraction is seen in this case and both O-ray and E-ray are separated by an angle depending upon the distance travelled in crystal.

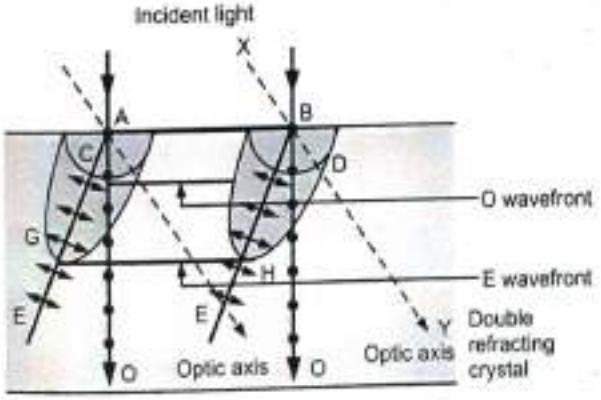
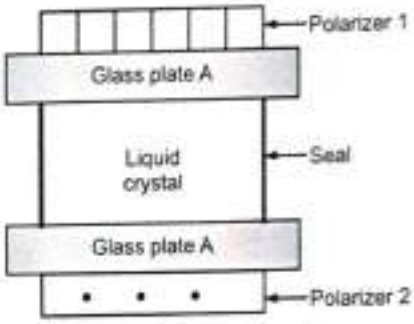


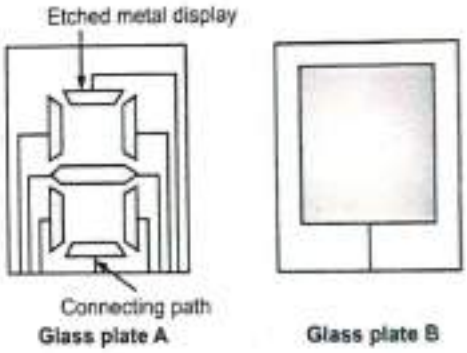
Fig. 1.42

1.19 APPLICATIONS OF POLARIZATION

- Liquid Crystal Display (LCD) is a passive device i.e. does not emit light of its own, and works on the principle of polarization. The common applications of LCD are wrist watch calculator, clock and general displays. Recently, LCDs have also replaced Cathode Ray Tube (CRT) in the displays of computers, laptops and TVs.
- A LCD display consists of liquid crystal sandwiched between two thin glass plates with transparent conducting coating on the inner faces. The conducting plate is etched in the form of 7-segment display (to display a digit or alphabet) as shown in Fig. 1.43 (b). The whole assembly is then placed between two polaroid sheets with crossed plane of polarization.



(a) Assembly



(b) Etched conducting coating

Fig. 1.43: LCD

- Basically, liquid crystals are optically active, which rotates the plane of polarization of plane polarized light. Therefore, the thickness of liquid crystal is selected in such a way that it rotates the plane of polarization by 90°.

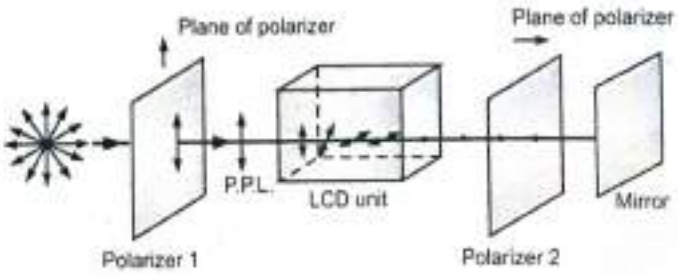


Fig. 1.44: Working of LCD

- Fig. 1.44 shows the schematic working of LCD. When the unpolarized light is incident on polarizer 1, it gets linearly polarized. Upon passing through the liquid crystal it gets rotated by 90° and therefore passes through the polarizer 2 whose transmission axis is perpendicular to that of front polarizer.
- The mirror on back, reflects back the liquid which emerges unobstructed from the front polarizer. This gives impression of uniform illumination.
- When an external voltage is applied the molecules within the electrodes will get aligned in the direction of the field. Therefore, the plane of polarization will not change in this region and will be absorbed by the polarizer. This gives a dark digit or alphabet.

SOLVED PROBLEMS

Problem 1.75: *Two polarizing plates have polarizing directions parallel so as to transmit maximum intensity of light. Through what angle must either plate be turned if the intensity of the transmitted beam is to drop to one third ?*

Data: $I = \frac{I_0}{3}$

Formula: From Law of Malus,

$$I = I_0 \cos^2 \theta$$

Solution:

Substituting, $\frac{I_0}{3} = I_0 \cos^2 \theta$

$$\cos^2 \theta = \frac{1}{3}$$

Or $\cos \theta = \pm \frac{1}{\sqrt{3}}$

$$\boxed{\theta = 54° 41' \text{ or } \pm 144° 40'}$$

Problem 1.76: *At a certain temperature, the critical angle of incidence of water for total internal reflection is 48° for a certain wavelength. What is the polarizing angle and the angle of refraction for light incident on the water that gives maximum polarization of the reflected light ?*

Data: Critical angle C = 48°

Formulae: (i) $\mu = \frac{1}{\sin C}$

(ii) $\mu = \tan i_p$

Solution:

(i) Substituting,

$$\mu = \frac{1}{\sin 48°}$$

$$\boxed{\mu = 1.345}$$

(ii) From Brewster's law,

$$\mu = \tan i_p$$

$$1.345 = \tan i_p$$

$$i_p = \tan^{-1} (1.345)$$

$$i_p = 53° 22'$$

But $i_p + r = 90°$

$\therefore$ $r = 90° - i_p$

$90° - 53° 22' = r$

$\therefore$ $\boxed{r = 36° 38'}$

Problem 1.77: *Two Nicol prisms are oriented with their principal planes making an angle of 60°. What percentage of incident unpolarized light will pass through the system ?*

Data: $\theta = 60°$

Formulae: (i) For unpolarized light,

$$I = \frac{I_0}{2}$$

(ii) For plane polarized light,

$$I_T = I \cos^2 \theta$$

$$= \frac{I_0}{2} \cos^2 \theta$$

Solution: $I_T = \frac{I_0}{2} \cos^2 60°$

$$I_T = 0.125\, I_0$$

$\therefore$ The percentage of incident unpolarized light transmitted through the system is

$$\% I_T = 0.125 \times 100$$

$\therefore$ $\boxed{\% I_T = 12.5\%}$

Problem 1.78: *A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. How can the analyzer be oriented so that the transmitted light is reduced to (1) 0.75, (2) 0.25 ?* **(04) (Dec. 06, 12, 14)**

Data: (1) $I = 0.75\ I_0$,

(2) $I = 0.25\ I_0$

Formula: $I = I_0 \cos^2\theta$

Solution:

Substituting $0.75\ I_0 = I_0 \cos^2\theta$

$$\frac{3}{4} = \cos^2\theta$$

$$\pm\frac{\sqrt{3}}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 30^\circ,\ \pm 120^\circ}$$

$$0.25\ I_0 = I_0 \cos^2\theta$$

$$\frac{1}{4} = \cos^2\theta$$

$$\pm\frac{1}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 60^\circ,\ \pm 150^\circ}$$

Problem 1.79: *A polarizer and an anlayzer are oriented so that the maximum of light is transmitted. To what fraction of its maximum value and intensity of transmitted light reduced when the analyzer is rotated through (i) 30°, (ii) 45° and (iii) 60°?*

Solution: Law of Malus:

$$I = I_m \cos^2\theta \quad \therefore \quad \frac{I}{I_m} = \cos^2\theta$$

(i) $\theta = 30^\circ, \frac{I}{I_m} = (\cos^2 30^\circ) = 0.75$

(ii) $\theta = 45^\circ; \frac{I}{I_m} = (\cos^2 45^\circ) = 0.50$

(iii) $\theta = 60^\circ, \frac{I}{I_m} = (\cos^2 60^\circ) = 0.25$

Problem 1.80: *If the plane of vibration of incident beam makes an angle of 30° with the optic axis, compare the intensities of the extra-ordinary and ordinary light.*

*(**Hint:** Amplitude of E-ray = A cos θ, Amplitude of O-ray = A sin θ).*

Solution: We know,

$I \propto A^2$ and according to law of Malus, $I \propto \cos^2\theta$

For E-ray: $I_E = A^2\cos^2\theta = A^2\cos^2 30^\circ = 0.75\ A^2$

For O-ray: $I_O = A^2\sin^2\theta = A^2\sin^2 30^\circ = 0.25\ A^2$

$$\therefore \quad \frac{I_E}{I_O} = \frac{0.75}{0.25} = 3$$

$$\boxed{I_E = 3I_O}$$

Problem 1.81: *Polarizer and analyzer are set with their polarizing directions parallel so that the intensity of transmitted light is maximum. Through what angle should either be turned so that the intensity is reduced to (i) 1/2 and (ii) 25% of the maximum intensity?* **(04) (Dec. 03, 12)**

Data: (i) $I = 0.5\ I_0$

(ii) $I = 0.25\ I_0$

Formula: $I = I_0 \cos^2\theta$

Solution:

(i)

$$I = I_0 \cos^2\theta$$

$$0.5\ I_0 = I_0 \cos^2\theta$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = 0.707$$

$$\boxed{\theta = 45^\circ}$$

(ii)

$$0.25\ I_0 = I_0 \cos^2\theta$$

$$\cos^2\theta = 0.25$$

$$\cos\theta = 0.5$$

$$\boxed{\theta = 60^\circ}$$

Problem 1.82: *At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light.* **(03) (May 13)**

(Data Given: Critical angle for diamond = 24.5°)

Data: $i_c = 24.5^\circ$

Formula: (i) $\mu = \frac{1}{\sin i_c}$

(ii) $\mu = \tan i_p$

Solution (i) $\mu = \frac{1}{\sin 24.5}$

$\mu = 2.41$

(ii) $i_p = \tan^{-1}(2.41)$

$i_p = 67^\circ 28$

SUMMARY

- **Unpolarized Light :** Has the electric vector vibrating along all possible directions at right angles to the direction of propagation of light.
- **Types of Polarization :** (i) plane, (ii) circular and (iii) elliptical.
- **Partially Polarised Light :** Mixture of plane polarized light and unpolarized light is partially polarized light.
- If the vibrations of the electric vector in a light wave are confined to a single plane, then the light wave is plane polarized or linearly polarized.
- **Polarization :** The phenomenon of confining vibrations of the light beam to a particular plane is called as polarization.
- **Plane Polarized Light :** Can be produced from unpolarized light by reflection, refraction, scattering, selective absorption and double refraction.
- **Law of Malus :** States that the intensity of the transmitted light is proportional to the square of the cosine of the angle between the plane of transmission of the analyzer and that of the polarizer i.e. $I = I_0 \cos^2\theta$.
- **Double Refraction :** When light passes through anisotropic crystals, it splits up into two rays, O-ray and E-ray. This phenomenon is known as double refraction or birefringence.
- **Positive Critical :** Birefringence of the crystal is given by, $\Delta\mu = \mu_e - \mu_o$.
- If the velocity of ordinary ray is greater than that of the extraordinary ray ($\mu_e > \mu_o$) then the crystals are positive.
- **Negative Crystal :** If the velocity of extraordinary ray is greater than that of the ordinary ray ($\mu_o > \mu_e$) then the crystals are negative.
- **Optic Axis :** Along the optic axis, O-ray and E-ray travel with the same velocity.
- LCD is the display which uses the principle of polarization.

IMPORTANT FORMULAE

- $I = I_0 \cos^2\theta$
- $\mu = \dfrac{1}{\sin c}$
- $\mu_o = \dfrac{\sin i}{\sin r_O} = \dfrac{c}{v_O}$
- $\mu_e = \dfrac{\sin i}{\sin r_e} = \dfrac{c}{v_e}$

EXERCISE

1. Explain the term polarization of light.
2. Define plane of polarization and plane of vibration. Explain a method to show that light waves are transverse.
3. Distinguish between polarized and unpolarized light.
4. Explain how you would obtain plane polarized light by reflection.
5. What is pile of plates ? Explain how it can be used for producing plane polarized light.
6. What is polarizing angle ? Explain.
7. State and explain the law of Malus.
8. Explain the phenomenon of double refraction in calcite.
9. Describe the construction and working of a Nicol prism.
10. What is a Nicol prism ? Explain how a Nicol prism can be used as an analyzer and polarizer.
11. Explain giving diagrams the nature of refraction observed in the case of calcite crystal when:
    (a) optic axis is parallel to the refractive surface and lying in the plane of incidence (normal incidence).
    (b) optic axis is perpendicular to the refracting surface and lying in the plane of incidence (normal incidence).
12. Give Huygen's construction for ordinary and extraordinary wavefronts when the beam of light is refracted through a doubly refracting crystal when the optic axis is inclined to the crystal surface and lying in the plane of incidence (normal incidence).

13. Mention the different methods of producing plane polarized light. Describe any one of them.

14. What is the basic principle of LCD ? Explain the working of LCD displays.

UNSOLVED PROBLEMS

1. If the plane of vibrations of the incident beam makes an angle of 30^0 with the optic axis, compare the intensities of extraordinary and ordinary light.

$$\left(\textbf{Ans. } \frac{I_e}{I_o} = 3\right)$$

3. A beam of light travelling in water strikes a glass plate which is also immersed in water. When the angle of incidence is 51^0, the reflected beam is found to be polarized. Calculate the refractive index of glass.

(**Ans.** 1.235)

4. A glass plate is used as a polarizer. Find the angle of polarization for it. Also find the angle of refraction, given μ for glass = 1.54.

(**Ans.** 57^0, 33^0)

5. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is maximum. Through what angle must either sheet be turned so that the intensity becomes one half the initial value ?

(**Ans.** 45^0, 135^0)

6. The refractive index for plastic is 1.25. Calculate the angle of refraction for a ray of light inclined at polarizing angle.

(**Ans.** 38.6^0)

8. A beam of light is passed through two Nicol prisms in series. In a particular setting, maximum light is passed by the system and it is 500 units. If one of the Nicols is now rotated by 20^0, calculate the intensity of transmitted light.

(**Ans.** 441.5 units)

9. Two Nicol prisms are oriented with their principal planes making an angle of 30^0. What percentage of incident unpolarized light will pass through the system ?

(**Ans.** 37.5 %)

10. A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through (i) 45^0, (ii) 90^0 ?

(**Ans.** 0.5, 0)

UNIVERSITY QUESTIONS

December 2012

1. State the Phenomena of Double Refraction. Hence, explain Huygen's Wave Theory of Double Refraction. **[6]**

May 2013

1. Define Double refraction. Explain Huygen's Theory of Double refracting crystal with diagram. **[6]**

November 2013

1. Explain the propagation of light through a quartz crystal plate for normal incidence. When **[6]**
   (i) Optic axis is parallel to the crystal surface and lying in the plane of incidence.
   (ii) Optic axis is perpendicular to the crystal surface and lying in the plane of incidence.
   (iii) Optic axis is inclined to the crystal surface and lying in the plane of incidence.

May 2014

1. Explain double refraction and hence give Huygen's theory of double refraction. **[6]**

December 2014

1. Explain propagation of light in a doubly refracting crystal when the optic axis is parallel to the crystal surface, with the help of neat diagram. **[3]**

May 2015

1. State and prove Law of Malus. **[3]**

November 2015

1. State and prove Malus law. **[3]**

May 2016

1. What is double refraction? Explain this phenomenon on the basis of Huygen's theory. **[6]**

December 2016

1. What is Double refraction ? Explain Huygens's theory of double refraction. **[6]**

May 2017

1. What is double refraction ? Draw neat and labelled diagram (either for positive or negative crystal) showing propagation of light within a doubly refracting crystal when optic axis is : **[3]**

   (i) parallel to crystal surface

   (ii) perpendicular to crystal surface

Nov. 2018

2. Light is incident on a quartz crystal plate at normal incidence. Explain with suitable diagram the propagation of light in following cases when optic axis is lying in the plane of incidence and is **[6]**

   (i) Parallel to the crystal surface

   (ii) Perpendicular to the crystal surface

   (iii) Inclined to the crystal surface

May 2019

1. What is double refraction ? Explain Huygen's theory of double refraction. **[6]**

✠ ✠ ✠

UNIT II - LASER AND OPTIC FIBRE

LASER

2.1 INTRODUCTION

- The term laser stands for **'light amplification by stimulated emission of radiation'**.
- Laser is a light source which is highly coherent i.e. radiation emitted by all the emitters (atoms or molecules) in source agree in phase, direction of emission, polarisation and are essentially of one wavelength or colour (monochromatic).
- Due to coherence, a beam of laser light can travel many miles with only a negligible divergence. This makes it different from the conventional light sources which emit many wavelengths with phase and direction widely varying.
- Around 1917, Einstein first predicted the existence of two different kinds of processes by which an atom can emit radiation by (i) Spontaneous emission, (ii) Stimulated emission.
- In a laser, the process of stimulated emission is used for amplifying the light waves. The fact that stimulated emission process could be used in the construction of coherent optical sources was first put forward by Townes and Schawlow.
- The energy of an atom in any atomic system can change by
  (i) Spontaneous emission
  (ii) Absorption
  (iii) Stimulated emission.

2.2 BASIC OF LASER AND ITS MECHANISM

2.2.1 Spontaneous Emission

- When a photon of energy $E = h\upsilon$ is incident on an atom, the atom absorbs the energy to rise to an excited state. The difference of the energy levels should be equal to the absorbed energy ($E_2 - E_1 = h\upsilon$). Though the electron stays in the ground state for an infinite amount of time (being a stable state), but it remains in the excited state for a short time called the ***'life time'*** of the excited state ($\approx 10^{-8}$ sec). Energy levels for which the life time is greater than 10^{-8} sec are called as ***'metastable states'***.

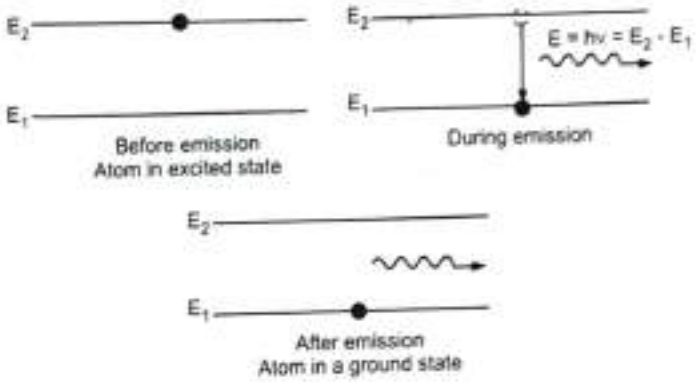


Fig. 2.1: Spontaneous emission

- The electron which is in an excited state E_2, spontaneously decays back to the lower energy level and radiates an energy equal to $E_2 - E_1$ in the form of photon of energy $h\upsilon$. Such an emission which is random in behaviour, depending only on the type of atom and the type of transition is known as ***'spontaneous emission'***.
- The emitted radiations are not coherent and have a large range of wavelengths. Only those transitions occur which are permitted by selection rules.

2.2.2 Absorption

- When a photon of energy $h\upsilon$ is incident on an atomic system, the atom gets excited from a lower energy E_1 to a higher energy E_2, if the energy of photon equals the difference in the energy levels i.e. $h\upsilon = E_2 - E_1$. In this process, the photon gets absorbed and the atom is said to be excited.

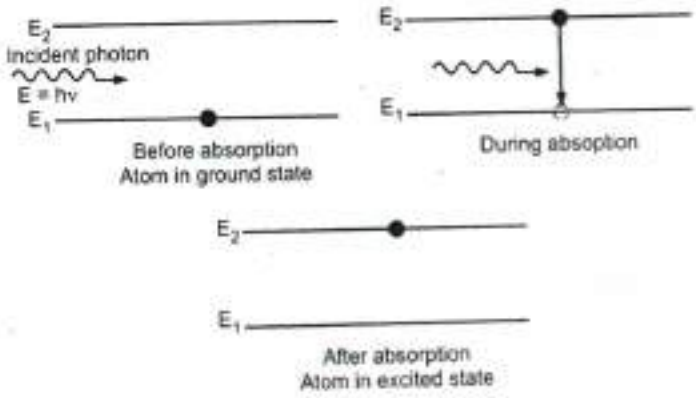


Fig. 2.2: Absorption

2.2.3 Requirement for Lasing Action

Stimulated Emission **(Nov. 13, May 16)**

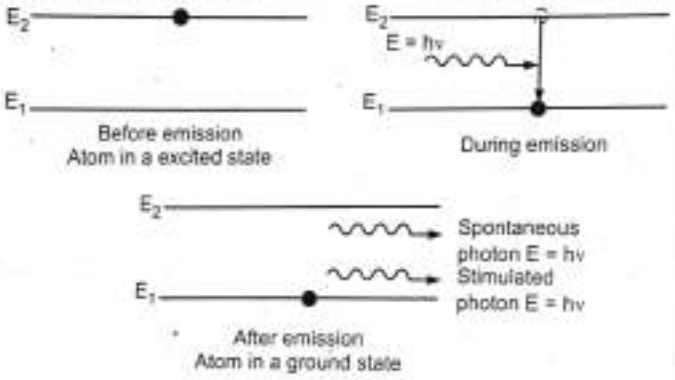


Fig. 2.3: Stimulated emission

- Consider Fig. 2.3 where the electrons are initially in the excited energy level and emission is stimulated before the spontaneous emission occurs. The excited atom is stimulated by a photon of exactly the same energy as the photon to be emitted. In such a case, two photons are emitted, one by the stimulated emission and the other stimulating photon.
- Both the photons travel in the same direction, have the same frequency and are in phase i.e. they are coherent.
- The emission of two photons with an input of only one photon implies amplification. The occurrence of spontaneous emission is directly proportional to the number of atoms in the specified energy level, whereas in stimulated emission, the rate of occurrence is proportional not only to the number of atoms in the excited state but also to the number of incident stimulating photons.

2.2.4 Population Inversion **(Nov. 13)**

- The process of getting a large percentage of atoms into an excited state is called as ***'population inversion'***. If a large number of atoms can be excited to upper energy levels, then the probability of stimulated emission and hence light amplification becomes greater.
- The states of the system, in which the population of the higher energy state is more than the population of the lower energy state, are called as ***'negative temperature states'*** (negative indicates a non-equilibrium state, not the physical state of the system).
- In any atomic system, the number of particles in a higher energy state is normally less than the number of particles in a lower energy state. If N_2 denotes the number of particles in higher energy level E_2, and N_1 denotes the number of particles in lower energy level E_1, then $N_2 < N_1$ i.e. the population of higher energy level is less than the population of lower energy level. This means that under normal conditions, the ground state E_1 is heavily populated than the excited state E_2.
- If photons of energy $h\upsilon = E_2 - E_1$ are incident on the atoms, a few of the incident photons get absorbed and some of the atoms get excited to the state E_2. This process of stimulated absorption depopulates level E_1. The rate at which this process occurs is expressed as

$$R_{12} = P_a N_1 \quad \text{... (1)}$$

where P_a is the probability of stimulated absorption and N_1 is the population of state E_1.

- Similarly, the stimulated emission depopulates energy level E_2 resulting in the emission of photons. The rate at which this process occurs is expressed as

$$R_{21} = P_e N_2 \quad \text{... (2)}$$

where P_e is the probability of the process of stimulated emission and N_2 is the population of state E_2.

- At thermal equilibrium, these probabilities are equal i.e. $P_a = P_e$. Then, on comparing the two rates, it is observed that more energy is absorbed than emitted.

i.e. from (1) and (2),

$$P_a N_1 > P_e N_2 \text{ because } N_1 > N_2.$$

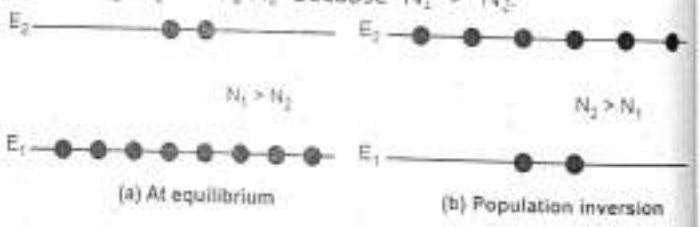


Fig. 2.4

- To produce more emission, it is essential to have $N_2 > N_1$ i.e. the number of particles in higher energy level must be made more than the number of particles in lower energy level. This is called as ***'population inversion'***.
- If this inversion is achieved, there can be more emission and incoming light will be amplified coherently. A system in which population inversion is achieved is called an ***'active system'***.
- The method of raising atoms from lower energy levels to higher energy levels is called as ***'pumping'***. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light, etc. A more common method of pumping is ***'optical pumping'***.

2.2.5 Metastable State (Nov. 15)

- The electron in an excited state has certain probability to decay or jump to a lower energy level. Generally, these probabilities are such that the jump occurs within 10^{-8} sec of excitation.
- However, there are some excited states, called '***metastable states***', which have a very low probability of decay i.e. electrons stay for longer time.
- Electrons may stay in the metastable excited states for seconds, minutes or even hours. In stimulated emission, the electrons must remain in excited level and wait for stimulating photon.
- Therefore, the active medium must have a metastable state. The population inversion can be obtained by using metastable states as the electrons rest in metastable state for long time.

2.2.6 Active Medium

- A medium in which the population inversion takes place is called the ***active medium***. The active medium is responsible for the light amplification and hence LASER. The active medium may be a solid, liquid or gas and accordingly the lasers are classified as solid state or gas lasers.
- Out of the total active medium, only small number of atoms are responsible for lasing action and remaining atoms help only in hosting active atoms or in population inversion. The atoms which particulate in stimulated emission are called ***active centres***.

2.2.7 Resonant Cavity

- A cavity can be constructed using mirrors such that the light rays return to their original position after travelling through the cavity for a certain number of times. Such cavities are known as ***resonant cavities***.
- Fig. 2.5 shows cavity formed by two parallel mirrors M_1 and M_2. One of the mirrors is completely silvered (M_1) and the other is partially silvered (M_2). The laser beam emerges from the resonant cavity through the partially silvered mirror M_2.

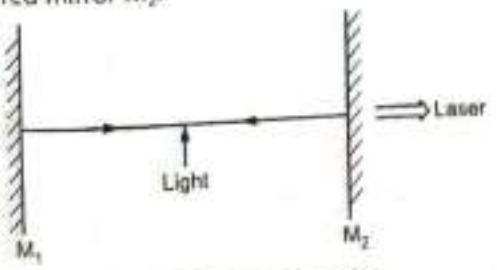


Fig. 2.5: Resonant cavity

- The active system is placed in the resonant cavity, the photon emitted will keep on reflecting back and forth within the cavity. The light which is incident parallel to the axis of optical cavity will only leak out as a laser. That is why, laser is highly directional.

2.2.8 Pumping (May 14)

The method of raising atoms from lower energy levels to higher energy levels is called as '***pumping***'. The pumping is used for achieving population inversion which is necessary for optical amplification to take place. There are several methods for pumping electrons. They are as follows:

1. Optical Pumping

- In optical pumping, an external light source (flash lamp) is used to produce a high population in some particular energy level E_2 (say) by selective absorption as shown in Fig. 2.6.

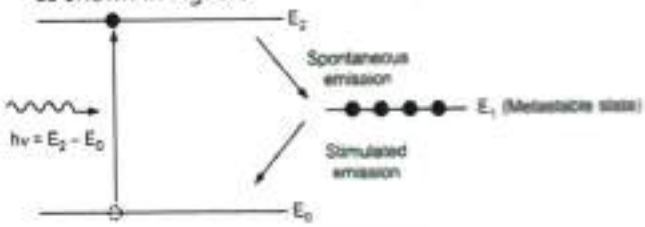


Fig. 2.6: Optical pumping

- When a flash of light falls on electrons in ground state, they absorb incident photons and get excited. After staying there for some time, some of the atoms make spontaneous transition to metastable state E_1. As the probability of spontaneous decay is less in metastable state, a large population accumulates in this level. This results in a population inversion between E_0 and E_1.
- Generally, this method is used in solid-state lasers, such as ruby laser.

2. Inelastic Atom-Atom Collisions

- Here suitable mixtures of gases are used. The gases are selected in such a way that their excited states are almost same. This makes the energy exchange possible between the atoms of the gases. If two gases A and B have same excited state, A^* and B^* then

$$A^* + B \rightarrow A + B^*$$

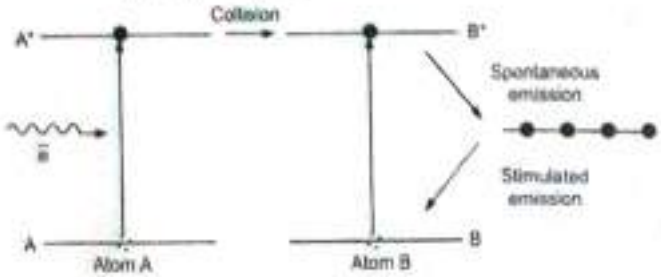


Fig. 2.7: Inelastic atom-atom collision

- The atom of gas A is excited by electric discharge. In collision with B, the energy is transferred to B. As a result, the excited level of atom B becomes more populated than lower level to which B can decay, as shown in Fig. 2.7.
- The best example is the He-Ne gas laser.

3. Forward Biasing of a p-n Junction

- If a p-n junction is formed with degenerate (heavily doped) semiconductors, the bands under forward bias appear as shown in Fig. 2.8.

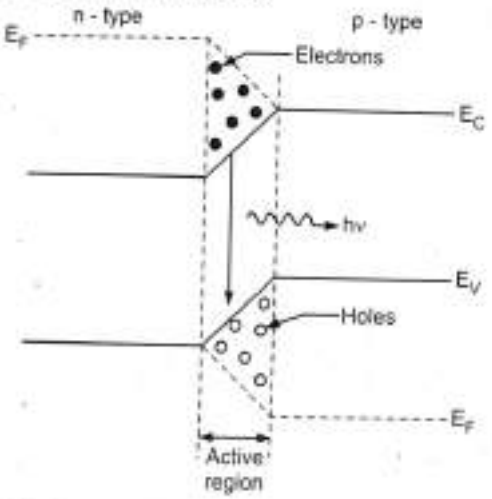


Fig. 2.8: Forward biasing of a p-n junction diode

- If the bias voltage is large enough, electrons and holes are injected into the active region. As a result, the depletion layer now contains a large number of electrons in conduction band and holes in valence band. If the population density is high enough, it gives population inversion. For a population inversion, the applied voltage should be selected in such a way that $eV > h\upsilon$ ($= E_g$).
- The other methods of pumping are electron excitation, chemical reactions, etc. These methods will not be discussed in detail as they are beyond the scope of the text.

2.3 CHARACTERISTICS OF LASER

Laser is basically a light source, but many of its properties are special or extreme. These properties make it different from ordinary sources.

The basic properties of lasers which make different from ordinary light source are:

(a) Monochromaticity

- When the laser produces only one wavelength then it is fully monochromatic. This is impossible, in principle and in practice also.
- If $\Delta\lambda$ is the range of wavelengths included in a laser beam of wavelength λ, then $\Delta\upsilon$ is the corresponding frequency. This frequency band $\Delta\upsilon$ is called the ***'line width'***.
- For white light consisting of all visible frequencies, the line width is $\sim 10^{14}$ Hz, while for a good laser it is about 10^{2} Hz.

(b) Coherence

- The light emerging from a laser is coherent both in space and time. The existence of finite bandwidth $\Delta\upsilon$ means that the different frequencies present in laser can get out of phase width each other.
- If two waves differ by frequency $\Delta\upsilon$, the time required to get out of phase by a full cycle is $1/\Delta\upsilon$. This is called as ***'coherence time'*** of a beam and is denoted by $\Delta\tau$.

i.e. $$\Delta\tau = \frac{1}{\Delta\upsilon} \quad \text{... (1)}$$

- For a laser with $\Delta\upsilon = 1$ MHz, $\Delta\tau = 1$ μs. On the other hand, sunlight has a bandwidth $\Delta\upsilon = 10^{14}$ Hz, therefore, $\Delta\tau = 10^{-14}$ s which is much smaller than laser.
- The speed of light is so large that it travels a very large distance for a short coherence time. The distance $\Delta L = c\Delta\tau$ is called ***'coherence length'*** of the beam. For a laser beam with $\Delta\upsilon = 1$ MHz, the coherence length $\Delta L = 300$ m.
- Only the portions of the same beam, separated by a distance less than coherence length are capable of producing stable interference pattern.

(c) Directionality

- A laser has high degree of directionality and can travel very large distances without deviation. For a typical laser, the divergence is about 10^{-3} radians. This means that the laser beam diverges by ~1 mm for every meter that it travels.
- The reason is that the active material is placed in a resonant cavity. The light is reflected back and forth in the cavity and light travelling parallel to the axis gets emitted as the laser beam. The light travelling in the other direction is reflected back in the cavity.

(d) Brightness

- A laser radiates light into a narrow beam and its energy is concentrated in a small region. This is due to the fact that the divergence is very less. This results into extremely high intensity.
- Even a 1 W laser appears more intense than a ordinary 100 W lamp. This high concentration is used in many applications such as drilling, welding and cutting.

2.4 SEMICONDUCTOR LASER – TWO LEVEL LASER SYSTEM

- A semiconductor diode laser is a p-n junction device that emits coherent light when forward biased. A p-n junction laser is also called ***'injection laser'*** as the charge carriers are injected into depletion region.
- The diode lasers are preferred over other lasers as they are compact, reliable and low cost.

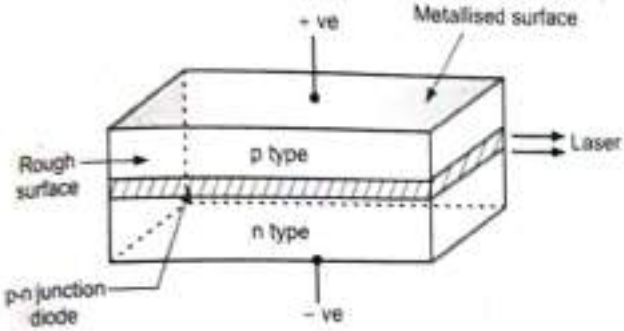


Fig. 2.9

Construction

- A laser diode is extremely small in size with sides of the order of 1 mm. The p-n junction of the diode lies in the horizontal plane. The top and bottom faces are metallised for making electrical contacts.
- Two plane mirrors are placed at front and back surface to form optical cavity. Sometimes the faces itself are polished to form optical cavity. The other two faces are roughened to prevent lasing action in that direction.

Working

(a) Under Equilibrium

- In semiconductor diode laser, the p-n semiconductor is heavily doped (degenerate semiconductor). Fig. 2.10 shows the energy band of highly doped p-region and n-region.

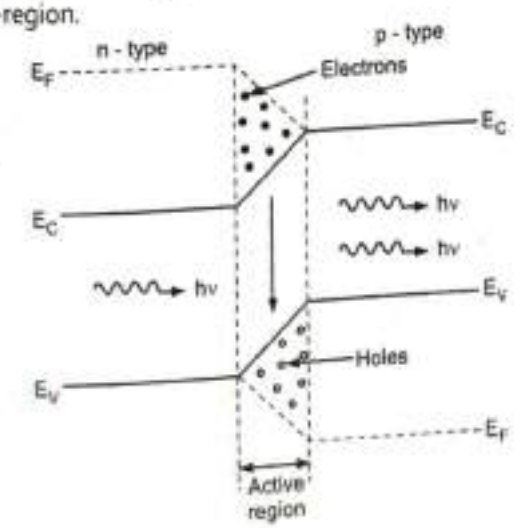


Fig. 2.10

- Due to very high doping on n-side, the donor level as well as a portion of conduction band is occupied by electrons. Hence, the Fermi level lies within the conduction band. Similarly, on the heavily doped p-side, holes exist in the valence band and hence the Fermi level lies in the valence band.

(b) Forward Bias, Pumping and Recombination

- At low forward current, the electron-hole recombinations cause spontaneous emission of photons and the junction acts as an LED. As the current is increased, the intensity of light increases.
- When the current reaches a threshold value, the carrier concentration in the depletion region reaches a very high value i.e. a large concentration of electrons in the conduction band and a large concentration of holes in the valence band.
- As a result, the higher energy levels in the depletion region are having high population density of electrons while the lower levels are vacant. This is a state of population inversion. The region where the population inversion is achieved is called as ***active region***. The forward bias acts as the pumping agent.

(c) Built-Up of Laser

- Due to forward bias, the band gap is very small and therefore electrons in the conduction band can recombine with holes in the valence band. During this process of recombination of electrons and holes, electrons loose their energy emitting a photon of energy equal to band gap.
- Some of the photons emitted during the process of recombination are reflected back from the polished ends and stimulate the electrons initiating the laser process. The highly polished ends continuously reflect the light back and forth. The photons parallel to the axis of resonator will leak out as laser beam.
- The frequency of radiated photon is $\upsilon = E_g/h$, where E_g is band gap and h is Planck's constant. The wavelength of laser beam depends upon the material used, for GaAs laser wavelength is 9000 A° in IR region and GaAsP it is 6500 A° in visible region at room temperature.

Advantage of Semiconductor Laser

- Very less power required, simple, compact, highly efficient, more divergent, less monochromatic and highly temperature sensitive.

2.5 SINGLE HETERO-JUNCTION LASER

Basically Diode Laser can be Classified in Two Categories :

- **Homojunction Semiconductor Laser:** A simple diode ;laser which makes use of the same semiconductor material on both sides of junction.
- **Heterojunction Semiconductor Laser:** A diode laser which makes use- of different semiconductor materials on the two sides of junction.
- In single hetero-junction diode, on one side of the active layer is a material with different energy gap which cause a difference in the refractive index of the materials. This can build wave guide structures that confine the photons to a specific area.

Construction

- The simplest of the heterostructures is the single heterojunction laser.
- **Active Layer of GaAs**
- It consists of an active layer of GaAs where the p-n junction is located.
- **Additional Layer of (AlGa)As**
- A layer of (AlGa)As is grown on top of the p-region.
- This layer has a higher band gap and also offer a lower refractive index than GaAs
- The layer act as a barrier for the injected carriers as it has a and provides better waveguide properties
- The top and bottom faces have metal contacts to pass current
- Front and rear faces are polished which works as resonator

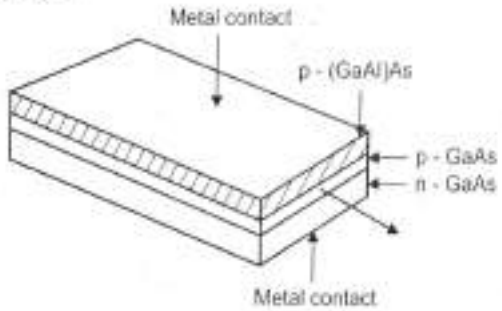


Fig. 2.11

Working

- The basic principle of working is similar to that of homojunction laser diode.
- **Active Layer:** Lasing action occurs in p-GaAs layer at the centre
- **Pumping Mechanism:** At forward bias, electrons and holes are injected in active region in high concentration. When diode current reaches threshold value, carrier concentration in active layer reaches maximum value.
- **Advantage of Heterojunction:** The electrons are injected from n-GaAs layer into active layer of p-GaAs. Due to change in refractive index at the boundary p-GaAs and p-(GaAl)As, electrons are reflected back into active region.
- Thus, the presence of hetero structure confines charge carriers into active region.

Population Inversion:

- Due to forward bias, active region contains large concentration of electrons in conduction band and simultaneously a large number of holes in valence band; which creates population inversion.

Lasing Action:

- Recombination of electrons and holes in active region leads to spontaneous emission of photons
- **Stimulated Emission:** These spontaneous photons propagating in active layer stimulate conduction electrons to jump into vacant states in valence band and produces stimulated photons.
- The reflection of photons from polished ends within the active layer provides feedback for lasing action and laser oscillations starts
- After attaining sufficient strength, laser beam emerges out from diode.

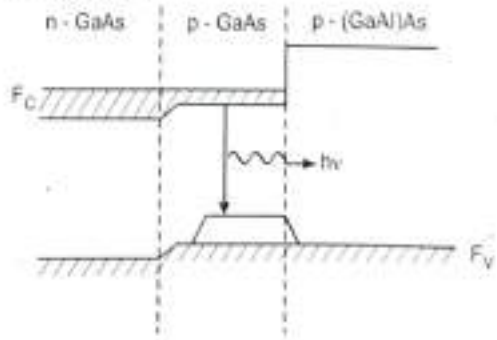


Fig. 2.12

Advantage of Heterojunction:

- Due to change in refractive index at the boundary p-GaAs and, p-(GaAl)As, the laser light is trapped within the active region and travels in one direction only.

Advantages of Heterojunction Laser:

- High efficiency even at room 'temperature'.

- Less operating current is required
- Threshold current density is less, due to which continuous laser output is possible

2.6 GAS LASER (CO_2) LASER

- The Carbon dioxide (CO_2) laser is a molecular gas laser based on a gas mixture carbon dioxide (CO_2), Helium (He) and Nitrogen (N_2) as the gain medium.
- Depending on manufacturing, continuous output power varies between a few watts to several hundred watts.

Principle and Vibration Modes of CO_2 Laser

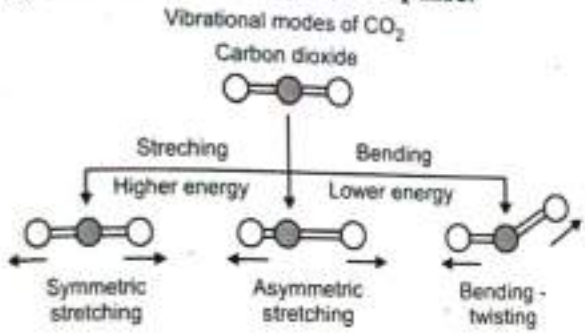


Fig. 2.13

- A CO_2 molecule is a linear molecule and with carbon atom in the middle. It undergoes three independent vibration oscillations.
- Transitions to the symmetric stretching mode and bending modes correspond to wavelengths of 10.6 μm and 9.6 μm respectively.

Construction of CO_2 Laser

- The output power of CO_2 laser depends on its diameter.
- A mixture of CO_2, Nitrogen and Helium is taken into a discharge tube powered by a by a power supply
- Gases are filled inside the tube with pressure 10% CO_2, 10% Nitrogen and 80% Helium.
- Fully reflecting mirror and partially reflecting mirror are fitted on the either side of the tube for re-circulation of the laser light inside the tube.

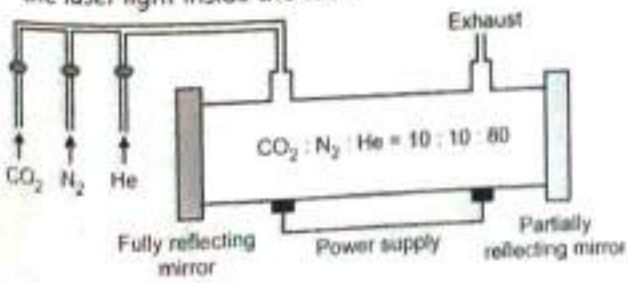


Fig. 2.14

Pumping

- Electric discharge is created in the laser tube. The energy of accelerating electrons is transferred to Nitrogen and they accumulate to metastable state.
- N_2 molecules collide in elastically with CO_2 molecules by collision and CO_2 molecules are moved to excited E_5 level.

Population Inversion

- E_5 is a higher energy level, while energy levels E_3 and E_4 are lower metastable levels of CO_2. Due to buildup of CO_2 molecules in higher E_5 energy level, population inversion takes place in E_5, and E_3 and E_4 energy levels.

Lasing Action

- Lasing action takes place in CO_2 which is active medium. Nitrogen and Helium improves efficiency of lasing action.
- Random and spontaneous photons emitted by few atoms of E_5 level travel back and forth between the end mirrors and helps in stimulated emission of photons.
- These re-circulated photons build up along the axis of axis of discharge tube (optical cavity).
- Oscillations takes place in between two vibrational levels of CO_2.
- The transition between $E_5 \rightarrow E_4$ produces IR radiations of wavelength 10.6 μm.
- The transition between $E_5 \rightarrow E_3$ produces IR radiations of wavelength 9.6 μm.
- CO_2 molecules in E_4 and E_3 levels falls to lower energy level E_2 which is close to ground level Helium atoms colloids with CO_2 molecules in E_2 level and reduce their population.
- The de-exited CO_2 molecules are again excited due to collision of nitrogen and are taken into upper excited state E_5. This helps in maintaining continuous laser output.

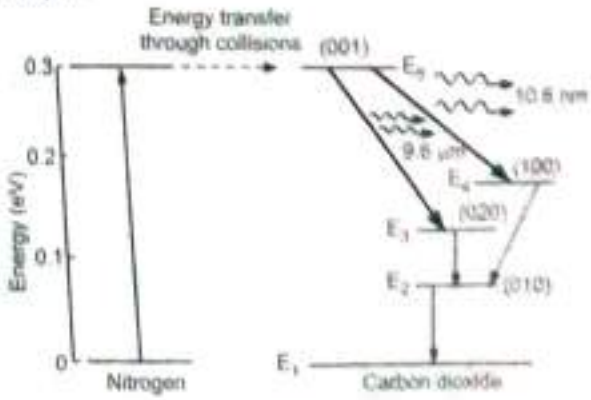


Fig. 2.15

Advantages of CO_2 Laser :

- Very high power output with relative efficiency of around 30-40%, which is better than HeNe and argon laser
- It operates in continuous mode
- CO_2 laser are low cost per watt and have good beam quality
- It generates high power output which ranges from few watts to 15 KW.
- Used primarily for materials processing applications.

2.7 APPLICATION OF LASER (Dec. 12, May 16)

2.7.1 Information Technology (Holography)

(May 13, 14)

Holography

- This is a technique of producing an interference pattern between a direct laser beam and a laser beam reflected from an object on a photographic plate. This pattern on the developed photographic plate, when illuminated with laser in a proper manner, produces a three-dimensional image of the object called a ***'hologram'***.
- Holography deals with three-dimensional image of the object whereas photography is a two-dimensional effect. In photography, the photographic plate records only the intensity of light due to the image formed on it. In holography, both the intensity and phase distribution are recorded simultaneously using interference technique. Due to this the image produced by the technique of holography has a true three-dimensional form and is as true as the object itself.

Basic Technique

The basic technique in holography is as follows:

(i) Hologram Recording

- The recording of hologram is achieved by superposition of ***'the object wave'*** with another wave called ***'the reference wave'***.
- The reference wave is usually a plane wave.
- The resulting interference pattern is recorded on a photographic plate. As the shape of the object is very irregular, it results in complicated fringe pattern. Thus a hologram is a record of complicated interference fringe pattern.
- Fig. 2.16 shows a typical configuration used for recording a hologram. A portion of a coherent beam (e.g. laser) is allowed to be scattered by the object and the other portion is reflected by a mirror. The former corresponds to the object wave and the latter to the reference wave.

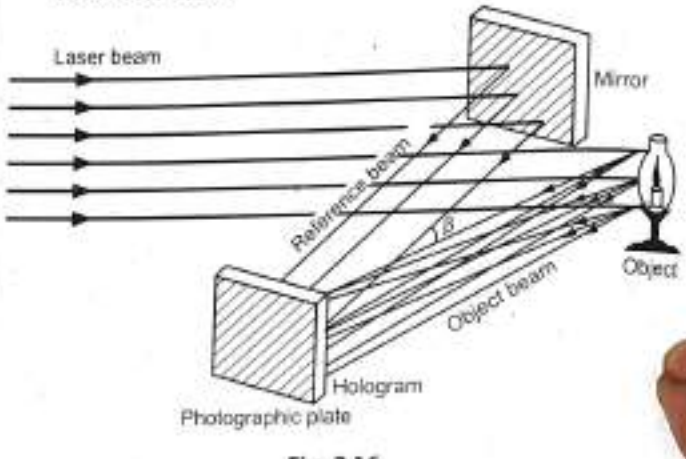


Fig. 2.16

- The object wave and the reference wave interfere and the resultant interference pattern is recorded on a photographic plate.
- When the photographic plate which has recorded the intensity variation is developed, one obtains a hologram of the object.
- The recorded interference pattern forms the hologram and contains information not only about the amplitude but also about the phase of the object wave. Unlike a photograph, the hologram bears little resemblance to the object.

(ii) Hologram Reconstruction

- To see the reconstructed image, the hologram is illuminated by the reference beam alone, maintaining the original alignment and orientation. This process is called as ***'reconstruction'***.
- The developed photographic film (hologram) will have alternate transparent and opaque part of very irregular shape. Thus, it will serve as a diffraction grating when illuminated by the light source.
- The light from the hologram will be diffracted inward and outward. One set of diffracted rays will converge to form real image while one set will diverge to form virtual image as shown in Fig. 2.17.
- Thus, the hologram can form real as well as virtual images. The geometry of construction can be changed, so as to give more emphasis on real or virtual image.

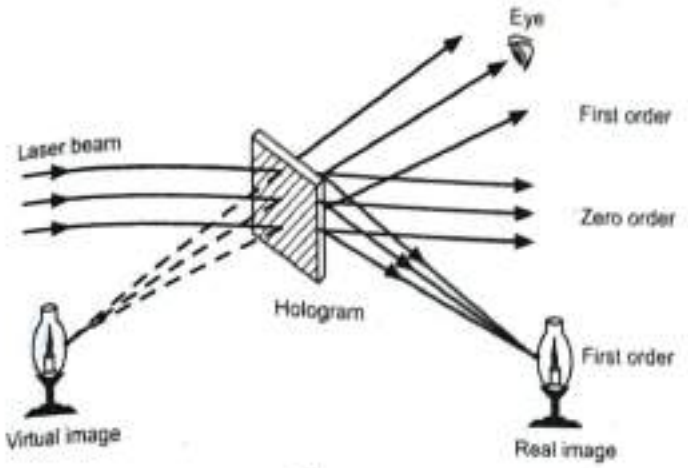


Fig. 2.17

2.7.2 Information Technology

Barcode Scanners

- Barcode is a machine-readable code in the form of numbers and a pattern of parallel lines of varying widths, printed on an object for quick identification. Laser scanners are used for decoding the barcode information on the products by moving pinpoint of laser to illuminate the barcode, and a single photocell receives the reflected light.

Optical Discs (CD/DVD/Blue-Ray)

- With a CD-RW, binary information is stored as areas of metal alloy that are either crystalline or amorphous. Crystalline areas have a regular structure that lets light pass through to the aluminum area and reflect back down again, thus storing ones.
- Amorphous areas have a random structure that scatters incoming laser light, so it can't reflect back, thus storing zeros. A CD-rewriter can change the metal alloy on the CD from one form to the other and back again, which is why this kind of disc can be erased and rewritten many times over.
- Audio CD use infrared lasers. DVDs use a shorter-wavelength red laser. Blue-Ray disc uses blue-light lasers to read and store data at an even higher density.

Fiber-Optic Communication Systems

- Optical fibers are widely used in communications for transmission over longer distances and at higher bandwidths. The advantages of fiber optic communication are: High data carrying capacity, High quality signal transmission, No pick up of stray signal i.e. noise, Cost effective, Small size, Light weight and flexible, Less power consumption, High Tolerance and corrosion resistant, Idle for carrying digital signal.

Lasers in Printer / Photocopy Machines

- A laser printer is a common type of computer printer that rapidly produces high quality text and graphics on plain paper. In a laser printer, a laser beam scan back and forth across a drum inside the printer, building up a pattern of static electricity.
- The static electricity attracts onto the page a kind of powdered ink called toner. Finally, as in a photocopier, a fuser unit bonds the toner to the paper.

2.7.3 Industrial

- Laser can be focused to a very high energy density into a small image (≈ 1 micron in diameter) with the help of suitable lenses. Due to the small size of the image and the control over the energy, lasers are used extensively for cutting, welding and drilling circuits.

1. **Drilling:** A laser beam is also used to drill holes of micron dimensions on printed circuit boards (PCBs). It is also used in resistance trimming in electric components industries. One can drill holes of the diameter of 10 μm through very hard substances like diamond. YAG laser is found to be very useful in such applications.
2. **Welding:** Lasers are used as a heat source in welding the joints of the metals. This type of precise welding is extremely important in micro-electronics in which thin films are used. Thermocouple wires can easily be welded with the help of high power laser beam.
3. **Micromachining:** Lasers are used for machining a surface in a slow and accurate manner to achieve an extraordinarily smooth finish.
4. **Cutting:** Another important industrial application is metal or fabric cutting. A finely focussed laser beam can cut thick and hard metal sheets with high precision and accuracy. It is also used in tailoring industries to cut thousands of layers of cloth at one instant.
5. Due to its intensity and directionality, laser is used in surveying. When tunnels are to be constructed, engineers use the laser beam as a reference, to check that it is being constructed along a straight line. Similarly, it can be used to dig a ditch to a certain prescribed depth. Its most interesting use in surveying has been in measuring the distance from the earth to the moon. This distance was measured to an accuracy of 600 ft, and with the aid of reflectors to within six inches. This accuracy will allow to determine the location of the north pole to within six inches. It is further believed that a laser could be used to check

whether the gravitational constant is actually a constant.

6. A laser beam can determine precisely the distance, velocity and direction as well as the size and form of distant objects by means of the reflected signal as in radar. A Lidar (Laser radar), which sends out beams of laser light and detects echoes even from atmospheric layers has been developed.

2.7.4 Medicine (Dec. 12)

- Bloodless cancer surgeries can be performed as the beam can be focussed on a small area, so that only the harmful tissue can be destroyed without damaging the surrounding region.
- Laser has been successfully used in ophthalmology, in the treatment of detached retinas, in welding cornea, etc. At the command of the physician, laser produces a beam of light which is directed onto the eye under treatment, to produce a minute coagulation. A series of these lesions weld the detached retina.
- Laser is used as a tool in the study of genetics. Lasers have been built into or are devised to be attached to microscopes. As a high density energy is achieved, it can be used in micro-surgery, micro-burning, etc. Such a microscopic laser can concentrate millions of watts of power per square millimeter into a selected area. For example, a focussed microscope laser can be used to make tiny openings (of 25 μ in diameter) in the cell walls, of say the nervous system, heart, retina, etc. without causing irreversible damage.
- Laser microprobes can be used as dental drills giving an advantage of no heating, no anaesthetic and no pain to the patient. They have also been successfully used for localized treatment of skin growths and blemishes in human beings. A large amount of energy can be transmitted through the skin to interact with deeper different biological materials or structures which are damaged.

SOLVED PROBLEMS

Problem 2.1 : *A laser light of wavelength 6328 A.U. falls normally on a grating which is 2 cm long. The first order spectrum is observed at an angle of 20°. Find the total number of slits on grating.* **(3) (May 15)**

Data: $\lambda = 6328$ A°

Length of grating = 2 cm

$n = 1$

$Q = 20°$

Formula:

(i) $(a + b) \sin\theta = n\lambda$

(ii) $N = \dfrac{1}{a + b}$

Solution:

$$(a + b) = \frac{1 \times 6328 \times 10^{-8}}{\sin 20}$$

$$= 1.850 \times 10^{-4} \text{ cm}$$

$$\therefore \quad N = \frac{1}{1.850 \times 10^{-4}}$$

$$= 5404 \text{ lines/cm}$$

Therefore, total lines on grating of length 2 cm = 5404 × 2 = 10808 lines.

SUMMARY

- The term LASER stands for light amplification by stimulated emission of radiation. It is an intense, coherent, monochromatic beam.
- **Absorption:** Absorption is a process in which a photon, of energy $h\nu$, gets absorbed by an atom and it goes from a lower energy state E_1 to a higher energy state E_2.
- **Emission: (i) Spontaneous Emission:** An electron which is raised to an excited state E_2 (due to absorption), spontaneously decays back to a lower energy level E_1 and radiates an energy equal to $E_2 - E_1$. Such an emission is called as spontaneous emission. This emission is random in nature and depends only on the type of atom and type of transition.

  (ii) Stimulated Emission: A photon of energy $h\nu = E_2 - E_1$ triggers an excited atom to drop to the lower energy state giving up a photon. This phenomenon of forced emission of photons is called as stimulated emission.
- **Population Inversion:** The process of getting a large percentage of atoms into an excited state is called as population inversion.
- **Active System:** A system in which population inversion is achieved is called an active system.

- **Pumping:** A method of raising atoms from lower energy levels to higher energy levels is called as pumping. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light (optical pumping) etc.
- **Metastable States:** Ordinary energy levels have a life time of 10^{-8} to 10^{-9} secs. Energy levels having a life time greater than ordinary energy levels ($\sim 10^{-6}$ to 10^{-3} secs) are called as metastable states.
- **Semiconductor Laser:** A semiconductor laser is a specially fabricated pn junction that emits coherent light when it is forward biased. The basic mechanism of producing laser in a semiconductor diode laser, is the electron-hole recombination at the pn junction when a current is passed through the diode.
- **Gas Laser (CO_2) :** Makes use of CO2 gas molecule as active medium.
- **Light** : From a laser is different from light from other sources in four basic ways:
  - (i) directionality,
  - (ii) monochromaticity,
  - (iii) coherence,
  - (iv) polarizability.
- **Applications :** Due to its unique properties, lasers are used in a variety of fields like welding, machining, surveying, communication, holography, cutting, drilling, information processing, surgery and related medical fields, in CD players, printers, etc.

IMPORTANT FORMULAE

- Rate of absorption, $R_{12} = P_a N_1$.
- Rate of stimulated emission, $R_{21} = P_e N_2$.
- At equilibrium, $P_a = P_e$.
- Population inversion, $N_2 > N_1$.
- Frequency of photons emitted, $\upsilon = \frac{E_2 - E_1}{h}$.
- For light transmission in fibre optics, $\theta_c = \sin^{-1}\frac{n2}{n1}$.

EXERCISE

1. Explain the following terms:
   - (i) Spontaneous emission
   - (ii) Stimulated emission
   - (iii) Population inversion
2. Explain action of gas laser. How does stimulated emission take place with exchange of energy between Helium and Neon atoms ?
3. Define and explain the terms:
   - (i) Pumping
   - (ii) Active systems.
4. What are the different uses to which laser beams are put in industry, medicine ?
5. What are the properties of laser ?
6. Write a note on use of lasers in fibre communication systems and information technology.
7. Write a note on semiconductor laser.

UNIVERSITY QUESTIONS

December 2012

1. Explain any one application of Laser in brief. **[3]**

May 2013

1. Explain the process of recording Holdgram with the help of LASER. **[3]**

November 2013

1. Explain the following:
   - (i) Stimulated emission.
   - (ii) Metastable state
   - (iii) Population inversion.

May 2014

1. What is difference between normal photography and holography? Why lasers are used to record hologram ? **[3]**

May 2016

1. What is stimulated emission of radiations? Explain its significance in production of laser. **[3]**
2. Explain any one engineering application of laser. **[3]**

December 2016

1. What is Holography ? Explain the process of hologram recording. **[3]**

May 2018

1. State the characteristics of laser beam and explain any one of them in brief. **[3]**

OPTIC FIBRE

2.8 INTRODUCTION

- Efforts to device communication systems for sending from one place to another distant place have been continuing by the human being. These systems have used as optical or acoustical means like signal lamps or horns, electrical codes like Morse Code (1938), Telephones (1878) so effort. A portion of electromagnetic waves is used in telephones, amplitude modulated and frequency modulated radio, television, CB (citizen's band radio), satellite link in the recent days.
- Optical region of the electromagnetic spectrum which contains wavelengths from 50 nm (500 A°) ultraviolet) to about 1000 nm (10000 A°) (far infrared) with the visible range from 400 nm to 700 nm (nanometre, 10^{-9} m).
- Advances in technology have made it possible to use optical fibres alongwith good optical sources, photodetectors and fibre cable connectors to transmit more data at high transmission rate from one place to a distant place.
- Optical fibre is a very thin and flexible medium having a cylindrical shape consisting of three sections: (i) The core, (ii) The cladding and (iii) The outer jacket.

Principle of Light Transmission

- The principle of light transmission through optical fibre is total internal reflection. For total internal reflection to take place at the fibre wall, the following conditions should be satisfied:
  - ➢ The refractive index of the core material (μ_1) must be greater than that of the cladding (μ_2).
  - ➢ At the core-cladding interface, the angle of incidence θ must be greater than the critical angle,

where $\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$.

- When a light ray travels from a denser to a rarer medium, the angle of refraction is greater than the angle of incidence. As the angle of incidence increases, the angle of refraction also increases and for a particular angle of incidence, the refracted ray grazes the interface between the core and the cladding. This angle of incidence is called as the **Critical Angle** θ_c.
- If angle of incidence is greater than θ_c, the ray will be reflected back into the core, i.e. it suffers **Total Internal Reflection**. For angles equal to or greater than the critical angle the light will be totally reflected and no light will be refracted. Fig. 2.18 shows total internal reflection.
- When light is incident on core of the fibre optics, it will be refracted and will travel in the core. After some time it will strike one of the core-cladding interface say upper surface. If the angle of incidence is greater than critical angle, it will be totally reflected and remain in the core.
- Now, the reflected light will travel to the lower surface. It is then incident on the lower surface where the same process is repeated and light gets transmitted from one end to the other end as shown in Fig. 2.18.

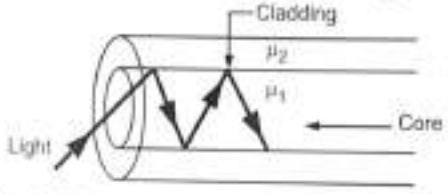


Fig. 2.18: Propagation of light in fibre optics

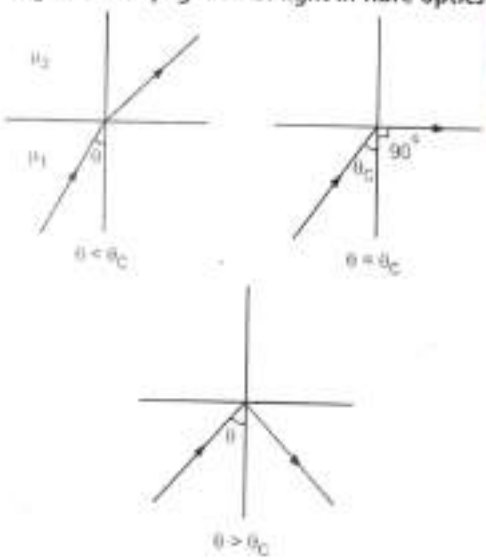


Fig. 2.19: Total internal reflection

2.8.1 Structure of Optical Fibre

- An Optical fibre has generally cylindrical, core of which has higher refractive index (μ_1) than that of the surrounding material (μ_2). The core and the surrounding dielectric together form an optical waveguide. Depending on the type of the waveguide, optical fibres are categorised into two steps as

1. Step Index Fibre and
2. Graded Index Fibre.

- The path through which light is propagated is called **Waveguide**. In case of optical fibre, core and cladding together work as optical waveguide.

1. Core-Cladding Fibre

Core : In this type of optical fibre, there is a single solid dielectric cylinder of radius a and refractive index μ_1 as shown in Fig. 2.20. This solid cylinder is known as the **Core** of the optical fibre.

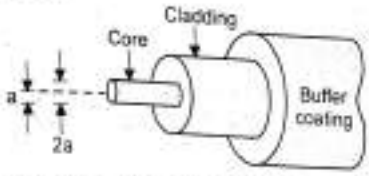


Fig. 2.20 : Construction of step index optical fibre

Cladding (as a Rarer Medium) :

- The core is surrounded by a solid dielectric (cone polymer) **Cladding** having a refractive index μ_2 which is less than μ_1. Cladding reduces the scattering losses due to dielectric discontinuities at the core surface.
- Also, it adds to the mechanical strength of the fibre and protects the core from absorbing the surface contamination which can come in contact with it. The cladding is made up of either glass or plastic materials.

Buffer (For Mechanical Strength) :

- Most fibres are encapsulated in an elastic abrasion resistant plastic material. This encapsulating material is called **Buffer Coating**. The buffer adds further mechanical strength to the fibre and keeps away the fibre from small geometrical irregularities, distortions or roughness of the surrounding surfaces. This also avoids random microscopic or sharp bends when the fibres are incorporated into cables or when supported on some other structures.
- The conventional optical fibre consists of a core region of refractive index, μ_1, which is surrounded by a cladding of lower refractive index, μ_2. The fibre or a bundle of fibres is sheathed in an outer protective covering.

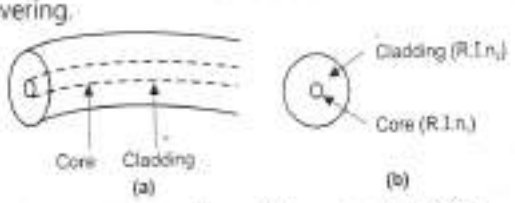


Fig. 2.21 : The fibre with core and cladding

There are two types of optical fibres, viz. (1) step index and (2) graded index.

2.9 PARAMETERS

2.9.1 Acceptance Angle and Numerical Aperture

(May 19)

Consider Fig. 2.22. ϕ is the angle of incidence at the core-cladding interface for a ray entering the core making an angle, i, with the fibre axis.

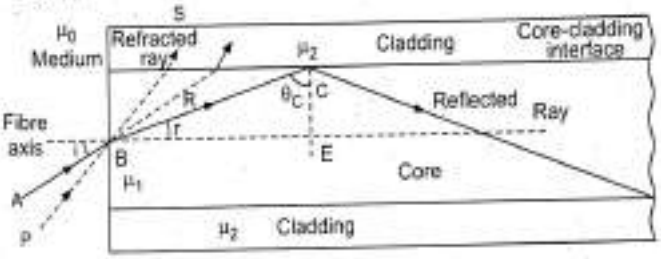


Fig. 2.22 : Ray propagation through step-index optical fibre

The condition for total internal reflection to take place is

$$\sin\phi \geq \frac{\mu_2}{\mu_1} \quad \text{... (2.1)}$$

We have, $\sin\phi = \sin(90 - r) = \cos r$ ... (2.2)

Also, $\frac{\sin i}{\sin r} = \mu_1, \quad \therefore \sin r = \frac{\sin i}{\mu_1}$ ... (2.3)

Now, $\sin r = \sqrt{1 - \cos^2 r}$

$$= \sqrt{1 - \sin^2\phi} \quad \text{... (2.4)}$$

The condition of total internal reflection, (2.1), therefore can be expressed as

$$\sin r \leq \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}} \quad \text{... (2.5)}$$

Using (2.4), we have,

$$\frac{\sin i}{\mu_1} \leq \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}}$$

$$\therefore \quad \sin i \leq \sqrt{\mu_1^2 - \mu_2^2} \quad \text{... (2.6)}$$

If i_m is the maximum angle of incidence for which total internal reflection can occur, we have

$$\sin i_m = \sqrt{\mu_1^2 - \mu_2^2} \quad \text{for } \mu_1^2 - \mu_2^2 < 1$$

$$= 1 \quad \text{for } \mu_1^2 - \mu_2^2 \geq 1 \text{... (2.7)}$$

Light incident within *the cone of half-angle* i_m at the input end of the fibre will undergo total internal reflection and be guided along the fibre. This, therefore, is a measure of the **Light Gathering Power** of the fibre. **sin i_m** is called the **Numerical Aperture** (N.A.) of the fibre.

The numerical aperture is a function of the refractive indices of core and the claddings.

2.9.2 Acceptance Cone

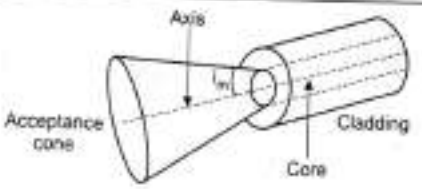


Fig. 2.23

The acceptance cone of an optical fibre decides its light gathering power and depends on acceptance angle. Larger the acceptance angle, larger is the light gathering power.

The acceptance cone is derived by rotating the acceptance angle about the fibre axis. The Fig. 2.23 shows the acceptance cone.

Problem 2.2: *Refractive index μ_1 = 1.48 and μ_2 = 1.45 in an optical fibre. Calculate numerical aperture and the maximum entrance angle $\theta_{0\,max}$ if the fibre is kept in air.*

Data : $\mu_1 = 1.48, \quad \mu_2 = 1.45$

Formula : Numerical aperture,

$$N.A. = \sqrt{\mu_1^2 - \mu_2^2}$$

Solution :

$$= \sqrt{1.48^2 - 1.45^2}$$

$$= \sqrt{2.1904 - 2.125} = \sqrt{0.0879}$$

$$= 0.2964$$

Also, $N.A. = n \cdot \sin\theta_{0\,max}$ (here $\mu = 1$ for air)

$\therefore$ $\theta_{0\,max} = \sin^{-1}(N.A.) = \sin^{-1}(0.2964)$

$= \boxed{17.24°}$ or $\boxed{17° 15'}$

Problem 2.3: *Numerical aperture of an optical fibre is 0.5. Find the refractive index of cladding if the refractive index of the core is 1.53. Also calculate index difference.*

Data : N.A. = 0.5, $\mu_1 = 1.53$

Formula : $N.A. = \sqrt{\mu_1^2 - \mu_2^2}$

Solution : $0.5 = \sqrt{1.53^2 - \mu_2^2}$

Squaring both sides,

$\therefore$ $0.25 = 1.53^2 - \mu_2^2$

$\therefore$ $\mu_2^2 = 2.34 - 0.25 = 2.09$

$\therefore$ $\mu_2 = 1.446$ (taking square roots)

Now, index difference

Δ = change of refractive index per unit change of core refractive index.

$$\therefore \quad \Delta = \frac{\mu_1 - \mu_2}{\mu_1} = \frac{1.53 - 1.446}{1.53} = \boxed{0.055}$$

2.10 TYPES OF OPTICAL FIBRE

2.10.1 Classification of Optical-Fibre Based on Refractive Index

Optical fibres are classified into following types based on refractive index of core and cladding :

(1) Step index, and (2) Graded index

1. **Step-Index Optical Fibre :** The core has an **Uniform Refractive Index**, μ_1 and the cladding has an **Uniform Refractive Index** μ_2 ($\mu_2 < \mu_1$). Let the core radius be 'a' and the outer radius of cladding be 'b'.

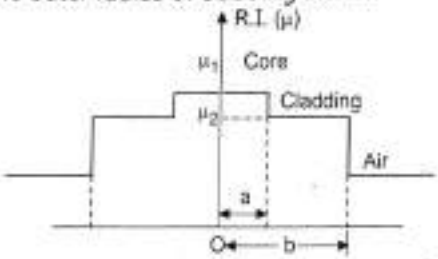


Fig. 2.24 : The refractive index profile of a step-index fibre

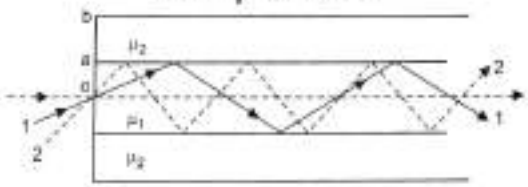


Fig. 2.25 : The paths of rays in step-index fibre

- In the step-index fibre, rays entering at different angles of incidence with the axis travel different path lengths and emerge out at different times. This results in **Pulse Dispersion**, i.e. an input pulse gets widened as it travels along the fibre.

2. **Graded-Index Optical Fibre :** The **Refractive Index of the Core Varies Continuously** from μ_1 at the centre to μ_2 at the core-cladding interface. The cladding has the constant refractive index μ_2.

- In the graded-index fibre, a ray is continuously bent and travels a periodic path along the axis. Rays entering at different angles follow different paths with the same period, both in space and time. This results into periodic **Self Focussing** of the rays as shown in Fig. 2.26. Therefore, the pulse dispersion is less as compared with the step-index fibre.

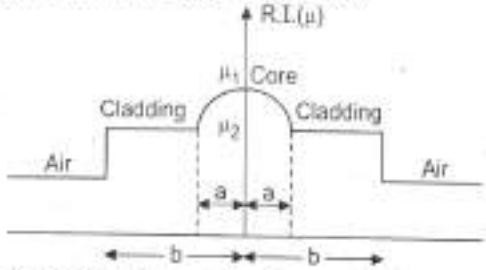


Fig. 2.26 : The R.I. profile of a graded-index fibre

Sr. No.	Description	Step Index Fibre	Graded Index Fibre
1.	Variation in RI of core and cladding.	Varies step-by-step	Varies with radial distance
2.	RI difference at core-cladding interface.	Larger difference	Smaller difference
3.	Transmission dispersion	Transmission of single and multimode signals.	Transmission of Multimode signal.
4.	Intermodal Dispersion	Larger	Less
5.	Bandwidth for multimode transmission	Lower bandwidth due to lower numerical aperture.	Higher bandwidth due to high numerical aperture.
6.	Propagation of rays	Light rays propagate as meridional rays since they cross axis of core.	Light rays propagated as skws rays as they do not pass through axis of core.
7.	Pattern of propagation of rays.	Zigzag manner	Either spherical or helical
8.	Applications	Long distance transmission.	Short distance transmission.

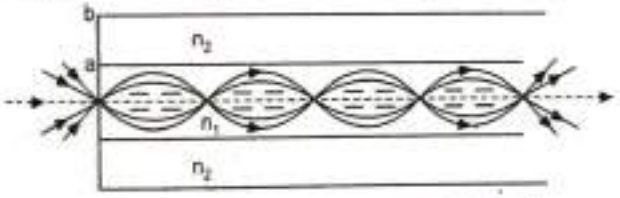


Fig. 2.27 : The paths of rays in graded-index fibre

- The core diameters (2a) in fibres in use may range between 4 μm to 100 μm. The core + cladding diameter (2b) usually ranges between 100 μm to 200 μm.

2.10.2 Classification of Optical Fibres Based on Number of Modes

Optical fibres are classified into two types based on number of modes transmitted :

(1) Single mode, and

(2) Multimode

- Fibres with narrow cores (about 10 μm) allow only one wave-mode to pass and are called **Monomode Fibres**; while those with core diameters about 50 μm and above allow different wavemodes and are called **Multimode Fibres**.

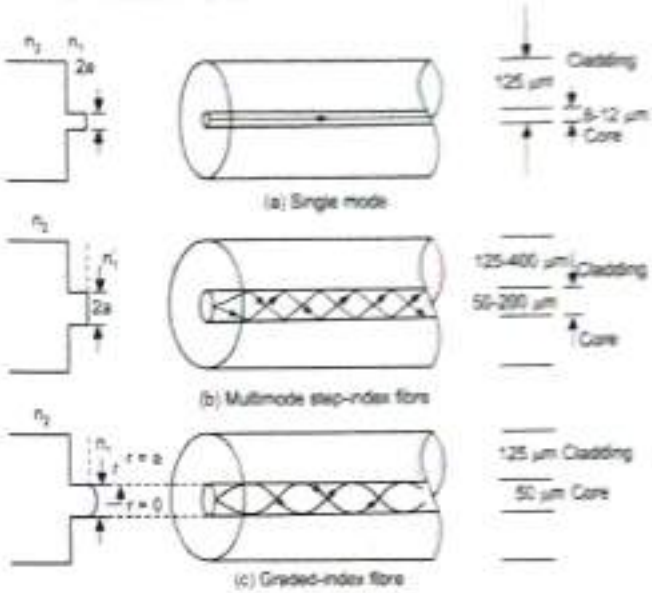


Fig. 2.28 : Types of fibre optics

- Optical fibres used commonly in telecommunication applications have 2b = 125 μm and (typical values for refractive indices are : $\mu_1 \cong 1.5$ and $\mu_2 = \mu_1 (1 - \Delta)$ where Δ = 0.01 to 0.02 (1 to 2 %)).

Sr. No.	Description	Single Mode	Multimode
1.	Number of modes	Transmit a single mode during transmission.	Transmit more than one mode during transmission.
2.	Diameter of single core	Very small (5 – 10 μm)	Larger than single mode (50 μm or more)
3.	Difference in refractive index of core and cladding	Very small	Very large
4.	End to end connections	Easy	Difficulty
5.	Intermodal dispersion (time difference between entry and arrival of optical signal)	There is no intermodal dispersion	They suffer from intermodal dispersion
6.	Applications	Long distance transmission	Short distance transmission

2.10.3 Index Difference (Core-Cladding Index Difference)

- In practical step-index fibres, the core with radius has refractive index μ_1. A typical value of μ_1 is 1.48. The cladding surrounding this core has a refractive index slightly lower than μ_1. The relation between μ_1 and μ_2 is given by

$$\mu_2 = \mu_1 (1 - \Delta)$$

where Δ is called core-cladding index difference or simply the index difference.

- The values of μ_2 are chosen so that the index difference Δ is equal to 0.01. As the core is having higher refractive index than that of cladding, electromagnetic waves at optical frequencies propagate along the fibre due to **Total Internal Reflection** at the core-cladding interface.
- An optical fibre may be either monomode or multimode in case of both the types i.e. step index and graded-index fibre.

2.11 ATTENUATION AND REASONS FOR LOSSES IN OPTIC FIBERS

- An optical signal propagating through a fiber will get attenuated progressively. It is the ratio of the optical output power from a fiber of length L to the input optical power.

If, P_i = power of optical signal at launch

P_o = power of optical signal emerging at other end

Then,

$$\text{Attenuation} = \frac{10}{L} \log \frac{P_i}{P_0}$$

The unit of measurement of attenuation is dB/km.

- The factors that affect the transmission of light waves through optical fibers are:

(a) Absorption (b) Dispersion, and

(c) Bending

(a) Attenuation Due to Absorption:

- When optical signal is transmitted into the fiber, energy is lost mainly due to electron absorption, Rayleigh scattering, material absorption (intrinsic absorption) and impurity absorption (extrinsic absorption).
- **Ravleigh Scattering :** Of light is due to small irregularities in the core structure. It is prominent if irregularities are of the order of wavelength of light. It can be minimized at longer wavelengths.
- **Material Absorption :** Is due to the loss of energy due to core material.
- **Electron Absorption :** Takes place when atoms in the core material absorb energy from incident light, which generally takes place at lower wavelengths.
- Attenuation due to impurities is higher than intrinsic absorption.

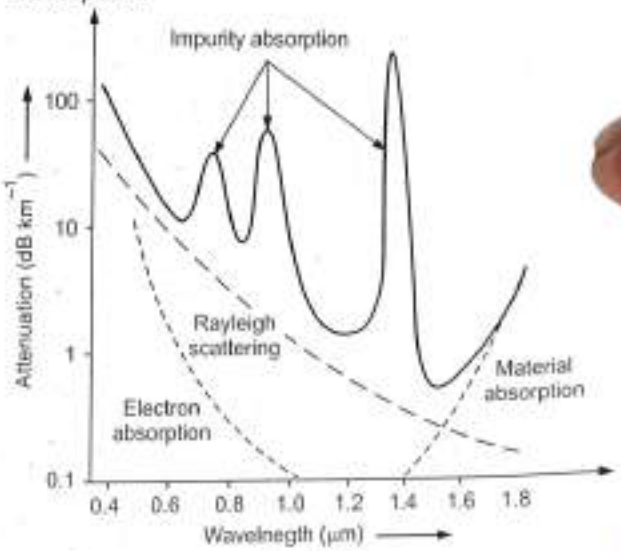


Fig. 2.29

(b) Attenuation Due to Dispersion:

- Dispersion is a phenomenon related to the variation in velocity of different frequencies (wavelengths) or different modes.
- Dispersion losses are the results of the distortion of optical signal when traveling along the fiber. The light waves spreads as a function of time when it propagates.
- Due to pulse broadening and overlapping, the output gets distorted.
- **Intermodal Dispersion:** A ray of light launched into fiber fallows a zigzag path. When many modes are propagating they travel with different net velocities and come out at different times which cause broadening of pulse. Low N.A fiber exhibit smaller dispersion also narrow spectral width of source limits the dispersion.
- **Waveguide Dispersion:** the effective refractive index for any mode varies with wavelength which cause pulse spread in. This is waveguide dispersion.
- **Material Dispersion:** It is based on the wavelength of optical signal. Light waves of different wavelengths travel at different speeds in a medium. The short wavelength waves travel slower than long wavelength wave. Thus, material dispersion can be reduced. By using more and more monochromatic source.

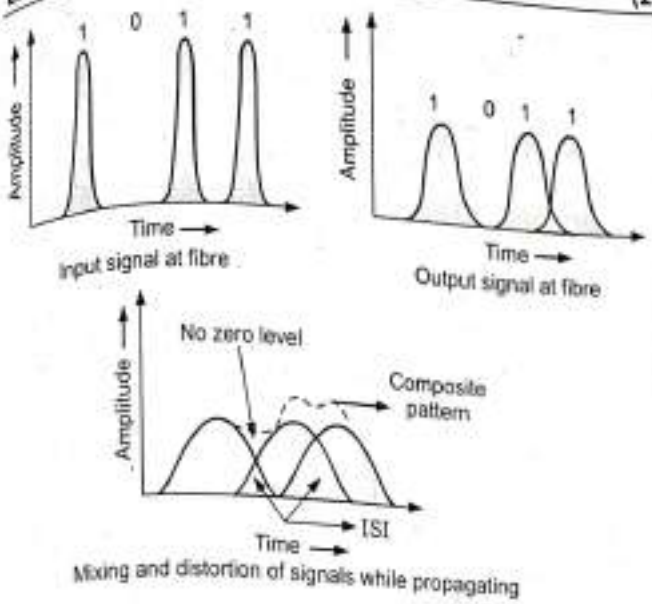


Fig. 2.30

(b) Bending

When an optical fiber is bent, radiative losses occur known as bending losses.

- **Macro Bending Loss :**
  - ➢ The radiative losses which arise when a fiber axis is bend during the cabling of the fiber. They mainly arise due to winding of the fiber cable.
  - ➢ Because of bending, light rays propagate into the cladding and hence are lost due to radiation.
  - ➢ The presence of thermal contraction between cladding and core materials, also lead to small amount of loss
  - ➢ These losses can be minimized by careful installation and improvement in manufacturing process

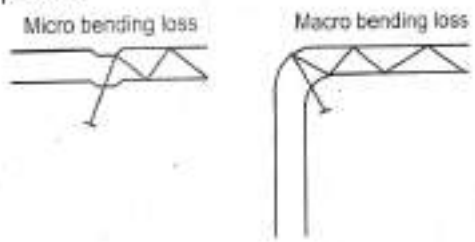


Fig. 2.31

- **Macro-Bending Losses :**
  - ➢ A macro-bend refers to the specified minimum bending radius.
  - ➢ Improper cabling, bending a fiber optic cable tighter than the specified bending radius, exerting external force may cause deformation in optic fiber and it can cause damage, even break the fiber carried in the cable.
  - ➢ The macro-bends result into loss of light radiation.
  - ➢ These losses can be minimized by careful handling and installation.

2.12 OPTICAL FIBRE COMMUNICATION LINK

Basically, the optical fibre communication link comprises of electric input signal, nsmitter, optical fibre and a receiver. Fig. 2.32 shows the essential elements of an optical fibre communication link.

A transmitter consists of a drive circuit and a light source. The receiver comprises of photodetector, amplifier and signal restorer.

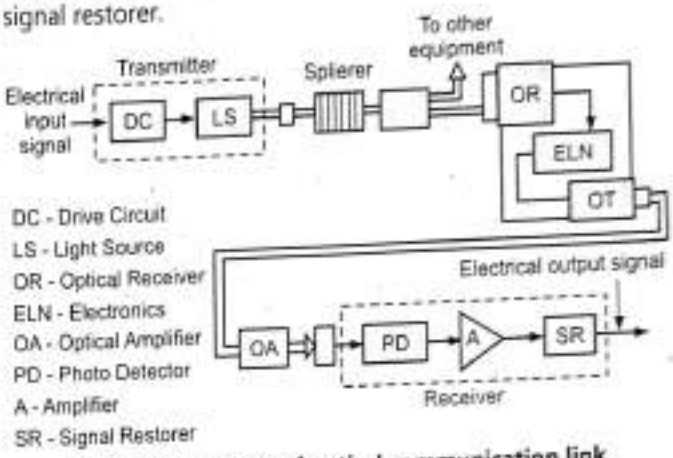


Fig. 2.32 : Elements of optical communication link

Working of Optical Fibre Communication Link :

Semiconductor Light-Emitting Diode (LED) and laser diode are suitable for transmitter source to launch optical power into the core of optical fibre. Transmitter (electronics) circuitry converts the electrical signals into optical signals. These launched signals emerge out at the receiver end, where they are received by Photodiode and are converted into electric current output.

An output signal becomes attenuated and distorted when it travels over a large distance. It is, therefore amplified and reshaped at the receiver end.

2.13 ADVANTAGES OF FIBRE OPTIC FIBRE COMMUNICATION OVER CONVENTIONAL METHODS

- **High Data Carrying Capacity**
  - ➢ Optical fiber can carry very high frequencies (~ 10^{14} Hz) which are helpful in carrying more information i.e. more phone lines or more TV programs or more data.
  - ➢ As optical fibers are thinner, more fibers can be bundled together for increasing data carrying capacity.
- **High Quality Signal Transmission**
  - ➢ Signals from one optic fiber do not interfere with those of other fibers which do not degrade the

signal quality. No leakage of information is possible.

- ➢ The signal degradation due to negligible as compared to conventional copper wires as there are no 12 R losses.
- ➢ Optic fibers are made from dielectric material which provides isolation between transmitter and receiver.

- **No Pick up of Stray Signal i.e. Noise**
  - ➢ As optic fibers are made up of either glass or plastic, they do not pick up stray signal or noise like copper wire does either due to transmitted signal or due to external factors.
  - ➢ Thus the effect of external electromagnetic interference, lightning, external magnetic field, nuclear radiation is minimal.
- **Cost Effective**
  - Optical fibers are typically made from silica. They are very cost effective as they are manufactured in bulk mass production
- **Small Size, Light Weight and Flexible**
  - ➢ Optic fibers are of small size, light weight and are flexible. Due to this advantage, they are very helpful in Medical imaging: in bronchoscopes, endoscopes, laparoscopes and mechanical imaging: inspecting mechanical welds in pipes and engines in airplane, rockets, space shuttle, etc
- **Less Power Consumption**
  - ➢ Optical fiber consumes less power for its operation as compared to metal wires.
- **High Tolerance and Corrosion Resistant**
  - ➢ Optic fibers can tolerate extreme temperature variations (in some cases even upto 800°C). Corrosion due to water or chemicals is minimal for optic fibers.
- **Idle for Carrying Digital Signal**
  - ➢ Optical fibers are very much suitable for carrying digital signals that are ideally suited for computer networks. As large amount of data can be transmitted they are very useful in broadband internet, LAN, etc.

Shortcomings of Optic Fiber

- Higher initial cost in installation
- Higher interfacing cost
- It requires remote electric power when transmitted through sea water
- Optic fibers are more expensive for repair/maintain

SUMMARY

- **Principle :** Total internal reflection.
- **Total Internal Reflection :** When light passes from denser to rarer medium and if angle of incidence is greater than critical angle, then light is totally reflected into the denser medium.
- **Optical Fibre :** It is a dielectric waveguide.
- **Waveguide:** The path through which light is propagated / guided. Optical waveguide consists of core and cladding.
- **Core :** A single dielectric cylinder of radius r and refractive index μ_1.
- **Cladding :** A solid dielectric material surrounding the core with refractive index μ_2 ($\mu_2 > \mu_1$).
- **Step-Index Optical Fibre :** The core has a uniform refractive index μ_1 and the cladding has a uniform refractive index μ_2.
- **Graded-Index Optical Fibre :** The refractive index of the core varies continuously from μ_1 at the centre to μ_2 at the core-cladding interface.
- **Acceptance Angle :** Light incident within the cone of half angle θ_0
- **Numerical Aperture :** $\sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$.
- Modes of propagation :
  (a) Single mode,
  (b) Multimode.

EXERCISE

1. Explain the elements of optical fibre communication link.
2. What are various parts in the optical fibre communication system ?
3. Explain total internal reflection and its relation with the working of optical fibre.
4. Give constructional features of optical fibre.
5. Describe various parts of optical fibre ?
6. Give name of different types of optical fibre and their structure.
7. What do you mean by
   (i) Monomode fibres,
   (ii) Multimode fibres.
8. Explain 'Numerical Aperture' and arrive at the expression for numerical aperture.

✠ ✠ ✠

UNIT III - QUANTUM MECHANICS

QUANTUM MECHANICS

3.1 INTRODUCTION

- The most outstanding development in modern science is the conception of quantum mechanics. The quantum mechanics is better than Newtonian classical mechanics in explaining the fundamental physics. There was big development in physics between the time of Newton and the time of quantum mechanics.
- Newton showed that the motion of planets and the free fall of an object on earth is governed by the same law. Thus, he unified terrestrial and celestial mechanics. This was in contrast to ancient belief that the world of the earth and heaven is governed by different laws.
- It was earlier believed that the heat is some peculiar substance called '***caloric***', which flows from a hot object to a cold object. But latter it was proved that the heat is the random motion or vibration of constituents of matter. Thus, thermodynamics and mechanics were unified.
- For a long time, the phenomena of electricity, magnetism and light were treated as independent branches and were unconnected. But in nineteenth century, Faraday and Maxwell along with others unified these independent branches of physics. They proved that all three phenomena are manifestations of electromagnetic field.
- The simplest example is the electric field of an electric charge that exerts a force an another charge when it comes in the range. An electric current produces a magnetic field that exerts a force on magnetic materials.
- Such fields can travel through space, independent of charge and magnet, in the form of electromagnetic wave. The best example of electromagnetic wave is light. Finally, Einstein unified space, time and gravity in his theory of relativity.
- Quantum mechanics also unified two branches of science: physics and chemistry.
- In previous developments in physics, fundamental concepts were not different from those of everyday experience, such as particle, position, speed, mass, force, energy and even field. These concepts are referred as '***classical***'.
- The world of atoms cannot be described and understood with these concepts. For atoms and molecules, the ideas and concepts used in dealing with objects in day to day life is not sufficient. Thus, it needed new concepts to understand the properties of atoms.
- A group of scientists W. Heisenberg, E. Schroedinger, P.A.M. Dirac, W. Pauli, M. Born and Neils Bohr, conceived and formulated these new ideas in the beginning of 20th century. This new formulation, a branch of physics, was named as ***quantum mechanics***.

3.1.1 Limitations of Classical Mechanics

- The classical physics is complete and beautiful in explaining daily experiences where big bodies are involved. But it breaks down severely at subatomic level and failed to explain some of the phenomenon totally.
- The phenomenas which classical physics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- In classical physics, a body which is very small in comparison with other body is termed as '***particle***'. Whereas in quantum mechanics, the body which cannot be divided further is termed as '***particle***'.
- The other main difference is the quantized energy state. In classical physics, an oscillating body can assume any possible energy. On the contrary, quantum mechanics says that it can have only descrete non-zero energy.

3.1.2 Need of Quantum Mechanics

- Classical mechanics successfully explained the motions of object which are observable directly or by instruments like microscope. But when classical mechanics is applied to the particles of atomic levels, it fails to explain actual behaviour. Therefore, the classical mechanics cannot be applied to atomic level, e.g. motion of an electron in an atom.

- Other phenomenas which classical mechanics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- The above problems were solved by Max Planck in 1900 by the introduction of the formula

$$E = nh\upsilon \quad \text{... (1)}$$

where, $n = 0, 1, 2, \ldots,$

h = Planck's constant = 6.63×10^{-34} J/s

- This is known as '***quantum hypothesis***' and marked the beginning of modern physics. The whole microscopic world obeys the above formula.

3.1.3 Wave Particle Duality of Radiation and Matter

The wave and particle duality of radiation can be easily understood by knowing what is a wave and what is a particle.

Wave

- A ***wave*** originates due to vibrations, and it is spread out over a large region of space. A wave cannot be located at a particular place and mass cannot be attached to a wave.
- Actually, a wave is a spread out disturbance specified by its amplitude a, frequency υ, wavelength λ, phase δ and intensity I.

Particle

- A ***particle*** is located at some definite point and it has mass. It can move from place to place. A particle gains energy when it is accelerated and it loses energy when it is slowed down.
- A particle is characterized by mass m, velocity v, momentum p, and energy E.
- Looking at the above facts, it might appear difficult to accept wave particle duality of radiation, i.e. radiation is a wave which is spread out over all space and that it is also a particle which is localised at a point in space.
- But this wave particle duality of radiation has to be accepted because, sometimes radiation has to be assigned the behaviour of a wave and at other times radiation is to be assigned particle nature as discussed below.
- The phenomena of interference, diffraction and polarisation require the presence of two or more waves at the same time and at the same position. It is very clear that two or more particles cannot occupy the same position at the same time. So one has to conclude that radiation behaves like waves.
- Spectra of black body radiation, production and scattering of X-rays, Compton effect, photoelectric effect, etc. could not be explained on wave nature of radiation. These phenomena established that radiant energy interacts with matter in the form of photons or quanta. With this, Planck's quantum theory came into being to conclude that radiation behaves like particles.
- Thus, radiation sometimes behaves as a wave and at some other times as a particle. It is to be noted that radiation cannot simultaneously exhibit its wave and particle properties. Now, wave-particle duality of radiation is universally accepted.

3.2 DE BROGLIE'S HYPOTHESIS (Dec. 12)

- Discrete particle nature of matter is very well established and now it is known that matter is composed of atoms and that electrons, protons and neutrons are the building blocks of all types of atoms.
- The electromagnetic wave theory explained the phenomena of interference, diffraction and polarisation. The quantum theory provided both qualitative and quantitative explanation of spectra of black body radiation, scattering of X-rays and Compton effect, emission of line spectra, photoelectric effect, etc.
- These two theories coupled together established wave-particle duality of radiation. But inspite of the success of this duality of radiation, the two fundamental postulates of Bohr's theory of atomic structure remained unexplained for a long time.
- In the mean time Einstein's mass-energy relation, $E = mc^2$, had been verified, establishing that radiation and mass are mutually convertible.
- On this background, in 1924 De Broglie extended the idea of dual nature of radiation to matter and proposed that matter possesses particle as well as wave characteristics. ***He believed that motion of electron within an atom is guided by a peculiar kind of waves called 'Pilot waves'.***
- While adverting the concept of matter waves, De Broglie was guided by wave - particle duality of radiation and the way in which nature manifests herself.
- **De Broglie put forth following arguments:**
  - ➢ Nature manifests herself mainly as matter and radiation, and nature loves symmetry. So wave

particle duality of radiation points to similar duality of matter.

- The principle of least action in mechanics and the principle of least time in optics imply similar conditions. This close analogy of these two principles from two different branches of physics, shows the probability of the behaviour of matter as a wave like entity under suitable circumstances. This close parallelism between mechanics and optics also indicates similarity between matter and radiation, i.e. if radiation has dual nature then matter must also have similar wave-particle duality.
- Bohr orbits are of definite size and are selected by quantum rules. The radii of these quantum orbits are proportional to the square of integral numbers $\left[r_n = \frac{h^2 \epsilon_0}{\pi m e^2} \cdot n^2\right]$ and electrons stay in these orbits for a considerable time without radiating energy. Thus, the stable non-radiating orbits of electrons in an atom are governed by integer rules. Now the only phenomena involving integers in physics are those of interference and modes of vibration of a stretched string, and both of them imply wave motion. So there must be a latent relationship between Bohr orbits and integers associated with them. This latent relationship can be understood by considering the length of a Bohr orbit as consisting of integral number of wavelengths (equal to the principal quantum number of the orbit) of the waves associated with electrons moving in that orbit (See Fig. 3.1).

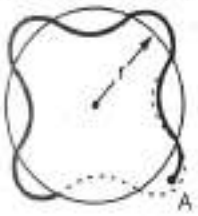


Fig. 3.1: If the orbit length $2\pi r$ is an integral number of wavelengths, the wave will reinforce itself when it returns to the starting point A. In the case shown, $2\pi r = 4\lambda$.

- Now, the quantum condition for Bohr orbits is $mvr = n \frac{h}{2\pi}$. So the principal quantum number $n = \frac{2\pi mvr}{h}$, and '***De Broglie wavelength***' can be obtained by dividing the circumference $2\pi r$ of the nth permissible orbit by its principal quantum number n.

So, $$\lambda = \frac{2\pi r}{n}$$

$$\lambda = \frac{2\pi r}{(2\pi mvr/h)}$$

$$\lambda = \frac{h}{mv}$$

- This expression for De Broglie wavelength, agrees with the one he postulated, namely $\lambda = \frac{\text{Planck's constant}}{\text{momentum of the particle}}$, on the basis of his hypothesis of matter waves.
- According to wave mechanics, electrons move around the nucleus as wave-packets which are formed in a somewhat similar manner as standing waves are formed.

3.2.1 Wavelength of Matter Waves or De Broglie Wavelength (Nov. 13)

- By De Broglie's hypothesis, a moving material particle is associated with a wave whose wavelength is called ***De Broglie wavelength*** and it is given by the ratio of Planck's constant to the momentum of the particle.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where p is the momentum of the particle, m its mass, and v the velocity.

- We can arrive at this expression for De Broglie wavelength, (1) by analogy with radiation and (2) by relativistic considerations.

3.2.2 De Broglie Wavelength by Analogy with Radiation

- By Einstein's mass-energy relation, we have

$$E = mc^2 \quad \text{... (1)}$$

where E is the energy equivalent of mass m, and c is the velocity of light.

- By Planck's quantum theory of radiation, the energy of a photon is given by

$$E = h\upsilon \quad \text{... (2)}$$

where h is Planck's constant and υ is the frequency of oscillations.

From equations (1) and (2), we get

$$h\upsilon = mc^2$$

But $$\upsilon = \frac{c}{\lambda},$$

(Since velocity = frequency × wavelength)

$$\therefore \quad h \cdot \frac{c}{\lambda} = mc^2$$

or $$\lambda = \frac{h}{mc} = \frac{h}{p} \quad \text{... (3)}$$

where p = mc is the momentum associated with the photon and λ is its wavelength.

3.2.3 De Broglie Wavelength in terms of K.E. of the Particle (Dec. 12, 14, May 14)

- The momentum of a particle of mass m moving with velocity v is given by p = mv; and the De Broglie wavelength associated with the particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{... (1)}$$

- The K.E. of the particle is given by

$$E = \frac{1}{2}mv^2$$

$$E = \frac{m^2 v^2}{2m} = \frac{p^2}{2m}$$

$$\therefore \quad p = \sqrt{2mE} \quad \text{... (2)}$$

From equations (1) and (2), we have

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \quad \text{... (3)}$$

In equation (3), De Broglie wavelength λ of a moving particle has been expressed in terms of K.E. of the particle.

3.2.4 De Broglie Wavelength for an Electron in Terms of Potential Difference

If an electron acquires velocity v on accelerating it through a potential difference of V volts, then the work done on the electron is eV, e being charge of the electron. This work is converted into kinetic energy of the electron,

$$\text{i.e.} \quad E = \frac{1}{2}mv^2 = eV \quad \text{... (1)}$$

If e is in coulombs, m in kg, V in volts, then velocity v will be in m/s.

$$\text{But} \quad \lambda = \frac{h}{\sqrt{2mE}}$$

From equation (1),

$$\lambda = \frac{h}{\sqrt{2meV}} \quad \text{... (2)}$$

Ignoring relativistic correction, we can take $m = m_o$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2m_o eV}} \quad \text{... (3)}$$

Now, h, m_o and e are universal constants with values

$$h = 6.625 \times 10^{-34} \text{ J-s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_o = 9.1 \times 10^{-31} \text{ kg.}$$

Substituting these values in equation (3), we get

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} \text{ metres}$$

$$= \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ metres}$$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{V}} \text{ A}^{\circ} \quad \text{... (4)}$$

3.3 CONCEPT OF PHASE VELOCITY AND GROUP VELOCITY (Dec. 12, Nov. 15, May 16)

3.3.1 Phase Velocity (May 13, 14, 15)

- As we know, with every material particle, a wave called De Broglie wave is associated, which becomes important at microscopic levels.
- The speed with which a crest or trough of a wave travels, is called the ***'phase velocity'*** of De Broglie waves. It is also called as ***'wave velocity'*** (u). For wave velocity, the phase remains constant.
- For a particle of mass m, moving with a velocity v, the De Broglie wavelength λ is given by

$$\lambda = \frac{h}{mv} \quad \text{... (1)}$$

and energy, $E = h\upsilon = mc^2$

$$\therefore \quad \upsilon = \frac{mc^2}{h} \quad \text{... (2)}$$

- The equation of motion of a plane wave of frequency υ and wavelength λ moving in x direction is,

$$y = a \sin(\omega t - kx)$$

where $\omega = 2\pi\upsilon$, the angular frequency ... (3)

and $k = \frac{2\pi}{\lambda}$, the propagation constant of the wave.

- For wave velocity, the phase remains constant i.e.

$$\omega t - kx = \text{constant} \quad \text{... (4)}$$

- The wave velocity,

$$u = \frac{dx}{dt} = \frac{\omega}{k} \text{ (by differentiating equation 4)}$$

$$\therefore \quad u = 2\pi\upsilon \times \frac{\lambda}{2\pi} = \upsilon\lambda \quad \text{... (5)}$$

where υ is the frequency of the matter waves.

Substitution of equations (1) and (2) in (5) gives

$$u = \frac{mc^2}{h} \times \frac{h}{mv}$$

$$u = \frac{c^2}{v}$$

i.e.

Wave velocity, $u = \frac{c^2}{v}$ ... (6)

- For any particle, as v is always less than c, the speed of light, we see that the phase velocity u comes to be greater than c, which is an unexpected result. The difficulty raised above can be overcome by understanding the wave group and group velocity v_g.

3.3.2 Group Velocity (May 13, Dec. 14)

- A material particle can be represented by a wave packet or the group of waves and the velocity of wave group is called as ***'group velocity'***. The individual waves travel inside the group with their phase velocities or wave velocities.
- Wave group (wave packet) can be obtained by interference of many waves of different frequencies and amplitudes so that resultant has a high value of amplitude near the vicinity of the particle and gets attenuated outside this region as shown in Fig. 3.2.
- We can say that the particle is somewhere in the wave packet and that wave packet moves with velocity of the particle carrying that particle in it. It can be shown that the group velocity v_g is the same as that of the particle velocity v.

Fig. 3.2: Group of waves

- Consider the superposition of two waves of equal amplitude a, different angular frequencies ω_1 and ω_2, and propagation constants k_1 and k_2.

Let $y_1 = a \sin(\omega_1 t - k_1 x)$

and $y_2 = a \sin(\omega_2 t - k_2 x)$

- The resultant will be

$$y = y_1 + y_2$$

$$y = a[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

$$y = 2a \cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right] \sin\left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2}\right]$$

$$\left[\because \sin A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}\right]$$

The resultant wave has the angular frequency $\frac{(\omega_1 + \omega_2)}{2}$ and its amplitude is

$$A = 2a \cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right]$$

- Thus, the amplitude of the wave group is modulated both in space and time. The velocity with which this envelope moves is given by

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{d\omega}{dk} \quad \text{... (1)}$$

This is the general formula for a group velocity.

3.4 PROPERTIES OF MATTER WAVES (Nov. 13)

Matter waves are generated by a moving matter particle. If a particle of mass m moves with velocity v, then the wavelength of matter waves associated with it is given by $\lambda = \frac{h}{mv}$ and these waves travel with velocity $u = \frac{c^2}{v}$.

From this, properties of matter waves can be stated as:

- Lighter the particle, greater would be the wavelength of the matter waves associated with it. $\left(\lambda \propto \frac{1}{m}, \text{ for v constant}\right)$.
- Smaller the velocity of the particle, greater would be the wavelength of the matter waves. $\left(\lambda \propto \frac{1}{v}, \text{ for m constant}\right)$.
- For $v = \infty$, λ becomes zero and for $v = 0$, λ becomes infinity i.e. the wave becomes indeterminate when $v = 0$. This simply means that matter waves are produced by particles moving with finite velocities.
- Matter waves are different from electromagnetic waves, because matter waves can be produced by a moving particle which may be charged or uncharged, whereas electromagnetic waves are produced only by a moving charged particle. As the De Broglie wavelength $\lambda = \frac{h}{mv}$ is independent of charge, it is evident that matter waves are not electromagnetic waves.
- The velocity of matter waves depends on the velocity of the particle generating them $\left(u = \frac{c^2}{v}\right)$ and it is not constant.

- Matter waves travel faster than light, because the particle velocity v cannot exceed the velocity of light c. So the velocity of matter waves $u = \frac{c^2}{v}$ is greater than c.
- A wave is spread out in space and it cannot be localised at any point. So the wave nature of matter introduces a certain uncertainty in the position of the particle.

SOLVED PROBLEMS

Problem 3.1: *What is the De Broglie wavelength of an electron when accelerated through a p.d. of 10,000 volts ?*

Data: $V = 10{,}000$ volts

Formula: $\lambda = \frac{12.27}{\sqrt{V}} A^\circ$

Solution: $\lambda = \frac{12.27}{\sqrt{10{,}000}}$

$= \boxed{0.1227\ A^\circ}$

Problem 3.2: *Compute the wavelength of the De Broglie waves associated with a proton moving with 5 % of the velocity of light. Proton has 1836 times the mass of one electron.*

Data: $m = 1836 \times 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ Joule-sec.,

$v = \frac{5}{100} \times 3 \times 10^8$ m/sec.

Formula: $\lambda = h/mv$

Solution: $\lambda = \frac{6.63 \times 10^{-34}}{1836 \times 9.1 \times 10^{-31} \times \frac{5}{100} \times 3 \times 10^8}$

$= \boxed{2.65 \times 10^{-14}\ m}$

Problem 3.3: *Compute the kinetic energy and velocity of an electron in terms of those of a neutron, when their De Broglie wavelengths are equal to 1 A°. Take neutron mass as 1835 times the mass of electron.*

Data: $\lambda_n = \lambda_e = 1\ A^\circ$; $m_n = 1835\ m_e$

Formula: $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2\,mE}}$

Solution: For electron,

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2\,m_e E_e}}$$

For neutron, $\lambda_n = \frac{h}{m_n v_n} = \frac{h}{\sqrt{2\,m_n E_n}}$

Taking ratios $\frac{\lambda_n}{\lambda_e} = \frac{m_n v_n}{m_e v_e} = 1$ (given)

We have, $\frac{v_n}{v_e} = \frac{m_e}{m_n} = \frac{1}{1835}$

or $v_e = 1835\, v_n$

and $\sqrt{\frac{2\,m_n E_n}{2\,m_e E_e}} = 1$

$\therefore \quad \frac{E_n}{E_e} = \frac{m_e}{m_n} = \frac{1}{1835}$

or $\boxed{E_e = 1835\, E_n}$

Problem 3.4: *De Broglie wavelength of electrons in a monoenergetic beam is 7.2×10^{-11} metres. Calculate the momentum and energy of electrons in the beam in electron volts.*

Data: $\lambda = 7.2 \times 10^{-11}$ m;

$h = 6.6 \times 10^{-34}$ J - s

Formulae: $p = \frac{h}{\lambda}$, $E = \frac{p^2}{2\,m}$

Solution: $p = \frac{6.6 \times 10^{-34}}{7.2 \times 10^{-11}}$

$= \boxed{0.916 \times 10^{-23}\ kg - m/sec}$

$E = \frac{(0.916 \times 10^{-23})^2}{2 \times 9.1 \times 10^{-31}}$ J

$= 0.0461 \times 10^{-15}$ J

$= \frac{0.0461 \times 10^{-15}}{1.6 \times 10^{-19}}$ eV

$= 0.0289 \times 10^4$ eV $= \boxed{289\ eV}$

Problem 3.5: *What is the De Broglie wavelength associated with a 5000 kg car having a constant speed of 20 m/sec. ?*

Data: $m = 5000$ kg, $h = 6.6 \times 10^{-34}$ J - sec, $v = 20$ m /sec.

Formula: $\lambda = \frac{h}{mv}$

Solution: $\lambda = \frac{6.6 \times 10^{-34}}{5000 \times 20} = \boxed{6.6 \times 10^{-39}\ m}$

As can be seen, the De Broglie wave associated with a macroscopic body is too small to be significant.

Problem 3.6: *The spacing between the atoms of a certain crystal is 1.2 A°. At what angle will the first-order Bragg reflection occur for thermal neutrons ?*

Kinetic energy = 0.025 eV, Mass of neutron is 1.67×10^{-27} kg

Data: $d = 1.2\ A°$, $n = 1$, $E = 0.025\ eV = 0.025 \times 1.6 \times 10^{-19}\ J$

Formulae: $\lambda = \dfrac{h}{\sqrt{2mE}}$ and $2d \sin\theta = n\lambda$

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 0.025 \times 1.6 \times 10^{-19}}}$$

$$= 18.057 \times 10^{-11}\ m = 1.8057\ A°$$

$$\sin\theta = \frac{\lambda}{2d} = \frac{1.8057 \times 10^{-11}}{2 \times 1.2 \times 10^{-11}} = 0.7524$$

$$\theta = \sin^{-1}(0.7524) = \boxed{48.79° = 48°48'}$$

Problem 3.7: *Find the K.E. of a neutron which has a wavelength of 3 A°. At what angle will such a neutron undergo first-order Bragg reflection from a calcite crystal for which the grating space is 3.036 A° ? Mass of neutron is 1.67 $\times 10^{-27}$ kg.*

Data: $\lambda = 3\ A° = 3 \times 10^{-10}$ m, $n = 1$, $d = 3.036\ A° = 3.036 \times 10^{-10}$ m, $m = 1.67 \times 10^{-27}$ kg

Formulae: (i) $\lambda = \dfrac{h}{\sqrt{2mE}}$

(ii) $2d \sin\theta = n\lambda$

Solution:

(i)
$$E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (3 \times 10^{-10})^2}$$

$$= \boxed{1.449 \times 10^{-21}\ J}$$

(ii)
$$\sin\theta = \frac{\lambda}{2d} = \frac{3 \times 10^{-10}}{2 \times 3.036 \times 10^{-10}} = 0.49407$$

$$\theta = \sin^{-1}(0.49407)$$

$$= \boxed{29.6085° = 29°\ 36'}$$

Problem 3.8: *An electron initially at rest is accelerated through a p.d. of 5000 V. Compute (i) the momentum, (ii) the De Broglie wavelength and (iii) the wave number of the electron. Also calculate the Bragg angle for its first-order reflection from (111) planes of nickel which are 2.04 A° apart.*

Data: $V = 5000$ V, $n = 1$, $d = 2.04\ A° = 2.04 \times 10^{-11}$ m

Formulae: (i) $p = \sqrt{2\ meV}$, (ii) $\lambda = \dfrac{12.27}{\sqrt{V}}\ A°$,

(iii) $\bar{\upsilon} = \dfrac{1}{\lambda}$, (iv) $2d \sin\theta = n\lambda$

Solution: (i)
$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 5000}$$

$$= \boxed{3.815 \times 10^{-23}\ kg\text{ - }m/sec}$$

(ii)
$$\lambda = \frac{12.27}{\sqrt{5000}}\ A°$$

$$= 0.17352\ A° = \boxed{0.17352 \times 10^{-10}\ m}$$

(iii) Wave number, $\bar{\upsilon} = \dfrac{1}{\lambda} = \boxed{5.76 \times 10^{10}\ m}$

(iv)
$$2d \sin\theta = n\lambda$$

$$\sin\theta = \frac{\lambda}{2d} = \frac{0.17352 \times 10^{-10}}{2 \times 2.04 \times 10^{-10}} = 0.0425$$

$$\theta = \sin^{-1}(0.0425) = \boxed{2.43° = 2°26'}$$

Problem 3.9: *Electrons from a heated filament accelerated by a p.d. of 10 kV are passed through a thin film of a metal for which the atomic spacing is 0.55 A. What is the angle of deviation of the first-order maximum ?* **(Dec. 08, 09)**

Solution: We have the relation

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}\ A°$$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{10 \times 10^3}} = \boxed{0.122\ A°}$$

Now, from Bragg's law,

$$2d \sin\theta = n\lambda$$

Here, $n = 1 \sin\theta = \dfrac{n\lambda}{2d}$

$$= \frac{1 \times 0.122}{2 \times 0.55} = 0.111$$

$$\boxed{\theta = 6°\ 22'}$$

Hence, required angle of deviation for the first-order maximum is 6° 22'.

Problem 3.10: *An electron is accelerated through a potential difference of 10 kV. Calculate the De-Broglie wavelength and momentum of the electron.* **(04) (Dec. 04)**

Data: $V = 10{,}000$ volts

Formulae: (i) $\lambda = \dfrac{12.27}{\sqrt{V}}\ A°$,

(ii) $p = \dfrac{h}{\lambda}$

Solution: (i) $\lambda = \frac{12.27}{\sqrt{10000}}$

$$\boxed{\lambda = 0.1227\ A^\circ}$$

(ii) $p = \frac{6.6 \times 10^{-34}}{0.1227 \times 10^{-10}}$

$$\boxed{p = 5.378 \times 10^{-23}\ \text{kg-m/sec}}$$

Problem 3.11: *Proton and alpha particles are accelerated through same potential difference. Compute the ratio of their De-Broglie wavelengths.* **(04) (May 05)**

Data: $m = 4\,m_p$, $e_\alpha = 2\,e_p$

Formula: $\lambda = \frac{h}{\sqrt{2meV}}$

Solution: For proton,

$$\lambda_p = \frac{h}{2m_p e_p V}$$

For alpha particle,

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha e_\alpha V}}$$

But, $m_\alpha = 4m_p$ and $e_\alpha = 2e_p$

$$\therefore \quad \lambda_\alpha = \frac{h}{\sqrt{16m_p e_p V}}$$

$$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \frac{h}{\sqrt{16m_p e_p V}} \cdot \frac{\sqrt{2m_p e_p V}}{h} = \boxed{\frac{1}{\sqrt{8}}}$$

Problem 3.12: *Find the De-Broglie wavelength of an electron accelerated through a potential difference of 100 volts.* **(02) (Dec. 05)**

Data: $V = 100$ volts

Formula: $\lambda = \frac{12.27}{\sqrt{V}}\ A^\circ$

Solution: $\lambda = \frac{12.27}{\sqrt{100}}$

$$\boxed{\lambda = 1.227\ A^\circ}$$

Problem 3.13: *What potential difference must be applied to an electron microscope to obtain electrons of wavelength 0.3 A° ?* **(04) (May 06)**

Data: $\lambda = 0.3\ A^\circ$

Formula: $\lambda = \frac{12.27}{\sqrt{V}}\ A^\circ$

Solution: $0.3 = \frac{12.27}{\sqrt{V}}$

$$\sqrt{V} = 40.9$$

$$\therefore \quad \boxed{V = 1672.8\ \text{volts}}$$

Problem 3.14: *Calculate the velocity and De-Broglie wavelength of an α-particle of energy 1 keV.* **(04) (May 07)**

Data: $E = 1$ keV, $m_\alpha = 4\,m_p$

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 1 \times 10^3 \times 1.6 \times 10^{-19}}}$

$$\lambda = 4.514 \times 10^{-13}\ m$$

$$\boxed{\lambda = 0.00451\ A^\circ}$$

Problem 3.15: *Which has a shorter wavelength of 1 eV photon or 1 eV electron ? Calculate the value and explain.* **(04) (May 07)**

Data: $E_p = 1$ eV

$E_e = 1$ eV

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda_p = \frac{h}{\sqrt{2m_p E_p}}$

$$\lambda_p = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1 \times 1.6 \times 10^{-19}}}$$

$$\lambda_p = 2.855 \times 10^{-11}\ m = \boxed{0.2855\ A^\circ}$$

$$\lambda_e = \frac{h}{\sqrt{2m_e E_e}}$$

$$\lambda_e = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1 \times 1.6 \times 10^{-19}}}$$

$$\lambda_e = 1.223 \times 10^{-9}\ m = \boxed{12.23\ A^\circ}$$

$$\lambda_e > \lambda_p$$

Because lighter the particle, greater will be the wavelength of the matter waves $\left(\lambda \propto \frac{1}{m}\right)$.

Problem 3.16: *An electron has kinetic energy equal to its rest mass energy. Calculate De-Broglie's wavelength associated with it.* **(04) (Nov. 07)**

Data: $E = m$

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{h}{\sqrt{2m^2}}$

$$\lambda = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31}\sqrt{2}}$$

$$\boxed{\lambda = 5.129 \times 10^{-4} \text{ m}}$$

Problem 3.17: *Find the De Broglie's wavelength associated with monoenergetic electron beam having momentum 10^{-23} kg m/s.* **(04) (May 09)**

Data: $p = 10^{-23}$ kg m/s

Formula: $\lambda = \frac{h}{p}$

Solution: $\lambda = \frac{h}{p}$

$$\lambda = \frac{6.63 \times 10^{-34}}{10^{-23}}$$

$$\lambda = 3.63 \times 10^{-11} \text{ m}$$

or $\boxed{\lambda = 0.663 \text{ A}^\circ}$

Problem 3.18: *Find the energy of neutron in unit of electron volt whose De-Broglie wavelength is 1 A°.*

(Given: $M_n = 1.674 \times 10^{-27}$ kg). **(04) (Dec. 09)**

Data: $M_n = 1.674 \times 10^{-27}$, $\lambda = 1$ A°

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $E = \frac{h^2}{2m\lambda^2}$

$$E = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$E = 1.316 \times 10^{-20} \text{ J}$$

$$\boxed{E = 8.225 \times 10^{-2} \text{ eV}}$$

Problem 3.19: *Calculate the De-Broglie wavelength of 10 keV protons in eV.* **(04) (May 10)**

Data: $E = 10$ keV, $m = 1.67 \times 10^{-27}$ kg

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$

$$\lambda = 2.868 \times 10^{-13} \text{ m}$$

$$\boxed{\lambda = 2.868 \times 10^{-3} \text{ A}^\circ}$$

Problem 3.20: *Calculate the wavelength of a photon and an electron both having an energy 1.0 eV. (**Given:** Planck's constant $h = 6.63 \times 10^{-34}$ J-s, Mass of electron $= 9.1 \times 10^{-31}$ kg).* **(04) (Dec. 11)**

Data: $E = 1.0$ eV

Formulae: (i) For electron, $\lambda = \frac{h}{\sqrt{2mE}}$ (ii) For photon, $E = h\nu$

i.e. $\lambda = \frac{hc}{E}$

Solution:

(i) For electron, $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1 \times 1.6 \times 10^{-19}}}$

$$\lambda = 1.228 \times 10^{-9} \text{ m} = \boxed{12.28 \text{ A}^\circ}$$

(ii) For photon, $\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 1.6 \times 10^{-19}}$

$$\lambda = 1.243 \times 10^{-6} \text{ m}$$

$$= \boxed{12430 \text{ A}^\circ}$$

Problem 3.21: *Calculate the De-Broglie wavelength associated with 1 Mev proton ($m_p = 1.67 \times 10^{-27}$ kg)* **(03) (May 13)**

Data: $E = 1$MeV

$m_p = 1.67 \times 10^{-27}$ kg

Formula: $\lambda = \frac{h}{\sqrt{2m_pE}}$

Solution: $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1 \times 10^6 \times 1.6 \times 10^{-19}}}$

$$\lambda = 2.86 \times 10^{-14} \text{m}$$

$$\lambda = 2.86 \times 10^{-4} \text{A}^\circ$$

Problem 3.22: *The position and momentum of 1 keV electron are simultaneously measured. If its position is located with 1 A°. Find the percentage of uncertainty in its momentum.* **(03) (Dec. 12)**

(Given: $h = 6.64 \times 10^{-34}$ J-sec., $m = 9.1 \times 10^{-31}$ kg)

Data: $h = 6.64 \times 10^{-34}$ J-sec., $m = 9.1 \times 10^{-31}$ kg, $\Delta x = 1$A° $= 1 \times 10^{-10}$ m.

Formulae: $\lambda = \frac{12.27}{\sqrt{V}}$ A°, $p = \frac{h}{\lambda}$ and $\Delta p_x = \frac{h}{\Delta x}$

$\therefore$ Wavelength, $\lambda = \frac{12.27}{\sqrt{V}}$ A°

$$= \frac{12.27}{\sqrt{10^3}} = 0.388 \text{ A}^\circ$$

For electron, Momentum,

$$p = \frac{h}{\lambda} = \frac{6.64 \times 10^{-34}}{0.388 \times 10^{-10}}$$

$$= 17.11 \times 10^{-24}$$

$$= 1.71 \times 10^{-23} \text{ kg-m/sec.}$$

Now, $\Delta p_x = \frac{h}{\Delta x}$

$$= \frac{6.64 \times 10^{-34}}{1 \times 10^{-10}}$$

$$= 6.64 \times 10^{-24} \text{ kg-m/sec.}$$

$$= 0.664 \times 10^{-23} \text{ kg-m/sec.}$$

Per cent change in momentum

$$= \frac{1.71 \times 10^{-23} - 0.664 \times 10^{-23}}{1.71 \times 10^{-23}} \times 100$$

$$= \frac{1.71 - 0.664}{1.71} \times 100$$

$$= 61\%$$

∴ Percentage of uncertainty in momentum of electron = 39%.

Problem 3.23: *What accelerating potential would be required for a proton with zero initial velocity to acquire a velocity corresponding to its de-Broglie wavelength of 10^{-10} m. [Given: $m_p = 1.67 \times 10^{-27}$ kg]* **(03) (May 14)**

Data: $m = 1.67 \times 10^{-27}$ kg, $\lambda = 10^{-10}$ m

Formula: $\lambda = \frac{h}{\sqrt{2mev}}$

Solution: $V = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times (10^{-10})^2}$

$V = 0.082$ V

Problem 3.24. : *Calculate the de Broglie wavelength of proton when it is accelerated by potential difference of 10 kV.* **(3) (May 15)**

Data: $V = 10$ kV, $m_p = 1.67 \times 10^{-27}$ kg.

Formula: $\lambda = \frac{h}{\sqrt{2meV}}$

Solution: $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times 10000}}$

$= 2.868 \times 10^{-13}$ m $= 2.868 \times 10^{-3}$ A°

Problem 3.25. : *Calculate the de Broglie wavelength of electron of energy 1 keV.* **(3) (Nov. 15)**

Data: E = 1 KeV

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^3 \times 1.6 \times 10^{-19}}}$

$= 3.87 \times 10^{-11}$ m $= 0.387$ A°

Problem 3.26. : *Calculate the de Broglie wavelength for a proton moving with velocity 1 percent of velocity of light.* **(3) (May 16)**

Data : $m_p = 1.673 \times 10^{-27}$ kg.

$V = \frac{1}{100} \times 3 \times 10^8$ m/sec.

Formula : $\lambda = \frac{h}{mv}$

Solution : $\lambda = \frac{6.63 \times 10^{-34}}{1.673 \times 10^{-27} \times \frac{1}{100} \times 3 \times 10^8}$

$= 1.32 \times 10^{-13}$ m

Problem 3.27: *Calculate the energy (in eV) with which a proton has to acquire de-Broglie wavelength of 0.1Å* **(3) (May 17)**

Data: $\lambda = 0.1$Å

$m_p = 1.67 \times 10^{-27}$ kg

Formula: $\lambda = \frac{h}{\sqrt{2m_pE}}$

Solution: $E = h^2/2m\lambda^2 = 1.3160 \times 10^{-18}$ J $= 8.225$eV

Problem 3.28 : *Calculate the de-Broglie wavelength for a 10 keV proton. ($m_p = 1.67 \times 10^{17}$ kg, $h = 6.63 \times 10^{-14}$ J.S, $e = 1.6 \times 10^{-14}$C)* **(3) (Nov. 17)**

Given : Data : E = 10 keV, $m = 1.67 \times 10^{-27}$ kg

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$

$\lambda = 2.868 \times 10^{-13}$ m

$\lambda = 2.868 \times 10^{-3}$ A°

Problem 3.29 : *Calculate the first energy eigen value of electron in eV trapped in rigid box of length 1 AU. ($e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J-sec, $m_e = 9.1 \times 10^{-31}$kg)* **(3) (May 18)**

Solution : Given : V = 10,000 volts

$$\lambda = \frac{h}{\sqrt{2\ MeV}} = \frac{12.27}{\sqrt{V}}\ A^{\circ}$$

$$\lambda = \frac{12.27}{\sqrt{10,000}} = 0.1227\ A^{\circ}$$

Problem 3.30 : *A proton and an alpha particle are accelerated by the same potential difference. Find the ratio of their de-Broglie wavelength (m_{α} = 6.68 0^{-27} charge on alpha particle = 2 × charge on electron, $m_p = 1.673 \times 10^{-27}$ kg)*

(3) (May 18)

Solution : Given : $M_{\alpha} = 4\ M_p$, $e_{\alpha} = 2\ e_p$

$$\lambda = \frac{h}{\sqrt{2\ mev}}$$

For proton,

$$\lambda_p = \frac{h}{2m_p e_p V}$$

$$\lambda_{\alpha} = \frac{h}{\sqrt{2m_{\alpha} e_{\alpha} V}}$$

but, $m_{\alpha} = 4\ m_p$ and $e_{\alpha} = 2e_p$

$$\lambda_{\alpha} = \frac{h}{\sqrt{16 m_p e_p V}}$$

$$\frac{\lambda_{\alpha}}{\lambda_p} = \frac{h}{\sqrt{16 m_p e_p V}} \cdot \frac{\sqrt{2 m_p e_p V}}{h}$$

$$= \frac{h}{\sqrt{16 \times 1.673 \times 10^{-27} \times e_p V}} \cdot \frac{\sqrt{2 \times 1.673 \times 10^{-27} \times e_p V}}{h}$$

$$= \frac{1}{\sqrt{8.014}} = \frac{1}{\sqrt{8}}$$

Problem 3.31 : *In a TV set electrons are accelerated by a potential difference of 10 KV. Calculate the de-Broglie wavelength matter waves associated with these electrons.*

($m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J.s, $e = 1.6 \times 10^{-19}$ C)

(3) (Nov. 18)

Data: V = 10,000 volts

Formulae: (i) $\lambda = \frac{12.27}{\sqrt{V}}\ A^{\circ}$, (ii) $p = \frac{h}{\lambda}$

Solution: (i) $\lambda = \frac{12.27}{\sqrt{10000}}$

$$\boxed{\lambda = 0.1227\ A^{\circ}}$$

(ii) $p = \frac{6.6 \times 10^{-34}}{0.1227 \times 10^{-10}}$

$$\boxed{p = 5.378 \times 10^{-23}\ kg\text{-}m/sec}$$

Problem 3.32 : *An electron initially at rest is accelerated through a potential difference of 3000 V. Calculate for the electron wave the following parameters.* **(3) (May 19)**

(i) The De-Broglie wavelength and

(ii) The momentum

(h : 6.63 × 10^{-34} Js)

Data : 12.28 V 3000 V

Formula : (i) k = T , **(ii)** P = q2mE V

Solution : (i) X ~ = 0.223 A° 3000

(ii) P = ~2meVq² × 9.1 × 10^{31} × 1.6 – 10^{19} 3000

= 2.959 × 10^{-23} Kg.m/s

Problem 3.33: *Calculate the wavelength of an electron of energy 291 eV.*

Data: E = 291 eV = 291 × 1.6 × 10^{-19} J

Formula: $\lambda = \frac{h}{\sqrt{2\ m\ E}}$

Solution: $\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 291 \times 1.6 \times 10^{-19}}}$

$$= \boxed{0.717\ A^{\circ}}$$

Problem 3.34: *Find the kinetic energy of a neutron in eV, whose De-Broglie wavelength is 1 A° (M_n = 1.67 × 10^{-27} kg).*

Solution: Given: λ_n = 1 A° = 1 × 10^{-10} m

M_n = 1.67 × 10^{-27} kg

h = 6.63 × 10^{-34} J.sec

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore\ E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$\therefore\ E = 1.3160 \times 10^{-20}\ J$$

$$\boxed{E = 8.225 \times 10^{-2}\ eV}$$

Problem 3.35: *Find the De-Broglie wavelength of an electron accelerated through a potential difference of 100 volts.*

Data: V = 100 volts

Formula: $\lambda = \frac{12.27}{\sqrt{V}}\ A^{\circ}$

Solution: $\lambda = \frac{12.27}{\sqrt{100}} = \boxed{1.227\ A^{\circ}}$

Problem 3.36 : *An electron beam is accelerated from rest through a potential difference of 200 V. Calculate the associated wavelength.* **(3) (Dec. 16)**

Data : $V = 2000\ V$

Formula : $\lambda = \frac{12.28}{\sqrt{V}}\ A^\circ$

Solution : $\lambda = \frac{12.28}{\sqrt{2000}} = 0.2745\ A^\circ$

Problem 3.37: *Find De Broglie wavelength of 10 keV electrons.*

Data: $E = 10\ keV = 10 \times 10^3 \times 1.6 \times 10^{-19}\ J$

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^4 \times 1.6 \times 10^{-19}}}$

$= 1.22 \times 10^{-11}\ m = \boxed{0.122\ A^\circ}$

Problem 3.38: *Proton and deuteron are accelerated by the same potential. Compare their De Broglie wavelengths. Assume mass of deuterium to be twice the mass of a proton.*

Data: $m_d = 2\ m_p$

Formula: $\lambda = \frac{h}{\sqrt{2\ meV}}$

Solution: For proton,

$$\lambda_p = \frac{h}{\sqrt{2\ m_p\ eV}}$$

For deuteron, $\lambda_d = \frac{h}{\sqrt{2\ m_d eV}} = \frac{h}{\sqrt{4\ m_p eV}}$

$$\frac{\lambda_p}{\lambda_d} = \frac{h/\sqrt{2\ m_p\ eV}}{h/\sqrt{4\ m_p\ eV}} = \sqrt{2}$$

$\therefore \boxed{\lambda_p : \lambda_d = \sqrt{2} : 1}$

3.5 HEISENBERG'S UNCERTAINTY PRINCIPLE

(May 13, 14, 15, 16 Nov. 13, 15)

- One of the tacit assumptions of classical physics, that the position of a mechanical system can be uniquely determined without disturbing its motion, is valid only for the motion of a body of ordinary size, like a cricket ball.
- But if one is considering the motion of an atomic particle, like an electron, a certain uncertainty is unavoidably introduced into the experimental measurement of its position and momentum.
- This uncertainty is not due to the imperfection of the measuring instruments but is something inherent in the nature of a moving body.
- The fact that a moving body must be regarded as a De Broglie wave group (packet) rather than as a localised entity suggests that, there is a fundamental limit to the accuracy with which we can measure its particle properties.
- A De Broglie wave group is shown in Fig. 3.3 (a). The particle may be anywhere within the group. For a very narrow wave group, as in Fig. 3.3 (b), the position of the particle can be readily found, but the wavelength λ, and hence the momentum $p = \frac{h}{\lambda}$, is impossible to establish.
- For a wide wave group, as in Fig. 3.3 (c), the wavelength and hence momentum estimate is satisfactory, but then the location of the position of the particle becomes uncertain.

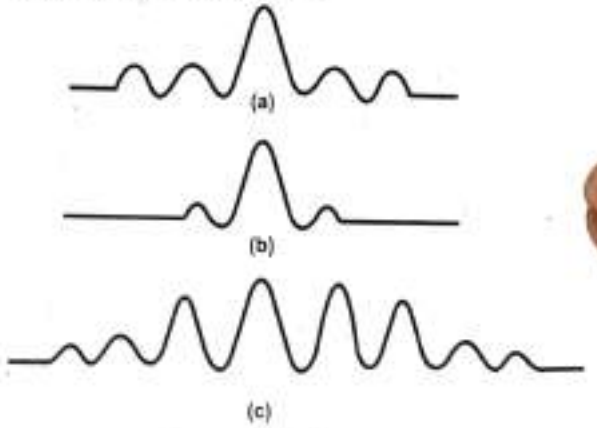


Fig. 3.3

- The answer to this question was given by Heisenberg in 1927, when he put forth the uncertainty principle.

Statement:

- Heisenberg's uncertainty principle states that, it is impossible to determine accurately and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables, like position x and momentum p; or energy E and time t, are called canonically conjugate variables.
- To examine the uncertainty principle, consider an electron of mass m associated with matter waves of wavelength λ. This electron can be found somewhere within this wave and therefore, the uncertainty in its position measurement Δx is equal to its wavelength λ.

$\therefore \quad \Delta x = \lambda \quad \ldots (1)$

- To observe the electron we have to illuminate it with light, say of wavelength λ, as in Fig. 3.4. In this process, photons of light strike the electron and bounce off it.

Each photon possesses the momentum $\frac{h}{\lambda}$ and when it collides with the electron, original momentum p of the electron is changed.

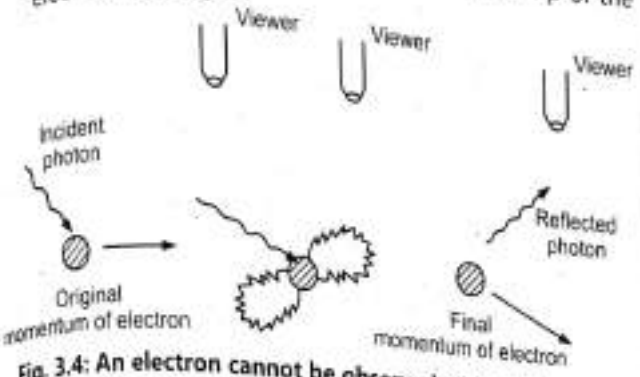


Fig. 3.4: An electron cannot be observed without changing its momentum by an indeterminate amount

- The precise change of the momentum of the electron cannot be predicted, but it is likely to be of the same order of magnitude as the photon momentum $\frac{h}{\lambda}$. Thus, the electron cannot be observed without changing its momentum by an indeterminate amount.
- The act of measurement of its position introduces an uncertainty in its momentum Δp and this uncertainty in the momentum is at least equal to the momentum of incident photon.

$$\therefore \quad \Delta p = \frac{h}{\lambda} \quad \text{... (2)}$$

From equations (1) and (2),

$$\Delta x \cdot \Delta p = \lambda \cdot \frac{h}{\lambda} = h \quad \text{... (3)}$$

- It is clear from equation (3) that if the position of the electron is known exactly at any given instant, i.e. if $\Delta x = 0$, then the momentum becomes indeterminate and vice-versa. Thus, both the position and the momentum cannot be determined accurately and simultaneously.
- The product of uncertainty in position measurement Δx of a body at some instant and the uncertainty in its momentum measurement Δp at the same instant is at best equal to the Planck's constant h (more correctly $\frac{h}{4\pi}$).

i.e. $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

This is Heisenberg's uncertainty principle.

Problem 3.39: *In an experiment, the wavelength of a photon is measured to an accuracy of one part per million. What is the uncertainty Δx in a simultaneous measurement of the position of the photon having a wavelength of 6000 A°?*

Data: $\lambda = 6000\ A^\circ = 6000 \times 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6}$$

Formulae: $\Delta p = \frac{h}{\Delta\lambda}$, $\Delta x \cdot \Delta p = h$

Solution:

$$\Delta p = \frac{6.6 \times 10^{-34}}{6000 \times 10^{-10}}$$

$$= 1.1 \times 10^{-21} \text{ kg-m/sec}$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{1.1 \times 10^{-21}} = \boxed{6 \times 10^{-13} \text{ m}}$$

Problem 3.40: *In order to locate the electron in an atom within a distance of 5×10^{-12} m using electromagnetic waves, the wavelength must be of the same order. Calculate the energy and momentum of the photon. What is the corresponding uncertainty in its momentum ?*

Data: $\lambda = \Delta x = 5 \times 10^{-12}$ m

Formulae: $p = \frac{h}{\lambda}$, $E = \frac{hc}{\lambda}$, $\Delta p_x = \frac{h}{\Delta x}$

Solution:

$$p = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}} = 1.32 \times 10^{-22} \text{ kg-m/sec}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{5 \times 10^{-12}}$$

$$= 3.96 \times 10^{-14} \text{ J}$$

$$\Delta p_x = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}}$$

$$= \boxed{1.32 \times 10^{-22} \text{ kg-m/sec}}$$

Problem 3.41: *Compute the uncertainty in the location of a 2 gram mass moving with a speed of 1.5 m/sec. and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/sec. Given, $\Delta p = 10^{-3}$ p.*

Data: $v_e = 0.5 \times 10^8$ m/sec, $\Delta p = 10^{-3}$ p, m = 2 grams $= 2 \times 10^{-3}$ kg,

v for the body = 1.5 m/sec

Formula: $\Delta x\, \Delta p = h$

Solution:

(i) For the body, $\Delta p = 10^{-3}\, p = 10^{-1}$ (mv)

$$\Delta x \cdot \Delta p = h$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{10^{-3} \times 2 \times 1.5 \times 10^{-3}}$$

$$= \boxed{2.2 \times 10^{-28} \text{ m}}$$

(ii) For the electron,

$$\Delta x = \frac{6.6 \times 10^{-34}}{10^{-3} \times 9.1 \times 10^{-31} \times 0.5 \times 10^{8}}$$

$$= \boxed{1.45 \times 10^{-8} \text{ m}}$$

It can be seen that the uncertainty associated with a microscopic body is very large and therefore, it plays a significant role in measurements.

Problem 3.42: *Assume that the uncertainty in the location of a particle is equal to its De Broglie wavelength. Show that the uncertainty in its velocity is equal to its velocity.*

(03) (Nov. 13)

Data: $\Delta x = \lambda$

Formula: $\Delta x \cdot \Delta p = h$

Solution: $\Delta x \cdot m\,\Delta v_x = h$

$$\Delta v_x = \frac{h}{\Delta x \cdot m} = \frac{h}{m\lambda}$$

Using $\lambda = \dfrac{h}{mv}$

we have $\Delta v_x = \dfrac{h\,m\,v}{mh} = v$

Problem 3.43: *An electron is confined to a box of length 1 A°. Calculate the minimum uncertainty in its velocity, given mass of electron = 9.1×10^{-31} kg, h = 6.6×10^{-34} J-sec.*

Data: $\Delta x = 1\ A^\circ = 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec, $m = 9.1 \times 10^{-31}$ kg

Formula: $\Delta x\,\Delta p_x = h$

Solution: $(\Delta x)_{max}\,(\Delta p_x)_{min} = h$

$$(\Delta x)_{max}\,(\Delta v)_{min} = h$$

$$(\Delta v)_{min} = \frac{h}{m\,\Delta x} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}}$$

$$= \boxed{0.725 \times 10^{7}\ \text{m/sec.}}$$

This is comparable to the speed of the electron and is therefore very large.

Problem 3.44: *What accelerating potential would be required for a proton with zero velocity to acquire a velocity corresponding to De Broglie wavelength of 10^{-14} m ?*

Data: $h = 6.62 \times 10^{-34}$ J-sec, $m = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C

Formula: $V = \dfrac{h^2}{2\,me\lambda^2}$

Solution: $V = \dfrac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times (10^{-14})^2}$

$$= 8.2 \times 10^{6}\ \text{volts} = \boxed{8.2\ \text{M volts}}$$

Problem 3.45: *Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A°.*

Data: $L = 10\ A^\circ = 10 \times 10^{-10}$ m

Formula: $\Delta x \cdot \Delta p_x = h$

Solution: $(\Delta x)_{max}\,(\Delta p_x)_{min} = h$

i.e. $(\Delta x)_{max}\,m(\Delta v_x)_{min} = h$

$$\Delta v_x = \frac{h}{m \cdot (\Delta x)_{max}} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-9}}$$

$$= \boxed{0.725 \times 10^{6}\ \text{m/sec.}}$$

Problem 3.46: *Electrons moving with a speed of 7.3×10^{7} m/sec have a wavelength of 0.1 A°. Calculate the Planck's constant.*

Data: $\lambda = 0.1\ A^\circ = 0.1 \times 10^{-10}$ m

$v = 7.3 \times 10^{7}$ m/sec.

Formula: $h = \lambda \cdot m\,v$

Solution: $h = 0.1 \times 10^{-10} \times 9.1 \times 10^{-31} \times 7.3 \times 10^{7}$

$$= \boxed{6.643 \times 10^{-34}\ \text{J-sec.}}$$

Problem 3.47: *An electron has a speed of 600 m/sec with an accuracy of 0.005 %. Calculate the uncertainty with which we can locate the position of the electron.*

Data: $v = 600$ m/sec

$\Delta v = 0.005\ \%$ of v

$$= \frac{0.005}{100} \times 600\ \text{m/sec}$$

Formula: $\Delta x \cdot \Delta p = h$

$$\Delta x = \frac{h}{\Delta p} = \frac{h}{m\,\Delta v}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31}} \times \frac{0.005}{100} \times 600$$

$$= \boxed{0.024\ \text{m}}$$

3.6 WAVE FUNCTION AND ITS SIGNIFICANCE

(Nov. 13, 15, May 15, 16)

- A wave motion appears in almost all branches of physics. A wave motion is defined as a *periodic disturbance travelling with finite velocity through a medium or space.*
- The simplest form of vibration is simple harmonic motion (S.H.M.) and a particle executing S.H.M. acts as a source which radiates waves.
- The wave motion provides a way for energy and momentum to move from one place to another without material particles making that journey.
- The waves can be classified according to their broad physical properties into mainly three categories:
  - ➢ Electromagnetic waves which need not require any medium to propagate.

- Matter waves which give the probability amplitude of finding a particle at a given position and time.
- **Mechanical Waves:** The mechanical waves are simplest one to understand because they are produced by some sort of mechanical vibrations which we can see.

- When a mechanical wave passes through a medium, the medium particles perform an S.H.M. given by equation

$$y = A \cos \omega t \quad \text{... (1)}$$

where A is the amplitude of the oscillation and $\omega = 2\pi\upsilon$, where υ is the frequency.

- This equation is applicable to all individual particles affected by the wave. Suppose the wave is progressing forward with velocity v. If P is the origin of the wave, then a particle at Q at a distance x from P will receive the wave x/v sec later than P did.

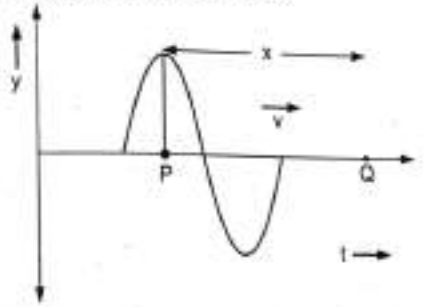


Fig. 3.5: Progressive wave moving with velocity v

- Hence, its displacement at time t and distance x from the origin will be

$$y = A \cos \omega \left(t - \frac{x}{v}\right) \quad \text{... (2)}$$

The wave equation of such a wave is

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{... (3)}$$

The solution of equation (3) is given by

$$y = Ae^{-i\omega(t - x/v)} \quad \text{... (4)}$$

- In a string we can represent the wave disturbance by the transverse displacement of y. Similarly, for light waves the field vectors E and B vary in space and time, for sound waves, pressure P varies in space and time. In the same way, for matter waves, the wave function ψ varies in space and time.
- So ψ in wave mechanics is analogous to electric field E in electromagnetic waves or to pressure P in the sound waves. However, ψ itself unlike E and P has no direct physical significance, but gives a measure of the probability of finding a particle at a particular position. Hence, it is called ***probability amplitude***.
- However, a probability amp is always real and positive, whereas ψ can be positive or negative. Therefore, ψ^2 is taken which is always positive. In general, ψ is complex, therefore, one takes $|\psi|^2$ instead of ψ^2, where $|\psi|^2 = \psi^* \psi$, ψ^* denoting the complex conjugate of ψ. In any case, $\psi^* \psi$ is always real and positive.
- If dv is a volume element located at a point, then the probability of finding the particle in the volume element at time t is proportional to $\psi^*\psi dv$. By analogy with ordinary mass density, the square of the wave function $\psi^*\psi$ is called the ***probability density*** i.e. ***probability per unit volume***.

3.6.1 Physical Significance of the Wave Function ψ

(Dec. 12, Nov. 13, May 15)

- Schroedinger interpreted ψ in terms of charge density. If A is the amplitude of an electromagnetic wave, then the energy per unit volume, i.e. energy density is equal to A^2. Also, the photon energy $h\upsilon$ is constant. So the number of photons per unit volume, i.e. the photon density is equal to $\frac{A^2}{h\upsilon}$, and it is proportional to the amplitude square.
- Similarly, if ψ is the amplitude of matter waves at any point in space, the particle density at that point may be taken as proportional to $|\psi|^2$. So $|\psi|^2$ is a measure of particle density and on multiplying this by the charge of the particle, we shall get the charge density. Thus, $|\psi|^2$ is a measure of '***charge density***'.
- According to Max Born, the value of $|\psi|^2$ at a point at a given time is related to the probability of finding the body described by its wave function ψ at that point at that instant. A large value of $|\psi|^2$ means a strong possibility of the presence of the body, while a small value of $|\psi|^2$ means a slight possibility of its presence. As long as $|\psi|^2$ is not actually zero somewhere, there is a definite chance, however small, of detecting the body there.
- Although the wave function ψ of a particle is spread out in space, this does not mean that the particle itself is also thus spread out. When an experiment is performed to detect a particle, an electron for instance, a whole electron is either found at a certain place and time, or it is not. There is nothing like 20 % of an electron. However, it is certainly possible that there is 20 % chance that the electron be found at that place and time, and it's likelihood that is specified by $|\psi|^2$ or $\psi\psi^*$, ψ^* being the complex conjugate of ψ.

- $|\psi|^2$ or $\psi\psi^*$ is taken as the probability density, i.e. the probability of finding the particle in unit volume. So the probability of the particle being present in a volume element dx·dy·dz is $|\psi|^2$ dx dy dz. Then, the wave function ψ is called the ***'probability density amplitude'***.
- Since the particle is certainly to be found somewhere in space, we must have,

$$\iiint |\psi|^2 \, dx\, dy\, dz = 1 \quad \text{... (1)}$$

the triple integral extending over all possible values of x, y, z.

- A function ψ satisfying this relation is called a ***'normalised wave function'*** and equation (1) is known as the ***'normalisation condition'***. Thus, ψ has to be a normalisable function.

  Besides being normalisable, ψ must also satisfy the following conditions:
  - ψ must be a single valued function, because ψ is related to the probability of finding the particle at a given place and time, and the probability can have only one value at a given point and time.
  - ψ must be finite, because the particle exists somewhere in space, and so integral over all space must be finite.
  - ψ and its derivatives $\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}$ must be continuous everywhere in the region where ψ is defined.

3.7 SCHROEDINGER'S WAVE EQUATION

(May 15, 16)

- Schroedinger started with De Broglie's idea of matter waves and developed it into a mathematical theory known as ***'wave mechanics'***. Schroedinger's wave equation is the mathematical representation of matter waves associated with a moving particle. There are two types of Schroedinger's wave equations:
  - Schroedinger's time independent wave equation
  - Schroedinger's time dependent wave equation.

3.7.1 Schroedinger's Time Independent Wave Equation

(Dec. 12, 14, Nov. 15, May 13, 14, 15)

- According to De Broglie's theory, a particle of mass m moving with a velocity v has a wave system of some kind associated with it, and its wavelength is given by $\lambda = \frac{h}{mv}$. The waves are produced only when something oscillates. Though we do not know the quantity that vibrates to produce the matter waves, but we can indicate that quantity by ψ.
- The periodic changes in ψ produce the wave system associated with the particle, just as the periodic changes in the displacement y of a string produce a wave system along the string.
- In quantum mechanics, ψ corresponds to the displacement y of wave motion in a string. However, ψ, unlike y, is not itself a measurable quantity and it may be complex.
- Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of matter waves at time t.
- By analogy with the wave equation

  $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$ of a two-dimensional wave (in xy plane), the wave equation for a three-dimensional wave with wave velocity u can be written as

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = u^2 \nabla^2 \psi \quad \text{... (1)}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the ***'Laplacian operator'***

The solution of equation (1) is

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} \quad \text{... (2)}$$

where $\psi_0(x, y, z)$ represents the amplitude of the wave at the point considered.

- The position vector of a point whose Cartesian coordinates are (x, y, z) is given by

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ being unit vectors along the axes. So equation (2) can be written as

$$\psi(\vec{r}, t) = \psi_0(\vec{r})\, e^{-i\omega t} \quad \text{... (3)}$$

Differentiating equation (3) twice with respect to time t, we get

$$\frac{\partial\psi}{\partial t} = -i\omega\, \psi_0(r)\, e^{-i\omega t}$$

and $$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)^2 \psi_0(\vec{r})\, e^{-i\omega t} = -\omega^2 \psi \quad \text{... (4)}$$

From equations (1) and (4), we get

$$u^2 \nabla^2 \psi = -\omega^2 \psi$$

$$\therefore \nabla^2 \psi + \frac{\omega^2}{u^2} \psi = 0 \quad \text{... (5)}$$

But $\omega = 2\pi\upsilon$, and $u = \upsilon\lambda$

$\therefore$ Equation (5) becomes

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \text{... (6)}$$

- The De Broglie wavelength of the waves associated with the particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{... (7)}$$

Substituting equation (7) in (6), we get

$$\nabla^2 \psi + \frac{4\pi^2 p^2}{h^2} \psi = 0 \quad \text{... (8)}$$

- The total energy E of the particle is the sum of it's K.E. $= \frac{1}{2}mv^2$ and potential energy V.

$$\therefore \quad E = \frac{1}{2}mv^2 + V$$

$$E = \frac{p^2}{2m} + V$$

This gives $p^2 = 2m(E - V)$ ... (9)

Substituting equation (9) in (8), we get

$$\nabla^2 \psi + \frac{8\pi^2 m (E - V)}{h^2} \psi = 0 \quad \text{... (10)}$$

- Equation (10) is called 'Schroedinger's time independent wave equation'.

Taking $\hbar = \frac{h}{2\pi}$, equation (10) becomes

$$\nabla^2 \psi + \frac{2m(E - V)}{\hbar^2} \psi = 0 \quad \text{... (11)}$$

3.7.2 Schroedinger's Time Dependent Wave Equation

- Schroedinger's time independent wave equation is

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \quad \text{... (1)}$$

- The time dependent wave equation is obtained by eliminating E from the time independent equation.

Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of the matter waves at time t. If u be the wave velocity, then the equation for a three-dimensional wave motion can be written as,

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = u^2 \nabla^2 \psi \quad \text{... (2)}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the ***'Laplacian operator'***

The solution of equation (2) is

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t}$$

$$= \psi_0(\vec{r})\, e^{-i\omega t} \quad \text{... (3)}$$

where $\psi_0(x, y, z)$ is the amplitude of the wave at the point considered.

- Differentiating equation (3) with respect to time t, we get

$$\frac{\partial \psi}{\partial t} = (-i\omega)\psi_0(\vec{r})\, e^{-i\omega t} = -i\omega\psi \quad \text{... (4)}$$

Now, $\omega = 2\pi\upsilon$ and $E = h\upsilon$ or $\upsilon = \frac{E}{h}$

$$\therefore \quad \omega = \frac{2\pi E}{h}$$

Putting this value of ω in equation (4), we get

$$\frac{\partial \psi}{\partial t} = -i \frac{2\pi E}{h} \psi \quad \text{... (5)}$$

Multiplying both sides of equation (5) by i,

$$i\frac{\partial \psi}{\partial t} = i^2 \frac{2\pi}{h} E\psi$$

$$\therefore \quad E\psi = i\frac{h}{2\pi}\frac{\partial \psi}{\partial t} = i\hbar\frac{\partial \psi}{\partial t} \quad \text{... (6)}$$

From equations (1) and (6), we get

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}\left(\frac{ih}{2\pi}\frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

Multiplying both sides of this equation by $\frac{-h^2}{8\pi^2 m}$, we get

$$-\frac{h^2}{8\pi^2 m}\nabla^2 \psi - \frac{ih}{2\pi}\frac{\partial \psi}{\partial t} + V\psi = 0$$

$$\left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t} \quad \text{... (7)}$$

$$\text{or} \quad \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t} \quad \text{... (8)}$$

Equation (8) is called 'Schroedinger's time dependent wave equation'.

Taking $H = \left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$

as ***'Hamiltonian operator'***

and $E = \frac{i h}{2\pi}\frac{\partial}{\partial t} = i \hbar \frac{\partial}{\partial t}$ as ***'Eigen operator'***,

equation (8) becomes

$$H \psi = E \psi \quad \text{... (9)}$$

3.8 APPLICATIONS OF SCHROEDINGER'S TIME INDEPENDENT WAVE EQUATION

- In quantum mechanics, the wave function of a system gives the description of that system. We apply Schroedinger's wave equation to a system, and then solve it to find the wave function of the system. We shall study how Schroedinger's time independent wave equation can be applied to a system and then solved to find the energy and wave function of the system under given conditions.
- We also aim at learning characteristic properties of solutions of this equation and comparing the predictions of quantum mechanics with those of Newtonian mechanics.
- As simple applications of Schroedinger's time independent wave equation, here we shall discuss the problems of :
  - Particle in a rigid box
  - Particle in a non-rigid box
  - Tunneling effect.

3.8.1 Particle Enforced in Infinite Deep Potential

(Nov. 13)

- Consider a particle confined to a rigid box and restricted to travelling along x-axis between x = 0 and x = L (See Fig. 3.6). Such a box has infinitely hard walls, and a particle does not lose energy when it collides with such walls. So the total energy E of the particle remains constant. The case under discussion is also called ***'infinite potential well'***.
- As shown in Fig. 3.6, the potential energy V of the particle is infinite on both sides of the box, while V is constant (say V = 0 for convenience) inside the box. This means that V (x) = 0 in the region $0 < x < L$; and $V(x) = \infty$ for $x \le 0$ and $x \ge L$.
- The particle cannot have an infinite amount of energy. So it cannot exist outside the box and hence particle wave function ψ is zero for $x \le 0$ and $x \ge L$. We now find the wave function ψ of the particle within the box, i.e. in the region $0 < x < L$. We use Schroedinger's time independent wave equation and solve it for this purpose.

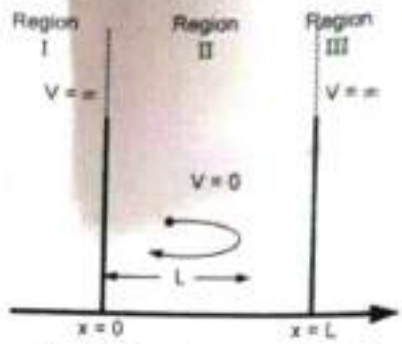


Fig. 3.6: Infinite potential well

- Schroedinger's time independent wave equation is

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \quad \text{... (1)}$$

- Now, the motion of the particle is along x-axis. So $\nabla^2\psi$ can be replaced by the total derivative $\frac{d^2\psi}{dx^2}$. Also V (x) = 0 inside the box. So for this problem, Schroedinger's equation (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{h^2}\psi = 0$$

i.e. $$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{... (2)}$$

where $$k^2 = \frac{8\pi^2 m E}{h^2} \quad \text{... (3)}$$

- Equation (2) is a total differential equation of second order with imaginary roots, and its general solution will involve two arbitrary constants. So the solution of equation (2) can be taken as

$$\psi(x) = A e^{ikx} + B e^{-ikx} \quad \text{... (4)}$$

- We now apply boundary conditions, namely $\psi(0) = 0$ and $\psi(L) = 0$. So from equation (4), we get

From first boundary condition, $\psi(0) = 0$,

$$\psi(0) = A + B = 0 \quad \text{i.e. } B = -A$$

and from second boundary condition, $\psi(L) = 0$,

$$\psi(L) = A e^{ikL} + B e^{-ikL} = 0$$

Taking $B = -A$, we get

$$Ae^{ikL} - Ae^{-ikL} = 0$$

Multiplying and dividing by 2i,

$$\therefore \quad 2i \cdot A \frac{(e^{ikL} - e^{-ikL})}{2i} = 0$$

or $$2iA \sin kL = 0 \quad \text{... (5)}$$

As $A \neq 0$, we infer from equation (5) that

$$\sin kL = 0$$

i.e. $k L = n\pi$, where $n = 1, 2, 3, \ldots\ldots$

$$\therefore \quad k = \frac{n\pi}{L} \quad \ldots (6)$$

From equations (3) and (6), we have

$$k^2 = \frac{8\pi^2 m E}{h^2} = \frac{n^2 \pi^2}{L^2}$$

$$\text{or} \quad E_n = \frac{n^2 \cdot h^2}{8\, mL^2} \quad \ldots (7)$$

- Equation (7) gives the energy values of the particle and it is evident that the energy of the particle can have only certain specific values as specified by equation (7). These energy values are called ***'Eigen values'***.
- Thus, the energy of a particle confined to a rigid box is quantized. It cannot have an arbitrary energy; the fact of its confinement leads to restrictions on its wave function that permit it to have only those energies as specified by equation (7).
- The integer n corresponding to the energy level E_n is called its ***'quantum number'***. n = 0 is not possible because the particle cannot have zero energy; and if it did, the particle wave function ψ would have to be zero everywhere in the box, and this means that the particle cannot be present there.
- The exclusion of E = 0 as a possible value for the energy of a trapped particle, like the limitation of E to a discrete set of definite values, is a quantum mechanical result. Classically, all energies, including zero, are presumed possible.

  Putting $m = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s and $L = 10^{-10}$ m in equation (7), we get

$$E_n = 6 \times 10^{-18}\, n^2 \text{ joules}$$

$$= \frac{6 \times 10^{-18}}{1.6 \times 10^{-19}}\, n^2 \text{ eV}$$

$$\cong 38\, n^2 \text{ eV}$$

- The energy levels of such electron trapped in a potential well of width 1 A° is shown in Fig. 3.7.

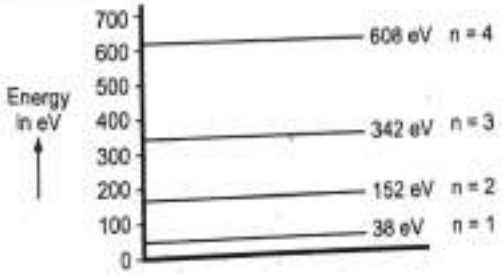


Fig. 3.7: Energy levels of electron in a rigid box of width 1 A°

Wave Function of a Particle in a Rigid Box :

- The wave function ψ (x) of a particle inside a rigid box of width L is given by

$$\psi(x) = 2\, i\, A \sin k x \quad \ldots (1)$$

$$\text{where} \quad k = \sqrt{\frac{8\pi^2 m E}{h^2}} = \frac{n\pi}{L} \quad \ldots (2)$$

From equations (1) and (2),

$$\psi(x) = 2\, i\, A \sin \frac{n\pi}{L} x \quad \ldots (3)$$

Equation (3) gives Eigen functions corresponding to energy Eigen values

$$E_n = \frac{n^2 h^2}{8\, m\, L^2} \quad \ldots (4)$$

The complex conjugate of ψ(x) is

$$\psi(x) = -2\, i\, A \sin \frac{n\pi}{L} x$$

To evaluate the constant A, we use the normalization condition.

$$\text{i.e.} \quad \int_0^L \psi\psi^* \, dx = 1$$

$$\int_0^L 4 A^2 \sin^2 \frac{n\pi}{L} x \, dx = 1$$

This gives $2 A^2 L = 1$ or $A = \dfrac{1}{\sqrt{2L}}$

- Putting this value of A in equation (3), we get the normalized wave function of the particle as

$$\psi_n = \frac{2\, i}{\sqrt{2L}} \sin \frac{n\pi}{L} x = i \sqrt{\frac{2}{L}} \cdot \sin \frac{n\pi}{L} x \quad \ldots (5)$$

- The normalized wave functions ψ_1, ψ_2 and ψ_3 together with the corresponding probability densities $|\psi_1|^2$, $|\psi_2|^2$ and $|\psi_3|^2$ are shown in Fig. 3.8 (a) and (b) respectively.

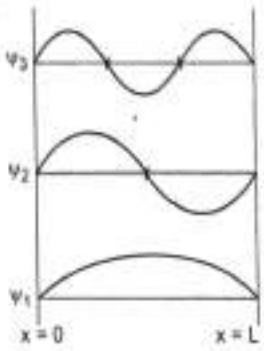


Fig. 3.8 (a): Wave function

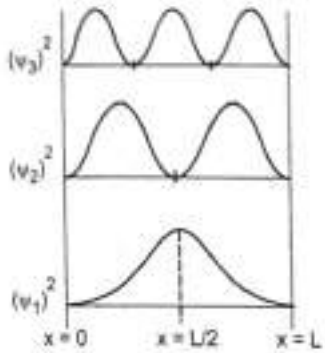


Fig. 3.8 (b): Probability densities

- While ψ_n may be negative as well as positive, $|\psi_n|^2$ is always positive. It is seen from Fig. 3.8 (b) that
  - In every case $|\psi_n|^2 = 0$ at $x = 0$ and $x = L$, the boundaries of the box.
  - The probability of the particle being present at a particular point in the box may be different for different quantum numbers. e.g. For $n = 1$, the probability is maximum at $x = \frac{L}{2}$. For $n = 2$ there is zero probability of the particle being present at $x = \frac{L}{2}$ and there are two maxima at $x = \frac{L}{4}$ and $x = \frac{3L}{4}$. Classical physics predicts the same probability for the particle to be present anywhere in the box.
- The wave function shown in Fig. 3.8 (a) resembles the possible vibrations of a string fixed at both ends. This is so because the waves in a stretched string and the waves representing a particle obey wave equation of the same form, and when similar restrictions are placed upon each kind of wave, the solutions are identical.

Comparison of Quantum Mechanical and Classical Mechanical Predictions

Quantum Mechanical	Classical Mechanical
1. The energy of the particle confined is quantized i.e. can have only discrete energy levels.	1. The energy of the particle has continuous values i.e. can have all possible values from 0 to ∞.
2. Energy E = 0 is not possible as the result of Heisenberg's uncertainty principle.	2. Can have energy E = 0.
3. Probability of finding the particle is different at different position.	3. Probability of finding the particle is same energy where.
4. Probability of finding the particle depends on the principle number and hence on energy state.	4. Probability of finding the particle is independent of energy level.
5. Probability is zero at the walls of the potential well.	5. Probability is maximum at the walls of the potential well i.e. at the boundaries.

3.8.2 Particle in Finite Potential Well

- Consider a particle trapped in a non-rigid box and restricted to travelling along x-axis between $x = 0$ and $x = L$ (See Fig. 3.9). The walls of the box are non-rigid. The potential outside the box is finite, say V_0. The potential V_0 is greater than the energy E of the particle inside the box. The potential energy of the particle inside the box is considered to be zero, i.e. $V = 0$. The case under discussion is also called ***'finite potential well'***.

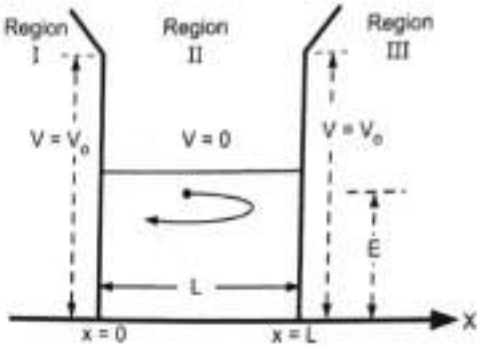


Fig. 3.9: Finite potential well

- As shown in Fig. 3.9, the potential outside the box is finite, say V_0, and inside the box $V = 0$. This means that $V(x) = 0$ in the region $0 < x < L$ and $V(x) = V_0$ for $x \le 0$ and $x \ge L$.
- Now, it is possible for the particle to have energy E that exceeds V_0. So the particle can exist outside the box and wave function ψ of the particle will not be zero at the boundaries of the box. We use Schroedinger's time independent wave equation to find wave function of the particle inside and on the two sides of the box.
- The Schroedinger's time independent wave equation is

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \quad \text{... (1)}$$

- Now, the motion of the particle is only along x-axis. So $\nabla^2\psi$ can be replaced by $\frac{d^2\psi}{dx^2}$. Also $V(x) = V_0$ outside the box. So the Schroedinger's equation (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi = 0$$

$$\text{i.e. } \frac{d^2\psi}{dx^2} - k'^2\psi = 0 \quad \text{... (2)}$$

$$\text{where} \quad k'^2 = \frac{8\pi^2 m}{h^2}(V_0 - E) \quad \text{... (3)}$$

- Let the wave functions be denoted by ψ_I, ψ_{II} and ψ_{III} respectively in regions I, II and III. Equation (2) is a total differential equation of second order with real roots, and its general solution will involve two arbitrary constants. So the solutions of equation (2) can be written as:

For region I, i.e. for $x < 0$,

$$\psi_I(x) = A e^{k'x} + B e^{-k'x} \quad \text{... (4)}$$

and for region III, i.e. for $x > L$,

$$\psi_{III}(x) = C e^{k'x} + D e^{-k'x} \quad \text{... (5)}$$

- If the wave function of the particle is not to be infinite as we go away from the boundaries of the box, the negative exponential term in equation (4) and the positive exponential term in equation (5) should be absent, i.e. $B = 0$ and $C = 0$.

$$\therefore \quad \psi_I(x) = A e^{k'x} \quad \text{... (6)}$$

$$\text{and} \quad \psi_{III}(x) = D e^{-k'x} \quad \text{... (7)}$$

- Equations (6) and (7) give wave functions of the particle on the two sides of the non-rigid box.
- Now, to find the wave function inside the box, we take $V(x) = 0$ inside the box. So the equation (1) becomes,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\psi = 0$$

$$\text{i.e.} \quad \frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{... (8)}$$

$$\text{where} \quad k^2 = \frac{8\pi^2 mE}{h^2} \quad \text{... (9)}$$

- Equation (8) is a total differential equation of second order with imaginary roots. So its solution can be taken as

$$\psi_{II}(x) = P e^{ikx} + Q e^{-ikx} \quad \text{... (10)}$$

- Equation (10) gives the wave function inside the box.
- The wave function of the particle will be known completely, both outside and inside the box, if we can evaluate the four constants A, D, P and Q of equations (6), (7) and (10).
- For this we need four independent equations among these constants, and they can be obtained by using the property of the wave function that ψ and $\frac{\partial\psi}{\partial x}$ must be continuous everywhere in the region where ψ is defined.

$$\text{So,} \quad \psi_I(0) = \psi_{II}(0)$$

$$\text{i.e.} \quad A = P + Q \quad \text{... (11)}$$

$$\text{and} \quad \psi_{II}(L) = \psi_{III}(L)$$

$$\text{i.e.} \quad P e^{ikL} + Q e^{-ikL} = D e^{-k'L} \quad \text{... (12)}$$

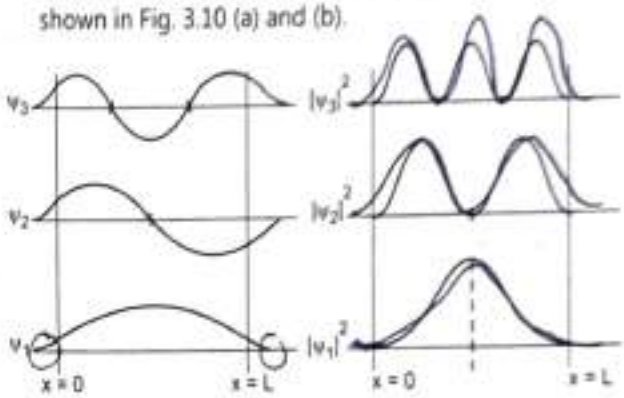
$$\text{Similarly,} \quad \left|\frac{\partial\psi_I}{\partial x}\right|_{x=0} = \left|\frac{\partial\psi_{II}}{\partial x}\right|_{x=0}$$

$$\text{i.e.} \quad Ak' = iPk - iQk \quad \text{... (13)}$$

$$\text{and} \quad \left|\frac{\partial\psi_{II}}{\partial x}\right|_{x=L} = \left|\frac{\partial\psi_{III}}{\partial x}\right|_{x=L}$$

$$\text{i.e.} \quad Pike^{ikL} - Qike^{-ikL} = -Dk'e^{-k'L} \quad \text{... (14)}$$

- The constants A, D, P and Q can be evaluated by solving equations (11), (12), (13) and (14). Thus, the wave function of the particle in a non-rigid box is known completely.
- The first few wave functions of a particle in a non-rigid box and the corresponding probability densities are shown in Fig. 3.10 (a) and (b).



(a) Wave function **(b) Probability densities**

Fig. 3.10

- It can be seen from Fig. 3.10 (a) that the wave functions ψ_n are not equal to zero outside the box. This means that even though the particle energy E is smaller than the value of potential outside the box, there is still a definite probability that the particle be found outside the box.
- In other words, even though the particle does not have sufficient energy to break through the walls of the box, it may nevertheless somehow penetrate them and leak out.
- The wave functions of a particle in a box, with rigid walls are zero at the walls. [See Fig. 3.10 (a)]. But when the confining box has non-rigid walls, the wave functions of the particle are not equal to zero at the walls. This means that the particle wave functions are somewhat longer in the case of a non-rigid box than the wave functions in the case of rigid box, i.e. wavelengths of the particle are larger when it is in a non-rigid box than the wavelengths when it is in a rigid box.
- A larger wavelength means a smaller frequency and hence smaller energy. Hence energy levels of a particle in a non-rigid box are lower than the corresponding energy levels of a particle in a rigid box. This is shown in Fig. 3.11 where full lines show energy levels of a particle in a rigid box and dotted lines show energy levels of a particle in a non-rigid box.

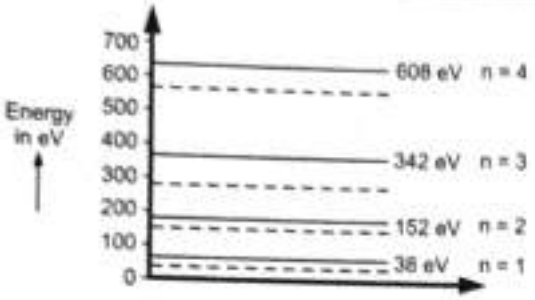


Fig. 3.11: Energy levels

- When the potential energy outside the box is finite, say V_0, it will be possible for a particle to have energy E that exceeds V_0. Such a particle is not trapped inside the box, for it always has sufficient energy to penetrate the walls of the box. Energy of the particle is not quantized but it may have any value above V_0. However, the K.E. of the particle outside the box is $(E - V_0)$ and it is always less than its K.E. inside the box.
- This is because whole energy E of the particle inside the box is K.E. as V = 0 there. Less energy means longer wavelength. So wave function ψ has a longer wavelength outside the box than inside.

3.9 TUNNELING EFFECT

- The phenomenon of the particles penetrating the potential barrier is called the ***'tunnel effect'*** or ***'tunneling'***. The examples of tunneling are tunnel diode, alfa decay, tunneling microscope, etc. The kinetic energy of α-particle is only few MeV but still it can escape from a nucleus whose potential wall is 25 MeV due to tunneling.
- Consider a particle approaching the potential barrier from the left i.e. from Ist region. If the particle has energy less than height of potential barrier V_0 i.e. $E < V_0$, classically, the particle will be always reflected back and hence will not penetrate the barrier. However, by quantum mechanics, there is some probability of penetrating to region III. The probability of penetration increases if $(V_0 - E)$ and L i.e. height and width of potential barrier are smaller.
- On the other hand, if $E > V_0$ classical mechanics predicts that the particle will always be transmitted. But quantum mechanics says that there is a finite probability of transmission and hence it is not certain that the particle will penetrate the barrier.
- Let us consider the one-dimensional potential barrier, where the height of the barrier is V_0 and width is L. The potential function is defined as $V(x) = 0$ for $x \le 0$ and $x \ge L$ and $V(x) = V_0$ for $0 < x < L$ as shown in Fig. 3.12.

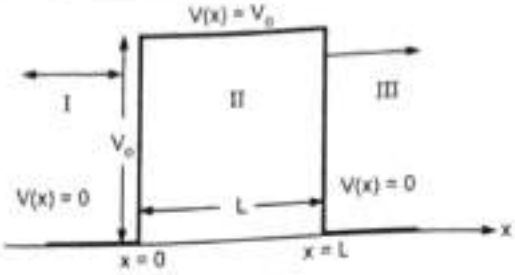


Fig. 3.12

- The Schroedinger's time independent equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \quad \text{... (1)}$$

- As the motion of the particle is in x-direction, so $\nabla^2\psi$ can be replaced by $\frac{d^2\psi}{dx^2}$. For regions I and III, $V(x) = 0$, therefore equation (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

Let $$k^2 = \frac{8\pi^2 m}{h^2}E \quad \text{... (2)}$$

$$\therefore \frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{... (3)}$$

- The equation (3) is a total differential equation of second order with imaginary roots and its general solution will involve two arbitrary constants. So the solution can be written as,

For region I, i.e. $x \le 0$,

$$\psi_I(x) = A e^{ikx} + B e^{-ikx} \quad \text{... (4)}$$

For region III, i.e. $x \ge L$

$$\psi_{III}(x) = C e^{ikx} + D e^{-ikx} \quad \text{... (5)}$$

- In equation (5), the first term represents the wave travelling along positive X-axis in region III i.e. the wave transmitted at x = L. The second term represents the wave travelling along negative X-axis in region III, but no wave travels back from infinity in region III. Therefore, taking D = 0 in equation (5),

$$\psi_{II}(x) = C e^{ikx} \quad \text{... (6)}$$

For region II i.e. $0 < x < L$ the $V(x) = V_0$, so the equation (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi = 0$$

Let $$\frac{8\pi^2 m}{h^2}(E - V_0) = k'^2 \quad \text{... (7)}$$

$$\therefore \frac{d^2\psi}{dx^2} + k'^2\psi = 0 \quad \text{... (8)}$$

The solution of above differential equation is given by

$$\psi_{II}(x) = P e^{ik'x} + Q e^{-ik'x} \quad \text{... (9)}$$

- The first term of the solution represents the wave travelling along positive X-axis in region II i.e. the wave transmitted at x = 0 and second term represents the wave travelling along negative X-axis in region II i.e. the wave reflected at x = L.
- For determining the constants A, B, C, P and Q, the boundary conditions are applied at x = 0 and x = L.

i.e. $\psi_I = \psi_{II}$

$$\frac{\partial \psi_I}{\partial x} = \frac{\partial \psi_{II}}{\partial x} \quad \text{at } x = 0$$

and $\psi_{II} = \psi_{III}$

$$\frac{\partial \psi_{II}}{\partial x} = \frac{\partial \psi_{III}}{\partial x} \quad \text{at } x = L$$

- The property of the barrier penetration is due to the wave nature of matter and is similar to the total internal reflection of light. If two glass plates are placed close to each other with a thin layer of air trapped between them, then the light coming from one plate will be transmitted by the second plate even if the angle is greater than the critical angle. But the intensity of the transmitted light will decrease exponentially with the thickness of the barrier.
- If a particle incidenting on the potential barrier with energy less than the height of the potential barrier, there is always some probability of transmission through the barrier. This phenomenon of crossing the barrier is called the ***'tunneling effect'***.

3.9.1 Examples of Tunneling Effect

1. Alpha Decay

Alpha decay is a type of radioactive decay in which an atomic nucleus emits an alpha particle. An alpha particle (α particle) consists of two protons and two neutrons bound together ($_2He^4$).

- An α-particle has energy of only around 4.8 MeV and it is bounded in the nucleus, with energy around 25 MeV (potential barrier). Thus classically it is impossible for α-particle to escape from nucleus.

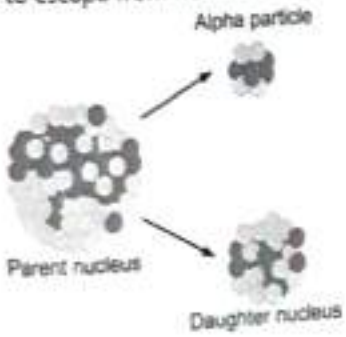


Fig. 3.13

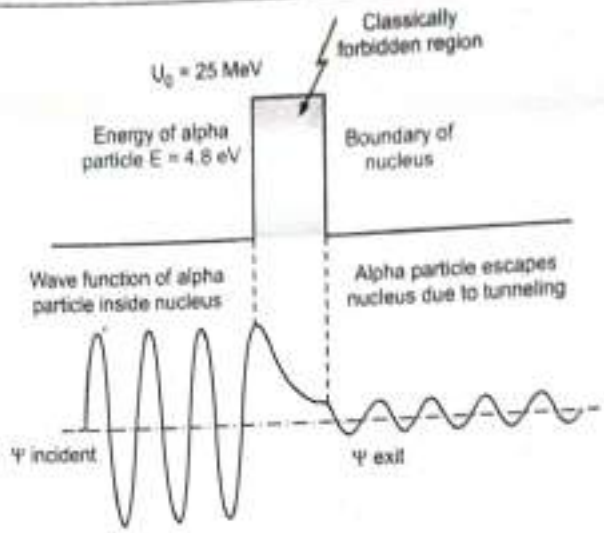
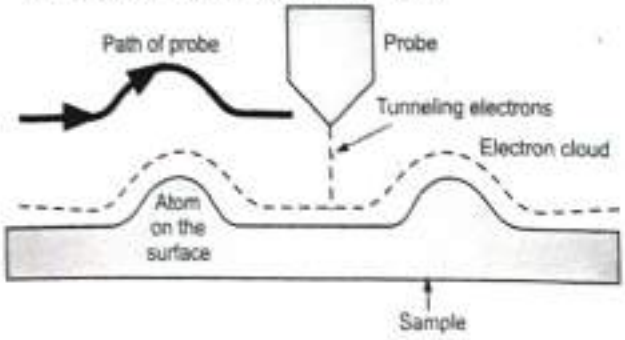


Fig. 3.14

- Quantum mechanically de-Broglie wave associated with alpha particle. The de-Broglie waves associated with alpha particle can penetrate the potential barrier (nucleus) and there is a non-zero probability that it will escape from the nucleus.
- The probability of escape of alpha particle is so small i.e. 1 in 10^{38} i.e. alpha particle has to strike potential well of nucleus 1038 or more times before it emerges, but it would definitely escape from the nucleus.

2. Scanning Tunneling Microscope (STM)

- A scanning tunneling microscope (STM) is an instrument for imaging surfaces at the atomic level by using the principle of tunneling effect. The resolution of an STM is 0.1 nm lateral and 0.0 nm depth measurement. With such high resolution, individual atoms of material can be imagined.
- The STM is based on the concept of quantum tunneling. When a conducting tip is brought close to the surface to be examined, the bias voltage applied between them will allow electrons to tunnel through the vacuum between them. The tunneling current between them will depend on the width of the barrier i.e. the distance between the probe and the surface if bias voltage between them is constant.



(a)

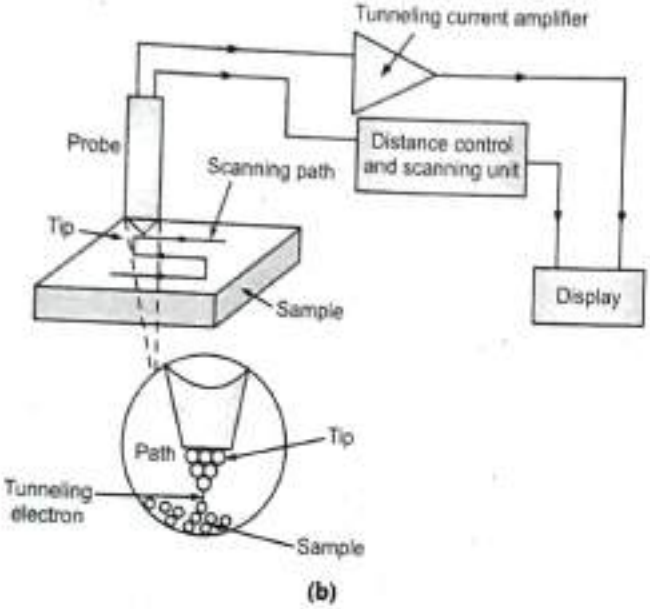
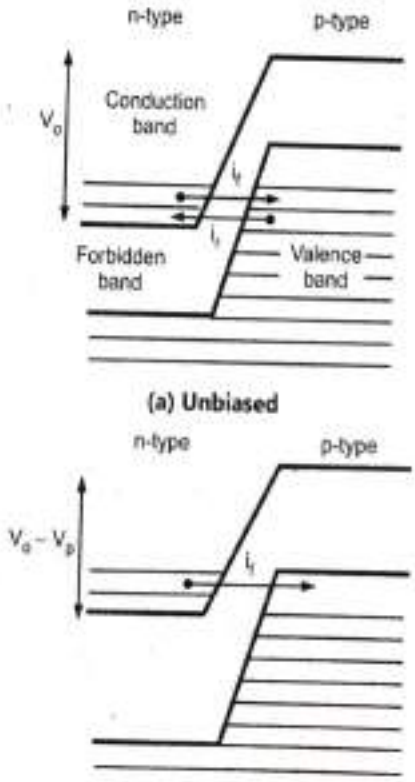


Fig. 3.15: Schematic diagram of STM

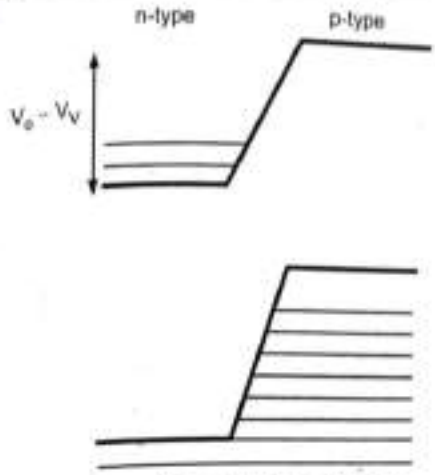
- This voltage can be recorded and displayed simulating the surface of the sample. The probe is moved vertically so that the distance between the probe and the surface remains constant as the probe scans laterally. Fig. 3.15 shows the basic mechanism of STM.

3. Tunnel Diode

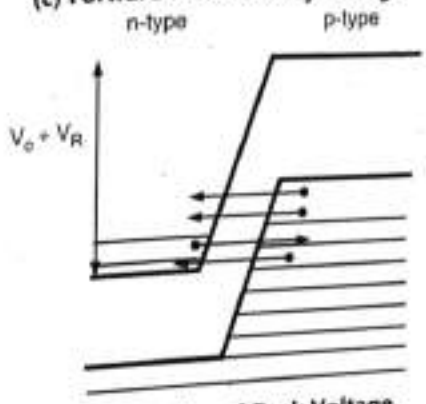
- A ***'tunnel diode'*** is a semiconductor device that uses the phenomenon of tunneling. The current flowing in the tunnel diode is produced by electrons tunneling through the potential barrier developed by depletion layer.
- A tunnel diode is constructed in the same way as ordinary diode but the concentration of impurity is very high. This results in high conductivity or low resistivity than ordinary diode. The semiconductors with low resistivity, approaching conductors are termed as ***'degenerate semiconductors'***. A p-n junction in a degenerate semiconductor is very thin ($\sim 10^{-6}$ cm) and potential barrier is about twice as high in case of ordinary diode.
- The potential barrier height in an ordinary diode is about one half of the forbidden bandwidth, while in tunnel diode it is wider than the forbidden band. As a result of the barrier layer being extremely thin, the electric field is very high ($\sim 10^{6}$ V/m) even in absence of an external voltage.
- In a tunnel diode, the charge carriers cross the junction by diffusion due to an electric field along with tunneling effect. In tunneling effect, there is certain probability of electron penetration through the barrier without a change in electron energy levels when the barrier is thin enough.
- The tunnel electron transfer takes place when the energies of electrons are less than the potential barrier height provided that on other side of the barrier unoccupied energy levels are available for the tunneling electrons. This electron transfer can take place in both directions.
- The tunnel currents originating in a p-n junction of a tunnel diode are shown in Fig. 3.16. Here diffusion and conduction currents are omitted to simplify the presentation of tunnel effect.
- Fig. 3.16 (a) shows energy level diagram of unbiased diode. The energy levels in valence and conduction bands occupied by electrons are represented by horizontal lines and the spacing between them indicate the energy levels unoccupied by electrons. As shown in the figure, the conduction band of n-type semiconductor and the valence band of p-type semiconductor show occupied energy levels corresponding to the same electron energies. Thus, tunnel transfer from the n-region to p-region (forward tunnel current i_f) and from the p-region to n-region (reverse tunnel current i_r) is possible. These currents are equal in magnitude and completely cancel each other, reducing the total current to zero.



(b) Forward Biased-Peak Voltage



(c) Forward Biased-Valley Voltage



(d) Reverse Biased Peak Voltage

Fig. 3.16: Tunnel diode

- When forward peak voltage V_p is applied, the potential barrier height is lowered to $(V_o - V_p)$ as shown in Fig. 3.16 (b). This increases tunnel transfer of electrons for the n-region into the p-region as there are free energy levels in the valence band in the p-region corresponding to occupied energy levels in the conduction band of the n-region.
- At the same time, electron transfer in the opposite direction is impossible, since the occupied energy levels of the p-region valence band correspond to the forbidden band of the n-region. Thus, reverse current is absent and forward current is maximum. The maximum current is called ***'peak current'*** and corresponding forward voltage is called ***'peak voltage'*** (V_p).
- In case of a higher forward voltage (V_v) as shown in Fig. 3.16 (c), the potential barrier height decreases to $(V_o - V_v)$. Under this condition, tunnel electron transfer becomes impossible, since occupied energy levels of one region correspond to the forbidden band of the other. This makes the tunnel current zero, but small current may be present due to other effects i.e. diffusion and leakage. This current is called ***"valley current"*** and voltage ***'valley voltage'*** V_v.
- A further rise in forward voltage results in an increase of the usual forward diffusion current as in ordinary diode. For forward voltages less than valley voltage, the diffusion current is negligible in comparison to the tunnel current. While for forward voltages more than valley voltage, diffusion current increases and attains characteristic of ordinary diode.
- In case of reverse biasing, the potential barrier height is increased to $(V_o + V_R)$ as shown in Fig. 3.16 (d). The number of occupied energy levels in the valence band of the p-region corresponding to free levels in the n-region conduction band increases considerably. This results in a significant rise in the reverse tunnel current, which can attain the same magnitude as forward tunnel current under forward voltage conditions.
- The I-V characteristics of a tunnel diode has been shown in Fig. 3.17. It can be seen that the current is absent at V = 0. A rise in forward voltage results in an increase in tunnel current and is maximum at a voltage V_p. A further increase in forward voltage reduces the tunnel current until it reaches a maximum current at V_v. This results in a negative resistance. If the voltage is increased further, the current increases due to usual diffusion current. The reverse current is of the same order i.e. many times higher than ordinary diode.

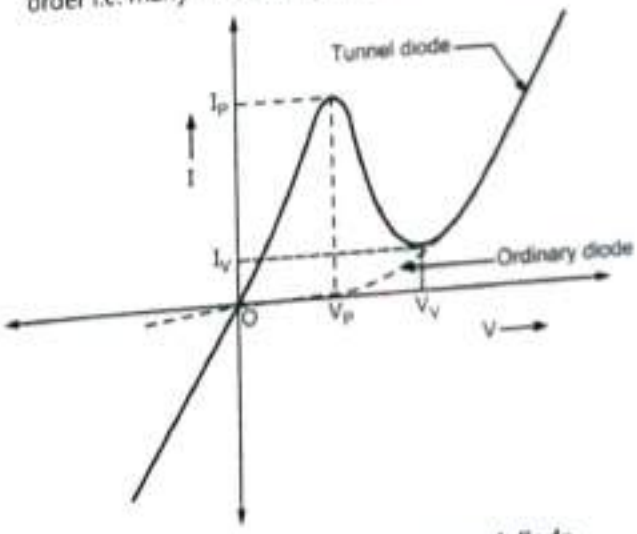


Fig. 3.17: I-V Characteristics of a tunnel diode

- The tunnel electron transfer can take place at very short interval i.e. 10^{-14} sec to 10^{-12} sec, therefore they can be used at microwave frequencies ($\sim 10^9$ Hz).

Problem 3.48: *Compute the permitted energy levels of an electron in an infinite potential well of width 1 A°.*

(03) (Dec. 05, 09, May 10, 11)

Data: $m = 9.1 \times 10^{-31}$ kg, $L = 1\ A^\circ = 10^{-10}$ m

Formula: $E_n = \frac{n^2 h^2}{8\,mL^2}$

Solution: $E_n = \frac{n^2 (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$

$= 6 \times 10^{-18}\, n^2$ J

$E_n = 38\, n^2$ eV

For $n = 1$, $\boxed{E_1 = 38 \text{ eV}}$

$n = 2$, $\boxed{E_2 = 152 \text{ eV}}$

$n = 3$, $\boxed{E_3 = 342 \text{ eV}}$ etc.

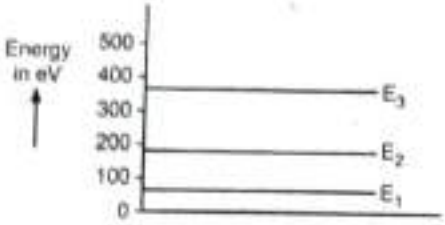


Fig. 3.18

Problem 3.49: *An infinite square well has a width of 1 A°. What is the fractional change in the lowest two permissible energies of one electron in this well if the width is increased to 2 A°?* **(04) (Dec. 08)**

Data: $L = 1\ A^\circ = 10^{-10}$ m, $L' = 2 \times 10^{-10}$ m

Formula: $E_n = \frac{n^2 h^2}{8\,mL^2}$

Solution: From previous numerical,

For $L = 1$, $E_n = 38\, n^2$ eV

i.e. $E_1 = 38$ eV and $E_2 = 152$ eV

For $L = 2$ the energies reduce by a factor of 4

$\therefore\ E_1' = 9.5$ eV and $E_2' = 38$ eV

Fractional change,

$\Delta E_1 = E_1 - E_1'$

$\Delta E_2 = E_2 - E_2'$

$\Delta E_1 = 38 - 9.5 = \boxed{28.5 \text{ eV}}$

$\Delta E_2 = 152 - 38 = \boxed{114 \text{ eV}}$

Problem 3.50: *Lowest energy level of an electron trapped in a potential well is 38 eV. Find the width of the well.* **(04) Dec. 04, (May 05)**

Data: $E = 38$ eV, $n = 1$.

Formula: $E = \frac{n^2 h^2}{8\,mL^2}$

Solution: $L^2 = \frac{n^2 h^2}{8\,mE}$

$L = \sqrt{\frac{(6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}}$

$= \boxed{10^{-10} \text{ m} = 1\ A^\circ}$

Problem 3.51: *An electron is trapped in a rigid box of width 1 A°. Find its lowest energy level and momentum. Hence find energy of 5th level.* **(Dec. 14)**

Data: $L = 1\ A^\circ = 10^{-10}$ m

Formula: $E_n = \frac{n^2 h^2}{8\,mL^2} = 38\, n^2$

Solution: $E_1 = 38$ eV

$E_1 = \frac{p_1^2}{2m}$

$\therefore\ p_1^2 = 2\,m\,E_1$

$= 2 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}$

$= 1106.56 \times 10^{-50}$

$\boxed{p_1 = 33.26 \times 10^{-25} \text{ kg - m/sec}}$

$E_5 = 25 \times 38$ eV $= \boxed{950 \text{ eV}}$

Problem 3.52: *At what kinetic energy an electron will have a wavelength of 5000 A°?* **(04) (Dec. 10)**

Data: $\lambda = 5000\ A^\circ$

Formula: $\lambda = \frac{h}{\sqrt{2mE}}$

Solution: $\lambda^2 = \frac{h^2}{2mE}$

$\therefore\ E = \frac{(6.63 \times 10^{-34})^2}{2\,(9.1 \times 10^{-31})\,(5000 \times 10^{-10})^2}$

$E = 9.66 \times 10^{-25}$ J

$\boxed{E = 6.038 \times 10^{-6} \text{ eV}}$

Problem 3.53: *Assuming atomic nucleus to be a rigid box (infinite potential well), calculate the ground state energy of an electron if it existed inside the nucleus.*

*(**Given:** Planck's constant = 6.63×10^{-34} J-s, Mass of the electron = 9.1×10^{-31} kg and size of the nucleus ~ 10^{-15} m. Using this result, argue that electron cannot exist inside the nucleus. Given maximum binding energy per nucleon = 8.8 MeV).* **(05) (Dec. 11)**

Data: $d = 10^{-15}$ m,

$$m = 9.1 \times 10^{-31} \text{ kg} \cdot \frac{8.1}{N} = 8.8 \text{ MeV}$$

Formulae: (i) $E_n = \frac{n^2 h^2}{8mL^2}$ (ii) $\Delta x \Delta p = h$

Solution: (i) $E_n = \frac{(1)^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-15})^2}$

$$E_n = 6.038 \times 10^{-10} \text{ J}$$

$$\boxed{E_n = 3.774 \times 10^{9} \text{ eV}}$$

(ii) $\Delta x \Delta p = h$

$$\Delta p = \frac{6.63 \times 10^{-34}}{10^{-15}}$$

$$\boxed{\Delta p = 6.63 \times 10^{-19} \text{ kg m/s}}$$

If this is the uncertainty in the momentum of electron in the nucleus, the momentum itself must be at least comparable to Δp i.e. $p \sim \Delta p$.

$$\therefore \quad E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31}}$$

$$E = 2.415 \times 10^{-7} \text{ J}$$

$$\boxed{E = 1.510 \times 10^{12} \text{ eV}}$$

This shows that if the electron exist in the nucleus, the energy must be at least 1.51×10^{12} eV, which is much more than B.E. per nucleon. Therefore, it cannot exist in the nucleus.

Problem 3.54: *An electron is trapped in a rigid box of width 2 A°. Find its lowest energy level and momentum. Hence, find energy of the 3rd energy level.* **(4) (May 06, 15)**

Data: L = 2 A°.

Formulae: $E_n = \frac{n^2 h^2}{8mL^2}$, $P_n = \sqrt{2mE_n}$

Solution: For lowest energy level, n = 1

$$E_1 = \frac{1^2 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$E_1 = 1.50 \times 10^{-18} \text{ J}$$

or $E_1 = 9.37$ eV

$$\therefore \quad P_1 = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.51 \times 10^{-18}}$$

$$P_1 = 1.66 \times 10^{-24} \text{ kg-m/sec}$$

For third energy level, n = 3.

$$\therefore \quad E_3 = \frac{3^2 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})}$$

$$\boxed{E_3 = 13.5 \times 10^{-18} \text{ J}}$$

or $\boxed{E_3 = 84.33 \text{ eV}}$

Problem 3.55: *Compare the lowest three energy states for an electron confined in an infinite potential well of width 10 A°.* **(04) (May 09)**

Data: L = 10 A°

Formula: $E_n = \frac{n^2 h^2}{8mL^2}$

Solution: $E_n = \frac{n^2 h^2}{8mL^2}$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$E_n = 0.603 \times 10^{-19} n^2 \text{ J}$$

or $E_n = 0.38 n^2$ eV

For first energy level,

n = 1

$$\therefore \quad \boxed{E_1 = 0.38 \text{ eV}}$$

For second energy level,

n = 2

$$\therefore \quad \boxed{E_2 = 1.52 \text{ eV}}$$

For third energy level,

n = 3

$$\therefore \quad \boxed{E_3 = 3.42 \text{ eV}}$$

Problem 3.56: *An electron is bounded by an infinite potential well of width 2×10^{-8} cm. Calculate the lowest two permissible energies of an electron.* **(03) (Dec. 12)**

(Given: $h = 6.64 \times 10^{-34}$ J-sec., $m = 9.1 \times 10^{-31}$ kg)

Data: $h = 6.64 \times 10^{-34}$ J-sec., $m = 9.1 \times 10^{-31}$ kg, n = 1, $L = 2 \times 10^{-10}$ m

Formula: $E_n = \frac{n^2 h^2}{8 mL^2}$

$$\therefore \quad E_n = \frac{n^2 (6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$\therefore \quad E_n = 1.5 \times 10^{-18} n^2 \text{ J}$$

$$\therefore \quad E_n = 9.44 n^2 \text{ eV} \approx 9.5 n^2 \text{ eV}$$

For n = 1, $E_1 = 9.5$ eV

For n = 2, $E_2 = 9.5 (2^2)$ eV

$$\therefore \quad E_2 = 38 \text{ eV}$$

Problem 3.57: *Calculate the energy and momentum of an electron confined in a rigid box of width 2A° for lowest energy state.* **(04) (May 13)**

Data: L = 2A°

Formula: (i) $E_n = \frac{n^2h^2}{8mL^2}$

(ii) $P = \sqrt{2mE}$

Solution: (i) $E_1 = \frac{1^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$E_1 = 1.509 \times 10^{-18} J$

$E_1 = 9.44e^{-V}$

(ii) $P_1 = \sqrt{2 \times 9/1 \times 10^{-31} \times 1.509 \times 10^{-18}}$

$P_1 = 1.663 \times 10^{-24}$ kg-m/sec

Problem 3.58: *A neutron is trapped in an infinite potential well of width 10^{-14} m. Calculate its first energy eigenvalue in eV.*

(3) (May 16)

Data : $m_n = 1.675 \times 10^{-27}$, $L = 10^{-14}$ m, n = 1,

Formula : $En = \frac{n^2h^2}{8mL^2}$

Solution : $En = \frac{(1)^2 (6.63 \times 10^{-34})^2}{8 \times 1.675 \times 10^{-27} \times (10^{-14})^2}$

$= 3.28 \times 10^{-13}$ J

$= 2.05$ MeV.

Problem 3.59: *Calculate the energy required to excite the electron from its ground state to fourth excited state in a rigid box of length 0.1 nm.* **(3) (Nov. 16)**

Data : $L = 10^{-80}$ cm

Formula : $E_n = \frac{n^2h^2}{8m L^2}$,

$E_1 = \frac{(1)^2h^2}{8m L^2}$

$E_4 = \frac{(4)^2h^2}{8m L^2}$

1 joule = 6.242×10^{18} electton volts

$E_4 - E_1 = \frac{16 h^2}{8m L^2} - \frac{h^2}{8 m L^2}$

$= \frac{15 h^2}{8 \times (9.1 \times 10^{-31}) \times (10^{-10})^2}$

$= \frac{15 \times (6.63 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (10^{-10})^2}$

$= 9.057 \times 10^{-7}$ J

$E_4 - E_1 = 565.341$

Problem 3.60 : *A neutron is trapped in an infinite potential well of width 1 Å Calculate the values of energy and momentum in its ground state.* **(3) (May 17)**

Data: $L = 1A^0$

$m_n = 1.675 \times 10^{-27}$ kg

n = 1

Formula: (i) $E_n = \frac{n^2h^2}{8mL^2}$

(ii) $P = \sqrt{2mE}$

Solution: (i) $E_1 = 1^2 \times (6.63 \times 10^{-34})^2/8 \times 1.675 \times 10^{-27} \times (1 \times 10^{-10})^2$

$E_1 = 3.280 \times 10^{-21} J$

$E_1 = 0.02050$eV

(ii) $P_1 = \sqrt{(2 \times 1.675 \times 10^{-27} \times 3.280 \times 10^{-21})}$

$P_1 = 3.3148 \times 10^{-24}$ kg-m/sec

Problem 3.61 : *Lowest energy of an electron trapped in a infinite potential well is 38 eV Calculate the width of the well.*

($e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J-sec, $m_e = 9.1 \times 10^{-31}$ kg) **(3) (Nov. 17)**

Data: E = 38 eV, n = 1.

Formula: $E = \frac{n^2 h^2}{8 mL^2}$

Solution: $L^2 = \frac{n^2 h^2}{8 mE}$

$L = \sqrt{\frac{(6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 38 \times 1.6 \times 10^{-19}}}$

$= 10^{-10}$ m = 1 A° ... **Ans.**

Problem 3.62 : *Calculate the lowest energy and corresponding momentum of an electron confined in a rigid box of width 2 A* **(3) (Nov. 18)**

($e = 1.6 \times 10^{-19}$C, $h = 6.63 \times 10^{-34}$ J-sec, $m_e = 9.1 \times 10^{-31}$ kg)

Solution : Given : L = 2A°

Formulae : $En = \frac{n^2h^2}{8mL^2}$, $Pn = \sqrt{2mEn}$

For lowest energy level n = 1

$E_1 = \frac{1^2 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$E_1 = 1.50 \times 10^{-18}$ J

Or $E_1 = 9.37$ ev

$\therefore$ $P_1 = 1.66 \times 10{-}24$ kg-m/sec

Problem 3.63: *Lowest energy of an electron trapped in a potential well is 38 eV. Calculate the width of the well.*

(h : 6.63×10^{-34} Js, m: 9.1×10^{-31} kg) **(3) (May 19)**

Data : n = 1, E = 38 eV

Formula : $E_n = \frac{n^2h^2}{8mL^2}$

Solution : $E_n = \frac{1 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{19}}$

$= 9.895 \times 10^{-34}$

$L = 0.994 \times 10^{-10}$ m = 1A°

Problem 3.64 : *The lowest energy of an electron trapped in a rigid box is 4.19 eV. Find the width of the box in A.U.*

(3) (Nov. 15)

Data: n = 1, E = 4.19 eV

Formula: $E = \frac{n^2h^2}{8mL^2}$

Solution: $L = \sqrt{\frac{n^2h^2}{8mE}}$

$L = \sqrt{\frac{(1)^2 (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 4.19 \times 1.6 \times 10^{-19}}}$

$= 2.987 \times 10^{-10}$ m

= 2.987 A°

3.10 INTRODUCTION TO QUANTUM COMPUTING

Principle and Limitation of Classical Computing

- The Classical computing relies on principles of Boolean algebra, operating with logic gate principle. Data is processed in an exclusive binary that is, either 0 (off / false) or 1 (on / true). These values are binary digits, or bits. The transistors and capacitors in CPU can only be in one state at any point either in 0 or 1.
- For smaller and faster circuits, we need to reach very smaller dimension of materials and ultimately a threshold is reached for application of classical laws of physics.

Principle of Quantum Computing

- The Quantum computing applies the properties of quantum physics to process information. A number of elemental particles such as electrons or photons can be used, with either their charge or polarization acting as a representation of 0 and/or 1. Each of these particles is known as a quantum bit, or qubit. A qubit is a unit of quantum information.

Superposition and Entanglement

- A qubit can hold both values (0 and 1) at the same time known as a superposition state. Thus, the number of computations that a quantum computer could undertake is 2^n, where n is the number of qubits used. For example, a quantum computer comprised of 500 qubits would have a potential to do 2500 calculations in a single step.
- Entanglement is correlation between particles acting as q-bits such as photons, electrons. By knowing the spin state of one entangled particle - up or down — we can know the spin of its correlated particle is in the opposite direction. Quantum entanglement allows qubits separated by distances to interact with each other instantaneously. When multiple qubits act coherently, they can process multiple options simultaneously. This allows them to process information within a fraction of the time.

Speed of Quantum Computer

- A classical computer has a clock speed measured in gigahertz, a processing speed of a few billion simple logic operations per second. The processing power of quantum computers is measured in teraflops or trillions of logic operations per second.
- In 2015, Google and NASA reported that their new 1097-qubit D-Wave quantum computers would be almost 100 million times faster than a regular computer chip. It is estimated that quantum computer would process the information within few seconds that classical computer would take 10,000 years to solve.

Potential Applications of Quantum Computing

- Currently quantum computers are in the stage of development. Based on estimation, it is predicted that quantum computers has tremendous potential to deal with many challenges that are otherwise almost impossible to cope up with existing classical computers.

Artificial Intelligence

- Artificial intelligence requires analysis of data from large datasets of image, video and text. This data is available in vast quantity. It is estimated that such unanalyzed data would be of around few petabytes. For analyzing and processing this huge data, traditional computers would require thousands years. Quantum computers would be able to process this data in few seconds.

Drug Design

- It requires a kind of trial and error method for many of the drugs to understand how they will react. These methods are very expensive, complex and require much processing time. Using quantum computers the process can be simulated more effectively.

Financial Optimization

- Currently classical computers are analyzing many financial tasks such as market analysis, estimated returns, risk assessment, financial transactions, etc are. It requires complex algorithms and tremendous computational time. By utilizing quantum technology great improvements could be achieved in terms of time saving and more accuracy.

Development of New Materials

- In materials science to develop new materials or to increase efficiency of existing materials, it requires lot of simulating. Although classical computers deal with these simulations, they have limitations in terms of speed, accuracy and time. Quantum computers would be able to deal with these challenges more effectively.

Logistics and Scheduling

- In industry optimizations are used in logistics and scheduling. Few examples to optimize route based on real time traffic analysis, industrial automation of process and production to maximize output cost effectively.
- At present classical computing is heavily used to optimize these tasks, some of the processes are very complicated for a classical computers. Quantum computing would be able to perform these tasks provide a solution in terms of less time and more accuracy.

Cyber Security

- In the era of deep penetration of internet, cyber security is one of the biggest challenges. Malware and viruses spread trough internet within fraction of seconds, which is difficult for classical computing to deal with.
- Various techniques to deal with cyber security threats can be developed using some of the quantum machine learning approaches to recognize the threats earlier and reduce the damage.

Dealing with Encryption

- In cyber age, security encryption methods are heavily deployed in areas such as defense, financial sectors, banks, user data security, etc. These organizations are under constantly under threat of cyber attack despite of heavy deployment of security measures using classical computing.
- The complex encryptions such as 2048 bit RSA encryptions are extremely difficult to deal with existing computing technologies. It has been demonstrated that quantum computing would deal with these tasks very easily It would be virtually impossible to break these encryptions.

Software Testing, Fault Simulation

- For very large software programs or large ASIC chips that have billions of transistors, it becomes difficult and expensive to verify them for correctness.
- There can be billions of different possibilities of error in codes and it is impossible for a classical computer to check every single one in simulation. Quantum computing can provide a much better dealing with these tasks with great improvement.

SUMMARY

- **De Broglie's Hypothesis :** States that a material particle of mass 'm' moving with a velocity v has a wave associated with it. This wave is called as a De Broglie wave and its wavelength is given by $\lambda = h/mv$.
- **De Broglie Wavelength :** In terms of kinetic energy of a particle is $\lambda = \frac{h}{\sqrt{2mE}}$.
- For a **Charged Particle** accelerated through a potential difference of V, the De Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mqV}}$.

  For an electron, the De Broglie wavelength is $\lambda = \frac{12.27}{\sqrt{V}}$ A°.
- **Wave Velocity :** The speed with which the De Broglie wave travels is called the phase velocity v_p. It is also called as the wave velocity (u).
- **Phase Velocity :** The phase velocity of matter waves is given by $v_p = u = \frac{c^2}{v}$, which is greater than the speed of light.
- **Heisenberg's Uncertainty Principle :** If states that it is impossible to determine precisely and simultaneously the values of both the members of a

pair of physical variables which describe the motion of an atomic system. Such pairs of variables are called as canonically conjugate variables.

$$\Delta x \cdot \Delta p \geq h$$

(Heisenberg's uncertainty principle for position and momentum.)

$$\Delta E \cdot \Delta t \geq h$$

(Heisenberg's uncertainty principle for energy and time.)

- **Schroedinger's Wave Equation** : It is the mathematical representation of matter waves associated with a moving particle. They are of two types:

  (i) Schroedinger's time independent wave equation:

  $$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

  (ii) Schroedinger's time dependent wave equation:

  $$\left(\frac{-h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial\psi}{\partial t}$$

  i.e. $H\psi = E\psi$

- **Wave Function :** The variable quantity characterizing De Broglie waves is denoted by ψ and it is called the wave function of the particle. This wave function contains all the information about the particle.
- **Probability :** The quantity $|\psi(x, y, z, t)|^2$, called the probability density or probability distribution function, determines the probability in unit volume of finding a particle at a given position at a given time.
- The probability of a particle being present in a volume element dx·dy·dz is $|\psi|^2 \cdot dx\, dy\, dz$.

  The probability of finding the particle in all space is

  $$\iiint_{\text{all space}} |\psi|^2\, dx\, dy\, dz = 1.$$

  This is the **Normalization Condition**. A wave function ψ satisfying this relation is called a normalized wave function.
- For a particle in a **Rigid Box**, the wave function is given by

  $$\psi_n(x, t) = i \cdot \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{n\pi}{L}\right)x$$

- The **Energy Level** of the particle is given by

  $E_n = \frac{n^2 h^2}{8mL^2}$ and is quantized. n = 1, 2, 3.

- For a **Non-Rigid Box**, the particle wave functions are longer than the wave functions in the case of rigid box.
- **Tuneling Effect :** Particles trapped in a box, despite having insufficient energy ($E < V_0$) to penetrate the walls some how leak out. This is a tunneling effect which is a behaviour unique to Quantum Mechanics.
- Alpha Decy, STM and Tunned diode are examples of tunneling effect.

IMPORTANT FORMULAE

- Quantum computing uses charge or polarizastion to represent 0 or 1.
- $\lambda = \frac{h}{mv} = \frac{h}{p}$
- $E = h\upsilon$, $E = \frac{p^2}{2m}$
- $E = mc^2$
- $\lambda = \frac{h}{\sqrt{2mE}}$
- $\lambda = \frac{h}{\sqrt{2meV}}$ and $\lambda \cong \frac{12.27}{\sqrt{v}}$ A°
- $u = \frac{h}{2m\lambda}$
- $\Delta x \cdot \Delta p \geq h$
- $\Delta E \cdot \Delta t \geq h$
- $\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$
- $\left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial\psi}{\partial t}$; $H\psi = E\psi$
- $E_n = \frac{n^2 h^2}{8mL^2}$; where n = 1, 2, ...
- Normalization condition $\iiint_{-\infty}^{+\infty} + \psi^* dx\, dy\, dz = 1.$

EXERCISE

1. Explain briefly the wave nature of matter and obtain an expression for the De-Broglie wavelength.
2. Starting from the De-Broglie concept, obtain Heisenberg's uncertainty principle. Give an illustration of the principle.
3. Show that De-Broglie wavelength of a charged particle is inversely proportional to the square root of the accelerating potential.

4. Write a short note on duality of matter and radiation.
5. Write a note on properties of matter waves. How are they different from electromagnetic and mechanical waves ?
6. What are group and phase velocities ? Explain them.
7. Discuss the properties of matter waves.
8. What are matter waves ? Show that the wavelength λ associated with an electron of mass m and kinetic energy E is given by $\lambda = \frac{h}{\sqrt{2\,mE}}$
9. Write a short note on Heisenberg's uncertainty principle.
10. State Heisenberg's uncertainty principle and explain it using the concept of De-Broglie wave groups.
11. Derive Schroedinger's time dependent and time independent wave equation.
12. Derive an expression for the energy levels and wave function of a particle enclosed in an infinite potential well.
13. Explain Harmonic oscillator classically and quantum mechanically. Compare between the two.
14. Explain the physical significance of ψ, ψ^2.
15. Explain particle in a non-rigid box. How is this situation different from a rigid box ?
16. Write a note on a particle in a potential well with infinite sides.
17. Write a short note on eigen values of particle in a rigid box.
18. Starting with the wave function for a particle moving in a one-dimensional potential well, show that the probability of existence within the well changes with quantum number.
19. Using Schroedinger's wave equation, find energy and wave function of a particle in a rigid box.
20. Explain De-Broglie's concept of matter waves. Show that the wavelength associated with electrons accelerated by a potential difference of V volts is given by $\frac{h}{\sqrt{2\,meV}}$.
21. Explaining De-Broglie's hypothesis of matter waves, describe an experiment in support of it.
22. State and explain the principle of uncertainty and illustrate it by an experiment on diffraction at a single slit.
23. State and explain Heisenberg's uncertainty principle. Illustrate it by the experiment for location of a particle by microscope.
24. Discuss properties of matter waves.
25. What is tunneling ? Explain working of tunneling diode.
26. Write a short note on STM.

UNSOLVED PROBLEMS

1. Calculate the De-Broglie wavelength of a 10 keV neutron.
   Given: Mass of the neutron = 1.67×10^{-27} kg.
   (**Ans.** $\lambda = 0.285$ A°)
2. The De-Broglie wavelength of electrons in a monoenergetic beam is 7.2×10^{-11} m. Calculate the momentum and energy of electrons in the beam in electron volts.
   (**Ans.** 0.92×10^{-23} kg-m/sec., E = 288 eV)
3. An electron is bound by a potential box of infinite height having a width 2.5 A°. Calculate the minimum uncertainty in its velocity. (**Ans.** 0.725×10^{7} m/sec.)
4. Calculate the wavelength associated with a particle of mass 1 g and moving with a velocity of 2×10^{2} cm/sec. Describe the possibility of performing a successful diffraction experiment with a beam of such particles.
   (**Ans.** 3.312×10^{-34} m, experiment not successful)
5. A bullet of mass 25 gm is moving with a speed of 400 m/sec. The speed is measured accurately upto 0.02 %. Calculate the certainty with which the position of the bullet can be located.
   Given h = 6.6×10^{-34} J.s. (**Ans.** 5.25×10^{-32} m)
6. Assume that a particle cannot be confined to a spherical volume of diameter less than the De-Broglie wavelength of the particle. Estimate the minimum K.E. a proton confined to a nucleus of diameter 10^{-14} m may have. What K.E. would an electron have to possess if it were confined to this nucleus ? (**Ans.** 8.2 MeV, 124 MeV)
7. What potential difference must be applied to an electron microscope to produce electrons of wavelength 0.4 A° ? (**Ans.** 937.9 volts)
8. The average time that an atom retains excess excitation energy before emitting it as electromagnetic radiation is 10^{-8} sec. Calculate the limit of accuracy with which the excitation energy of the emitted radiation can be determined.
   (**Ans.** 6.63×10^{-26} J)

9. Find the lowest K.E. permissible for an electron in (i) a cubical box of side 1 cm. (ii) same box of side 3 A°.
(**Ans.** 1.13×10^{-14} eV, 12.6 eV)

10. An electron is bounded by a potential that is approximated by an infinite square well of width 2×10^{-8} cm. Calculate the lowest two permissible energies of the electron. (**Ans.** 19 eV, 38 eV)

UNIVERSITY QUESTIONS

December 2012

1. Define Phase Velocity and Group Velocity. Hence obtain the relation between V_p and V_g for DeBroglie Wave. **[6]**
2. State DeBroglie's Hypothesis. Hence obtain the relation for DeBroglie's Wavelength in terms of Energy. **[4]**
3. Explain the physical significance of ψ and $|\psi|^2$. **[4]**
4. Derive Schrodinger's Time Independent Wave Equation. **[6]**

May 2013

1. Derive Schroedinger time independent wave equation. **[6]**
2. Define phase velocity, Group velocity and Derive their expressions **[4]**
3. Explain Heisenberg Uncertainty Principle and prove this principle using single slit Diffraction experiment **[6]**
4. Does the matter waves are electromagnetic waves? Explain. **[3]**
   (1) The matter waves are generated by charged as well as uncharged particles where as electromagnetic waves are generated only by charged particle.
   (2) The velocity of matter waves is not constant but depends on velocity of particles generating them i.e. $u = \frac{C^2}{V}$. Where as electromagnetic waves travel with uniform velocity C in a given medium.

November 2013

1. Derive an expression for energy of a particle trapped in an infinite potential wall. **[6]**
2. State and explain De Broglie's hypothesis of matter waves. State any two properties of matter waves.
3. The uncertainty in the location of the particle is equal to its De Brogile wavelength. Show that the uncertainty in the velocity of a particle is equal to the particle velocity itself. **[3]**
4. State and explain Heisenberg's uncertainty principle. Illustrate it by an experiment of electron diffraction at a single slit. **[6]**
5. What is wave function ψ ? Write down the conditions satisfied by wave function ψ. **[4]**
6. Calculate the energy difference between the ground state and first excited state of an electron in the rigid box of length 1 A°. **[3]**

May 2014

1. State and explain Heisenberg's uncertainty principle. Prove the same for pair of variables energy and time. **[6]**
2. Explain in brief, working of Scanning Tunneling Microscope (STM). **[4]**
3. Deduce Schrodinger's time independent wave equation. **[6]**
4. Define phase velocity of a matter wave. Show that phase velocity of matter wave is greater than velocity of light. **[4]**
5. Starting from $\lambda = \frac{h}{mv}$, obtain $\lambda = \frac{h}{\sqrt{2mE}}$ where E is KE of the particle. **[3]**

December 2014

1. Deduce Schrodinger time independent wave equation. **[6]**
2. Define group velocity. Show that the group velocity of matter wave is equal to particle velocity. **[4]**
3. Calculate the de Broglie wavelength of electron having kinetic energy 1 KeV. **[3]**
4. State Heisenberg's uncertainty principle and prove it by thought experiment of electron diffraction at a single slit. **[6]**
5. What is wave function ? Explain what is normalization of wave function. **[4]**
6. An electron is trapped in a rigid box of width 2 A°. Find its lowest energy level. **[3]**

May 2015

1. State and explain Heisenberg's Uncertainty principle. Illustrate the same with electron diffraction at a single slit. **[6]**
2. What is wave function ψ ? Give the physical significance of it. **[4]**
3. An electron is trapped in a rigid box of width 2 A.U. Find its lowest energy in eV. **[3]**
4. Deduce SchrÖdinger's time independent wave equation. **[6]**
5. Define phase velocity and prove that it is always greater than velocity of light. **[4]**

November 2015

1. Deduce Schrodinger's time independent wave equation. [6]
2. Define phase (wave) velocity. Show that the phase velocity of matter wave is greater than the velocity of light. [4]
3. State Heisenberg's Uncertainty principle and prove it by thought experiment of electron diffraction at a single slit. [6]
4. What is wave function ? Explain what is normalization of wave function. [4]

May 2016

1. Deduce Schrodinger's time independent wave equation. [6]
2. State and explain Heisenberg's uncertainty principle. [4]
3. Define phase velocity and group velocity. Show that group velocity is equal to particle velocity. [6]
4. Explain why probability of finding of a particle cannot be predicted by the interpretation of wave function ψ. Explain physical significance of $|\psi|^2$. [4]

December 2016

1. Derive an expression for energy of a particle trapped in an infinite potential well. [6]
2. Define phase velocity and group velocity. Derive the relation between them. [4]
3. What is De-Broglie's hypothesis of matter waves. Show that the De-Broglie's wavelength of a charged particle is inversely proportional to the square root of the accelerating potential. [6]
4. Write down the conditions which are to be satisfied by well behaved wave function. [4]

May 2017

1. What is de-Broglie hypothesis. Derive an expression for de-Broglie wavelength for an electron when it is accelerated by potential difference 'V'. [4]
2. Explain physical significance of wave function ψ and $(\psi)^2$. State the mathematical conditions that wave function ψ should satisfy. [4]

November 2017

1. State and explain Heisenberg's Uncertainty Principle. Show that it is also applicable for energy and time. [6]
2. State and explain de-Broglie hypothesis of matter waves. Explain in brief any two properties of matter waves. [4]
3. Starting from de-Broglie hypothesis, derive schrodinger's time independent wave equation. [6]

May 2018

1. State and explain Heisenberg's uncertainty principle. Illustrate it by an experiment of diffraction at a single slit. [6]
2. State de-Broglie hypothesis of matter waves. Derive the expression for matter waves for an accelerating particle in terms of its kinetic energy. [4]
3. Derive equation of energy when a particle is confined to an infinite potential well. Draw first three energy levels for an electron confined in it. [6]
4. Explain wave-function Ψ. Give the physical significance of $|\Psi^2|$. [4]

November 2018

1. Obtain an expression for Heisenberg's Uncertainty Principle for energy and time. [4]
2. State and explain Heisenberg's Uncertainty Principle. Show that it is also applicable for energy and time. [6]

May 2019

1. State and explain Heisenberg's uncertainty principle. Illustrate the principle of electron diffraction at t single silt. [6]
2. What is De-Broglie hypothesis? Derive an expression for De-Broglie wavelength for an electron when it is accelerated by potential difference V. [4]
3. Deduce Schrodinger's time independent wave equation. [6]
4. Write down the conditions which are to be satisfied by well behaved wave function. [3]

UNIT IV

SEMICONDUCTOR PHYSICS

4.1 INTRODUCTION TO SOLID STATE PHYSICS

- From the engineering point of view, knowledge of electric, magnetic and dielectric properties of materials is very essential.
- Basically, solids can be classified into different categories such as conductors, insulators and semiconductors. The classification of materials from electrical point of view into conductors, semiconductors and insulators is based according to their resistivity range.
- For conductors the resistivity ranges from 1.6×10^{-8} to 1.4×10^{-6} ohm-m. The resistivity range of semiconductors and insulators is respectively from 10^{-4} to 10^{6} ohm-m and from 10^{7} to 10^{16} ohm-m.
- Considering the rapid development of semiconductor electronics, stress has been given to discuss the semiconductor devices in this chapter. Semiconductor devices are highly compact, low power consuming and efficient. They have replaced vacuum tubes to a great extent.
- For a better knowledge of semiconductors, one should understand the properties of semiconductors on the basis of band theory of solids. For this, elementary knowledge of electronic configuration of atoms and quantum numbers is quite essential.

4.1.1 Electron Configuration of Atoms

- Different physical and chemical properties of various elements are attributed to different configuration of electrons in their atoms. The four quantum numbers that specify the electron state in an atom are n, l, m_l and m_s.
- As per ***'Pauli's exclusion principle'***, no two electrons in an atom can have the same set of quantum numbers n, l, m_l and m_s. This principle gives the arrangement of electrons in different orbits.
- The various subshells are filled in the following manner:
  - ➢ As we go from one atom to the next in the order of increasing atomic numbers, the electrons are added one by one to various shells.
  - ➢ Electrons enter different subshells in the order of increasing energy. The subshell of lower energy is filled up first while the higher energy subshell is filled up later.

The sequence of filling different subshells can be remembered from Fig. 4.1.

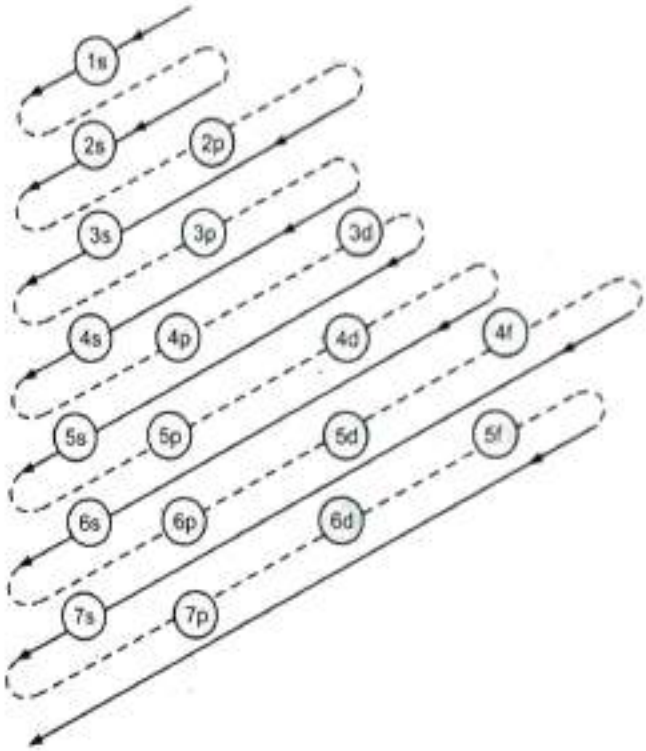


Fig. 4.1

  - ➢ As per Pauli's principle, an orbital cannot have more than two electrons.
  - ➢ According to Hund's rule, electron pairing does not take place in p, d, f orbitals until all the orbitals of the given subshell contain one electron each with parallel spin.

For example, the filling of various orbitals of nitrogen (Z = 7) can be

$$1s^2\ 2s^2\ 2p_x^1\ 2p_y^1\ 2p_z^1$$

↑ ↓	↑ ↓	↑ ↑ ↑
1s	2s	2p

4.1.2 Electron Energy States of an Isolated Atom

- An isolated atom of an element with atomic number Z and mass number A consists of a positively charged nucleus, with Z protons and (A – Z) neutrons around which Z electrons revolve in different orbitals. The orbits are characterized by a set of four quantum numbers n, l, m_l and m_s.
- The distribution of electrons in an atom i.e., energy states decide the properties of the element to which the atom belongs. The energy of an electron in an atom depends on n as well as l i.e. energy of the electron is a function of n, l, or E = E (n, l).
- As n and l can have only discrete values, the energy E will have discrete values. The energy states characterized by n, l numbers are generally degenerate i.e. electrons with different set of quantum numbers will have the same energy, due to different m_l values for a given l. The state with same n and l will be (2l + 1) degenerate.
- The number of electrons that can have the same energy E (n, l) with given n and l is 2 (2l + 1), the factor 2 is due to two possible values of m_s for each m_l. The state s is non-degenerate and has two electrons. But p, d, f states are respectively 3-fold, 5-fold, 7-fold degenerate and the number of electrons in those states are 6, 10, 14 respectively.
- As such the energy states of an isolated atom will be quite discrete. The energy states of an isolated lithium atom are shown in Fig. 4.2.

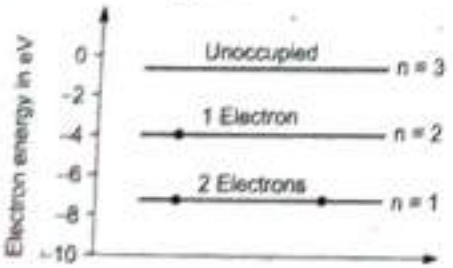


Fig. 4.2

4.2 CONCEPT OF ELECTRICAL CONDUCTIVITY

4.2.1 Concept of Free Electron

- The electrons in a partially filled conduction band are relatively free to move in the specimen as they are not bound to any atom but are in a band which is shared by the entire crystal. These electrons are called as ***'free electrons'***.
- The electrons well inside the specimen are surrounded by positive ion cores from all the sides, hence net force exerted by the iron cores (or lattice points) on electrons is zero. Due to the thermal energy, the electrons are in constant motion with average velocity of 10^5 m/s at room temperature. But the direction of motion of electron is totally random and take a very complex path. This results in net zero current.
- On the way electrons collide with the lattice points which are in vibration motion due to thermal energy. This reduces the velocity of electrons and comes to almost rest. This results in small amount of resistivity and the loss of energy is converted into thermal energy.
- The average time between collisions is called the ***'mean free time'*** and the length of the path during this free time is called the ***'mean free path'***. The mean free paths have different lengths. Fig. 4.3 (a) shows motion of electrons under equilibrium conditions.

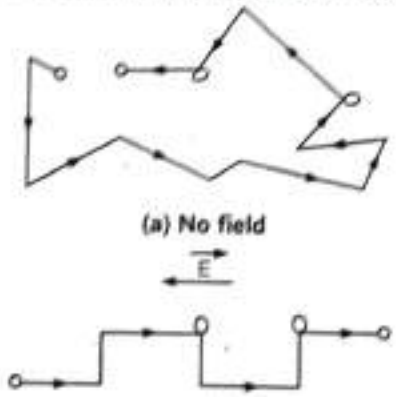


Fig. 4.3 : Motion of free electrons

- In presence of field $\vec{E}$, the direction of motion will be directed along the electric field. This results into net current flowing in a particular direction.
- As the electron tries to move in the field, the number of collisions increases. In fact, the external field simply bends the path of the electron between two collisions. In general, the conductivity depends on number of charge carriers and their freedom to move.

4.2.2 Free Electron Theory

- The free electron theory of metals was first proposed by Drude and later improved by Lorentz and hence the theory is called the Drude-Lorentz theory. Following are the basic assumptions made in the theory.
  - ➢ All metals contain a fixed number of valence electrons forming an ***electron gas***, which are free to move throughout the volume of the metal like molecules of a gas.

- The electron velocities in metals obey the classical ***Maxwell-Boltzmann distribution*** of velocities.
- The positive ions which can vibrate about their mean position, cannot move from one lattice site to another. The repulsive force between the negatively ***charged electron*** is ignored and the electric field due to the positive ions is assumed to be uniform.
- The electrons move from one point to another randomly with ***random velocity*** which is temperature dependent. At room temperature, this velocity is about 4×10^5 m/s.
- The ***kinetic energy*** of the electron is given by 3 kT/2, where k is Boltzmann's constant and T is absolute temperature.
- In absence of external electric field, the electrons move in ***random directions***, making collisions from time to time with positive ions, which are fixed in lattice. This makes net current zero.
- When an electric field is applied, free electrons move towards positive terminal of the supply. Thus, the electrons will experience two motions – random motion due to temperature and drift motion due to applied voltage. As a result the electron will move in ***opposite direction to the electric field*** while maintaining their random motion.
- While drifting towards positive of the supply, the electrons colloid with positive ions. During each collision the electron loses all its drift velocity and starts from rest once again. The average distance covered by an electron between collisions is known as ***mean free path*** 'λ' and time taken to cover this distance is termed as relaxation time 'τ'.
- As the temperature increases, the vibration of the ion core increases, this increases the probability of electron-core collision. As a result, ***resistivity increases with increase in temperature***.

4.2.3 Drawbacks of Classical Free Electron Theory

- The free electron theory, successfully established Ohm's law, showed that the resistivity is directly proportional to the temperature and the Wiedemann-Franz relation was proved. However, the theory has many drawbacks.

The Main Drawbacks are:

- The specific heat capacity value based on classical theory shows that it is independent of temperature. But as per quantum theory, it directly depends on temperature i.e. it increases with the increase in temperature.
- As per classical theory, the paramagnetic susceptibility is inversely proportional to the temperature. But experimental results show that it is almost independent of temperature.
- The classical theory failed to explain occurrence of long mean free paths (10^8 or 10^9 times interatomic spacing).
- Classification of solids on band theory i.e. metals, semimetals, semiconductors and insulators cannot be done by classical theory.
- The positive values of Hall coefficient of metals could not be explained by classical theory.
- Classical theory also failed to explain photoelectric effect, Compton effect and black body radiation.

4.3 OPENING OF BAND GAP DUE TO INTERNAL ELECTRON DIFFRACTION

- When an electron moves through a lattice of positive ions, it experiences varying potentials. The potential of an electron at the positive ion site is zero and is maximum in between two ions. (shown in Fig. 4.4 (a)).
- The potential experienced by an electron, when it passes along a line through the positive ions is as shown in Fig. 4.4 (b).
- Kronig and Penney approximated these potentials inside the crystal to the shape of rectangular steps as shown in Fig. 4.4 (c).

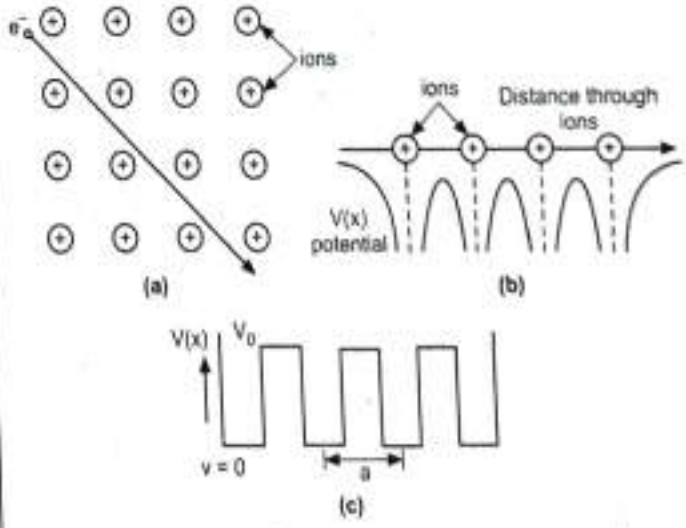


Fig. 4.4

- The energies of electrons can be known by solving Schrödinger's wave equation in such a lattice.
- By solving the Schrödinger's wave equation using Bloch theorem, we get Energy vs K (momentum) diagram, as shown in Fig. 4.4 (d).
- The dotted line is representing the motion for a free electron. As electron is moving in a periodic potential, it appears that there are gaps which open in energy spectrum at an integer of ka/π.
- These gaps correspond to energy bands inside the solids. Thus, the energy level of electron in the crystal lattice is discrete.

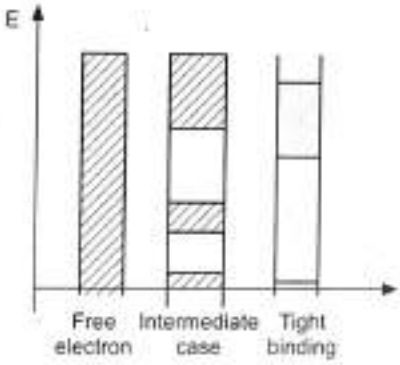


Fig. 4.5

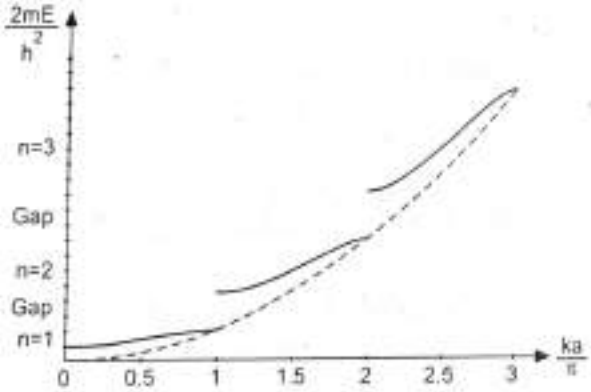


Fig. 4.6

4.3.1 Band Theory of Solids

- In a single isolated atom, the electrons in each orbit have definite energy associated with it. But in case of solids all the atoms are close to each other, so the energy levels of outermost orbit electrons are affected by the neighboring atoms.
- Band formation is preferred in solids as atomic spacing is very small. In liquids and gases as atomic spacing is much larger as compared to solids, the energy levels of outermost orbit electrons are not affected by neighboring atoms and band formation is not preferred.
- When two single or isolated atoms are bring close to each other then the outermost orbit electrons of two atoms are interact or shared with each other i.e. the electrons in the outermost orbit of one atom experience an attractive force from the nearest or neighboring atomic nucleus. Due to this the energies of the electrons will not be in same level, the energy levels of electrons are changed to a value which is higher or lower than that of the original energy level of the electron.
- The electrons in same orbit exhibit different energy levels. The grouping of these different energy levels is called energy band.
- However, the energy levels of inner orbit electrons are not much affected by the presence of neighboring atoms.

There are number of energy bands in solids but three of them are very important. These three energy bands are important to understand the behavior of solids. These energy bands are - Valence band, Conduction band, Forbidden band or forbidden gap

Valance Band:

- The energy band which is formed by grouping the range of energy levels of the valence electrons or outermost orbit electrons is called as valence band.
- In the solids, valance electrons are not bound to any particular atom therefore, the valance band arises from the splitting of the valance energy levels is common to the entire crystal.
- The valance electrons are available for conduction in valance band.

Conduction Band:

- The energy band which is formed by grouping the range of energy levels of the free electrons is called as conduction band.
- The band of allowed energy levels higher to the valence band will be vacant at absolute zero.
- At temperature, other than absolute zero, electrons are excited to this upper band. The excited atoms are said to be free and are responsible for electrical conduction in the solid.
- The conduction band comprises of the free electrons that are capable of electrical conduction
- The conduction band is also common to the entire crystal.

Forbidden Energy Gap/Bandgap:

- The energy gap which is present between the valence band and conduction band by separating these two energy bands is called as forbidden band or forbidden gap.
- Ideally it is the energy difference between top of the valence band and bottom of the conduction band.
- In solids, electrons cannot stay in forbidden gap because there is no allowed energy state in this region.
- Forbidden gap is the major factor for determining the electrical conductivity of a solid. The classification of materials as insulators, conductors and semiconductors is mainly depends on forbidden gap.

4.4 DENSITY OF STATE

- The energy band in a solid can accommodate a large number of electrons in the energy levels. Every energy level in a energy band consists of two states and each state can accommodate only one electron. Thus, an electron can have only two states with opposite spins.
- For understanding the properties of solids it is important how energy levels are distributed in a band and how electrons are distributed in the energy levels.
- It is also important to know the energy states available for accommodating the electrons. The number of states lying in the range of energies between E and E + dE is given by

$$g(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}\,dE \qquad \ldots (1)$$

The function g(E) is known as ***density of states function***.

- The ***density of states*** is defined as the number of available states per unit volume per unit energy interval centered around E. The variation of density of states with energy is shown in Fig. 4.7.

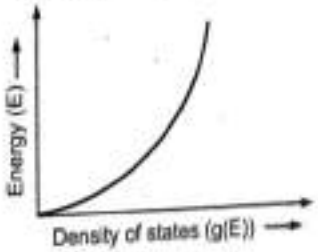


Fig. 4.7: Density of states

- It is seen from the graph that the number of available states decreases with energy and becomes zero at the bottom edge of the band.
- Although the density of states give the number of states that could be occupied by the charge carrier, but at the energy states may not be filled in an energy band.
- A particular energy level E is occupied or not is determined by the probability P(E) that a carrier can have the energy E. Therefore, the number of carriers per unit volume within a given energy range depends both on the number of available states within the range and probability that the carrier can occupy the states.
- The carrier concentration in energy range dE will be the product of the density states and probability of their occupation.

Therefore, the carrier concentration is given by

$$N(E)\,dE = P(E)\,g(E)\,dE$$

where N(E) is the carrier distribution function.

4.5 EFFECTIVE MASS OF ELECTRONS AND HOLES

- An isolated electron or free electron moving in vacuum has a well defined mass and obeys Newtonian mechanics when accelerated by an electric field. But inside the crystal an electron is not free to move.
- Consider an electron inside a material moving under the external electric or magnetic field. The electron will experience periodic variation of potential due to ions while moving in the material. Now, the electron moving through this changing potential under the action of external electric or magnetic field, the mass of electron is different than free electron in vacuum and is referred as ***effective mass of electron***. The effective mass of an electron is denoted by m*.
- The effective mass of an electron is given by the relation.

$$m^* = \left(\frac{h}{2\pi}\right)\left(\frac{d^2E}{dk^2}\right)^{-1} \qquad \ldots (1)$$

where, h = Planck's constant

k = Wave number

k = Number of complete wave cycles in one meter of linear space

$$k = \frac{2\pi}{\lambda}$$

- When electron moves inside the lattice, its position in terms of k varies and therefore the effective mass. The

effective mass cannot be only greater than m but also less than m. Sometimes, it can be negative also.

- Fig. 4.8 shows the variation of effective mass with wave number. When k = 0, the effective mass of electron approaches the mass of free electron ($m^* = m$). As the value of k increases m^* increases reaching a maximum value at the point of inflection.
- Above the point of inflection, m^* is negative and at $k \to \pi/a$, it decreases to a small negative value, where 'a' is lattice constant.

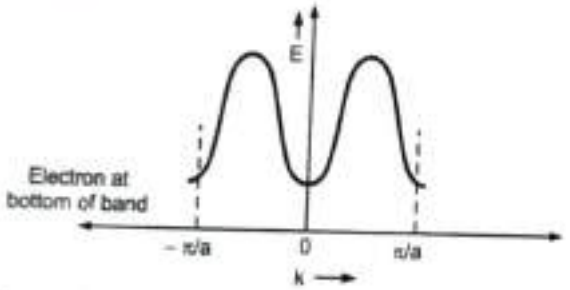


Fig. 4.8: Variation of effective mass with wave number

- The effective mass of an electron depends on its location. The electrons near the bottom of the conduction band have an effective mass equal to mass of the free electron while the electrons near the top of the valence band have negative effective mass.
- Near the bottom of the band, the effective mass m^* has a constant, value which is positive. But as k increases, m^* varies as it is a function of k. Beyond the inflection point, the mass m^* becomes negative, the region is close to the top of the band.
- The velocity of electron decreases for $k > k_o$, thus acceleration is negative. Therefore, the velocity will be opposite to the applied force, implying a negative mass.

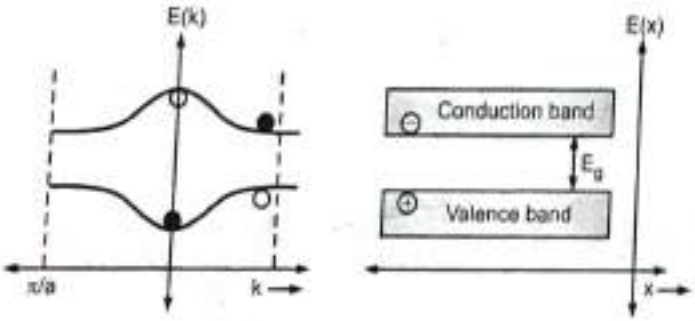


Fig. 4.9: Concept of electron and hole

- In this region of k-space, the lattice exerts such a large retarding force on the electron that it overcomes the applied force and produces a negative acceleration. Thus, in upper half of the band, the removal of a electron with a negative effective mass is identical to creating in its place a particle of positive effective mass. This is termed as ***hole*** and a hole is considered to have an effective mass m_h^*.

4.6 FERMI-DIRAC PROBABILITY DISTRIBUTION FUNCTION

(May 13, 14, 15, 16, Dec. 14, Nov. 15)

1. Fermi Level in Conductors or Metals

- The statement that a solid is composed of N atoms implies that each atomic level splits into N-energy levels and bands of energy are formed. The filling of the bands follows a simple rule. States of lowest energy are filled first, then the next lowest and so on, till all the electrons are accommodated.
- The highest filled state is called the **Fermi level** and its corresponding energy is called the '***Fermi energy***' E_F. The magnitude of E_F depends on the number of electrons per unit volume in the solid because the electron density determines how many electrons must go into the bands.
- At 0 K, all states upto E_F are full and all states above E_F are empty.
- At higher temperatures, the random thermal energy will empty a few states below E_F by elevating a few electrons to yet higher energy states. No transitions to states below E_F occur as they are full. Thus, an electron cannot change its state unless enough energy is provided to take it above E_F.
- **The highest filled state in the highest energy band which contains electrons in a metal, at 0 K, is called the Fermi level and its corresponding energy is called the Fermi energy E_F.**

2. Fermi Level in Semiconductors

- In semiconductors, the Fermi level is a reference level that gives the probability of occupancy of states in conduction band as well as in valence band.
- In case of intrinsic semiconductors, the band picture consists of a band of completely filled states called as the '***valence band***' separated from a band of unoccupied states called as the '***conduction band***', by an energy gap E_g. For an intrinsic semiconductor, the Fermi level lies at the centre of the forbidden band, indicating that the states occupied in conduction band are equal to the states unoccupied in valence band. In other words, for every electron in the conduction band, there is a hole in the valence band.

- **So Fermi level in the semiconductors may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes when 'weighted' according to their energies.**

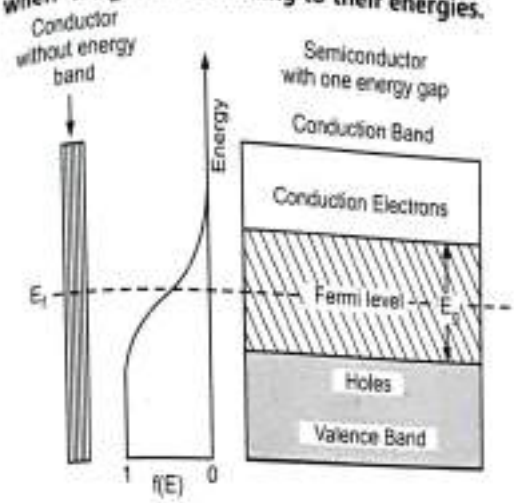


Fig. 4.10

- However, it is to be noted that Fermi level is only an abstraction. A hollow body can have a centre of gravity at the centre where there is no matter. Similarly, a material can have a Fermi level at an energy which is forbidden to all electrons. For example, in an intrinsic semiconductor, the Fermi level is at the centre of the forbidden band.
- An electrons in solids obey Fermi-Dirac statistics. In the development of this type of statistics, the wave nature of electrons and the Pauli's exclusion principle will have to be taken into consideration.
- The result of the statistical arrangement gives the distribution of electrons over a range of allowed energy levels at thermal equilibrium.

i.e. $$P(E) = \frac{1}{1 + e^{(E - E_f)/kT}} \quad ...(1)$$

where k is Boltzmann's constant.

- The Fermi-Dirac distribution function P(E) gives the probability that an energy state of energy E will be occupied by an electron at absolute temperature T. The quantity E_F is the Fermi energy.
  - For an energy E equal to the Fermi level E_F, the probability of occupation is given by

$$P(E_f) = [1 + e^{(E_F - E_f)/kT}]^{-1} \quad ...(2)$$

$$P(E_F) = \frac{1}{2}$$

Thus, an energy state at the Fermi level has a probability $\frac{1}{2}$ of being occupied by an electron for a temperature T > 0 K.

  - At T = 0 K for $E < E_F$, the term $e^{(E - E_F)/kT} = 0$ so that P (E) = 1; i.e. the probability of finding an electron with energy less than Fermi energy is unity. Or it can be said that at T = 0 K all energy states below E_F have a probability of occupancy of unity i.e. they are certainly occupied.
  - At T = 0 K for $E > E_F$, the term $e^{(E - E_F)/kT} = \infty$ so that P (E) = 0; i.e. the energy states above E_F have zero probability of occupancy and they are therefore empty. The probability function plotted for different temperatures is shown in Fig. 4.11.

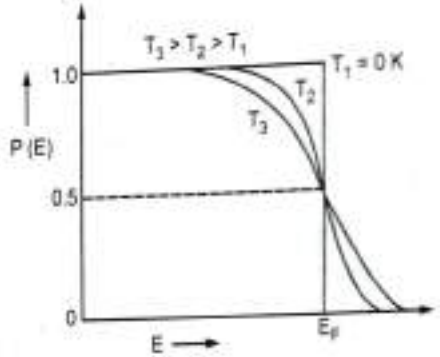


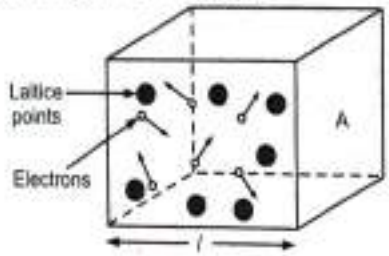
Fig. 4.11: The Fermi-Dirac distribution function

- The rectangular distribution implies that at 0 K every available energy state upto E_F is filled with electrons and all the states above E_F are empty.
- At temperatures higher than 0 K, some probability exists for states above Fermi level to be filled. When the temperature is raised, there is a greater possibility of electrons being found above the Fermi level with an equal probability of finding a hole below the Fermi level.

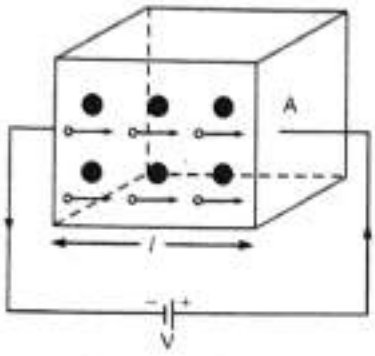
4.7 ELECTRICAL CONDUCTIVITY OF CONDUCTORS AND SEMICONDUCTORS

4.7.1 Conductivity of Conductors

- According to the free electron model of an atom, the valence electrons are not attached to individual atoms. They move about freely along all directions among the atoms. These free electrons are called as conduction electrons and they form the ***'free electron cloud'*** or ***'free electron gas'*** or ***'Fermi gas'***.



(a) Random motion



(b) Directed motion

Fig. 4.12: Current flow in conductors

- In the absence of an external electrical field, the electrons move randomly in all directions [See Fig. 4.12 (a)]. When an electric field is applied to the metal, the random motion becomes directed [See Fig. 4.12 (b)]. This type of directed motion is known as ***'drift'***. The drift velocity v of the electrons depends upon the electron mobility μ_e and the applied electric field E.
- The drift velocity v is given by

$$v = \mu_e E \quad \text{... (1)}$$

Let A = conductor cross-section area

n = electron density (i.e. number of free electrons per unit volume of the conductor)

l = length of the conductor

V = voltage applied across the two ends of the conductor

E = electric field applied.

- Then the charge crossing the cross-section 'A' of the conductor in unit time is equal to n × (v × A) e. This rate of flow of charge constitutes the current.

i.e. $$I = n v A e \quad \text{... (2)}$$

Substituting for v from equation (1), we get

$$I = n \mu_e E A e \quad \text{... (3)}$$

Now, substituting for $E = \frac{V}{l}$ in equation (3), we get

$$I = n \mu_e \frac{V}{l} A e \quad \text{... (4)}$$

$$\therefore \quad \frac{V}{I} = \frac{l}{A} \cdot \frac{1}{n \mu_e e} \quad \text{... (5)}$$

By Ohm's law, we have,

$$R = \frac{V}{I} \quad \text{... (6)}$$

$$\therefore \quad R = \frac{l}{A} \cdot \frac{1}{n \mu_e e} \quad \text{... (7)}$$

But $$R = \rho \frac{l}{A} \quad \text{... (8)}$$

where ρ is the resistivity of the conductor. Comparing equations (7) and (8), we get,

$$\rho = \frac{1}{n \mu_e e} \quad \text{... (9)}$$

The unit of ρ is ohm-m.

- Conductivity 'σ' is defined as the reciprocal of resistivity.

$$\therefore \text{Conductivity, } \sigma = \frac{1}{\rho} = n e \mu \text{ mho/m} \quad \text{... (10)}$$

- Now, current density J is defined as the current flowing across the unit cross section.

From (3), we have,

$$J = \frac{I}{A} = n e \mu_e E \quad \text{... (11)}$$

From (10) and (11), we have

$$J = \sigma E \quad \text{or} \quad \sigma = \frac{J}{E}$$

4.7.2 Conductivity in a Semiconductor

Fig. 4.13 shows the total current flow in a semiconductor. This current is a sum of current flow due to electron flow and hole flow.

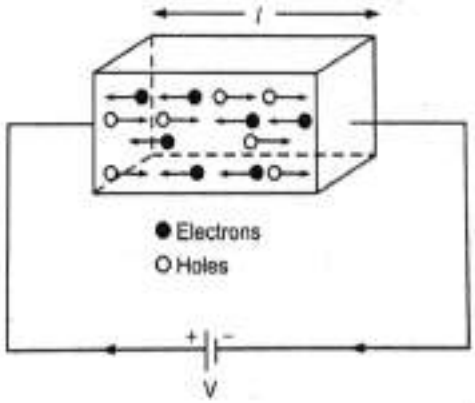


Fig. 4.13: Current flow in a semiconductor

In a semiconductor, let

n_e = electron density in the conduction band

n_p = hole density in valence band

μ_e = electron mobility

μ_p = hole mobility

v_e = drift velocity of electrons

v_p = drift velocity of holes

A = cross section of the semiconductor

V = voltage applied across the semiconductor of length l

The current due to electrons is given by

$$I_e = n_e v_e Ae \quad \text{... (1)}$$

and the current due to holes is given by

$$I_p = n_p v_p Ae \quad \text{... (2)}$$

Therefore, total current flowing through the semiconductor will be.

Total current, $I = I_e + I_p$

$$I = n_e v_e Ae + n_p v_p Ae$$

$$\therefore \quad I = Ae(n_e v_e + n_p v_p) \quad \text{... (3)}$$

The drift velocity of a charged particle in electric field E is,

$$v = \mu E$$

$\therefore$ For electrons,

$$v_e = \mu_e E$$

and for holes, $v_p = \mu_p E$

But $\quad E = \dfrac{V}{l}$

$$\therefore \quad v_e = \mu_e \frac{V}{l} \quad \text{... (4)}$$

$$v_p = \mu_p \frac{V}{l} \quad \text{... (5)}$$

Substituting equations (4) and (5) in equation (3), we get

$$I = Ae\left(n_e \mu_e \frac{V}{l} + n_p \mu_p \frac{V}{l}\right)$$

$$I = \frac{AeV}{l}(n_e \mu_e + n_p \mu_p) \quad \text{... (6)}$$

$$\therefore \quad R = \frac{V}{I} = \frac{l}{Ae(n_e \mu_e + n_p \mu_p)} \quad \text{... (7)}$$

But $\quad R = \rho \dfrac{l}{A} \quad \text{... (8)}$

$\therefore$ Resistivity of the given semiconductor is given by [comparing equations (7) and (8)],

$$\rho = \frac{1}{e(n_e \mu_e + n_p \mu_p)} \text{ ohm-m} \quad \text{... (9)}$$

The conductivity is reciprocal of resistivity.

$$\therefore \text{Conductivity } \sigma = \frac{1}{\rho} = e(n_e \mu_e + n_p \mu_p) \text{ mho/m} \quad \text{... (10)}$$

Hence, conductivity in a semiconductor is a sum of conductivity due to both electrons and holes.

Or $\quad \sigma_{tr} = \sigma_e + \sigma_p$

From equation (3), $\dfrac{I}{A} = e(n_e \mu_e + n_p \mu_p) E$

$\therefore$ The current density

$$J = \frac{I}{A} = e(n_e \mu_e + n_p \mu_p) E \quad \text{... (11)}$$

From (10) and (11),

$$J = \sigma E$$

Case (i): Intrinsic Semiconductor

For intrinsic semi conductors, number of electrons and holes are exactly same,

$$n_e = n_p = n_i$$

$\therefore$ Conductivity of an intrinsic semiconductor is

$$\sigma_i = e n_i (\mu_e + \mu_p)$$

Case (ii): N-type Extrinsic Semiconductor

- For N-type semiconductors, electron concentration is much greater than the hole concentration.

  $\therefore \quad n_e >> n_p \quad$ or $\quad n_e \mu_e >> n_p \mu_p$

  Hence $\quad \sigma_N = e n_e \mu_e$

- If n_d is electron concentration or concentration of donor atoms,

  then, $\quad \sigma_N = e n_d \mu_e \quad$ (as $n_e = n_d$)

Case (iii): P-type Extrinsic Semiconductor

- In P-type semiconductor, electron concentration is negligibly small in comparison to hole concentration.

  Then $\quad n_p >> n_e \quad$ or $\quad n_p \mu_p >> n_e \mu_e$

  $\therefore \quad \sigma_p = e n_p \mu_p$

- If n_a is acceptor atom concentration then $\sigma_p = e n_a \mu_p$ (as $n_p = n_a$)

SOLVED PROBLEMS

Problem 4.1: *Calculate the current produced in a small Germanium plate of area 1 cm² and of thickness 0.3 mm when a P.D. of 2 V is applied across the faces.* **(04) (Dec. 09)**

Given: $\quad n_i = 2 \times 10^{19} /m^3$

$e = 1.6 \times 10^{-19}$ C

$\mu_e = 0.36$ m²/volt-sec

$\mu_h = 0.17$ m²/volt-sec

Solution: Data: $\quad A = 1 \times 10^{-4}$ m³

$V = 2$ volts

$l = 0.3$ mm $= 0.3 \times 10^{-3}$ m

Formula: $I = n_i e (\mu_e + \mu_h) \frac{V}{l} \cdot A$

Solution: $I = 2 \times 10^{19} \times 1.6 \times 10^{-19} (0.36 + 0.17) \frac{2 \times 10^{-4}}{0.3 \times 10^{-3}}$

$= \boxed{1.13 \text{ amp.}}$

Problem 4.2: *Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is $1.5 \times 10^{16} / m^3$ and the mobilities of electrons and holes are 0.12 and 0.05 m^2/V - sec respectively at room temperature.*

Data: $n_i = 1.5 \times 10^{16} /m^3$,

$\mu_e = 0.12 \text{ m}^2/\text{V-sec}$, $\mu_h = 0.05 \text{ m}^2/\text{V-sec}$.

Formula: $\sigma_{in} = \sigma_n + \sigma_p$

$\sigma_{in} = n_i e(\mu_e + \mu_h)$

Solution: $\sigma_{in} = 1.5 \times 10^{16} \times 1.6 \times 10^{-19} (0.12 + 0.05)$

$= \boxed{4.1 \times 10^{-4} \text{ mho/m}}$

Problem 4.3: *Calculate the conductivity of the Germanium specimen if a donor impurity is added to the extent of one part in 10^8 Germanium atoms in room temperature.*

Given: Avogadro number = 6.02×10^{23} atoms/moles

At. wt. of Ge = 72.6

Density of Ge = 5.32 g/cm^3

Mobility μ_e = 3800 cm^2/ V-sec

Formula: $\sigma = e n_d \mu_e$

Solution: Concentration of Ge atoms

$= \frac{6.02 \times 10^{23}}{72.6} \times 5.32$

$= 4.41 \times 10^{22} /\text{cm}^3$

Since there is one donor atom per 10^8 Germanium atoms then

$n_d = \frac{4.41 \times 10^{22}}{10^8} = 4.41 \times 10^{14} / \text{cm}^3$

In N-type semiconductor, n > p

then $\sigma = e n_d \mu_e$

$= 1.6 \times 10^{-19} \times 4.41 \times 10^{14} \times 3800$

$= \boxed{0.268 \text{ mho/cm.}}$

Problem 4.4: *The resistivity of an n-type semiconductor is 10^{-6} Ω cm. Calculate the number of donor atoms which must be added to obtain the resistivity.* **(04) (Nov. 07)**

Given: $\mu_e = 1000 \text{ cm}^2/\text{V-sec.}$

Data: $\rho = 10^{-6} \Omega \text{ cm}$

$\mu_e = 1000 \text{ cm}^2/\text{V-sec}$

Formula: Resistivity

$\rho = \frac{1}{n_d e \mu_e}$

Solution: $n_d = \frac{1}{\rho e \mu_e}$

$\therefore \quad n_d = \frac{1}{10^{-6} \times 1.6 \times 10^{-19} \times 1000}$

$= \boxed{6.25 \times 10^{21} \text{ atoms.}}$

Problem 4.5: *Calculate the conductivity of extrinsic silicon at room temperature if the donor impurity added is 1 in 10^8 silicon atoms.*

Given: At room temperature,

$n_i = 1.5 \times 10^{10}$ per cm^3

$\mu_e = 1300 \text{ cm}^2 / \text{volt-sec}$

and number of silicon atoms per unit volume = 5×10^{22}

Formula: $\sigma_n = ne \mu_e$

Solution: If there is 1 donor atom per 10 silicon atoms then the number of donor atoms per cm^3

$n_d = \frac{\text{number of silicon atoms/unit volume}}{10^8}$

$= \frac{5 \times 10^{22}}{10^8} = 5 \times 10^{14}$

Assuming all the donors are ionised and n >> p, hole conduction can be neglected.

$\therefore \quad \sigma_n = n e \mu_e$

$\sigma_n = n_d e \mu_e$

$= 5 \times 10^{14} \times 1.6 \times 10^{-19} \times 1300$

$= \boxed{0.104 \text{ mho/cm.}}$

Problem 4.6: *In Germanium, the energy gap is 0.75 eV. What is the wavelength at which Germanium starts to absorb light ?*

Data: $E_g = 0.75$ eV

Formula: $E_g = h\upsilon = \frac{hc}{\lambda}$

Solution: Energy gap in a semiconductor is the minimum energy required to shift an electron from the top of valence band to the bottom of the conduction band. If photons of minimum energy $h\upsilon$ are absorbed by a material to enable electrons to cross the energy gap, then

$$h\upsilon = E_g$$

$$\therefore \quad E_g = h\upsilon = h\frac{c}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{\lambda} J$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times \lambda} eV$$

i.e. $$E_g = \frac{12400}{\lambda} eV, \text{ if } \lambda \text{ is in A.}$$

$$\therefore \quad \lambda = \frac{12400}{E_g} = \frac{12400}{0.75}$$

$$\therefore \quad \boxed{\lambda = 1653 \text{ A}^\circ}$$

Problem 4.7: *Calculate the average thermal velocity, the drift velocity and the mobility of electrons in copper in an electric field of 100 V/cm. Calculate also the density of the electric currents. The resistivity of copper is 1.72×10^{-8} ohm-m at 25°C. Boltzmann constant is 1.38×10^{-23} J/K, density of copper is 8.9×10^3 kg/m^3 and At. wt. is 63.54.*

Data: $E = 100$ V/cm, $\rho = 1.72 \times 10^{-8}\ \Omega$-m, $k = 1.38 \times 10^{-23}$ J/K, density $= 8.9 \times 10^3$ kg/m^3, At. wt. = 63.54.

Formulae: (i) $v = \sqrt{\frac{3kT}{m}}$, (ii) $v_d = \mu E$, (iii) $\sigma = ne\mu = \frac{1}{\rho}$

Solution: At equilibrium, the electrons follow the Maxwell-Boltzmann distribution. So their average K.E. for each degree of freedom is $\frac{1}{2}$ kT. For particles moving in three dimensions, we can write,

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$\therefore v = \left(\frac{3\,kT}{m}\right)^{1/2} = \left(\frac{3 \times 1.38 \times 10^{-23} \times 298}{9.1 \times 10^{-31}}\right)^{1/2}$$

$$v = 1.16 \times 10^5 \text{ m/sec.}$$

Since each copper atom contributes one valence electron to the conduction band, the number of electrons/m^3 will be equal to the number of copper atoms/m^3.

$$\therefore \text{ No. of electrons/m}^3 = n$$

$$= \frac{6.02 \times 10^{26} \times 8.9 \times 10^3}{63.54}$$

$$= 0.84 \times 10^{29} \text{ atoms/m}^3$$

$$\text{Mobility } \mu = \frac{1}{\rho \cdot n \cdot e}$$

$$\mu = \frac{1}{1.72 \times 10^{-8} \times 0.84 \times 10^{29} \times 1.6 \times 10^{-19}}$$

$$= 4.33 \times 10^{-3} \text{ m}^2\text{/volt-sec}$$

$$\text{Drift velocity } v_d = \mu \cdot E$$

$$= 4.33 \times 10^{-3} \times 100$$

$$\therefore \quad \boxed{v_d = 0.433 \text{ m/sec}}$$

Problem 4.8: *Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is 1.6×10^{10}/cm^3.* **(May 04, Dec. 14)**

$\mu_e = 1500$ cm^2/volt-sec

$\mu_h = 500$ cm^2/volt-sec at room temperature

Data: $n_i = 1.6 \times 10^{10}$/cm^3

$\mu_e = 1500$ cm^3/V-sec

$\mu_h = 500$ cm^3/V-sec

Formula: $\sigma_{in} = \sigma_n + \sigma_p$

Solution: $\sigma_{in} = n_i e (\mu_e + \mu_h)$

$$= 1.6 \times 10^{10} \times 1.6 \times 10^{-19} (1500 + 500)$$

$$= \boxed{5.12 \times 10^{-6} \text{ mho/cm}}$$

Problem 4.9: *Calculate the conductivity of a Germanium sample if a donor impurity is added to the extent of one part in 10^7 Ge atoms at room temperature.* **(04) (May 06)**

Given: Avogadro number

$$N_a = 6.02 \times 10^{23} \text{ atoms/gm-mole}$$

Atomic weight of Ge = 72.6

Density of Ge = 5.32 gm/cc

Mobility $\mu_e = 3800$ cm^2/V-sec

Formula: $\sigma = e\, n_d\, \mu_e$

Solution: Concentration of Ge atoms

$$= \frac{6.02 \times 10^{23}}{72.6} \times 5.32$$

$$= 4.41 \times 10^{22} \text{ /cm}^3$$

Since there is one donor atom per 10^7 Ge atoms then

$$n_d = \frac{4.41 \times 10^{22}}{10^7} = 4.41 \times 10^{15} \text{ /cm}^3$$

$$\therefore \quad \sigma = 1.6 \times 10^{-19} \times 4.41 \times 10^{15} \times 3800$$

$$\boxed{\sigma = 2.68 \text{ mho/cm}}$$

Problem 4.10: *Calculate the energy gap of silicon, given that it is transparent to radiation of wavelength greater than 11000 A°.* **(04) (May 07)**

Given: $\lambda = 11000$ A°

Formula: $E_g = h\upsilon = \frac{hc}{\lambda}$

Solution: $E_g = \frac{hc}{\lambda}$

$$E_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{11000 \times 10^{-10}}$$

$$E_g = 1.808 \times 10^{-19} J$$

$$\boxed{E_g = 1.13 \text{ eV}}$$

Problem 4.11: *Calculate the number of acceptors to be added to a Germanium sample to obtain the resistivity of 10 Ω-cm.* **(04) (May 08)**

Given: $\mu = 1700 \text{ cm}^2/\text{volt-sec}$

Data: $\rho = 10 \text{ cm}$

$\mu = 1700 \text{ cm}^2/V\text{-sec}$

Formula: $\rho = \frac{1}{n_a \mu_e e}$

Solution: $n_a = \frac{1}{\rho \mu_e e}$

$$n_a = \frac{1}{10 \times 1700 \times 1.6 \times 10^{-19}}$$

$$\boxed{n_a = 3.6 \times 10^{14} / \text{cm}^3}$$

Problem 4.12: *Calculate the band gap energy in Germanium. Given that it is transparent to radiation of wavelength greater than 17760 A°.* **(04) (May 10)**

Data: $\lambda = 17760 \text{ A}°$

Formula: $E_g = h\upsilon = \frac{hc}{\lambda}$

Solution: $E_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{17760 \times 10^{-10}}$

$$E_g = 1.12 \times 10^{-19} J$$

$$E_g = \frac{1.12 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\boxed{E_g = 0.7 \text{ eV}}$$

Problem 4.13: *Intrinsic silicon is doped with phosphorus, with the atomic ratio of 10^8 (Si): 1 (P). Calculate the conductivity of N type of silicon thus formed. Given mobility of electrons in silicon $\mu_e = 1400 \text{ cm}^2 \text{ Vs}^{-1}$. Atomic weight of intrinsic silicon = 28.085, Avogadro's number = 6.022×10^{23} atoms per mole, Density of silicon = 2.33 gm/cm^3.* **(04) (Dec. 11)**

Data:

Avogadro number = 6.022×10^{23} atoms/moles

At. wt. of Si = 28.085

Density of Si = 5.32 gm/cm^3

Mobility $\mu_e = 1400 \text{ cm}^2 \text{ Vs}^{-1}$

Formula: $\sigma = e\, n_d\, \mu_e$

Solution: Concentration of Si atoms

$$= \frac{6.022 \times 10^{23}}{28.085} \times 2.33$$

$$= 4.996 \times 10^{22} / \text{cm}^3$$

Since there is one donor atom per 10^8 Si atoms,

then, $n_d = \frac{4.996 \times 10^{22}}{10^8} = 4.996 \times 10^{14} / \text{cm}^3$

The conductivity is,

$$\sigma = e\, n_d\, \mu_e$$

$$= 1.6 \times 10^{-19} \times 4.996 \times 10^{14} \times 1400$$

$$= 1.119 \times 10^{-1} \text{ mho/cm}$$

$$= 0.1119 \text{ mho/cm}$$

Problem 4.14 : *Calculate the conductivity of Ge sample if the donor impurity is added to an extent of one part in 10^8 Ge atoms at room temperature (Data Given: $N_o = 6.023 \times 10^{23}$ atoms/gn-mole. At. Wt. of Ge = 72.6).* **(3) (May 13)**

(Density of Ge = 5.32gm/cc. $\mu = 3800 \text{ cm}^2/\text{v-s}$.)

Data : At. Wt. Of Ge = 72.6

Density of Ge = 5.32 gm/cc

$\mu = 3800 \text{ cm}^2/\text{V.S.}$

Formula: $\sigma = nd\mu e$

Solution:

$$\text{Number of Ge atoms/unit volume} = \frac{6.023 \cdot 10^{23} \cdot 5.32}{72.6}$$

$$= 4.41 \times 10^{22}/\text{cm}^3$$

As ratio of donor to pure semiconductor for is 1: 10^8

$$\therefore \quad n_d = \frac{4.41 \cdot 10^{22}}{10^8} = 4.41 \times 10^{14}/\text{cm}^3$$

Conductivity

$$\sigma = 4.41 \times 10^{14} \times 1.6 \times 10^{-19} \times 3800$$

$$\therefore \quad \sigma = 0.268 \text{ mho/cm.}$$

Problem 4.15: *Calculate the number of acceptors to be added to a germanium sample to obtain the resistivity of 10 Ω cm.* **(3) (Dec. 14)**

($\mu = 1700 \text{ cm}^2/\text{V.sec.}$)

Data: $\rho = 10\ \Omega\text{.cm}$, $\mu = 1700 \text{ cm}^2/\text{V.sec.}$

Formula: $\rho = \frac{1}{n_a e \mu}$

Solution: $n_a = \frac{1}{10 \times 1.6 \times 10^{-19} \times 1700}$

$= 3.676 \times 10^{14} /cm^3$

Problem 4.16: *Calculate the number of acceptors to be added to a germanium sample to obtain the resistivity of 20 Ω cm.* **(3) (Nov. 15)**

Given: $\mu = 1700\ cm^2/V.sec$

Data: $\rho = 20\ \Omega\ cm$

$\mu_p = 1700\ cm^2/V.sec$

Formula: Resistivity

$$\rho = \frac{1}{n_a \mu_p e}$$

Solution: No. of acceptor

$$N_a = \frac{1}{20 \times 1700 \times 1.6 \times 10^{-19}}$$

$= 1.838 \times 10^{14}$ atoms

Problem 4.17: *Calculate the number of acceptor atoms that need to be doped in germanium sample to obtain the resistivity of 8 Ω cm.*

[Given: mobility $\mu = 1600\ cm^2/V.s$] **(3) (May 16)**

Ans. Data : $\rho = 8\ \Omega\ cm$, $\mu_a = 1600\ cm^2/V\text{-}sec$.

Formula : $\rho = \frac{1}{N_a \mu_a e}$

Solution: $n_a = \frac{1}{8 \times 1600 \times 1.6 \times 10^{-19}}$

$= 4.88 \times 10^{14}/cm^3$

Problem 4.18: *A sample of intrinsic germanimum at room temperature has a carrier concentration $4.41 \times 10^{22}\ cm^3$. Donor impurity is added in the ratio 1 donor atom per 10^8 atoms/cm^3 of germanium. Determine the resistivity of the material thus formed. (Given : m_e 3800 $cm^2/V.s$)* **(3) (May 17)**

Solution : Carrier concentration, $n = 4.41 \times 10^{22}\ /cm^3$

Mobility $\mu_e = 3800\ cm^2/V\text{-}sec$

Formula: $\sigma = e\, n_d\, \mu_e$

Since there is one donor atom per 10^8 Germanium atoms therefore, the concentration of donor atoms

$n_d = 4.41 \times 10^{22} / 10^8$

$n_d = 4.41 \times 10^{14}$

Therefore the conductivity,

$\sigma = e\, n_d\, \mu_e$

$= 1.6 \times 10^{-19} \times 4.41 \times 10^{14} \times 3800$

$= \boxed{0.268\ mho/cm.}$

Problem 4.19: *Calculation the band gap energy. (in eV) in silicon, given that it is transparent to radiation of wavelength the greater than 1000 AU. $h = 6.63 \times 10^{-34}$ J-sec. $C = 3 \times 10^8$ m/s.* **(3) (Nov. 17)**

Given Data : $\lambda = 11000$ AU

$h = 6.63 \times 10^{-34}$ J-sec

Formula : $E_g = h\gamma = \frac{hc}{\lambda}$

Solution : $E_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{11000 \times 10^{-10}}$

$E_g = 1.808 \times 10^{-19}$ J

$E_g = 1.13$ eV

Problem 4.20 : *Calculate the number of acceptors to be added in germanium sample to obtain the resistively of 20 Ω cm.*

Given $\mu_h = 1700 cm^2/V.sec$, $e = 1.6 \times 10^{-19}$C **(3) (May 18)**

Solution : Given : $\mu = 1700\ cm^2$/volt-sec

$\rho = 20$ cm

$\mu = 1700\ cm^2$

$\rho = \frac{1}{n_a \mu_a e}$

$n_a = \frac{1}{\rho \mu_a e} = \frac{1}{20 \times 1700 \times 1.6 \times 10^{-19}}$

$n_a = 1.8382 \times 10^{14}$ atoms

Problem 4.21: *Calculate the number of acceptor to be added to a germanium sample to obtain the resistivity of 10 Ω-cm.* **(3) (May 19)**

Data : $\rho = 10\ \Omega$-cm, $\mu = 1700\ cm^2/V$ dec.

Formula : $\rho = \frac{1}{6} = \frac{1}{n\mu e}$

Solution : $n = \frac{1}{10 \times 1.6 \times 10^{-19} \times 1700} = 5.882 \times 10^{14}\ /cm^3$

Problem 4.22: *Find the drift velocity for the electron in silver wire of radius 1.00 mm and carrying a current of 2 amperes. Density of silver is 10.5 g/cm^3.*

Data: r = 1.00 mm, I = 2 amp, density = 10.5 g/cc.

Formula: $v = \frac{I}{q \cdot n \cdot A}$

Solution: $I = q n v A$

Silver is monovalent. So each atom may be assumed to contribute one electron. One gram atomic weight of silver, 108 g, has 6×10^{23} atoms (Avogadro's number).

The density of silver is 10.5 g/cm^3. So 108 g will occupy 108/10.5 = 10.3 cm^3.

$\therefore$ Number of electrons per unit volume, $n = \frac{6 \times 10^{23}}{10.3} = 6 \times 10^{22}$

or $n = 6 \times 10^{28}$ per m^3

The cross-sectional area of wire,

$$A = \pi r^2 = \pi (10^{-3})^2 = 3 \times 10^{-6} \text{ m}^2$$

Now, $$v = \frac{I}{q \times n \times A}$$

$$= \frac{2}{(1.6 \times 10^{-19}) \times (6 \times 10^{28}) \times (3 \times 10^{-6})}$$

$$\boxed{v = 7 \times 10^{-4} \text{ m/sec}}$$

4.8 POSITION OF FERMI LEVEL IN INTRINSIC SEMICONDUCTORS

- The Fermi-Dirac probability function is

$$P(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \quad \text{... (1)}$$

and it gives the probability of an electron occupying a state of energy E.

- The derivation of the position of Fermi level in intrinsic semiconductors is based on the following assumptions:
  - The widths of the valence band and conduction band are small when compared to the forbidden gap E_g.
  - As band widths are small, all levels in the band have the same energy. The levels in the conduction band have energy E_c while levels in valence band have energy E_v.
  - At 0 K, the solid is like an insulator i.e. no conduction is possible as valence band is completely filled and conduction band is completely empty.
- At any other temperature say T,

Let n_c = number of electrons in the conduction band.

n_v = number of electrons in the valence band.

$N = n_c + n_v$ = number of electrons in both the bands

From the probability theory, we have

$$P(E_c) = \frac{n_c}{N}$$

i.e. $$n_c = N P(E_c) \quad \text{... (2)}$$

where $P(E_c)$ is the probability of an electron having energy E_c in the conduction band.

- So, according to Fermi-Dirac probability distribution function defined in equation (1), we have,

$$P(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \quad \text{... (3)}$$

From equations (2) and (3),

$$n_c = \frac{N}{1 + e^{(E_c - E_F)/kT}} \quad \text{... (4)}$$

Similarly, $$n_v = \frac{N}{1 + e^{(E_v - E_F)/kT}} \quad \text{... (5)}$$

Substituting (4) and (5) in

$$N = n_c + n_v$$

$$\therefore \quad N = \frac{N}{1 + e^{(E_c - E_F)/kT}} + \frac{N}{1 + e^{(E_v - E_F)/kT}}$$

Or $[1 + e^{(E_c - E_F)/kT}][1 + e^{(E_v - E_F)/kT}]$

$$= 1 + e^{(E_c - E_F)/kT} + 1 + e^{(E_v - E_F)/kT}$$

This gives $e^{(E_c + E_v - 2E_F)/kT} = 1$

i.e. $$\frac{E_c + E_v - 2E_F}{kT} = 0$$

or $E_c + E_v - 2E_F = 0$

$$\therefore \quad E_F = \frac{E_c + E_v}{2}$$

- Thus, Fermi level in intrinsic semiconductors is exactly in the middle of the forbidden gap as shown in Fig. 4.14.

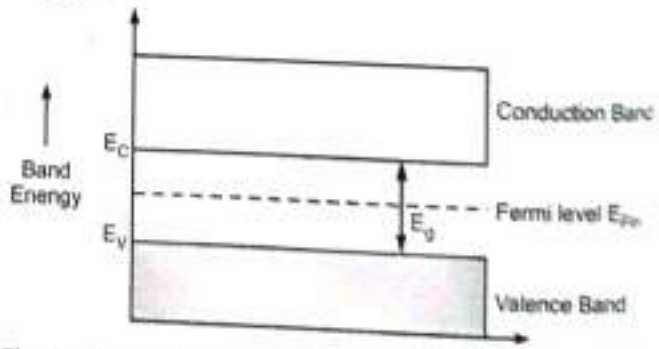


Fig. 4.14: Position of Fermi level in intrinsic semiconductor

4.9 POSITION OF FERMI LEVEL IN EXTRINSIC SEMICONDUCTORS (May 15)

1. Position of Fermi Level in N-type Semiconductor

- The N-type or pentavalent impurity semiconductor has more conduction electrons than holes. This moves the

'centre of gravity' up so that the Fermi level is above the middle of the forbidden band.

- Donors represent isolated energy levels located very close to the bottom of the unfilled conduction band. (Normally for Ge, 0.01 eV below the lower edge of conduction band). Hence, very little energy is required to raise an electron from the donor level into the conduction band where it is free for conduction of electricity.
- In Fig. 4.15, the energy levels of the impurity atom (donor) are shown as isolated circles not as a band because here atoms are isolated from each other. E_{Fn} represents the position of Fermi level in the case of intrinsic semiconductors.

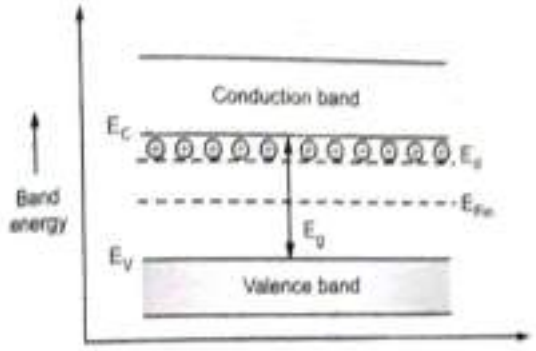


Fig. 4.15: Indicates donor level

- The position of Fermi level in the case of extrinsic semiconductor depends both on the doping and on the temperature. At 0 K all the donor levels are occupied and there are no electrons in the conduction band. Since P (E) = 1 upto the donor levels and P(E) = 0 at the conduction band, Fermi level E_f must be somewhere in the range $E_d \le E_f \le E_c$. (See Fig. 4.16).

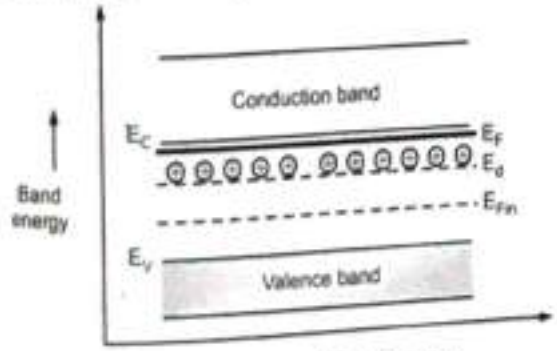


Fig. 4.16: Position of E_f at T = 0 K

2. **Position of Fermi Level in P-type Semiconductor**

- In P-type semiconductor, the concentration of holes is greater than that of electrons.
- This makes the ***'centre of gravity'*** move down, so that Fermi level is below the middle of the forbidden band. Acceptors represent isolated energy levels, and lie close to the top of filled valence band.

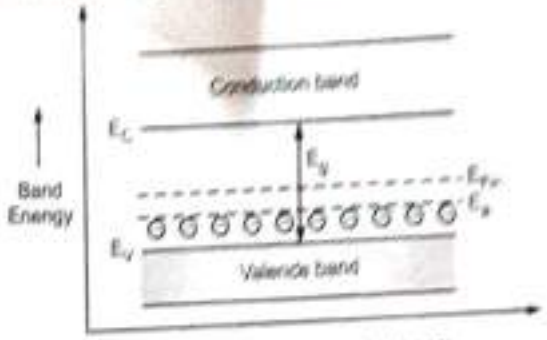


Fig. 4.17: Indicates acceptor level

- In Ge, this discrete energy level is only 0.01 eV above the valence band. Hence, a very small amount of energy is required for an electron to leave the valence band and occupy the acceptor energy level. Thus, holes are created in the valence band. Fig. 4.17 indicates the acceptor level in a P-type semiconductor.
- At T = 0 K the Fermi level lies somewhere in the range between $E_v \le E_f \le E_a$ (See Fig. 4.18). At this temperature, the conduction band is empty.

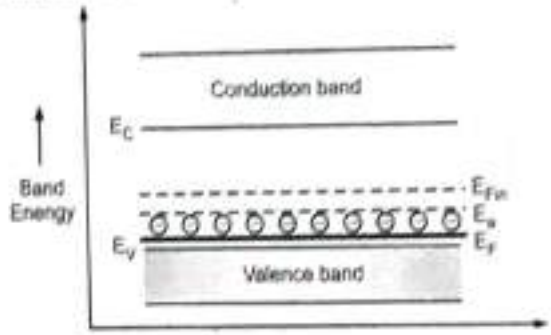


Fig. 4.18: Indicates position of E_f at T = 0 K

4.10 P-N JUNCTION DIODE

- When a crystal of a pure semiconductor is doped so that one half of it is P-type and the other half is N-type, then the border between P-type and N-type is called ***'P-N junction'***. The P-N junction have non-linear resistance which is the basis for all solid state electronic devices.
- A junction diode is a two terminal device having one P-N junction. The junction diode passes a large current in one direction and almost no current in the other direction. Therefore, such a diode can be used as a rectifier.
- The Fermi level in P-type material is located close to the top of the valence band, whereas in N-type material Fermi level lies close to the bottom of the conduction band.

- The P-type material has more holes than free electrons and the N-type material has more free electrons than holes. When the junction is made between these materials, the holes would tend to move from P-type material into the N-type material and electrons would tend to move from the N-type material into the P-type material, due to the difference in the concentration of holes and electrons on either side of a P-N junction. This process is called as ***'diffusion'***.
- The diffused charge carriers combine at the junction to neutralize each other. Due to this neutralisation, a charge free space called ***'depletion layer'***, of width of the order of a few microns, is formed near the junction.

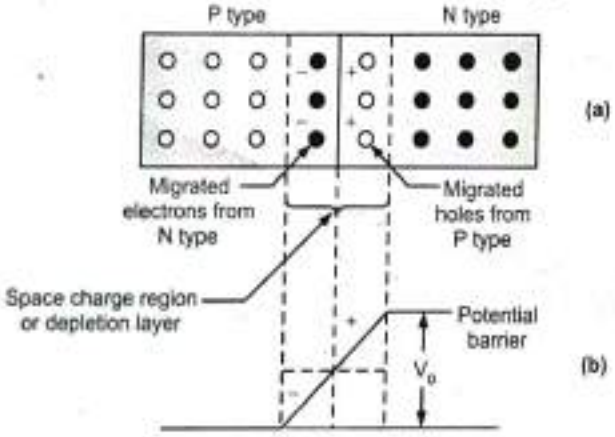


Fig. 4.19: (a) p-n Junction (After joining), (b) Potential barrier

- Due to the diffusion of holes from P to N region, negative ions are produced in P region. Similarly, due to the diffusion of electrons from N into P region, positive ions are produced in N-region. Both these negative and positive ions are immobile and form parallel rows of opposite charges facing each other across the depletion layer.
- Because of this charge separation, an electric potential V_0 develops across the junction under an equilibrium condition, i.e. with the junction externally isolated. This potential is called as the ***'junction potential'***, or ***'barrier potential'*** and it can be represented by a battery called as the ***'space charge equivalent battery'***. Once the potential barrier is established, further diffusion of majority charge carriers across the junction is prevented.

4.11 WORKING OF A P-N JUNCTION DIODE

(Dec. 12, May 16)

1. Zero Bias

- Consider a P-N junction formed by fusing a P-type and an N-type semiconductor under unbiased condition. Under this condition, the Fermi level will have to realign such that it exists as a single energy level for the entire specimen.
- This Fermi level in P-type is located close to the top of the valence band whereas in N-type it is close to the bottom of the conduction band. Because the Fermi level is lower on the P-side relative to the N-side, electrons move across the boundary to the P-side and thereby equalize the Fermi levels.
- The band edges in the two specimen shift themselves to make the alignment of Fermi levels possible and the energy band diagram remains no more the same (See Fig. 4.20) but assumes a shape as shown in Fig. 4.21.

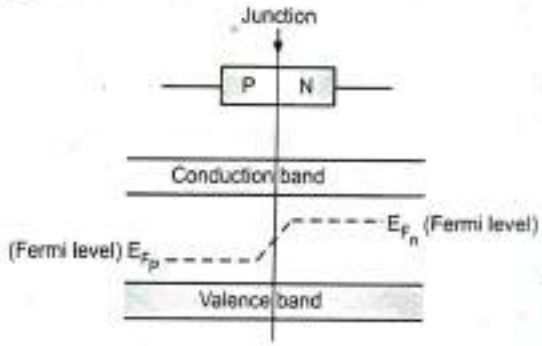


Fig. 4.20: Non-equilibrium energy band picture of a p-n junction

- The conduction band of P-type is shifted upwards by eV_B over the conduction band of N-type where V_B is the ***'potential barrier'*** across the junction.
- The minority electrons in the conduction band of P-type are at a higher energy than the majority electrons in the conduction band of N-type. Hence, the electrons crossing the junction from P-region will not encounter the potential barrier while the electrons crossing the junction from N-region side will face the potential barrier. The band picture for a zero bias diode at equilibrium is shown in Fig. 4.21.

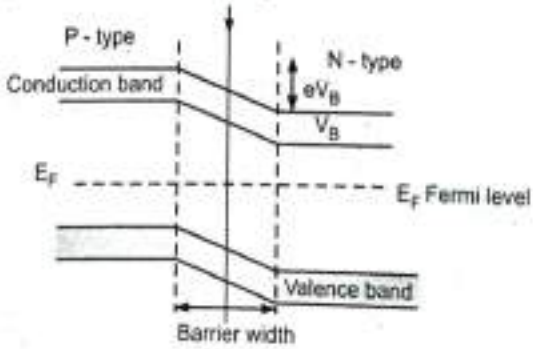


Fig. 4.21: Shifting of the band in P and N-type semiconductors

2. Forward Bias

- Under forward biasing the P-side is connected to the positive terminal and N-side to the negative terminal of the battery.
- Due to the forward bias, equilibrium conditions are disturbed and therefore energy bands and the Fermi levels are altered. Since in forward bias, negative terminal of the battery is connected to N-type side, the energy of the electrons in the N-side increases by an amount eV, where V is the externally applied voltage.
- Consequently, Fermi level raises by eV and the energy bands adjust their positions so as to suit the elevation of Fermi level.
- Due to the increase in energy in N-type side, the potential barrier is reduced to $e(V_B - V)$ and the barrier width is reduced. Hence the electrons crossing the junction from N-side will now face a low potential barrier and they can easily cross the junction.

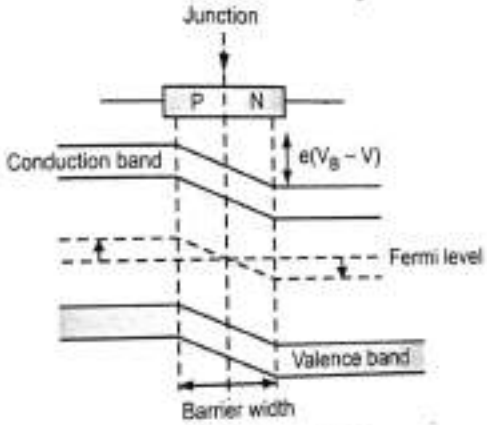


Fig. 4.22: Forward bias

- For conduction to take place in a P-N diode, the forward bias potential should be greater than the barrier potential. Band picture of P-N junction diode in forward bias condition is shown in Fig. 4.22.

3. Reverse Bias

- In this case, N-side is connected to the positive terminal and P-side to the negative terminal of the battery.
- This lowers the Fermi level on N-side by an amount eV raising the barrier height to $e(V_B + V)$ and thereby increasing the width of the depletion layer. See Fig. 4.23.
- The electrons which are the majority carriers in the N-side will now face a greater potential barrier in crossing the junction. Therefore the number of electrons crossing from N-side to P-side decreases and hence the current is very much reduced.

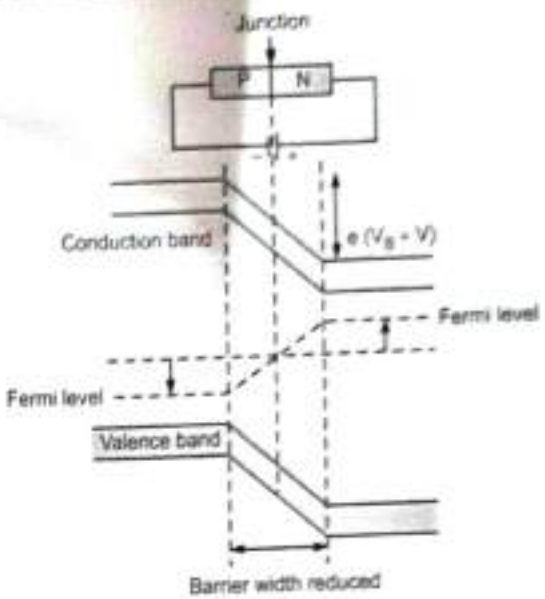


Fig. 4.23: Reverse bias

4.12 EXPRESSION FOR BARRIER POTENTIAL

The internal potential barrier V0 can be calculated from the electron concentrations in p and n regions of the diode.

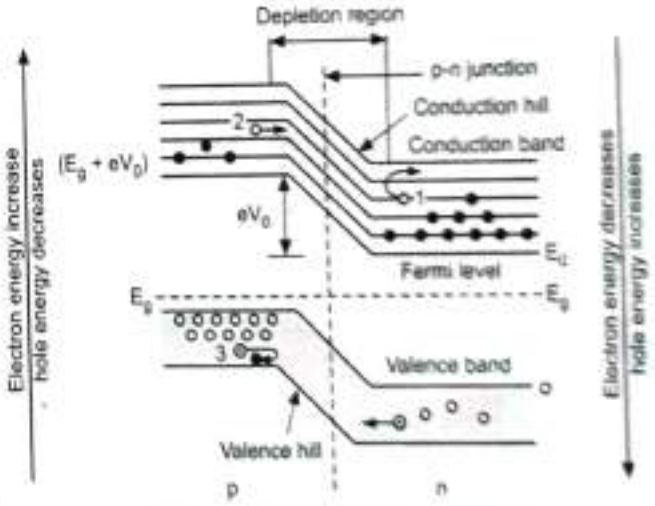


Fig. 4.24 : Energy band diagram of p-n junction at equilibrium

The electron concentration in the conduction band on the n-side can be written as

$$n_n = N_c e^{-(E_g - E_f)/kT} \quad \text{... (1)}$$

Due to the barrier potential V_0, the band gap energy on p-region increases by (Eg+V0). The electron concentration on p-side can be written as

$$n_p = N_v e^{-(E_g + eV_0) - E_f/kT} \quad \text{... (2)}$$

Dividing the two equations, we get

$$\frac{n_n}{n_p} = e^{\frac{eV_0}{kT}} \quad \text{... (3)}$$

Above equation shows that at thermal equilibrium, concentrations of electrons on both sides of the junction are related through Boltzmann factor.

Taking log of both sides,

$$V_0 = \frac{kT}{e} \ln \frac{n_0}{n_p} \quad \text{... (4)}$$

This equation can be written as

$$V_0 = \frac{kT}{e} \ln \frac{n_n p_p}{n_p p_n} \quad \text{... (5)}$$

At room temperature, all the impurities are ionized, hence:

$$n_n = N_D, \quad p_p = N_A$$

Also,

Thus, equation (5) can be written as

$$V_0 = \frac{kT}{e} \ln \frac{N_D N_A}{n_i^2}$$

The factor $\frac{kT}{e}$ is of the dimensions of voltage and is denoted by VT. Thus $V_0 = V_T \ln \frac{N_D N_A}{n_i^2}$

Thus, the barrier potential equation indicates that barrier potential in a junction diode depends on the equilibrium concentration of the impurities in p and n regions and does not depend on the charge density in the depletion region.

4.13 IDEAL DIODE EQUATION

When the diode is forward-biased with potential V_f, the potential barrier is lowered by an amount of energy eV_f and the probability of a majority carrier crossing the junction and diffusion current density is increased by a factor of $e^{eV_f/kT}$

Thus, diffusion current density components $\dot{J}_{hp}$ and $\dot{J}_{en}$ in forward bias diode are given by

$$\dot{J}_{hp} = J_{hp} e^{eV_f/kT} = J_{hn} e^{eV_f/kT} \quad \text{and}$$

$$\dot{J}_{en} = J_{en} e^{eV_f/kT} = J_{ep} e^{eV_f/kT}$$

Where, J_{hp} and J_{en} are diffusion current densities in unbiased diode.

The drift current density components have not changed and have the same magnitude as in equilibrium case.

Therefore, the net hole current density across the forward bias junction is

$$J_h = \dot{J}_{hp} - J_{hn} = J_{hn} (e^{\frac{eV_f}{kT}} - 1)$$

Similarly, the net electron current density across the forward bias junction is

$$J_e = \dot{J}_{en} - J_{ep} = J_{ep} (e^{\frac{eV_f}{kT}} - 1)$$

The total current density across the forward bias junction is the sum of electron and hole current density components. Thus,

$$J = J_e + J_h = (J_{hn} + J_{ep}) (e^{\frac{eV_f}{kT}} - 1) = J_0 (e^{\frac{eV_f}{kT}} - 1)$$

J = I/A is current density. Thus, above equation can be written as

$$I = I_0 (e^{\frac{eV_f}{kT}} - 1)$$

Where, I = ~~J/A~~ and I_0 = ~~J_0/A~~

The above equation can generally be written as

$$I = I_0 (e^{\frac{eV_f}{kT}} - 1)$$

Which is called diode equation. Where, V denotes voltage ($V = V_F$ is for forward bias and $V = -V_R$ for reverse bias) and I_0 denotes reverse saturation current.

4.14 APPLICATIONS OF P-N JUNCTION DIODE : SOLAR CELLS

- Solar cell is a type of photo-voltaic cell. Photo-electric cells are of three types:
  1. Photo-emissive cell. 2. Photo-voltaic cell.
  3. Photo-conductive cell
- All these are based on the principle of photo-electric effect i.e. photon energy is being converted into electrical energy.
- The photo-voltaic cells are based on the photo-voltaic effect which is the conversion of light into an electric current without the aid of an external battery. In this case, voltage is developed in the cell. Solar cells are the best example of this type.

4.14.1 Action of a Solar Cell

- When a P-N junction is exposed to sunlight, photons of energy hv are absorbed, if hv is greater than the band gap E_g. Electron-hole pairs are then generated in both the P-side and N-side of the junction. The electrons and holes that are produced within a small distance of the junction reach the space charge region by diffusion.

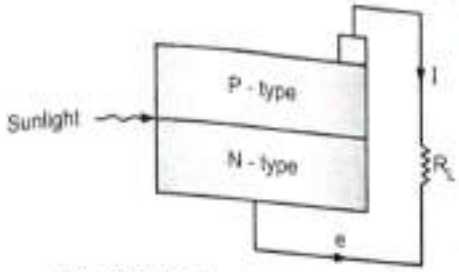
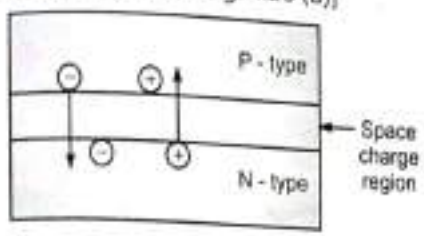
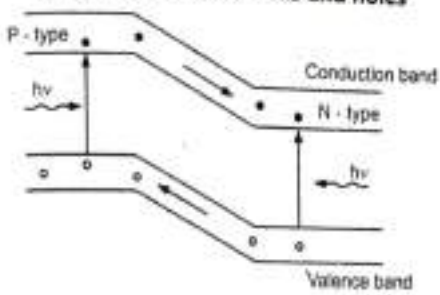


Fig. 4.25 : Solar cell p-n junction with load resistance R_L

- The electron-hole pairs are separated by the strong barrier field existing across the space charge region. Electrons in the P-side slide down the barrier potential to move to the N-side while holes in N-side move towards the P-side. [See Fig. 4.26 (b)]



(a) Diffusion of electrons and holes



(b) Energy band diagram corresponding to diffusion of electrons and holes

Fig. 4.26

- When the P-N junction is open circuited, the accumulation of electrons and holes on the two sides of the junction gives rise to an ***'open-circuited voltage'*** V_{oc}. When a load resistance is connected across the diode, current flows in the circuit. This effect is known as ***'photovoltaic effect'***.

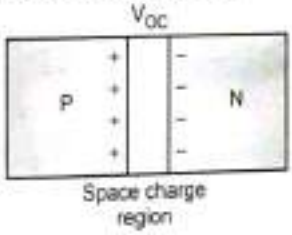


Fig. 4.27

- When the diode terminals are short circuited, the maximum current obtained is called the short ***'circuit current'*** I_{sc}.
- The current flows as long as the diode is illuminated by sunlight. The magnitude of current flowing is proportional to the light intensity. The electrical energy generated in the solar cell is in the form of dc voltage of approximately 0.5 V. The current varies with the surface area of photo-voltaic cell and with the intensity of the incident light falling on it. The efficiency of the solar cell is given by,

$$\eta = \frac{\text{Power output}}{\text{Incident solar power}}$$

4.14.2 I-V Characteristics of a Solar Cell

- Fig. 4.28 shows I-V characteristics of a solar cell. The I-V characteristics of solar cell is studied by changing the load resistance R_L and measuring current flowing through and voltage appearing across it. When load R_L is zero maximum current flows through it but voltage will be zero, this current is called the ***'short-circuit current I_{sc}'***. As voltage is zero it will come on current axis.
- As load increase at a particular stage when R_L becomes infinity, the current will be zero and voltage will reach the maximum value. This maximum voltage is called ***open-circuit voltage V_{oc}*** and is plotted on voltage axis. For maximum voltage V_{OC}, current is zero.
- So the product V_{OC} I_{SC} will not give the maximum power drawn from the solar cell.
- For getting maximum power from the solar cell, draw a line at 45° passing from origin, the point (V_m, I_m) where the line will cut the curve will give maximum usable power. The ratio of maximum usable power to ideal power is called as ***fill factor***.

$$\text{Fill factor 'f'} = \frac{\text{Usable power}}{\text{Ideal power}}$$

$$\text{i.e. fill factor 'f'} = \frac{V_m \cdot I_m}{V_{oc} \cdot I_{sc}}$$

where,

I_m and V_m are the maximum usable current and voltage obtained experimentally.

I_{sc} = short-circuit current

V_{oc} = open-circuit voltage

$V_{oc} \cdot I_{sc}$ = theoretically obtained maximum power of the solar cell.

$V_m \cdot I_m$ = experimentally obtained maximum power of the cell.

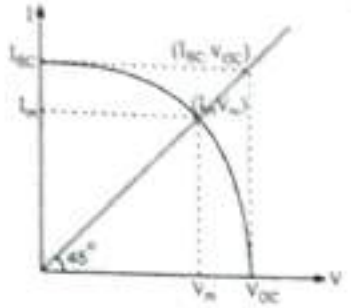


Fig. 4.28

- As the potential difference increases, the recombination of electrons and holes also increases and hence potential difference cannot be increased beyond certain limits. To reduce the recombination, the upper region is of smaller width so that carriers can reach the junction before they recombine.
- When photovoltaic cells are connected in series or parallel, they form a solar battery.

4.14.3 Advantages

- It is an environmentally clean source of energy.
- It is free and available in adequate quantities.
- It is used in satellite communication.
- It is used as power source in artificial satellites.
- It can be used to supply power to places in remote areas and in fuel starved areas.

4.14.4 Disadvantages

- *Dilute source*: Even in the hottest region on earth, radiation flux rarely exceeds 1 kW/m^2 which is the value for technological utilization. It requires large collecting areas in moving application, which results in excessive cost. Thus, use is restricted because of its high price.
- Solar energy varies with time because of day-night cycle.

4.14.5 Solar Cell Applications

- Solar cells are used extensively in satellites and space vehicles for long duration power supply. Their first application was in 1958 to power Vangaurd I, a space satellite. Since then solar cells have remained as an important source of power in space applications due to their low weight, reliability and durability.
- Success of solar cells in space led to their terrestrial applications. For terrestrial use, there are three broad categories, namely: industrial, social and consumer applications.

1. Industrial Applications

- **Navigational Aids:** Marine beacons, remote light beacons near the airport and navigational lights around the world were earlier powered by kerosene/batteries having several maintenance problems. Now-a-days, they are powered by simple solar cells which are reliable and cost effective.
- **Alarm Systems:** Railway signals, alarm systems for fire, flood warnings, traffic lights, highway telephones, etc. are all being powered by solar cells.
- **Defence Equipments:** Many defence equipments for example, mobiles, telephones, remote radar, large instruments used in remote areas, etc. are now being effectively powered by solar cell.
- **Telecommunications:** Telecommunication equipments are often located in remote and inaccessible areas, for example, islands, deserts, etc. They now consume reduced power due to the use of solid-state devices. This has facilitated the use of solar cell as an economical and reliable power source.
- **Emergency Equipment:** Charging of batteries on rafts, boats and for providing essential services after natural disasters like earthquakes, floods, etc. can be done efficiently by a solar cell.
- **Automatic Meteorological Stations:** A meteorological station collects meteorological data at fixed time intervals at several locations and analyses them to predict weather forecasting accurately. A solar powered meteorological weather station is reliable, economical and is relatively free of maintenance problems.

2. Social Applications

- Providing electric power to remote villages and islands, specially in developing countries, by solar cell. These villages are unconnected to the main grid. A small stand-alone type solar cell can provide a rural house enough power for lighting tubes, TV and a small refrigerator.
- Solar cell powered pumps are being installed to provide potable water and for irrigation purposes.
- Special solar cell powered portable refrigerators are being used for the transportation and storage of vaccines. These vaccines are used for mass immunisation programmes for improving rural standards and reducing infant mortality rate.
- Providing electricity to TV, for schools, for educational and recreational purposes.

3. **Consumer Applications**

- A large number of consumer items which are low powered are now being powered by solar cells. Some of these products are pocket calculators, clocks, torches, watches, radios, lights, electric fans, toys, battery chargers, etc.

4.14.6 Ways of Improving Efficiency of Solar Cell

There are number of technologies being developed for improving efficiency of solar cells. The efficiency of solar cell could be upto 45% depending on the technology used for its development. Higher the efficiency higher is the production cost or problem of scaling to mass level. The solar cells that are used for commercial purpose are developed using crystalline silicon technology with efficiencies ranging from 15 to 21.5%. Following are the ways of improving efficiency of solar cell:

Fill Factor

- Fill factor refers to the utilization of available surface area and it ranges from about 70 to 90%. The fill factor of solar panels depends on their shape e.g. round or moon shaped (lower fill factor), square cells (more fill factor).

Solar Cells Glazing

- For long life, solar cells must be protected from the elements (rain, snow, hail, bird dropping etc). Polycarbonate or low-iron glass is generally used due to high optical transmissivity (around 90%). Surface coating treatments reduce reflections for even higher transmissivity.

Solar Panel Orientation

- For highest output, solar panels must be perpendicular to the sun's rays. However, for roof-top installations solar panels are fitted as per the roof pitch and orientation. For other types of fixed installations, the azimuth is oriented to the south and tilt adjusted for the winter period sun as intensity is low in winter.

Solar Tracker

- Solar tracker is the rotation of the panel or array of solar cells so that they always directly face the sun. However, the larger the array, the more difficult will be the mechanics. The optimum tilt angle changes slowly as the earth rotates on its axis.

Light Concentrators

- Solar panel output power may be increased to almost 50% via a light concentrator such as a Fresnel lens or mirror. However, installing concentrators for a large array of solar cells and orientation of the mirror creates an additional tracking problem.

Solar Charge Controls

- Since the solar panel may not have the correct voltage to charge a battery, it must be controlled via a solar charge controller to prevent battery overcharge. The excess power needs to be dissipated using heat sink.

MPPT Controller

- MPPT stands for Maximum Power Point Tracking. The MPPT control does not turn the excess power into heat but it turns it into additional charge current. The control senses both input voltage and current and then adjust output to maximize power transfer.

4.15 HALL EFFECT AND HALL COEFFICIENT

(May 13, Nov. 15)

4.15.1 Hall Effect

(Dec. 14, May 15)

- It often becomes necessary to determine whether a material is an N-type or a P-type semiconductor. Measurement of conductivity alone does not give this information as no distinction can be made between hole and electron conduction.
- Hall effect is used to differentiate between the two types of carriers. It provides a means of determining the density and mobility of charge carriers and gives information about the sign of the predominant charge carrier.
- If a piece of conductor (metal or semiconductor) carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field. This phenomenon is known as '***Hall effect***' and the voltage so generated is called as '***Hall voltage***'.

Explanation of the Effect

- Assume that the sample material is an N-type semiconductor. The current flow consists, almost entirely, of electrons moving from right to left. This movement corresponds to the direction of conventional current from left to right as shown in Fig. 4.29 (a).
- If v is the drift velocity of the electrons moving perpendicular to the magnetic field B, there is a downward force Bev acting on each electron. This causes the electrons to be deflected in the downward

direction. This makes negative charges to accumulate on the bottom face of the slab [See Fig. 4.29 (b)] leaving positive ions on the top surface.

- This gives rise to a potential difference along the top and bottom faces of the specimen across points M and N with the bottom face being negative. This potential difference causes a field E_H in the negative y-direction and so a force eE_H acts on the electrons in the upward direction.
- Under equilibrium, the upward force due to the electric field just balances the downward force due to the magnetic field.

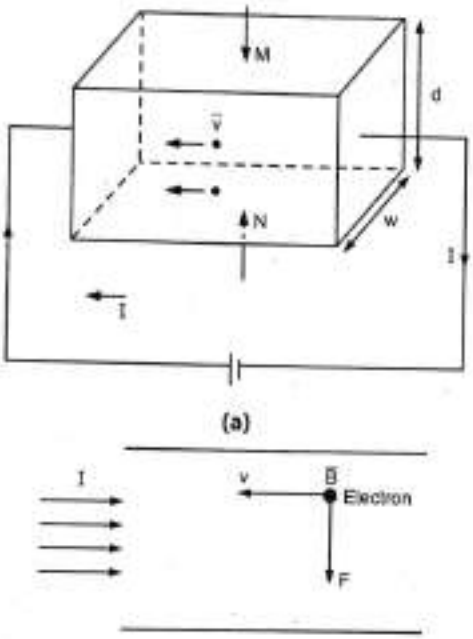


Fig. 4.29 : Hall effect

Thus, $e E_H = e B v$

$\therefore \quad E_H = v B$ ... (1)

- If I is the current in the x-direction then,

$$I = n v A e$$

or $$v = \frac{I}{neA} \quad ...(2)$$

where n is the concentration of charge carriers.

$$\therefore \quad E_H = \frac{B I}{neA} \quad ... (3)$$

Also $$E_H = \frac{V_H}{d}$$

where V_H is the Hall voltage named after the scientist Hall who first predicted and measured the Hall voltage.

$$\therefore \quad V_H = E_H d \quad ... (4)$$

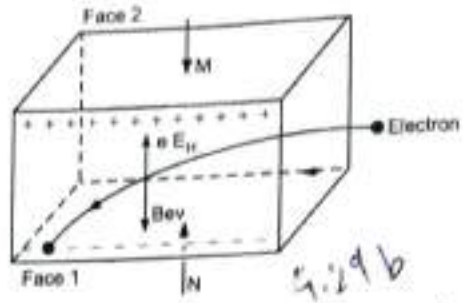


Fig. 4.30: Motion of electrons in an n-type semiconductor

Substituting this in expression (3),

$$V_H = \frac{1}{ne} \cdot \frac{BId}{A} \quad ... (5)$$

or $$V_H = R_H \frac{BId}{A} \quad ... (6)$$

where,

$R_H = \frac{1}{ne}$ is the Hall coefficient for any charge e. ... (7)

- If J_x is the current density of charge carriers in x-direction then,

$$V_H = \frac{1}{ne} \cdot B J d \quad \left(\text{as } J = \frac{I}{A}\right) \quad ... (8)$$

In this specimen, as the dominant charge carriers are electrons,

$$\therefore \quad V_H = -\frac{1}{ne} B J d \quad ... (9)$$

- In expression (8), all three quantities V_H, B and J can be measured. Hence, Hall coefficient and current density can be found.

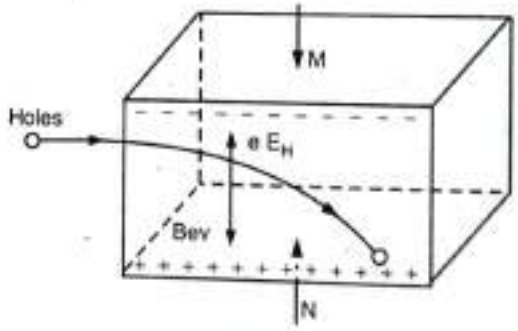


Fig. 4.31: Motion of holes in p-type semiconductor

- Similarly, formulae can be derived for P-type semiconductors. All the formulae are same except that the Hall coefficient will be positive.
- The sign of the Hall voltage gives the sign of the charge carrier and this provides one of the few methods by which the sign of the charge carrier can be ascertained.

∴ Hall voltage,

$$V_H = R_H \cdot \frac{BId}{A} = R_H B J d \quad \ldots (10)$$

4.15.2 Hall Coefficient (R_H)

- The Hall coefficient R_H is determined by measuring the Hall voltage that generates the Hall field. If V_H is the Hall voltage across the sample of thickness d then

$$V_H = E_H d \quad \ldots (1)$$

Also, the Hall voltage is given by,

$$V_H = R_H \frac{BId}{A} \quad \ldots (2)$$

If w is the width of the sample, then its cross-section will be d × w.

$$\therefore \quad V_H = R_H \frac{BId}{dw} = R_H \frac{BI}{w} \quad \ldots (3)$$

$$\text{or} \quad R_H = \frac{w}{BI} V_H = \frac{1}{nq} \quad \ldots (4)$$

- As all quantities in relation (4) are measurable except for n, this relation is used to find the number of charge carriers per unit volume. For metals such as Na, Cu, Ag and Au, the value of n given by this equation is close to the number of valence electrons per unit volume.
- In the case of semiconductors, the interpretation becomes more complex. However, it should be noted that the Hall voltage varies inversely as n, so one would expect it to be larger for semiconductors than for metals.

4.15.3 Applications of Hall Effect

Determination of Type of Semiconductor

- For an N-type semiconductor, the Hall coefficient is negative whereas for a P-type semiconductor, it is positive. Thus, the sign of Hall coefficient is used to determine whether a given semiconductor is N or P-type.

Calculation of Charge Carrier Concentration

- The Hall voltage V_H is measured by placing two probes at the centres of the top and bottom faces of the sample as shown in Fig. 4.29. If $\vec{B}$ is the magnetic flux density, then

$$n = \frac{1}{e} \cdot \frac{BId}{A} \cdot \frac{1}{V_H}$$

- Current I is measured using a current measuring device. Therefore, R_H and hence n can be calculated.

Determination of Mobility

- If conduction is due to one type of charge carriers, for example electrons, then

$$\sigma = ne\mu_e$$

$$\mu_e = \frac{\sigma}{ne} = \sigma R_H$$

$$\mu_e = \sigma \cdot \left(\frac{V_H A}{B I d}\right)$$

Knowing σ, and measuring other parameters as in the above applications, the mobility of electrons μ_e can be determined.

SOLVED PROBLEMS

Problem 4.23: *A silver wire is in the form of a ribbon 0.50 cm wide and 0.10 mm thick. When a current of 2A passes through the ribbon perpendicular to a 0.80 T magnetic field, how large a hall voltage is produced along the width ? The density of silver is 10.5 g/cm³.*

(May 07, Dec. 12)

Data: d = 0.50 cm, t = 0.10 mm, I = 2 amp, B = 0.80 T, density = 10.5 g/cc.

Formula: $V_H = \frac{1}{nq} \cdot \frac{BId}{A}$

Solution: The atomic weight of silver is 108, so the number of atoms in 1 cm³ is

$$n = (6 \times 10^{23}) \left(\frac{10.5}{108}\right) = 6 \times 10^{22} \text{ per cm}^3$$

Silver is monovalent and we can assume that each atom contributes one electron.

∴ Number of electrons per m³ = 6×10^{28}

$$A = 0.05 \times 0.001 = 5 \times 10^{-5} \text{ m}^2$$

$$\text{Hall voltage, } V_H = \frac{1}{nq} \cdot \frac{BI \cdot d}{A}$$

$$= \frac{1}{6 \times 10^{28} \times 1.6 \times 10^{-19}} \times \frac{0.80 \times 2.0 \times 0.05}{5 \times 10^{-5}}$$

$$= \boxed{1.67 \times 10^{-7} \text{ volt.}}$$

Problem 4.24: *A copper specimen having length 1 metre, width 1 cm and thickness 1 mm is conducting 1 amp current along its length and is applied with a magnetic field of 1 Tesla along its thickness. It experiences a Hall effect and a Hall voltage of 0.074 microvolts appears along its width. Calculate the Hall coefficient and the mobility of electrons in copper. Conductivity of copper is $\sigma = 5.8 \times 10^7 (\Omega m)^{-1}$.*

(04) (May 06)

Data: $l = 1$ m, $d = 1$ cm $= 10^{-2}$ m, $t = 1$ mm $= 10^{-3}$ m, $B = 1$ Tesla, $I =$ amp, $V_H = 0.074 \times 10^{-6}$ volts, $\sigma = 5.8 \times 10^{7}$ $(\Omega m)^{-1}$

Formulae: (i) $V_H = R_H \cdot \frac{BId}{A}$, (ii) $\sigma = \frac{\mu}{R_H}$.

Solution: (i) $R_H = \frac{V_H \cdot A}{BId} = \frac{0.074 \times 10^{-6} \times 10^{-2}}{1 \times 1 \times 10^{-2}}$

$= \boxed{0.074 \times 10^{-6}\ m^3/coulomb}$

(ii) $\mu = \sigma R_H = 5.8 \times 10^{7} \times 0.074 \times 10^{-6}$

$= \boxed{4.292\ m^2/volt\text{-}sec.}$

Problem 4.25: *A specimen having length 1.00 cm, width 1.00 mm and thickness 0.1 mm is made to conduct with 1.00 mA current and is placed in a magnetic field of 1.0 Wb/m², acting along the thickness. Calculate the Hall voltage in case of (i) N type semiconductor with Hall coefficient of -3.44×10^{-8} m³/C and (ii) Aluminium with Hall coefficient of -0.3×10^{-10} m³/C. Which of these materials is more sensitive to Hall effect ? Why ?* **(05) (Dec. 11)**

Data: $l = 1.00$ cm, $d = 1.00$ mm, $t = 0.1$ mm, $I = 1.00$ mA, $B = 1.0$ Wb/m². For semiconductor, $R_H = -3.44 \times 10^{-8}$ m³/C.

For aluminium, $R_H = -0.3 \times 10^{-10}$ m³/C

Formula: $V_H = R_H \cdot \frac{BId}{A}$

Solution: (1) For N-type semiconductor,

$$V_H = -3.44 \times 10^{-8} \times \frac{1 \times 1 \times 10^{-3} \times 1 \times 10^{-3}}{0.1 \times 10^{-3} \times 1 \times 10^{-3}}$$

$$\therefore \quad \boxed{V_H = -3.44 \times 10^{-7}\ V}$$

(2) For aluminium,

$$V_H = -0.3 \times 10^{-10} \times \frac{1 \times 1 \times 10^{-3} \times 1 \times 10^{-3}}{0.1 \times 10^{-3} \times 1 \times 10^{-3}}$$

$$\boxed{V_H = -0.3 \times 10^{-9}\ V}$$

N-type semiconductor is more sensitive to Hall effect than aluminium. Because for same experimental setup, it gives more Hall voltage.

Problem 4.26: *A slab of silicon 2 cm in length 1.5 cm wide and 2mm thick is applied with magnetic field of 0.4 T along its thickness. When a current of 75 A flows along the length, the voltage measured across the width is 0.81 mV. Calculate the concentration of mobile electrons in silicon.* **(03) (May 14)**

Data: $h = 2$ cm, $d = 1.5$ cm, $t = 2$mm, $I = 75$ A, $B = 0.4$ T, $V = 0.81$ mV

Formula: $V_H = \frac{1}{nq} \frac{BId}{A}$

$$n = \frac{1}{0.81 \times 10^{-3} \times 1.6 \times 10^{-19}} \frac{0.4 \times 75 \times 2 \times 10^{-2}}{2 \times 10^{-2} \times 2 \times 10^{-3}}$$

$$n = 1.157 \times 10^{26}\ /\ m^3$$

Problem 4.27: *The hall coefficient of a specimen of a doped Silicon is found to be 3.66×10^{4} m³/C. The resistivity of the specimen is 8.93×10^{-3} Ωm. Determine the mobility of the charge carriers.* **(03) (Nov. 13)**

Data: $R_H = 3.66 \times 10^{4}$ m³/C

$\rho = 8.93 \times 10^{-3}$ Ωm

Formula: $R_H = \frac{1}{n_e}$ and $\sigma = n_e\mu$

$\therefore \quad \rho = \frac{1}{n_e\mu} = \frac{R_H}{\mu}$

or $\mu = \frac{R_H}{\rho} = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}}$

$\mu = 4.098 \times 10^{-2}$ m²/V-sec

Problem 4.28: *The hall coefficient of a specimen of a doped Silicon is found to be 3.66×10^{4} m³/C. The resistivity of the specimen is 8.93×10^{-3} Ωm. Determine the mobility of the charge carriers.* **(3) (Nov. 13)**

Data:

$R_H = 3.66 \times 10^{4}$ m³/C

$\rho = 8.93 \times 10^{-3}$ Ωm

Formula: $R_H = \frac{1}{n_e}$ and $\sigma = n_e\mu$

Solution : $\therefore \quad \rho = \frac{1}{n_e\mu} = \frac{R_H}{\mu}$

or $\mu = \frac{R_H}{\rho} = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}}$

$\mu = 4.098 \times 10^{-2}$ m²/V-sec

Note: As there are misprints in the paper received by colleges. The value of answer may be different for different data pointed in paper.

Problem 4.29: *A slab of silicon 2 cm in length 1.5 cm wide and 2mm thick is applied with magnetic field of 0.4 T along its thickness. When a current of 75 A flows along the length, the voltage measured across the width is 0.81 mV. Calculate the concentration of mobile electrons in silicon.* **(3) (May 14)**

Ans. Data: h = 2 cm, d = 1.5 cm, t = 2mm, I = 75 A, B = 0.4 T, V = 0.81 mV

Formula: $V_H = \frac{1}{nq} \cdot \frac{BId}{A}$

$$n = \frac{1}{0.81 \times 10^{-3} \times 1.6 \times 10^{-19}} \cdot \frac{0.4 \times 75 \times 2 \times 10^{-2}}{2 \times 10^{-2} \times 2 \times 10^{-3}}$$

$$n = 1.157 \times 10^{26} / m^3$$

Problem 4.30: *The Hall coefficient of a specimen of doped silicon is found to be 3.66×10^{-3} m^3/C. The resistivity of the specimen is 8.93 Ω-m. Determine the mobility of charge carriers.*

(3) (Nov. 17)

Given : Data : $R_H = 3.66 \times 10^{-3}$ m^3/C

$\rho = 8.93\ \Omega m$

Formula : $R_H = \frac{1}{ne}$

and $\sigma = ne\mu$

$$\rho = \frac{1}{ne\mu}$$

$$= \frac{RH}{\mu}$$

Solution : $\mu = \frac{R_H}{\rho}$

$$= \frac{3.66 \times 10^{-3}}{8.93}$$

$$\mu = 4.098 \times 10^{-4}\ m^2/V\text{-sec}$$

Problem 4.31: *Calculate the mobility of charge carriers in a doped silicon whose conductivity is 100 per Ω-m and the Hall coefficient is 3.6×10^{-4} m^3/Coulomb*

(04) (May 09, Nov. 16)

Data: $\sigma = 100$ per Ω-m

$R_H = 3.6 \times 10^{-4}$ m^3/Coulomb

Formula: $\mu = 6\ R_H$

Solution: $\mu = 6\ R_H$

$\mu = 100 \times 3.6 \times 10^{-4}$

$\boxed{\mu = 0.036\ m^2/V\text{-sec}}$

SUMMARY

- The **Energy Levels** of an isolated atom are discrete.
- **Energy Band :** In crystals or solids, the allowed energy levels of an atom are modified by the proximity of other atoms in such a way that the discrete energy levels of the individual atoms become bands. Each band contains as many discrete levels as there are atoms in the material.
- Elements are **classified** as (i) conductors, (ii) semiconductors and (iii) insulators.
- **Valance Band :** The band formed by a series of energy levels containing the valence electrons is known as valence band.
- **Conductive Band :** The lowest unfilled permitted energy band is called the conduction band.
- **Band Gap :** The energy required for an electron to jump from the valence band to the conduction band is called the 'band gap' or forbidden gap of the semiconductor.
- **Semiconductors :** Materials having properties intermediate between those of conductors and insulators are known as semiconductors.
- **Semiconductors** : are of two types: (i) intrinsic and (ii) extrinsic.
- **Intrinsic semiconductors :** Are those which are pure (free from electroactive and crystalline defects).
- **Doping :** Is the process of adding an impurity to intrinsic semiconductors to increase its conductivity.
- **Extrinsic Semiconductors :** Are obtained by doping an intrinsic semiconductor. They are of two types: (i) p-type extrinsic semiconductor, (ii) n-type extrinsic semiconductor.
- **P-Type :** An extrinsic semiconductor formed by doping a trivalent impurity is called a p-type semiconductor. In this type, holes are the majority charge carriers and electrons are the minority charge carriers.
- **N-Type :** An extrinsic semiconductor formed by doping a pentavalent impurity is called as n-type semiconductor. In this type, electrons are the majority charge carriers and holes are the minority charge carriers.
- **Hall Effect :** When a current carrying specimen (I) is placed in a transverse magnetic field (B) an electric field 'E' is induced in the specimen perpendicular to

both I and B. This phenomenon is called as Hall effect and the voltage hence developed is called as Hall voltage.

- **Fermi Energy :** The highest filled state in the highest occupied energy band at 0 K is called the Fermi level for a metal. The corresponding energy is called the Fermi energy (E_f).
- The **Fermi-Dirac Probability Distribution Function** P(E) gives the probability that an energy state of energy E is occupied by an electron at T K.

$$P(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

- The **Fermi Level** in intrinsic semiconductors is exactly in the middle of the forbidden gap.

$$E_f = \frac{E_c + E_v}{2}$$

- The **Position of the Fermi** level in a p-type extrinsic semiconductor is close to the valence band as holes are the majority charge carriers.
- The **Fermi Level** in an n-type extrinsic semiconductor is close to the conduction band as electrons are the majority charge carriers.
- When a pn junction is illuminated with light, it has the property of producing an e.m.f. This effect is called as **Photovoltaic Effect**.
- Solar cell is a semiconducting device that converts sunlight into electricity. It works on the principle of photovoltaic effect.
- Due to their low weight, reliability and durability, solar cells are used widely in industrial, social and consumer applications.

IMPORTANT FORMULAE

- $v = \mu_e E$
- $I = n \mu_e \frac{V}{l} \cdot A \cdot l$ (for metals)
- $\rho = \frac{1}{n e \mu_e}$
- $\sigma = \frac{1}{\rho} = n \cdot e \mu_e$ (for conductors/metals)
- $I = l \cdot A \cdot \frac{V}{\rho} (n \mu_e + p \mu_h)$ (for semiconductors)
- $\rho = \frac{1}{e (n \mu_e + p \mu_h)}$ (for semiconductors)
- $\sigma = \frac{1}{\rho} = e (n \mu_e + p \mu_h)$ (for semiconductors)
- $\sigma_i = e n_i (\mu_e + \mu_h)$ (for intrinsic semiconductors)
- $\sigma_n = n_e e \mu_e = n_d e \mu_e$ (for n-type semiconductors)
- $\sigma_p = n_h e \mu_h = n_a e \mu_h$ (for p-type semiconductors)
- $V_H = R_H \cdot \frac{BId}{A}$
- $R_H = \frac{1}{nq}$
- $\mu = \sigma R_H$

EXERCISE

1. Describe in brief the formation of energy bands in solids.
2. Explain the terms valence band, conduction band and forbidden energy gap.
3. Classify the elements into conductors, insulators and semiconductors on the basis of band theory of solids.
4. Give the energy band picture of lithium, beryllium, sodium, diamond and silicon.
5. Derive an expression for conductivity in a metal.
6. Derive an expression for conductivity in an intrinsic and extrinsic semiconductor.
7. Discuss dependence of conductivity on temperature.
8. Explain Hall effect and Hall coefficient.
9. What is Fermi energy ? Show the location of Fermi energy levels in intrinsic and extrinsic semiconductors.
10. What is Fermi function ? Show that the Fermi level lies at the centre of the energy gap in an intrinsic semiconductor.
11. Explain why a potential difference develops across an open circuited P-N junction.
12. Give the energy band picture of a P-N junction diode and explain the effect of biasing on the band picture.
13. "P-N junction is a unidirectional device". Explain.
14. Explain the working of a P - N junction diode under forward and reverse bias.
15. Explain the process that takes place in and around the depletion layer.
16. Write a note on construction and characteristics of a solar cell.

17. Explain the working of a solar cell. Give the significance of the cell parameters I_{SC}, V_{OC} and fill factor.

18. Explain free electron theory in short. Give its short comings.

UNSOLVED PROBLEMS

1. The mobilities of carriers in intrinsic germanium sample at room temperature are μ_e = 3600 cm^2/volt-sec and μ_h = 1700 cm^2/volt-sec. If the density of electrons is same as holes and is equal to 2.5×10^{13} per cm^3, calculate the conductivity.

   (**Ans.** 2.12 mho/m)

2. Calculate the number of acceptors to be added to a Germanium sample to obtain the resistivity ρ = 10 ohm. cm. Given μ = 1700 cm^2/volt-sec.

   (**Ans.** 3.676×10^{14} per cm^3)

3. At room temperature the conductivity of a silicon crystal is 5×10^{-4} mho/cm. If the electron and hole mobilities are 0.14 m^2/volt-sec and 0.05 m^2/volt-sec, determine the density of carriers.

   (**Ans.** 1.64×10^{18} / m^3)

4. The specific density of tungsten is 18.8 g/cm^3 and its atomic wt. is 184.0. Assume that there are two free electrons per atom. Calculate the concentration of free electrons. Av. No. = 6.025×10^{23} / gmole.

   (**Ans.** 2.5×10^{23}/cm^3)

5. Compute the conductivity of copper for which μ_e = 34.8 cm^2/volt-sec and d = 8.9 gm/cm^3. Assume that there is one free electron per atom.

   Av. No. = 6.025×10^{23} / g mole, at. wt. of Cu = 63.5.

   If an electric field is applied across such a copper bar with an intensity of 10 V/cm, find the average velocity of free electrons.

   (**Ans.** 47.02×10^{-4} mho/cm, 348 cm/sec.)

6. The resistivity of copper wire of diameter 1.03 mm is 6.51 ohm per 300 m. The concentration of free electrons in copper is 8.4×10^{28} / m^3. If the current is 2 A, find (a) mobility, (b) drift velocity, (c) conductivity.

   (**Ans.** 0.413 m^2/volt-sec, 0.286×10^{-20} m/sec, 55.5×10^8 mho/m)

7. Calculate the energy gap in silicon if it is given that it is transparent to radiation of wavelength greater than 11000 A°.

   (**Ans.** 1.13 eV)

8. An N-type semiconductor is to have a resistivity of 10 ohm-cm. Calculate the number of donor atoms which must be added to achieve this.

   (**Ans.** 12.5×10^{23})

   Assume μ_e = 500 cm^2/volt-sec.

UNIVERSITY QUESTIONS

December 2012

1. Draw energy band picture for P-N junction in case of (i) Zero Bias, (ii) Forward Bias, (iii) Reverse Bias. **[3]**
2. Derive an expression for Conductivity in Semiconductor. **[6]**

May 2013

1. Define Fermi level. Plot the variation of Fermi level with the increase of temperature for n-type and p-type semiconductor. **[3]**
2. Define Hall effect. Derive the expression of Hall coefficient, Hall Voltage and discuss their applications. **[6]**

November 2013

1. What is the effect of following factors on the conductivity of semiconductors ? **[3]**
   (i) Increase in impurity of concentration.
   (ii) Increase in temperature.
   (iii) Increase in intensity of light.

May 2014

1. Explain Fermi-Dirac distribution function specifying the meaning of each term in it. **[3]**
2. Derive the expression for the conductivity of intrinsic and extrinsic semiconductor. **[6]**

December 2014

1. Explain Fermi dirac probability distribution function with the meaning of each symbol in it. **[3]**
2. Explain Hall effect. Derive the equation of Hall voltage and Hall coefficient. **[6]**

May 2015

1. What is Fermi level ? Show the position of Fermi level in P-type semiconductor at temperature T = 0 K and T > 0 K. **[3]**
2. What is Hall effect ? Derive the equation of Hall voltage. **[6]**

November 2015

1. What is Fermi level ? Explain Fermi-Dirac probability distribution function. [3]
2. Explain Hall effect. Derive the equation of Hall voltage and Hall coefficient. [6]

May 2016

1. What is Fermi energy in semiconductor? With the help of labelled diagram show the position of Fermi level in the case of a diode that is connected in forward bias. [3]
2. Derive an expression for conductivity in case of intrinsic and extrinsic semiconductors. [6]

November 2016

1. Derive an expression for conductivity in intrinsic and extrinsic Semiconductors. [6]

May 2017

1. What is hall effect ? Derive the expression for Hall voltage and Hall coefficient. State applications of Hall effect. [6]

November 2017

1. Explain the working of P-N junction diode in [6]
   (i) Zero Bias
   (ii) Forward Bias
   (iii) Reverse Bias on the basis of energy level diagram.

May 2018

1. What is photovoltaic effect? Explain the construction and working of solar cell. Also draw the IV characteristics of solar cell and define fill factor. [6]

November 2018

1. Explain Hall effect. Derive the expression for Hall voltage and Hall co-efficient. [6]

May 2019

1. Define Fermi level in conductors and semiconductors. Show that the Fermi level lies at the centre of energy gap in an intrinsic semiconductor. [6]

✠ ✠ ✠

MAGNETISM

5.1 ORIGIN OF MAGNETISM

- The Magnetism originates from the spin and orbital magnetic moment of an electron.

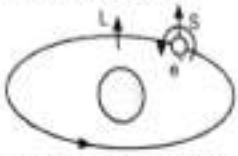


Fig. 5.1 : Spin and Orbital motion of an electron

- The orbital motion of an electron around the nucleus is analogous to the current in a loop of wire.
- The magnetic moment of a current carrying conductor is given by
  $\mu = I.A$ where I is the current in A and A is the area in m^2.

Orbital Magnetic Moment

- The magnetic moment of an electron in orbit is given by

$$\alpha^2 = \pi r^2 \frac{ev}{2\pi r} = \frac{evr}{2} \quad \text{... (1)}$$

where r is the radius of orbit, e - charge and v is the velocity

- The angular momentum of an electron must be an integral multiple of Plancks const.

$$mvr = \frac{nh}{2p} \quad \text{... (2)}$$

where m is the mass and h is the Planck's const.

- If the electron revolves in the first orbit then n = 1. Therefore orbital magnetic moment of an electron is given by from (1) and (2)

$$\alpha^2 = \frac{eh}{4\pi m} \quad \text{... (3)}$$

Which is known as ***Bohr Magneton***, the smallest possible orbital magnetic moment

Spin Magnetic Moment

- Similarly the smallest possible magnetic moment due to spin of the electron is

$$\alpha^2 = \frac{eg}{4\pi m}$$

- According to quantum theory the spin of electrons have only two possibilities +1/2 or -1/2.
  Similar to eqn. (3) we can write in the form

$$\alpha^2 = \frac{e}{2m} S \quad \text{... (4)}$$

where S is the spin quantum number here given by $\frac{1}{2} \cdot \frac{h}{2\pi}$.

In short

$$\alpha^2 = g \frac{e}{2m} S \quad \text{... (5)}$$

Here g is the term known as g- factor.

When g = 2, the spin contribution arises and when g = 1 the orbital contribution arises.

- The mass of the nucleus is so large that the magnetic moment contribution can be neglected compared to the electronic magnetic moment.
- The gryomagnetic ratio is proportional to the g-factor and 'g' arises due to the precession of the electrons similar to the precession of a top in a gravitational force. The value of g tells us whether the origin of magnetic moment is spin or orbital motion of electrons.

5.1.1 Magnetic Parameters

- **Magnetic Field Strength 'H'** : The magnitude of the force experienced by a unit north pole at any point in the field is called the strength of the magnetic field at that point. The unit of field is called the gauss or oersted (C.G.S. unit). In M.K.S. system the intensity of magnetic field is expressed in Newton/Ampere-metre or weber/meter^2.
- **Magnetic Induction or Flux Density 'B'** : It is defined as the number of hypothetical induction lines passing normally through unit area. It is measured in Tesla (T) or weber/meter^2.
  $\therefore B = \phi/A$ where ϕ is the normal flux and A the area of cross-section.
- **Intensity of Magnetisation 'I'** : It measures the degree of magnetisation of a magnetised specimen and is defined as the magnetic moment per unit volume.

$$I = \frac{\text{Magnetic moment (M)}}{\text{Volume (V)}}$$

- **Magnetic Susceptibility 'χ'** : It measures the ease with which the specimen can be magnetised. It is defined as the ratio of the intensity of magnetisation induced in it to the magnetising field strength

$$\chi = \frac{I}{H}$$

- **Magnetic Permeability 'μ'** : The measure of the degree to which the lines of magnetic force can

penetrate the medium is called the absolute permeability of the medium. It is denoted by μ_a.

- It is also defined as the ratio of the magnetic induction B produced in a material to the magnetising or induced field H.

$$\mu_a = \frac{B}{H}$$

Also $\mu_a = \mu_o \mu_r$

where μ_o is the permeability of free space and is equal to $4\pi \times 10^{-7}$ Henry/meter or is equal to 1 (in C.G.S. units). μ_r is the relative permeability which is measured by the ratio of the number of lines of force per unit area in the medium to the number of lines per unit area if the medium were replaced by vacuum. For free space $\mu_r = 1$ and $B = \mu_o H$.

5.1.2 Relationship Between μ and χ

Consider a magnetic material of cross-sectional area A and relative permeability μ_r to be placed in a uniform magnetic field of strength H.

Two Types of Lines of Induction Pass Through it :

1. Due to the magnetic field.
2. Due to magnetisation by induction.

Hence, $B = \mu_o H + \mu_o I$ ... (1)

where μ_o is the permeability of free space.

But $B = \mu_o \mu_r H$ ... (2)

From (A) and (B)

$$\mu_o \mu_r H = \mu_o H + \mu_o I$$

i.e. $(\mu_r - 1) H = I$

$$\mu_r - 1 = \frac{I}{H} = \chi$$

(χ being the susceptibility of the material).

$\mu_r = 1 + \chi$ gives the relation between the magnetic susceptibility and the magnetic permeability. In free space $\mu_a = 1 + \chi$.

5.2 TYPES OF MAGNETIC MATERIALS

- The most materials interact only slightly with an impressed magnetic field. A few substances, however, greatly alter any magnetic field in which they are placed.
- Consider a simple experiment to indicate the way in which material objects affect magnetic fields. An alternating potential difference is applied across the terminals of a toroidal coil so that current through it varies sinusoidally

$$i = i_o \sin 2\pi ft \quad \text{... (1)}$$

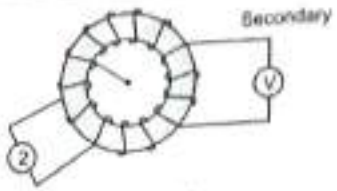


Fig. 5.2

- In Fig. 5.2 the voltage induced in the secondary coil can be used to determine the magnetic properties of the material inside the toroid.
- The flux inside such a coil is found to be

$$\phi = \frac{\mu_o N i a}{2} ln\left(1 + \frac{a}{b}\right)$$

- N is the total number of loops on the toroid. By Faraday's law, the induced e.m.f. in the secondary coil of N_2 loops wound on the toroid is

$$\epsilon_2 = -\frac{\mu_o N N_2 a}{2\pi} ln\left(1 + \frac{a}{b}\right)\frac{di}{dt}$$

$$\epsilon_2 = -M_\delta \frac{di}{dt} \quad \text{... (2)}$$

where $M_\delta = \frac{\mu_o N N_2 a}{2\pi}$ is the mutual inductance.

Since we known how the current in the toroid changes with time, we can rewrite (2) as

$$\epsilon_2 = -2\pi f M_\delta i_o \cos 2\pi ft \quad \text{... (3)}$$

By using an appropriate voltmeter in the secondary coil, the maximum value of ϵ_2, namely, $2\pi f M_\delta i_o$ can be measured.

- In obtaining (3) it has been assumed that the space in the interior of the coil is empty.
- Experiments can be performed when the interior of the coil is completely filled with some material, and the effect on-the induced voltage ϵ_2 caused by changing the material within the coil can be found. It is found that the value of ϵ_2 does not change if the coil is filled with such very diverse substances as air, water, oil, wood, Al, Cu, plastic etc.
- Only a very special class of substances, change the induced voltage, by large amount. These are the so called ferromagnetic materials, and they are almost always pure or alloy composition containing iron, cobalt and nickel.
- In 1845 Faraday concluded that all substances were, to a greater or lesser extent affected by a magnet a few being attracted, but most are repelled. This gave rise to the grouping of different substances into
  - Ferromagnetic like iron, which are strongly attracted
  - Paramagnetic, feebly attracted and
  - Diamagnetic which are repelled by a magnet.

Materials can also be classified on the following basis :

- Sign of χ and its value.
- Value of permeability μ_r.
- Presence or absence of permanent magnetic dipoles.

5.2.1 Classification on the Basis of χ and μ_r

- **Paramagnetic :** Substances are those for which I varies linearly with H and χ has a small positive value. Also μ_r is slightly greater than 1. Examples are, platinum solutions of salts of iron, oxygen, manganese, palladium etc. For platinum $\mu = 1.00002$ and $\chi = 1.71 \times 10^{-6}$. Also, χ not only decreases with increase of magnetising force but it also depends on the temperature. Curie discovered that the susceptibility of some paramagnetics varies inversely as the absolute temperature.
- **Diamagnetic :** Substances have $\mu < 1$ and χ is constant and has a negative value of the order of 10^{-4} to 10^{-6}. Example, bismuth, antimony, Zn, Ag, Cu, Sb, Au, Fb, water, alcohol, air hydrogen. When placed in a magnetic field they have a tendency to move away from the field.
- **Ferromagnetic :** Substances are those which can be magnetised to a great extent. They have an abnormally high value of χ e.g. steel, iron, cobalt, nickel and alloys of these substances. In these substances the magnetisation is not proportional to the magnetising force, hence χ and μ vary with the magnetising force considerably. Also χ varies with temperature. When a ferromagnetic substance is heated, its χ varies inversely as the absolute temperature. This is called as Curies law and is expressed as. χT = Constant (T being absolute temperature). Thus, χ steadily decreases with increase in temperature, until a critical temperature is reached, at which ferromagnetism disappears and the substance becomes paramagnetic. This absolute temperature is called the Curie temperature. The susceptibility of a ferromagnetic substance above its Curie point is inversely proportional to the amount its temperature is above the Curie temperature.

  i.e. $\chi \propto \frac{1}{(T - T_c)}$, T_c being Curie temperature.

  This law is called the Curie - Weiss law.

  T_c for cobalt is about 1100°C, for nickel 4000 G and for iron 770°C.

  Ferromagnetics have non linear variation of μ_r with H i.e. $B \neq \mu_o H$ and hysteresis effect is exhibited.

5.2.2 Classification on the Basic of Presence or Absence of Permanent Magnetic Dipoles

- Magnetic materials can also be classified on-the basis of the presence or absence of permanent magnetic dipoles in them. Materials which lack permanent magnetic dipoles are called diamagnetic. Magnetisation of such materials occurs when the applied field induces a magnetic moment in the individual atoms. This is due to the external magnetic field causing changes in the electron orbits. In the absence of the external field, the net magnetic moment of the orbit is zero.
- If permanent dipoles are present in the atoms of a material, it could be paramagnetic, ferromagnetic, antiferromagnetic or ferrimagnetic. Differentiation is done on the basis of interaction between individual dipoles.
  - The substance is diamagnetic if dipole interaction is zero and orientation of individual dipole moment is random. Point the magnetic moments will align with an applied magnetic field.
  - For ferromagnetics, dipoles interact to line up parallel to each other, to give a large resultant magnetisation. This is known as ordered magnetism because of the stronger interatomic interactions.
  - When magnetic moments of equal magnitude are present and neighbouring Moments are aligned antiparallel, the substance is antiferromagnetic. In these substances, the magnetisation vanishes.
  - When unequal magnetic moments are present-and are aligned antiparallel to each otter, the substance is ferrimagnetic. In these substances a net magnetisation exists.

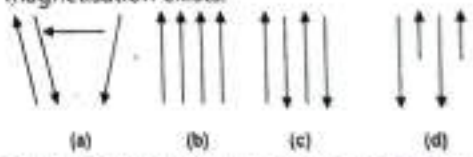


Fig. 5.3 : Paramagentic, ferromagnetic, antiferromagnetic and ferrimagnetic arrangement of spins

- Diamagnetism is a universal property of all materials. However, diamagnetic properties are weaker than paramagnetic ones and still weaker than ferromagnetic properties. The presence of a permanent magnetic moment in atoms is a necessary condition for the existence of ferromagnetic properties.
- The pecularities of ferromagnetism are due to the formation of vast regions or domains. In the domains the magnetic moments of a large number of atoms are arranged parallel to one another giving magnetic saturation.

5.3 APPLICATIONS

5.3.1 Transformer Core

The electrical power transformer features primary, secondary, and tertiary windings. The transformer is driven by the flux between the windings. Magnetic cores are used in transformers, which serve as the pathway of the flux. The core can be made from various materials.

Materials Used for Building Magnetic Cores in Transformers

The magnetic core is basically a material with magnetic permeability, which helps confine magnetic fields in transformers. The following are the types materials used for producing magnetic cores for the transformers:

- **Amorphous Steel:** This is one of the popular options for creating magnetic cores in transformers. These cores are made from several paper-thin metallic tapes, which help reduce the flow of eddy currents. Amorphous steel cores have fewer losses than other magnetic cores, *and can easily* operate at *high* temperatures than standard lamination stacks. The amorphous steel cores are most commonly used in high efficiency transformers that operate at medium frequencies.
- **Solid Iron Core:** These cores provide magnetic flux, and helps retain high magnetic fields without iron saturation. The cores are not recommended for transformers operating at AC applications because large eddy currents are produced by the magnetic field. These eddy currents produce heat at high frequencies.
- **Amorphous Metals:** Also known as vitreous metals, these metals are glassy or non-crystalline. These metals are used for creating high-performance transformers. The materials have low conductivity, which helps reduce eddy currents.
- **Ferrite Ceramics:** Ferrite ceramics are a class of ceramic compounds made from iron oxide, and one or multiple metallic elements. The magnetic cores made from ferrite ceramics are used in high-frequency applications. The ceramic materials are produced in different specifications to meet diverse electrical requirements. These ceramic materials serve as efficient insulators, and help decrease eddy currents.
- **Laminated Magnetic Cores:** These cores feature thin iron sheets covered with an insulating layer. These insulators prevent eddy currents, and confine them to narrow loops within each single lamination layer. The thinner lamination minimizes eddy current effects.
- **Carbonyl Iron Cores:** These magnetic cores are made from powdered carbonyl iron, and provide stable service across a wide range of magnetic flux and temperature levels. The carbonyl iron powder cores feature small iron spheres coated with a thin insulating layer. These cores help reduce the effects of eddy currents at high temperatures.
- **Silicon Steel:** Silicon steel has high electrical resistivity. The silicon steel core provides stable performance over the years. Silicon steel offers high saturation flux density. A few years ago, characteristics of silicon steel was altered with chemical changes, and today, the new product is known as AN type M6. The M6 steel has high permeability and low losses, and it is used in high-performance applications.

5.3.2 Magnetic Storage

Principle of Magnetic recording and Reproduction

- Magnetic recording is storage of the sound pressure variations in the form of elementary magnets (formed in a magnetic material), whose length and strength depend on audio signals. It was discovered by Poulsen of Denmark in 1898.
- Magnetic recording is based on the principle (shown in Fig. 5.3) that certain materials (like iron oxide) when brought in a magnetic field, get magnetised and retain that magnetism permanently until altered. The various steps involved in magnetic recording are described below.
- Sound pressure variations are converted into electrical variations (audio signal) by a microphone. The audio output of the microphone is amplified and fed to the coil of an electromagnet. The electromagnet (called 'Head') has a minute gap through which magnetic lines of force cannot pass easily due to high reluctance of air. When a tape with a coating of a magnetic material (like iron oxide) is made to pass across the gap, the lines of force get an easy path through the iron oxide which is formed into elementary magnets. The magnetic strength of electromagnet, and hence, through the gap covered by
- The iron oxide of the tape, depends on the audio current. Thus, the coating of iron oxide on the tape is magnetised in accordance with the audio current and hence, in accordance with the sound pressure variations. The magnetism is the iron oxide can remain retained for long time. This means that sound has been recorded in the form of varying magnetic field.

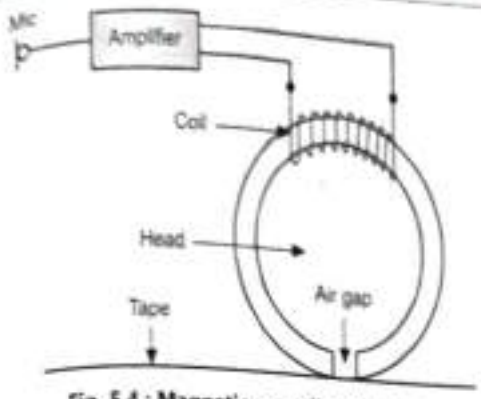


Fig. 5.4 : Magnetic recording on tape

- In reproducing the recorded sound, the tape is again made to pass through a similar gap (or even the same gap), and it causes change of lines of force through the coil. This induces e.m.f. (audio signal) in the coil, which is in accordance with the rate of change of magnetic flux in the tape. The induced e.m.f. is amplified and is fed to a loudspeaker which converts the audio signal into sound.
- Fig. 5.5 shows a magnetised tape for 3 cycles of audio signal. The Fig. 5.5 shows that as the wavelength decreases. Each cycle gives rise to two bar magnets.

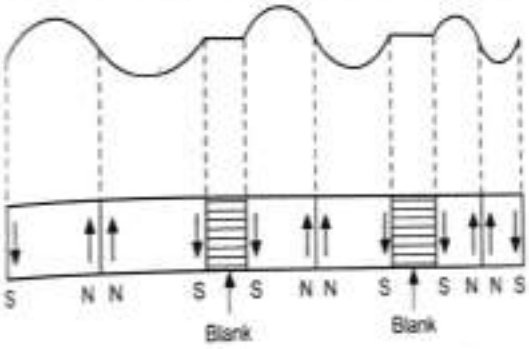


Fig. 5.5 : Magnetisation of tape by three audio cycles of different frequencies

Magneto-Optical Recording

- **MO** or **Magneto-Optical Recording** is a method of storing and retrieving data using a laser and a magnet. The laser in the drive heats the magnetic surface to a high enough temperature to allow the polarization of the disk to be modified with a magnet for more precisely written data. When data needs to be read from the disk a laser of less intensity is used.
- The data is written onto magnetic optical disks on the recording layer in the size of the laser beam as shown in the Fig 5.6 The top layer is thin film of magnetic material (like gadolinium, iron, cobalt alloy) formed by sputtering. This material has the characteristics to facilitate copying the magnetic domain onto the top layer by warming the top layer with the laser.

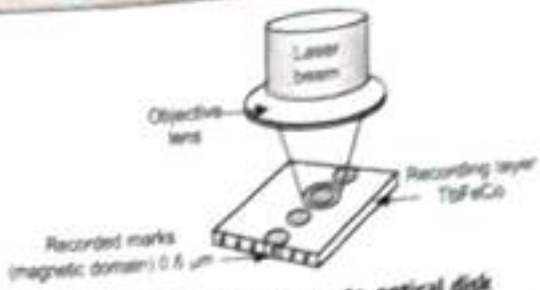


Fig. 5.6 : Conventional magneto-optical disk

- At the same time, applying a positive external magnetic field, as in Fig. 5.7, helps the magnetic domain quickly grow larger to just the size of the light spot. Also, the magnetic orientation of the recording layer will not be affected within this temperature range so high density is kept as it is. This phenomenon is called ***Magnetic Amplifying***. The amplified magnetic domain is the size of the light spot, allowing clear, large signals to be easily picked up. For continuous reading of data, the magnetic moments must be read precisely one by one.

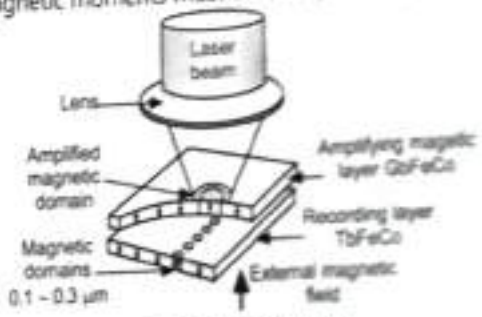


Fig. 5.7 : MAMMOS

- So, to make sure the amplified magnetic domain will not interfere with the reading of adjacent domains, the direction of the external magnetic field must be reversed, causing the amplified magnetic domain to shrink and disappear instantly. The magnetic amplifying layer will return to its original magnetic orientation and be prepared for the next amplification.

SUMMARY

- **Dipole :** The torque experienced by a magnet when placed in an external field is called dipole moment.
- **Types of Magnetic Materials :** Depending upon the permanent magnetic dipole present a magnetic material can be classified as (i) Paramagnetic (ii) Ferromagnetic (iii) Antiferromagnetic and (iv) Ferromagnetic.
- The magnetic materials have got wide range of applications in the field of electrical engineering and storage devices.

EXERCISE

1. What is magnetic material? Give the types of magnetic materials?
2. Sate and explain any two applications of magnetic materials.

SUPERCONDUCTIVITY

5.4 INTRODUCTION

(Dec. 12, Nov. 13, May 14)

- Superconductivity is one of the most exciting phenomena because of its special feature when compared to conductors and has wide range of applications in industry. Experimentally, superconductivity was discovered by Dutch physicist, Heike Kamerlingh Onnes in 1911. While studying the temperature dependence of resistance of metals, he discovered that the electrical resistance in certain metals such as lead and mercury suddenly disappears at around 4 K.
- In 1933, W. Hans Meissner revealed that the superconductors repel the magnetic lines of force when it is placed in an external magnetic field.
- The theoretical explanation of superconductivity was given by John Bardeen, Leon N. Cooper and J. Robert Schreiffer in 1957. The theory is famous as ***BCS theory*** which explained superconductivity based on pairing of electrons called '***Cooper pair***'.
- Some important features of superconductivity are as follows:
  - The electrical resistance drops to zero at critical temperature.
  - The magnetic flux lines are excluded from the bulk of superconductor.
- It is a known fact that the resistivity of pure metals decreases with decreasing temperature. When the temperature falls below a certain value (the exact value depending on the substance), the resistivity vanishes entirely.

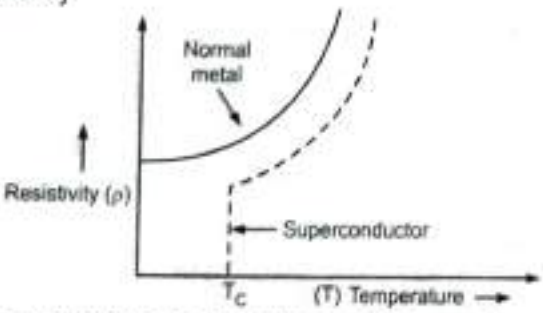


Fig. 5.8: Variation of resistance with temperature

- In metals, both the thermal vibrations of atoms and the presence of impurities or imperfections scatter the moving conduction electrons. This gives rise to electrical resistivity. The variation of resistivity for a pure metal and superconductor is shown in Fig. 5.8.
- At the beginning of the twentieth century, in 1908, H. Kamerling Onnes, a Dutch Physicist, successfully liquified helium. As helium boils at 8.2 K, it therefore became possible to study the properties of materials at low temperature.
- In 1911, he observed that the electrical resistivity of pure mercury dropped suddenly to zero at about the boiling point of helium. He concluded that mercury had passed into a new state, which he called the ***superconducting state*** due to its remarkable electrical properties.
- The temperature at which the material changes its state from a state of normal resistivity to a superconducting state, is called the ***transition or critical temperature*** T_c.
- A conductor having zero (or almost zero) electrical resistance is called a ***superconductor*** and the phenomenon is called as ***superconductivity***.

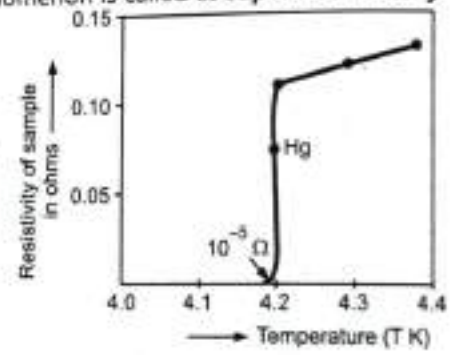


Fig. 5.9: Resistance of mercury as a function of temperature showing a transition from normal state to superconducting state at a critical temperature of 4.2 K

- The superconducting transition is found to be very sharp for a pure metal and it is broad for an impure metal. The zero magnetic induction in a superconductor is responsible for levitation effects.
- In a famous levitation experiment, a horizontal bar magnet was suspended from a chain. It was lowered over a sheet of lead, which had been cooled to the superconducting state. As the magnet came nearer to the superconducting state, the magnet remained floating horizontally over the lead sheet.
- The field of the approaching magnet induces a current on the surface of the superconductor. As the resistance is zero in the superconductor, the current persisted and the field due to the current repelled the bar magnet.
- This persistence of currents is found uniquely in superconductors. Certain experiments on the study of decay of these supercurrents in a solenoid found decay time to be greater than 10^5 years.

5.5 PROPERTIES OF SUPERCONDUCTORS

Following are the properties of superconductors:

5.5.1 Zero Electrical Resistance

- A superconductor is characterized by zero electrical resistance. The temperature below which the resistance of the material vanishes is called as the '***transition temperature***' or '***critical temperature***'. It is referred as T_c.
- As it is not possible to test experimentally whether the resistance is zero, the specimen is connected in a circuit as shown in Fig. 5.10.

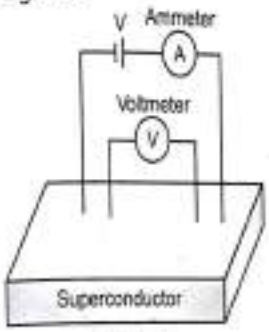


Fig. 5.10

- When the material is in normal conducting state, a voltage drop is measured across its ends. As the material is cooled below its transition temperature T_c, the voltage drop disappears as its resistance drops to zero ($R = V/I$).
- A more sensitive method devised by K. Onnes consists in measuring the decrease of current in a closed ring of superconducting wire.

Table 5.1: A List of Some Superconductors Alongwith their Critical Temperature

Sr. No.	Material	T_c in K
1.	Copper, silver, gold	Non-superconducting
2.	Rhodium	240×10^{-4}
3.	Aluminium	1.1
4.	Tin	3.72
5.	Mercury	4.15
6.	Lead	7.2
7.	Niobium	9.3
8.	Niobium-titanium alloys	9-11
9.	Lead molybdenum sulphide	14
10.	Niobium-tin	18.3
11.	Vanadium-gallium	15.4
12.	Niobium-germanium	23.3

- It has been observed that traces of paramagnetic elements in the specimen can lower the transition temperature. Hence, it becomes necessary to remove these traces completely. Non-magnetic impurities have no marked effect on the transition temperature.

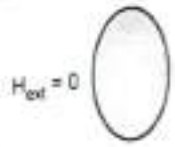
5.5.2 Meissner Effect

(Dec. 12, 14, Nov. 15, May 13, 14, 15, 16)

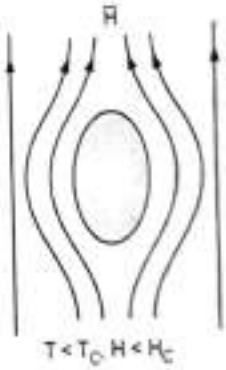
- Meissner and Ochsenfeld discovered in 1933 that a superconductor completely expels any magnetic field lines that were initially penetrating it in its normal state. This property is independent of the path by which the superconducting state is reached.

Path 1

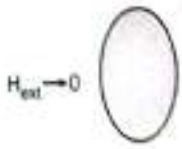
- The sample is in superconducting state and is brought to the magnetic field. It is found that the magnetic flux is totally expelled from the sample.



(a) Superconductor is initially outside the magnetic field



(b) When the superconductor is transported into the magnetic field, it pushes the field lines aside



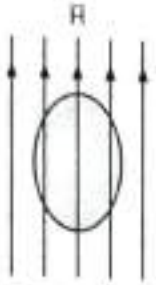
(c) When the magnetic field is switched off

Fig. 5.11

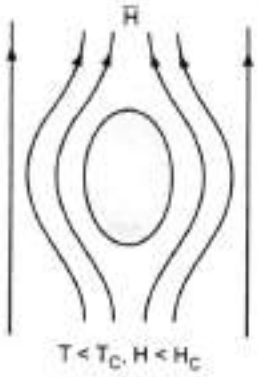
Path 2

- The magnetic field is applied first to the sample in the normal state. Then the material be cooled to below T_c in the presence of the magnetic field. Meissner and Ochsenfeld found that the magnetic flux is totally

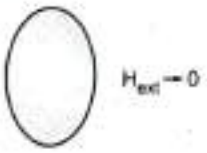
expelled from the sample as it becomes superconducting. This expulsion of magnetic flux during the transition from normal to superconducting state is called as **'Meissner effect'**.



(a) Superconductor in a magnetic field at T > T_c, magnetic field penetrates the body in normal state



(b) On cooling the superconductor in the presence of magnetic field, it expels the flux lines within the specimen at T = T_c. This is Meissner effect



(c) When the field is switched off, field is not trapped within the superconductor. Superconductivity is thus characterized by perfect diamagnetism

Fig. 5.12

Explanation of Meissner Effect

- When a superconducting sample is placed in a magnetic field, it induces currents which circulate on the surface of the specimen in a manner that it creates a magnetic field everywhere equal and opposite to the applied magnetic field.

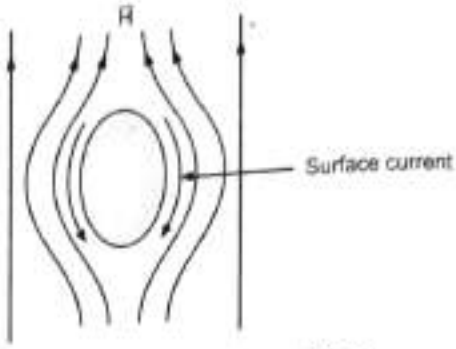
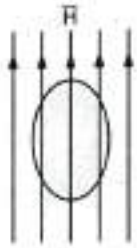
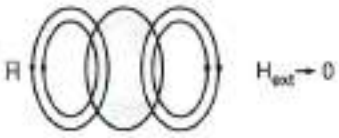


Fig. 5.13: Meissner effect

- Meissner effect cannot be explained by the assumption that a superconductor is a resistanceless conductor. A superconductor is not just a perfect conductor but has an additional property. A material in the superconducting state does not permit any magnetic flux to exist within the body of the material.
- When a perfect conductor is cooled in a magnetic field until its resistance becomes zero, the magnetic field in the material is frozen or trapped in the material. It cannot change subsequently, irrespective of the applied field. Therefore, a conductor does not exhibit diamagnetic behaviour even slightly.



(a) Conductor held in magnetic field is cooled to the state of zero electrical resistance



(b) When magnetic field H_{ext} is switched off, magnetic field is trapped in the ideal conductor

Fig. 5.14

- The magnetic induction inside the specimen is given by,

$$B = \mu_o (H + M) \qquad \text{(Normal state } T > T_c)$$

where, H – external applied field

M – magnetisation produced within the specimen

For $T < T_c$ $\quad B = 0$

$$\therefore \quad \mu_o (H + M) = 0 \qquad \text{(Superconducting state)}$$

$$H = -M$$

The susceptibility of the material,

$$\chi = \frac{M}{H} = -1 \qquad \text{(Perfect diamagnetism)}$$

- Thus, the superconducting state is characterized by perfect diamagnetism. Meissner effect conclusively proves whether a particular material has become a superconductor or not. Because of Meissner effect, superconducting materials strongly repel external magnets, it leads to both **'levitation effect'** and **'suspension effect'**.

5.5.3 Critical Magnetic Field (May 13, 16, Nov. 15)

- K. Onnes discovered in 1913 that, when a superconductor is placed in an increasing magnetic field, it loses superconductivity at a certain value H_c of the field. The magnetic field strength at which superconductivity gets destroyed is called the **'critical magnetic field'** H_c. This value is a characteristic of the metal and depends on its orientation in the magnetic field and the temperature.
- The relation between superconductivity and magnetic field plays an important role in the study of properties of superconductors. Obviously, the value of H_c varies with temperature. Fig. 5.15 shows the variation of H_c with temperature for a typical superconductor.

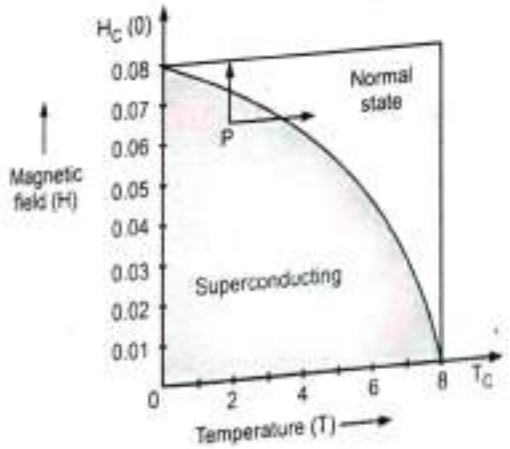


Fig. 5.15

- From Fig. 5.15, consider point P, where the temperature and the magnetic field are within the shaded region, the metal is in the superconducting state. On increasing either the temperature or the field, it can be driven into the normal state. Hence, it can be seen that a superconductor has two possible states: (i) the superconducting one which is resistanceless and perfectly diamagnetic and (ii) a normal state which is the same as a normal metal.
- At any temperature $T < T_c$, the material remains superconducting until a corresponding critical magnetic field is applied. When the magnetic field exceeds the critical value, the material goes into the normal state. The critical field required to destroy the superconducting state decreases progressively with increase in temperature.
- For example, a magnetic field of 0.04 T will destroy the superconductivity of mercury at T = 0 K, whereas a field of 0.02 T is sufficient to destroy its superconductivity at T = 3 K.

The variation of critical field with temperature is given by the relation

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

where $H_c(0)$ is the critical magnetic field at 0 K.

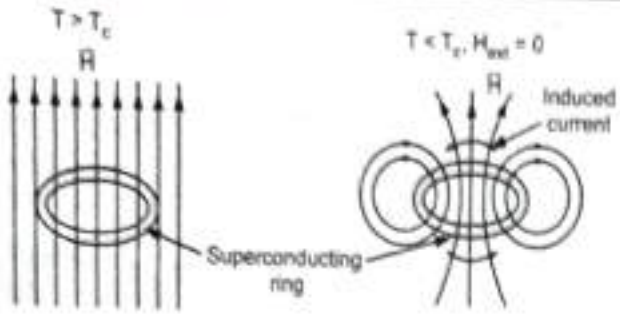
5.5.4 Persistent Currents (Dec. 12)

- Consider a superconducting ring placed in a magnetic field. When cooled to below the critical temperature, it becomes superconducting. The external field induces a current in the ring. When switched off, the current will continue to keep flowing, on its own accord, around the loop, as long as the loop is held below the critical temperature.
- Such a steady current flowing with undiminished strength is called **persistent current**. This current does not need external power to maintain it as there does not exist I^2R losses. If the superconducting ring has a finite resistance R, the current circulating in the ring would decrease according to the relation,

$$I(t) = I(0)\ e^{-Rt/L}$$

where L is the inductance of the ring.

- Calculations show that once the current flow is initiated, it persists for more than 10^5 years. Persistent current is one of the most important properties of a superconductor.
- Superconductor coils with persistent currents produce magnetic fields. They can therefore be used as magnets which do not require a power supply to maintain its magnetic field.



(a) Superconducting ring is cooled in the presence of a magnetic field

(b) At T < T_c, the magnetic field is switched off. Persistent current is induced in the ring

Fig. 5.16

5.6 TYPE I AND TYPE II SUPERCONDUCTORS

(May 13, 15, Nov. 15)

- There are two types of superconductors: type I and type II. There is no difference in the mechanism of superconductivity in both the types. Both have similar thermal properties at the transition temperature in zero magnetic field.
- The difference lies in their behaviour in a magnetic field, particularly in Meissner effect.

5.6.1 Type-I Superconductors

- In a type-I superconductor, the transition from a superconducting state to normal state, in the presence of a magnetic field, occurs sharply at the critical value H_c. At this point, the field penetrates completely.
- Below H_c, type-I superconductors are perfectly diamagnetic. They completely expel the magnetic field from the interior of the specimen. Upto the critical field strength, magnetization of the material grows in proportion to the external field. At the transition temperature, it suddenly drops to zero to the normal conducting state.
- The magnetic field penetrates only the surface layer and current flows only in this layer. Aluminium and lead are examples of type-I superconductors.
- As superconductivity gets destroyed at low values of critical field, type-I superconductors cannot be used in solenoids for producing large magnetic fields. Such superconductors are also called as ***'soft superconductors'***.

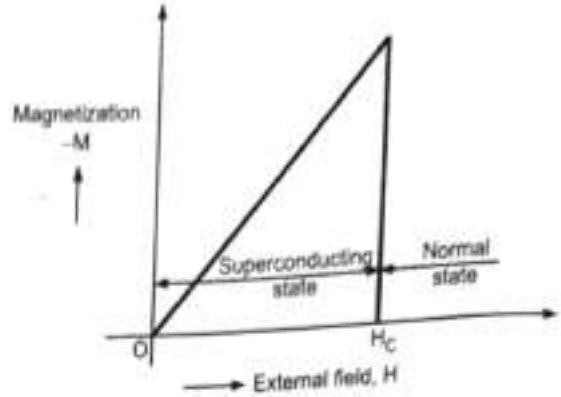


Fig. 5.17: Magnetization curve for a type-I superconductor

5.6.2 Type-II Superconductors

(Dec. 14)

- Type-II superconductor, also known as ***'hard superconductor'*** is characterized by two critical fields H_{c1} and H_{c2}. ($H_{c1} < H_c < H_{c2}$). It exists in three states: superconducting, mixed and normal.

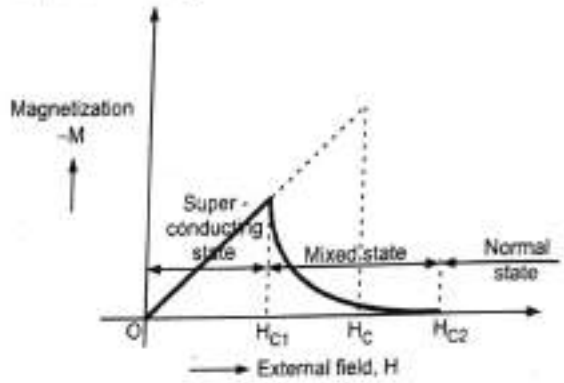


Fig. 5.18: Magnetization curve in type-II superconductor

Superconducting State

- This occurs upto a critical field H_{c1}. The magnetization increases with the applied magnetic field and the external magnetic flux is completely expelled from the interior of the material.

Mixed State

- This region extends from H_{c1} to H_{c2}. At H_{c1}, the magnetic flux penetrates the material. Between H_{c1} and H_{c2}, the material is in a mixed state magnetically but electrically it is a superconductor. Meissner effect is incomplete. In this region, the superconductor is threaded by flux lines and is said to be in a ***'vortex state'***. Value of H_{c2} may be 100 times higher than H_c (~20 to 50 Wb/m^2).
- As superconductivity is retained upto high values of magnetic fields, type-II superconductors are found useful in applications where high magnetic fields are created. Commercial solenoids wound with type-II

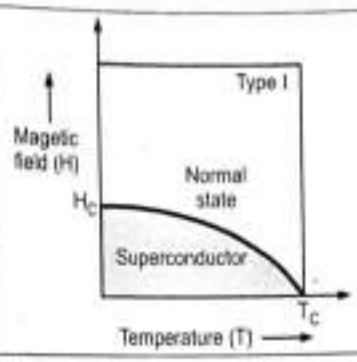
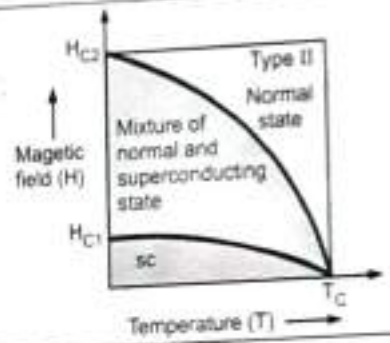
superconductors produce high, steady magnetic fields above 10 T.

- Once the magnetic field is created by a superconductor solenoid, it does not require electrical power to maintain it. But the solenoid must be kept below critical transition temperature.

Normal State

- When the magnetic field exceeds critical field strength H_{c2}, magnetization vanishes completely. The sample is penetrated by the external field and superconductivity is destroyed. The specimen reverts from superconducting state to normal state.
- Type-II superconductors have a distinguishing feature. The supercurrents arising in an external magnetic field can flow not only on the surface but also in its bulk. The magnitude of the currents carried is also large when the magnetic field is between H_{c1} and H_{c2}.

Table 5.2: Types of Superconductor - Differences

Sr. No.	Property	Type-I Superconductor	Type-II Superconductor
1.	Variation of magnetic field with temperature		
2.	Critical magnetic field	Has one critical magnetic field H_c.	Has two critical magnetic fields H_{c1} and H_{c2}.
3.	Transition from superconducting to normal state	Transition from superconducting state to normal state in the presence of a magnetic field occurs sharply at the critical value H_c.	If external magnetic field is less than H_{c1}, material remains superconductor. When external magnetic field increases above H_{c2}, their superconductivity is destroyed.
4.	Magnetization below and above critical magnetic field	They are perfectly diamagnetic below H_c and completely expel magnetic field from interior of the superconducting phase.	For $H_{c1} < H < H_{c2}$ they exist in magnetically mixed and electronically superconducting state.
5.	Change in magnetization with external magnetic field	Upto H_c magnetization of the material grows in proportion to the external field and then abruptly drops to zero at the transition to the normally conducting state.	The magnetization of Type-II superconductors grows in proportion to the external field upto H_{c1}. The external magnetic flux is expelled from the interior of the material till then. At H_{c1}, magnetic field lines begin penetrating the material. As magnetic field increases further, the magnetic flux through the material increases. At H_{c2}, magnetization vanishes completely. External magnetic field penetrates completely and superconductivity is destroyed.
6.	Current carrying capacity	They are poor carriers of electrical current.	They are good carriers of electrical current.
7.	Magnetic field generation capacity	About 0.01 to 0.2 Wb/m^2 (value of H_c).	About 20 to 50 Wb/m^2 (Value of H_{c2}).
8.	Applications as magnets	Not much useful due to low H_c.	Useful due to high H_{c2}.
9.	Examples	Aluminium, lead, indium	Transition metals and alloys consisting of niobium, silicon and vanadium, Nb-Ti alloys, Nb_3Sn, etc.

5.7 LOW AND HIGH TEMPERATURE SUPERCONDUCTORS

- Superconductivity is a phenomenon of exactly zero electrical resistance and the expulsion of magnetic flux fields that occurs in certain materials, called ***Superconductors***, when cooled below a chrematistics critical temperature. Depending upon the temperature at which the superconductor properties are exhibited they can be classified in two categories, low temperature and high temperature superconductors.
- The ***Low Temperature Superconductor*** are the Nb-based ALLOY (MOST COMMONLY Nb-47wt.%Ti) and A15 (Nb3Sn and Nb3A1) superconductors in use prior to the discovery of "***High Temperature***" oxide superconductors on 1986. For LTS superconductors that temperature is usually well below 20 K (–253 °C). Nb-47wt.%Ti alloy has become the dominant commercial superconductor because it can be economically manufactured in a ductile from with the prerequisite nano-structure needed for high critical current.
- These superconductors are often termed "***Technical Superconductors***" because of their applicability to engineering tasks. All these conductors require cooling to 4 K (liquid He is the most common coolant). The LTS superconductors are preferred to use at LTS superconductor applications, including MRI and NMR devices.
- Superconductors whose critical temperature (T_C) is greater than 77 K are called '***High Temperature Superconductors***'. This temperature is much higher than critical temperature 4.2 K for mercury. Hence, they are termed as high temperature superconductors. This temperature can be achieved by liquid nitrogen which is less expensive and easy to handle. Also, liquid nitrogen has a larger heat capacity and is therefore, a better coolant than liquid helium.
- In general, high T_C superconductors are ceramic materials. Several new oxide superconductors have T_C above 90 K. The superconductivity has not yet been achieved at room temperature, but experiments are going on to achieve it. The superconductivity at room temperature has not been ruled out theoretically.
- High T_C superconductors are prepared from the oxides of barium, copper, yttrium, etc. The unit cells of these types of superconductors are complicated. The unit cell consists of one rare earth metal atom, two barium atoms and three copper atoms and seven oxygen atoms. The number of atoms of each metal element give name 1-2-3 superconductors to this class. The 1-2-3 superconductors form layer of copper and oxygen atoms sandwiched between layers of other elements in the compound.
- Fig. 5.19 shows $YBa_2Cu_3O_7$ (YBCO), a ceramic oxide superconductor.

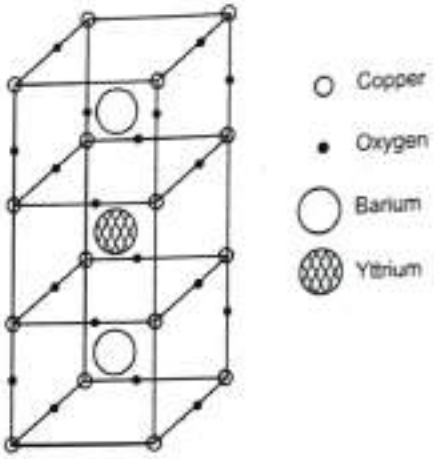


Fig. 5.19 : Unit cell of $YBa_2Cu_3O_7$

- The rare earth oxide systems La-Sr-Cu-O and Y-Ba-Cu-O has stable superconducting transition temperatures ranging from 80 K to 100 K.

5.8 JOSEPHSON EFFECT

Josephson Junction

- Two superconductors connected by a thin layer of insulating material (~ 1-2 nm) is called a '***Josephson junction***'. Under suitable conditions, Josephson found that remarkable effects were associated with the tunneling of superconducting electron pairs from a superconductor, through a layer of an insulator, into another superconductor. This junction is called a ***weak link***. The effect found to be associated with the pair tunneling is called ***Josephson effect***.

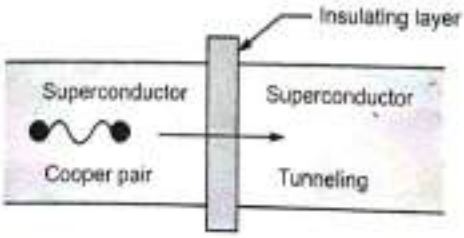


Fig. 5.20: Josephson junction

5.8.1 DC Josephson Effect (Nov. 13)

- When two superconductors are separated by a thin insulating layer, Cooper pairs tunnel through the junction and current flows across the junction without any external applied voltage. If this current does not exceed critical current I_c, voltage across the junction is zero. This effect is known as ***DC Josephson effect***.
- The Cooper pairs on each side of the junction can be represented by a wave function.

  The dc current obtained due to the tunneling of Cooper pairs through the insulating layer is given by $I_j = I_c \sin \phi$, where ϕ is the phase difference between wave functions of Cooper pairs on either side.

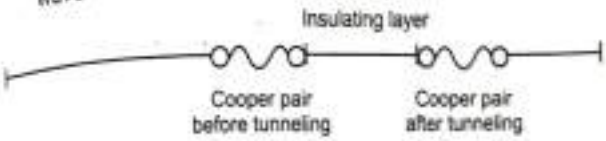


Fig. 5.21: DC effect

5.8.2 AC Josephson Effect (Nov. 13)

- When a dc voltage is applied to the Josephson junction and the current I_j through the junction exceeds a critical value I_c, a potential difference V appears across the Josephson junction and the junction current I_j varies sinusoidally with time. This effect is known as ***AC Josephson effect***.
- In such a case, the energies of the Cooper pair on both the sides of the barrier differ by 2 eV. The alternating supercurrents are accompanied by the emission or absorption of electromagnetic radiation.

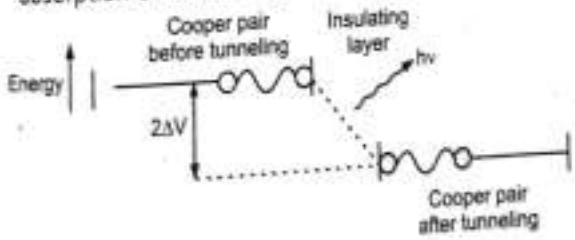


Fig. 5.22: AC effect

- If ΔV is the finite potential difference between the superconductors, the electron pairs on opposite sides of the barrier differ in energy by an amount $2\Delta V = 2$ eV. Hence, frequency υ of the associated photon will be given by,

$$h\upsilon = 2\,eV \quad \text{or} \quad \upsilon = 2\left(\frac{e}{h}\right)V.$$

- Josephson suggested the determination of h/e from this relation after measuring applied voltage and frequency of emitted radiation. This experiment was carried out between 1967 and 1968. It is one of the simplest methods available to measure the fundamental constant.

5.9 SQUID (SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE)

- This is a sensitive magnetometer which is constructed using one or more Josephson junctions. There are two types of SQUID circuits : (i) dc and (ii) ac. An ac device is the more common commercially available device. It uses one Josephson junction in a ring as shown in Fig. 5.23.

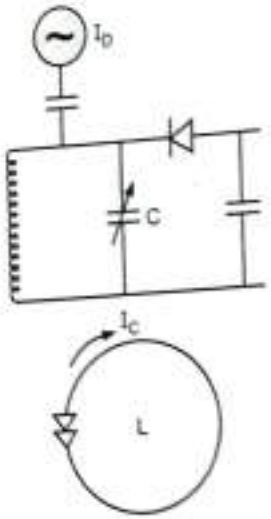


Fig. 5.23

- SQUID is basically a superconducting loop with a 'weak link' to measure magnetic flux charges within the loop. A weak link is a region that has much lower critical current than the rest of the superconducting ring. When the current in the link exceeds the critical current, the link becomes normal. It allows flux to penetrate the link. When flux penetrates the link, the current falls to critical value and the link reverts to superconducting state. The weak link thus acts as a switching device.
- The critical current in the weak link varies periodically as the total flux through the area enclosed by the superconducting loop changes. The changing magnetic field due to changes in squid current induce e.m.f. and current in the sensing coil. This e.m.f. induced in the coil can be used to drive an electronic counting circuit. The timing of the switching will give a measure of the magnitude of the circulating current. This gives a method for an extremely accurate measurement of the magnetic flux density.

5.9.1 Applications of SQUID

- SQUIDs act as very sensitive magnetometers. They are used in a variety of electrical and magnetic measurement applications.

- Heart and brain generate extremely small electric currents which set up weak magnetic fields of the order of 10^{-14} to 10^{-15} T. (Several tens of a pico tesla for the heart and around 100 femto tesla for the brain.)

  SQUIDs are used to measure these fields and to produce magneto-cardiographs and magneto-encephalographs. This makes it possible to provide medical data (rapidly and in detail) by detecting varying magnetic fields produced by the heart and the brain.

- SQUIDs are used in magnetometry, which involves the study of geomagnetic abnormalities magnetically for locating valuable ore bodies or deposits, even oil. Minerals are located deep within the earth, hidden from view by layers of surface rock. However, different rock layers show different responses to the earth's magnetic field. Superconducting magnetic maps are, therefore, found useful in locating these bodies.

  Also, earthquake scientists study magnetic changes caused by stresses and shearing motion in rock formation. Scientists hope to predict likely earthquake periods by magnetically monitoring strain build up within the earth.

- To locate submarines, light, radio waves and sound waves have been used but with severe limitations. Neither radio waves nor light penetrate more than a few metres in sea water. Sound travels further than light or radio waves under water, but changes in water density reflect back sound waves. Magnetic detection now offers a viable alternative to sonar. The presence of tons of iron-laden steel in today's submarines strongly distorts the earth's magnetic field as the submarine passes by. This can be measured by a SQUID.

- SQUID can be used as a voltage standard and a voltmeter with a sensitivity as high as 10^{-16} volts.

- It can be used as a highly sensitive detector of infrared radiation.

5.10 APPLICATIONS OF SUPERCONDUCTIVITY

(Nov. 15, May 13, Dec. 14)

- The phenomenon of superconductivity finds numerous applications which can be broadly classified into two types.

1. **Large-Scale Applications**

   These are applications requiring large currents, long lengths of superconductors in environments where the magnetic field may be several tesla (1 tesla = 10^4 Oersted). Examples include magnets and power transmission lines, transformers and generators, where current densities of atleast 10^5 amps/cm^2 are required. Superconductors are more advantageous than normal conductors because of their lower resistance and hence smaller power loss.

2. **Small-Scale Applications**

   These are applications involving minute amounts of current or fields. Examples are detection systems like SQUIDS.

5.10.1 Large-Scale Applications

- The cost of energy consumption in the world and the electrical energy in particular are staggering. It is said that about one-fifth the power generated is lost due to I^2R losses. The elimination of even a small fraction of the resistive load will have a staggering impact.

- Another important area of application is the use of high temperature superconductors in the production of strong magnetic fields above the 2 Tesla level. This will eliminate the use of iron cores in motors, generators and transformers resulting in reduced size, weight and losses from iron cores.

Wires and Superconducting Magnets

- As R = 0 for a superconductor, there are no I^2R losses. There is no energy dissipation associated with the flow of a current through a superconductor. A current set up in a closed loop of a superconductor persists, almost forever, without decay.

- Superconducting wires could be used for very economical long distance power transmission, as energy dissipation is low and electrical power transmission can be done at a lower voltage level. Electric generators made with superconducting wire are more efficient than conventional generators wound with copper wire.

Magnetic Levitation (Maglev)

- The zero magnetic induction in a superconductor is responsible for levitation effects.
- This phenomenon has led to one of the most spectacular applications, maglev or magnetically levitated train. Superconducting magnetic coils produce the magnetic repulsion required to levitate the train. Maglev trains will not slide over the rails but will float on an air cushion over a magnetised track. As there is no mechanical friction, speeds upto 500 km/hr can be achieved easily. As these trains are capable of very high speeds, they can compete with short hop plane flights in crowded air corridors.
- There are several maglev train test strips and there is talk about a 13 mile commercial line in the Orlando-Florida area and a longer one between Los Angeles and Las Vegas. One proposal is to use an on-board electromagnet to levitate the train above the laminated iron rail in the guide with ~1 cm air gap.
- A second proposal is to use superconducting wire coils in the vehicle to produce a magnetic field of the same polarity as coils in the guides, the repulsive force lifts the vehicle above the track (about 10-15 cm). As iron is not required for the magnetic field, the vehicle could be much lighter.

Electronics Industry

- Superconductors will change the face of the electronics industry, particularly IC fabrication. Currently, due to large amounts of heat generated (I^2R losses) there is a limit to the number of components that can be placed on a single chip. With the use of superconductors, more densely packed chips may be used.
- With the use of superconducting chips in digital electronics, logic delays of 13 pico seconds and switching times of 9 pico seconds have been achieved. By using basic Josephson junctions (refer small-scale applications), sensitive microwave detectors, magnetometers and stable voltage sources have been manufactured.

Computer Industry

- Currently, logic elements operate at speeds of nanoseconds. By using Josephson junctions, information can be transmitted more rapidly and by several orders of magnitude. Research is being conducted on 'petaflop' computers. A petaflop is a thousand-trillion floating point operations per second. Today's fastest computer has only achieved 'teraflop' speeds - trillions of operations per second.

Superconducting Magnets

- The most important use of superconductivity has been in the production of high magnetic fields (> 10^5 Gauss or 10 Tesla) over large volumes without a large consumption of electrical power.
- As superconductors are capable of carrying, without energy loss, about 100 times larger current densities as compared to normal conductors like copper, they can be used for building light weight, high intensity, compact magnets useful in various applications. Relatively small superconducting magnets have very economically replaced gigantic water-cooled copper conductor magnets which dissipate several megawatts of electrical power. Superconducting magnets (SCM) find application in many areas in technology, including energy storage devices for electrical power industry, electric motor windings, electromagnetic pumps, etc.
- Superconducting magnets are also used in the field of medicine for NMR (Nuclear Magnetic Resonance) imaging particularly for producing NMR tomography. This is of particular importance for investigating pathological changes in the brain. By applying a strong magnetic field from a superconducting magnet across the body, hydrogen atoms inside the body are forced to take up energy from the magnetic field. This energy is then released at a frequency that can be detected and displayed on a computer. This method is called as Magnet Resonance Imaging (MRI) and is widely used in hospitals.
- Superconducting magnets are also used in high energy physics experiments. Large particle accelerators employ magnets producing high fields for bending and guiding the accelerated particles. Controlled nuclear fusion requires confining high temperature plasma within a closed region. This is done by using superconducting magnets. Superconducting magnets have also been employed for magnetically separating refining ores, isotopes and chemicals.

Military Applications

- Superconductors have found a wide variety of applications in the military. HTSC (high temperature superconductors) are being used to detect mines and submarines.
- Smaller motors are being built by Navy ships using superconducting wires.

- 'E-bombs' have been used by the US army in March 2003 when US forces attacked Iraq. These are devices that use strong superconducting magnets to create a fast, high intensity electromagnetic pulse to disable an enemy's electronic equipment.

5.10.2 Small-Scale Applications of Superconductivity

- Brian D. Josephson, a graduate student at Cambridge University, in 1962, predicted that electrical current would flow between two superconducting materials even when they are separated by a non-superconductor or insulator. This tunneling phenomenon is called as the ***'Josephson Effect'***.
- It has been applied to electronic devices such as the SQUID, an instrument capable of detecting and measuring extremely weak magnetic fields.

SUMMARY

- **Superconductors** : A conductor having zero electrical resistance is called a superconductor and this phenomenon is called as superconductivity.
- **Critical Temperature :** The temperature below which superconductivity is exhibited is called as critical transition temperature (T_c).
- **Superconductivity Vanishes** : If temperature, magnetic field and current density exceed the critical value. For superconducting state, $T < T_c$, $H < H_c$ and $J < J_c$.
- **Meissner Effect** : The expulsion of magnetic field/flux from the interior of the specimen, when cooled below the critical temperature is called as Meissner effect.
- Variation of critical magnetic field with temperature is given by

$$H_c(T) = H_c(0)\left[1-\frac{T}{T_c}\right]^2$$

where $H_c(0)$ is the critical magnetic field at 0 K.

- **Persistent Currents :** When a current is induced in a superconducting ring or loop held below the critical temperature, it persists undiminished as long as the temperature remains below the critical temperature T_c.
- **Type-I Superconductors** : Are pure specimens which expel completely magnetic field lines. They exhibit perfect diamagnetism. They are also called as soft superconductors.
- **Type-II Superconductors :** Are characterized by two critical fields. Between the two critical fields, the magnetic flux partially penetrates the material. Above the upper critical field flux, penetration is total. They are also called as hard superconductors.
- Tunneling of current between two superconductors separated by an insulator is known as **Josephson Effect**.
- The flow of a dc current across the Josephson junction, in the absence of any electric or magnetic field is known as **dc Josephson Effect**.
- When a dc voltage is applied across the Josephson junction, RF current oscillations are setup across the junction along with the emission or absorption of electromagnetic radiation. This is known as **ac Josephson Effect**.
- **SQUID** is a sensitive magnetometer which is constructed one or more Josephson junctions.

IMPORTANT FORMULAE

- Isotope effect, $M^{1/2}\,T_c$ = constant.
- $\lambda = \dfrac{\lambda_0}{\left[1-\left(\dfrac{T}{T_c}\right)^4\right]}$.
- $H_c = H_0\left[1-\left(\dfrac{T}{T_c}\right)^2\right]$.

EXERCISE

1. What is superconductivity ? What are the characteristics of superconductors ?
2. Explain Meissner effect, isotope effect, critical temperature and critical field.
3. What are the types of superconductors ? Where do they find application ?
4. Enumerate the different applications of superconductors. How are they advantageous as compared to normal conductors ?
5. Explain some properties of type-I and type-II superconductors.
6. Write a short note on SQUID.

UNIVERSITY QUESTIONS

December 2012

1. Explain the Phenomena of Super-conductivity. Explain Type - I and Type - II Super-conductors. [6]
2. State and explain : [3]
   (a) Meissner Effect.
   (b) Persistent Current.

May 2013

1. Differentiate between Type-I and Type-II Superconductor with diagram. [4]
2. Explain two applications of Superconductivity. [3]
3. Explain Meissner effect and Critical magnetic field for Superconductivity. [6]

November 2013

1. Explain D.C. and A.C. Josephson effect. [4]
2. Distinguish between Type-I and Type-II superconductors. (Any 3 points) [3]

May 2014

1. State Meissner effect. Why materials in superconducting state exhibit diamagnetism. [4]
2. State any six applications of superconductors. [3]

December 2014

1. Explain critical field of a superconductor and give any three points to differentiate type – I and type – II superconductors. [6]
2. Explain the Meissner effect. What important property of superconductor it explain. [4]
3. Explain two applications of superconductivity. [3]

May 2015

1. State and explain Meissner effect and hence show that superconductivity is influenced by perfect diamagnetism. [6]
2. Distinguish between type-I and type-II superconductors. [4]
3. State any six applications of superconductivity. [3]

November 2015

1. Explain: [6]
   (i) Critical field (ii) Meissner effect.
2. Explain in brief the BCS theory of superconductivity. [4]
3. Give any six applications of superconductivity. [3]

May 2016

1. Explain the applications of superconductors in the field of electronics. [3]

Dec. 2016

1. What is superconductivity ? Explain the Meissner effect in superconductors. [6]
2. Explain zero electrical resistance property and isotope effect in superconductor. [4]
3. Explain any one application of superconductors in brief. [3]

May 2017

1. Explain critical magnetic field of superconductor. Differentiate between type-I and type-II superconductors (four points). [6]
2. Explain Meissner effect and show that superconductors exhibit perfect diamagnetism. [4]
3. State applications of superconductors. Explain any one application. [3]

Nov. 2017

1. Explain the following terms of superconductivity with the help of necessary figure. Give formula and graph wherever necessary. [6]
   (i) Meissner effect.
   (ii) Critical Magnetic Pield.
2. Give the statement of Meissner effect and show that supper conductors are perfectly diamagnetic. [4]

May 2018

1. Give any four points to distinguish between Type I and Type II superconductors. [4]

Nov. 2018

1. What is superconductivity? Explain Meissner effect and show that superconductors are perfectly diamagnetic. **[6]**

2. Explain the following terms of superconductivity. **[4]**

   (i) Critical Magnetic Field

   (ii) Persicstent Current

May 2019

1. State applications of nano-particle. Explain any one application. **[2]**

✠ ✠ ✠

NON DESTRUCTIVE TESTING

6.1 INTRODUCTION

The basic purpose of non-destructive test is to know the quality of the material so that it may be considered suitable for a particular application or may be rejected if it is not upto the required specification. The tests take a lot of time for getting conclusive results which may be applied for development of appliance using the sample. Hence, getting a faster, reliable and cheaper methods of testing is challenge for engineers. Because use of materials with creep, fatigue stress etc. reduces life of the appliance and sometimes may result into accident. So these defects must be detected, if present, in the materials. Following are the methods of testing the materials.

- **Destructive Testing** : In this method, the sample under test is destroyed and cannot be used for any purpose after the test. Examples are tensile test, impact test etc. The tests are performed on samples of material and its results are used for similar material as the original sample is destroyed in the process. The destructive testing is cheap, faster and simple to perform. But disadvantage is that the sample under investigation is destroyed. So, use of this method is restricted to cheaper material which is available in abundance. The destructive tests are used to find out mechanical properties like hardness, toughness, compressive strength, yield strength and tensile strength.

- **Non-Destructive Testing (NDT)** : This type of testing does not change the structural properties of the material under investigation so the material can be used in future. Examples are ultrasonic, radiology etc. The main advantage is that the sample under investigation is not destroyed and hence can be performed on finished components. The main disadvantage is, the technique is expensive and time consuming. Non-destructive tests are generally used to find out the defects in the materials.

6.2 OBJECTIVES OF NDT

The non-destructive testing has following salient features

- It has no effect on future serviceability.
- Safety norms may be applied as per the requirements after the tests.
- Profits or productivity may be increased after the test.

6.3 CHARACTERISTICS OF NON DESTRUCTIVE TESTING METHODS

- Non-destructive tests can be classified into two groups. The first group consists of tests used to ***locate defects***. The methods in this group are simple visual examination on the surface as well as the interior of the body by drilling holes, using liquid penetrants to locate cracks, studying propagation of sound i.e. hammering at one end and studying behaviour of sound at other end. Some other methods are magnetic methods, electrical methods, radiographic methods and ultrasonic methods.
- The second group of non-destructive test consists of methods used for ***determining the dimensional***, physical or mechanical characteristics of a material. In this method, one can find thickness of paint coating where only one surface is available for measurement. The strength of material surface or roughness can also be determined.

6.4 PRINCIPLES OF PHYSICS IN NON DESTRUCTIVE TESTING

In visual inspection method, the surface is examined with naked eyes.

1. **Visual Inspection Method** : This method is used to locate the defects. In visual inspection method, it can be used to find out surface defects. Sometimes, the defects in interior of the body can be inspected for surface properties by drilling holes. This method can be used to check surface defects such as roughness, cracks and corrosion. The method is widely used to find defect in pipes, assemblies and interior of ducts.

2. **Liquid Penetrant Methods :** In liquid penetrant method, liquids are used for detecting crack or flaws present on the surface of metals or non-porous materials such as ceramics, plastics and glass. This method is employed at different stages such as in-process, final stage and for maintenance.

3. **Magnetic Methods :** In this method, the specimen is magnetised and the distortion of magnetic field due to defect is studied. The magnetic field distortion may be studied in many ways, but the most widely used method employs magnetic particles such as iron oxide. The flux distortion due to a defect will change the induced voltage. This method is used to identify the defects that break the surface and defects close to the surface. Specifically, this method is used for ferromagnetic material to find the surface and sub-surface defects such as cracks, flaws, cavities etc.

4. **Electrical Methods:** These methods are based on the principle that different materials have different resistivities. In electrostatic method, variation of electric fields can be detected by motion of charged particles.

   When a coil carrying high frequency alternating current is moved near a conductor, eddy currents are induced in the conductor which developes a magnetic field. In defects, physical or chemical, the structural variations cause change in resistivity, hence changing the eddy currents. This as a result changes the induced magnetic field. Various techniques used for non-destructive testing are :

   - Change of electrical resistance.
   - Thermoelectric effect.
   - Triboelectric effect.
   - Variation of static field.

   This method can be used to find defects on surface or just below the surface of any metallic sample.

5. **Radiographic Methods :** Radiographic method uses the technique of X-rays or γ-radiations. Now-a-days, even electron and neutron beams are being used. In this method, a beam of X-ray, γ-ray, electron or neutrons are passed through the sample. On passing through the sample, these high energy beams are modified by various defects present in the material. The phenomenon of differential absorption forms the basis of use of radiology. The intensity difference on the developed film between the image of an area having a defect and image of defect free part of the specimen allows us to know the defect present.

   This method can be employed to identify the internal defects along with surface defects. Any type of material can be tested by this method provided thickness of the specimen is small.

6. **Acoustic Emission Testing (AET) :** Acoustic Emission Testing is performed by applying a localized external force such as an abrupt mechanical load or rapid temperature or pressure change to part being tested. The resulting stress waves in turn generate short-lived, high frequency elastic waves in the form of small material displacements, or plastic deformation, on the part surface that are detected by sensors that have been attached to the part surface. When multiple sensors are used, the resulting data can be evaluated to locate discontinuities in the part.

7. **Ultrasonic Methods :** The ultrasonic waves can be used in the detection of cracks or cavities in metal casting by the property of reflection of sound waves when a change in medium takes place. For detecting the defects, a strong ultrasonic beam is send through the object. Due to crack or cavity there is a change of medium and so the beam gets reflected. By knowing the velocity of sound waves in the medium, position of defect can be detected.

   This method can be used to determine the internal as well as the surface defects. An important advantage of this method is that it can test large size castings without destroying the material.

8. **Thermal Methods :** Basic principle in the thermal test is to apply heat to the specimen and measure the heat distribution. In case, there are flaws, heat distribution and consequently temperature measurement will differ as the flaw changes the heat distribution. The heat may be supplied directly or by electrical heating or by infrared sources and temperature can be measured by thermocouples. For proper measurement thermal resistance of the material under investigation should be high.

9. **Thermograph :** This corresponds to optical thermal testing using non-destructive method. This technique involves scanning of thousand of points per second in the form of a picture, known as ***thermogram***. In case there is any flaw, the intensity of radiation from that surface will change. Thermography is a good and effective tool for diagnostic evaluation of electronic circuits. It helps to identify the defective component and separate it without any change in the circuit. Thermistors are very sensitive to temperature variations and are used in many fields of thermography.

6.5 ADVANTAGES OF NONDESTRUCTIVE METHODS

Nondestructive testing has been around a lot longer than you might think. In fact, the first "nondestructive" test, visual testing, is as old as mankind.

These are the reasons why nondestructive testing is useful in industries ranging from automobile to oil and gas.

1. **Safety**

- Nondestructive testing is conducted to determine whether a component is damaged or in need of repair. The aim of nondestructive testing is to identify and resolve damages that otherwise would lead to disaster.
- The NDTs are completely harmless to humans (radiographic testing must be conducted under strict settings) and all tests leave tested products completely undamaged.
- Additionally, in industries with volatile or high-pressure equipment and machinery, nondestructive testing as a preventative measure saves lives.

2. **Reliability**

- When it comes to accurate results, nondestructive testing is reliable because of the variety of available and complementary options. Any given piece of equipment or machinery can be subject to a number of nondestructive tests, which eliminates the risk of oversight or inaccuracy.

3. **Affordability**

- Different industries have different safety standards and different types of machinery have to be regularly inspected at different intervals. But regardless of what has to be inspected, a nondestructive test will always be the most affordable option. Destructive testing methods, like automobile crash tests, will be costly.
- Additionally, nondestructive tests can result in insights that lead to the replacement or repair of equipment before malfunction can occur.

4. **Peace of Mind**

- The final benefit of nondestructive testing may be hard to quantify, but it's the most important of all. Knowing that your equipment is functioning the way it should (and that future accidents can be prevented with simple checkups) adds peace of mind.

6.6 ACOUSTIC EMISSION TESTING

- **Acoustic Emission Testing** (AET) is a non-destructive testing and monitoring method to detect and locate hidden defects. Acoustic Emission analysis provides overall information on the physical condition and leakproofness of the tested object.
- **Acoustic** Emission analysis provides overall information on the physical condition and leakproofness of the tested object. In order to detect and locate possible defects and leakages in the tested structure, AT applies a classical physical principle that is used to the industry's advantage by means of state-of-the-art data reception and processing systems.

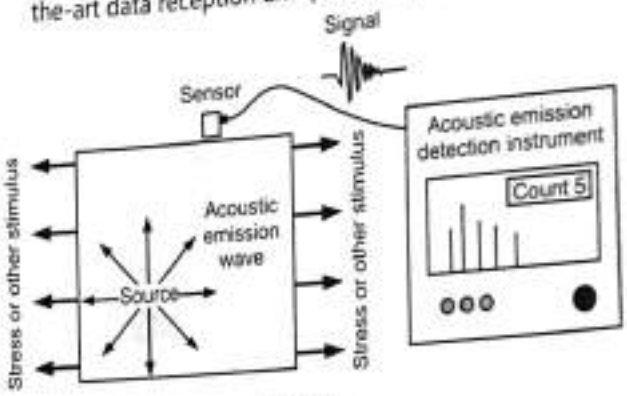


Fig. 6.1

- **Acoustic Emission (AE)** refers to the generation of transient elastic waves produced by a sudden redistribution of stress in a material. When a structure is subjected to an external stimulus (change in pressure, load, or temperature), localized sources trigger the release of energy, in the form of stress waves, which propagate to the surface and are recorded by sensors. With the right equipment and setup, motions on the order of picometers (10^{-12} m) can be identified. Sources of AE vary from natural events like earthquakes and rockbursts to the initiation and growth of cracks, slip and dislocation movements, melting, twinning, and phase transformations in metals. in composites, matrix cracking and fiber breakage and debonding contribute to acoustic emissions. AE's have also been measured and recorded in polymers, wood, and concrete, among other materials.
- Detection and analysis of AE signals can supply valuable information regarding the origin and importance of a discontinuity in a material. Because of the versatility of Acoustic Emission Testing (AET), it has many industrial applications (e.g. assessing structural integrity, detecting Flaws, testing for leaks, or monitoring weld quality) and is used extensively as a research tool.

- Acoustic Emission is unlike most other nondestructive testing (NDT) techniques in two regards. The first difference pertains to the origin of the signal. Instead of supplying energy to the object under examination, AET simply listens for the energy released by the object. AE tests are often performed on structures while in operation, as this provides adequate loading for propagating defers and triggering acoustic emissions.
- The second difference is that AET deals with dynamic processes, or changes, in a material. This is particularly meaningful because only active features (e.g. crack growth) are highlighted. The ability to discern between developing and stagnant defects is significant. However, it is possible for flaws to go undetected altogether if the loading is not high enough to cause an acoustic event. Furthermore, AE testing usually provides an immediate indication relating to the strength or risk of failure of a component. Other advantages of AET include fast and complete volumetric inspection using multiple sensors, permanent sensor mounting for process control, and no need to disassemble and clean a specimen.
- Unfortunately, AE systems can only qualitatively gauge how much damage is contained in a structure. In order to obtain quantitative results about size, depth, and overall acceptability of a pa other NDT methods (often ultrasonic testing) are necessary. Another drawback of AE stems from loud service environments which contribute extraneous noise to the signals. For successful applications, signal discrimination and noise reduction are crucial.

Applications

Weld Monitoring

- Doing the welding process, temperature changes induce stresses between the weld and the base metal. These stresses are often relieved by heat treating the weld. However, in some cases tempering the weld is not possible and minor cracking occurs. Amazingly, cracking can continue for up to 10 days after the weld has been completed.

Bridges

- Bridges contain many welds, joints and connections, and a combination of load and environmental factors heavily influence damage mechanisms such as fatigue cracking and metal thinning due to corrosion. Bridges receive a visual inspection about every two years and when damage is detected, the bridge is either shut down, its weight capacity is lowered, or it is singled so for more frequent monitoring. Acoustic Emission is increasingly being used for bridge monitoring applications because it can continuously gather data and detect changes that may be it to damage without requiring lane closures or bridge shutdown. In fact, traffic flow is commonly used to load or stress the bridge for the AE testing.

Aerospace Structures

- Most aerospace structures consist of complex assemblies of components that have been design to carry significant loads while being as light as possible. This combination of requirements leads ID many parts that can tolerate only a minor amount of damage before failing.
- This fact makes detection of damage extremely important but components are often packed tightly together making access for inspections difficult. AET has found applications in monitoring the health of aerospace structures because sensors can be attached in easily accessed areas that are remotely located from damage prone sites. AET has been used in laboratory structural tests, as well as in Bit test applications.

6.7 ULTRASONIC

- Ultrasonics are the acoustic waves having frequency between 20 kHz to 20 MHz. These frequencies are non-audible to human being, but many animals, like bats, can hear the ultrasonic waves. The ultrasonic waves are like sound waves but having frequency above 20 kHz. The other properties which make them different are, they are highly direction, having high energy concentration. Due to this they can travel large distances in air, liquid and solids.
- As a result of these special properties they have found application in non-destructive testing for detection of flaws, cracks, holes etc. The ultrasonic inspection uses ultrasonic waves for detection of defects.

  Principle : The basic principle of ultrasonic method, is that wherever there is change of density the waves will be partially reflected and partially refracted. In detection depending upon the requirement either reflected or transmitted waves are detected. The presence of defect will change the density of medium and hence reflected as well as transmitted wave will change the behaviour whenever they come across the defects.

- The main set-up for ultrasonic method consists of a transmission transducer, receiving transducer and a display unit. Transducer is a device which converts electrical energy to sound energy or vice versa. The transmission transducer converts the a.c. electrical signal into ultrasonic waves and receiving transducer converts the ultrasonic wave energy into electrical energy. Many a times same transducer is used for transmission and reception of ultrasonic waves.
- The transducers are kept in contact with the specimen to be tested. A thin layer of fluid is used for perfect contact between the transducer and specimen. The fluid layer between the transducer and specimen removes the air and provides a medium for proper transfer of the sound waves.
- Ultrasonic method can be used for thickness measurement, flow detection etc.

6.7.1 Thickness Measurement

- Ultrasonic thickness measurement is based on the **Echo Principle**. A piezoelectric transducer attached to the test piece converts the electric pulse into ultrasonic waves. The transducer can be attached to the test piece directly or it can be coupled to the piece by an incompressible medium such as oil or water.
- The ultrasonic waves propagate into the test piece. They travel through the sample and are reflected back from the opposite surface. The same transducer then receives the reflected echo and converts it to an electrical pulse. The time taken for the pulse to travel through the sample is related to the thickness and the velocity in the material. The thickness T can be expressed by the formula,

$$\text{Thickness} = \frac{\text{Velocity of sound in specimen} \times \text{time}}{2}$$

$$T = \frac{vt}{2}$$

where, v is the velocity of the ultrasound in the material and t is the time between the pulse being transmitted and the echo being received.

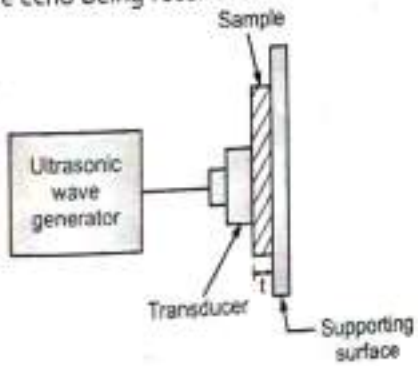


Fig. 6.2

- If the velocity of ultrasonic waves in the material is not known, then the value has to be determined experimentally by using the same material whose thickness is already known. From the known values of T and t, the velocity v of ultrasound in the material can be calculated.
- The advantage of using ultrasonic waves for thickness measurement is that the thickness can be measured from one side of the test piece. There is no need for drilling holes or otherwise inflicting damage to the piece.

6.7.2 Flow Detection

1. Pulse Echo System

- In this method, a ultrasonic pulse is produced by exciting the transducer with an electric pulse from signal generator. The pulse of ultrasonic wave is made to travel through the specimen under investigation. The reflected ultrasonic waves are received by the same transducer. The time of transit and amplitude is displayed on a CRO. Fig. 6.3 shows arrangement for pulse echo system.
- In pulse echo system, pulses of ultrasonic waves are transmitted at regular intervals of time. The pulses will be reflected by defects like flaw or cracks if present, otherwise it will be reflected from bottom surface of the specimen.
- As the pulse is transmitted from the top surface of the specimen, a peak 'A' will appear on CRO. If a defect is present there will be another peak B, before peak C, which is due to reflected waves from the bottom surface. The time delay between two pulses can be measured and if velocity of sound wave in that medium is known, the position of the defect can be calculated from the formula $d = \frac{vt}{2}$, where d is the depth of defect from top surface, v is velocity of wave in the specimen and t is the time lapse between peak A and peak B.

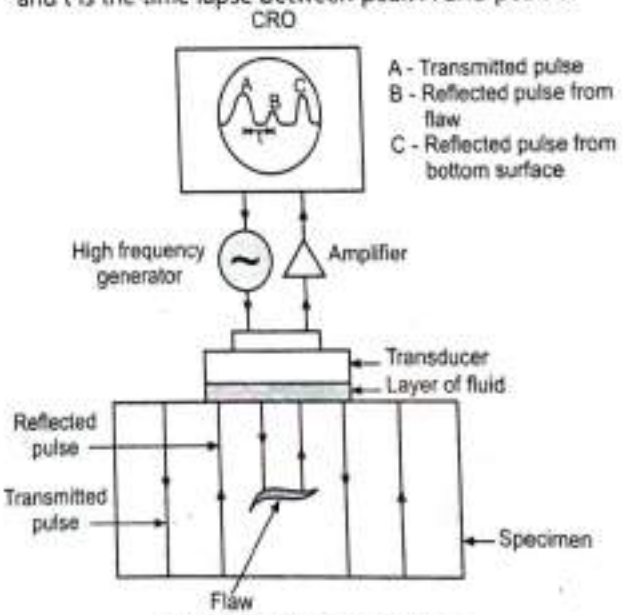


Fig. 6.3 : Pulse echo system

The Advantages of Pulse Echo System are :

- Can be used for specimen of any size and shape.
- Can be used even if one face of specimen is available for testing.
- Only one coupling is required between transducer and specimen and hence chances of error are less.
- The distance of the defect can be measured.

2. Through Transmission System

- In this method, two ultrasonic transducers are used, one as transmitter and one as receiver. The electrical signal is connected to transducer and transducer is attached in the one side of the specimen. The receiver transducer is placed on the other side of the specimen. Both the transducers are coupled to the specimen with a liquid layer between them as shown in Fig. 6.4.
- Pulses of the ultrasonic waves are transmitted at regular intervals of time and the transmitted pulses are received from the other side of the specimen. If there is no defect within the material, a fixed strength of signal will reach the receiver and the output of the CRO will be constant. If a defect is present in the specimen then there will be reduction in the strength of the received signal because a part of signal will be reflected from the defect. This will form a ditch on the CRO screen. Thus, the presence of the defect can be detected.

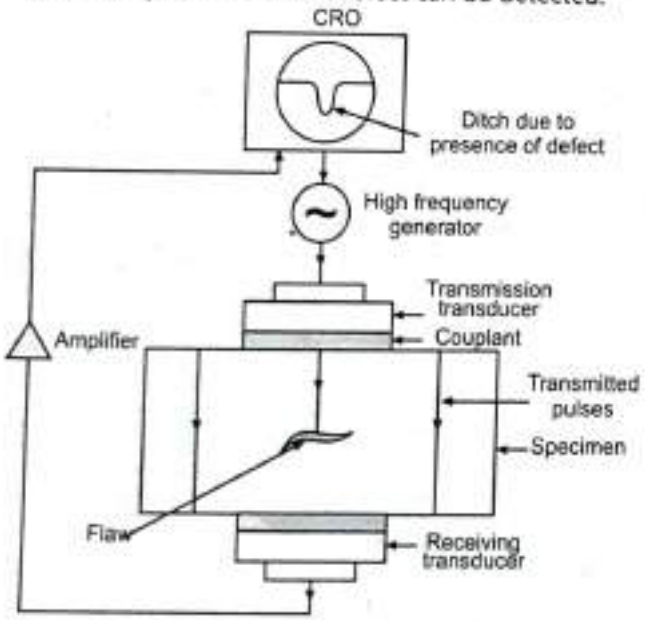


Fig. 6.4 : Through transmission system

The Disadvantages of the Through Transmission System are :

- Both the sides of the specimen should be available for detection.
- Both the sides of the specimen should be parallel.
- Two transducers and two points of coupling increase the cost and error.
- The depth of the defect cannot be detected.

6.8 RADIOGRAPHY TESTING

- The X-ray and γ-ray radiography is used to detect defects such as crack, flaws, cavities present in metals casting. Detection and treatment of such defects are of special importance because they may lead to failure or accident. The detection of defects are important in components designed to withstand high temperature and high pressure in power plants, nuclear reactors, chemical vessels and oil refining equipments.
- In radiographic methods, X-rays or γ-rays are passed through the specimen. On passing through the material they are partially absorbed by the medium and partially from the defects, if present. The amount of absorption will depend upon the density and thickness of the specimen. Along with this it will also depend upon the characteristics of the radiation.
- Due to the defects the beam intensity gets modified. The difference in the intensity on the developed film between an area containing defects and area free of defects allows us to know the defect.
- The basic radiography involves passing of radiation through the specimen and taking photograph on a photo-sensitive paper or fluorescent screen or electronic sensing equipment.
- The word radiography means producing an image on a film using electromagnetic radiation. When the image is produced on a photosensitive paper/film, the process is known as paper ***radiography***. If the image of the specimen is obtained on fluorescent screen, the process is known as ***fluoroscopy***. And when the radiation after passing through the specimen is sensed by an electronic equipment, the process is termed as ***radiation gauging***.

Principle :

- The basic principle of radiography is the absorption of radiations by material and scattering from the defects. The difference between the intensities between the transmitted radiation from part free of defects and having defects will give us position of the defect.
- The experimental arrangement of the technique consists of a source of radiation and a photographic plate. The sample under investigation is placed in the path of the radiation and the photographic film is placed on the other side of the specimen. The distance between specimen and photographic film is very small. Fig. 6.5 shows experimental set-up for radiography.

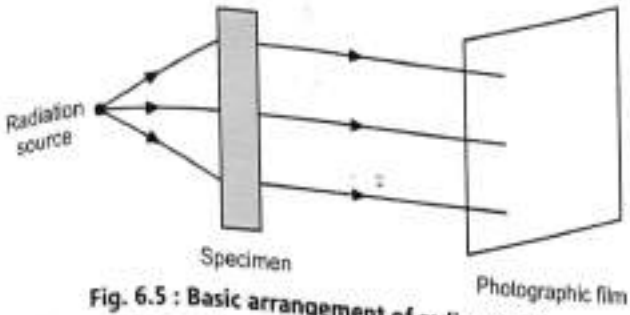


Fig. 6.5 : Basic arrangement of radiography

- The diverging beam of the radiation falls on the specimen. Then depending on the thickness and absorption characteristics of the material, some amount of radiation will be absorbed. The absorption in the part having defect and free of defect is different. When the film is developed, there will be an area of different image density which correspond to the defect in the material. With a single radiogram only presence of defect can be detected, but for getting position, the radiation should be passed through different angles. From a set of radiogram obtained from different angle will give the position of defects.

Sources of Radiation :

- The X-rays can be produced by Coolidge tube. Different wavelength X-rays can be obtained by using different target material. The X-rays have wavelength range from 0.001 A° to 40 A°. The γ-rays cannot be obtained artificially. So one has to use natural radioactive substance. The major radioactive substances giving γ-rays are Caesium^{137}, Cobalt^{60}, Iridium^{192} etc. The γ-ray source is normally well protected by lead shielding for leakage. The γ-rays have wavelength range from 0.005 A° to 3 A°. Both X-rays and γ-rays have energies varying from 50 eV to 10^7 eV.

Radiographic Film

- The film which is used for radiography is different than that used for normal photography. The film base used in radiography is thicker (0.018 mm) than photographic film. The radiographic film has emulsion of photosensitive material on both sides. The silver salt imulsions are very sensitive to X-rays and γ-rays.

Difference between X-ray Radiography and γ-ray Radiography

- For small defects in castings, less than 50 mm, X-rays give better results than γ-rays.
- X-rays give quick results in seconds while γ-rays may take many hours.
- For defects in thick castings, γ-rays give better results than X-rays.
- Due to minimum scattering, γ-rays scan better than X-rays. Moreover, γ-rays give better results for varying thickness whereas the X-rays are more suitable for uniform thickness.
- The γ-ray apparatus is small and portable, so it is convenient for on site inspections.
- Unlike the X-ray method, the γ-ray technique can be used to test a number of castings at the same time.

6.8.1 X-Ray Radiography

- In X-ray radiography, the X-rays are produced in a Coolidge tube. When a high energy electron strike a target with high atomic number. This incident electron may dislodge one of the electrons in the innermost shell. For filling up the vacancy, electron from higher energy travel jump to lower energy level and radiate characteristic X-rays. Some of the high energy electrons may not collide with the electrons in the shell. These electrons get de-accelerated, giving continuous X-rays.
- The casting which is to be tested is exposed to the X-rays. The X-rays are made to incident on the casting in the perpendicular direction. The X-ray photosensitive film is kept on backside and in contact with the casting. Fig. 6.6 shows the set-up for X-ray radiography. During exposure, the X-rays pass through the casting and affect the X-ray film. Since the defects like cracks, flaws, cavities, porosity etc. have less density than the part of casting which is free of defects. Thus, defective portion transmits X-rays better than defect free portion of metal. Therefore, the film appears darker where the defects are present in comparison to other part of the film.

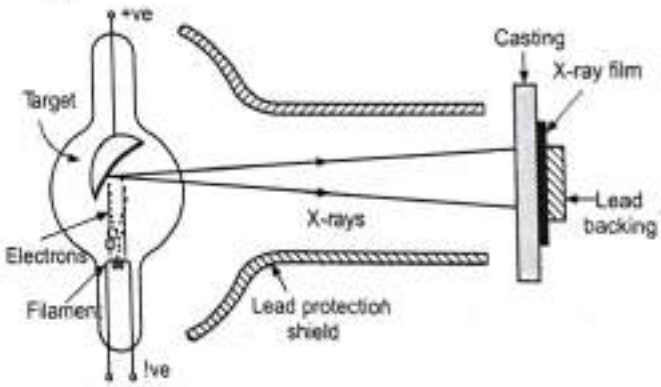


Fig. 6.6 : X-ray radiography

- The developed X-ray film having light and dark areas in the film is called ***radiograph*** or ***exograph***.
- Fig. 6.7 shows the radiographs of defect free casting and castings with holes and porosity.

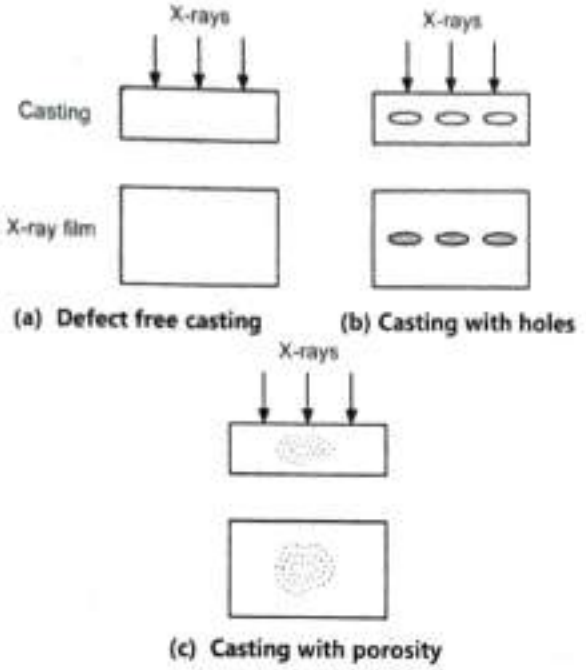


Fig. 6.7 : Radiographs of different types of casting

Displacement Method

- The above method just gives an idea of presence of defect in the casting, but does not give the position or depth at which the defect is present. For getting the position of defect two or more radiograms should be obtained from different angle. One of the methods of getting the position of the defect is the displacement method. Fig. 6.8 shows the displacement method of X-ray radiography.

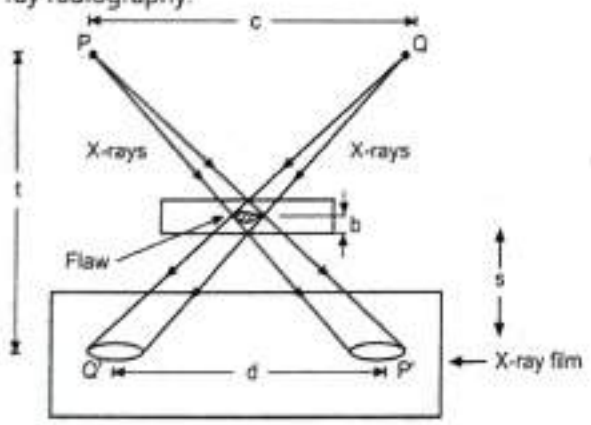


Fig. 6.8

- Suppose, the casting has a flaw at distance b from the bottom surface of the specimen. By the displacement method of radiography this distance can be measured. As shown in Fig. 6.8, the casting is exposed to the X-rays from two different positions P and Q. The distance between two positions i.e. PQ = C. When the X-ray source is at point P it will form an image of flaw at point P' on the X-ray film. Similarly, when the source is at Q, the image of same flaw will be formed at Q'. The distance between the two images i.e. P'Q' = d.

- If t is the distance between the source and the film, s is the distance between casting and film, then the position of the flaw is given by the relation,

$$b = \frac{t \times d}{d + c} - s$$

6.8.2 Gamma Ray Radiography

- The γ-rays are emitted in all directions from natural radioactive materials such as Cobalt 60, unlike X-rays which are radiated just in one direction. Therefore, a large number of separate castings can be tested simultaneously. For testing, each casting having a photosensitive film on its back is placed in a circle around the source placed at the centre. Fig. 6.9 shows arrangement for γ-ray radiography. The castings are exposed to the radiations for a long time.

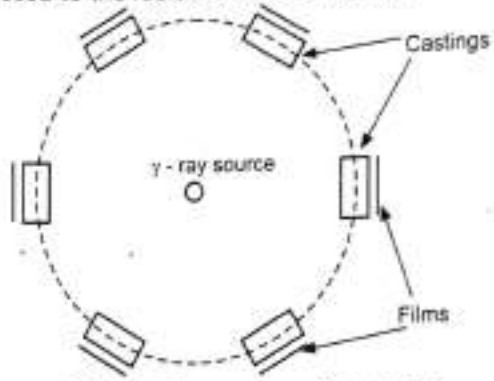


Fig. 6.9 : Gamma ray radiography

- After exposure for several hours the film is developed and separated radiogram is obtained for each casting simultaneously.

6.8.3 Fluoroscopy

- In radiography the image is recorded on a X-ray film. But in ***fluoroscopy***, the radiation after passing through the casting is allowed to fall on a screen coated with a fluorescent material on a base. When the radiations strike the fluorescent screen they convert it into visible light. The amount of light radiated depends upon the intensity of radiations falling on it. So the part of fluorescent screen receives more radiations will radiate more light. Fig. 6.10 shows schematic diagram of fluoroscopy.

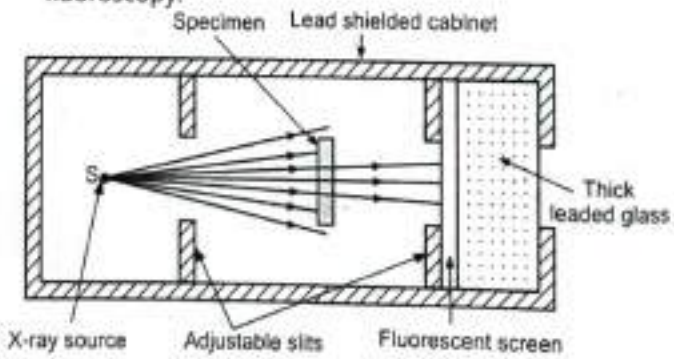


Fig. 6.10 : Fluoroscopy

- The appearance of the fluoroscope image is the reverse of that in developed radiograph. The defects in radiograph show dark spot where defects are present but on fluorescent screen it will appear brighter. Because the point where defects are present absorption will be less and more radiations will be transmitted. These transmitted radiations strike the screen. As intensity of light radiated by the fluorescent screen depends on the intensity of radiations falling on it hence, the point where defect is present will be brighter than other parts.
- The major advantage of fluoroscopy is that a real time image is obtained. Hence, the method is faster than radiography and can be applied at larger-scale production level.

1. Uses of Radiography

- Radiography can be used for sample of different thickness and density. The defects such as porosity or holes may be detected easily.
- Plane defects such as cracks are not always detectable and ability to locate crack will depend upon orientation of the sample with the beam.
- Radiography is appropriate in locating flaws when flaws are non-planar.
- Radiography can be used for testing welds and castings.
- Radiography can be used to check assemblies for proper placement of components.
- Radiography can be used to check liquid level in a sealed liquid container.
- Radiography can be used to test electrical and electronic components to detect discontinuities in the electrical path.

2. Disadvantages of Radiography

- The investment cost is very high in comparison with other systems.
- It requires larger area for installation.
- The X-ray and γ-ray radiations are harmful for human being. It can cause cancer in human beings. So especial protection should be provided to the workers working on it.
- The instrument should be used carefully.
- Cost of operation is high.

SUMMARY

- In **Non-Destructive Testing** the sample is not destroyed and can be used for further application.
- In **Destructive Testing** the sample under investigation is destroyed.
- **Radiography :** In radiography X-rays and γ-rays are used to detect the defects.
- In **X-Ray Radiography**. X-ray along with photographic film is used.
- Natural radioactive materials are used as source of radiation in **γ-Ray Radiography**.
- For **Getting Depth** of the defect more than one radiogram from different angle should be obtained.
- **Fluoroscopy :** In radiology when image is formed on fluorescent screen rather than photographic film, it is called **Fluoroscopy**.
- **Pulses of Ultrasonic** waves are send in the specimen and reflected beam is monitored on CRO which can be used for thickness measurement and flow detected.

EXERCISE

1. What is non-destructive testing ? Compare it with destructive testing.
2. What are the objectives of NDT.
3. What is different methods of NDT ? Explain them in short.
4. What are the steps involved in liquid penetration method ?
5. Give advantages and disadvantages of liquid penetration method.
6. Write a short note on radiography ?
7. What are the differences in X-ray and γ-ray radiography.
8. Explain the arrangement of X-ray radiography.
9. Write a short note on γ-ray radiography.
10. What is fluoroscopy ?
11. What are advantages and disadvantages of radiology?
12. How pulse echo system is used to detect defects.
13. Give different methods of detecting defects using ultrasonic waves.
14. What are different types of visual display units used in ultrasonic method.
15. Give advantages and disadvantages of NDTs.

NANOTECHNOLOGY

6.9 INTRODUCTION

- Nano, Greek for '***dwarf***', means one billionth. The measurement at this level is in nanometer (abbreviated "nm") – billionth of a meter.
- To get a sense of nanoscale, a human hair measures roughly 75,000 nm, a bacterial cell measures a few hundred nanometers. On the other side, ten hydrogen atoms lined up end-to-end make up 1 nm. The smallest thing which can be seen with naked human eye is of the order of 10,000 nm.
- '***Nanoscience***' is the study of the fundamental principles of molecules and structures with at least one dimension is in the size range of 1 to 100 nm. These structures are known as '***nanostructures***'. The research and application of the nanostructures into nanoscale devices is called '***nanotechnology***'.

6.9.1 Nanoparticles

- An atom or small molecule in the form of vapour is smaller than a nanometer in size. But as they are in gaseous form and their molecules are not in arranged manner, hence do not fall in category of nanoscience.
- The nanostructures are the smallest solid things that is possible to make. At nanoscale, most of the physical properties like conductivity, hardness or melting point are totally different than when they are in gaseous or crystal form. At nanoscale these properties depend on not only on the material but also the size of the nanostructure.
- At such a size, the classical Newtonian mechanics or thermodynamics is not able to explain the observed properties. So one has to apply quantum mechanics to explain the properties of nanoscale materials.
- The basic nanoscience is not new, the chemist have been doing nanoscience for hundreds of years.
- The stained glass windows in medieval charges contain different size gold nano particles. The different size gold particles created different colours as orange, purple, red or greenish in the glass. The new about current nanoscience is aggressive focus on developing applied technology and the right tools for doing it.
- Here, we will be studying different properties, methods of synthesis and applications of nanotechnology.

6.10 QUANTUM CONFINEMENT AND SURFACE TO VOLUME RATIO

6.10.1 Quantum Confinement

- Quantum confinement is change of electronic and optical properties when the material sampled is of sufficiently small size - typically 10 nanometers or less. The bandgap increases as the size of the nanostructure decreases. Specifically, the phenomenon results from electrons and holes being squeezed into a dimension that approaches a critical quantum measurement, called the exciton Bohr radius.
- The first experimental evidence of the quantum confinement effects in clusters came from crystalline CuCl nanoparticles grown in silicate glasses. When the size of the cluster is smaller, its band gap is larger, consequently the absorption peak of UV-VIS spectra is shifted closer to the blue. The 13andgap of ZnS nanoparticles was found to be 4.03eV, which is higher than. that of bulk ZnS which is 3.7eV is due to quantum confinement.

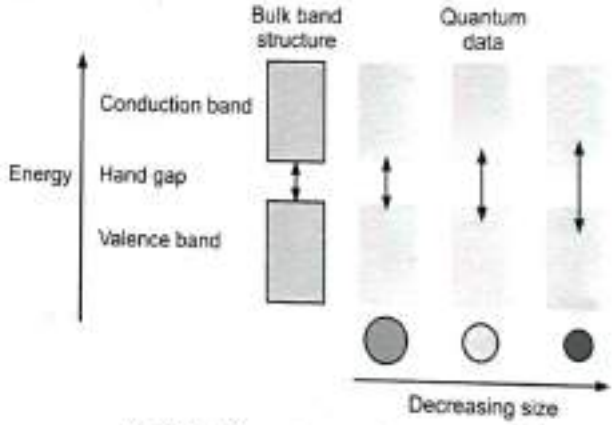


Fig. 6.11 : Quantum confine effect

6.10.2 Surface Area to Volume Ratio in Nanoparticles

- Surface area to volume ratio in nanoparticles have a significant effect on the nanoparticles properties. Firstly, nanoparticles have a relative larger surface area when compared to the same volume of the material.

  For example, let us consider a sphere of radius r:

  The surface area of the sphere will be $4\pi r^2$.

  The volume of the sphere

$$= \frac{4}{3}(\pi r^3)$$

Therefore the surface area to the volume ratio will be

$$\frac{4\pi r^2}{\frac{4}{3}} = 3/r$$

- It means that the surface area to volume ratio increases with the decreases in radius of the sphere and vice versa. It can also be conclude here that when given volume is divided into smaller piece, the surface area increases. Therefore as particle size decreases, a greater portion of the atoms are found at the surface compared to those inside. For example, a particle of size 3 nm has 40% of its atoms on its surface, at 10 nm 20% of its atoms and at 30 nm has 5 % of its atoms on its surface.
- Therefore nanoparticles have a much greater surface area per unit volume compared with the larger particles. It leads to nanoparticles more chemically reactive. As growth and catalytic chemical reaction occurs at surfaces, therefore a given mass of nanomaterial will be much more reactive than the same mass of material made up of large particles. It is also found that materials which are inert in their bulk form are reactive when produced in their nanoscale form. It can improve their properties.

6.11 PROPERTIES OF NANOPARTICLES

(Dec. 12, Nov. 15, May 15, 16)

- The properties of nanoparticles are different from the properties when they are in bulk form (crystal) or in vapour form (gas). The nanoparticles are the smallest solid things possible to make. The properties of the materials are size dependent when it is below critical size (usually less than 100 nm).
- At such a small size, the shape of the nanostructure also decides the property of the material. By making nanomaterials of different size and shape, one can obtain desired property. These properties can be used in many applications in the fields of science, engineering, medicine and environment. Some of the major properties are as follows.

6.11.1 Optical

(May 13, 16, Nov. 13)

- The stained glasses are made by mixing small amount of metal particles like gold, cobalt, nickel, etc. Basically, glass is transparent and the colour appearing on the glass is due to nanoparticles of metals of different sizes.
- The colour of nanoparticles are different from the colour of bulk material. When nanoparticles of gold are formed, they give bright red colour instead of yellow as it appears in bulk form.

  In 1908, G. Mie explained the phenomenon by using classical electromagnetic theory. When a beam of light of intensity I_0 and wavelength λ passes through a medium, the transmitted intensity is given by,

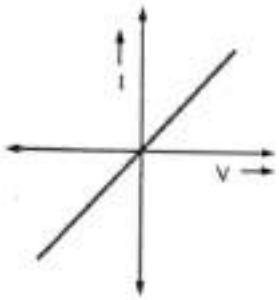
$$I = I_0 e^{-\mu x} \quad \text{...(1)}$$

  where μ is the ***'extinction coefficient'*** and depends on number of particles in medium, volume of colloidal particles and extinction cross-section of a particle.
- When the light is passed through a medium, a fraction is absorbed and a part is scattered, hence the extinction cross-section is the sum of ***'absorption extinction'*** and ***'scattered extinction cross-section'***. The scattering coefficient of the light depends on wavelength λ and the absorption coefficient depends inversely on the volume of the colloidal particles i.e $1/V$ or $1/R^3$ and dielectric constant.
- Thus, the absorption is independent of particle size. This theory explained the observation of absorption of light for metal nanoparticle in visible range. But for particles of size less than ~10 nm failed to explain the size dependency of the optical properties.
- The Drude model explained the particle size dependency of optical properties. For explaining it he assumed that the dielectric constant not only depends on frequency but also on particle size. According to the Drude model, the electrons can be considered as plasma.

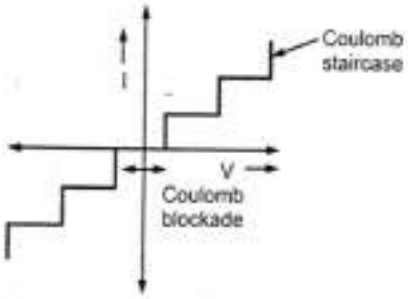
6.11.2 Electrical

(May 13, 14, 16, Nov. 13, Dec. 14)

- The ease with which the material can conduct electricity is called ***'conductivity'***. The conductivity of any material depends on the number of charge carriers, charge, mass of charge carrier and the relaxation time (time between two collisions with ion core). The inverse of conductivity is called ***'resistivity'***.
- When a voltage V is applied across the conductor, current flowing through it is given by Ohm's law and gives a linear graph as shown in Fig. 6.12 (a). But for nanoparticles, the variation of current with changing voltage is as shown in Fig. 6.12 (b).



(a) A typical metal resistor



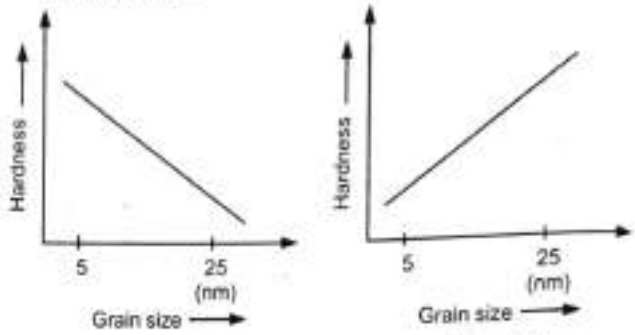
(b) A quantum dot

Fig. 6.12: Voltage versus current

- If the dimensions of metal piece is reduced to 100 nm or less, there appears a region around zero voltage for which there is no current.
- The electrons are transferred when the voltage is ± e/2C. A single electron is transferred by tunneling when the voltage is ± e/2C. Therefore, when the voltage is less than this, electrons cannot be transferred. This gives a region of zero current at low bias voltage and is known as ***'Coulomb blockade region'***. The repeated tunneling of single electron produces ***'Coulomb staircase'***.
- In general, electrical resistivity of materials having nanosized grain is larger than the polycrystalline materials. When electrons are moving, they get scattered at grain boundaries. This results in higher resistivity. The materials having nanosized grains have larger number of grain boundaries than polycrystalline materials. This results in higher resistivity in materials having nanosized grains.

6.11.3 Mechanical

- The mechanical properties of a material depend upon the composition and bonds between the atoms. The mechanical properties like elasticity, hardness, ductility, etc. are result of this.
- The presence of impurities and imperfections in crystalline forms change these properties. For a nanoscale material, the material tends to form a single crystal. The nanocrystals are highly pure and free of imperfections.



(a) Nanoparticles **(b) Bulk crystal**

Fig. 6.13: Hardness variation with grain size

- It has been observed that in metallic nanocrystals, Young's modulus reduces drammatically. For example, magnesium nanocrystal has Young's modulus equal to 3900 N/mm^2 against 4100 N/mm^2 for polycrystalline form.
- For bulk material, the hardness increases linearly with the grain size. But in nanomaterials, the hardness increases linearly with decrease of the particle size as shown in Fig. 6.13.
- But the density of nanocrystalline pellet is often low as some pores are left when the powder is compressed to form pellets. When the deposition is done at high temperature, the densities approach the density of polycrystalline materials.

6.12 APPLICATIONS OF NANOTECHNOLOGY

(Dec. 12, May 14, Nov. 15)

- At nanoscale the materials change their physical properties such as colour, resistance, strength etc. drammatically. These special properties can be used in many fields for making new devices, instruments and consumer goods. These nanoscale devices are extremely small in dimensions and have unique features. Some of the applications are as mentioned below.

6.12.1 Medical (Targeted Drug Delivery)

(May 13, Nov. 13, May 15)

- The nanoparticles can be used for drug delivery, detection of cancer or tumor and treating them. As nanoparticles are very small in size, they can be

injected easily and can be guided towards specific part in the body. Recently, gold nanorods which have strong scattering and absorption property in the infrared are used to detect and destroy cancer cells in rats. The infrared laser beam can be used to detect and destroy cancer cells without affecting the healthy cells. This is possible because healthy cells require twice the laser power as compared to cancer cell for destruction and infrared light can get easily transmitted through cells and muscles. Thus, by using low power infrared laser, only cancer cells can be killed without affecting the healthy cells.

- Drugs can be encapsulated in nanocapsules and can be guided towards desired part of the body. Then drug can be delivered in controlled manner, fastly and slowly, by opening the capsule in desired way. The opening of capsule can be controlled externally by magnetic field, infrared light or physiologically. This will help in treating diabetic or HIV affected patients.
- The scientists are also working to develop better body implants. The body implants should be strong and biocompatible. The body implant should be strong enough so that it does not get deformed easily. Also, it should be biocompatible so that once implanted body cells should be able to grow.
- Some nanotechnology-based tests are developed which are simple and fast. These tests can be used for detection of viruses, DNA, proteins and antibodies. Porous silicon and carbon nanotubes-based sensors can also be used in medical field.

6.12.2 Electronics (May 13, Nov. 13, Dec. 14)

- The discovery of semiconductor transistor replaced the heavy and power consuming vacuum tubes. The transistors are power saving, small and light weight. The Integrated Circuits (ICs) made the instruments still smaller by fabricating a number of large components on a small chip. But the size of components cannot be reduced to atomic level, because after some level the properties of materials become size dependent. At this level, the '***nanoscience***' or '***nanotechnology***' comes into the picture.
- Single Electron Transistor (SET), Spin Valves and Magnetic Tunnel Junctions (MTJ) are the new devices based on nanotechnology. These devices are small, faster and relatively cheaper. The spin valve type devices are used in personal computers to read disk, which have enabled to increase data storage capacity of hard disks. These are the devices based on charge and spin. Earlier, devices were based on charge only and spin was neglected. The spin-based electronics is called as '***spintronic***' or '***magnetoelectronics***'.
- Using an external magnetic field, spin transport can be controlled. The advantage with spin is that it cannot be easily destroyed by scattering from collisions with other charges, impurities or imperfections. Some of the spin-based devices are Spin FET, Spin LED, Spin RTD etc.
- Nanotechnology can also be used in computers for designing '***nonvolatile memory***', smaller and faster microprocessor and better quality monitors. The nanoparticle coating on screen of TV or monitors will improve quality and resolution.

6.12.3 Space and Defence

- In space and defence also, scientists are trying to replace the conventional materials by nanomaterials. A nanoporous material called '***aerogels***' have extremely low density ranging between 0.01 to 0.8 gm/cm^3. Aerogels have small nanosized pores in them and can be of various materials. Basically, aerogels are poor conductors of heat. Therefore, aerogels can be used in spacecrafts to reduce the weight. Even special light-weight suits and jackets can be made using aerogels.
- In satellites or spacecrafts, we use solar energy. But solar cells have low efficiency resulting in large surface area and heavy weight. Nanoparticles can be used to reduce the size and weight of solar cells. Space vehicles also need materials which can withstand harsh and extreme conditions during launching and in space. Polymer composites using silica fibres and nanoparticles have larger Young's modulus, low temperature coefficient of expansion and high impact strength. The nanoparticles in polymer composites are better radiation protectors in comparison to microparticle-based composites.

6.12.4 Automobiles (Dec. 14, May 16)

- The body of a car is made up of steel and some alloys. The body structure should be strong and non-deformable. The nanotube composites have mechanical strength better than steel and attempts are made to make composites that can replace steel.
- Currently, manufacturing of nanotubes is expensive but scientists are trying to make it economical. Nanoparticle paints provide smooth, thin and attractive coating. Research is going on to change the colour of car by applying a small voltage.
- Self-cleaning glass can be made by mixing small amount of titania (TiO_2) nanoparticles while manufacturing it. The titania is capable of dissociating organic dust in presence of UV light present in sunlight. Once dissociated it may fall down or evaporate. Water droplets on glass give hazy look, but titania glass can spread the water evenly giving clear sight. Such special glass can be used to make window glass of a car.
- By using nanoparticles light weight and less rubber consuming, thinner tyres can be made. This will reduce the weight, price of the car and will increase the mileage as a result of reduced weight. Nanoparticles can be used as catalysts to convert harmful emissions into less harmful gases. With the help of nanocarbon tubes, hydrogen fuel can be stored safely and can be used for running a car. The hydrogen fuel is eco-friendly and ever lasting.

SUMMARY

- **Nano :** The Greek word nano means one billionth.
- **Nanoscience** is the study of the fundamental principles of molecules and structures with at least one dimension of nanosize.
- The research and application of the nanostructures into nanoscale devices is called **Nanotechnology**.
- The widening of band gap at nano level is due to quantium confinement effect.
- The nanpparticles have large surface to volume ratio which is the factor affecting most of its physical properties.
- Most of the **Physical Properties** of nanomaterials are different than when they are in bulk form.
- **Optical Property :** The colour of nanoparticles are different from the colour of bulk material.
- **Electrical Property :** The resistivity of materials having nanosized grain is larger than polycrystalline materials.
- **Mechanical Property :** The nanocrystalline pellet have low density as some pores are left when the powder is compressed.
- The nanoparticles have vast applications in almost all the fields.
- Most of the applications are still in trial stage and very few are commercially available.

EXERCISE

1. What is nanoscience ? Why classical mechanics cannot be applied to nanoparticles ?
2. Why properties of nanoparticles are different from what they are in bulk form ? Explain optical and electrical properties of nanoparticles.
3. State and explain any three properties of nanoparticles.
4. Explain the following properties of nanoparticles:
   (i) Electrical
   (ii) Mechanical properties.
5. Explain the applications of nanoparticles in the field of
   (a) automobile,
   (b) medical.
6. Give advantages of nanoparticles over bulk material in the field of (a) automobile, (b) space and defence.
7. Nanoparticles will reduce environmental pollution. Justify.

UNIVERSITY QUESTIONS

December 2012

1. Explain any two applications of Nano-technology. [4]
2. Explain any two properties of Nano-particle. [3]
3. Explain Synthesis of Metal Nano-particle by Colloidal Route Method. [3]

May 2013

1. Explain the optical and electrical properties of nanoparticles. [4]
2. Explain the applications of nanoparticles in medical and electronic Industry. [3]

November 2013

1. Explain following properties of nano-particles: [4]
   (i) Optical property.
   (ii) Electrical property.
2. Explain the applications of nanoparticles in medical and electronic field. [3]

May 2014

1. Discuss the electrical and structural properties of nano-materials. [6]
2. Explain any one application of nanotechnology. [3]

December 2014

1. Explain the applications of nanoparticles in automobile and electronic industry. [4]
2. Explain electrical properties of nano-particles. [3]

May 2015

1. Discuss applications of nanotechnology in medical field. [3]

November 2015

1. Explain any two properties of nano-particles in brief. [4]
2. Explain the applications of nano-particles in electronic industry. [3]
3. Explain the synthesis of nano-particles by chemical method in colloidal form with diagram and example. [6]

May 2016

1. Explain the applications of nano-particles in the field of automobiles. [3]
2. Why are the properties of nano-particles different from that of the bulk materials? Explain any two properties of non-particles. [6]

November 2016

1. Explain the following properties of Nano-particles [4]
   (i) Magnetic Property
   (ii) Mechanical Property.

May 2017

1. State applications of nano-particles. Explain any one applications. [3]
2. What is nanotechnology ? Explain optical and electrical properties of nano-particles. [6]

November 2017

1. Give brief explanation of the optical properties of nanoparticles with the help of quantum confinement effect and G Mie equation. [4]
2. Explain the mechanical properties of Nanoparticles. [3]

May 2018

1. Explain the optical and electrical properties of nanoparticles. **[6]**
2. List the applications of nanotechnology in the field of automobile. Explain any one application in brief. **[3]**

November 2018

1. Give brief explanation of the optical properties of nanoparticles with the help of quantum confinement effect and G Mie equation. **[4]**

May 2019

1. State applications of nano-particle. Explain any one application. **[3]**

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QUESTION BANK FOR IN-SEM. EXAMINATION (UNIT I & UNIT II)

QUESTIONS ON UNIT I : INTERFERENCE

6 Marks

1. A thin film of uniform thickness is illuminated by monochromatic light. Obtain the conditions for darkness and brightness of the film as observed in reflected light.
2. Draw a neat and labeled diagram and write the conditions for constructive and destructive interference in case of thin film of (a) uniform thickness, (b) wedge shape. Why the interference pattern in the reflected and transmitted system is complementary
3. Write the conditions for constructive and destructive interference in thin film of wedge shaped. Explain how this setup can be used for testing the flatness of a glass plate.

3/4 Marks

1. Explain in brief (a) Electromagnetic waves (b) Electromagnetic spectrum
2. Draw diagram showing interference in reflected light in a thin wedge shaped film. Write down the mathematical conditions for maximum and minimum intensity of light in reflected system.
3. Why excessively thin films appear to be perfectly black in reflection when illuminated by white light?
4. Explain why a thin film, when illuminated by white light, appears colored when observed in reflected light.
5. Explain with diagram how interference Principle is used to design anti reflection coating. Derive an expression for its thickness.

QUESTIONS ON UNIT I : DIFFRACTION

6 Marks

1. Explain Fraunhoffer diffraction at a single slit and obtain the condition for principal maxima and minima. Draw the intensity distribution curve.
2. For a plane diffraction grating, starting from the equations of resultant amplitude and intensity, derive conditions for maxima and minima of the diffraction pattern.
3. What is Raileigh's criterion for resolving power? Obtain an expression for resolving power of grating.

3/4 Marks

1. What is diffraction of light? Explain in brief Fraunhoffer diffraction.
2. The resultant amplitude of a wave when monochromatic light is diffracted from a single slit is $E_\theta = E_m \frac{\sin \alpha}{\alpha}$. Specify the terms involved in it and derive the condition for minima.
3. What is Raileigh's criterion for resolving power? Write a short note on resolving power of telescope.

QUESTIONS ON UNIT I : POLARIZATION

6 Marks

1. What is double refraction? Explain in the phenomenon of it on the basis of Huygen's theory.

3/4 Marks

1. What is polarized light? Define (a) plane of polarization (b) plane of vibration.
2. State and explain Law of Malus.
3. Explain how the phenomenon of polarization of light is used in LCD displays.

QUESTIONS ON UNIT II : LASER

6 Marks

1. With the help of energy band diagram explain construction and working of single heterojunction semiconductor laser
2. With the help of energy band diagram explain construction and working of CO_2 laser.
3. What is holography? State its principle. Explain the process of holography recording and reconstruction.

3/4 Marks

1. Define: (a) Stimulated emission (b) population inversion (c) pumping (d) metastable state.
2. What is stimulated emission of radiations? Explain its significance in production of laser.
3. What is an optical resonator? What is its role in lasing?
4. State the characteristics of a laser beam and explain any one of them in brief.
5. What is the difference between normal photography and holography? Which property of laser is most useful to record a holograph and why?

6. Explain the process of recording Hologram with the help of LASER.
7. State applications of laser in the field of information technology. Explain any one in brief.
8. State industrial applications of laser. Explain any one in brief.
9. State medical applications of laser. Explain any one in brief.

QUESTIONS ON UNIT II : OPTIC FIBERS

6 Marks

1. Describe attenuations in optic fiber viz. absorption, dispersion and bending and ways to overcome the losses.
2. Explain in brief (a) single mode and (b) multimode optic fibers.
3. Explain in brief (a) step-index and (b) graded-index optic fibers.

3/4 Marks

1. What are optic fibers? Explain its construction and principle in brief.
2. Define the following and give equations wherever necessary (a) Critical angle (b) total internal reflection (c) acceptance angle
3. Define the following and give equations wherever necessary (a) acceptance angle (b) acceptance cone (b) Numerical aperture.
4. Define acceptance angle and acceptance cone. What is their significance in optic fiber?
5. What is total internal reflection? Explain its significance in optic fiber communication.
6. Define critical angle. Explain its significance in optic fiber communication.
7. Define numerical aperture. Explain its significance in optic fiber communication.
8. Differentiate between single mode and multimode optic fibers.
9. Differentiate between step-index and graded-index optic fibers.
10. Explain basic building blocks of a communication system.
11. State the advantages of optic fiber communication over conventional systems.

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$2\mu t \cos r = 3\lambda$

$2\mu t \cos r = \lambda$