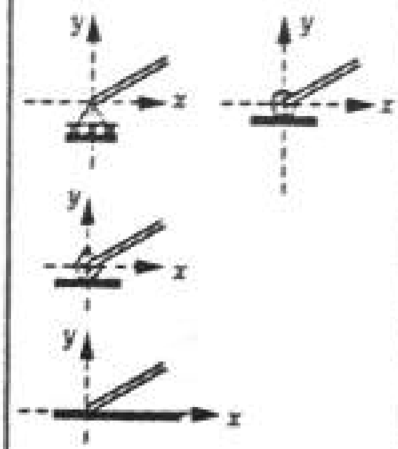
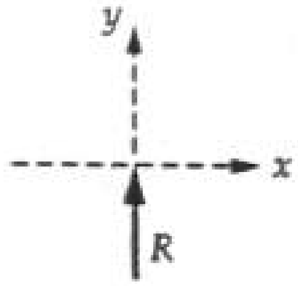
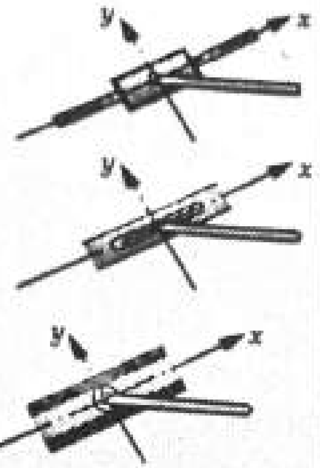
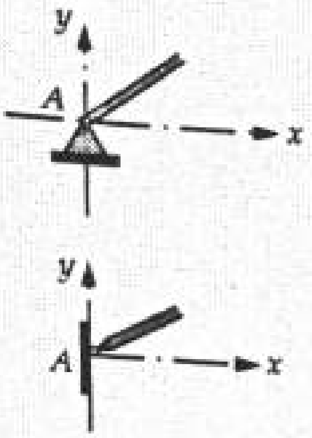
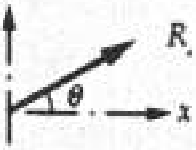
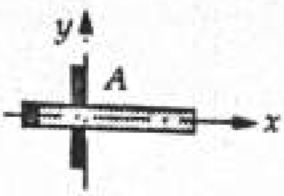
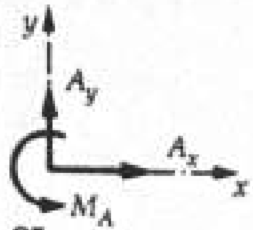
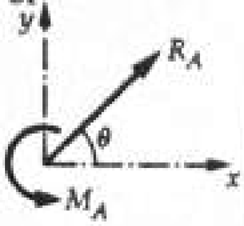
UNIT - III

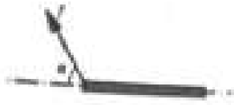
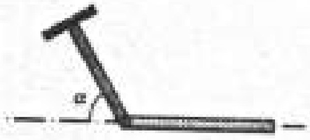
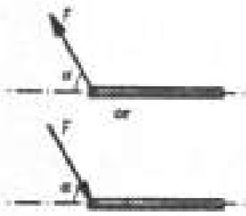
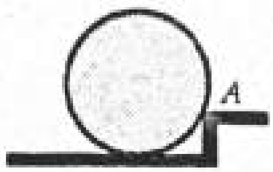
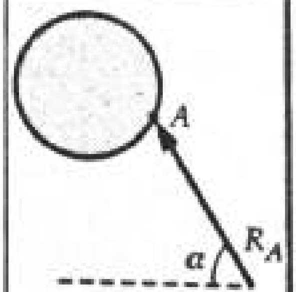
6 Equilibrium of Coplanar Force System

Important Points to Remember

- Free body diagram is the isolated diagram of an object/system of objects/any point in the system in which all forces and couple moments acting on it are shown including support reactions.
- Different types of supports and their corresponding reactions are given in Table 6.1.

Sr. No.	Type of support	Symbolic representation	Reaction components
1.	a) Roller b) Rocker c) Object on frictionless surface		 Reaction is perpendicular to the surface on which object is resting.

2.	a) Collar on frictonless rod. b) Pin in a slot without friction c) Roller in a slot		 Reaction is perpendicular to the rod or the slot.
3.	Pin or hinge		 or  where $R_A = \sqrt{A_x^2 + A_y^2}$ $\theta = \tan^{-1} \frac{A_y}{A_x}$
4.	Fixed		 or 

5.	Cable		 There is tensile force in the cable.
6.	Weightless link		 Link (i.e. rod) can have tensile or compressive force when subjected to forces only at the ends.
7.	Roller at a sharp edge		 The direction of reaction is towards the roller but the angle 'α' depends on the other applied forces.

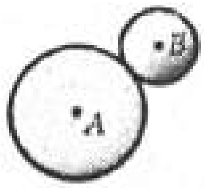
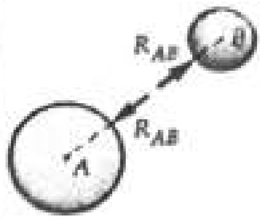
8.	Corner		
9.	Two rollers		 Reaction is along the line joining the two centres.

Table 6.1 Types of supports and corresponding reactions

- The conditions for equilibrium of a concurrent force system are

$$\sum F_x = 0 \quad , \quad \sum F_y = 0$$

- If there are three forces in equilibrium, Lami's theorem can be used.
- When an object is in equilibrium under the action of three forces which are not parallel, then the three forces must be concurrent.
- The conditions for equilibrium of coplanar non-concurrent force system are

$$\sum F_x = 0$$
$$\sum F_y = 0$$

and $$\sum M = 0$$

Q.1 State conditions of equilibrium for -
i) Co-planer concurrent forces
ii) Concurrent forces in space
iii) Co-planer non-concurrent forces
iv) Non-concurrent forces in space. [SPPU : Dec.-09, Marks 4]

Ans. :

i) $\sum F_x = 0$ and

$\sum F_y = 0$

ii) $\sum F_x = 0$

$\sum F_y = 0$ and

$\sum F_z = 0$

iii) $\sum F_x = 0$

$\sum F_y = 0$

$\sum M = 0$

iv) $\sum F_x = 0 ; \sum F_y = 0 ; \sum F_z = 0$

$\sum M_x = 0 ; \sum M_y = 0 ; \sum M_z = 0$

Q.2 State and explain active forces, reactive forces and free body diagram with suitable example. [SPPU : May-13, Marks 5]

Ans. : Active forces are the forces which try to produce motion of the body. Reactive forces arise as a result of interaction of the body with the supports.

It is the isolated diagram of an object/system or any point in the system in which all forces and couple moments acting on it are shown including support reactions.

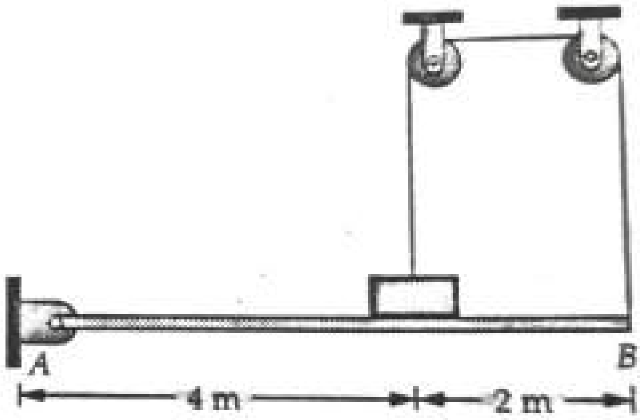


Fig. Q.2.1

When objects are supported by some supported mechanisms, the supporting mechanism exerts a force or couple moment on the object which tries to restrict the motion of the object. These forces or couple moements which oppose or restrict the motion by objects are called support reactions.

For example, consider the system shown in Fig. Q.2.1. Weight of rod is W_1 and weight of block is W_2. Their free body diagrams are as showm om Fig. Q.2.2.

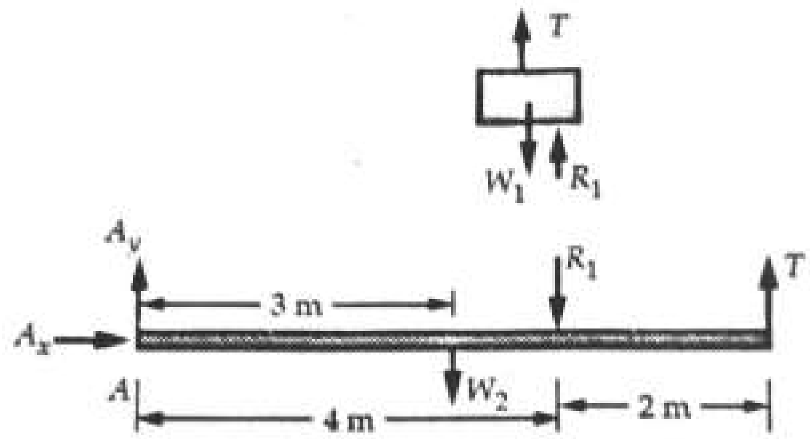


Fig. Q.2.2

Q.3 State and explain two force and three force principle for equilibrium with sketches. **[SPPU : Dec.-13, Marks 5]**

Ans. : If there are two forces in equilibrium, they must be equal in magnitude, opposite in direction and acting along the same line.

When an object is in equilibrium under the action of three forces which are not parallel, then the three forces must be concurrent.

If two forces F_1 and F_2 are not parallel, their lines of action will intersect at some point A as shown in Fig. Q.3.1. If line of action of the third force F_3 does not pass through A, the combined moment of all the forces about A will be non-zero and the object will not remain in equilibrium. Hence, for equilibrium of the object, the forces must be concurrent. Note that this condition is

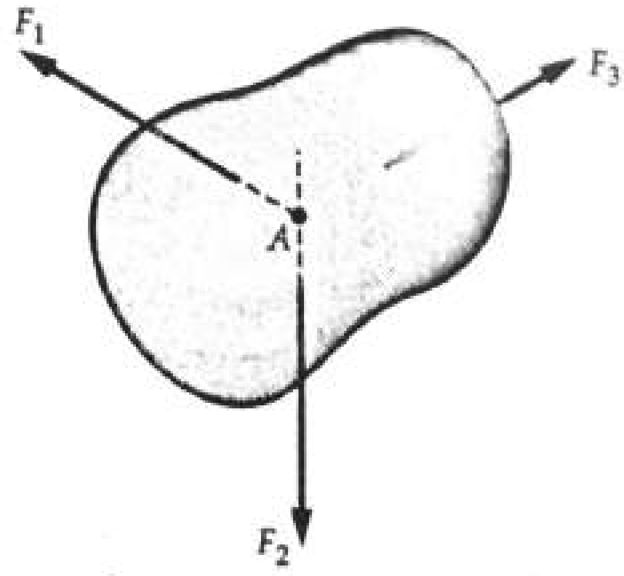


Fig. Q.3.1

applicable only for three non-parallel forces. For more than three forces, the moment of F_3 about A can be nullified by moment of F_4 about A without the force system becoming concurrent as shown in Fig. Q.3.2.

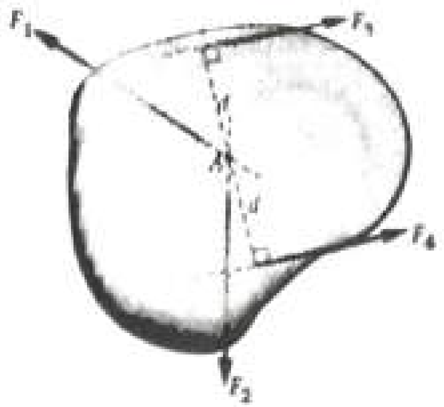


Fig Q.3.2

Q.4 Two sphere P and Q each of weight 50 N and a radius of 100 mm rest in horizontal channel of width 360 mm as shown in Fig. Q.4.1. Determine the reaction at the point of contact A, B and C.

[SPPU : Dec.-12, Marks 7]

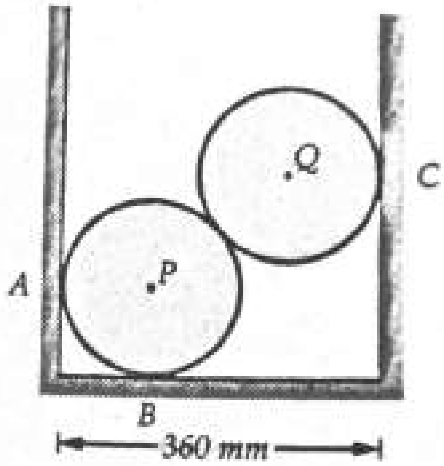


Fig. Q.4.1

Ans. : The angle of the reaction between P and Q is obtained by joining centres of P and Q drawing horizontal from P and vertical from P as shown in Fig. Q.4.1 (a).

$$\cos\alpha = \frac{360 - r_P - r_Q}{r_P + r_Q}$$

$$= \frac{360-100-100}{100+100}$$

$$\therefore \quad \alpha = 36.87^\circ$$

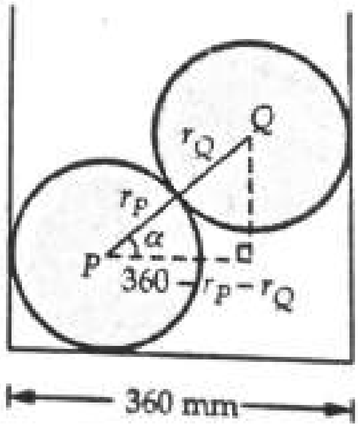


Fig. Q.4.1 (a)

The free body diagrams of the two spheres are shown in Fig. Q.4.1 (b).

Using Lami's theorem for Q,

$$\frac{R_C}{\sin 126.87} = \frac{R_{PQ}}{\sin 90} = \frac{50}{\sin(180-36.87)}$$

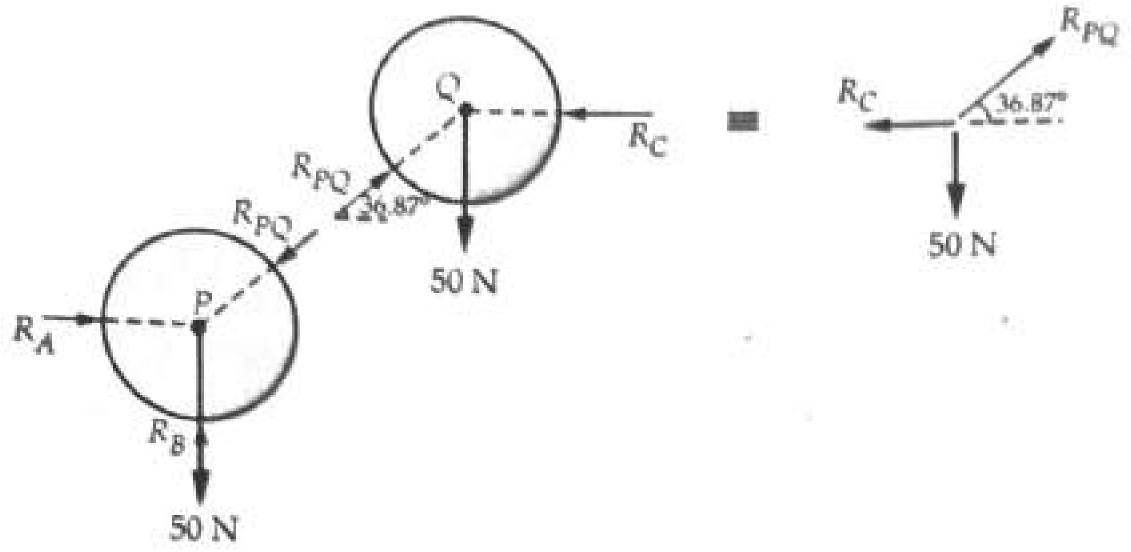


Fig. Q.4.1 (b)

$$\boxed{R_C = 66.67 \text{ N}} \quad \text{... Ans.}$$

$$R_{PQ} = 83.33 \text{ N}$$

For P, $\sum F_x = 0$;

$$R_A - R_{PQ} \cos 36.87 = 0$$

$$\therefore \quad \boxed{R_A = 66.67 \text{ N} \rightarrow} \quad \text{... Ans.}$$

$$\sum F_y = 0$$

$$R_Q - 50 - R_{PQ} \sin 36.87 = 0$$

$$\therefore \quad \boxed{R_B = 100 \text{ N} \uparrow} \quad \text{... Ans.}$$

Q.5 Determine the magnitude of F_1 and F_2 so that the particle is in equilibrium. Refer Fig. Q.5.1. ☞ [SPPU : Dec.-13, Marks 6]

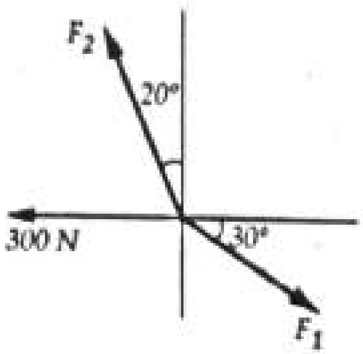


Fig. Q.5.1

Ans. : $\sum F_x = 0 : F_1 \cos 30 - F_2 \sin 20 - 300 = 0$

$\therefore \ F_1 \cos 30 - F_2 \sin 20 = 300$ ...(1)

$\sum F_y = 0 : \quad - F_1 \sin 30 + F_2 \cos 20 = 0$ ...(2)

From equations (1) and (2)

$\therefore \quad \boxed{F_1 = 438.57 \text{ N}}$ ... Ans.

and $\quad \boxed{F_2 = 233.36 \text{ N}}$ ... Ans.

Q.6 Determine the magnitude and direction θ of force F so that the particle is in equilibrium. Refer Fig. Q.6.1. [SPPU : Dec.-14, Marks 5]

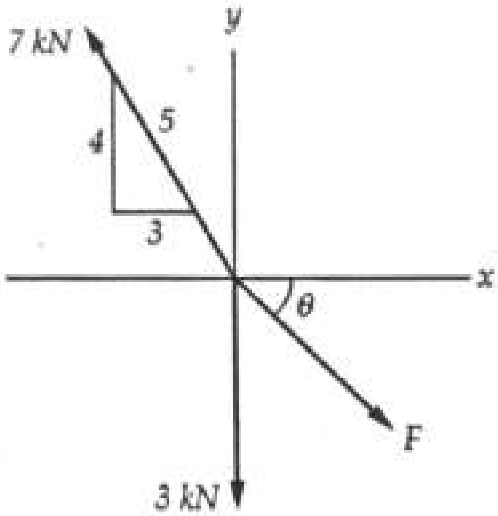


Fig. Q.6.1

Ans. : $\tan \alpha = \frac{4}{3} \quad \therefore \alpha = 53.13°$

$\sum F_x = 0 : \quad F \cos \theta - 7 \times \cos 53.13 = 0$

$\therefore \quad F \cos \theta = 4.2$ ... (1)

$\sum F_y = 0 : \ - F \sin \theta + 7 \times \sin 53.13 - 3 = 0$

$\therefore \quad F \sin \theta = 2.6$ ... (2)

Dividing equation (2) by (1),

$$\tan \theta = \frac{2.6}{4.2}$$

$$\therefore \quad \theta = 31.76°$$

From equation (1),

$$F \cos 31.76 = \frac{21}{5}$$

$$\therefore \quad F = 4.94 \text{ kN}$$

$$\therefore \quad \boxed{F = 4.94 \text{ kN}, 31.76^\circ} \quad \text{... Ans.}$$

Q.7 The 20 kg homogeneous smooth sphere rests on the two inclines as shown in Fig. Q.7.1. Determine the contact forces at A and B.

[SPPU : May-15, Marks 6]

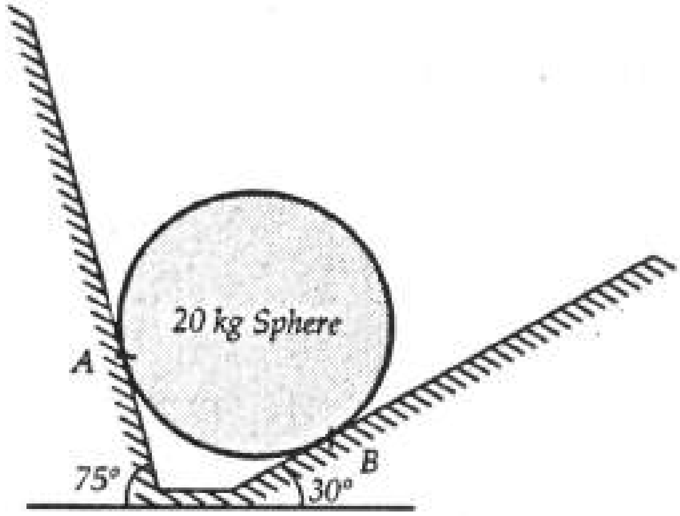


Fig. Q.7.1

Ans. : The FBD of sphere is shown in Fig. Q.7.1 (a). Using Lami's theorem,

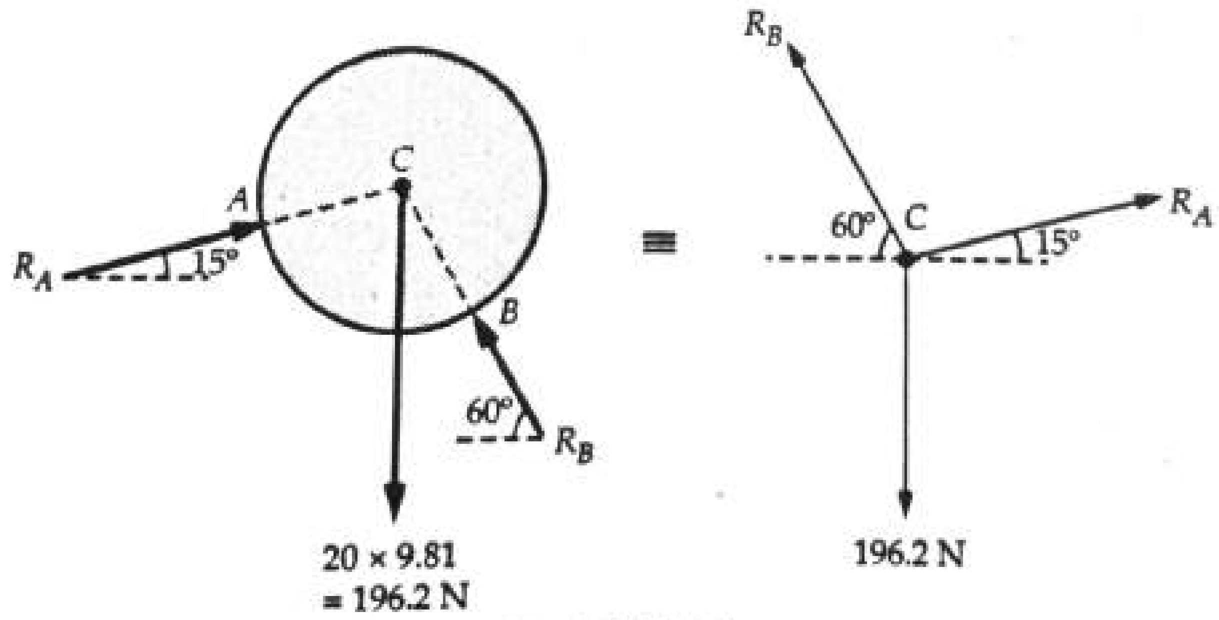


Fig. Q.7.1 (a)

$$\frac{R_A}{\sin 150} = \frac{R_B}{\sin 105} = \frac{196.2}{\sin 105}$$

$\therefore$ $\boxed{R_A = 101.56 \text{ N}, 15°}$ ... Ans.

$\therefore$ $\boxed{R_B = 196.2 \text{ N}, 60°}$ ... Ans.

Q.8 A sphere weighing 1000 N is placed in a wrench as shown in Fig. Q.8.1, find the reactions at the point of contacts.

[SPPU : Dec.-15, Marks 5]

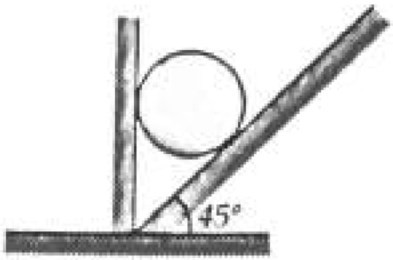


Fig. Q.8.1

Ans. : The FBD of sphere is shown in Fig. Q.8.1 (a).

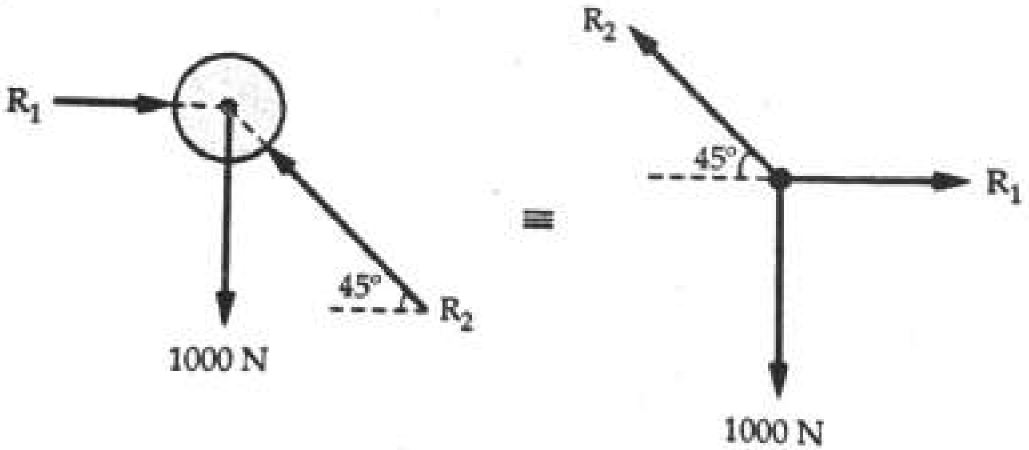


Fig. Q.8.1 (a)

Using Lami's theorem,

$$\frac{R_1}{\sin 135} = \frac{R_2}{\sin 90} = \frac{1000}{\sin 135}$$

$\therefore$ $\boxed{R_1 = 1000 \text{ N} \rightarrow}$ ...Ans.

$\therefore$ $\boxed{R_2 = 1414.2 \text{ N}, 45°}$ ...Ans.

Q.9 Determine the magnitude and position of force F so that the force system shown in Fig. Q.9.1 maintain equilibrium.

[SPPU : Dec.-15, Marks 6]

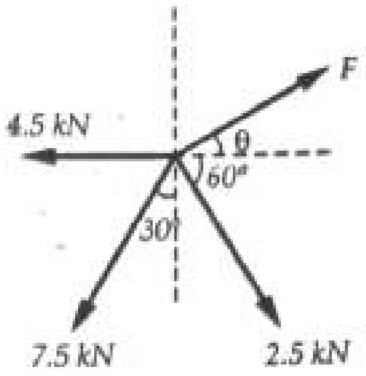


Fig. Q.9.1

Ans. : $\sum F_x = 0$:

$F \cos\theta - 4.5 - 7.5 \sin 30 + 2.5 \cos 60 = 0$

$\therefore \quad F\cos\theta = 7$ ... (1)

$\sum F_y = 0$:

$F \sin\theta - 7.5 \cos 30 - 2.5 \sin 60 = 0$

$\therefore \quad F\sin\theta = 8.66$ ... (2)

Divide equation (2) by equation (1).

$$\tan\theta = \frac{8.66}{7}$$

$\therefore \quad \boxed{\theta = 51.05^\circ}$ ...Ans.

Substituting θ in equation (1),

$\therefore \quad \boxed{F = 11.135 \text{ kN}}$ ... Ans.

Q.10 The 30 kg pipe is supported at A by a system of five cords as shown in Fig. Q.10.1. Determine the force in each cord for equilibrium.

☞ [SPPU : May-16, Marks 6]

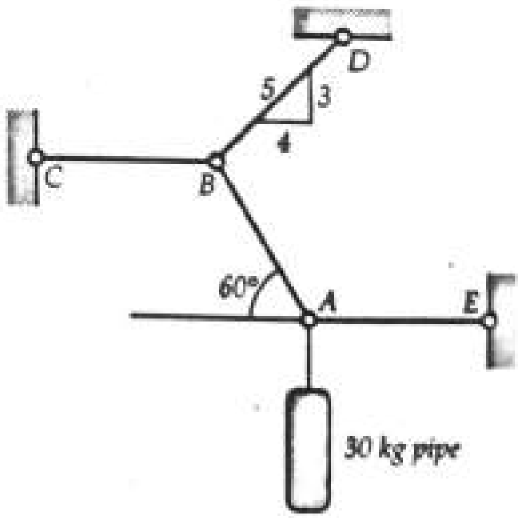


Fig. Q.10.1

Ans. : The free body diagrams of *A* and *B* are shown in Fig. Q.10.1 (a).

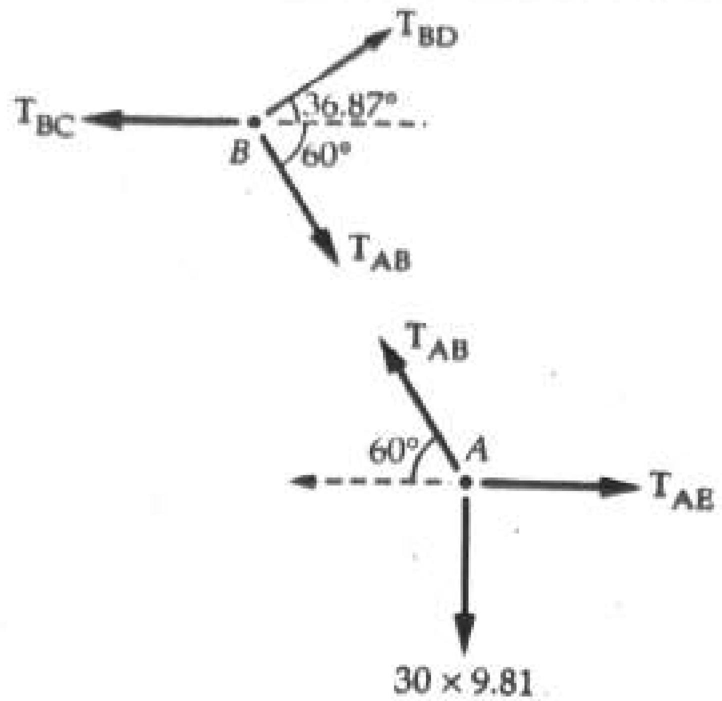


Fig. Q.10.1 (a)

Using Lami's theorem for A,

$$\frac{T_{AB}}{\sin 90} = \frac{T_{AE}}{\sin 150} = \frac{30 \times 9.81}{\sin 120}$$

$\therefore$ $\boxed{T_{AB} = 339.83 \text{ N}}$ ...Ans.

$\boxed{T_{AE} = 169.91 \text{ N}}$ ...Ans.

Using Lami's theorem for B,

$$\frac{T_{BC}}{\sin 96.87} = \frac{T_{BD}}{\sin 120} = \frac{T_{AB}}{\sin 143.13}$$

Using value of T_{AB},

$\boxed{T_{BC} = 562.32 \text{ N}}$ ...Ans.

$\boxed{T_{BD} = 490.5 \text{ N}}$ ...Ans.

Q.11 Two identical rollers, each of weight 700 N are supported by an inclined plane and a vertical wall as shown in Fig. Q.11.1. Find the reactions exerted by the wall and inclined plane at C, D and E.

Fig. Q.11.1

Ans. : The F.B.D. of each roller is shown in Fig. Q.11.1 (a). Th reaction between the two rollers makes angle 30° with horizontal a the line joining the centres is parallel to the inclined plane.

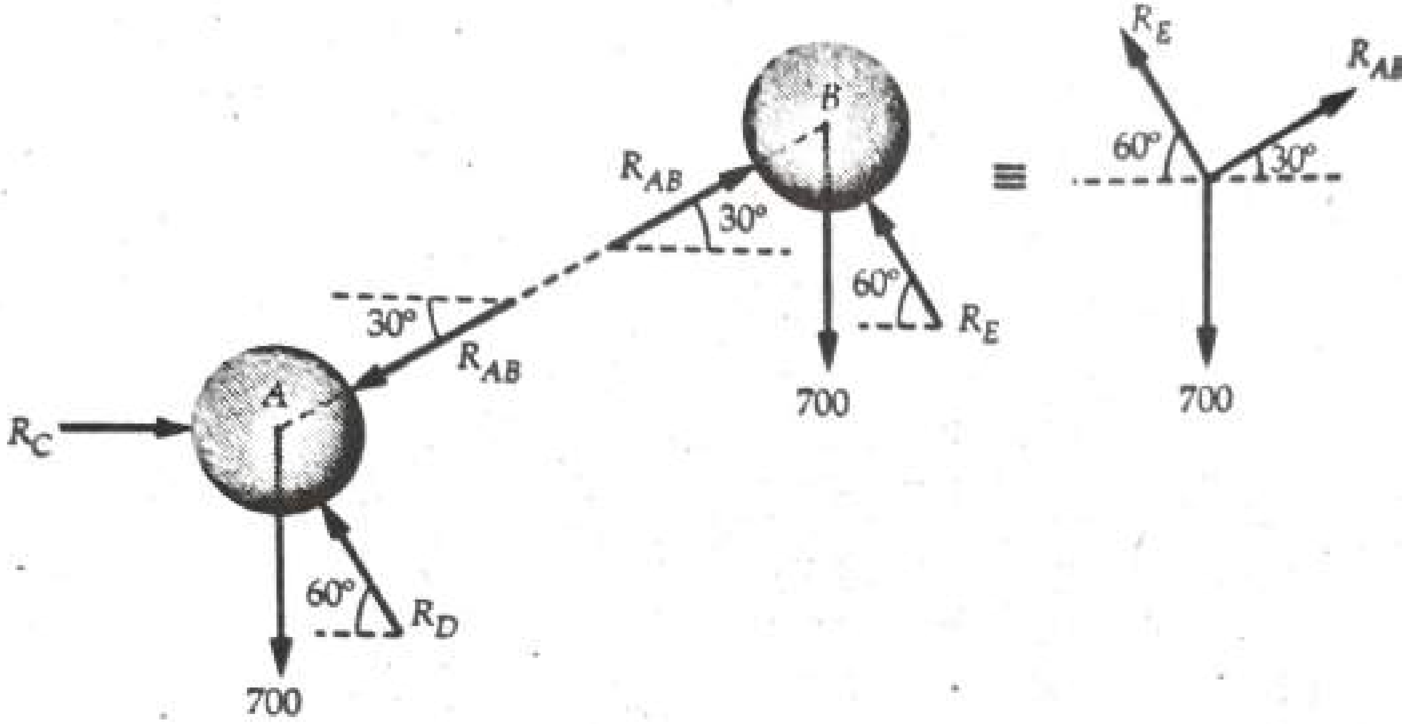


Fig. Q.11.1 (a)

For B, using Lami's theorem,

$$\frac{R_{AB}}{\sin 150} = \frac{R_E}{\sin 120} = \frac{700}{\sin 90}$$

$$\therefore \quad R_{AB} = 350 \text{ N}$$

$$\boxed{R_E = 606.22 \text{ N}, 60°} \quad \text{...Ans.}$$

For A,

$$\Sigma F_y = 0$$

$$R_D \sin 60 - R_{AB} \sin 30 - 700 = 0$$

$\therefore$ $\boxed{R_D = 1010.36 \text{ N}, 60°}$ ...Ans.

$\therefore$ $\Sigma F_x = 0$

$$R_C - R_{AB} \cos 30 - R_D \cos 60 = 0$$

$\therefore$ $\boxed{R_C = 808.3 \text{ N} \rightarrow}$ ...Ans.

Q.12 Two spheres A and B of diameter 80 mm and 120 mm respectively are held in equilibrium by separate strings as shown in Fig. Q.12.1. Sphere B rests against vertical wall. If masses of spheres A and B are 10 kg and 20 kg, determine the tension in the string and reactions at point of contact.

[SPPU : Dec.-16, Marks 6]

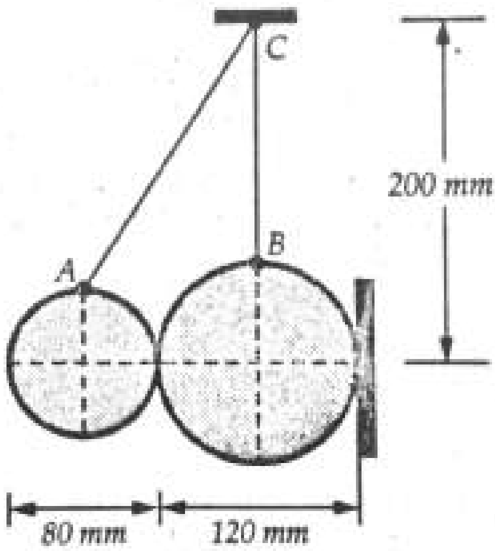


Fig. Q.12.1

Ans. : To find angle of string AC with horizontal, draw horizontal at A as shown in Fig. Q.12.1 (a).

$$\tan\alpha = \frac{160}{100}$$

$\therefore$ $\alpha = 58°$

The two free body diagrams are shown in Fig. Q.12.1 (b)

For FBD of sphere 1, $\sum F_y = 0$: $T_{AC} \sin 58 - 98.1 = 0$

$\therefore$ $\boxed{T_{AC} = 115.68 \text{ N}}$ ...Ans.

$$\sum F_x = 0$$

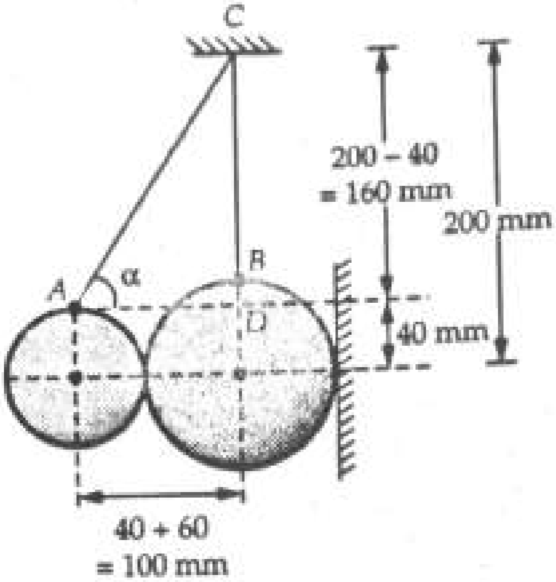


Fig. Q.12.1 (a)

$$T_{AC} \cos 58 - R_{12} = 0$$

$\therefore$ $\boxed{R_{12} = 61.3 \text{ N}}$ ...Ans.

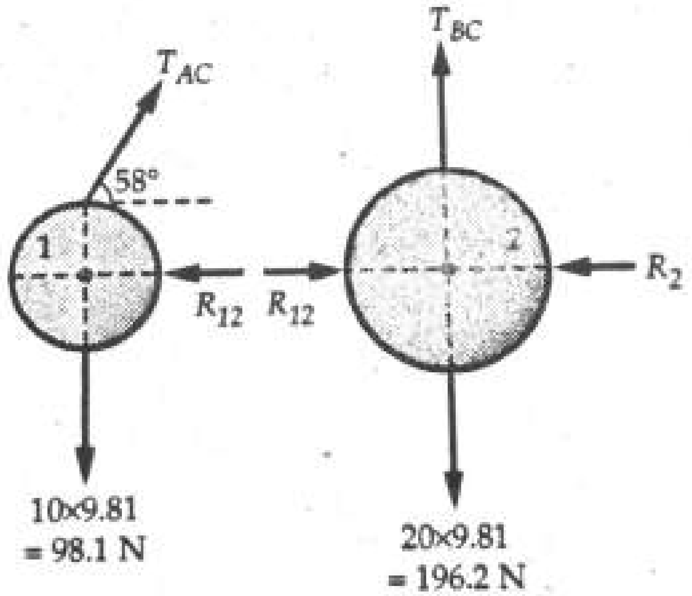


Fig. Q.12.1 (b)

For sphere 2,

$$\sum F_x = 0 :$$

$$R_{12} - R_2 = 0$$

$\therefore$ $\boxed{R_2 = 61.3 \text{ N} \leftarrow}$ ...Ans.

$$\sum F_y = 0 : T_{BC} - 196.2 = 0$$

$\therefore$ $\boxed{T_{BC} = 196.2 \text{ N}}$ ...Ans.

Q.13 A cylinder of mass 100 kg rest between the inclined plane as shown in Fig. Q.13.1 (a). Determine the normal reaction at A and B.

[SPPU : May-17, Marks 6]

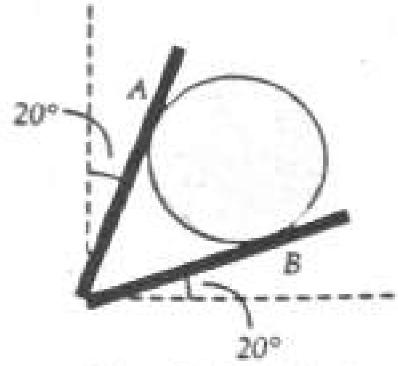


Fig. Q.13.1 (a)

Ans. : The FBD of cylinder is shown in Fig. Q.13.1 (a).

Using Lami's theorem,

$$\frac{R_A}{\sin 160} = \frac{R_B}{\sin 70} = \frac{981}{\sin 130}$$

$\therefore$ $\boxed{R_A = 438 \text{ N}, 20^\circ}$ ...Ans.

$\therefore$ $\boxed{R_B = 1203.4 \text{ N}, 70^\circ}$ ...Ans.

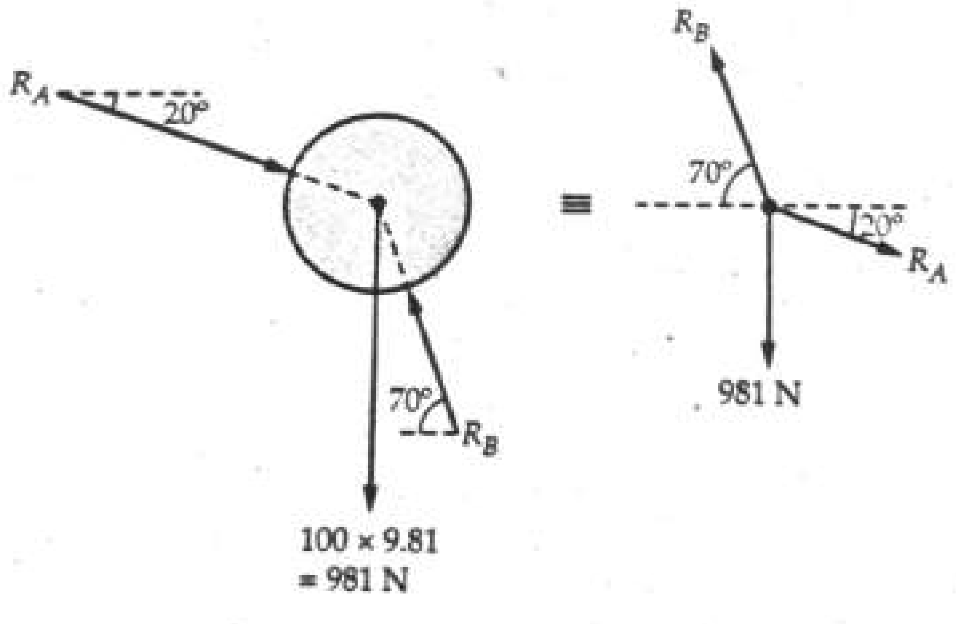


Fig. Q.13.1 (b)

Q.14 Determine the tension developed in wires CA and CB required for equilibrium of 10 kg cylinder as shown in Fig. Q.14.1.

[SPPU : May-18, Marks 6]

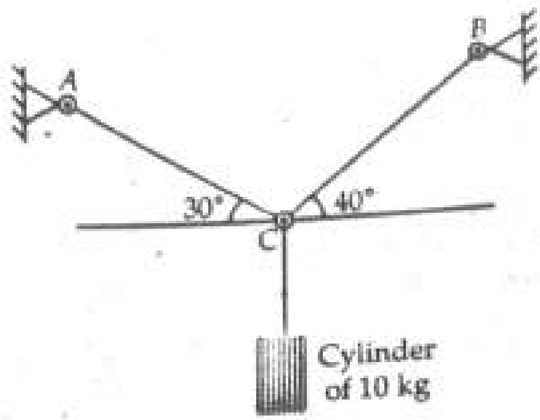


Fig. Q.14.1

Ans. : The FBD of point C is shown in Fig. Q.14.1 (a).

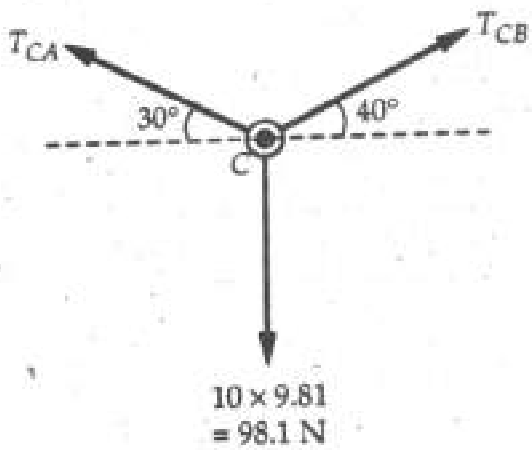


Fig. Q.14.1 (a)

Using Lami's theorem, $\frac{T_{CA}}{\sin 130} = \frac{T_{CB}}{\sin 120} = \frac{98.1}{\sin 110}$

$\therefore$ $\boxed{T_{CA} = 80\ N}$ ... **Ans.**

$\therefore$ $\boxed{T_{CB} = 90.4\ N}$ ... **Ans.**

Q.15 Determine the reactions at all the point of contacts for a sphere of 200 N kept in a trough as shown in the Fig. Q.15.1.

[SPPU : Dec.-18, Marks 6]

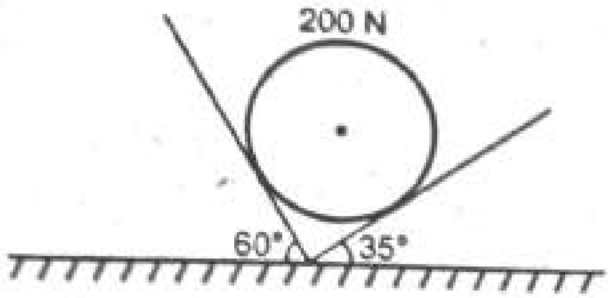


Fig. Q.15.1

Ans. : The FBD of sphere is shown in Fig. Q.15.1 (a).

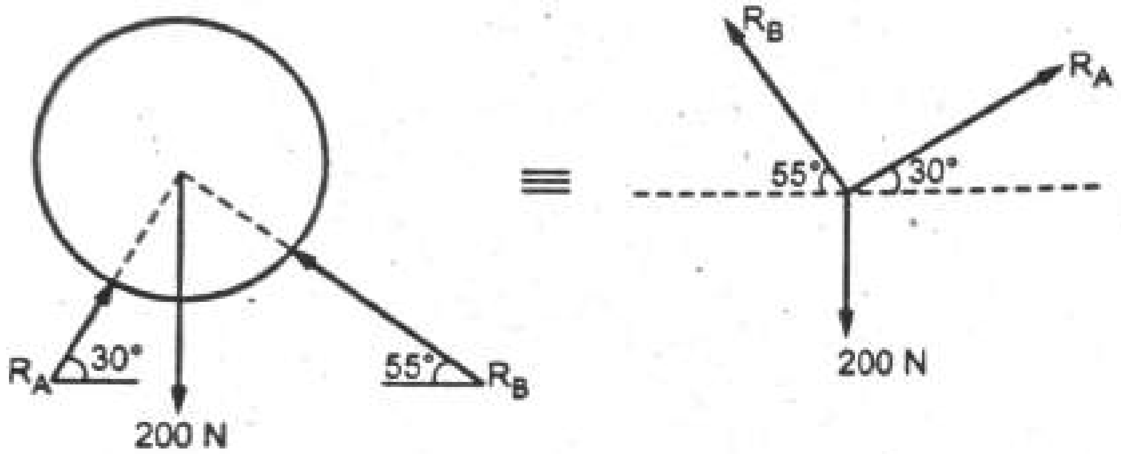


Fig. Q.15.1 (a)

Using Lami's theorem

$$\frac{R_A}{\sin 145} = \frac{R_B}{\sin 120} = \frac{200}{\sin 95}$$

$$\boxed{R_A = 115.153 \text{ N}} \quad \text{... Ans.}$$

$$R_B = 173.866 \text{ N} \quad \text{... Ans.}$$

Q.16 Two cables tied together at C are loaded as shown. Knowing that W = 190 N, determine the tension a) in cable AC, b) in cable BC.

[SPPU : May-09, Marks 6]

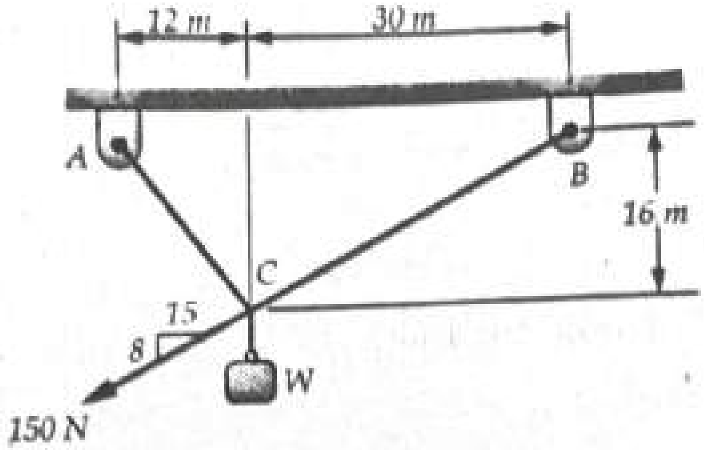


Fig. Q.16.1

Ans. : The F.B.D. of point C is as shown in Fig. Q.16.1 (a).

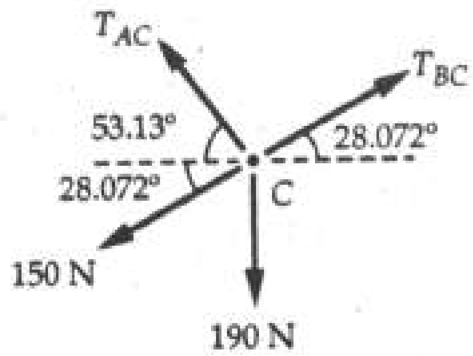


Fig. Q.16.1 (a)

$\Sigma F_x = 0$:

$$T_{BC} \cos 28.072 - T_{AC} \cos 53.13 - 150 \cos 28.072 = 0 \quad \text{... (1)}$$

$\Sigma F_y = 0$:

$$T_{BC} \sin 28.072 + T_{AC} \sin 53.13 - 150 \sin 28.072 - 190 = 0 \quad \text{... (2)}$$

From equations (1) and (2),

$\therefore$ $\boxed{T_{BC} = 265.36 \text{ N}}$ ...Ans.

$\therefore$ $\boxed{T_{AB} = 169.64 \text{ N}}$ ...Ans.

Q.17 The 500 N crate is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension of 2500 N before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted. Refer Fig. Q.17.1.

[SPPU : May-12, Marks 6]

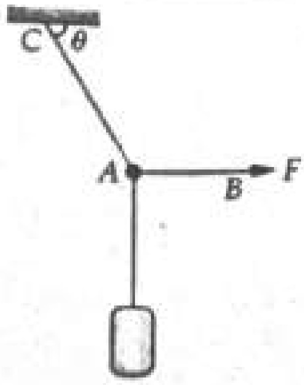


Fig. Q.17.1

Ans. : The F.B.D. of point A is shown in Fig. Q.17.1 (a).

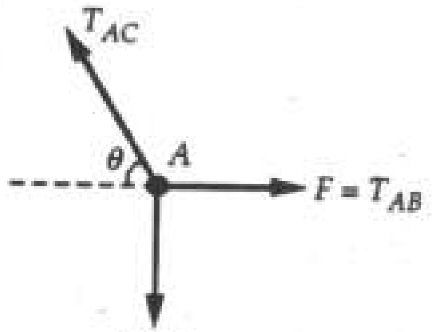


Fig. Q.17.1 (a)

Using Lami's theorem,

$$\frac{T_{AB}}{\sin(90+\theta)} = \frac{T_{AC}}{\sin 90} = \frac{500}{\sin(180-\theta)}$$

$$\therefore \quad \frac{T_{AB}}{\cos\theta} = \frac{500}{\sin\theta} \quad \text{and} \quad \frac{T_{AC}}{1} = \frac{500}{\sin\theta}$$

$$\therefore \quad T_{AB} = \frac{500}{\tan\theta} \quad \text{and} \quad T_{AC} = \frac{500}{\sin\theta}$$

For $\quad T_{AB} = 2500$ N,

$$2500 = \frac{500}{\tan\theta}$$

$$\therefore \quad \theta = 11.31°$$

and $\quad T_{AC} = \frac{500}{\sin 11.31} = 2549.5 \text{ N} > 2500 \text{ N}$

$$\therefore \quad \theta \neq 11.31°$$

For $\quad T_{AC} = 2500$ N, $\quad 2500 = \frac{500}{\sin\theta}$

$\therefore \quad \theta = 11.537°$

and $\quad T_{AB} = \dfrac{500}{\tan 11.537} = 2449.48 \text{ N} < 2500 \text{ N}$

$\therefore \quad \boxed{\theta = 11.537°}$ ...Ans.

Q.18 The motor at B winds up the cord attached to the 65 N crate with a constant speed as shown in Fig. Q.18.1. Determine the force in cord CD supporting the pulley and the angle θ for equilibrium. Neglect the size of pulley at C. **[SPPU : May-13, Marks 6]**

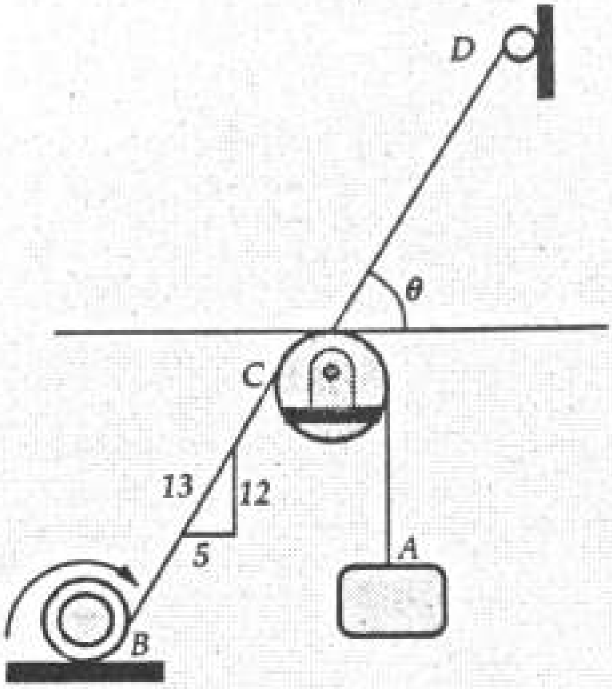


Fig. Q.18.1

Ans. : $T_{AC} = T_{BC} = 65$ N

The F.B.D. of pulley C is shown in Fig. Q.18.1 (a).

$$\tan \alpha = \frac{5}{12} \quad \therefore \alpha = 22.62°$$

$\sum F_x = 0$:

$$T_{CD} \cos\theta - 65 \times \sin 22.62 = 0$$

$$\therefore \quad T_{CD} \cos\theta = 25 \qquad \text{...(1)}$$

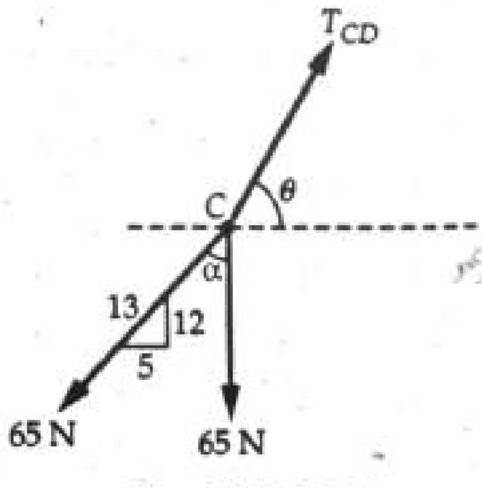


Fig. Q.18.1 (a)

$\sum F_y = 0$:

$$T_{CD} \sin\theta - 65 - 65 \times \cos 22.62 = 0$$

$$\therefore \quad T_{CD} \sin\theta = 125 \qquad ...(2)$$

Dividing equation (2) by (1),

$$\tan\theta = \frac{125}{25}$$

$$\therefore \quad \boxed{\theta = 78.69^\circ} \qquad ...Ans.$$

From equation (1),

$$T_{CD} = \frac{25}{\cos 78.69}$$

$$\therefore \quad \boxed{T_{CD} = 127.47 \text{ N}} \qquad ...Ans.$$

Q.19 Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder refer Fig. Q.19.1.

[SPPU : May-14, Marks 6]

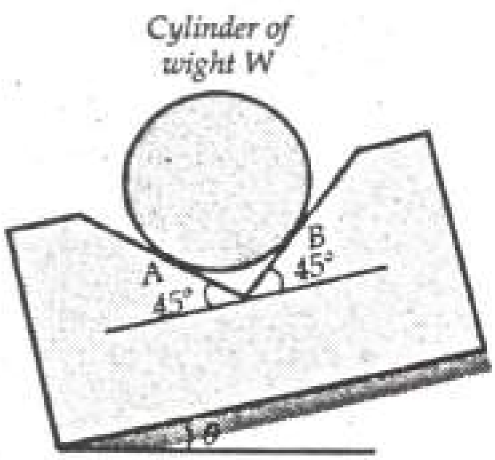


Fig. Q.19.1

Ans. : The FBD of cylinder is shown in Fig. Q.19.1 (a).

$$\sum F_x = 0$$

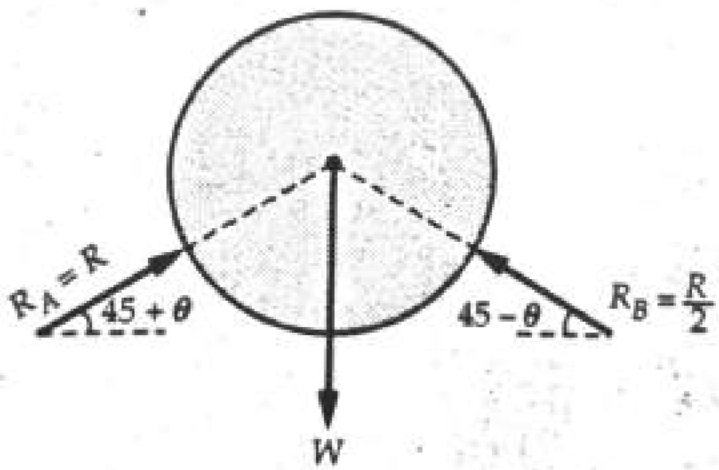


Fig. Q.19.1 (a)

$$R \cos (45 + \theta) - \frac{R}{2} \cos (45 - \theta) = 0$$

$$\cos (45 + \theta) = \frac{1}{2} \cos (45 - \theta)$$

$$\cos 45 \cos \theta - \sin 45 \sin \theta = \frac{1}{2} [\cos 45 \cos \theta + \sin 45 \sin \theta]$$

$$\frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2} \times \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{2} \times \frac{1}{\sqrt{2}} \sin \theta$$

$$\frac{1}{\sqrt{2}} \left(1 - \frac{1}{2}\right) \cos \theta = \frac{1}{\sqrt{2}} \left(1 + \frac{1}{2}\right) \sin \theta$$

$$\frac{1}{2} \cos \theta = \frac{3}{2} \sin \theta$$

$$\cos \theta = 3 \sin \theta$$

$$\therefore \quad \tan \theta = \frac{1}{3}$$

$$\therefore \quad \boxed{\theta = 18.435^\circ}$$ ...Ans.

Q.20 Three cables are joined at the junction C as shown in Fig. Q.20.1. Determine the tension in cable AC and BC caused by the weight of the 30 kg cylinder.

[SPPU : May-14, Marks 5, Dec.-16, 17, Marks 6]

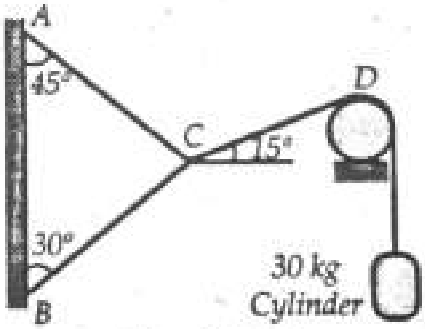


Fig. Q.20.1

Ans. : The tension in CD equals the weight of the cylinder. All forces are concurrent at C.

The FBD of C is shown in Fig. Q.20.1 (a).

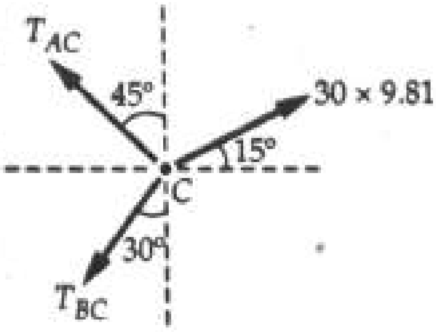


Fig. Q.20.1 (a)

Using Lami's theorem,

$$\frac{T_{AC}}{\sin 135} = \frac{T_{BC}}{\sin 120} = \frac{30 \times 9.81}{\sin 105}$$

$\therefore \quad \boxed{T_{AC} = 215.44 \text{ N}}$ ...Ans.

$\therefore \quad \boxed{T_{BC} = 263.86 \text{ N}}$ ...Ans.

Q.21 A 300 mm wooden beam weighing 540 N is supported by a pin and bracket at A and by cable BC. Find the reaction at A and tension in cable BC. Refer Fig. Q.21.1.

[SPPU : Dec.-13, May-19, Marks 6]

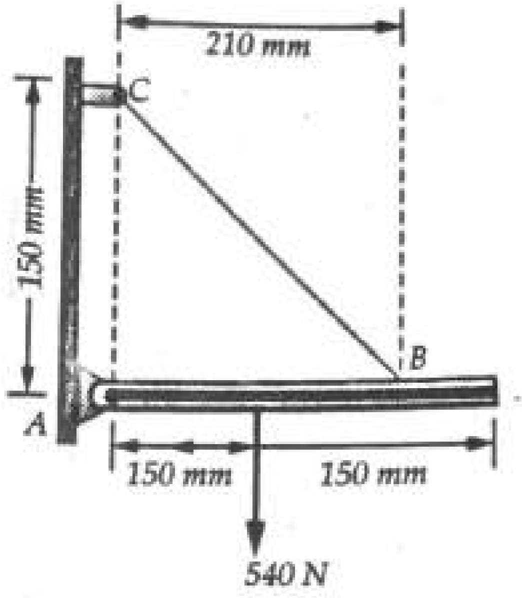


Fig. Q.21.1

Ans. : The F.B.D. of beam is shown in Fig. Q.21.1 (a).

The angle made by cable with horizontal is

$$\alpha = \tan^{-1}\left(\frac{150}{210}\right)$$

$$= 35.54°$$

$\sum M_A = 0$:

$$-(540)(150) + (T \sin 35.54)(210) = 0$$

$\therefore \quad \boxed{T = 663.57 \text{ N}}$ ...Ans.

$\sum F_x = 0 : \quad A_x - T\cos 35.54 = 0$

$\therefore \quad A_x = 540 \text{ N}$

$\sum F_y = 0$:

$$A_y - 540 + T \sin 35.54 = 0$$

$\therefore \quad A_y = 154.3 \text{ N}$

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{540^2 + 154.3^2}$$

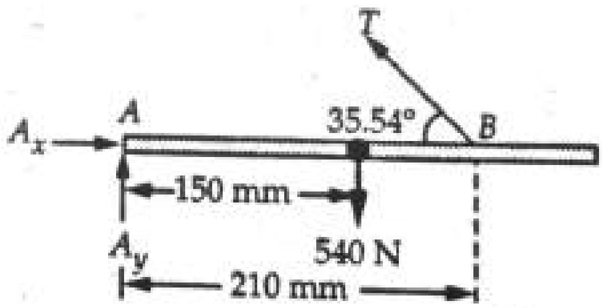


Fig. Q.21.1 (a)

$\therefore \quad \boxed{R_A = 561.6 \text{ N}}$ ...Ans.

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{154.3}{540}\right)$$

$\therefore$ $$\boxed{\theta_A = 15.95^\circ}$$ ...Ans.

Q.22 The uniform 18 kg bar OA is held in position as shown in Fig. Q.22.1 by the smooth pin at O and the cable AB. Determine the tension in the cable and the reaction at O.

[SPPU : May-14, Dec.-17, Marks 6]

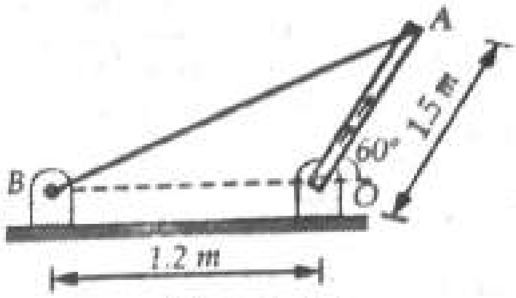


Fig. Q.22.1

Ans. : The angle made by AB with horizontal is

$$\alpha = \tan^{-1}\left[\frac{1.5\sin 60}{1.2+1.5\cos 60}\right] = 33.67^\circ$$

The FBD of bar OA is shown in Fig. Q.22.1 (a)

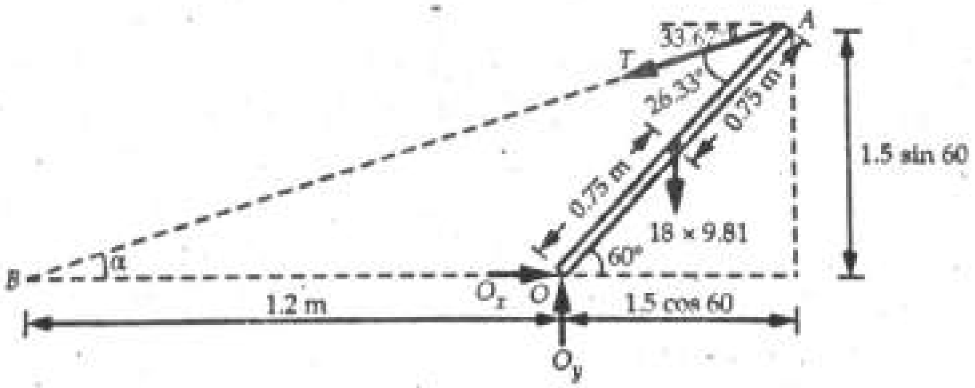


Fig. Q.22.1 (a)

$$\sum M_O = 0 :$$

$$-18 \times 9.81 \times 0.75\cos 60 + T\sin 26.33 \times 1.5 = 0$$

$\therefore$ $$\boxed{T = 99.53 \text{ N}}$$ ... Ans.

$$\sum F_x = 0 :$$

$$O_x - T\cos 33.67 = 0$$

$$\therefore \quad O_x = 82.83 \text{ N}$$

$$\sum F_y = 0 :$$

$$O_y - 18 \times 9.81 - T \sin 33.67 = 0$$

$$\therefore \quad O_y = 231.76 \text{ N}$$

$$\therefore \quad R_O = \sqrt{O_x^2 + O_y^2} = \sqrt{82.83^2 + 231.76^2}$$

$$\therefore \quad \boxed{R_O = 246.116 \text{ N}} \quad \text{...Ans.}$$

$$\theta_O = \tan^{-1}\left(\frac{O_x}{O_y}\right) = \tan^{-1}\left(\frac{231.76}{82.83}\right)$$

$$\therefore \quad \boxed{\theta_O = 70.33^\circ} \quad \text{...Ans.}$$

Q.23 A force of 150 N acts on the end of beam ABD as shown in Fig. Q.23.1. Determine the magnitude of tension in cable BC to maintain equilibrium. **[SPPU : Dec.-14, Marks 6]**

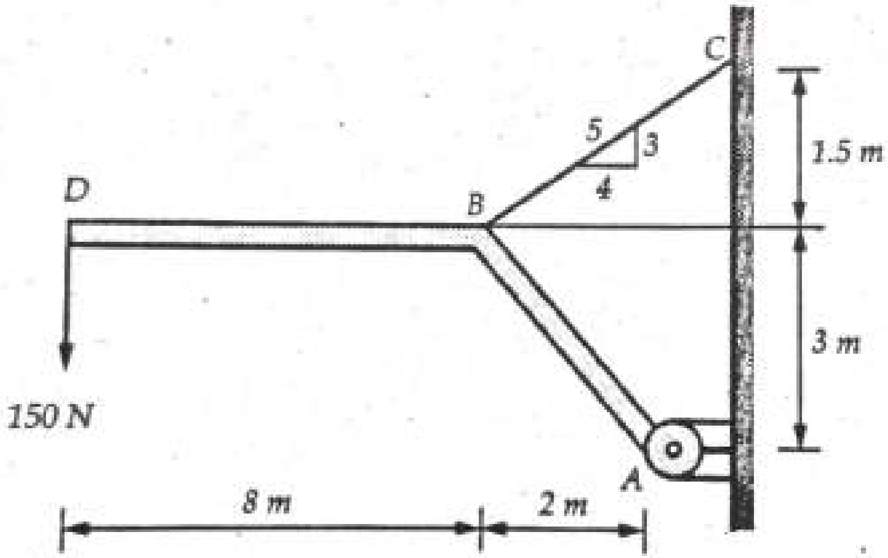


Fig. Q.23.1

Ans. : The FBD of beam is shown in Fig. Q.23.1 (a).

$$\tan \alpha = \frac{3}{4} \qquad \therefore \alpha = 36.87^\circ$$

$$\sum M_A = 0 :$$

$$(150)(10) - (T \times \cos 36.87)(3) - (T \times \sin 36.87)(2) = 0$$

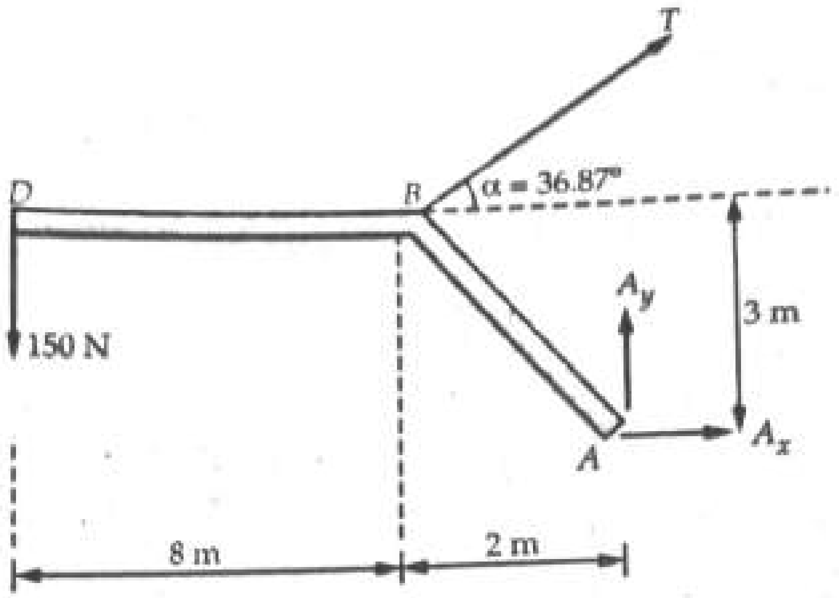


Fig. Q.23.1 (a)

$$1500 = \frac{18T}{5}$$

$$\therefore \quad \boxed{T = 416.67 \text{ N}} \quad \text{...Ans.}$$

Q.24 Determine the reactions at roller A and pin B for equilibrium of the memeber ACB as shown in Fig. Q.24.1.

[SPPU : Dec.-15, Marks 5]

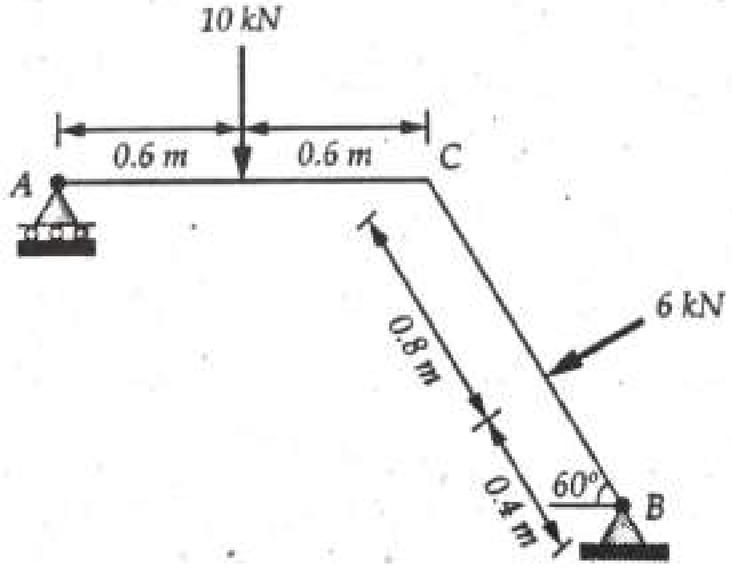


Fig. Q.24.1

Ans. : The FBD of member ACB is shown in Fig. Q.24.1 (a).

$$\sum M_B = 0 : \ (6)(0.4) + (10)(1.2 \cos 60 + 0.6)$$

$$-R_A(1.2 \cos 60 + 1.2) = 0$$

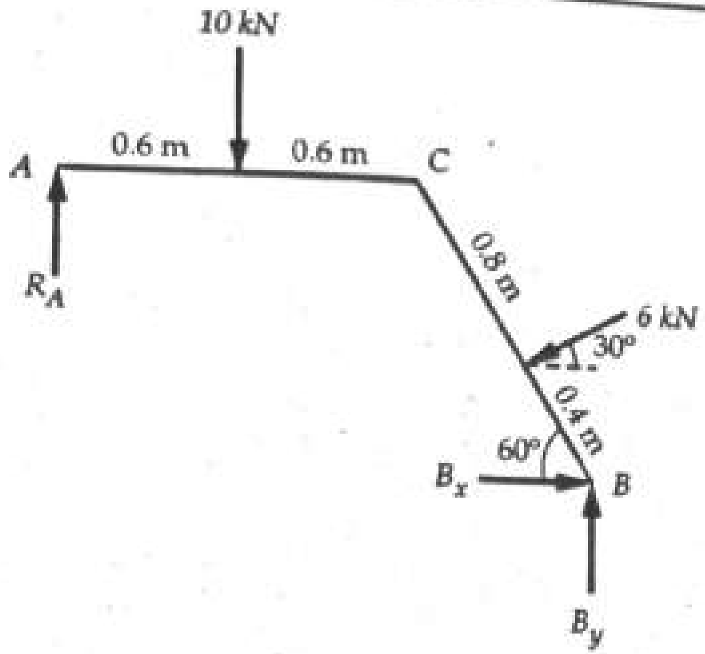


Fig. Q.24.1 (a)

$\therefore \quad \boxed{R_A = 8 \text{ kN} \uparrow}$ ...Ans.

$$\sum F_x = 0: \quad B_x - 6\cos 30 = 0$$

$$B_x = 5.196 \text{ kN}$$

$$\sum F_y = 0: \quad R_A - 10 - 6\sin 30 + B_y = 0$$

$$B_y = 5 \text{ kN}$$

$$R_B = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{5.196^2 + 5^2}$$

$\therefore \quad \boxed{R_B = 7.21 \text{ kN}}$ ...Ans.

$$\theta = \tan^{-1}\left(\frac{|B_y|}{|B_x|}\right) = \tan^{-1}\left(\frac{5}{5.196}\right)$$

$\therefore \quad \boxed{\theta = 43.9^\circ \angle}$ ...Ans.

Q.25 A joist of length 4 m and weighing 200 N is raised by pulling a rope as shown in Fig. Q.25.1. Determine the tension T induced in the rope and reaction at end A of joist.

[SPPU : Dec.-09, Marks 6]

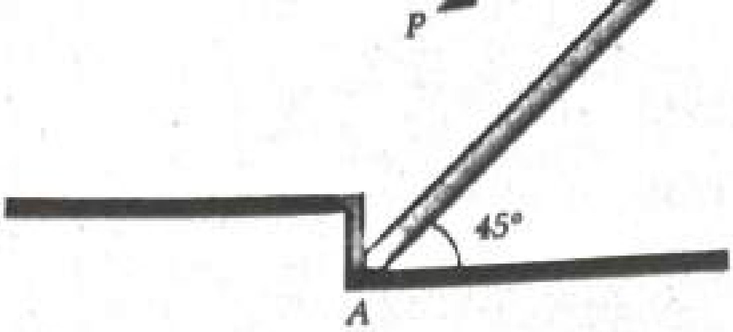


Fig. Q.25.1

Ans. : F.B.D. of joist is shown in Fig. Q.25.1 (a).

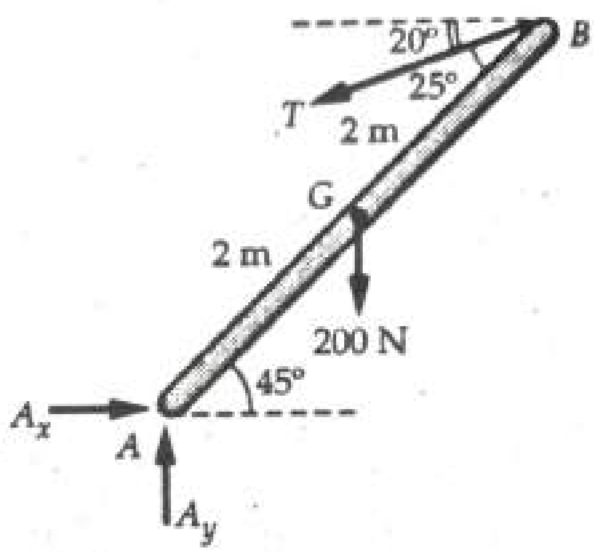


Fig. Q.25.1 (a)

$\sum M_A = 0$

$(T \sin 25)(4) - (200)(2 \cos 45) = 0$

$\therefore \quad \boxed{T = 167.32 \text{ N}}$ ... Ans.

$\sum F_x = 0:$

$A_x - T \cos 20 = 0$

$A_x = 157.23 \text{ N}$

$\sum F_y = 0:$

$A_y - 200 - T \sin 20 = 0$

$\therefore \quad A_y = 257.23 \text{ N}$

$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{157.23^2 + 257.23^2}$

$\therefore \quad \boxed{R_A = 301.48 \text{ N}}$ ...Ans.

$\theta = \tan^{-1}\left(\frac{Ay}{Ax}\right) = \tan^{-1}\left(\frac{257.23}{157.23}\right)$

$\therefore \quad \boxed{\theta = 58.56^\circ}$ ...Ans.

Q.26 The L-shaped member ACB is supported by a pin support at C and by an inextensible cord attached at A and B and passing over a frictionless pulley at D. Determine the tension in the cord and the reaction at C. Refer Fig. Q.26.1. [SPPU : May-11, Marks 7]

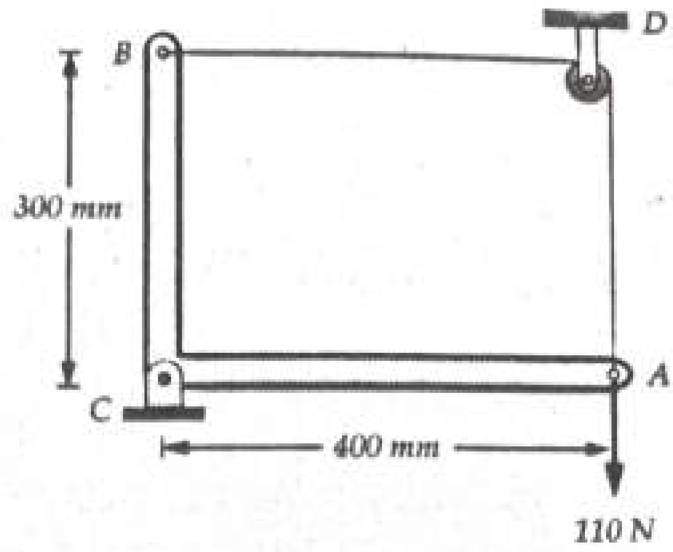


Fig. Q.26.1

Ans. : The F.B.D. of member *ACB* is shown in Fig. Q.26.1 (a).

$\sum M_C = 0$:

$-(T)(300)+(T)(400)-(110)(400) = 0$

$\therefore \quad \boxed{T = 440 \text{ N}}$ ...**Ans.**

$\sum F_x = 0$:

$F_x + T = 0$

$\therefore \quad C_x = -440 \text{ N}$

$\sum F_y = 0$:

$C_y + T - 110 = 0$

$\therefore \quad C_y = -330 \text{ N}$

$R_C = \sqrt{C_x^2 + C_y^2}$

$= \sqrt{440^2 + 330^2}$

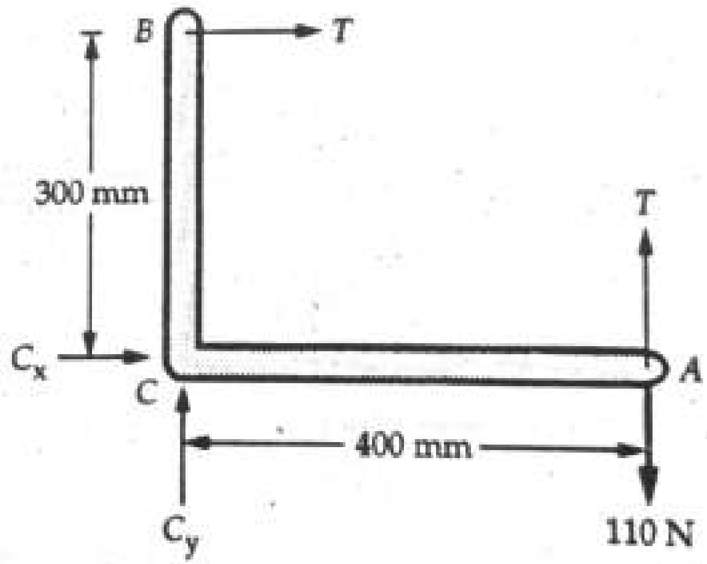


Fig. Q.26.1 (a)

$\therefore \quad \boxed{R_C = 550 \text{ N}}$ ...**Ans.**

$$\theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{330}{440}\right)$$

$\therefore$ $\boxed{\theta_C = 36.87^\circ}$ ...Ans.

Q.27 Determine the reaction at A and B for the member ACB loaded and supported as shown in Fig. Q.27.1 when $\alpha = 30^\circ$.

[SPPU : May-12, Marks 6]

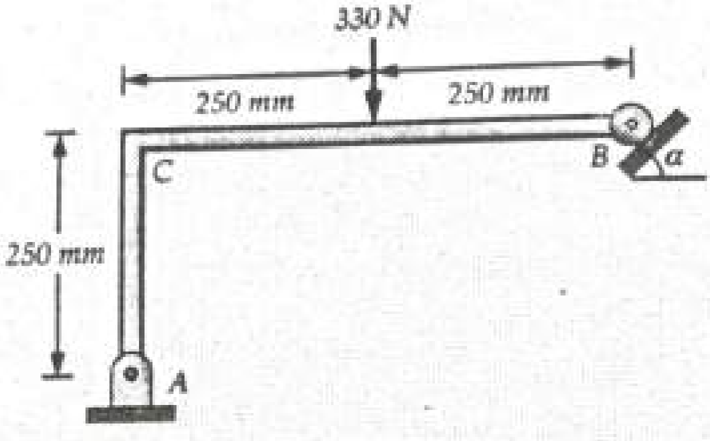


Fig. Q.27.1

Ans. : The F.B.D. of member *ACB* is shown in Fig. Q.27.1 (a).

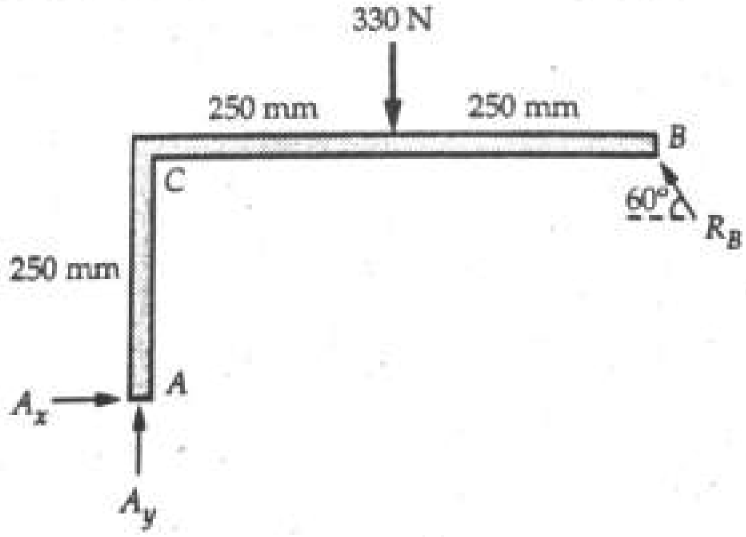


Fig. Q.27.1 (a)

$$\sum M_A = 0:$$

$$(R_B \cos 60)(250) + (R_B \sin 60)(500) - (330)(250) = 0$$

$\therefore$ $\boxed{R_B = 147.85 \text{ N}}$ ...Ans.

$$\sum F_x = 0 :$$

$$A_x - R_B \cos 60 = 0$$

$$\therefore \quad A_x = 73.925 \text{ N}$$

$$\sum F_y = 0 :$$

$$A_y - 330 + R_B \sin 60 = 0$$

$$\therefore \quad A_y = 201.96 \text{ N}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{73.925^2 + 201.96^2}$$

$$\therefore \quad \boxed{R_A = 215.06 \text{ N}}$$

$$\theta_A = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right) = \tan^{-1}\left(\frac{201.96}{73.925}\right)$$

$$\therefore \quad \boxed{\theta_A = 69.9 \measuredangle}$$ ...Ans.

Q.28 Three loads are applied as shown in Fig. Q.28.1 to a light beam supported by cables attached at B and C. Neglecting the weight of the beam, determine the range of values of Q for which neither cable becomes slack when P = 0. **[SPPU : Dec.-12, Marks 7]**

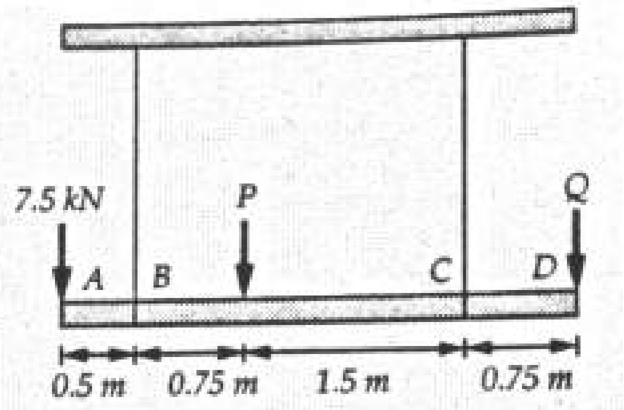


Fig. Q.28.1

Ans. : The F.B.D of beam is shown in Fig. Q.28.1 (a).

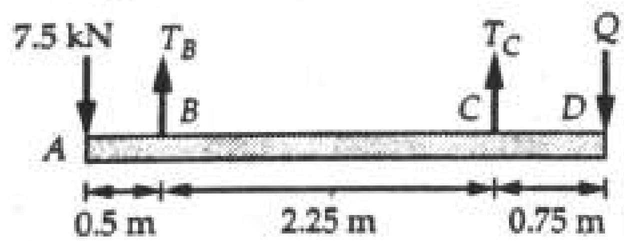


Fig. Q.28.1 (a)

For maximum Q, $T_B \to 0$. Using $\sum M_C = 0$,

$$(7.5)(2.75) - (Q)(0.75) = 0$$

$\therefore \qquad Q = 27.5 \text{ kN}$

For minimum Q, $T_c \to 0$. Using $\sum M_B = 0$,

$(7.5)(0.5)-(Q)(3) = 0$

$\therefore \qquad Q = 1.25 \text{ kN}$

$\therefore$ **Range of Q is 1.25 kN ≤ 27.5 kN** ...**Ans.**

Q.29 The boom supports the two vertical loads P_1 = 800 N and P_2 = 350 N as shown in Fig. Q.29.1. Determine the tension in cable BC and component of reaction at A.

[SPPU : May-13, Marks 6]

Ans. : The F.B.D. of beam is shown in Fig. Q.29.1 (a).

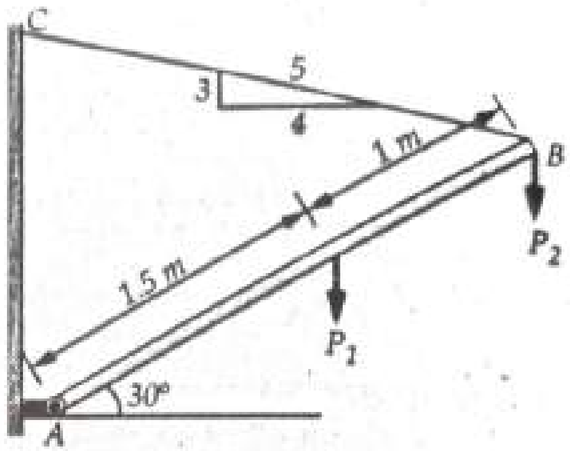


Fig. Q.29.1

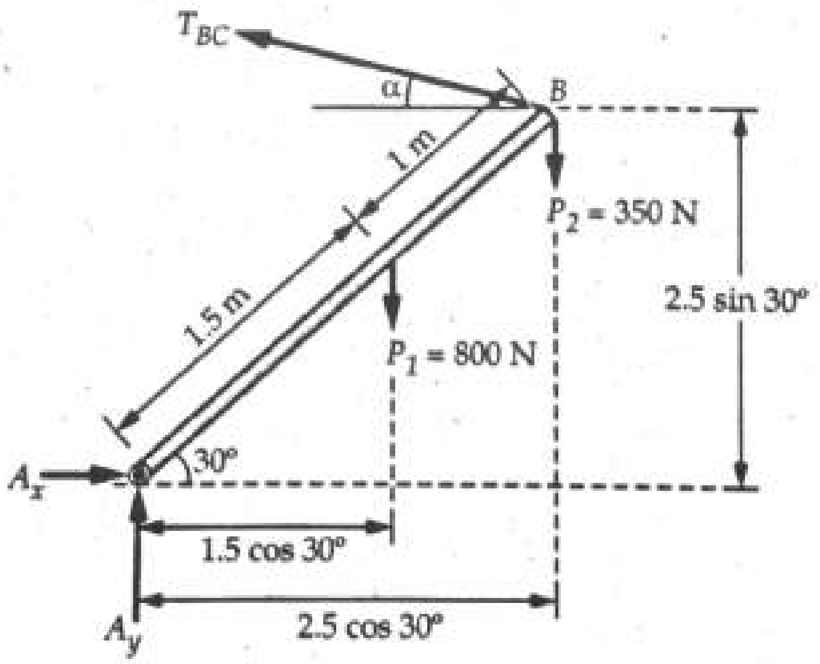


Fig. Q.29.1 (a)

$$\tan \alpha = \frac{3}{4} \qquad \therefore \alpha = 36.87°$$

$\sum M_A = 0$:

$-(800)(1.5 \cos 30) - (350)(2.5 \cos 30)$

$+ (T_{BC} \times \cos 36.87)(2.5 \sin 30)$

$+ (T_{BC} \times \sin 36.87)(2.5 \cos 30) = 0$

$\therefore \quad \boxed{T_{BC} = 781.63 \text{ N}}$ ...Ans.

$\sum F_x = 0$:

$A_x - T_{BC} \times \cos 36.87 = 0$

$\therefore \quad A_x = 781.63 \times \cos 36.87$

$\therefore \quad \boxed{A_x = 625.3 \text{ N} \rightarrow}$ ...Ans.

$\sum F_y = 0$:

$A_y - 800 - 350 + T_{BC} \times \sin 36.87 = 0$

$\therefore \quad \boxed{A_y = 774.82 \text{ N} \uparrow}$ ...Ans.

Q.30 The wall crane is supported by smooth collar at B and pin at A as shown in Fig. Q.30.1. If the vertical component of reaction at A is 10 kN, determine the force P, normal reaction at B and tangential component of reaction at A. [SPPU : May-17, Marks 6]

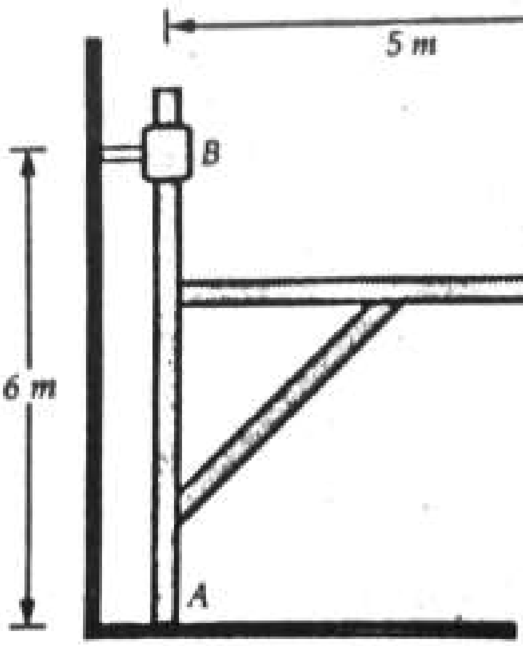


Fig. Q.30.1

Ans. : The F.B.D of wall crane is shown in Fig. Q.30.1 (a).

$\sum F_y = 0 : \ 10 - P = 0$

$\therefore \quad \boxed{P = 10 \text{ kN}} \quad$ ...Ans.

$\sum M_A = 0 :$

$R_B \times 6 - P \times 5 = 0$

$\therefore \quad \boxed{R_B = 8.33 \text{ kN} \leftarrow} \quad$ ...Ans.

$\sum F_x = 0 : \ A_x - R_B = 0$

$\therefore \quad \boxed{A_x = 8.33 \text{ kN} \rightarrow} \quad$ ...Ans.

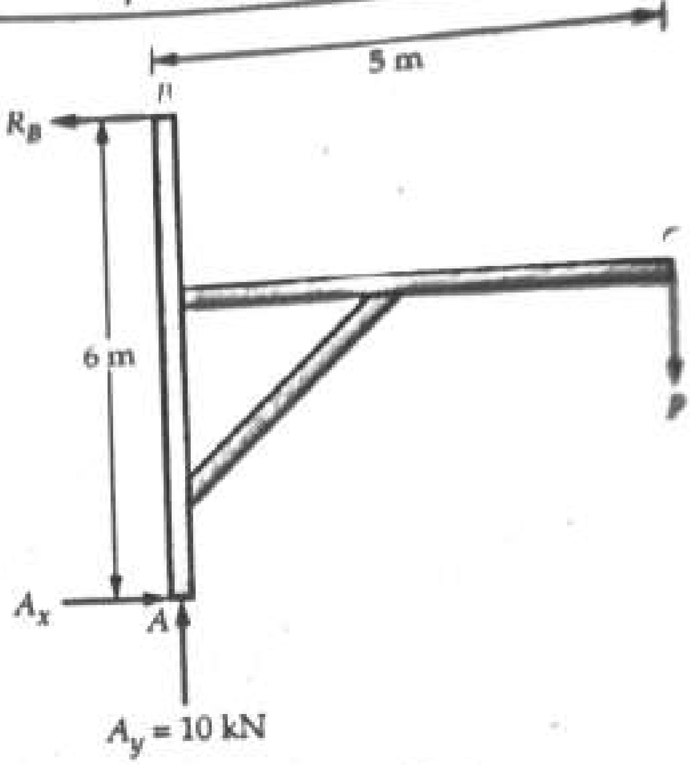


Fig. Q.30.1 (a)

Q.31 A cylinder of weight 1000 N is rest on the stair as shown in Fig. Q.31.1. Determine the minimum magnitude of force P to raise the cylinder over the step.

[SPPU : Dec.-14, Marks 6]

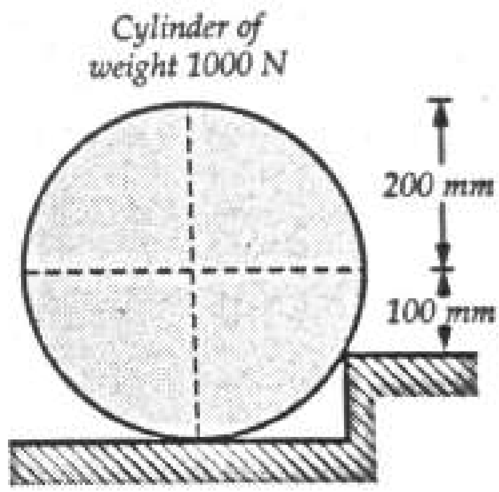


Fig. Q.31.1

Ans. : To raise the cylinder, reaction at D is zero.

From Fig. Q.31.1 (a),

$$AE = \sqrt{200^2 - 100^2} = 173.2 \text{ mm}$$

$\sum M_A = 0 :$

$1000 \times 173.2 - P \sin\alpha \, (100 + 200 \sin \alpha)$

$- P\cos\alpha(173.2 + 200\cos\alpha) = 0$

$P[100 \sin\alpha + 200 \sin^2 \alpha + 173.2 \cos\alpha + 200 \cos^2 \alpha] = 1000 \times 173.2$

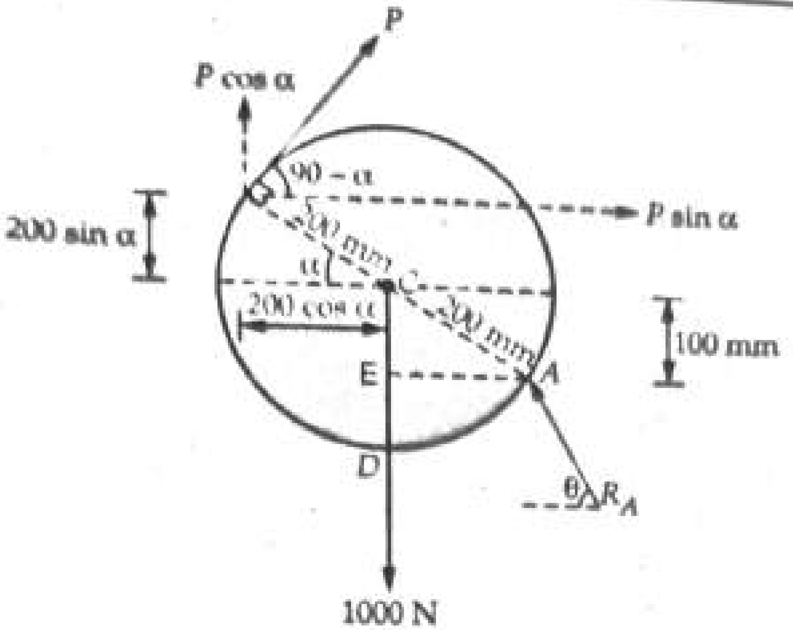


Fig. Q.31.1 (a)

$$P[100 \sin\alpha + 173.2 \cos\alpha + 200] = 1000 \times 173.2$$

$$\therefore \qquad P = \frac{1000 \times 173.2}{100 \sin\alpha + 173.2 \cos\alpha + 200} \qquad \text{...(1)}$$

For minimum P, $\frac{dP}{d\alpha} = 0$

$$\therefore -\frac{1000 \times 173.2 \times [100 \cos\alpha - 173.2 \sin\alpha]}{[100 \sin\alpha + 173.2 \cos\alpha + 200)^2} = 0$$

$$100 \cos\alpha - 173.2 \sin\alpha = 0$$

$$\tan\alpha = \frac{100}{173.2}$$

$$\therefore \qquad \alpha = 30°$$

Substitute in equation (1)

$$P = \frac{1000 \times 173.2}{100 \sin 30 + 173.2 \cos 30 + 200}$$

$$\boxed{P = 433 \text{ N}} \qquad \text{...Ans.}$$

Q.32 The boom is intended to support two vertical loads F_1 and F_2 as shown in Fig. Q.32.1. If the cable CB can sustain a maximum load of 1500 N before it fails. Determine the critical loads if $F_1 = 2F_2$. Also determine the reaction at pin support A.

[SPPU : May-15,16, Marks 6]

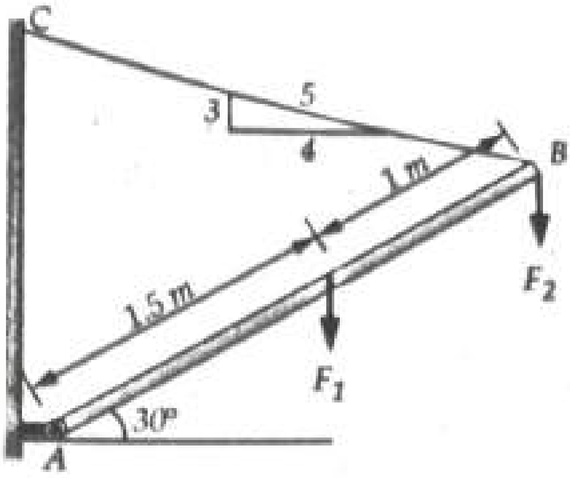


Fig. Q.32.1

Ans. : The FBD of boom is shown in Fig. Q.32.1 (a)

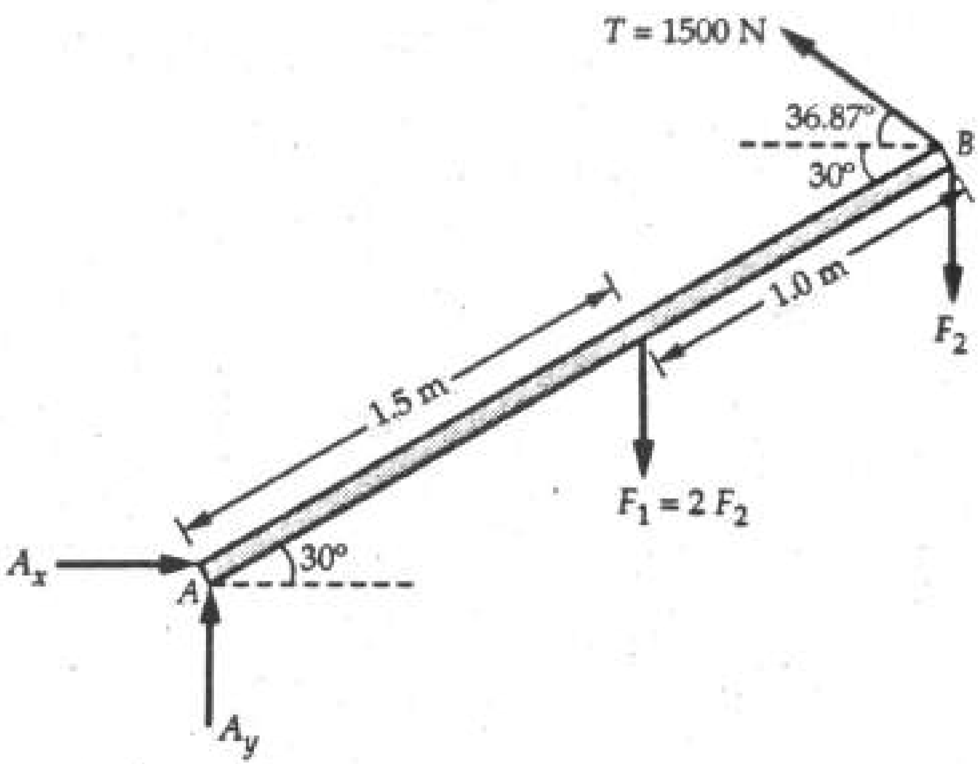


Fig. Q.32.1 (a)

$$\tan \alpha = \frac{3}{4}$$

$$\therefore \quad \alpha = 36.87^\circ$$

$\Sigma M_A = 0$:

$$-2F_2 \times 1.5 \cos 30 - F_2 \times 2.5 \cos 30 + 1500 \sin 66.87 \times 2.5 = 0$$

$$\therefore \quad \boxed{F_2 = 724 \text{ N}} \quad \text{...Ans.}$$

$$F_1 = 2F_2 = 2 \times 724$$

$$\therefore \quad \boxed{F_1 = 1448 \text{ N}} \quad \text{...Ans.}$$

$$\Sigma F_x = 0 : \quad A_x - 1500 \cos 36.87 = 0$$

$$\therefore \quad A_x = 1200 \text{ N}$$

$$\Sigma F_y = 0 : \quad A_y - 1448 - 724 + 1500 \sin 36.87 = 0$$

$$\therefore \quad A_y = 1272 \text{ N}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{1200^2 + 1272^2}$$

$$\therefore \quad \boxed{R_A = 1748.71 \text{ N}} \quad \text{...Ans.}$$

$$\theta = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) = \tan^{-1}\left(\frac{1272}{1200}\right)$$

$$\therefore \quad \boxed{\theta = 46.67^\circ \angle} \quad \text{...Ans.}$$

END...

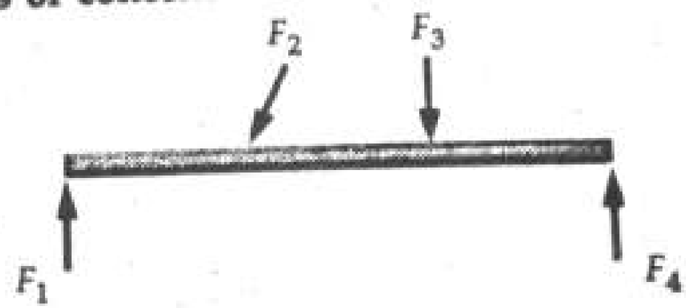
7 Beams

Important Points to Remember

- Beams are structural members which have their length large compared to their cross sectional dimensions.

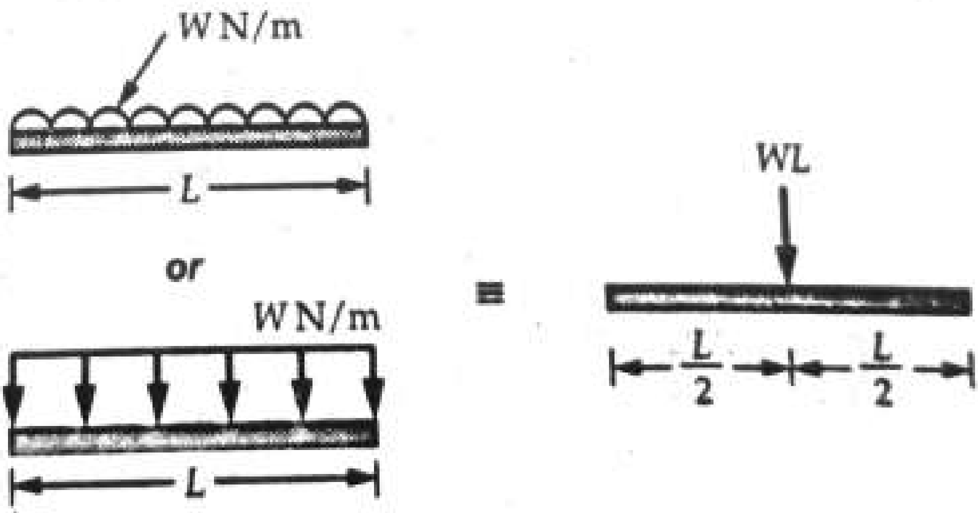
Types of Loads on Beams

i) Point loads or concentrated loads

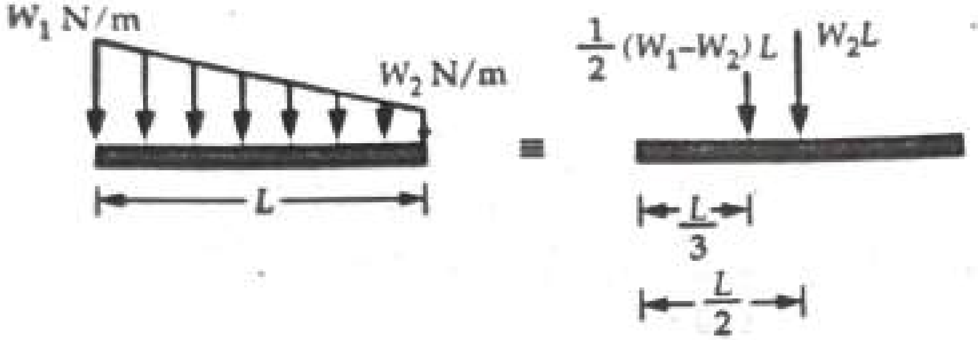


ii) Couple moments

iii) Uniformly distributed load (u.d.l)



iv) Uniformly varying load (u.v.l)



- While drawing F.B.D. of beams, all distributed loads must be converted to corresponding point load.

General Method for Solving Beams

1. Draw free body diagram of the beam. Reduce all distributed loads to point loads.
2. Use equations of static equilibrium.

$$\sum F_x = 0$$
$$\sum F_y = 0 \quad \text{and}$$
$$\sum M = 0$$

to solve for the unknowns.

3. For compound beams draw separate F.B.D. of each beam showing equal and opposite reactions at common supports. Use equations of static equilibrium for each F.B.D. to solve for unknowns. Always start with that F.B.D. which has least number of unknowns.

Q.1 Determine reaction at A and B for the beam loaded and supported as shown in Fig. Q.1.1. **[SPPU : May-16, Marks 5]**

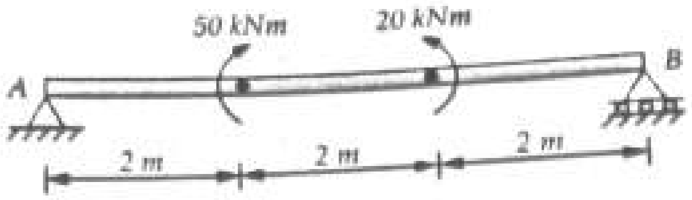


Fig. Q.1.1

Ans. : The FBD of beam is shown in Fig. Q.1.1 (a).

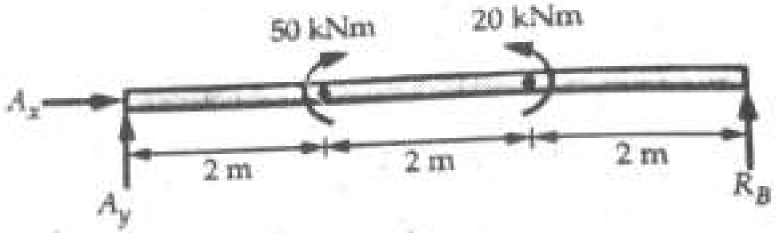


Fig. Q.1.1 (a)

$$\sum F_x = 0 : \; A_x = 0$$

$$\sum M_A = 0 : \; -50 + 20 + R_B \times 6 = 0$$

$$\therefore \quad \boxed{R_B = 5 \text{ kN} \uparrow} \qquad \text{...Ans.}$$

$$\sum F_y = 0 : \; A_y + R_B = 0$$

$$\therefore \quad A_y = -5 \text{ kN}$$

$$\therefore \quad \boxed{R_A = 5 \text{ kN} \downarrow} \qquad \text{...Ans.}$$

Q.2 A simply supported beam AB of span 6 m is loaded and supported as shown in Fig. Q.2.1. Find the reactions at support A and B. **[SPPU : Dec.-15, Marks 6]**

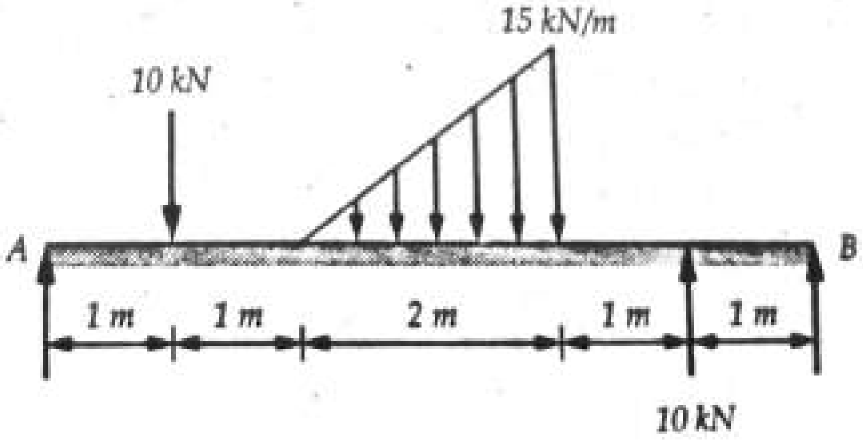


Fig. Q.2.1

Ans. : The FBD of beam is shown in Fig. Q.2.1 (a).

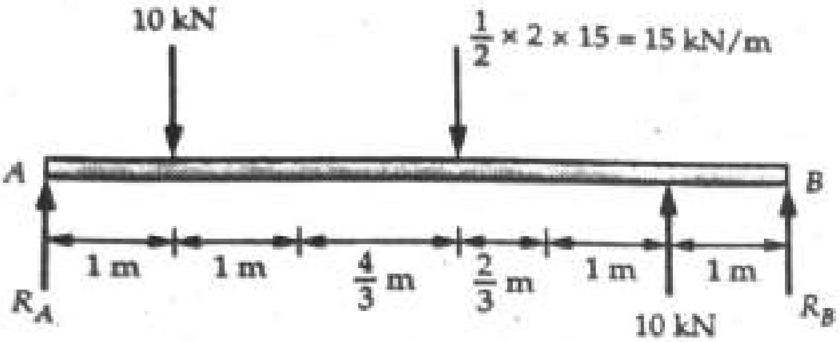


Fig. Q.2.1 (a)

$$\sum M_A = 0 : -(10)(1)-(15)\left(2+\frac{4}{3}\right)+(10)(5)+(R_B)(6) = 0$$

$\therefore$ $\boxed{R_B = 1.667 \text{ kN} \uparrow}$ ...Ans.

$$\sum F_y = 0 : \; R_A - 10 - 15 + 10 + R_B = 0$$

$\therefore$ $\boxed{R_A = 13.333 \text{ kN} \uparrow}$ ...Ans.

Q.3 Determine the support reaction for the beam loaded and supported as shown in Fig. Q.3.1. **[SPPU : Dec.-16, Marks 5]**

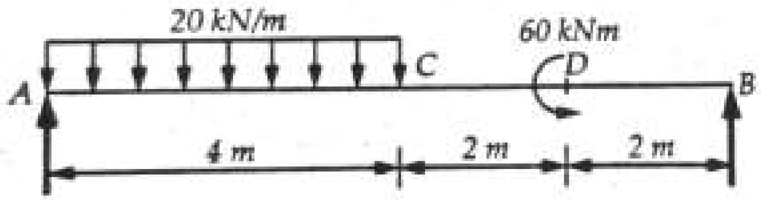


Fig. Q.3.1

Ans. : The FBD of beam is shown in Fig. Q.3.1 (a).

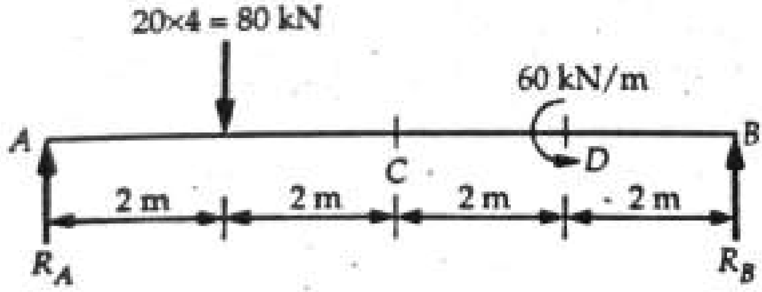


Fig. Q.3.1 (a)

$$\sum M_A = 0 : -80 \times 2 + 60 + R_B \times 8 = 0$$

$\therefore$ $\boxed{R_B = 12.5 \text{ kN} \uparrow}$ ...Ans.

$$\sum F_y = 0 : R_A + R_B - 80 = 0$$

$\therefore$ $\boxed{R_A = 67.5 \text{ kN} \uparrow}$ ...Ans.

Q.4 Determine the component of reaction at hinge A and tension in the cable BC as shown in Fig. Q.4.1. [SPPU : Dec.-16, Marks 5]

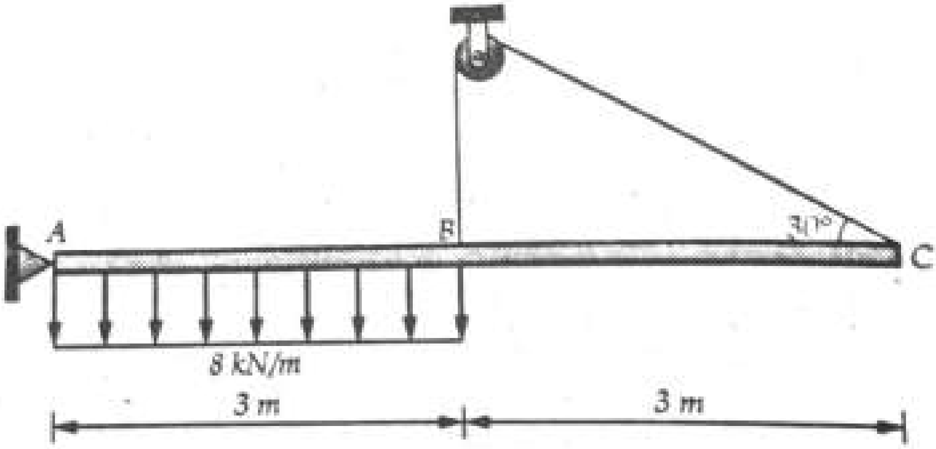


Fig. Q.4.1

Ans. : The FBD of beam is shown in Fig. Q.4.1 (a).

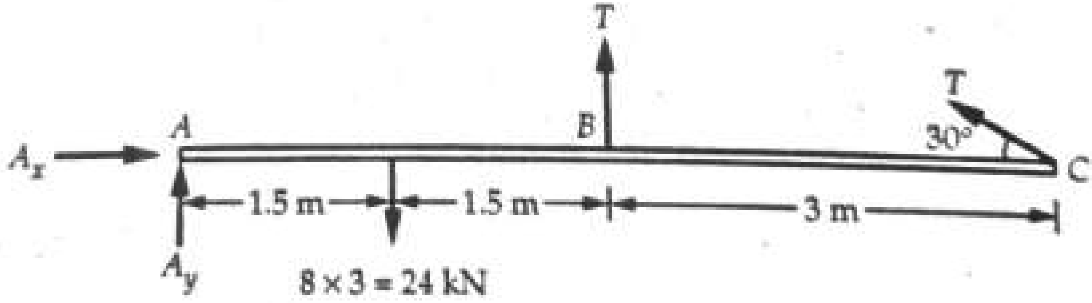


Fig. Q.4.1 (a)

$$\Sigma M_A = 0 : -24 \times 1.5 + T \times 3 + T \sin 30 \times 6 = 0$$

$\therefore$ $\boxed{T = 6 \text{ kN}}$ ...Ans.

$$\Sigma F_x = 0 \quad A_x - T \cos 30 = 0$$

$$\therefore \quad \boxed{A_x = 5.196 \text{ kN} \rightarrow} \quad \text{...Ans.}$$

$$\Sigma F_y = 0 : A_y - 24 + T + T\sin 30 = 0$$

$$\therefore \quad \boxed{A_y = 15 \text{ kN} \uparrow} \quad \text{...Ans.}$$

Q.5 Determine reaction at A and B for the beam loaded and supported as shown in Fig. Q.5.1. Moments are act at point C, D and E. **[SPPU : May-17, Marks 5]**

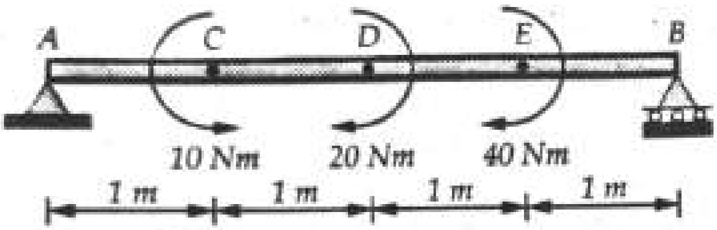


Fig. Q.5.1

Ans. : The FBD of beam is shown in Fig. Q.5.1 (a).

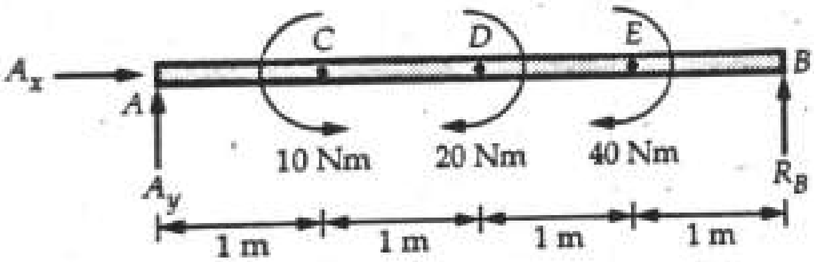


Fig. Q.5.1 (a)

$$\sum F_x = 0 : A_x = 0$$

$$\sum M_A = 10 - 20 - 40 + R_B \times 4 = 0$$

$$\therefore \quad \boxed{R_B = 12.5 \text{ N} \uparrow} \quad \text{...Ans.}$$

$$\sum F_y = 0 : A_y + R_B = 0$$

$$A_y = -12.5 \text{ N}$$

$$\therefore \quad \boxed{R_A = 12.5 \text{ N} \downarrow} \quad \text{...Ans.}$$

Q.6 Determine the component of reaction at hinge A and tension in the cable BC as shown in Fig. Q.6.1. [SPPU : May-17, Marks 5]

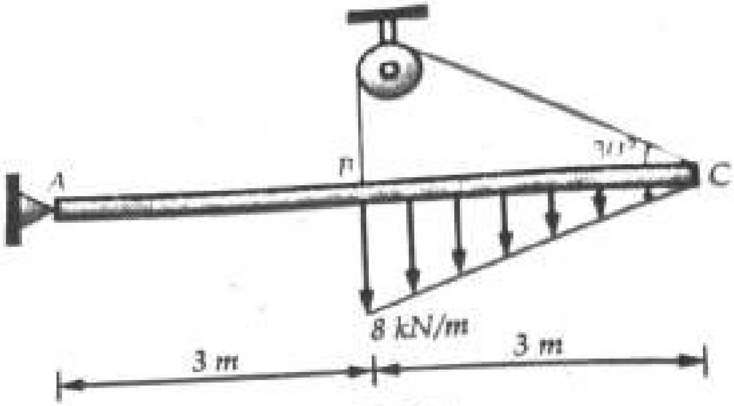


Fig. Q.6.1

Ans. : The FBD of beam is shown in Fig. Q.6.1 (a).

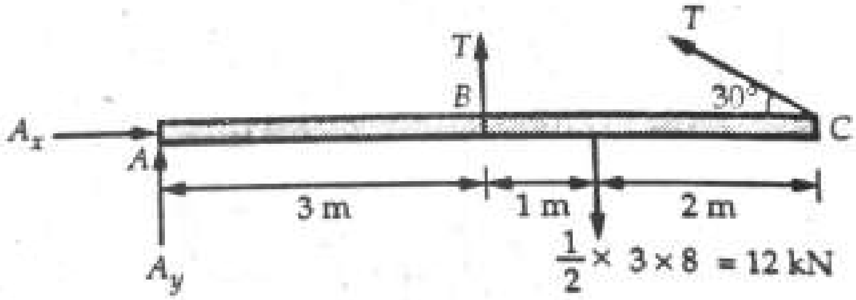


Fig. Q.6.1 (a)

$$\sum M_A = 0 \; : \; T \times 3 + T \sin 30 \times 6 - 12 \times 4 = 0$$

$\therefore$ $\boxed{T = 8 \text{ kN}}$ ...Ans

$$\sum F_x = 0 \; : \; A_x - T \cos 30 = 0$$

$\therefore$ $\boxed{A_x = 6.93 \text{ kN} \rightarrow}$ ...Ans

$$\sum F_y = 0 \; : \; A_y + T + T \sin 30 - 12 = 0$$

$\therefore$ $\boxed{A_y = 0}$ ...Ans

Q.7 A beam supports a load varying uniformly from an intensity of w_1 kN/m at left end to w_2 kN/m at the right end as shown in Fig. Q.7.1. If the reactions R_L = 6 kN and R_R = 12 kN, determine the intensity of loading w_1 and w_2 : [SPPU : May-15, Marks 5]

Scanned by CamScanner

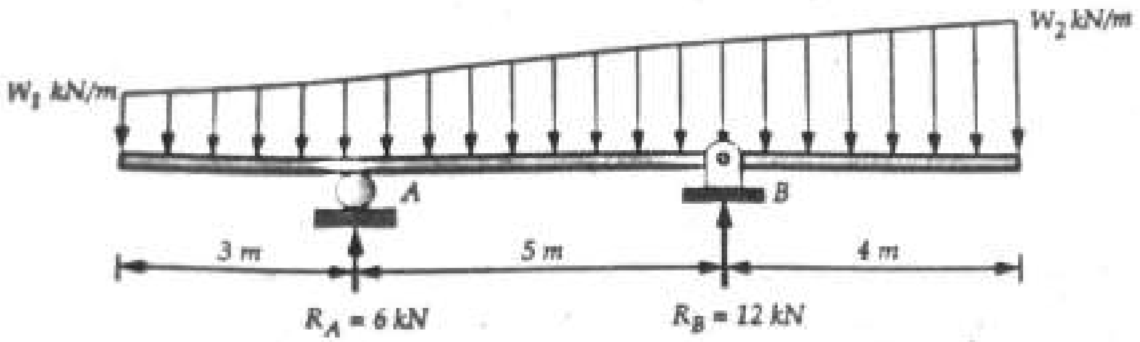


Fig. Q.7.1

Ans. : The FBD of beam is shown in Fig. Q.7.1 (a).

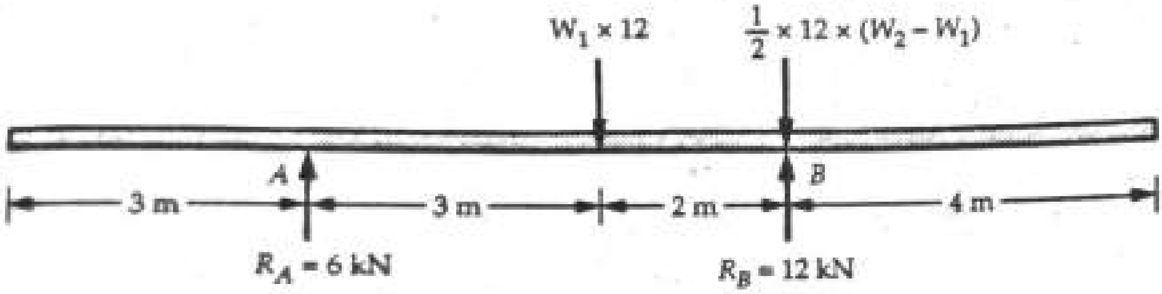


Fig. Q.7.1 (a)

$$\Sigma M_B = 0 : -6 \times 5 + W_1 \times 12 \times 2 = 0$$

$\therefore$ $\boxed{W_1 = 1.25 \text{ kN/m}}$ ...Ans.

$$\Sigma F_y = 0 : 6 - W_1 \times 12 + 12 - \frac{1}{2} \times 12 \, (W_2 - W_1) = 0$$

$$6 - 1.25 \times 12 + 12 - 6 \, (W_2 - 1.25) = 0$$

$\therefore$ $\boxed{W_2 = 1.75 \text{ kN/m}}$ ...Ans.

Q.8 A simply supported beam loaded and supported is as shown in Fig. Q.8.1. If the reactions at supports are equal in magnitude, determine the overhang a. [SPPU : May-16, Marks 5]

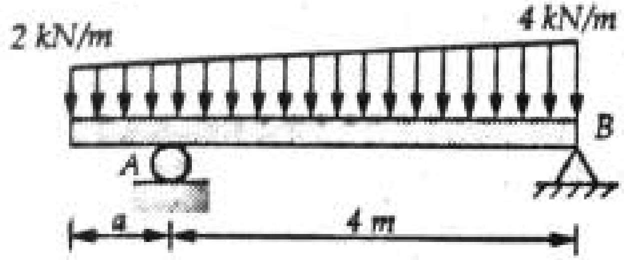


Fig. Q.8.1

Ans. : The FBD of beam is shown in Fig. Q.8.1 (a).

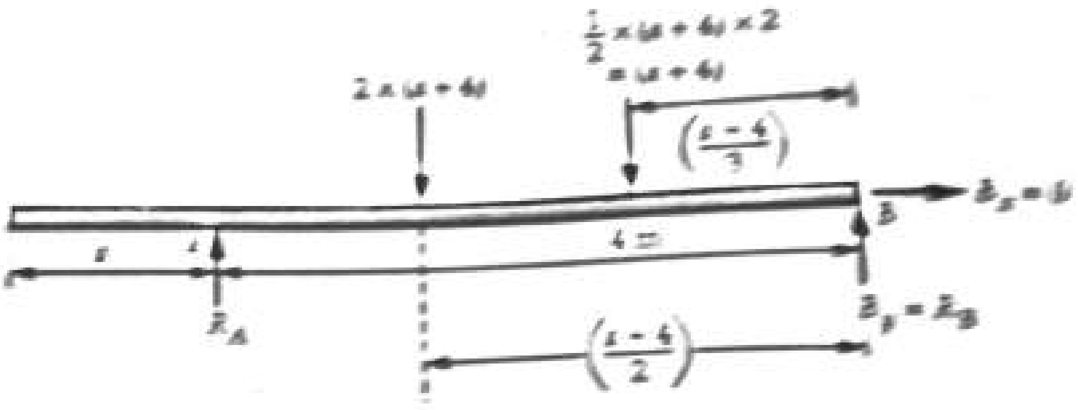


Fig. Q.8.1 (a)

$$\sum M_B = 0:$$

$$(x+4)\times\left(\frac{x+4}{3}\right)+2(x+4)\times\left(\frac{x+4}{2}\right)-R_A\times 4=0$$

$$R_A\times 4 = \frac{(x+4)^2}{3}+(x+4)^2=\frac{4(x+4)^2}{3}$$

$$\therefore \quad R_A = \frac{(x+4)^2}{3}$$

$$\sum F_y = 0: R_A+R_B-2(x+4)-(x+4)=0$$

$$\frac{(x+4)^2}{3}+R_B-3(x+4)=0$$

$$R_B = 3(x+4)-\frac{(x+4)^2}{3}$$

$$\therefore \quad R_B = (x+4)\left[3-\frac{x+4}{3}\right]$$

As reactions are equal in magnitude,

$$R_A = R_B$$

$$\therefore \quad \frac{(x+4)^2}{3} = (x+4)\left[3-\frac{x+4}{3}\right]$$

$$\frac{x+4}{3} = 3-\frac{x+4}{3}$$

$$\frac{2(a+4)}{3} = 3$$

$$a + 4 = \frac{9}{2}$$

$\therefore$ $\boxed{a = 0.5 \text{ m}}$ ...Ans.

Q.9 Determine the support reaction of the beam loaded and supported as shown in Fig. Q.9.1 : [SPPU : May-15, Marks 5]

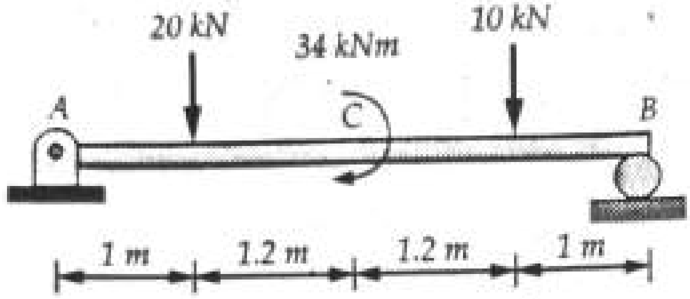


Fig. Q.9.1

Ans. : The FBD of beam is shown in Fig. Q.9.1 (a).

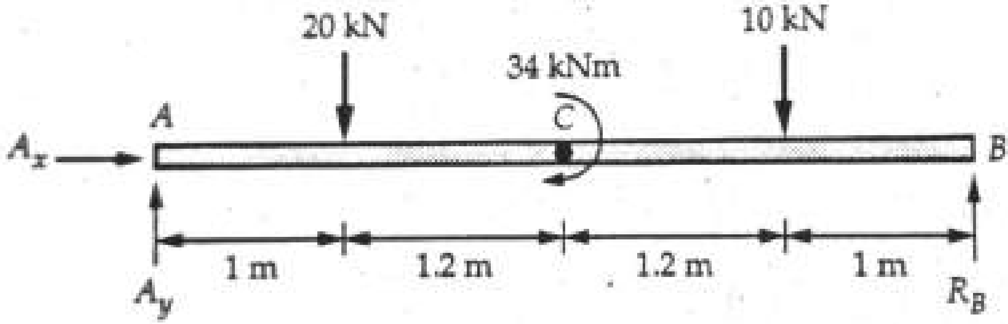


Fig. Q.9.1 (a)

$\Sigma F_x = 0 : A_x = 0$

$\Sigma M_A = 0 : -20 \times 1 - 34 - 10 \times 3.4 + R_B \times 4.4 = 0$

$\therefore$ $\boxed{R_B = 20 \text{ kN} \uparrow}$ ...Ans.

$\Sigma F_y = 0 : A_y - 20 - 10 + R_B = 0$

$\therefore$ $A_y = 10 \text{ kN}$

$\therefore$ $\boxed{R_A = 10 \text{ kN} \uparrow}$ ...Ans.

Q.10 Determine the length a of overhang so that the reaction at B is twice of the reaction at A for the beam loaded and supported as shown in Fig. Q.10.1. [SPPU : Dec.-14, Marks 5]

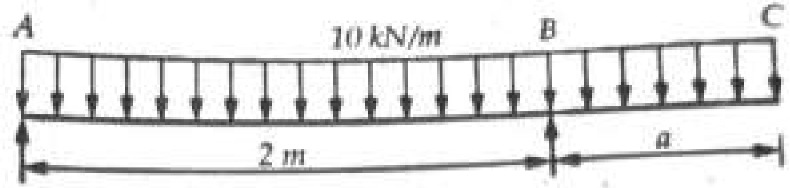


Fig. Q.10.1

Ans. : Draw FBD of beam as shown in Fig. Q.10.1 (a). Replace the *udl* from *A* to *B* by a point load of 20 kN at 1 m from *A* and *udl* from *B* to *C* by point load of 10 a at distance $\frac{a}{2}$ to the right of *B*.

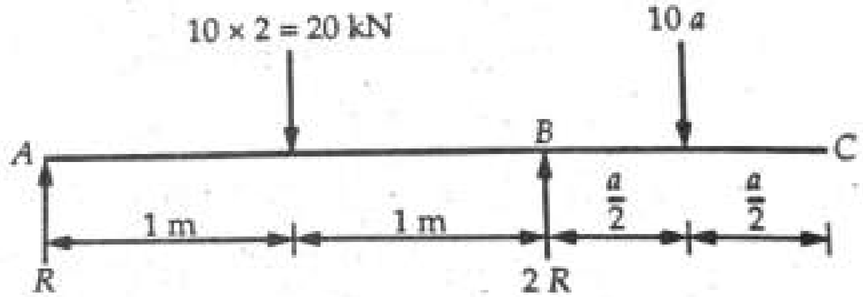


Fig. Q.10.1 (a)

Let $\quad R_A = R$

Then $\quad R_B = 2R$

$$\sum M_A = 0 :$$

$$-(20)(1)+(2R)(2)-(10a)\left(2+\frac{a}{2}\right) = 0$$

$$4R = 20+20a+5a^2 \qquad \text{... (1)}$$

$$\sum F_y = 0 :$$

$$R - 20 + 2R - 10a = 0$$

$$3R = 20 + 10a \qquad \text{... (2)}$$

Divide equation (1) by (2)

$$\therefore \quad \frac{4}{3} = \frac{20+20a+5a^2}{20+10a}$$

$$80 + 40a = 60 + 60a + 15a^2$$

$$15a^2 + 20a - 20 = 0$$

$$\therefore \quad \boxed{a = 0.67 \text{ m}} \quad \text{...Ans.}$$

Q.11 Determine the support reaction for the beam loaded and supported as shown in Fig. Q.11.1. 50 kN force is inclined at 30° to horizontal. [SPPU : May-14, Marks 5]

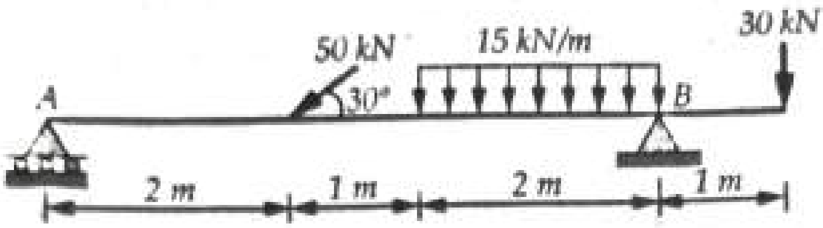


Fig. Q.11.1

Ans. : The FBD of beam is shown in Fig. Q.11.1 (a)

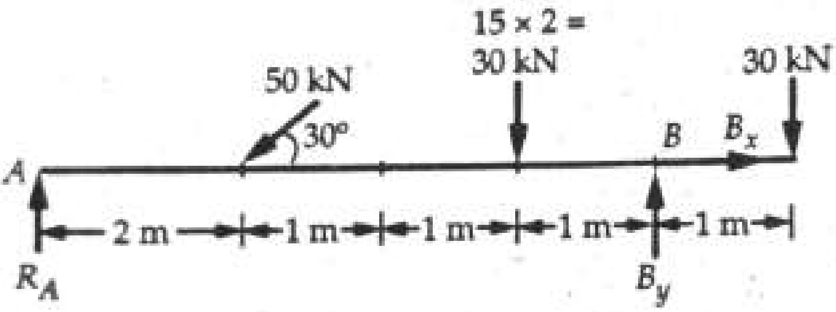


Fig. Q.11.1 (a)

$$\sum M_B = 0 :$$

$$-R_A(5) + (50 \sin 30)(3) + (30)(1) - (30)(1) = 0$$

$$\therefore \quad \boxed{R_A = 15 \text{ kN} \uparrow} \quad \text{...Ans.}$$

$$\sum F_x = 0 : - 50 \cos 30 + B_x = 0$$

$$B_x = 43.3 \text{ kN}$$

$$\sum F_y = 0 : R_A - 50 \sin 30 - 30 + B_y - (30) = 0$$

$$B_y = 70 \text{ kN}$$

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{433^2 + 70^2}$$

$\therefore$ $$\boxed{R_B = 82.31 \text{ kN}}$$ ...Ans.

$$\theta_B = \tan^{-1}\left(\frac{|B_y|}{|B_x|}\right) = \tan^{-1}\left(\frac{70}{43.3}\right)$$

$\therefore$ $$\boxed{\theta_B = 58.26^\circ}$$ ...Ans.

Q.12 Determine the horizontal and vertical components of reaction at the supports for the beam as shown in Fig. Q.12.1.

[SPPU : May-18, Marks 7]

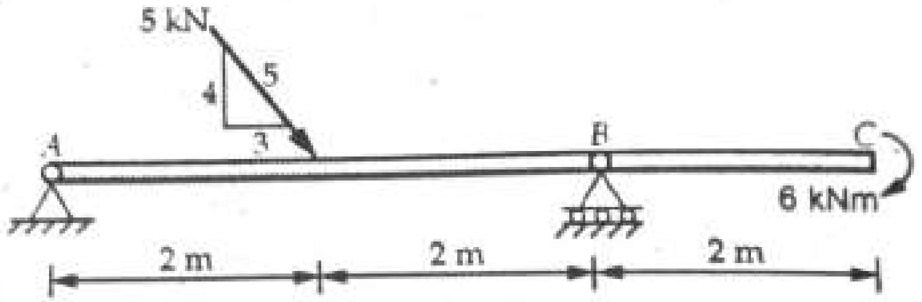


Fig. Q.12.1

Ans. : The FBD of beam is shown in Fig. Q.12.1 (a).

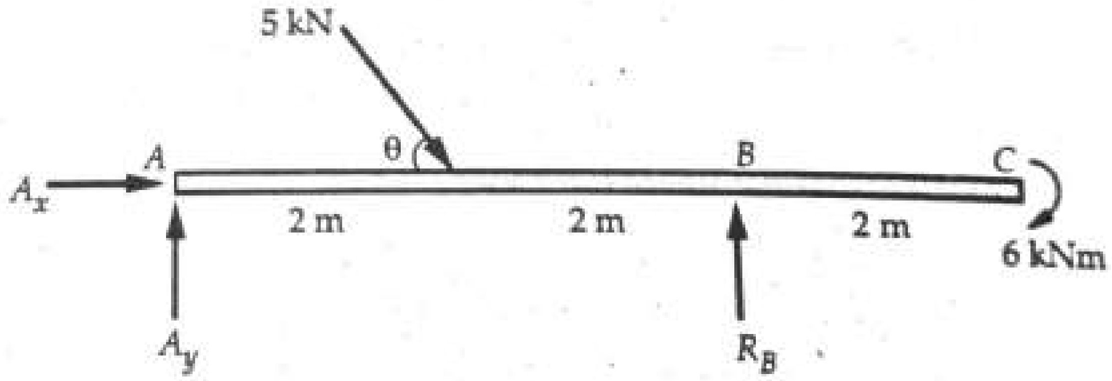


Fig. Q.12.1 (a)

For 5 kN force, $\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$

$$\sum M_A = 0 : -(5\sin\theta)(2) + (R_B)(4) - 6 = 0$$

$$\therefore \quad 4\,R_B = 5 \times \frac{4}{5} \times 2 + 6$$

$\therefore$ $$\boxed{R_B = 14 \text{ kN} \uparrow}$$ ...Ans.

$$\sum F_x = 0: \quad A_x + 5\cos\theta = 0$$

$$A_x = -5 \times \frac{3}{5} = -3 \text{ kN}$$

$\therefore$ $\boxed{A_x = 3 \text{ kN} \leftarrow}$ ...Ans.

$$\sum F_y = 0: \quad A_y - 5\sin\theta + R_B = 0$$

$$A_y = 5 \times \frac{4}{5} - 14 = -10 \text{ kN}$$

$\therefore$ $\boxed{A_y = 10 \text{ kN} \downarrow}$ ...Ans.

Q.13 The beam AB with pin at 'A' and roller at 'B' loaded as shown in the Fig. Q.13.1. Determine the reactions at the supports A and B. **[SPPU : Dec.-18, Marks 6]**

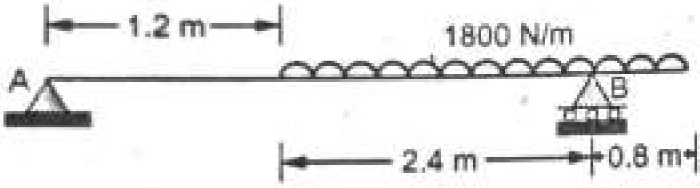


Fig. Q.13.1

Ans. : The FBD of beam is shown in Fig. Q.13.1 (a).

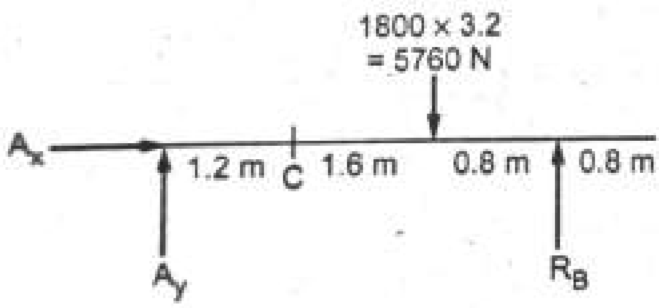


Fig. Q.13.1 (a)

$$\sum F_x = 0: A_x = 0$$

$$\sum M_A = 0: -5760 \times (1.2 + 1.6) + R_B \times 3.6 = 0$$

$\therefore$ $\boxed{R_B = 4480 \text{ N} \uparrow}$ ...Ans.

$$\sum F_y = 0: A_y + R_B - 5760 = 0$$

$\therefore \quad A_y = 1280 \text{ N} \uparrow$

$\therefore \quad \boxed{R_A = 1280 \text{ N} \uparrow}$ ...Ans.

Q.14 The beam AB with pin at 'B' and roller 'A' is loaded as shown in the Fig. Q.14.1. Determine the reaction at the supports A and B. **[SPPU : May-19, Marks 6]**

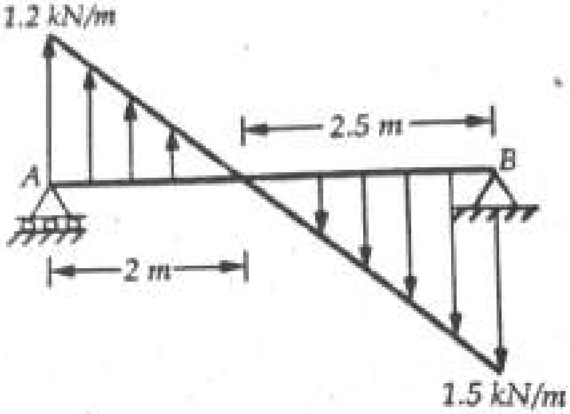


Fig. Q.14.1

Ans. : The F.B.D. of beam is shown in Fig. Q.14.1 (a).

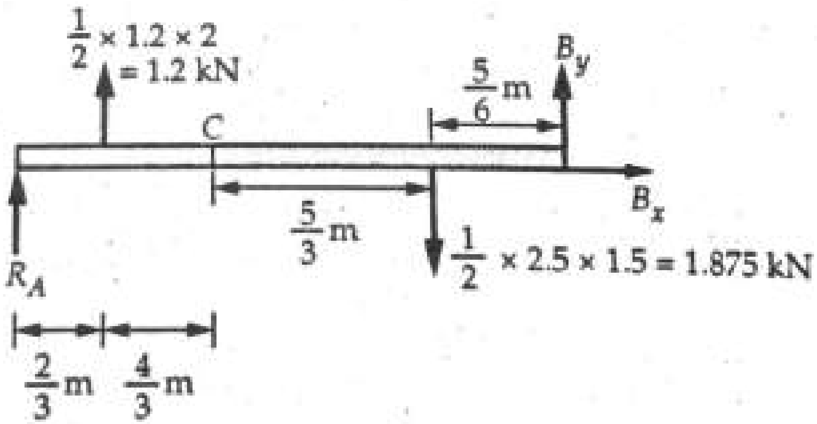


Fig. Q.14.1 (a)

$$\sum M_A = 0 : 1.2 \times \frac{2}{3} + B_y \times 4.5 - 1.875 \times \left(2 + \frac{5}{3}\right) = 0$$

$\therefore \quad \boxed{B_y = 1.35 \text{ kN} \uparrow}$ ...Ans.

$\therefore \quad \sum F_x = 0 \quad \boxed{B_x = 0}$ ...Ans.

$$\sum F_y = 0 : R_A + 1.2 - 1.875 + B_y = 0$$

$\therefore$ $\boxed{R_A = -0.675 \text{ kN} = 0.675 \text{ kN} \downarrow}$ ...Ans.

Q.15 Determine the reactions at support for the beam loaded and supported as shown in Fig. Q.15.1. [SPPU : Dec.-13, Marks 5]

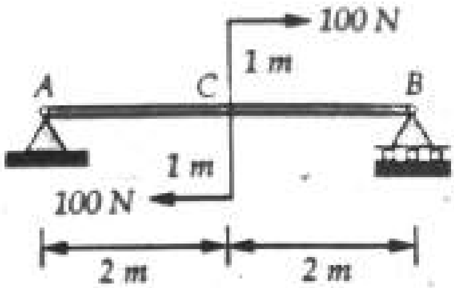


Fig. Q.15.1

Ans. : The F.B.D. of beam is shown in Fig. Q.15.1 (a).

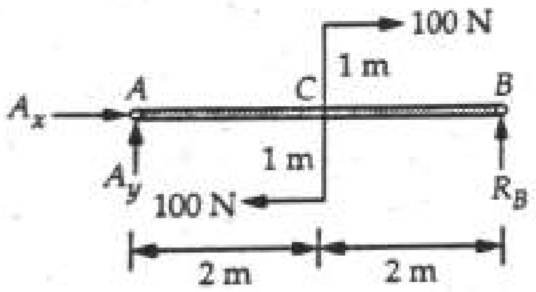


Fig. Q.15.1 (a)

$$\sum F_x = 0 : A_x + 100 - 100 = 0$$

$\therefore$ $A_x = 0$

$$\sum M_A = 0 : -(100)(1) - (100)(1) + (R_B)(4) = 0$$

$\therefore$ $\boxed{R_B = 50 \text{ N} \uparrow}$ ...Ans.

$$\sum F_y = 0 : A_y + R_B = 0$$

$\therefore$ $A_y = -50 \text{ N}$

$\therefore$ $A_y = 50 \text{ N} \downarrow$

$\therefore$ $\boxed{R_A = 50 \text{ N} \downarrow}$ ...Ans.

Q.16 The rope BC will fail when the tension becomes 50 kN as shown in Fig. Q.16.1. Determine the greatest load P that can be applied to the beam at B and reaction at A for equilibrium.
[SPPU : May-13, Marks 6]

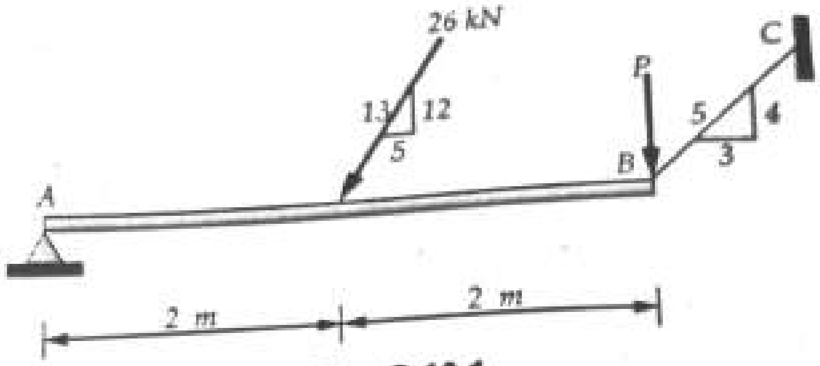


Fig. Q.16.1

Ans. : The F.B.D. of beam *AB* is shown in Fig. Q.16.1 (a).

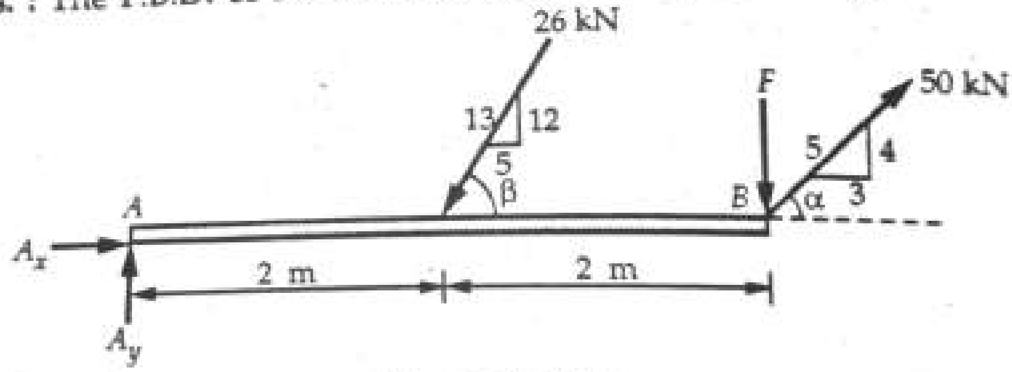


Fig. Q.16.1 (a)

$$\tan \alpha = \frac{4}{3} \qquad \therefore \alpha = 53.13°$$

$$\tan \beta = \frac{12}{5} \qquad \therefore \beta = 67.38°$$

$\sum M_A = 0$:

$$-(26 \times \sin 67.38)(2) - (P)(4) + (50 \times \sin 53.13)(4) = 0$$

$$\therefore \qquad \boxed{P = 28 \text{ kN}}$$ ...Ans.

$\sum F_x = 0$:

$$A_x - 26 \times \cos 67.38 + 50 \times \cos 53.13 = 0$$

$$\therefore \qquad A_x = -20 \text{ kN}$$

$$\therefore \qquad \boxed{A_x = 20 \text{ kN} \leftarrow}$$ ...Ans.

$\sum F_y = 0$:

$A_y - 26 \times \sin 67.38 - P + 50 \times \sin 53.13 = 0$

$\therefore \quad \boxed{A_y = 12 \text{ kN} \uparrow}$ ...Ans.

Q.17 Determine the support reactions for beam AB loaded and supported as shown in Fig. Q.17.1. [SPPU : May-10]

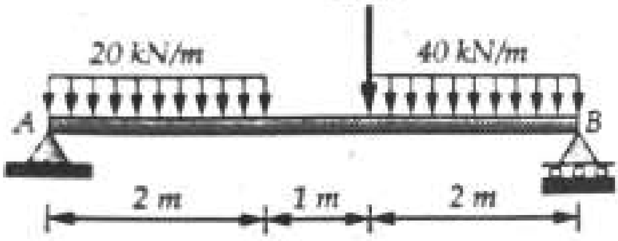


Fig. Q.17.1

Ans. : The F.B.D. of the beam is shown in Fig. Q.17.1 (a).

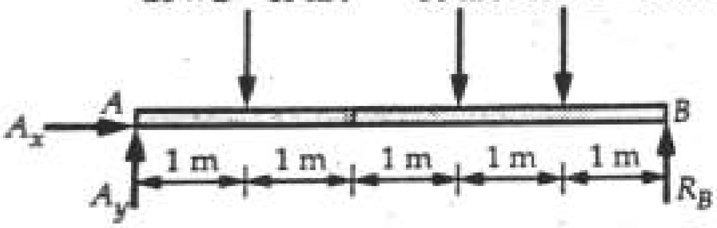


Fig. Q.17.1 (a)

$\sum F_x = 0 : A_x = 0$

$\sum M_A = 0$:

$-(40)(1) - (30)(3) - (80)(4) + (R_B)(5) = 0$

$\therefore \quad \boxed{R_B = 90 \text{ kN} \uparrow}$ ...Ans.

$\sum F_y = 0$:

$A_y - 40 - 30 - 80 + R_B = 0$

$\therefore \quad A_y = 60 \text{ N}$

$\therefore \quad \boxed{R_A = 60 \text{ N} \uparrow}$ ...Ans.

Q.18 A beam 'ABCD' having self weight 2 kN/m is subjected to additional load as shown in Fig. Q.18.1. Find the support reaction at 'B' and 'C'. **[SPPU : May-02, 09]**

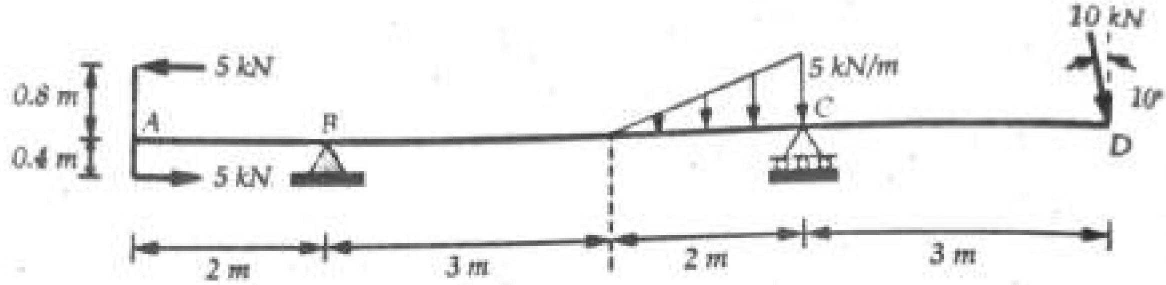


Fig. Q.18.1

Ans. : The two forces of 5 kN form a couple of moment $5 \times 1.2 = 6$ kNm. The weight of beam is $2 \times 10 = 20$ kN acting at the centre. The F.B.D. of beam is shown in Fig. Q.18.1 (a).

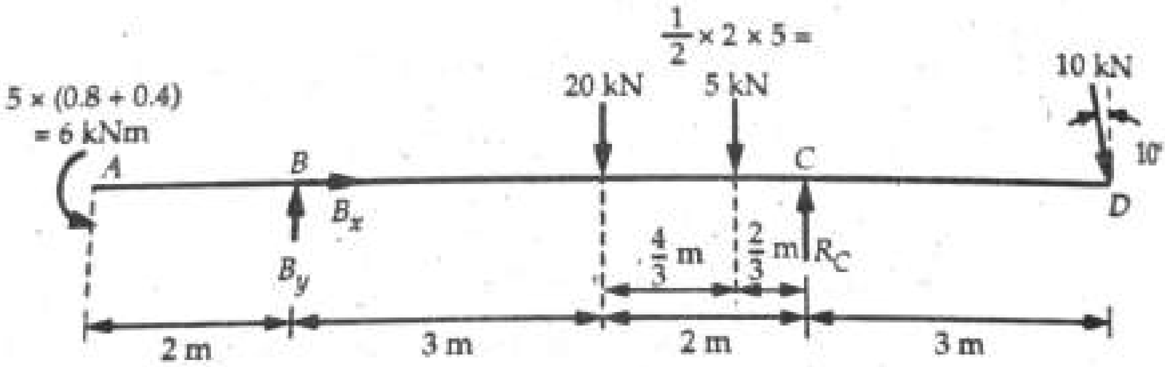


Fig. Q.18.1 (a)

$$\Sigma M_B = 0$$

$$6-(20)(3)-(5)\left(3+\frac{4}{3}\right)+(R_C)(5)-(10\cos 10)(8) = 0$$

$$\therefore \qquad \boxed{R_C = 30.89 \text{ kN} \uparrow} \qquad \text{...Ans}$$

$$\Sigma F_x = 0$$

$$B_x + 10\sin 10 = 0$$

$$\therefore \qquad B_x = -1.74 \text{ kN}$$

$$\Sigma F_y = 0 \qquad B_y - 20 - 5 + R_C - 10\cos 10 = 0$$

$$\therefore \qquad B_y = 3.96 \text{ kN}$$

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.74^2 + 3.96^2}$$

$\therefore$ $\boxed{R_B = 4.325 \text{ kN}}$ ... Ans.

$$\theta = \tan^{-1}\left(\frac{|B_y|}{|B_x|}\right) = \tan^{-1}\left(\frac{3.96}{1.74}\right)$$

$\therefore$ $\boxed{\theta = 66.28^\circ}$ ...Ans.

Q.19 The maximum allowable value of each of the reactions is 360 N neglecting the weight of the beam; determine range of values of distance 'd' for which the beam is safe. Refer Fig. Q.19.1.

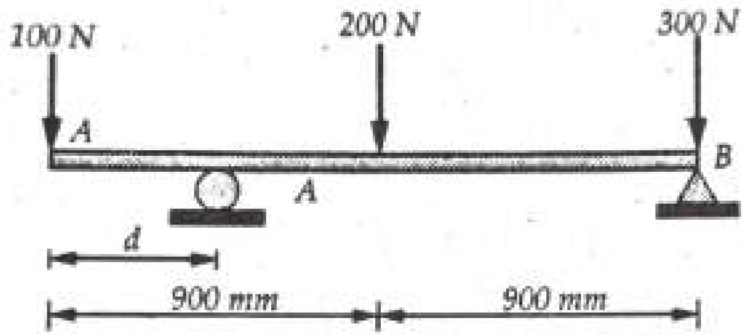


Fig. Q.19.1

Ans. : As all given forces are vertical, the horizontal component of reaction at B is zero. Let the vertical components of reaction be R_A and R_B at A and B respectively.

For minimum 'd', $R_B > R_A$

$\therefore$ $R_B = 360$ N

$\sum M_A = 0$:

$(100)(d) - (200)(900 - d) - (300)(1800 - d) + (360)(1800 - d) = 0$

$100\,d - 200 \times 900 + 200\,d - 300 \times 1800 + 300\,d + 360 \times 1800 - 360\,d = 0$

$\therefore$ $\boxed{d_{min} = 300 \text{ mm}}$ ...Ans.

For maximum 'd', $R_A > R_B$

$\therefore \quad R_A = 360$ N

$\sum M_B = 0:$

$(200)(900) + (100)(1800) - (360)(1800 - d) = 0$

$\therefore \quad \boxed{d = 800 \text{ mm}}$ ...Ans.

Q.20 For the given loading of the beam AB, determine the range of values of the mass 'm' of the crate for which the system will be in equilibrium, knowing that the maximum allowable value of the reactions at each support is 2.5 kN and the reaction at E must be directed downward. Refer Fig. Q.20.1. [SPPU : May-11, Marks 6]

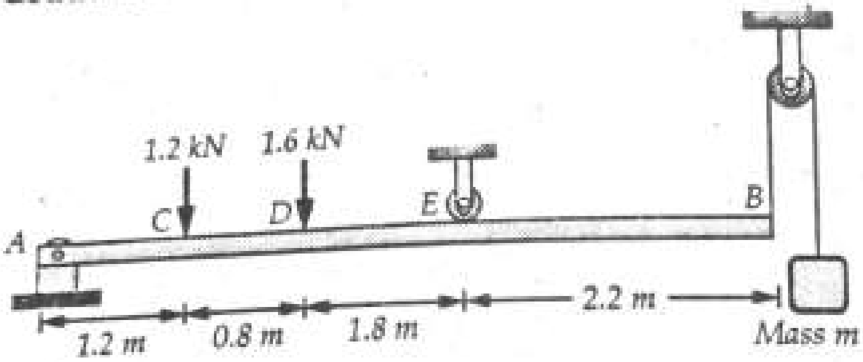


Fig. Q.20.1

Ans. : The tension in cable at B is equal to the weight of the crate.

$\therefore \quad T_B = m \times 9.81$

The F.B.D. of beam AB is shown in Fig. Q.20.1 (a).

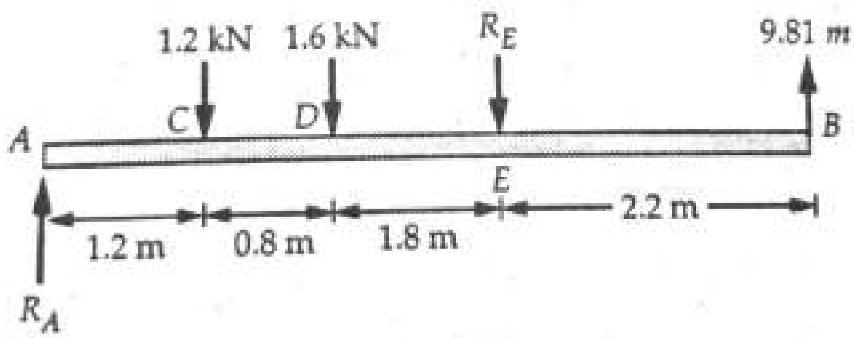


Fig. Q.20.1 (a)

For minimum 'm', $R_A = 2.5$ kN

$\sum M_E = 0$

$(1.6)(1.8) + (1.2)(2.6) + (9.81\,m)(2.2) - (R_A)(3.8) = 0$

$$6+21.582\,m-2.5\times3.8 = 0$$

$\therefore \quad m = 0.162$ kg

For maximum 'm', $R_E = 2.5$ kN

$$\sum M_A = 0$$

$$-(1.2)(1.2)-(1.6)(2)-(R_E)(3.6)+(9.81\,m)(6) = 0$$

$$-(1.2)(1.2)-(1.6)(2)-(2.5)(3.6)+(9.81\,m)(6) = 0$$

$\therefore \quad m = 0.232$ kg

$\therefore$ Range of m is

$$\boxed{0.162 \text{ kg} \leq m \leq 0.232 \text{ kg}}$$ ...Ans.

Q.21 Find support reactions at support 'D' and E' for the beam system as shown in the Fig. Q.21.1. [SPPU : Dec.-10, Marks 6]

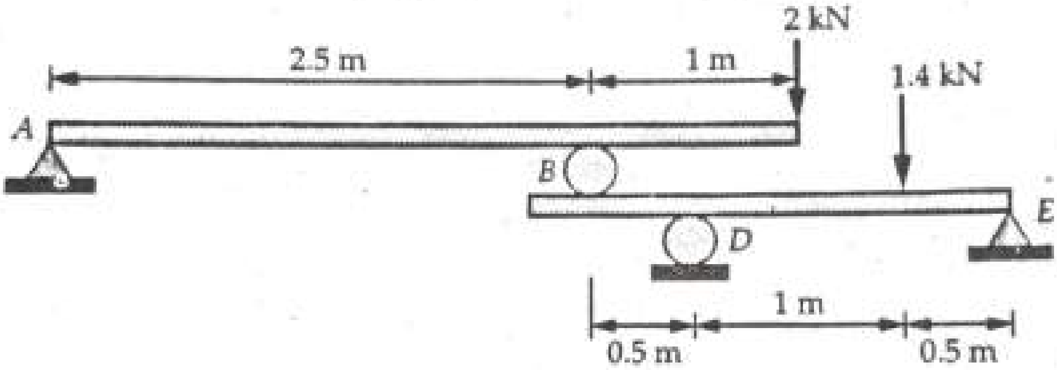


Fig. Q.21.1

Ans. :

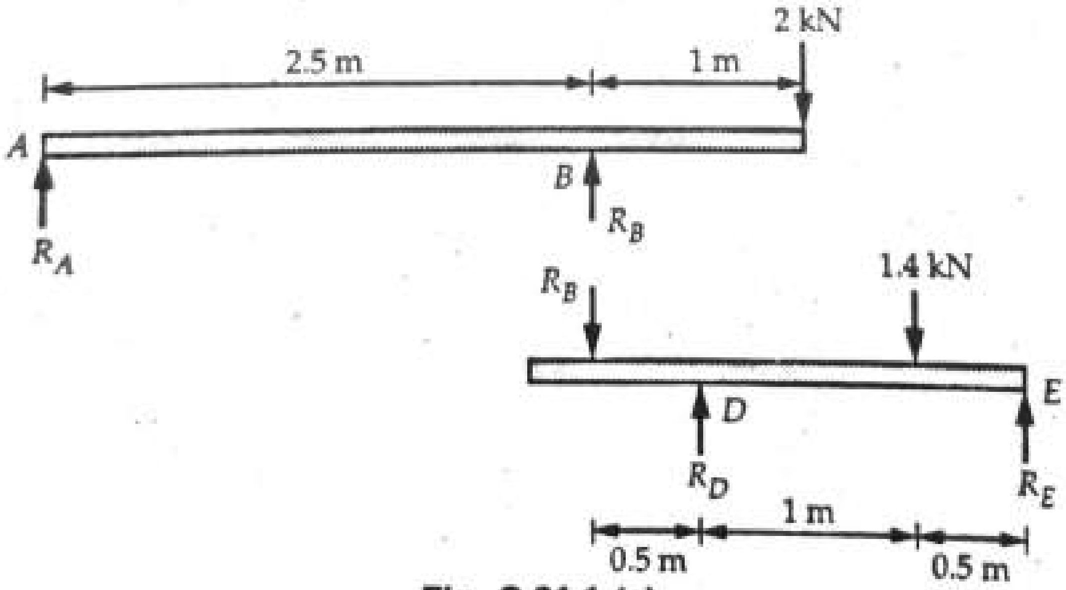


Fig. Q.21.1 (a)

The free body diagrams of the two beams are shown in Fig. Q.21.1 (a). The horizontal components of reaction at A and E are zero as there is no horizontal component of applied force.

For beam AB,

$\sum M_A = 0$:

$(R_B)(2.5) - (2)(3.5) = 0$

$\therefore \quad R_B = 2.8 \text{ kN}$

For beam BDE,

$\sum M_E = 0$:

$(R_B)(2) - (R_D)(1.5) + (1.4)(0.5) = 0$

$\therefore \quad \boxed{R_D = 4.2 \text{ kN} \uparrow}$ ... Ans.

$\sum F_y = 0$:

$-R_B + R_D - 1.4 + R_E = 0$

$\therefore \quad R_E = 1.4 + 2.8 - 4.2$

$\boxed{R_E = 0}$ ... Ans.

END...

8 Space Forces

Important Points to Remember

- If a force is parallel to one of the coordinate axes, it can be directly written in terms of unit vector along that direction.
- If angle made by force with one of the axes is given and the angle made by its projection in a plane with one of the axis in that plane is given, then resolve the force into two mutually perpendicular components :
  i) A component along the axis with which angle of force is given and
  ii) The perpendicular component which is the projection of force in the plane of the other two axes.

Then resolve the projection along the two axes in the plane of the projection.

- If a force of magnitude F is directed from point $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$, then

$$\vec{F} = F\hat{e}_{AB} = F\left[\frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

- If θ_x, θ_y and θ_z are the angles made by the force with x, y and z - directions respectively, then $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are called the direction cosines. The components of force can be obtained using

$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

- The relation between these direction cosines is

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

- If components F_x, F_y and F_z of force F are known in three dimensions, its magnitude and direction (defined by the three angles $\theta_x, \theta_y, \theta_z$) can be obtained using

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$$

- The resultant of concurrent force system is a force acting through the point of concurrence of the given forces. The x, y and z components of resultant can be obtained using

$$R_x = \sum F_x \quad ; \quad R_y = \sum F_y \quad ; \quad R_z = \sum F_z$$

- Magnitude of resultant can be obtained using

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

- The direction of resultant is defined by the three angles with coordinate axes given by

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) \; ; \quad \theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) ; \quad \theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

- The conditions for equilibrium of a concurrent force system are

$$\sum F_x = 0 ; \sum F_y = 0 ; \sum F_z = 0$$

- The resultant of a parallel force system in space is a force parallel to the given force system.
- The position of the resultant force can be obtained using Varignon's theorem about two different axes perpendicular to the plane of the forces.

- For equilibrium of parallel force system, one of the equations can be obtained by equating summation of all forces, along an axis parallel to the forces to zero.
- Additional equations can be obtained by equating moments of all forces about different lines in a plane perpendicular to the forces to zero.

Q.1 Three cables AB, AC and AD hold down a balloon as shown in Fig. Q.1.1. Find vertical force exerted at the base of balloon A, knowing that tension in cable AB is 259 N. [SPPU : Dec.-03]

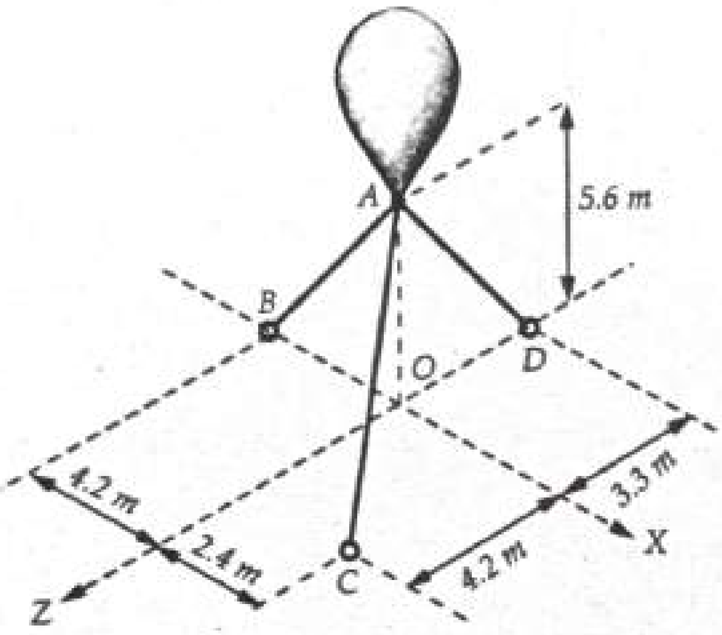


Fig. Q.1.1

Ans. : The required co-ordinates are A (0, 5.6, 0), B (– 4.2, 0, 0), C (2.4, 0, 4.2), D (0, 0, – 3.3). Let magnitude of force acting at A = P then

$$\vec{P} = P\hat{j}$$

$$\vec{T}_{AB} = 259\left[\frac{-4.2\hat{i} - 5.6\hat{j} + 0\hat{k}}{\sqrt{4.2^2 + 5.6^2}}\right] = -155.4\hat{i} - 207.2\hat{j} + 0\hat{k}$$

$$\vec{T}_{AC} = T_{AC}\left[\frac{2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k}}{\sqrt{2.4^2 + 5.6^2 + 4.2^2}}\right]$$

$$= T_{AC} \times \frac{2.4}{7.4} \hat{i} - \frac{T_{AC} \times 5.6 \hat{j}}{7.4} + \frac{T_{AC} \times 4.2}{7.4} \hat{k}$$

$$\vec{T}_{AD} = T_{AD} \left[\frac{0\hat{i} - 5.6\hat{j} - 3.3\hat{k}}{\sqrt{5.6^2 + 3.3^2}} \right] = \frac{-T_{AD} \times 5.6 \hat{j}}{6.5} - \frac{T_{AD} \times 3.3}{6.5} \hat{k}$$

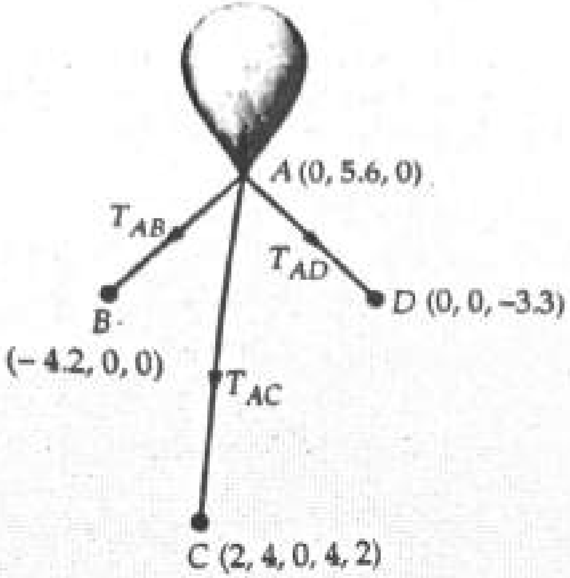


Fig. Q.1.1 (a)

$$R_x = \sum F_x = 0 \Rightarrow -155.4 + T_{AC} \times \frac{2.4}{7.4} = 0 \quad \text{... (1)}$$

$$\sum F_y = 0 \Rightarrow P - 207.2 - T_{AC} \times \frac{5.6}{7.4} - T_{AD} \times \frac{5.6}{6.5} = 0 \quad \text{... (2)}$$

$$\sum F_z = 0 \Rightarrow T_{AC} \times \frac{4.2}{7.4} - T_{AD} \times \frac{3.3}{6.5} = 0 \quad \text{... (3)}$$

From (1) : $T_{AC} = 479.15$ N

From (3) : $T_{AD} = 535.66$ N

From (4) : $\boxed{P = 1031.29 \text{ N}}$ ... Ans.

Q.2 A vertical mast OD is having base 'O' with ball and socket. Three cables DA, DB and DC keep the mast similar in equilibrium. If tension in the cable DA is 100 kN, find tensions in the cables DB and DC and force in the mast. Refer Fig. Q.2.1.

[SPPU : Dec.-06, Similar in May-19, Marks 7]

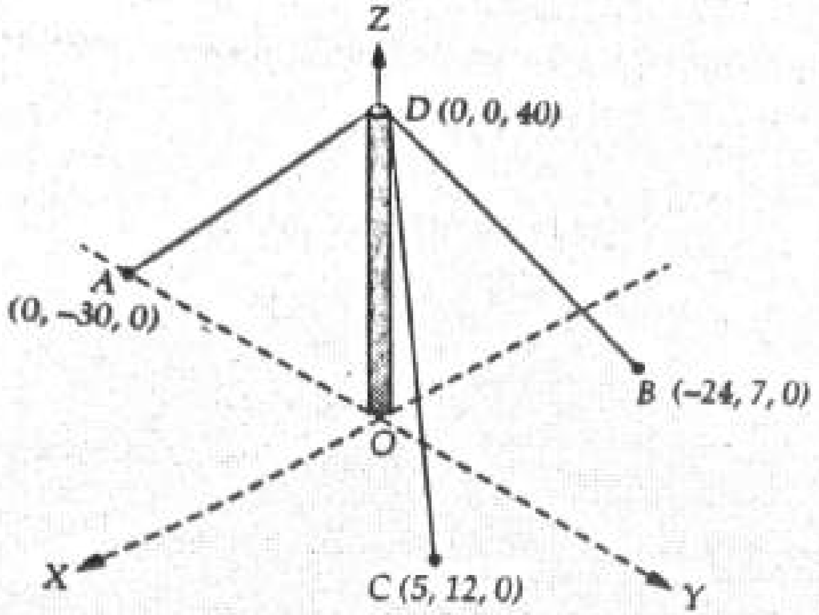


Fig. Q.2.1

Ans. : All the forces are concurrent at point D.

$$\vec{T}_{DA} = 100\left[\frac{-30\hat{j}-40\hat{k}}{\sqrt{30^2+40^2}}\right] = -60\hat{j}-80\hat{k}$$

$$\vec{T}_{DB} = T_{DB}\left[\frac{-24\hat{i}+7\hat{j}-40\hat{k}}{\sqrt{24^2+7^2+40^2}}\right] = \frac{T_{DB}}{47.17}(-24\hat{i}+7\hat{j}-40\hat{k})$$

$$\vec{T}_{DC} = T_{DC}\left[\frac{5\hat{i}+12\hat{j}-40\hat{k}}{\sqrt{5^2+12^2+40^2}}\right] = \frac{T_{DC}}{42.06}(5\hat{i}+12\hat{j}-40\hat{k})$$

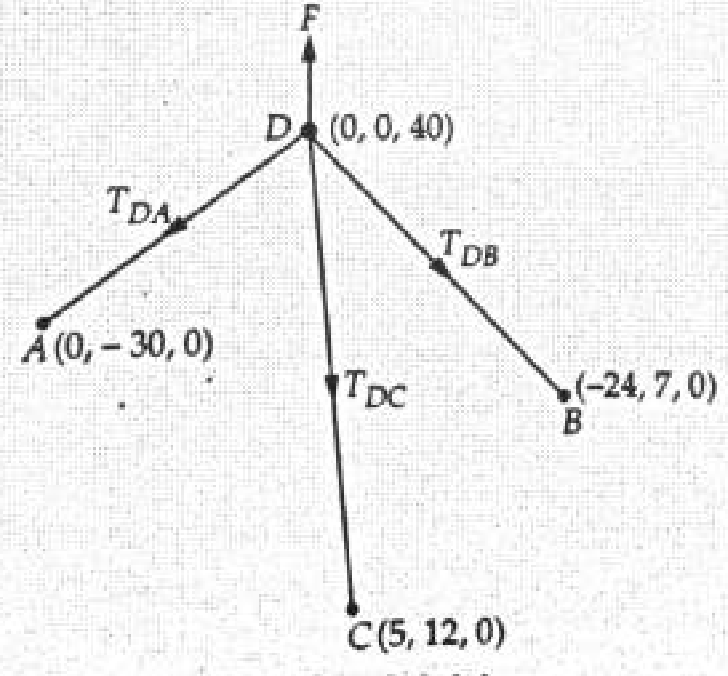


Fig. Q.2.1 (a)

Let the force in mast be $\vec{F} = F\hat{k}$ (As the mast is subjected to forces at the ends only, it has axial force.)

$$\sum F_x = 0 : \frac{-24\,T_{DB}}{47.17} + \frac{5\,T_{DC}}{42.06} = 0 \quad \text{... (1)}$$

$$\sum F_y = 0 : -60 + \frac{7\,T_{DB}}{47.17} + \frac{12\,T_{DC}}{42.06} = 0 \quad \text{... (2)}$$

$$\sum F_z = 0 : -80 - \frac{40\,T_{DB}}{47.17} - \frac{40\,T_{DC}}{42.06} + F = 0 \quad \text{... (3)}$$

From (1), (2) and (3),

$$\boxed{T_{DB} = 43.81 \text{ kN}}$$

$$\boxed{T_{DC} = 187.5 \text{ kN}}$$

$$\boxed{F = 295.48 \text{ kN}} \quad \text{... Ans.}$$

Q.3 A 200 kg cylinder is hung by means of two cables AB and AC which are attached to the top corners B and C of a vertical wall. A horizontal force P perpendicular to the wall holds the cylinder 1.2 m away from the wall. Determine the value of P and the tension in the cables AB and AC. [SPPU : Dec.-02,12, May-09]

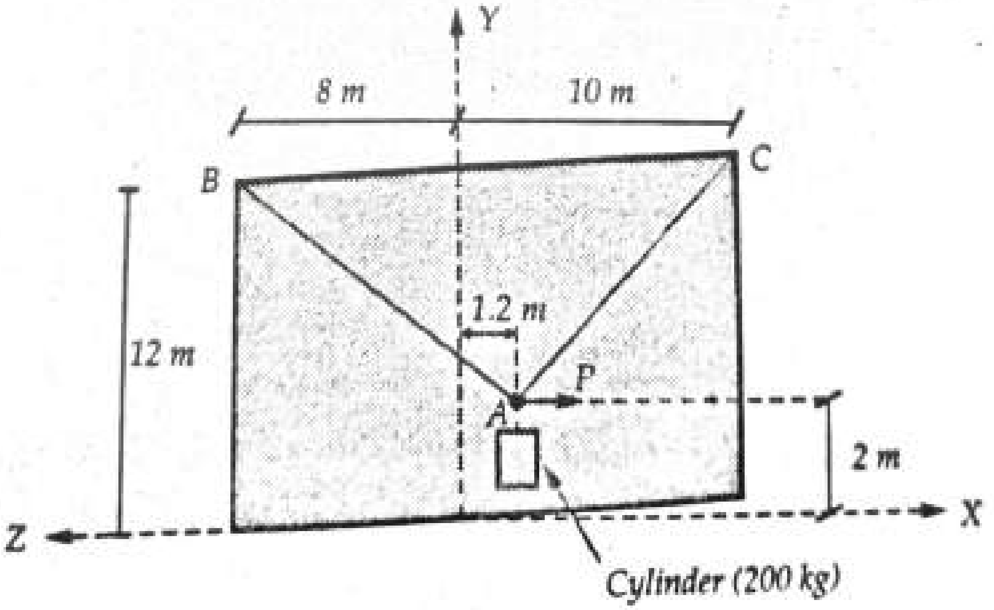


Fig. Q.3.1

$$\boxed{T_{AB} = 1401.96 \text{ N}}$$ ... Ans.

$$\boxed{T_{AC} = 1237.63 \text{ N}}$$ ... Ans.

Q.4 The cable exerts forces F_{AB} = 100 N and F_{AC} = 120 N on the ring at A as shown in Fig. Q.4.1. Determine magnitude of the resultant force acting at A. **[SPPU : May-10, 14, Dec.-17, Marks 6]**

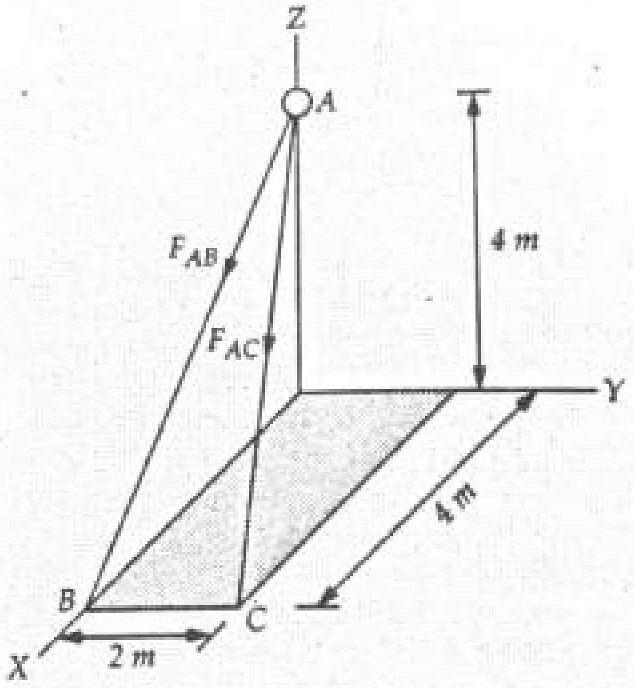


Fig. Q.4.1

Ans. : The co-ordiantes of A, B and C are

$A(0, 0, 4)$, $B(4, 0, 0)$ and $C(4, 2, 0)$

F_{AB} = 100 N is directed from A to B

$$\therefore \quad \vec{F}_{AB} = 100\left[\frac{4\hat{i}+0\hat{j}-4\hat{k}}{\sqrt{4^2+4^2}}\right]$$

F_{AC} = 120 N is directed from A to C

$$\therefore \quad \vec{F}_{AC} = 120\left[\frac{4\hat{i}+2\hat{j}-4\hat{k}}{\sqrt{4^2+2^2+4^2}}\right]$$

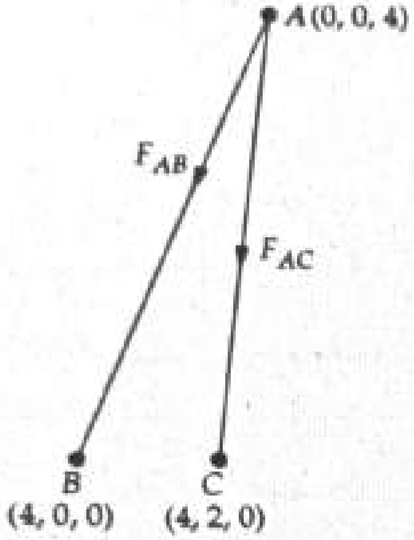


Fig. Q.4.1 (a)

$$R_x = \sum F_x = \frac{100 \times 4}{\sqrt{4^2 + 4^2}} + \frac{120 \times 4}{\sqrt{4^2 + 2^2 + 4^2}}$$

$$\therefore \quad R_x = 150.71 \text{ N}$$

$$R_y = \sum F_y = 0 + \frac{120 \times 2}{\sqrt{4^2 + 2^2 + 4^2}}$$

$$\therefore \quad R_y = 40 \text{ N}$$

$$R_z = \sum F_z = \frac{-100 \times 4}{\sqrt{4^2 + 4^2}} - \frac{-120 \times 4}{\sqrt{4^2 + 2^2 + 4^2}}$$

$$\therefore \quad R_z = -150.71 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{150.71^2 + 40^2 + 150.71^2}$$

$$\therefore \quad \boxed{R = 216.86 \text{ N}} \quad \text{... Ans.}$$

Q.5 A vertical load of 50 kg is supported by three rods as shown in Fig. Q.5.1. Determine the force in each rod for the co-ordinates of points as below.
A (– 4, – 1, 0), B (3, 3, 0), C (3, – 2, 0) and D (0, 0, 6)

[SPPU : May-11, Marks 7]

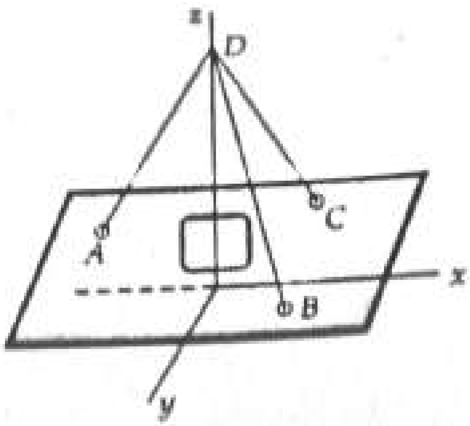


Fig. Q.5.1

Ans. : Each rod has compressive force. Hence forces in rods are directed towards D.

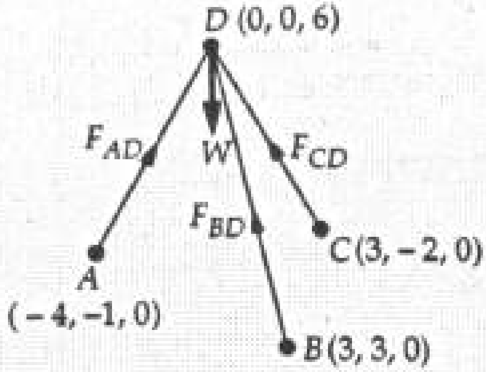


Fig. Q.5.1 (a)

$$\vec{F}_{AD} = F_{AD}\,\hat{e}_{AD} = F_{AD}\left[\frac{4\hat{i}+\hat{j}+6\hat{k}}{\sqrt{4^2+1^2+6^2}}\right]$$

$$\vec{F}_{AD} = \frac{F_{AD}}{\sqrt{53}}(4\hat{i}+\hat{j}+6\hat{k})$$

$$\vec{F}_{BD} = F_{BD}\,\hat{e}_{BD} = F_{BD}\left[\frac{-3\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{3^2+3^2+6^2}}\right]$$

$$\therefore \quad \vec{F}_{BD} = \frac{F_{BD}}{\sqrt{54}}(-3\hat{i}-3\hat{j}+6\hat{k})$$

$$\vec{F}_{CD} = F_{CD}\,\hat{e}_{CD} = F_{CD}\left[\frac{-3\hat{i}+2\hat{j}+6\hat{k}}{\sqrt{3^2+2^2+6^2}}\right]$$

$$\therefore \qquad \vec{F}_{CD} = \frac{F_{CD}}{7}(-3\hat{i}+2\hat{j}+6\hat{k})$$

$$\vec{W} = -50\times 9.81\,\hat{k} = -490.5\,\hat{k}$$

$\sum F_x = 0$:

$$\frac{4F_{AD}}{\sqrt{53}} - \frac{3F_{BD}}{\sqrt{54}} - \frac{3F_{CD}}{7} = 0 \qquad \text{... (1)}$$

$\sum F_y = 0$:

$$\frac{F_{AD}}{\sqrt{53}} - \frac{3F_{BD}}{\sqrt{54}} + \frac{2F_{CD}}{7} = 0 \qquad \text{... (2)}$$

$\sum F_z = 0$:

$$\frac{6F_{AD}}{\sqrt{53}} + \frac{6F_{BD}}{\sqrt{54}} + \frac{6F_{CD}}{7} - 490.5 = 0 \qquad \text{... (3)}$$

From equations (1), (2) and (3),

$$\boxed{F_{AD} = 255.06 \text{ N (C)}} \qquad \text{... Ans.}$$

$$\boxed{F_{BD} = 188.8 \text{ N (C)}} \qquad \text{... Ans.}$$

$$\boxed{F_{CD} = 147.15 \text{ N (C)}} \qquad \text{... Ans.}$$

Q.6 The support assembly shown in Fig. Q.6.1 is bolted in place 'B', 'C' and 'D' supporting a downward force of 45 N applied at 'A'. Determine the forces in the members AB, AC and AD.

[SPPU : Dec.-11, Marks 7]

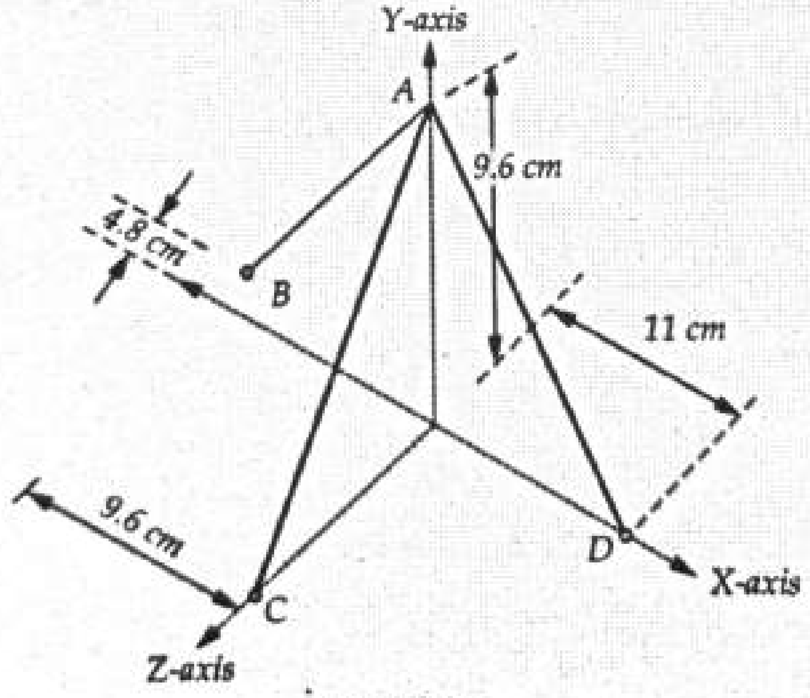


Fig. Q.6.1

Ans. : Distance of C from origin is not given. Assume this distance to be 11 cm.

The co-ordinates of points are :

A (0, 9.6, 0), B(−9.6, 0, − 4.8), C (0, 0, 11), D (11, 0, 0)

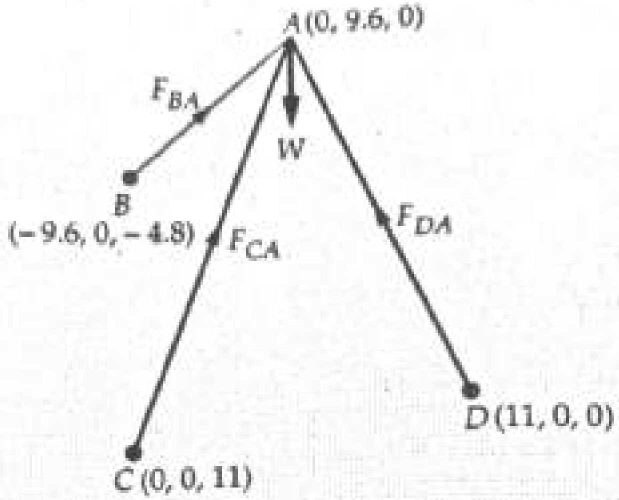


Fig. Q.6.1 (a)

All forces are concurrent at A. Forces in AB, AC and AD are compressive and hence are directed towards A.

$$\vec{W} = -45\hat{j}$$

$$\vec{F}_{BA} = F_{BA}\,\hat{e}_{BA} = F_{BA}\left[\frac{9.6\hat{i}+9.6\hat{j}+4.8\hat{k}}{\sqrt{9.6^2+9.6^2+4.8^2}}\right]$$

$$\therefore \quad \vec{F}_{BA} = \frac{F_{BA}}{14.4}(9.6\,\hat{i}+9.6\,\hat{j}+4.8\,\hat{k})$$

$$\vec{F}_{CA} = F_{CA}\,\hat{e}_{CA} = F_{CA}\left[\frac{0\hat{i}+9.6\hat{j}-11\hat{k}}{\sqrt{9.6^2+11^2}}\right]$$

$$\therefore \quad \vec{F}_{CA} = \frac{F_{CA}}{14.6}(0\hat{i}+9.6\hat{j}-11\hat{k})$$

$$\vec{F}_{DA} = F_{DA}\,\hat{e}_{DA} = F_{DA}\left[\frac{-11\hat{i}+9.6\hat{j}+0\hat{k}}{\sqrt{11^2+9.6^2}}\right]$$

$$\therefore \quad \vec{F}_{DA} = \frac{F_{DA}}{14.6}(-11\hat{i}+9.6\hat{j}+0\hat{k})$$

$\sum F_x = 0$:

$$\frac{9.6\,F_{DA}}{14.4} - \frac{11F_{DA}}{14.6} = 0 \qquad \text{... (1)}$$

$\sum F_y = 0$:

$$-45 + \frac{9.6\,F_{BA}}{14.4} + \frac{9.6\,F_{CA}}{14.6} + \frac{9.6\,F_{DA}}{14.6} = 0 \qquad \text{... (2)}$$

$\sum F_z = 0$:

$$\frac{4.8\,F_{BA}}{14.4} - \frac{11F_{CA}}{14.6} = 0 \qquad \text{... (3)}$$

From equations (1), (2) and (3),

$\therefore$ $\boxed{F_{BA} = 29.23 \text{ N}}$

$\therefore$ $\boxed{F_{CA} = 12.93 \text{ N}}$

$\therefore$ $\boxed{F_{DA} = 25.87 \text{ N}}$ ... Ans.

Q.7 Determine the force developed in cable AB, AC and AD used to support the 40 N crate as shown in Fig. Q.7.1.

[SPPU : May-12, Marks 7]

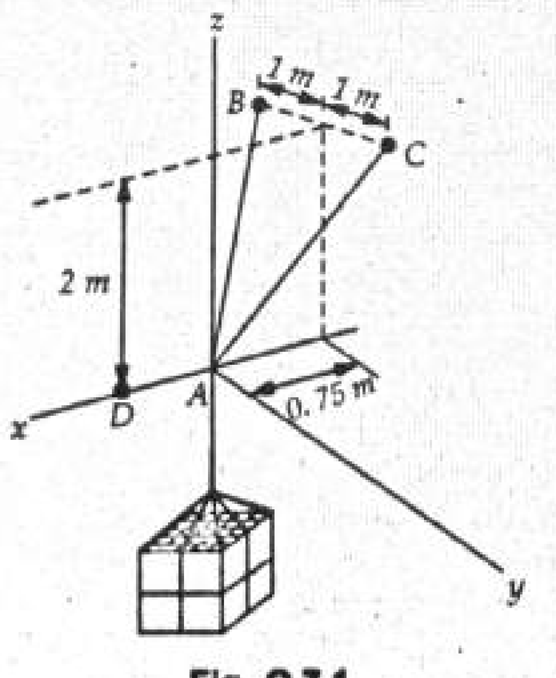


Fig. Q.7.1

Ans. : All forces are concurrent at A and directed away from A.

The co-ordinates are A (0, 0, 0),

B (– 0.75, –1, 2) and C (– 0.75, 1, 2)

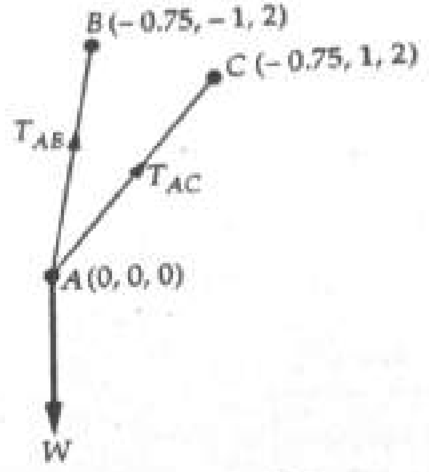


Fig. Q.7.1 (a)

The weight of the crate is,

$$\vec{W} = -40\hat{k}$$

$$\vec{T}_{AD} = T_{AD}\,\hat{i}$$

($\vec{T}_{AD}$ is directed along positive x - axis)

$$\vec{T}_{AB} = T_{AB}\left[\frac{-0.75\hat{i}-\hat{j}+2\hat{k}}{\sqrt{0.75^2+1^2+2^2}}\right]$$

$$= \frac{T_{AB}}{\sqrt{5.5625}}[-0.75\hat{i}-\hat{j}+2\hat{k}]$$

$$\vec{T}_{AC} = T_{AC}\left[\frac{-0.75\hat{i}+\hat{j}+2\hat{k}}{\sqrt{0.75^2+1^2+2^2}}\right]$$

$$= \frac{T_{AC}}{\sqrt{5.5625}}[-0.75\hat{i}+\hat{j}+2\hat{k}]$$

$$\sum F_x = 0:$$

$$T_{AD} - 0.75\frac{T_{AB}}{\sqrt{5.5625}} - 0.75\frac{T_{AC}}{\sqrt{5.5625}} = 0 \qquad \dots (1)$$

$\sum F_x = 0:$

$$\frac{-T_{AB}}{\sqrt{5.5625}} + \frac{T_{AC}}{\sqrt{5.5625}} = 0$$

$\therefore$

$$-T_{AB} + T_{AC} = 0 \quad \text{... (2)}$$

$\sum F_z = 0:$

$$-40 + 2\frac{T_{AB}}{\sqrt{5.5625}} + 2\frac{T_{AC}}{\sqrt{5.5625}} = 0$$

$\therefore$

$$T_{AB} + T_{AC} = 20\sqrt{5.5625} \quad \text{... (3)}$$

From equations (2) and (3),

$$\boxed{T_{AB} = 23.585 \text{ N}} \quad \text{... Ans.}$$

$$\boxed{T_{AC} = 23.585 \text{ N}} \quad \text{... Ans.}$$

From equation (1),

$$\boxed{T_{AD} = 15 \text{ N}} \quad \text{... Ans.}$$

Q.8 The three cables are used to support the 800 N lamp as shown in Fig. Q.8.1. Determine the force developed in each cable for equilibrium. **[SPPU : May-13, Marks 6]**

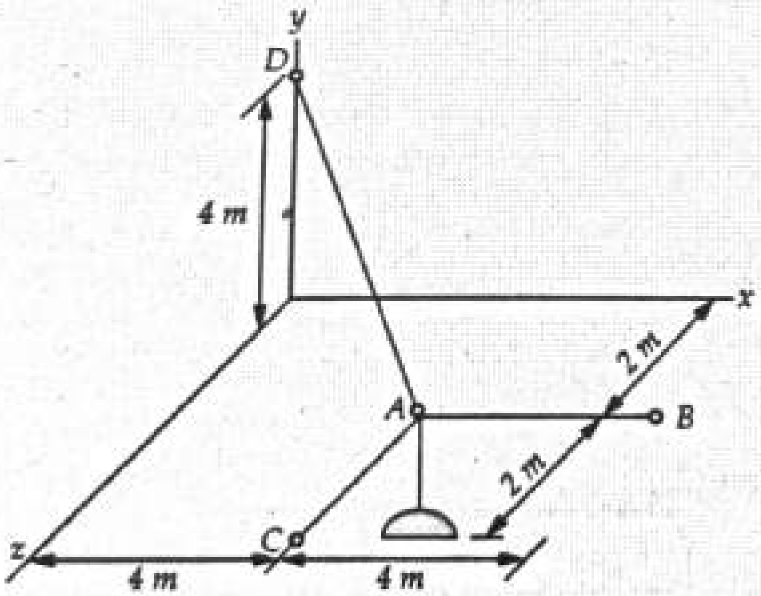


Fig. Q.8.1

Ans. : The force system is concurrent at A. The coordinates of points are $A(4, 0, 2)$ and $D(0, 4, 0)$.

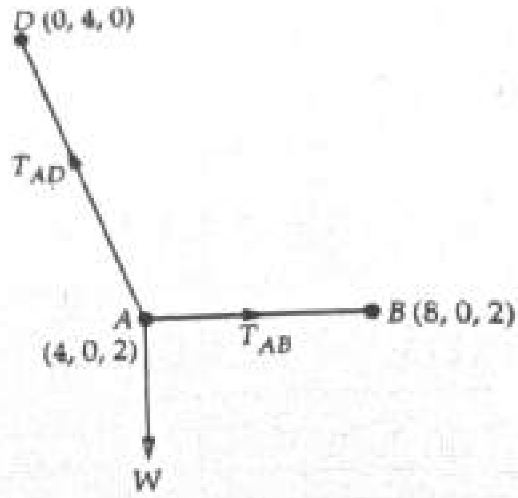


Fig. Q.8.1 (a)

$$\vec{W} = -800\,\hat{j}$$

$$\vec{T}_{AB} = T_{AB}\,\hat{i}$$

$$\vec{T}_{AC} = T_{AC}\,\hat{k}$$

$$\vec{T}_{AD} = T_{AD}\left[\frac{-4\hat{i}+4\hat{j}-2\hat{k}}{\sqrt{4^2+4^2+2^2}}\right] = \frac{T_{AD}}{6}(-4\hat{i}+4\hat{j}-2\hat{k})$$

$\sum F_x = 0:$

$$T_{AB} - 4\frac{T_{AD}}{6} = 0 \qquad \text{...(1)}$$

$\sum F_y = 0:$

$$-800 + \frac{T_{AD}\times 4}{6} = 0 \qquad \text{...(2)}$$

$$\sum F_z = 0: \quad T_{AC} - \frac{2T_{AD}}{6} = 0 \qquad \text{...(3)}$$

From equation (2),

$$\boxed{T_{AD} = 1200\text{ N}} \qquad \text{... Ans.}$$

From equation (1),

$$\boxed{T_{AB} = 800 \text{ N}} \quad \text{... Ans.}$$

From equation (3),

$$\boxed{T_{AC} = 400 \text{ N}} \quad \text{... Ans.}$$

Q.9 A rectangular plate is supported by three cables at A as shown in Fig. Q.9.1. Knowing that the tension in cable AD is 120 N, determine the weight of the plate. [SPPU : Dec.-13, Marks 6]

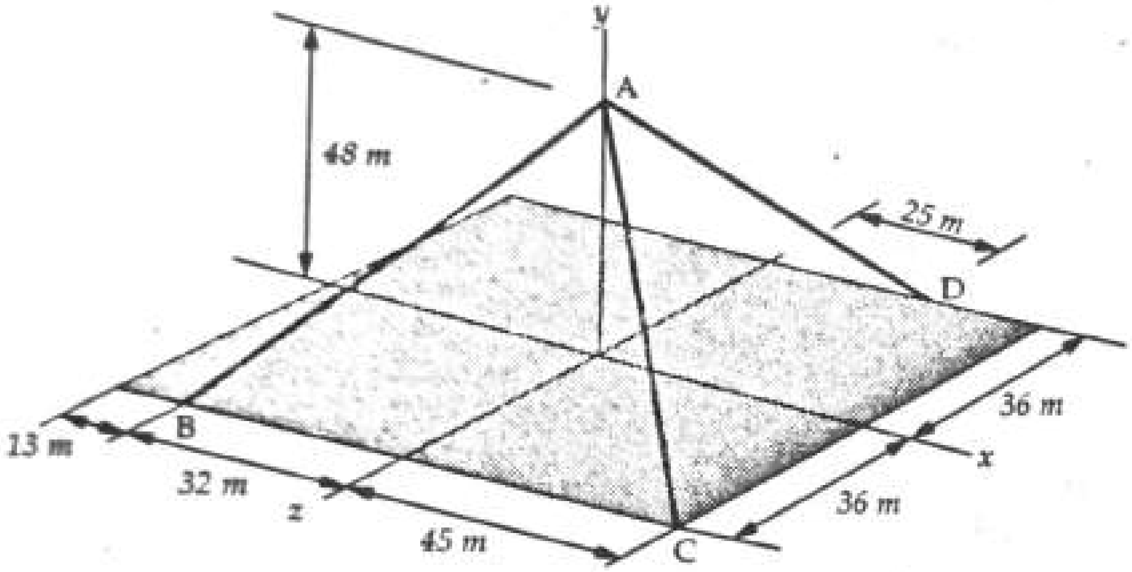


Fig. Q.9.1

Ans. : The required co-ordinates are A (0, 48, 0), B (– 32, 0, 36), C (45, 0, 36), D (25, 0, – 36). All forces are concurrent at A. The weight of the plate acts vertically downward through its C.G.

$\therefore$ The force at A is W N ↑

$$\vec{W} = +W\hat{j}$$

The three tensile forces act away from A.

$$\therefore \quad \vec{T}_{AB} = T_{AB}\left[\frac{-32\hat{i} - 48\hat{j} + 36\hat{k}}{\sqrt{32^2 + 48^2 + 36^2}}\right]$$

$$= \frac{-T_{AB} \times 32}{68}\hat{i} - \frac{T_{AB} \times 48}{68}\hat{j} + \frac{T_{AB} \times 36}{68}\hat{k}$$

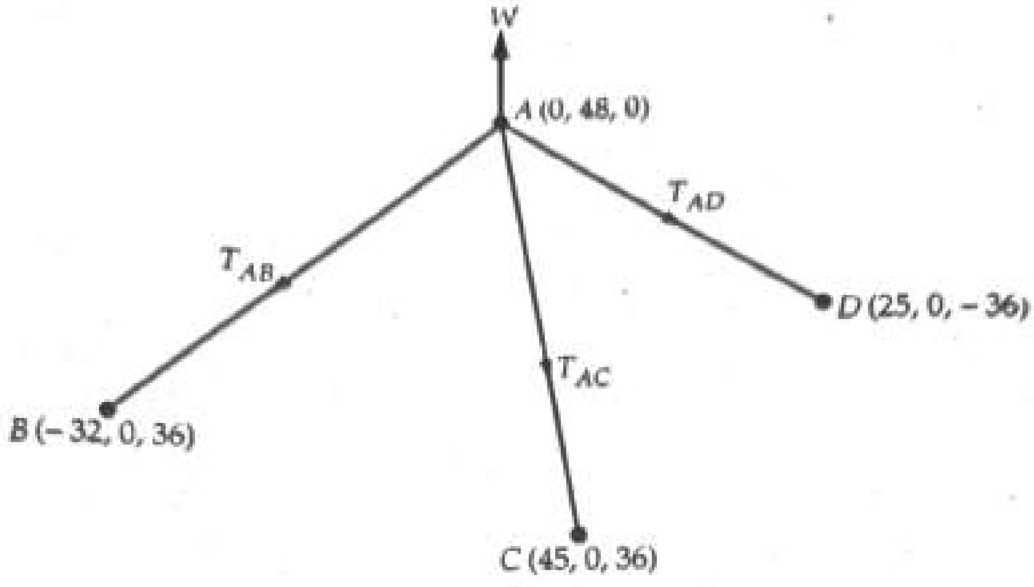


Fig. Q.9.1 (a)

$$\vec{T}_{AC} = T_{AC}\left[\frac{45\hat{i} - 48\hat{j} + 36\hat{k}}{\sqrt{45^2 + 48^2 + 36^2}}\right]$$

$$= \frac{T_{AC} \times 45}{75}\hat{i} - \frac{T_{AC} \times 48}{75}\hat{j} + \frac{T_{AC} \times 36}{75}\hat{k}$$

$$\vec{T}_{AD} = 120\left[\frac{25\hat{i} - 48\hat{j} - 36\hat{k}}{\sqrt{25^2 + 48^2 + 36^2}}\right]$$

$$= 120 \times \frac{25}{65}\hat{i} - 120 \times \frac{48}{65}\hat{j} - 120 \times \frac{36}{65}\hat{k}$$

For equilibrium,

$$R_x = \sum F_x = 0 \Rightarrow -T_{AB} \times \frac{32}{68} + T_{AC} \times \frac{45}{75} + 120 \times \frac{25}{65} = 0 \quad \text{... (1)}$$

$$R_y = \sum F_y = 0 \Rightarrow W - T_{AB} \times \frac{48}{68} - T_{AC} \times \frac{48}{75} - 120 \times \frac{48}{65} = 0 \quad \text{... (2)}$$

$$R_z = \sum F_z = 0 \Rightarrow T_{AB} \times \frac{36}{68} + T_{AC} \times \frac{36}{75} - 120 \times \frac{36}{65} = 0 \quad \text{... (3)}$$

From equations (1), (2) and (3),

$$\therefore \qquad \boxed{W = 177.23 \text{ N}} \qquad \text{... Ans.}$$

Q.10 The ball is suspended from the horizontal ring using three spring each having a stiffness of k = 50 N/m and an unstretched length of 1.5 m. If h = 2 m, determine the weight of ball. Refer Fig. Q.10.1.

[SPPU : Dec.-14, Marks 6]

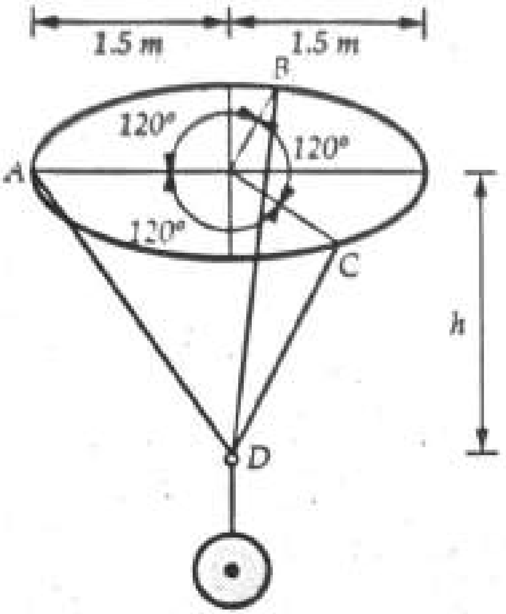
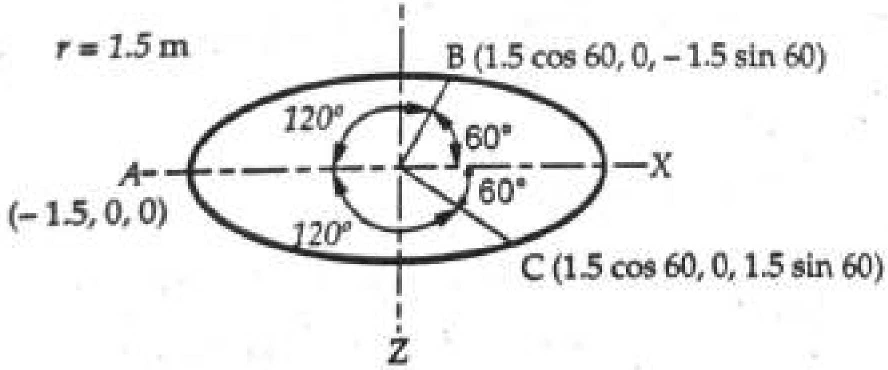


Fig. Q.10.1

Ans. : Taking x-axis horizontal, *y*-axis vertical, *z*-axis outward from the plane and the origin at the centre of the ring, the coordinates are :

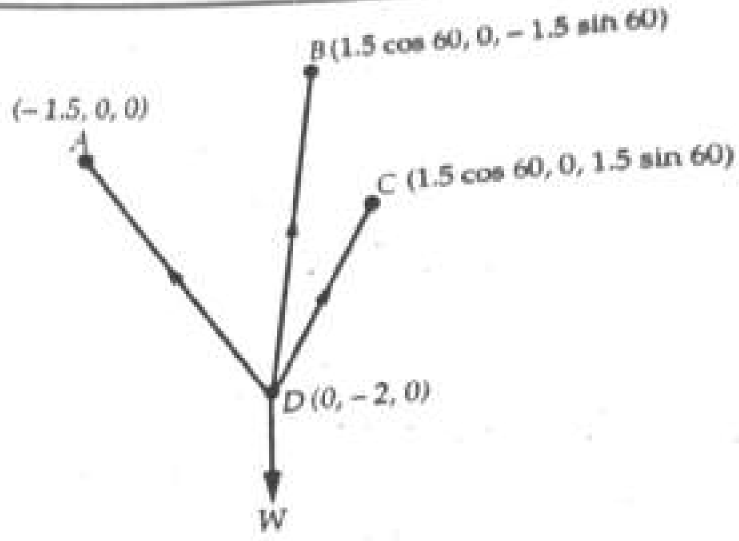
A (–1.5, 0, 0), *B* (1.5 cos 60, 0, – 1.5 sin 60),

C (1.5 cos 60, 0, 1.5 sin 60), *D* (0, –2, 0)



(a) Top view of ring

Fig. Q.10.1 (a)



(b)

Fig. Q.10.1 (b)

All the springs are of same length.

$$L_{DA} = \sqrt{(-1.5-0)^2 + (0+2)^2 + 0^2} = 2.5 \text{ m}$$

Deformation of spring DA = 2.5 – 1.5 = 1.0 m spring force $F = kx$

$$\therefore \quad F_{DA} = F_{DB} = F_{DC} = 50 \times 1 = 50 \text{ N}$$

All forces are concurrent at D. The force in DA, DB and DC will be directed away from D.

$$\vec{F}_{DA} = 50\left[\frac{-1.5\hat{i} + 2\hat{j} + 0\hat{k}}{\sqrt{1.5^2 + 2^2}}\right] = -30\hat{i} + 60\hat{j}$$

$$\vec{F}_{DB} = 50\left[\frac{1.5\cos 60\hat{i} + 2\hat{j} - 1.5\sin 60\hat{k}}{\sqrt{(1.5\cos 60)^2 + 2^2 + (1.5\sin 60)^2}}\right]$$

$$= 15\hat{i} + 60\hat{j} + 25.98\hat{k}$$

$$\vec{F}_{DC} = 50\left[\frac{1.5\cos 60\,\hat{i} + 2\,\hat{j} - 1.5\sin 60\,\hat{k}}{\sqrt{(1.5\cos 60)^2 + 2^2 + (1.5\sin 60)^2}}\right]$$

$$= 15\,\hat{i} + 60\,\hat{j} + 25.98\,\hat{k}$$

$$\vec{W} = -W\,\hat{j}$$

Hence reaction at D will be equal to weight of plate in upward direction.

Taking G as origin, x axis horizontally towards right, y-axis upward and z-axis outward from the plane,

$$\vec{W} = 1500 \times 9.81 \hat{j} = 14715 \hat{j}$$

The required coordinates are,

A (–1.2, 0, 1.2), B (1.2, 0, 1.2), C (0, 0, –1.2), D (0, 2.4, 0)

The tensile forces act away from D.

$$\vec{T}_{DA} = T_{DA}\left[\frac{-1.2\hat{i} - 2.4\hat{j} + 1.2\hat{k}}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}}\right]$$

$$\therefore \quad \vec{T}_{DA} = \frac{T_{DA}}{2.94}(-1.2\hat{i} + 2.4\hat{j} + 1.2\hat{k})$$

$$\vec{T}_{DB} = T_{DB}\left[\frac{1.2\hat{i} - 2.4\hat{j} + 1.2\hat{k}}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}}\right]$$

$$= \frac{T_{DB}}{2.94}(1.2\hat{i} - 2.4\hat{j} + 1.2\hat{k})$$

$$\vec{T}_{DC} = T_{DC}\left[\frac{0\hat{i} - 2.4\hat{j} - 1.2\hat{k}}{\sqrt{2.4^2 + 1.2^2}}\right]$$

$$= \frac{T_{DC}}{2.683}(0\hat{i} - 2.4\hat{j} - 1.2\hat{k})$$

$$\Sigma F_x = 0 : \frac{-1.2 T_{DA}}{2.94} + \frac{1.2 T_{DB}}{2.94} = 0 \quad \text{...(1)}$$

$$\Sigma F_y = 0 : 14715 - \frac{2.4 T_{DA}}{2.94} - \frac{2.4 T_{DB}}{2.94} - \frac{2.4 T_{DC}}{2.683} = 0 \quad \text{...(2)}$$

$$\Sigma F_z = 0 : \frac{1.2 T_{DA}}{2.94} + \frac{1.2 T_{DB}}{2.94} - \frac{1.2 T_{DC}}{2.683} = 0 \quad \text{...(3)}$$

From equations (1), (2) and (3),

$$\therefore \quad \boxed{T_{DA} = 4506.5 \text{ N}} \quad \text{... Ans.}$$

∴ $\boxed{T_{DB} = 4506.5 \text{ N}}$ ... Ans.

∴ $\boxed{T_{DC} = 8225.07 \text{ N}}$ ... Ans.

Q.12 If each cable can sustain a maximum tension of 600 N, determine the greatest weight of the bucket and its contents that can to supported. Refer Fig. Q.12.1. [SPPU : Dec.-15, Marks 6]

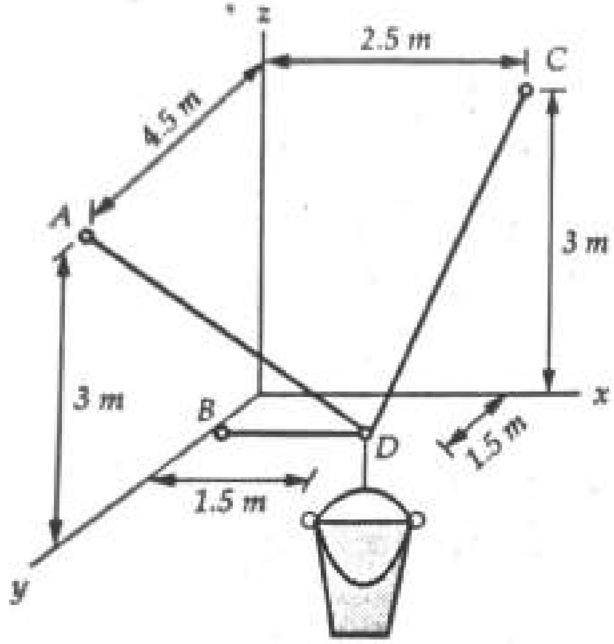


Fig. Q.12.1

Ans. : Let weight of bucket be W.

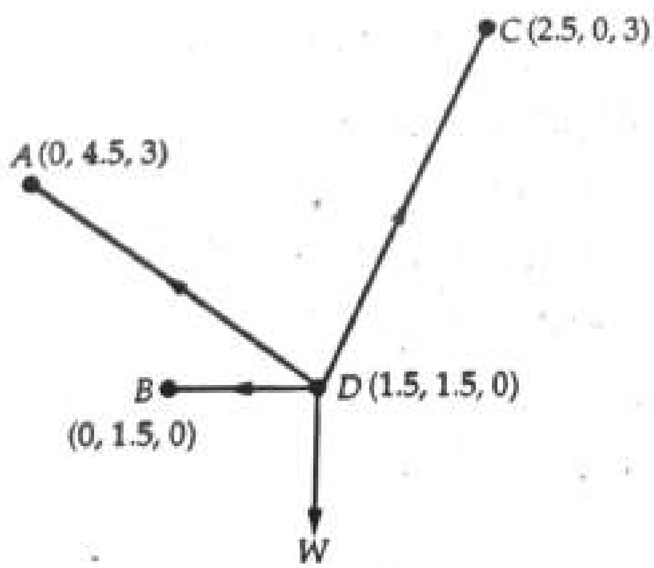


Fig. Q.12.1 (a)

$\therefore \quad \vec{W} = -W\hat{k}$

The co-ordinates of points are

A (0, 4.5, 3), B (0, 1.5, 0), C (2.5, 0, 3), D (1.5, 1.5, 0)

All forces are concurrent at D.

The tensile forces are directed away from D.

$$\vec{T}_{DB} = -T_{DB}\,\hat{i}$$

$$\vec{T}_{DA} = T_{DA}\left[\frac{-1.5\,\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{1.5^2+3^2+3^2}}\right]$$

$$= \frac{T_{DA}}{4.5}(-1.5\,\hat{i}+3\hat{j}+3\hat{k})$$

$$\vec{T}_{DC} = T_{DC}\left[\frac{1\hat{i}-1.5\hat{j}+3\hat{k}}{\sqrt{1^2+1.5^2+3^2}}\right]$$

$$= \frac{T_{DC}}{3.5}(\hat{i}-1.5\hat{j}+3\hat{k})$$

$$\sum F_x = 0 : \frac{-1.5T_{DA}}{4.5} - T_{DB} + \frac{T_{DC}}{3.5} = 0 \qquad \text{... (1)}$$

$$\sum F_y = 0 : \frac{3T_{DA}}{4.5} - \frac{1.5T_{DC}}{3.5} = 0 \qquad \text{... (2)}$$

$$\sum F_z = 0 : \frac{3T_{DA}}{4.5} + \frac{T_{DC}}{3.5} - W = 0 \qquad \text{... (3)}$$

From equations (1), (2) and (3),

$$T_{DA} = 0.5\text{ W}$$

$$T_{DB} = 0.0556\text{ W}$$

$$T_{DC} = 0.778\text{ W}$$

The largest tension is T_{DC}.

$\therefore \quad T_{DC} = 600\text{ N}$

$\therefore \quad 0.778\text{ W} = 600$

$\therefore \quad \boxed{\text{W} = 771.21\text{ N}}$ ... Ans.

Q.13 A 90 N load is suspended from the hook shown in Fig. Q.13.1. The load is supported by two cables and a spring having stiffness k = 500 N/m. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x-y plane and cable AC lies in the x-z plane :

[SPPU : May-16, Dec.-17, Marks 6]

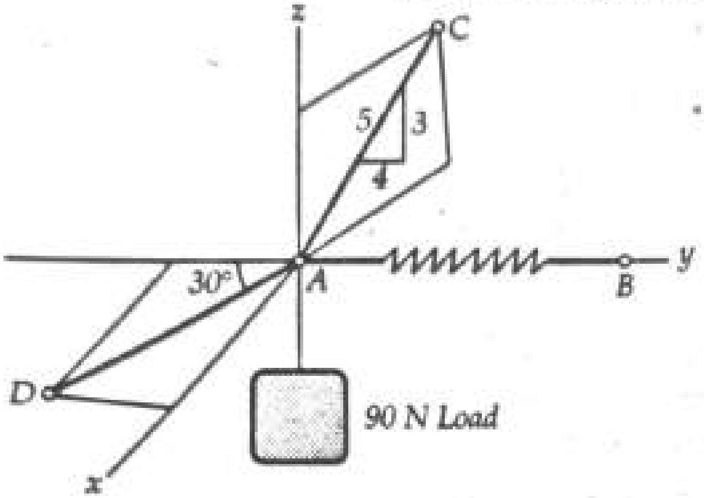


Fig. Q.13.1

Ans. :

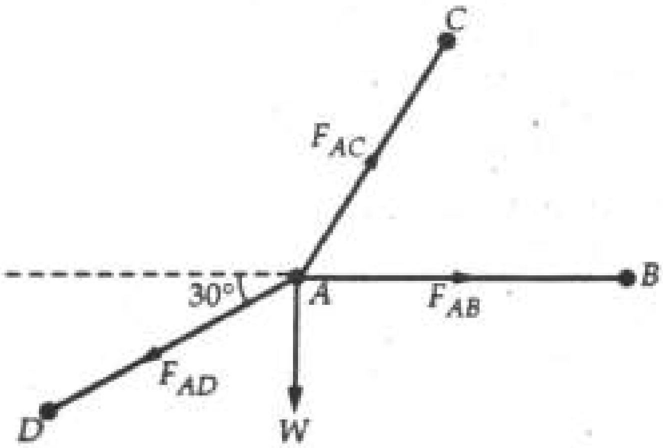


Fig. Q.13.1 (a)

$$\vec{F}_{AB} = F_{AB}\,\hat{j}$$

$$\vec{F}_{AC} = -F_{AC}\cos\theta\,\hat{i} + F_{AC}\sin\theta\,\hat{k}$$

$$\cos\theta = \frac{4}{5}, \quad \sin\theta = \frac{3}{5}$$

$\therefore$
$$\vec{F}_{AC} = -F_{AC}\times\frac{4}{5}\hat{i} + F_{AC}\times\frac{3}{5}\hat{k}$$

$$\vec{F}_{AD} = F_{AD}\sin 30\,\hat{i} - F_{AD}\cos 30\,\hat{j}$$

$$\vec{W} = -90\hat{k}$$

$$\sum F_x = 0 : -F_{AC} \times \frac{4}{5} + F_{AD} \sin 30 = 0 \quad ...(1)$$

$$\sum F_y = 0 : F_{AB} - F_{AD} \cos 30 = 0 \quad ...(2)$$

$$\sum F_z = 0 : +\frac{3}{5} F_{AC} - 90 = 0 \quad ...(3)$$

From equation (3).

$$\boxed{F_{AC} = 150 \text{ N}} \quad \text{... Ans.}$$

Substitute in equation (1).

$$\boxed{F_{AD} = 240 \text{ N}} \quad \text{... Ans.}$$

From equation (2),

$$F_{AB} = 207.85 \text{ N}$$

$$F_{AB} = k\,x \text{ where } k = 500 \text{ N/m}$$

$$\therefore \quad 207.85 = 500\,x$$

$$\therefore \quad \boxed{x = 0.416 \text{ m}} \quad \text{... Ans.}$$

Q.14 The square steel plate has mass of 1800 kg with mass center G as shown in Fig. Q.14.1. Determine the tension in each cable so that the plate remains horizontal. [SPPU : Dec.-16, May-17, Marks 6]

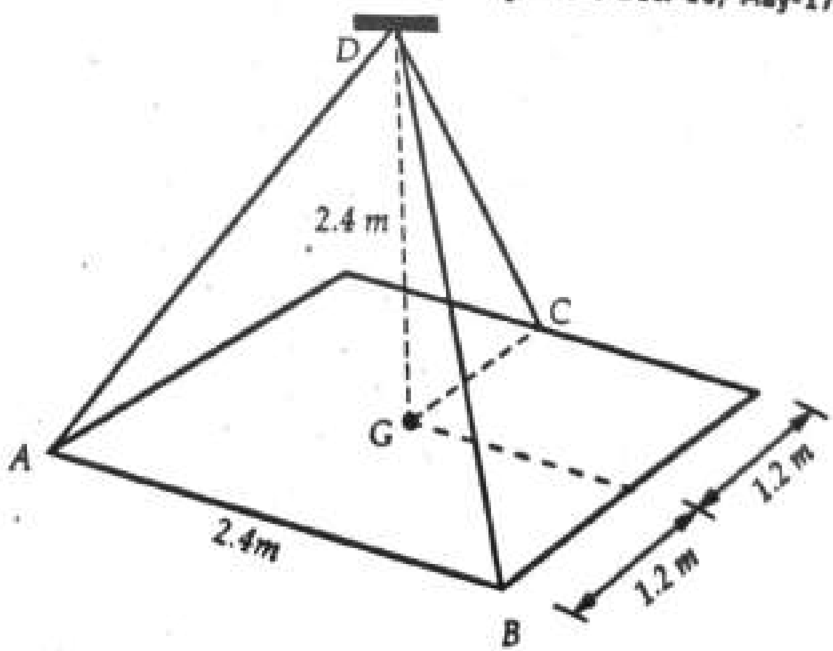


Fig. Q.14.1

Ans. : The weight of plate acts at G in downward direction.

Hence reaction at D will be equal to weight of plate in upward direction.

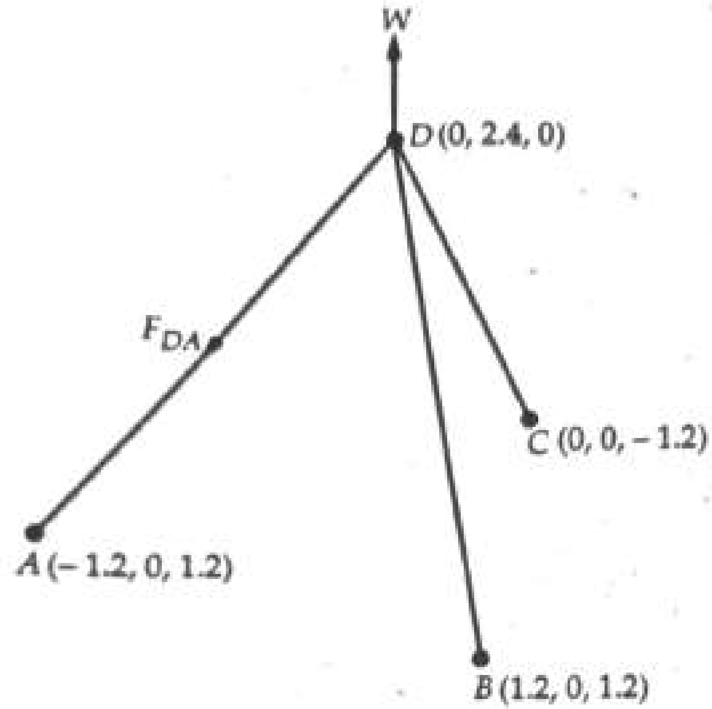


Fig. Q.14.1 (a)

Taking G as origin, x axis horizontally towards right, y-axis upward and z-axis outward from the plane,

$$\vec{W} = 1800 \times 9.81 \hat{j} = 17658 \hat{j}$$

The required coordinates are,

A (–1.2, 0, 1.2), B (1.2, 0, 1.2), C (0, 0, –1.2), D (0, 2.4, 0)

The tensile forces act away from D.

$$\vec{T}_{DA} = T_{DA}\left[\frac{-1.2\,\hat{i} - 2.4\hat{j} + 1.2\hat{k}}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}}\right]$$

$$\therefore \quad \vec{T}_{DA} = \frac{T_{DA}}{2.94}(-1.2\,\hat{i} + 2.4\hat{j} + 1.2\hat{k})$$

$$\vec{T}_{DB} = T_{DB}\left[\frac{1.2\,\hat{i} - 2.4\ \hat{j}\ + 1.2\,\hat{k}}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}}\right]$$

$$= \frac{T_{DB}}{2.94}(1.2\,\hat{i} - 2.4\hat{j} + 1.2\,\hat{k})$$

$$\vec{T}_{DC} = T_{DC}\left[\frac{0\,\hat{i} - 2.4\,\hat{j} - 1.2\,\hat{k}}{\sqrt{2.4^2 + 1.2^2}}\right]$$

$$= \frac{T_{DC}}{2.683}(0\,\hat{i} - 2.4\,\hat{j} - 1.2\,\hat{k})$$

$$\Sigma F_x = 0 : \frac{-1.2\,T_{DA}}{2.94} + \frac{1.2\,T_{DB}}{2.94} = 0 \quad \text{...(1)}$$

$$\Sigma F_y = 0 : 17658 - \frac{2.4\,T_{DA}}{2.94} - \frac{2.4\,T_{DB}}{2.94} - \frac{2.4\,T_{DC}}{2.683} = 0 \quad \text{...(2)}$$

$$\Sigma F_z = 0 : \frac{1.2\,T_{DA}}{2.94} + \frac{1.2\,T_{DB}}{2.94} - \frac{1.2\,T_{DC}}{2.683} = 0 \quad \text{...(3)}$$

From equations (1), (2) and (3),

$\therefore$ $\boxed{T_{DA} = 5407.8 \text{ N}}$ ... Ans.

$\therefore$ $\boxed{T_{DB} = 5407.8 \text{ N}}$ ... Ans.

$\therefore$ $\boxed{T_{DC} = 9870 \text{ N}}$ ... Ans.

Q.15 The 10 kg lamp shown in Fig. Q.15.1 is suspended from three equal length cords. Determine its smallest vertical distances from the ceiling if the force developed in any cord is not allowed to exceed 50 N. [SPPU : May-18, Marks 7]

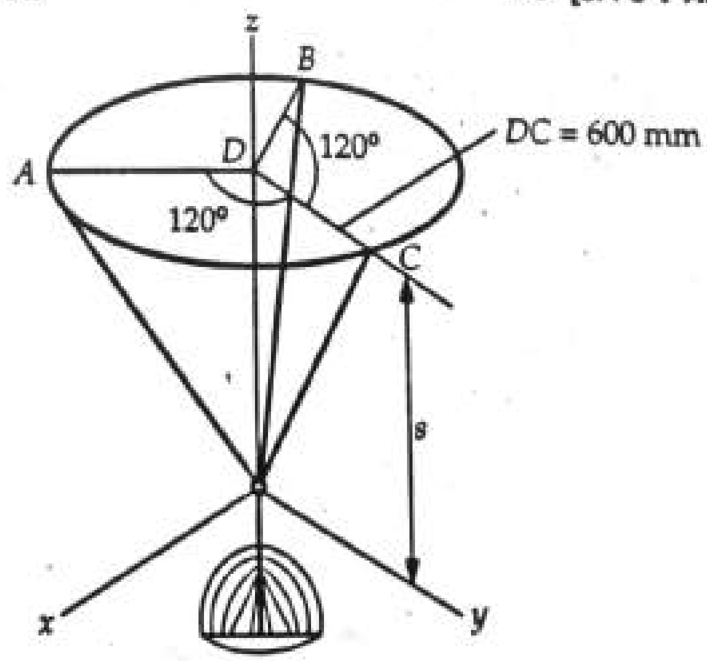


Fig. Q.15.1

Ans. : Let θ be the angle made by each cord with z-axis. Then,

$$\tan\theta = \frac{600}{s} \qquad \ldots(1)$$

Vertical component of each force is 50 cos θ.

The total vertical component of the three tensile forces balances the weight of lamp.

$$\therefore \quad 3\times 50\cos\theta = 10\times 9.81$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{98.1}{150}\right) = 49.16°$$

From equation (1), $\tan 49.16 = \frac{600}{s}$

$$\therefore \quad \boxed{s = 518.64 \text{ mm}}$$

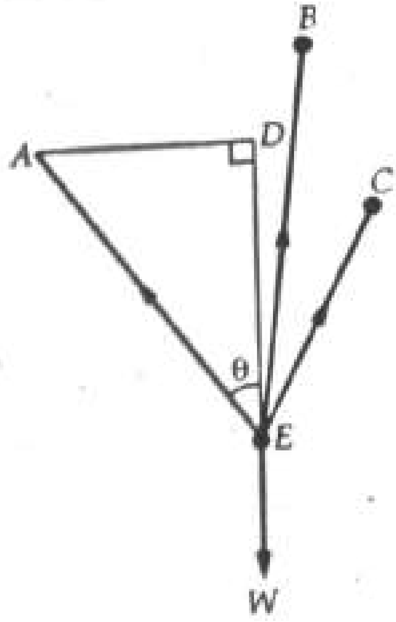


Fig. Q.15.1 (a)

Q.16 Three cables are used to support a container as shown in the Fig. Q.16.1. Determine the tension in the cables AB, AC and AD if the weight of the container is 1000 N. [SPPU : Dec.-18, Marks 7]

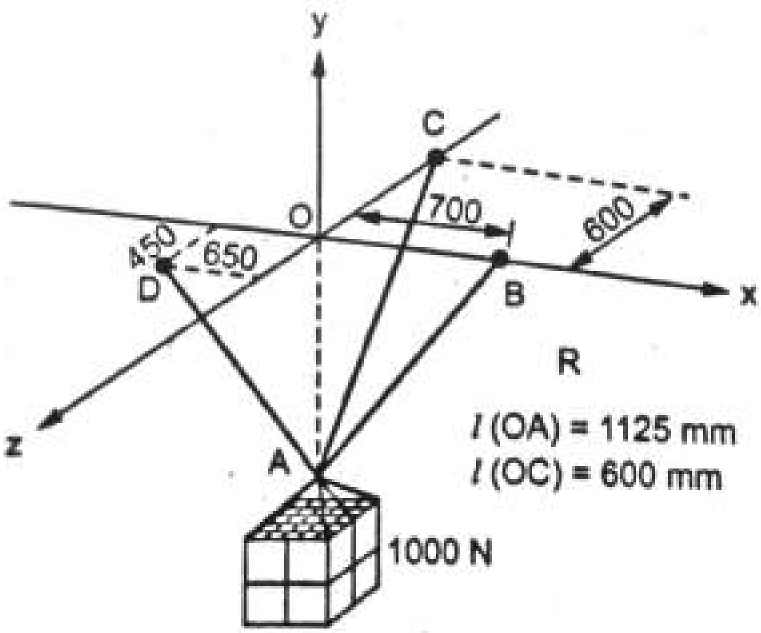


Fig. Q.16.1

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Ans. : W = 1000 N

The coordinates of point A, B, C and D are

A (0, – 1125, 0) B (700, 0, 0)

C (0, 0, – 600) D (– 650, 0, 450)

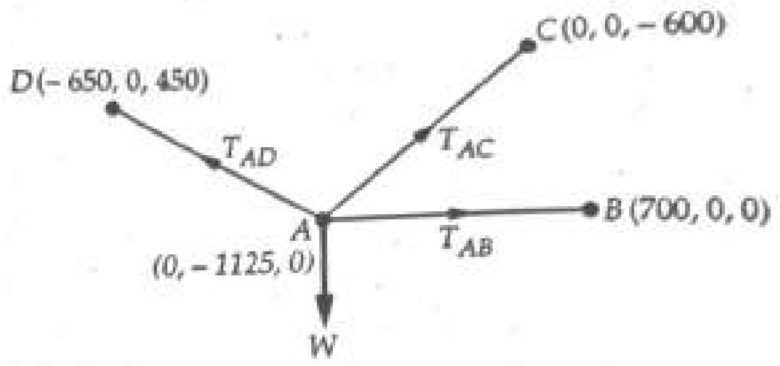

Fig. Q.16.1 (a)

i) $$\bar{T}_{AC} = T_{AC}\,\bar{e}_{AC}$$

$$= T_{AC}\left[\frac{(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}\right]$$

$\therefore$ $$\bar{T}_{AC} = T_{AC}\left[\frac{0i+(0-(-1125))j+(-600-0)k}{\sqrt{1125^2+600^2}}\right]$$

$\therefore$ $$\bar{T}_{AC} = T_{AC}(0.882j-0.471k)$$

ii) $$\bar{T}_{AB} = T_{AB}\,\bar{e}_{AB}$$

$$= T_{AB}\left[\frac{(700-0)i+(0-(-1125))j+0k}{\sqrt{700^2+1125^2}}\right]$$

$\therefore$ $$\bar{T}_{AB} = T_{AB}[0.528i+0.849j]$$

iii) $$\bar{T}_{AD} = T_{AD}\bar{e}_{AD}$$

$$= T_{AD}\left[\frac{-650i+(0-(-1125))j+450k}{\sqrt{650^2+1125^2+450^2}}\right]$$

$\therefore$ $$\bar{T}_{AD} = T_{AD}(-0.473i+0.818j+0.327k)$$

iv) $$\bar{W} = -1000\,j$$

Now applying conditions of equilibrium

$\sum F_x = 0 = 0.528 T_{AB} - 0.473 T_{AD}$

$\sum F_y = 0 = 0.882 T_{AC} + 0.849 T_{AB} + 0.818 T_{AD} - 1000$

$\sum F_z = 0 = -0.471 T_{AC} + 0.327 T_{AD}$

Solving above equations we get,

$$\therefore \quad \boxed{\begin{aligned} T_{AC} &= 316.885 \text{ N} \\ T_{AB} &= 408.887 \text{ N} \\ T_{AD} &= 456.432 \text{ N} \end{aligned}} \quad \text{...Ans.}$$

Q.17 The uniform concrete slab has a weight of 5500 N. Determine tension in each of the three parallel supporting cables when the slab is held in horizontal plane as shown in Fig. Q.17.1.
[SPPU : May-10]

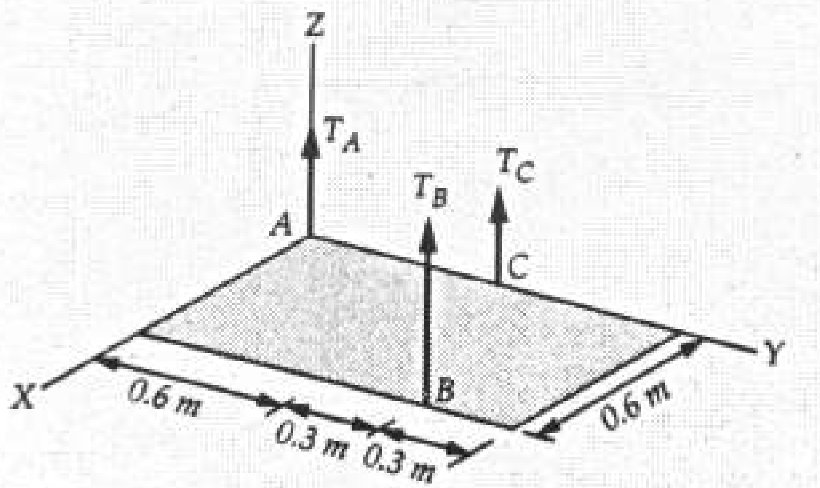

Fig. Q.17.1

Ans. : Weight of slab acts at its centre .

Using $\sum M_{y\text{-}axis} = 0$

$\therefore (T_B)\ (0.6) + (5500)\ (0.3) = 0$

$$\therefore \quad \boxed{T_B = 2750 \text{ N}} \quad \text{... Ans.}$$

$\sum M_{x\text{-}axis} = 0$

$\therefore (T_B)\ (0.9) + (T_C)\ (0.6) - (5500)\ (0.6) = 0$

$$\therefore \qquad \boxed{T_C = 1375 \text{ N}} \qquad \text{... Ans.}$$

$$\sum F_z = 0$$

$$T_A + T_B + T_C - 5500 = 0$$

$$\therefore \qquad \boxed{T_A = 1375 \text{ N}} \qquad \text{... Ans.}$$

Q.18 Determine the magnitude of the resultant and its location with respect to origin O as shown in Fig. Q.18.1.

[SPPU : May-11, Marks 6]

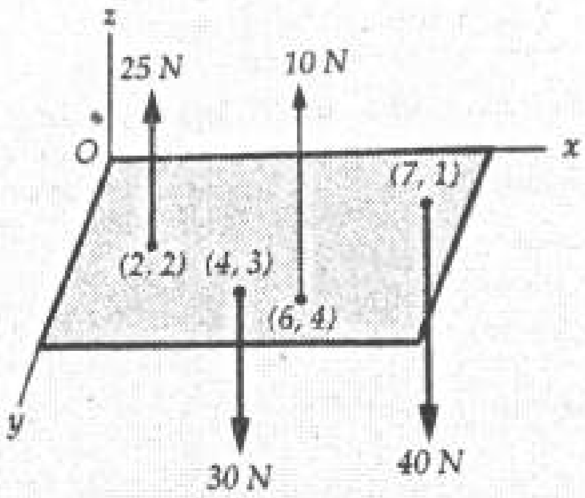

Fig. Q.18.1

Ans. :

$$R_x = R_y = 0$$

$$R_z = \sum F_z = 25 + 10 - 30 - 40$$

$$\therefore \qquad R_z = -35 \text{ N}$$

$$\therefore \qquad \boxed{\vec{R} = -35\ \hat{k} \text{ N}} \qquad \text{... Ans.}$$

Let the resultant act at point $(x, y, 0)$ in the X-Z plane.

Using Varignon's theorem about Y-axis,

$$(\sum M)_y = (M_R)_y$$

$$(25)(2) + (10)(6) - (30)(4) - (40)(7) = -(35)(x)$$

$$\therefore \qquad \boxed{x = 8.29} \qquad \text{... Ans.}$$

Using Varignon's theorem about X-axis,

$$(\sum M)_x = (M_R)_x$$

$$-(25)(2)-(10)(4)+(30)(3)+(40)(1) = (35)(y)$$

$$\therefore \quad \boxed{y = 1.143} \quad \text{... Ans.}$$

∴ The resultant acts at point (8.29, 1.143, 0)

Q.19 Three parallel bolting forces act on the rim of the circular cover plate of radius r = 0.8 m as shown in Fig. Q.19.1. Determine the magnitude and direction of a resultant force equivalent to the given force system and locate its point of application on the cover plate.

[SPPU : May-12, Marks 7, Dec.-14, May-16, Marks 6]

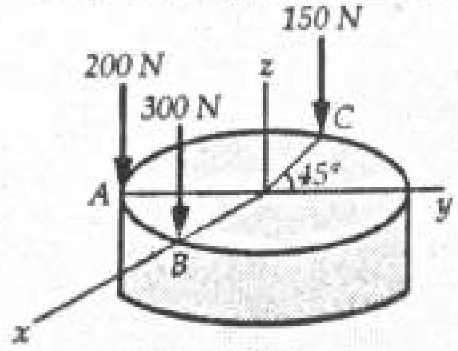

Fig. Q.19.1

Ans. :

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$R_z = \sum F_z = -200-300-150$$

$$\therefore \quad R_z = -650 \text{ N}$$

$$\therefore \quad \boxed{R = 650 \text{ N}\downarrow} \quad \text{... Ans.}$$

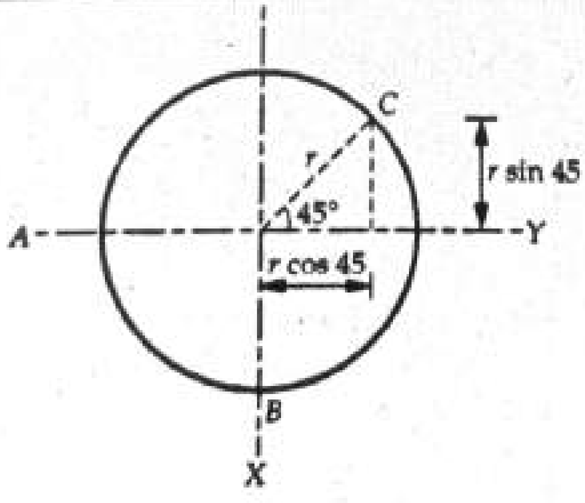

Fig. Q.19.1 (a)

Let the resultant force act at point (x, y, 0) on the cover plate.

Using Varignon's theorem about y-axis,

$$(\sum M)_y = (M_R)_y$$

$$(300)(0.8) - (150)(0.8 \sin 45) = (650)(x)$$

$$\therefore \quad \boxed{x = 0.24 \text{ m}} \quad \text{... Ans.}$$

Using Varignon's theorem about x-axis,

$$(\sum M)_x = (M_R)_x$$

$$(200)(0.8) - (150)(0.8 \cos 45) = -(650)(y)$$

$$\therefore \quad \boxed{y = -0.116 \text{ m}} \quad \text{... Ans.}$$

The point of application is (0.24, – 0.116, 0)

Q.20 A square foundation supports four loads as shown in Fig. Q.20.1. Determine magnitude, direciton and point of application of resultant of four forces. **[SPPU : Dec.-12, Marks 6]**

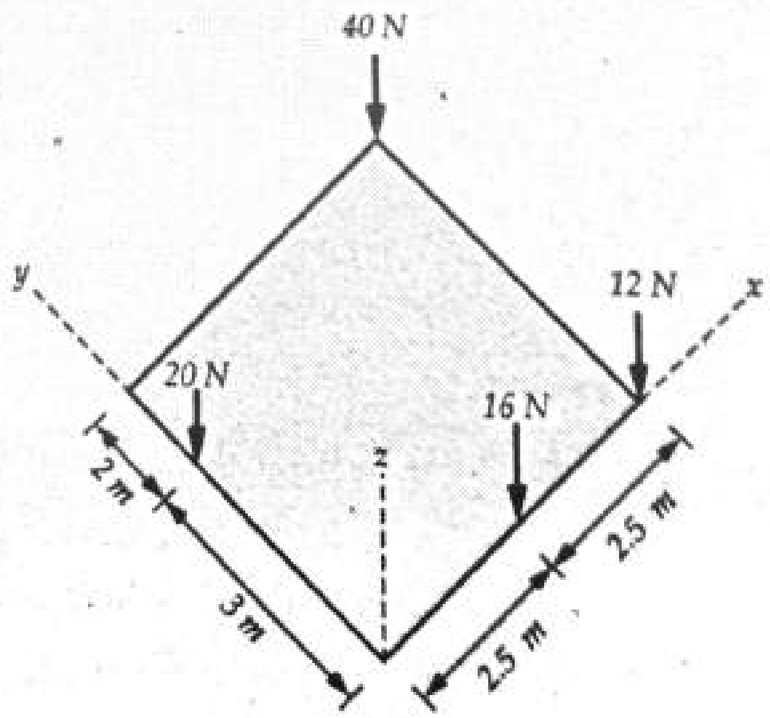

Fig. Q.20.1

Ans. :

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$R_z = \sum F_z = -16 - 12 - 20 - 40$$

$$\therefore \quad R_z = -88 \text{ N}$$

$$\boxed{R = 88 \text{ N in negative z-direction}} \quad \text{... Ans.}$$

Let the resultant act at point $(x, y, 0)$.

Using Varignon's theorem about X-axis,

$$\left(\sum M\right)_x = (M_R)_x$$

$$-(20)(3) - (40)(5) = -(88)(y)$$

$$\boxed{y = 2.955 \text{ m}}$$

Using Varignon's theorem about Y-axis,

$$\left(\sum M\right)_y = (M_R)_y$$

$$(16)(2.5) + (12)(5) + (40)(5) = (88)(x)$$

$$\therefore \quad \boxed{x = 3.41 \text{ m}}$$

$\therefore$ Point of application is (3.41, 2.955, 0) m ... Ans.

Q.21 A concrete foundation mat in the shape of regular hexagon with 3 m side support column loads as shown in Fig. Q.21.1. Determine the magnitude of the additional loads P_1 and P_2 that must be applied at B and F if resultant of all six loads is to pass through the centre of the mat. [SPPU : May-13, Marks 5]

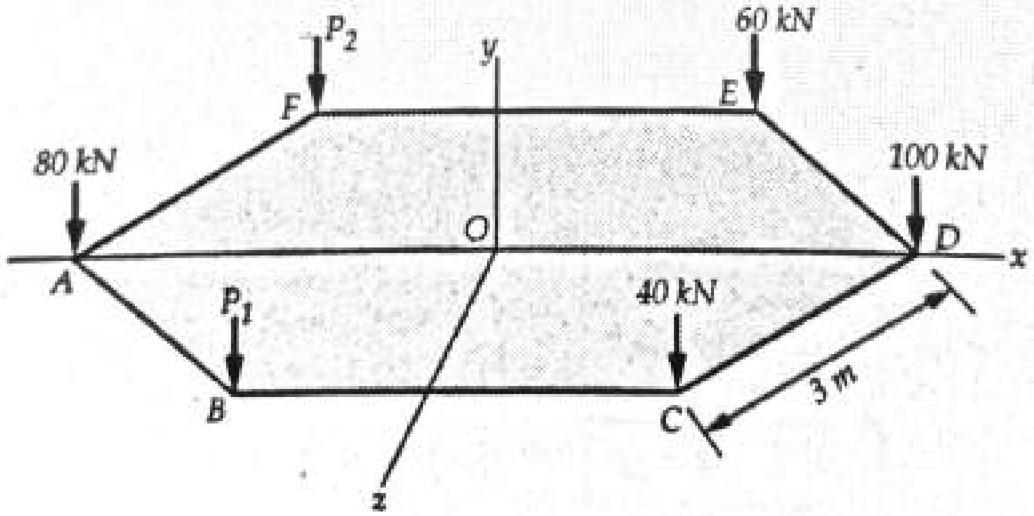

Fig. Q.21.1

Ans. : The distances of points B, C, E and F are shown in Fig. Q.21.1 (a).

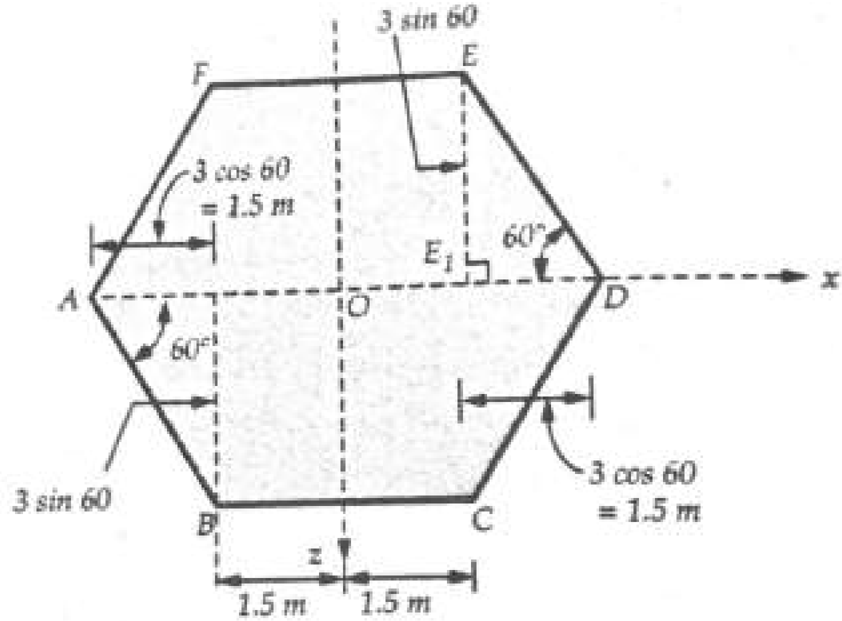

Fig. Q.21.1 (a)

Using Varignon's theorem about x-axis,

$$(\sum M)_x = (M_R)_x$$

$$(P_1)(3 \sin 60) - (P_2)(3 \sin 60) + (40)(3 \sin 60) - (60)(3 \sin 60) = 0$$

$$\therefore \quad P_1 - P_2 = 20 \qquad \text{...(1)}$$

Using Varignon's theorem about z-axis,

$$(\sum M)_z = (M_R)_z$$

$$(P_1)(1.5) + (P_2)(1.5) + (80)(3)$$
$$-(60)(1.5) - (40)(1.5) - (100)(3) = 0$$

$$\therefore \quad P_1 + P_2 = 140 \qquad \text{...(2)}$$

From equations (1) and (2),

$$\boxed{P_1 = 80 \text{ kN}} \qquad \text{... Ans.}$$

$$\boxed{P_2 = 60 \text{ kN}} \qquad \text{... Ans.}$$

Q.22 A concrete foundation mat of 5 m radius supports four equally spaced columns, each of which is located 4 m from the centre of the mat. Determine the magnitude and point of application of the resultant of the four loads as shown in Fig. Q.22.1. [SPPU : Dec.-13, May-19, Marks 6]

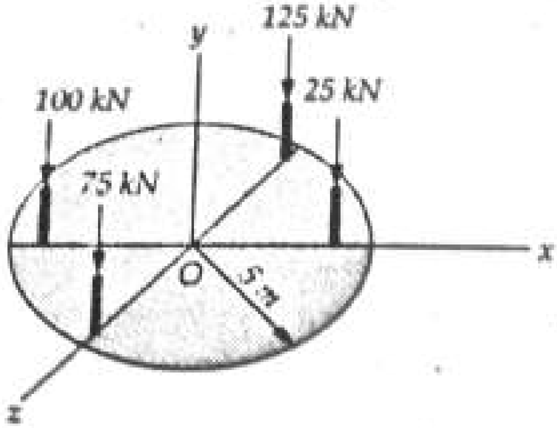

Fig. Q.22.1

Ans. :

$$R_x = 0, \quad R_z = 0$$

$$R_y = \sum F_y = -25 - 125 - 100 - 75$$

$\therefore$ $$R_y = -325 \text{ kN}$$

$\therefore$ $$\boxed{R = 325 \text{ kN}}$$ ... Ans.

Let the resultant act at point $(x, 0, z)$.

Using Varignon's theorem about Z-axis,

$$\left(\sum M\right)_z = (M_R)_z$$

$$(100)(4) - (25)(4) = -(325)(x)$$

$\therefore$ $$\boxed{x = -0.923 \text{ m}}$$ ... Ans.

Using Varignon's theorem about X-axis,

$$\left(\sum M\right)_x = (M_R)_x$$

$$(75)(4) - (125)(4) = (325)(z)$$

$\therefore$ $$\boxed{z = -0.615 \text{ m}}$$ ... Ans.

$\therefore$ Point of application of resultant is

$\therefore$ $$\boxed{(-0.923, 0, -0.615)}$$ ... Ans.

Q.23 A 10 kN platform is supported vertically at A, B and C as shown in Fig. Q.23.1. If it carries a load of 6 kN, determine the reaction at each support. [SPPU : May-14, Marks 6]

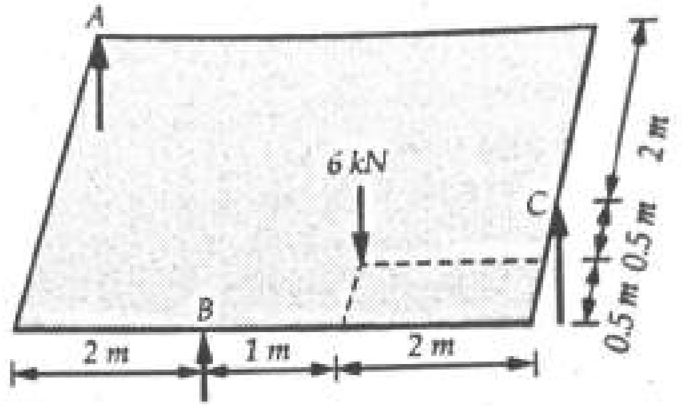

Fig. Q.23.1

Ans. : The weight of platform acts through the centre of gravity G as shown in Fig. Q.23.1 (a). The reactions at *A*, *B* and *C* are vertically upwards as shown in Fig. Q.23.1 (a).

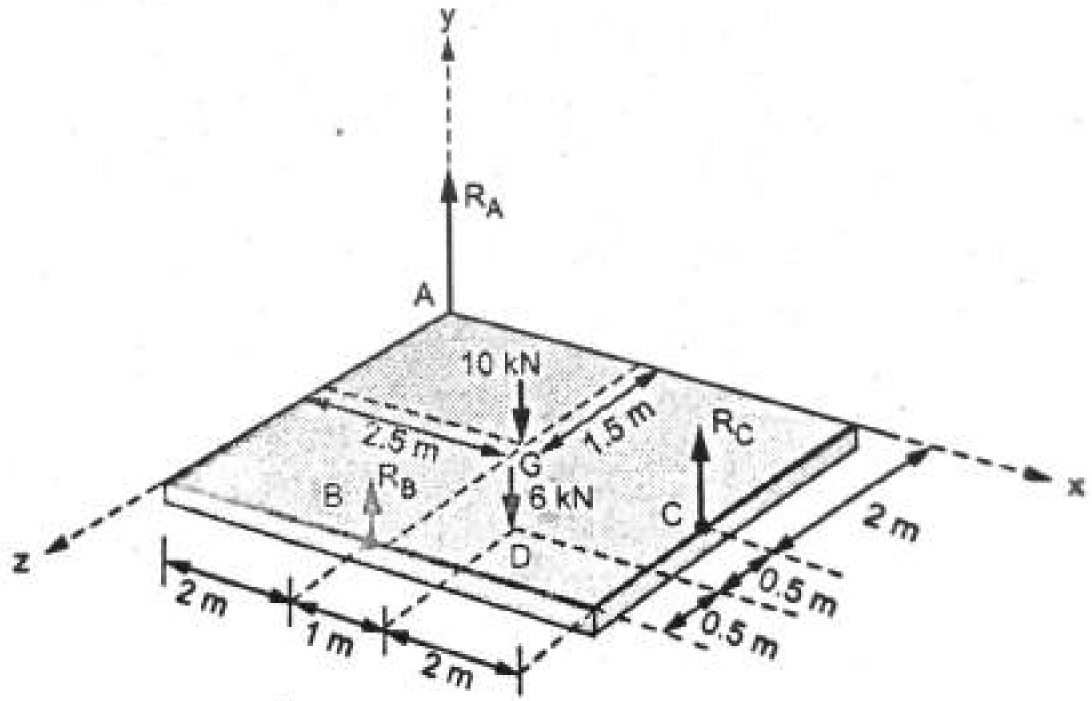

Fig. Q.23.1 (a)

$\sum M_{x\text{-axis}} = 0$:

$(10)(1.5) + (6)(2.5) - (R_C)(2) - (R_B)(3) = 0$

$\therefore \quad 3 R_B + 2 R_C = 30$ ... (1)

$$\sum M_{x\text{-axis}} = 0 \ :$$

$(R_B)(2) + (R_C)(5) - (10)(2.5) - (6)(3) = 0$

$\therefore \quad 2R_B + 5R_C = 43$ ... (2)

From (1) and (2)

$\therefore \quad \boxed{R_B = 5.818 \text{ kN} \uparrow}$ ... Ans.

$\therefore \quad \boxed{R_C = 6.273 \text{ kN} \uparrow}$ ... Ans.

$$\sum F_y = 0 \ :$$

$R_A + R_B + R_C - 10 - 6 = 0$

$\therefore \quad \boxed{R_A = 3.909 \text{ kN} \uparrow}$ ... Ans.

Q.24 Determine the resultant of the parallel force system which act on the plate as shown in Fig. Q.24.1. [SPPU : May-15, Marks 6]

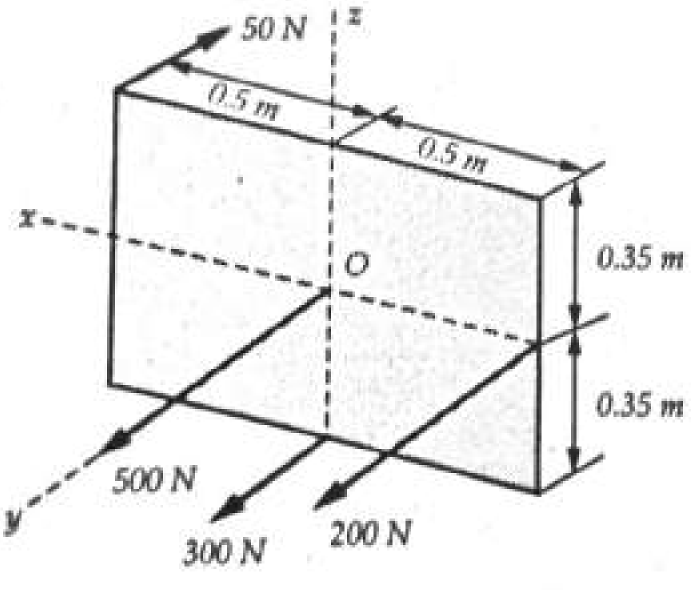

Fig. Q.24.1

Ans. : $R_x = R_z = 0$

$R_y = \Sigma F_y = 500 + 300 + 200 - 50$

$\therefore \quad R_y = 950 \text{ N}$

$\therefore \quad \boxed{\bar{R} = 950 \hat{j} \text{ N}}$ ... Ans.

Let the resultant act at point (x, 0, z) in first quadrant of x-z plane.

Using Varignon's theorem about z-axis,

$$(\sum M)_z = (M_R)_z$$

$$-200 \times 0.5 - 50 \times 0.5 = +950 \times x$$

$$\therefore \quad \boxed{x = -0.1316 \text{ m}} \quad \text{... Ans.}$$

Using Varignon's theorem about x-axis,

$$(\sum M)_x = (M_R)_x$$

$$300 \times 0.35 + 50 \times 0.35 = -950 \times z$$

$$\therefore \quad \boxed{z = -0.129 \text{ m}} \quad \text{... Ans.}$$

∴ Resultant acts at point (–0.1316, 0, –0.129) m.

Q.25 Determine the magnitude and direction of a resultant force of a given force system as shown in Fig. Q.25.1 and locate its point of application on the slab. **[SPPU : Dec.-15, Marks 6]**

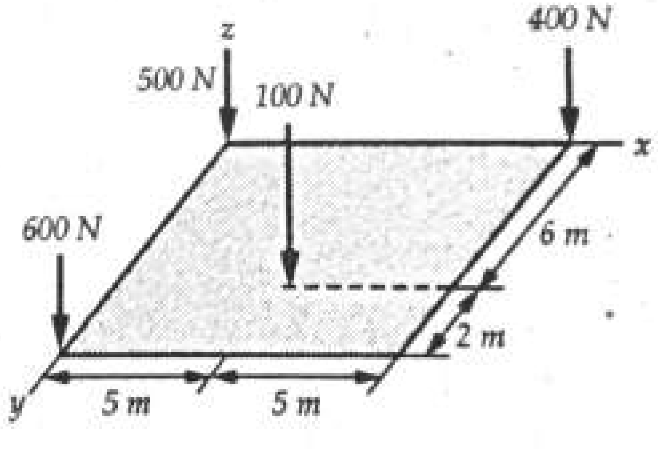

Fig. Q.25.1

Ans. :

$$R_x = R_y = 0$$

$$R_z = \sum F_z$$

$$\therefore \quad R_z = -600-500-100-400 = -1600 \text{ N}$$

$$\therefore \quad \boxed{\mathbf{R = 1600\ N \downarrow}} \quad \text{... Ans.}$$

Let the resultant act at point (x, y, 0) m. Using Varignon's theorem about y-axis,

$$(\sum M)_y = (M_R)_y$$

$$-(100)(5)-(400)(10) = -(1600)(x)$$

$\therefore$ $$\boxed{x = 2.8125 \text{ m}}$$ ... Ans.

Using Varignon's theorem about x-axis,

$$(\sum M)_x = (M_R)_x$$

$$(100)(6)+(600)(8) = (1600)(y)$$

$\therefore$ $$\boxed{y = 3.375 \text{ m}}$$ ... Ans.

The point of application of resultant force is (2.8125, 3.375, 0) m.

Q.26 Four parallel bolting forces act on the rim of the circular cover plate as shown in Fig. Q.26.1. If the resultant force 750 N is passing through (0.15 m, 0.1 m) from the origin 0, determine the magnitude of forces P_1 and P_2. [SPPU : Dec.-16, Marks 6]

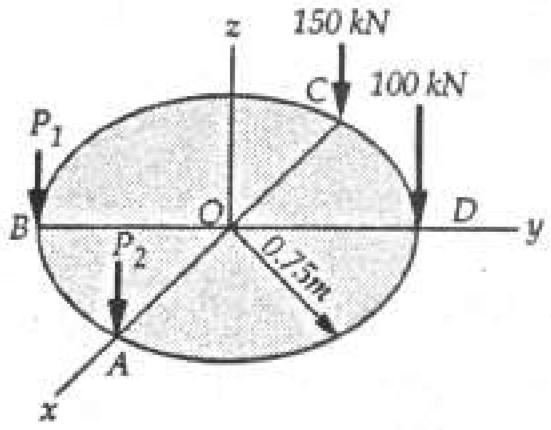

Fig. Q.26.1

Ans. : The resultant will act downward at the given point.

Using Varignon's theorem about X-axis,

$$(\sum M)_x = (M_R)_x$$

$$P_1 \times 0.75 - 100 \times 0.75 = 750 \times 0.1$$

$\therefore$ $$\boxed{P_1 = 0}$$ ... Ans.

Using Varignon's theorem about Y - axis

$$(\sum M)_y = (M_R)_y$$

From equations (1), (2) and (3).

$$\boxed{F_1 = 60 \text{ kN}}$$ ... Ans.

$$\boxed{F_2 = 90 \text{ kN}}$$ ... Ans.

Q.29 The square mat foundations supports four columns as shown in the Fig. Q.29.1. Determine the magnitude and position of the resultant force w.r. to origin 'O'. **[SPPU : Dec.-18, Marks 7]**

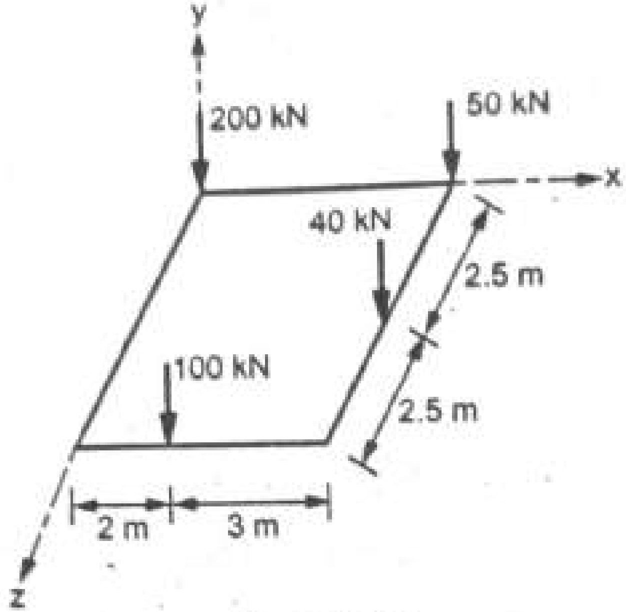

Fig. Q.29.1

Ans. :

$$R_x = R_z = 0$$

$$R_y = \sum F_y = -200 - 100 - 40 - 50$$

$\therefore$ $$R_y = -390 \text{ kN} = 390 \text{ kN} \downarrow$$ ...Ans.

Let the resultant act at point (x, 0, z) m. Using Varignon's theorem about z - axis,

$$-50 \times 5 - 100 \times 2 - 40 \times 5 = -390 \times x$$

$\therefore$ $$\boxed{x = 1.67 \text{ m}}$$

Similarly, moment about x - axis,

$$40 \times 2.5 + 100 \times 5 = 390 \times z$$

$\therefore$ $$\boxed{z = 1.538 \text{ m}}$$

Q.30 A uniform rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown in Fig. Q.30.1. Determine the tension in each wire. **[SPPU : May-10]**

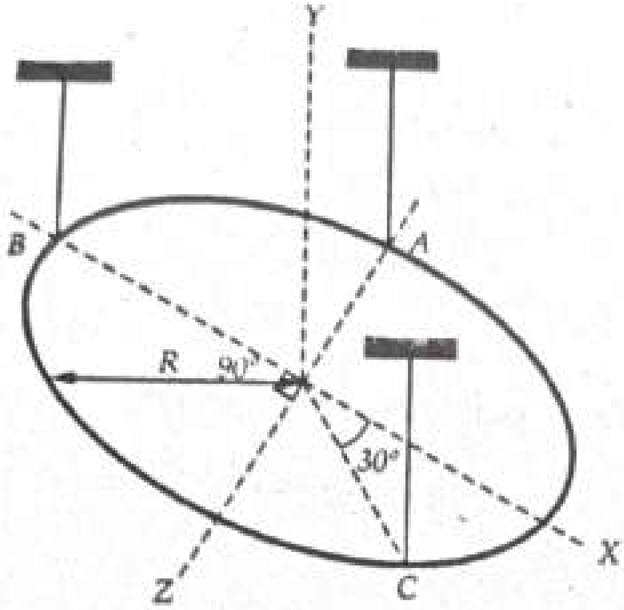

Fig. Q.30.1

Ans. : The tensile forces act vertically upwards on the ring and its weight acts downwards through the centre.

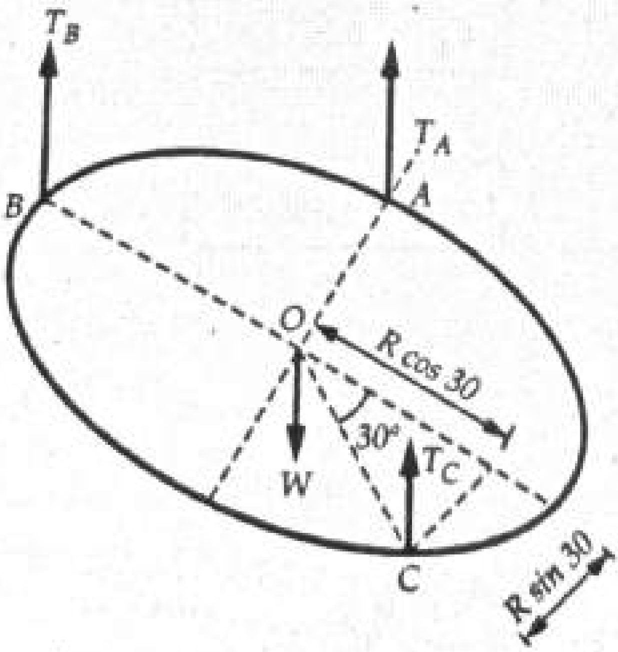

Fig. Q.30.1 (a)

$$P_2 \times 0.75 - 150 \times 0.75 = 750 \times 0.15$$

$\therefore$ $$\boxed{P_2 = 300 \text{ kN}}$$ ... Ans.

Q.27 Four parallel bolting forces act on the rim of the circular cover plate as shown in Fig. Q.27.1. If the resultant force 750 N is passing through (0.15 m, 0.1 m) from the origin O, determine the magnitude of forces P_1 and P_2. **[SPPU : May-17, Marks 6]**

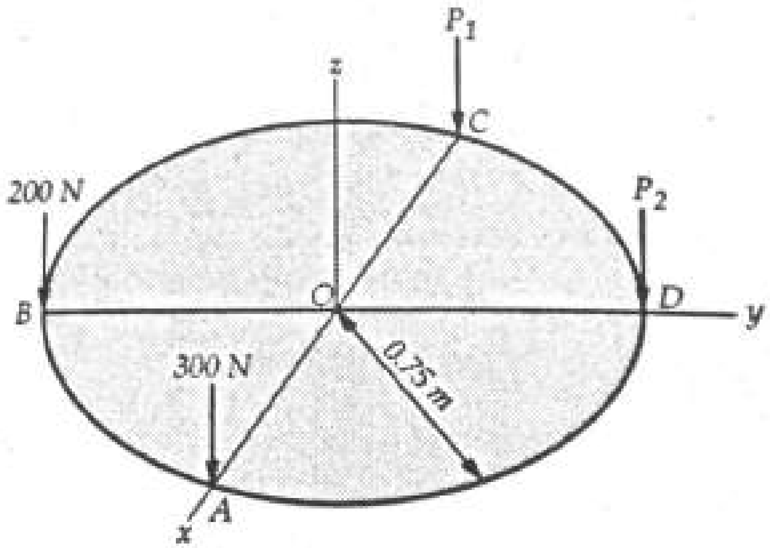

Fig. Q.27.1

Ans. : The resultant will act downward at the given point.

Using varignon's theorem about x - axis,

$$(\sum M)_x = (M_R)_x$$

$$200 \times 0.75 - P_2 \times 0.75 = -750 \times 0.1$$

$\therefore$ $$\boxed{P_2 = 300 \text{ N}}$$ ... Ans.

$\sum F_z = R_z$:

$$P_1 + P_2 + 200 + 300 = 750$$

$\therefore$ $$P_1 = -50 \text{ N}$$

$\therefore$ Magnitude of P_1 is

$$\boxed{P_1 = 50 \text{ N}}$$ ... Ans.

Note : The answers are different if Varignon's theorem is used first about y - axis to find F_1. This is due to incorrect position of resultant.

Q.28 The building slab is subjected to four parallel column loading shown in Fig. Q.28.1. Determine F_1 and F_2 if the resultant force acts through point (12 m, 10 m). [SPPU : May-18, Marks 6]

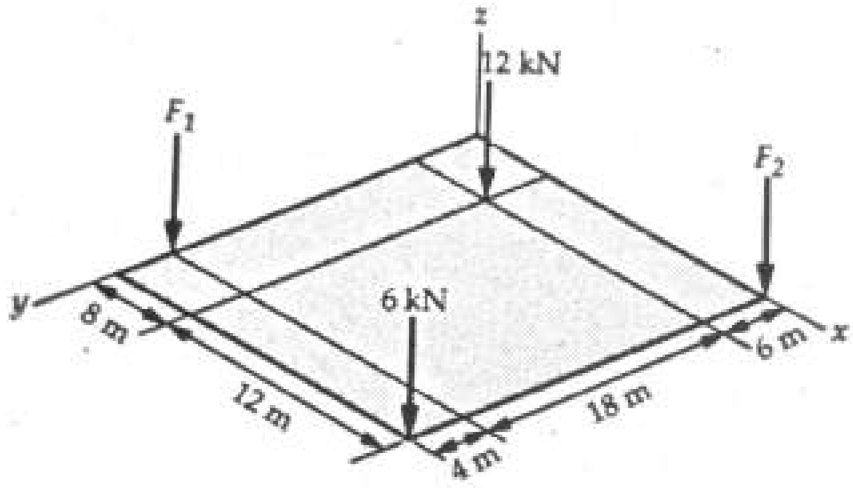

Fig. Q.28.1

Ans. : Using Varignon's theorem about x-axis,

$$(\sum M)_x = (M_R)_x$$

$$(12)(6)+(6)(28)+(F_1)(24) = R \times 10$$

$$\therefore \quad 10R - 24F_1 = 240 \qquad ...(1)$$

Using Varignon's theorem about y-axis,

$$(\sum M)_y = (M_R)_y$$

$$-12\times 8 - 6\times 20 - F_2 \times 20 = -R\times 12$$

$$\therefore \quad 12R - 20F_2 = 216 \qquad ...(2)$$

$$\sum F_z = R_Z$$

$$\therefore \quad -F_1 - F_2 - 12 - 6 = -R$$

$$\therefore \quad R - F_1 - F_2 = 18 \qquad ...(3)$$

$$\sum F_y = 0:$$

$$T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = W \qquad \ldots(1)$$

$$\sum M_{x\text{-axis}} = 0:$$

$$(T_A)(R) - (T_C)(R \sin 30) = 0$$

$$\therefore \quad T_A = 0.5\, T_C \qquad \ldots(2)$$

$\sum M_{z\text{-axis}} = 0:$

$$-(T_B)(R) + (T_C)(R \cos 30) = 0$$

$$\therefore \quad T_B = T_C \cos 30 \qquad \ldots(3)$$

Substitute (2) and (3) in (1)

$$\therefore \quad 0.5\, T_C + T_C \cos 30 + T_C = W$$

$$\therefore \quad \boxed{T_C = 0.423\, W} \qquad \ldots \text{Ans.}$$

From equation (2),

$$\therefore \quad \boxed{T_A = 0.211\, W} \qquad \ldots \text{Ans.}$$

From equation (3),

$$\therefore \quad \boxed{T_B = 0.366\, W} \qquad \ldots \text{Ans.}$$

END...

UNIT - IV

9 Trusses

Important Points to Remember

- Trusses are structures consisting of straight slender rods connected only at their ends.

If m = Number of members

j = Number of joints,

then for a perfect truss,

$$m = 2j - 3$$

Rules for finding zero force member :

i) If there are three members at a joint out of which two are collinear and no external force acts on that joint, the third member is a zero force member as shown in Fig. 9.1. The other two members have same force.

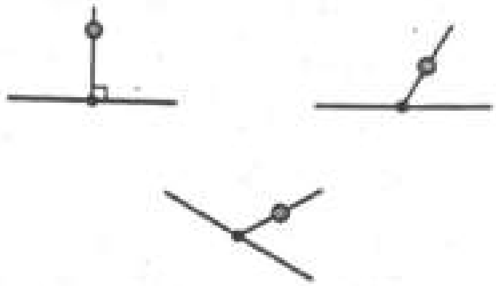

Fig. 9.1

ii) If there are only two members at a joint and there is no external force on that joint, both members are zero force members (Fig. 9.2).

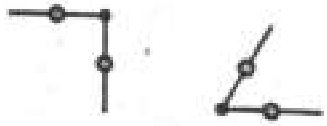

Fig. 9.2

iii) If there are only two members at a joint and the external force is along one of the members, then the other is a zero force member (Fig. 9.3).

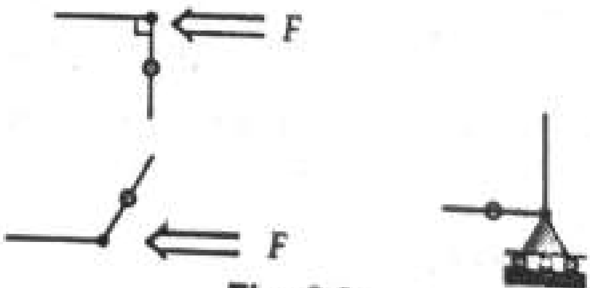

Fig. 9.3

Method of Joints

i) Use equilibrium of complete truss to find support reactions.

ii) Identify zero force members by inspection.

iii) Draw F.B.D. of a joint where there are at the most two unknowns. Assume unknown forces in members to be tensile, i.e., directed away from the point.

iv) Use $\sum F_x = 0$ and $\sum F_y = 0$ to find the two unknown forces. If the answer is positive, the force is tensile and compressive if it is negative.

v) Proceed to the next joint where there are at the most two unknowns.

vi) At the last joint, verify $\sum F_x = 0$ and $\sum F_y = 0$.

Method of Sections

i) Use equilibrium of complete truss to find support reactions.

ii) Identify zero force members by inspection.

iii) Divide the truss into two parts by passing a section through it which cuts at the most three members in which forces are unknown.

iv) Draw F.B.D. of any one part of the truss assuming tensile forces in members cut by the section.

v) Use $\sum F_x = 0, \sum F_y = 0, \sum M = 0$ to find the three unknowns.

Q.1 State the condition for perfect truss explaining the symbols used if any. Also define redundant and deficient truss.

[SPPU : May-09, Marks 4]

Ans. : If m = Number of members,

j = Number of joints,

then, the condition for a perfect truss is,

$$m = 2j - 3$$

For a redundant truss,

$$m > 2j - 3$$

For a deficient truss,

$$m < 2j - 3$$

Q.2 Name different methods of finding out the forces in members of a truss. When do you use these methods ?

[SPPU : Dec.-09, Marks 4]

Ans. : The two methods to find forces in members of a truss are : method of joints and method of sections.

Method of joints is used to find forces in all members of the truss. Method of sections is used to find forces in only a few members of the truss.

Q.3 i) State the assumptions for the analysis of plane truss.
ii) Define deficient, perfect and redundant truss.

[SPPU : May-14, Marks 5]

Ans. : i) The following assumptions are made in analysis of simple trusses :

1) All members have negligible weight.
2) All members have uniform cross-section.
3) Members are connected at the joints through pin connections which are frictionless.
4) Couple moments, which produce bending (non-axial force), do not act on members of the truss.
5) All members have only axial force.

ii) Refer Q.1.

Q.4 A pin jointed truss in loaded and supported as shown in Fig. Q.4.1. Identify the zero force members. Also find the forces in other members and components of reaction at A and D.

[SPPU : May-09 (Old), Marks 8]

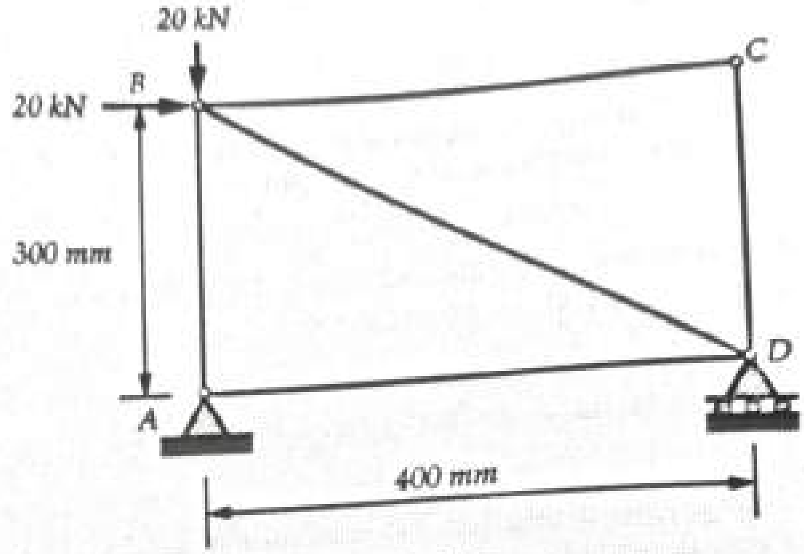

Fig. Q.4.1

Ans. :

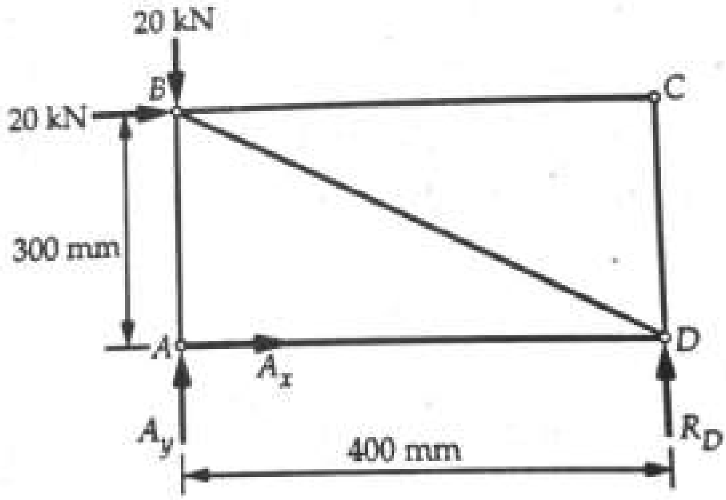

Fig. Q.4.1 (a)

By inspection, the zero force members are BC and CD. i.e.,

$\therefore$ $\boxed{F_{BC} = F_{CD} = 0}$ ...Ans.

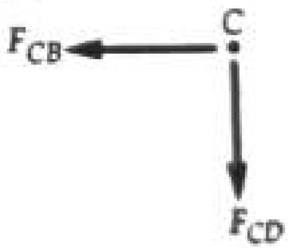

Fig. Q.4.1 (b)

For the complete truss,

$\sum M_A = 0 : R_D \times 400 - 20 \times 300 = 0$

$\therefore$ $\boxed{R_D = 15 \text{ kN} \uparrow}$ ...Ans.

$\sum F_x = 0$:

$$20 + A_x = 0 \qquad \therefore A_x = -20 \text{ kN}$$

$$\therefore \quad \boxed{A_x = 20 \text{ kN} \leftarrow} \qquad \text{...Ans.}$$

$\sum F_y = 0$: $\quad A_y + R_D - 20 = 0$

$$A_y = 20 - 15$$

$$\therefore \quad \boxed{A_y = 5 \text{ kN} \uparrow} \qquad \text{...Ans.}$$

From F.B.D of joint *B* shown in Fig. Q.4.1 (c)

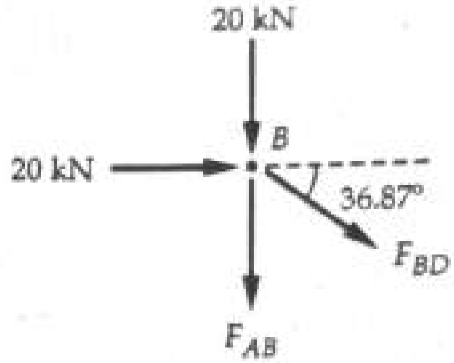

Fig. Q.4.1 (c)

$$\tan \theta_D = \frac{300}{400} \quad \therefore$$

$$\theta_D = 36.87°$$

$\sum F_x = 0$:

$$F_{BD} \cos 36.87 + 20 = 0$$

$$F_{BD} = -25 \text{ kN}$$

$$\therefore \quad \boxed{F_{BD} = 25 \text{ kN (C)}}$$

$\sum F_y = 0$:

$$-20 - F_{AB} - F_{BD} \sin 36.87 = 0$$

$$-20 - F_{AB} - (-25) \sin 36.87 = 0$$

$$\therefore \quad F_{AB} = -5 \text{ kN}$$

$$\therefore \quad \boxed{F_{AB} = 5 \text{ kN (C)}} \qquad \text{...Ans.}$$

From F.B.D of joint *A* shown in Fig. Q.4.1 (d)

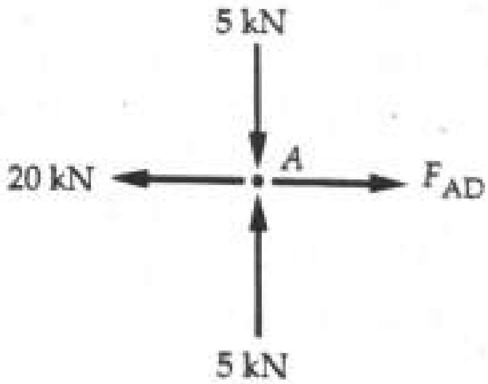

Fig. Q.4.1 (d)

$\sum F_x = 0$:

$$F_{AD} - 20 = 0$$

$$\therefore \quad \boxed{F_{AD} = 20 \text{ kN (T)}} \qquad \text{...Ans.}$$

Member	Magnitude of force (kN)	Nature
AB	5	C
AD	20	T
BD	25	C
BC	0	-
CD	0	-

Q.5 Determine the forces in each member of the truss loaded and supported as shown in Fig. Q.5.1. **[SPPU : May-10]**

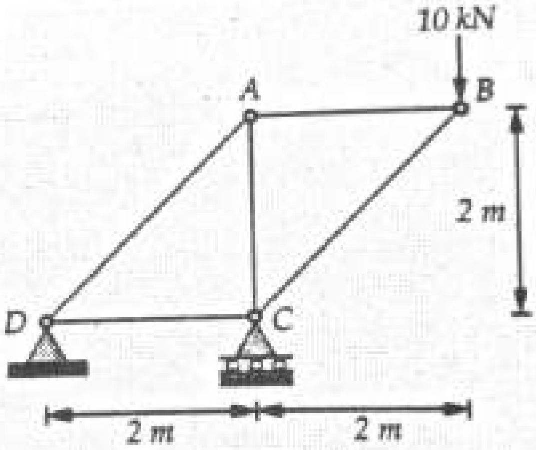

Fig. Q.5.1

Ans. : The horizontal component of reaction at D is zero as there is no applied force in horizontal direction. Let R_D and R_C be the vertical reaction at D and C respectively.

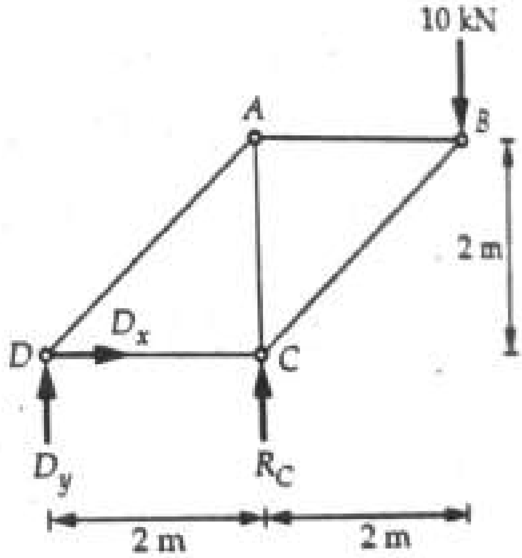

Fig. Q.5.1 (a)

$\sum M_D = 0$:

$(R_C)(2) - (10)(4) = 0$

$\therefore \quad R_C = 20 \text{ kN} \uparrow$

$\sum F_Y = 0$:

$R_D + R_C - 10 = 0$

$\therefore \quad R_D = -10 \text{ kN} = 10 \text{ kN} \downarrow$

Consider **F.B.D. of joint *B*** shown in Fig. Q.5.1 (b)

$\sum F_Y = 0$:

$-10 - F_{BC} \sin 45 = 0$

$$F_{BC} = -14.142 \text{ kN}$$

$\therefore$ $\boxed{F_{BC} = 14.142 \text{ kN (C)}}$ **...Ans.**

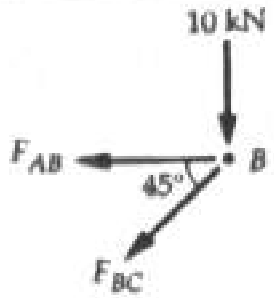

Fig. Q.5.1 (b)

$\sum F_x = 0$:

$-F_{AB} - F_{BC} \cos 45 = 0$

$\therefore$ $\boxed{F_{AB} = 10 \text{ kN (T)}}$ **...Ans.**

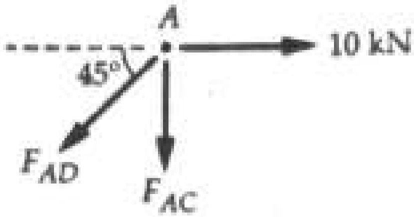

Fig. Q.5.1 (c)

For **F.B.D. of joint *A*** shown in Fig. Q.5.1 (c)

$\sum F_x = 0$:

$-F_{AD} \cos 45 + 10 = 0$

$\therefore$ $\boxed{F_{AD} = 14.142 \text{ kN (T)}}$ **...Ans.**

$\sum F_y = 0$:

$-F_{AD} \sin 45 - F_{AC} = 0$

$\therefore$ $F_{AC} = -10 \text{ kN}$

$\therefore$ $\boxed{F_{AC} = 10 \text{ kN (C)}}$ **...Ans.**

For **F.B.D. of joint *D*** shown in Fig. Q.5.1 (d)

$\sum F_x = 0$:

$F_{CD} + 14.142 \cos 45 = 0$

$\therefore$ $F_{CD} = -10 \text{ kN}$

$\therefore$ $\boxed{F_{CD} = 10 \text{ kN (C)}}$ **...Ans.**

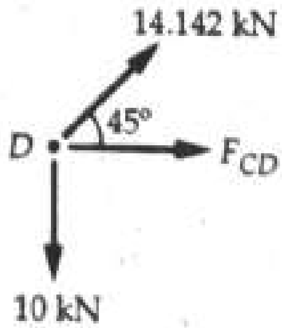

Fig. Q.5.1 (d)

Member	Magnitude of force (kN)	Nature
AB	10	T
AC	10	C
AD	14.142	T
BC	14.142	C
CD	10	C

Q.6 Identify zero force members and determine forces in the members of the truss as shown in Fig. Q.6.1. **[SPPU : May-10]**

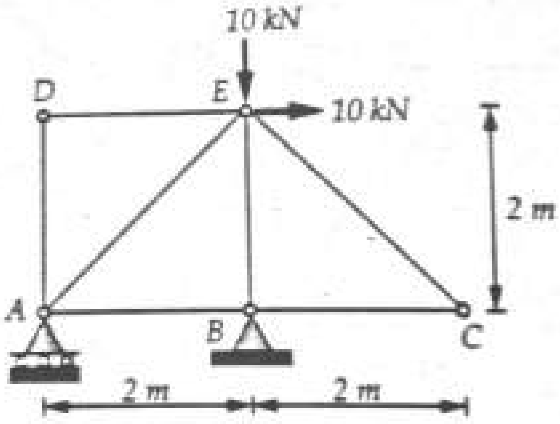

Fig. Q.6.1

Ans. : As there are two members at joint D and no applied force,

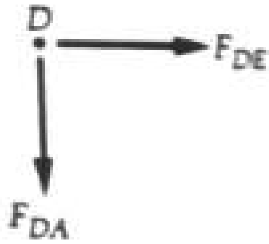

Fig. Q.6.1 (a)

$\therefore$ $$\boxed{F_{DE} = F_{AD} = 0}$$ ...Ans.

Similarly from joint C,

$\therefore$ $$\boxed{F_{BC} = F_{CE} = 0}$$ ...Ans.

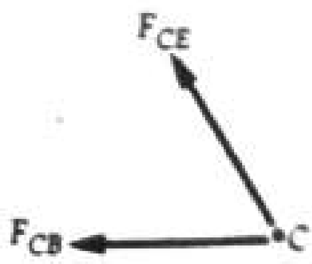

Fig. Q.6.1 (b)

Let B_x and B_y be the reaction components at B and R_A be the vertical reaction at A.

$\sum F_x = 0$:

$$B_x + 10 = 0$$

$\therefore$ $$B_x = -10 \text{ kN} = 10 \text{ kN} \leftarrow$$

$\sum M_A = 0$:

$(B_y)(2) - 10(2) - (10)(2) = 0$

$\therefore \quad B_y = 20$ kN ↑

$\sum F_y = 0$:

$R_A - 10 + B_y = 0$

$\therefore \quad R_A - 10 + 20 = 0$

$\therefore \quad R_A = -10$ kN

$= 10$ kN ↓

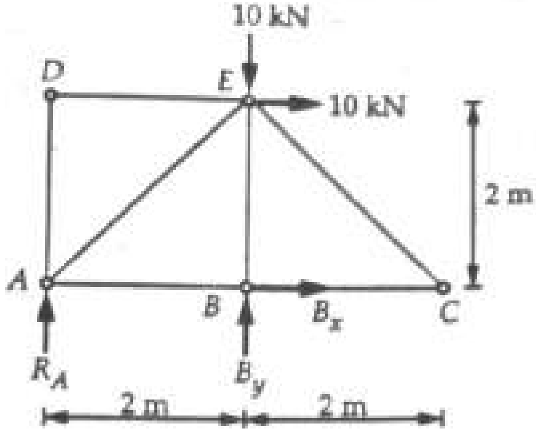

Fig. Q.6.1 (c)

The F.B.D. of joint *A* is shown in Fig. Q.6.1 (d)

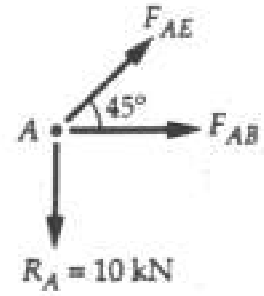

Fig. Q.6.1 (d)

$\sum F_y = 0$:

$F_A \sin 45 - 10 = 0$

$\therefore \quad \boxed{F_A = 14.142 \text{ kN (T)}}$ ...Ans.

$\sum F_x = 0$:

$F_{AE} \cos 45 + F_{AB} = 0$

$\therefore \quad F_{AB} = -10$ kN

$\therefore \quad \boxed{F_{AB} = 10 \text{ kN (C)}}$ ...Ans.

The F.B.D. of joint *B* is shown in Fig. Q.6.1 (e)

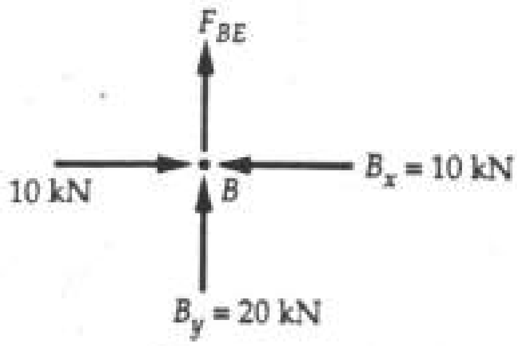

Fig. Q.6.1 (e)

$\sum F_y = 0$:

$F_{BE} + 20 = 0$

$\therefore \quad F_{BE} = -20$ kN

$\therefore \quad \boxed{F_{BE} = 20 \text{ kN (C)}}$ ...Ans.

Member	Magnitude of force (kN)	Nature
AB	10	C
AD	0	-
AE	14.142	T

BC	0	-
BE	20	C
CE	0	-
DE	0	-

Q.7 Identify zero force members and find magnitude and nature of forces in remaining members of the truss as shown in Fig. Q.7.1.
[SPPU : May-11, Marks 6]

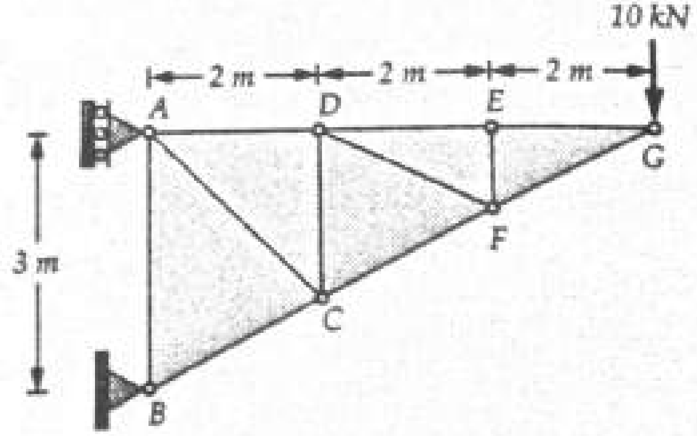

Fig. Q.7.1

Ans. : The zero force members are *EE, FD, DC, AB* and *AC*.

Note	The truss is not stable with roller at A.

For F.B.D. of joint G shown in Fig. Q.7.1 (a),

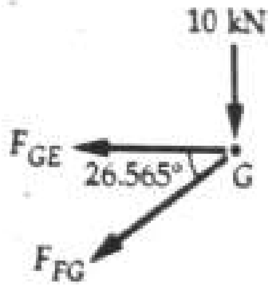

Fig. Q.7.1 (a)

$$\tan \theta_G = \frac{3}{6} \quad \therefore \theta_G = 26.565^\circ$$

$\sum F_y = 0$:

$$-10 - F_{FG} \sin 26.565 = 0$$

$$\therefore \quad F_{FG} = -22.36 \text{ kN} = 22.36 \text{ kN (C)}$$

$\sum F_x = 0$:

$$-F_{GE} - F_{FG} \cos 26.565 = 0$$

$$\therefore \quad F_{GE} = 20 \text{ kN (T)}$$

$$\boxed{F_{AD} = F_{DE} = F_{GE} = 20 \text{ kN (T)}} \quad \text{...Ans.}$$

$$\boxed{F_{BC} = F_{CF} = F_{FG} = 22.36 \text{ kN (C)}}$$ ...Ans.

Member	Magnitude of force (kN)	Nature
AB	0	-
AC	0	-
AD	20	T
BC	22.36	C
CF	22.36	C
DC	0	-
DE	20	T
EF	0	-
FD	0	-
FG	22.36	C
GE	20	T

Q.8 Determine the force in each member of the truss and state if the members are in tension or compression. Refer Fig. Q.8.1. Assume L = 2 m and P = 10 kN.
[SPPU : May-12, Marks 7]

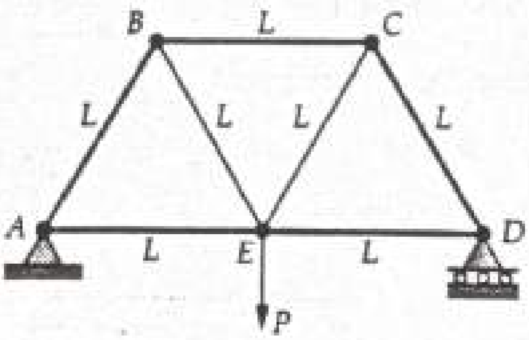

Fig. Q.8.1

Ans. : As the truss is symmetrically loaded,

$$A_y = R_D = \frac{10}{2} = 5 \text{ kN}\uparrow$$

and $A_x = 0$

All the three triangles in the truss are equilateral. Hence all angles are 60°. For F.B.D. of joint *A* shown in Fig. Q.8.1 (a).

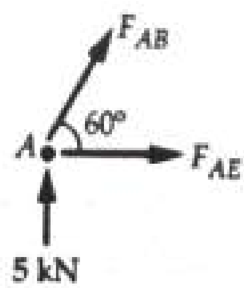

Fig. Q.8.1 (a)

$$\sum F_y = 0 :$$

$$F_{AB} \sin 60 + 5 = 0$$

$\therefore \quad F_{AB} = -5.7735 \text{ kN}$

$\therefore \quad \boxed{F_{AB} = 5.7735 \text{ kN (C)}}$ ...Ans.

$$\sum F_x = 0 :$$

$$F_{AB} \cos 60 + F_{AE} = 0$$

$$(-5.7735) \cos 60 + F_{AE} = 0$$

$\therefore \quad \boxed{F_{AE} = 2.887 \text{ kN (T)}}$ ...Ans.

For F.B.D of joint B shown in Fig. Q.8.1 (b),

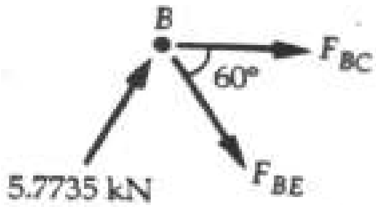

Fig. Q.8.1 (b)

$\sum F_y = 0 :$

$$5.7735 \sin 60 - F_{BE} \sin 60 = 0$$

$\therefore \quad F_{BE} = 5.7735 \text{ kN (T)}$

$\sum F_x = 0 :$

$$F_{BC} + F_{BE} \cos 60 + 5.7735 \cos 60 = 0$$

$\therefore \quad F_{BC} = -5.7735 \text{ kN}$

$\therefore \quad \boxed{F_{BC} = 5.7735 \text{ kN (C)}}$ ...Ans.

The truss is symmetric

$\therefore \quad \boxed{F_{CE} = F_{BE} = 5.7735 \text{ kN (T)}}$ ...Ans.

$\boxed{F_{CD} = F_{AB} = 5.7735 \text{ kN (C)}}$ ...Ans.

and $\quad \boxed{F_{DE} = F_{AE} = 2.887 \text{ kN (T)}}$ ...Ans.

Member	Magnitude of force (kN)	Nature
AB	5.7735	C
AE	2.887	T
BC	5.7735	C
BE	5.7735	T

CD	5.7735	C
CE	5.7735	T
DE	2.887	T

Q.9 A plane truss is loaded and supported as shown in Fig. Q.9.1. Determine the magnitude and nature of forces in all the members.
[SPPU : Dec.-12, Marks 6]

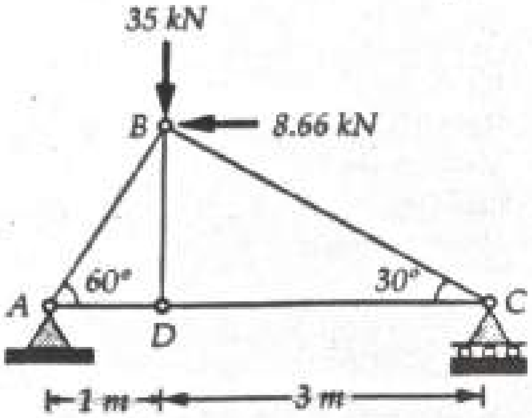

Fig. Q.9.1

Ans. : The F.B.D. of complete truss is shown in Fig. Q.9.1 (a).

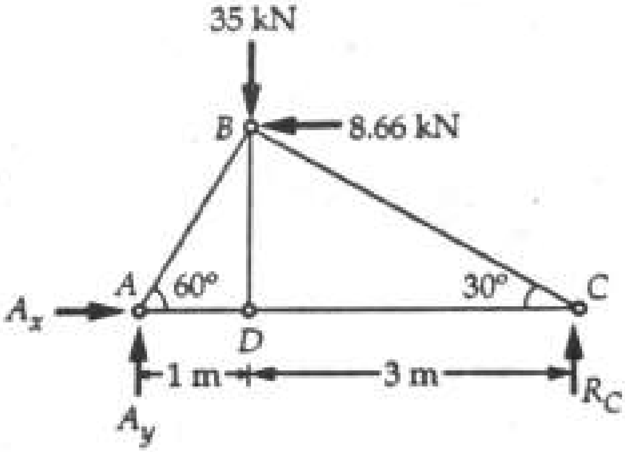

Fig. Q.9.1 (a)

$$\tan 60 = \frac{BD}{1}$$

$$\therefore \quad BD = 1 \cdot \tan(60)$$

$\sum F_x = 0$:

$$A_x - 8.66 = 0$$

$$\therefore \quad A_x = 8.66 \text{ kN} \rightarrow$$

$\sum M_A = 0$:

$$(R_C)(4) + (8.66)(1 \tan 60) - (35)(1) = 0$$

$$\therefore \quad R_C = 5 \text{ kN} \uparrow$$

$\sum F_y = 0$

$$A_y + R_C - 35 = 0$$

$$\therefore \quad A_y = 30 \text{ kN} \uparrow$$

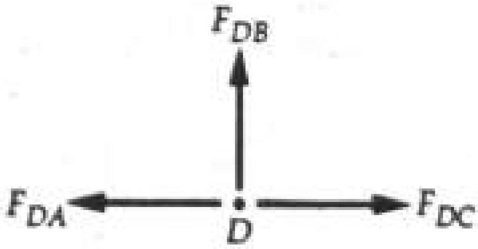

Fig. Q.9.1 (b)

BD is a zero force member i.e.,

$\therefore \quad \boxed{F_{BD} = 0}$ ...Ans.

and $F_{AD} = F_{CD}$

For F.B.D of joint *A* shown in Fig. Q.9.1 (c).

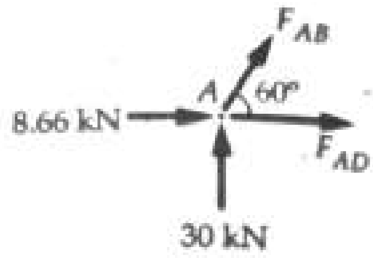

Fig. Q.9.1 (c)

$\sum F_y = 0$

$$F_{AB} \sin 60 + 30 = 0$$

$$\therefore \quad F_{AB} = -34.64 \text{ kN}$$

$\therefore \quad \boxed{F_{AB} = 34.64 \text{ kN (C)}}$ ...Ans.

$\sum F_x = 0:$

$$F_{AB} \cos 60 + F_{AD} + 8.66 = 0$$

$$-34.64 \cos 60 + F_{AD} + 8.66 = 0$$

$\therefore \quad \boxed{F_{AD} = 8.66 \text{ kN (T)}}$ ...Ans.

$\therefore \quad \boxed{F_{CD} = 8.66 \text{ kN (T)}}$ ...Ans.

For F.B.D. of joint *B* shown in Fig. Q.9.1 (d)

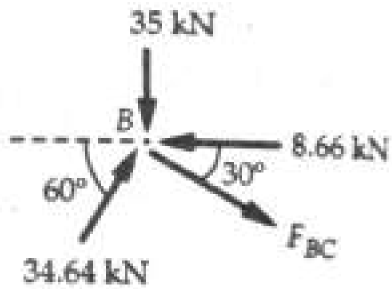

Fig. Q.9.1 (d)

$\sum F_x = 0:$

$$F_{BC} \cos 30 - 8.66 + 34.64 \cos 60 = 0$$

$$F_{BC} = -10 \text{ kN}$$

$\therefore \quad \boxed{F_{BC} = 10 \text{ kN (C)}}$ ...Ans.

Member	Magnitude of force (kN)	Nature
AB	34.64	C
AD	8.66	T
BC	10	C

BD	0	-
CD	8.66	T

Q.10 Determine the forces in each member of the truss and state if the members are in tension or compression. Refer Fig. Q.10.1.

[SPPU : May-13, Marks 6]

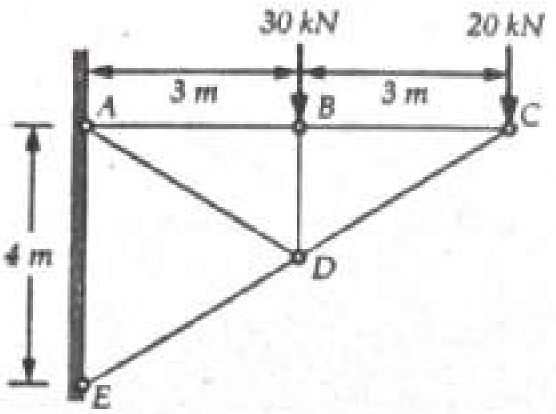

Fig. Q.10.1

Ans. : For the **F.B.D. of joint** C shown in Fig. Q.10.1 (a).

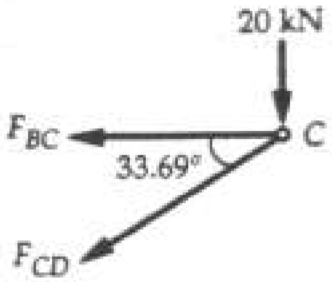

Fig. Q.10.1 (a)

$$\tan \theta_C = \frac{4}{6} \quad \therefore \theta_C = 33.69°$$

$\sum F_y = 0:$

$$-20 - F_{CD} \sin 33.69 = 0$$

$$\therefore \quad F_{CD} = -36.056 \text{ kN}$$

$$\therefore \quad \boxed{F_{CD} = 36.056 \text{ kN (C)}}$$ ...Ans.

$\sum F_x = 0:$

$$-F_{BC} - F_{CD} \cos 33.69 = 0$$

$$-F_{BC} + 36.056 \cos 33.69 = 0$$

$$\therefore \quad \boxed{F_{BC} = 30 \text{ kN (T)}}$$ ...Ans.

For the **F.B.D. of joint** B shown in Fig. Q.10.1 (b),

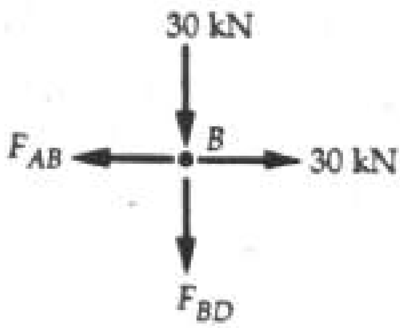

Fig. Q.10.1 (b)

By inspection, at B

$\therefore \quad \boxed{F_{AB} = 30 \text{ kN (T)}}$ ...Ans.

and $\quad F_{BD} = -30$ kN

$\therefore \quad \boxed{F_{BD} = 30 \text{ kN (C)}}$ ...Ans.

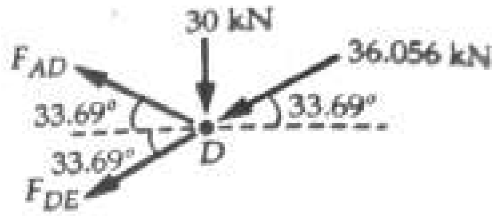

Fig. Q.10.1 (c)

For the F.B.D. of joint D shown in Fig. Q.10.1 (c),

$\sum F_x = 0$:

$$-F_{AD} \cos 33.69 - F_{DE} \cos 33.69 - 36.056 \cos 33.69 = 0 \quad ...(1)$$

$\sum F_y = 0$:

$$F_{AD} \sin 33.69 - F_{DE} \sin 33.69 - 36.056 \sin 33.69 - 30 = 0 \quad ...(2)$$

From equations (1) and (2),

$$\boxed{F_{AD} = 27.042 \text{ kN (T)}}$$

$$F_{DE} = -63.1 \text{ kN}$$

$$\therefore \quad \boxed{F_{DE} = 63.1 \text{ kN (C)}}$$

Member	Magnitude of force (kN)	Nature
AB	30	T
AD	27.042	T
BC	30	T
BD	30	C
CD	36.056	C
DE	63.1	C

Q.11 Determine the magnitude and nature of forces in the members AB, AH and GC of the truss loaded and supported as shown in Fig. Q.11.1. **[SPPU : Dec.-13, Marks 6]**

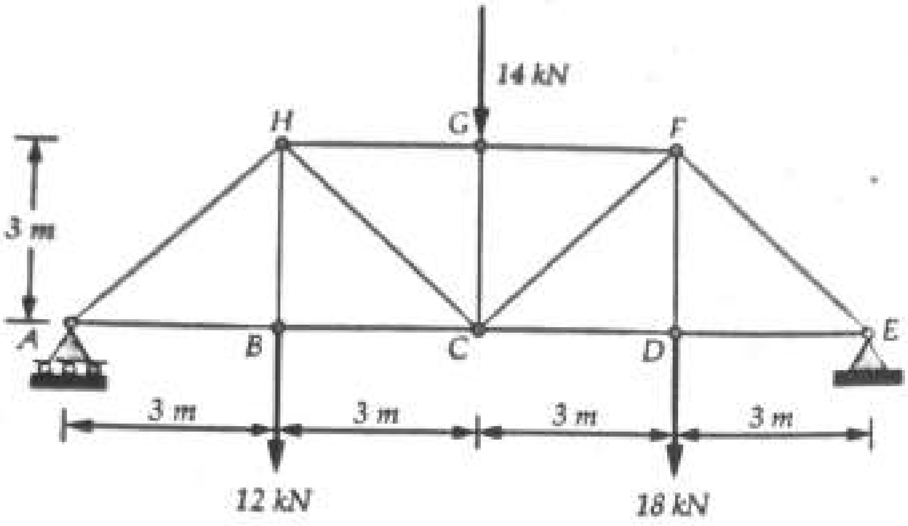

Fig. Q.11.1

Ans. : The vertical reaction at A can be obtained using $\sum M_E = 0$.

$$(18)(3) + (14)(6) + (12)(9) - (R_A)(12) = 0$$

$$R_A = 20.5 \text{ kN} \uparrow$$

From the **F.B.D.** of joint A shown in Fig. Q.11.1 (a),

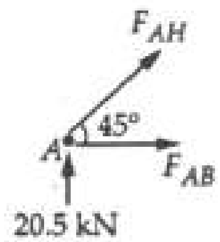

Fig. Q.11.1 (a)

$$\tan \theta_A = \frac{3}{3} \quad \therefore \theta_A = 45°$$

$\sum F_y = 0$: $F_{AH} \sin 45 + 20.5 = 0$

$\therefore \quad F_{AH} = -29$ kN

$\therefore \quad \boxed{F_{AH} = 29 \text{ kN (C)}}$ ...Ans.

$\sum F_x = 0$: $F_{AH} \cos 45 + F_{AB} = 0$

$\therefore \quad \boxed{F_{AB} = 20.5 \text{ kN (T)}}$ ...Ans.

Force in GC can be obtained using **F.B.D.** of joint G shown in Fig. Q.11.1 (b).

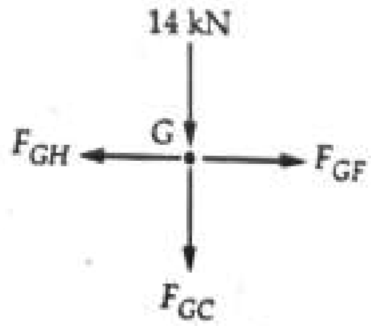

Fig. Q.11.1 (b)

$\sum F_y = 0$: $-F_{GC} - 14 = 0$

$\therefore \quad F_{GC} = -14$ kN

$\therefore \quad \boxed{F_{GC} = 14 \text{ kN (C)}}$

Member	Magnitude of force (kN)	Nature
AB	20.5	T
AH	29	C
GC	14	C

Q.12 Determine the axial forces in each member of the plane truss as shown in Fig. Q.12.1. **[SPPU : May-14, Marks 6]**

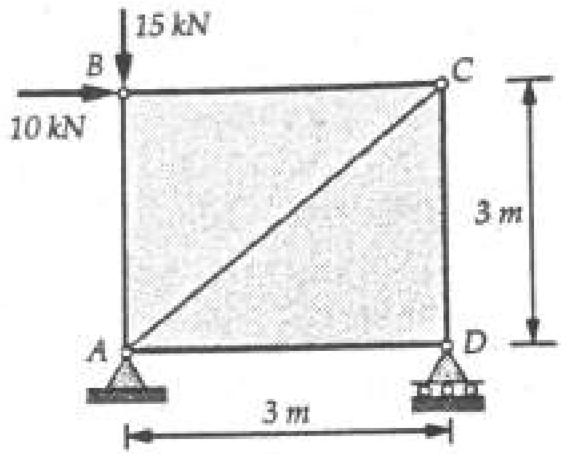

Fig. Q.12.1

Ans. :

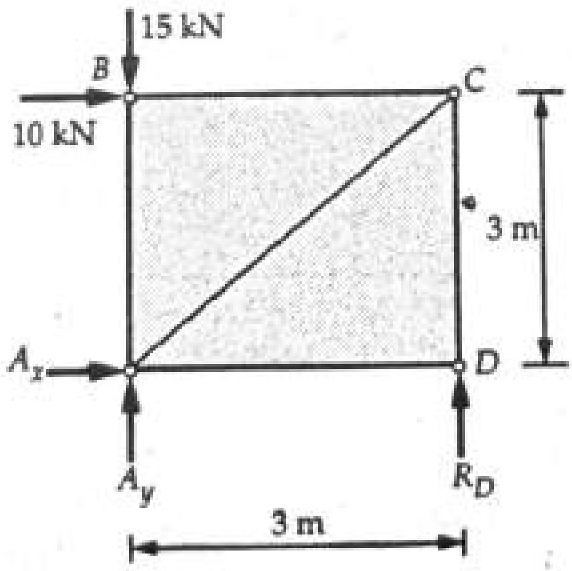

Fig. Q.12.1 (a)

Using $\sum M_A = 0$ for the complete truss,

$(R_D)(3) - 10\,(3) = 0$

$$\therefore \quad R_D = 10 \text{ kN} \uparrow$$

$$\sum F_x = 0 \; : \; 10 + A_x = 0$$

$$A_x = -10 \text{ kN} = 10 \text{ kN} \leftarrow$$

$$\sum F_y = 0 \; : \; A_y + R_D - 15 = 0$$

$$\therefore \quad A_y = 5 \text{ kN} \uparrow$$

By inspection, at B

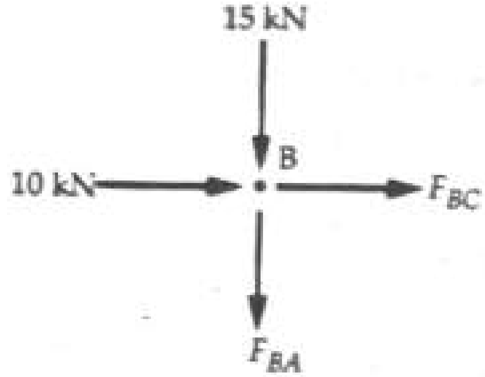

Fig. Q.12.1 (b)

$\therefore$ $\boxed{F_{BC} = 10 \text{ kN (C)}}$ ...Ans.

$\therefore$ $\boxed{F_{AB} = 15 \text{ kN (C)}}$ ...Ans.

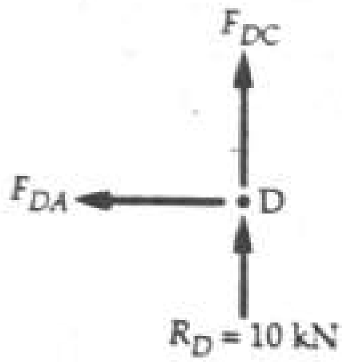

Fig. Q.12.1 (c)

By inspection at D,

$\therefore$ $\boxed{F_{AD} = 0}$ ...Ans.

$\therefore$ $\boxed{F_{CD} = 10 \text{ kN (C)}}$ ...Ans.

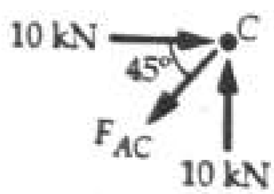

Fig. Q.12.1 (d)

For the FBD of joint C shown in Fig. Q.12.1 (d).

$$\sum F_x = 0 \; : \; 10 - F_{AC} \cos 45 = 0$$

$\therefore$ $\boxed{F_{AC} = 14.142 \text{ kN (T)}}$ ...Ans.

Member	Magnitude of force (kN)	Nature
AB	15	C
AC	14.142	T
AD	0	-
BC	10	C
CD	10	C

Q.13 Determine the forces in each member of the plane truss as shown in Fig. Q.13.1 in terms of the external loading and state if the members are in tension or compression. Use θ = 30°, L = 2 m and P = 100 N.

[SPPU : Dec.-14, Marks 6]

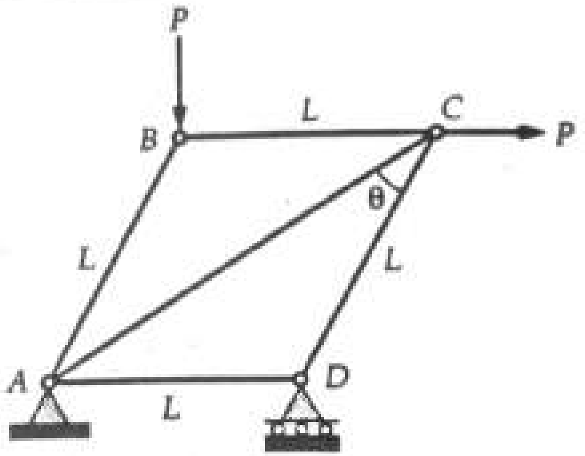

Fig. Q.13.1

Ans. : The FBD of complete truss is shown in Fig. Q.13.1 (a).

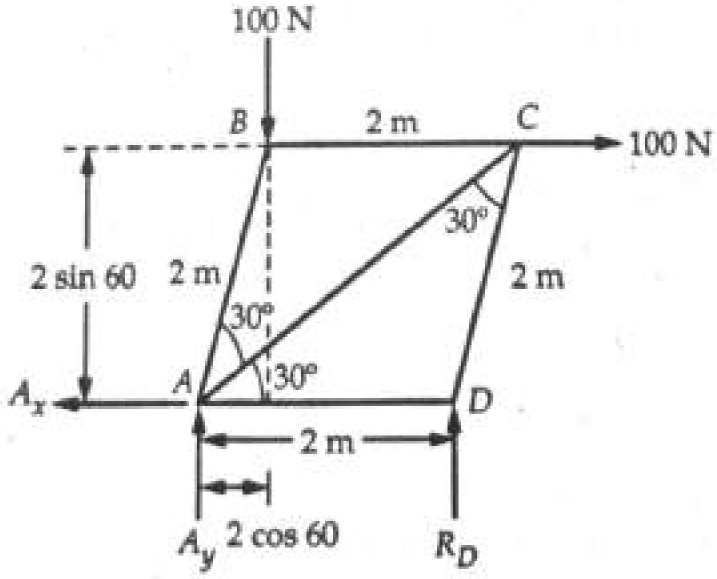

Fig. Q.13.1 (a)

$$\sum F_x = 0 : -A_x + 100 = 0$$

$$\therefore \quad A_x = 100 \text{ N} \leftarrow$$

$$\sum M_A = 0:$$

$(R_D)(2) - (100)(2 \cos 60) - (100)(2 \sin 60) = 0$

$$\therefore \quad R_D = 136.6 \text{ N}$$

$$\sum F_y = 0:$$

$$A_y - 100 + R_D = 0$$

$$A_y = -36.6 \text{ N} = 36.6 \text{ N} \downarrow$$

For FBD of joint D shown in Fig. Q.13.1 (b),

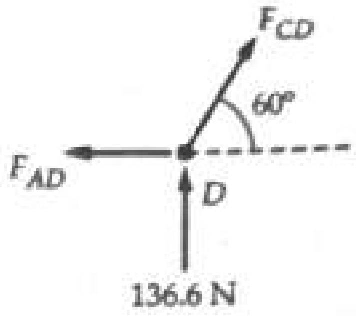

Fig. Q.13.1 (b)

$$\sum F_y = 0:$$

$F_{CD} \sin 60 + 136.6 = 0$

$$F_{CD} = -157.732 \text{ N}$$

$$\therefore \quad \boxed{F_{CD} = 157.732 \text{ N} \quad (C)}$$ ...Ans.

$$\sum F_x = 0: F_{CD} \cos 60 - F_{AD} = 0$$

$$-157.732 \cos 60 - F_{AD} = 0$$

$$F_{AD} = -78.866 \text{ N}$$

$$\therefore \quad \boxed{F_{AD} = 78.866 \text{ N (C)}}$$ ...Ans.

For the FBD of joint C shown in Fig. Q.13.1 (c),

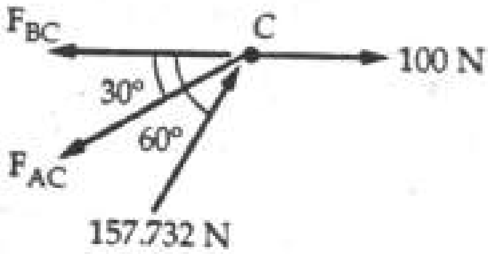

Fig. Q.13.1 (c)

$$\sum F_y = 0:$$

$$-F_{AC} \sin 30 + 157.732 \sin 60 = 0$$

$$\therefore \quad \boxed{F_{AC} = 273.2 \text{ N (T)}}$$ ...Ans.

$$\sum F_x = 0:$$

$$-F_{BC} - F_{AC} \cos 30 + 100 + 157.732 \cos 60 = 0$$

$$F_{BC} = -57.732 \text{ N}$$

$$\therefore \quad \boxed{F_{BC} = 57.732 \text{ N (C)}}$$ ...Ans.

For FBD of joint B shown in Fig. Q.13.1 (d),

$$-F_{AB} \cos 60 - 57.732 = 0$$

$$F_{AB} = -115.464 \text{ N}$$

$\therefore$ $\boxed{F_{AB} = 115.464 \text{ N (C)}}$ ...Ans.

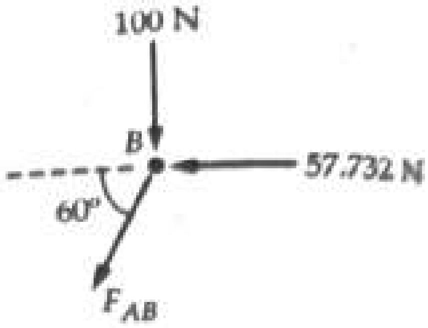

Fig. Q.13.1 (d)

Member	Magnitude of force (N)	Nature
AB	115.464	C
AC	273.2	T
AD	78.866	C
BC	57.732	C
CD	157.732	C

Q.14 Members AB and BC can support a maximum compressive force of 800 N and members AD, DC and BD can support a maximum tensile force of 2000 N. Determine the greatest load P the truss can support. Refer Fig. Q.14.1.

[SPPU : Dec.-15, Marks 6]

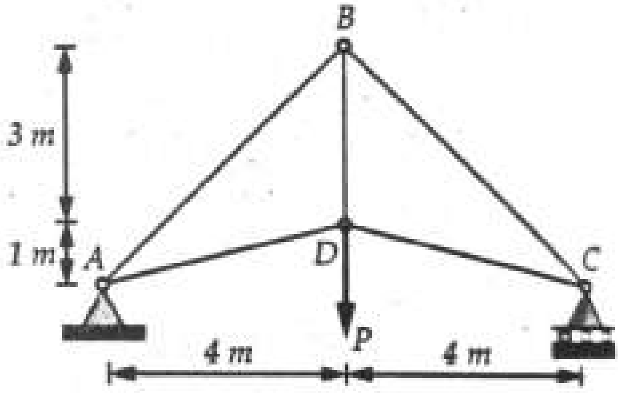

Fig. Q.14.1

Ans. : The FBD of truss is shown in Fig. Q.14.1 (a).

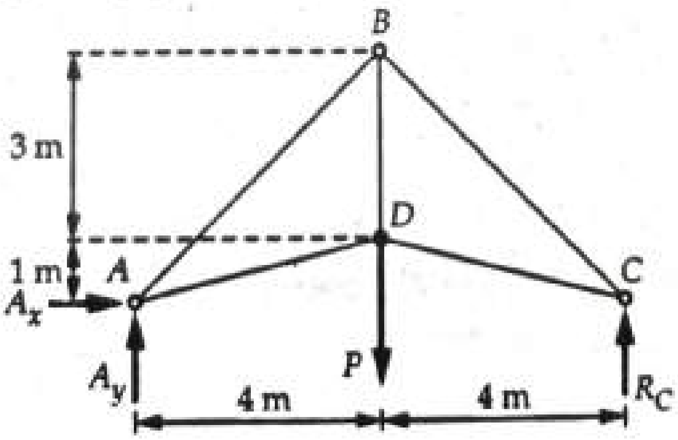

Fig. Q.14.1 (a)

$\sum M_A = 0 : \quad -P \times 4 + R_C \times 8 = 0$

$\therefore \quad R_C = \frac{P}{2}$

$\sum F_x = 0 : \quad A_x = 0$

$\sum F_y = 0 : \quad A_y + R_c - P = 0$

$\therefore \quad A_y = \frac{P}{2}$

For **FBD of joint A** shown in Fig. Q.14.1 (b).

$$\tan \theta_{AB} = \frac{4}{4} \quad \therefore \theta_{AB} = 45°$$

$$\tan \theta_{AD} = \frac{1}{4} \quad \therefore \theta_{AD} = 14.036°$$

$\sum F_x = 0 : F_{AD} \cos 14.036 + F_{AB} \cos 45 = 0$ ... (1)

$\sum F_y = 0 : F_{AD} \sin 14.036 + F_{AB} \sin 45 + \frac{P}{2} = 0$ ... (2)

From equations (1) and (2),

$$F_{AD} = 0.687\ P(T)$$

$$F_{AB} = -0.943\ P = 0.943\ P\ (C)$$

By symmetry,

$$F_{BC} = F_{AB} = 0.943\ P\ (C)$$

$$F_{CD} = A_{AD} = 0.687\ P\ (C)$$

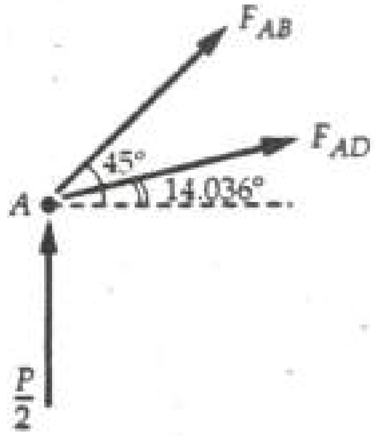

Fig. Q.14.1 (b)

For **FBD of joint B** shown in Fig. Q.14.1 (c),

$\sum F_y = 0 :$

$0.943 P \sin 45 + 0.943 P \sin 45 - F_{BD} = 0$

$\therefore \quad F_{BD} = 1.334\ P\ (T)$

$F_{AB} = F_{BC} = 800\ N\ (C)$

$\therefore \quad 0.943\ P = 800$

$\therefore \quad P = 848.36\ N$ ... (3)

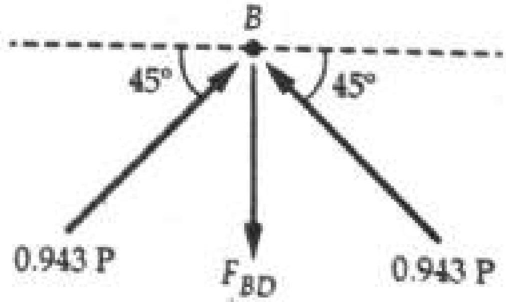

Fig. Q.14.1 (c)

As $F_{BD} > F_{CD}$ and $F_{BD} > F_{AD}$

$\therefore \quad F_{BD} = 2000 \text{ N}$

$\therefore \quad 1.334\, P = 2000$

$\therefore \quad P = 1500 \text{ N} \quad \text{... (4)}$

The smaller value of P from equations (3) and (4) satisfied to all the criteria.

$\therefore \quad \boxed{P = 848.36 \text{ N}}$ ...Ans.

Select such value of P which does not affect the other members.

Q.15 Determine the force in the members BE and BD of the truss which supports the load as shown in Fig. Q.15.1. All interior angles an 60 ° and 120 °. [SPPU : Dec.-15, Marks 6]

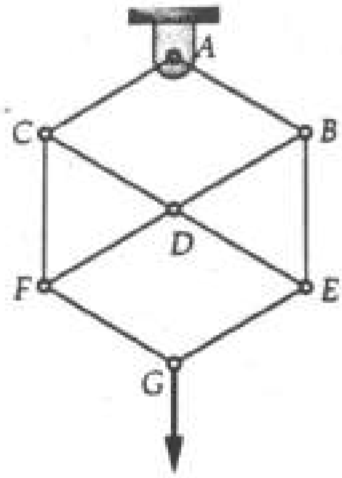

Fig. Q.15.1

Ans. : From given Fig. Q.15.1,

$R_A = 100 \text{ N} \uparrow \quad (\because A_x = 0)$

For FBD of joint A shown in Fig. Q.15.1 (a),

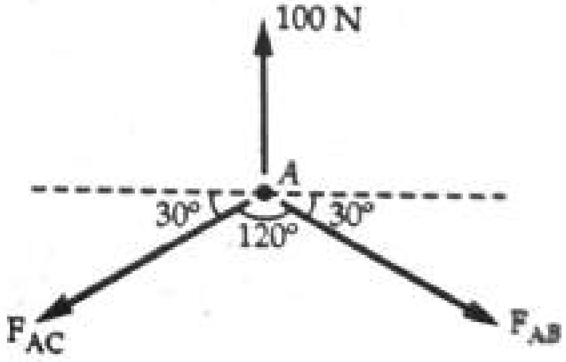

Fig. Q.15.1 (a)

$\sum F_x = 0:$

$-F_{AC}\cos 30 + F_{AB}\cos 30 = 0$

$\therefore \quad F_{AC} = F_{AB}$

$\sum F_y = 0:$

$100 - F_{AC}\sin 30 - F_{AB}\sin 30 = 0$

$$100 = 2F_{AB}\sin 30$$

$\therefore \quad F_{AB} = 100 \text{ N (T)}$

For FBD of joint B shown in Fig. Q.15.1 (b),

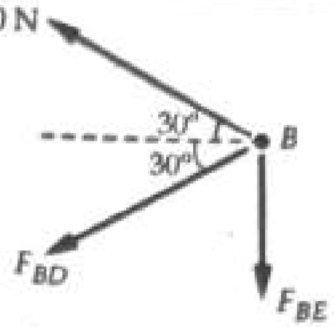

Fig. Q.15.1 (b)

$\sum F_x = 0:$

$$-100\cos 30 - F_{BD}\cos 30 = 0$$

$$F_{BD} = -100 \text{ N}$$

$\therefore \quad \boxed{F_{BD} = 100 \text{ N (C)}}$ ...Ans.

$\sum F_y = 0:$

$$100\sin 30 - F_{BD}\sin 30 - F_{BE} = 0$$

$\therefore \quad \boxed{F_{BE} = 100 \text{ N (T)}}$ ...Ans.

Member	Magnitude of force (N)	Nature
BD	100	C
BE	100	T

Q.16 Determine the force in each member of the truss as shown in Fig. Q.16.1 and tabulate the result with magnitude and nature of force in the members.

[SPPU : May-16, Marks 6]

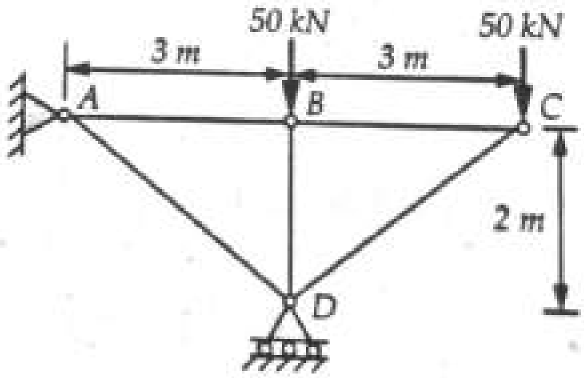

Fig. Q.16.1

Ans. : The FBD of truss is shown in Fig. Q.16.1 (a).

$$\sum F_x = 0: \ A_x = 0$$

$$\sum M_A = 0:$$

$$-50\times 3 - 50\times 6 + R_D \times 3 = 0$$

$\therefore \quad R_D = 150 \text{ KN} \uparrow$

$$\sum F_y = 0:$$

$$A_y - 50 - 50 + R_D = 0$$

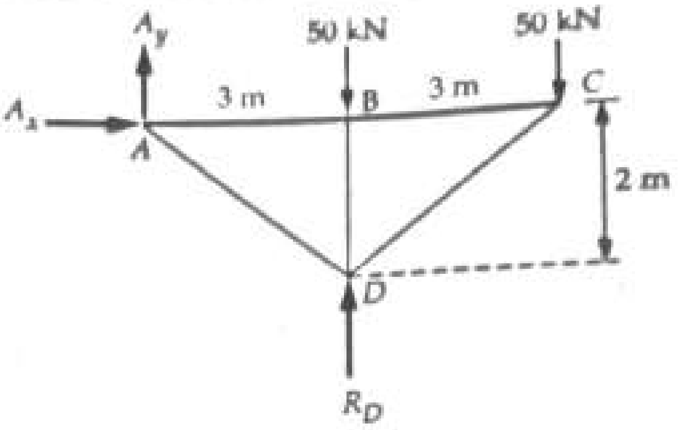

Fig. Q.16.1 (a)

$$\therefore \quad A_y = -50 \text{ kN} = 50 \text{ kN}\downarrow$$

For FBD of joint C shown in Fig. Q.16.1 (b),

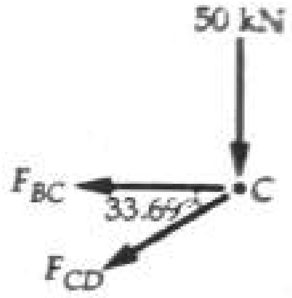

Fig. Q.16.1 (b)

$$\tan\theta_C = \frac{2}{3} \quad \therefore \theta_C = 33.69°$$

$$\sum F_y = 0:$$

$$-50 - F_{CD}\sin 33.69 = 0$$

$$\therefore \quad F_{CD} = -90.14 \text{ kN}$$

$$= 90.14 \text{ kN (C)}$$

$$\sum F_x = 0:$$

$$-F_{BC} - F_{CD}\cos 33.69 = 0$$

$$\therefore \quad F_{BC} = 75 \text{ kN (T)}$$

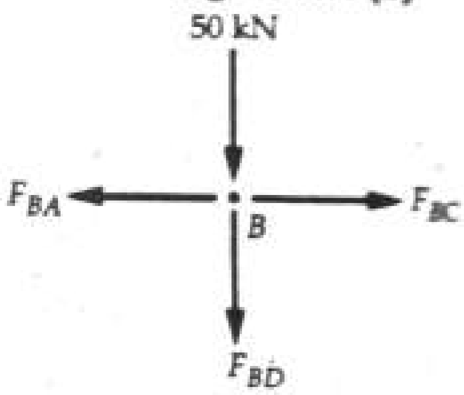

Fig. Q.16.1 (c)

By inspection of joint *B*,

$$F_{AB} = F_{BC} = 75 \text{ kN (T)}$$

$$F_{BD} = 50 \text{ kN (C)}$$

For FBD of joint D shown in Fig. Q.16.1 (d),

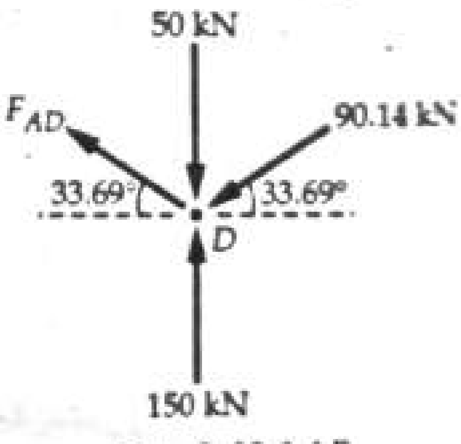

Fig. Q.16.1 (d)

$$\sum F_x = 0:$$

$$-F_{AD}\cos 33.69 - 90.14\cos 33.69 = 0$$

$$\therefore \quad F_{AD} = -90.14 \text{ kN}$$

$$\therefore \quad F_{AD} = 90.14 \text{ kN (C)}$$

The result are tabulated as follows :

Member	Magnitude of force (kN)	Nature
AB	75	T
AD	90.14	C
BC	75	T
BD	50	C
CD	90.14	C

Q.17 Determine the forces in the members of the truss loaded and supported as shown in the Fig. Q.17.1. Tabulate the result with magnitude and nature of force in the members.

[SPPU : Dec.-16, Marks 5]

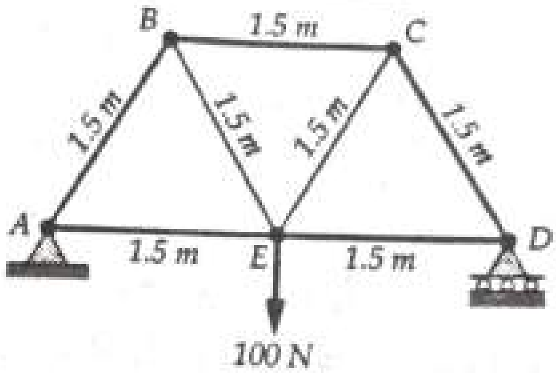

Fig. Q.17.1

Ans. : As the truss is symmetrically loaded,

$$A_y = R_D$$

$$= \frac{100}{2} = 50 \text{ N} \uparrow$$

$$A_x = 0$$

All the three triangles in the truss are equilateral. Hence all angles are 60°.

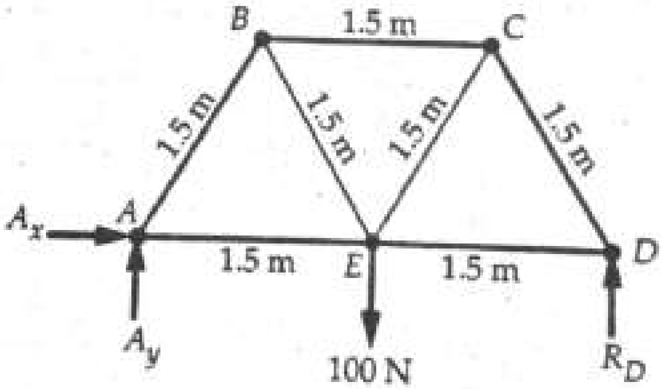

Fig. Q.17.1 (a)

For FBD of joint A shown in Fig. Q.17.1 (b)

$$\Sigma F_y = 0 : F_{AB} \sin 60 + 50 = 0$$

$$F_{AB} = -57.735 \text{ N} = 57.735 \text{ N (C)}$$

$$\Sigma F_x = 0 : F_{AB} \cos 60 + F_{AE} = 0$$

$$\therefore \quad F_{AE} = 28.87 \text{ N (T)}$$

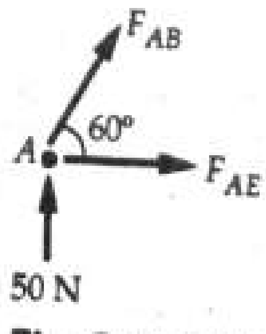

Fig. Q.17.1 (b)

For FBD of joint B shown in Fig. Q.17.1 (c),

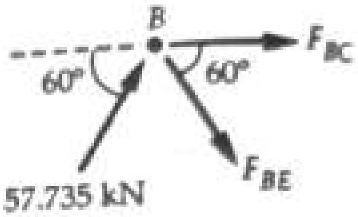

Fig. Q.17.1 (c)

$$\Sigma F_y = 0:$$

$$57.735 \sin 60 - F_{BE} \sin 60 = 0$$

$$\therefore \quad F_{BE} = 57.735 \text{ N (T)}$$

$$\Sigma F_x = 0:$$

$$F_{BC} + F_{BE} \cos 60 + 57.735 \cos 60 = 0$$

$$\therefore \quad F_{BC} = -57.73 \text{ N}$$

$$\therefore \quad F_{BC} = 57.735 \text{ N (C)}$$

The truss is symmetric

$$\therefore \quad F_{CE} = F_{BE} = 57.735 \text{ N (T)}$$

$$F_{CD} = F_{AB} = 57.735 \text{ N (C)}$$

$$F_{DE} = F_{AE} = 28.87 \text{ N (T)}$$

The calculations are tabulated as follows :

Member	Magnitude of force (N)	Nature
AB	57.735	C
AE	28.87	T
BE	57.735	T
BC	57.735	C
CE	57.735	T
CD	57.735	C
DE	28.87	T

Q.18 Determine the forces in the members of the truss loaded and supported as shown in the Fig. Q.18.1. Tabulate the result with magnitude and nature of force in the members.

[SPPU : May-17, Marks 5]

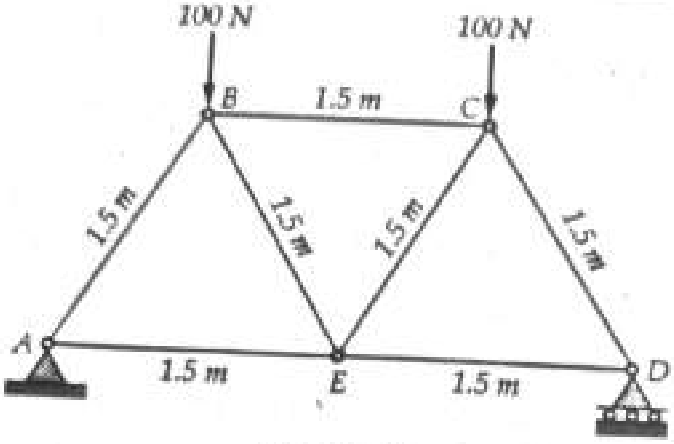

Fig. Q.18.1

Ans. : The F.B.D of complete truss is shown in Fig. Q.18.1 (a).

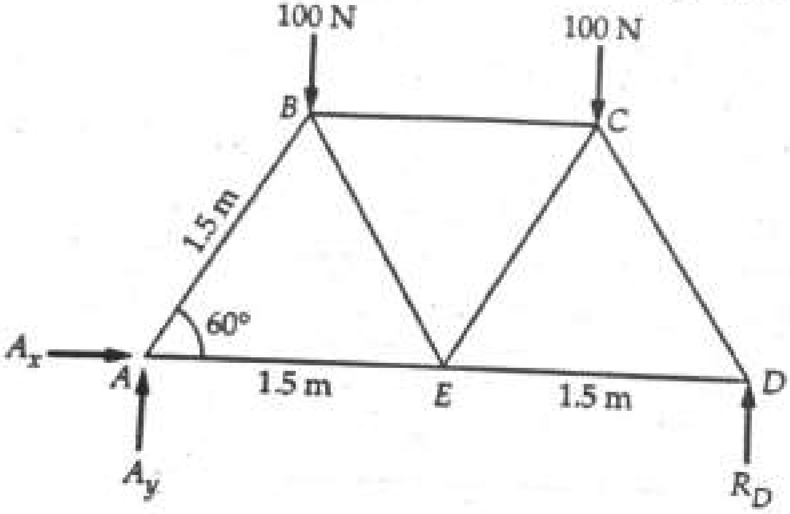

Fig. Q.18.1 (a)

$\therefore$ $$\sum F_x = 0: \; A_x = 0$$

As the truss is symmetric,

$$A_y = R_D = \frac{100+100}{2} = 100 \text{ N}$$

For F.B.D of joint A shown in Fig. Q.18.1 (b).

$$\sum F_y = 0 : 100 + F_{AB} \sin 60 = 0$$

$$F_{AB} = -115.47 \text{ N}$$

$\therefore$ $$F_{AB} = 115.47 \text{ N (C)}$$

$$\sum F_x = 0 : F_{AB} \cos 60 + F_{AE} = 0$$

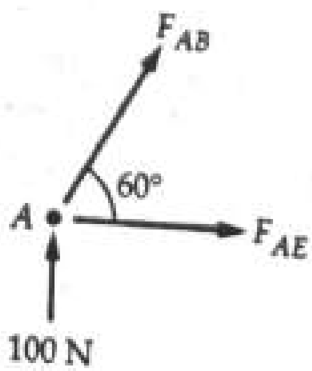

Fig. Q.18.1 (b)

$- 115.47 \cos 60 + F_{AE} = 0$

$\therefore \quad F_{AE} = 57.735$ N (T)

$F_{DE} = F_{AE} = 57.735$ N (T)

$F_{CD} = F_{AB} = 115.47$ N (C)

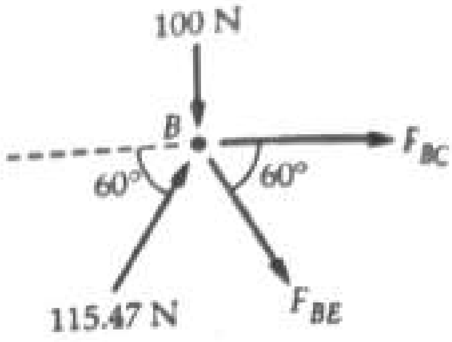

Fig. Q.18.1 (c)

For FBD of joint B shown in Fig. Q.18.1 (c).

$$\sum F_y = 0 : 115.47 \sin 60 - 100 - F_{BE} \sin 60 = 0$$

$\therefore \quad F_{BE} = 0$

$$\sum F_x = 0 : 115.47 \cos 60 + F_{BC} = 0$$

$\therefore \quad F_{BC} = 57.735$ (C)

$F_{CE} = F_{BE} = 0$

The results are tabulated as follows :

Member	Magnitude of Force (kN)	Nature
AB	115.47	C
AE	57.735	T
BE	0	-
BC	57.735	C
CE	0	-
CD	115.47	C
DE	57.735	T

Q.19 Determine the forces in each member of the truss shown in Fig. Q.19.1. State if the members are in tension or compression.

[SPPU : May-18, Marks 6]

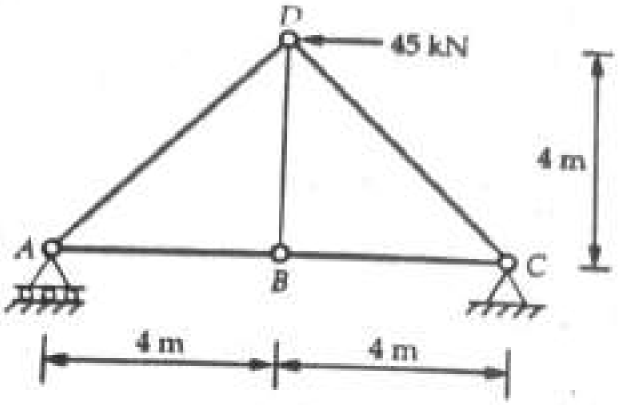

Fig. Q.19.1

Ans. : The FBD of complete truss is shown in Fig. Q.19.1 (a).

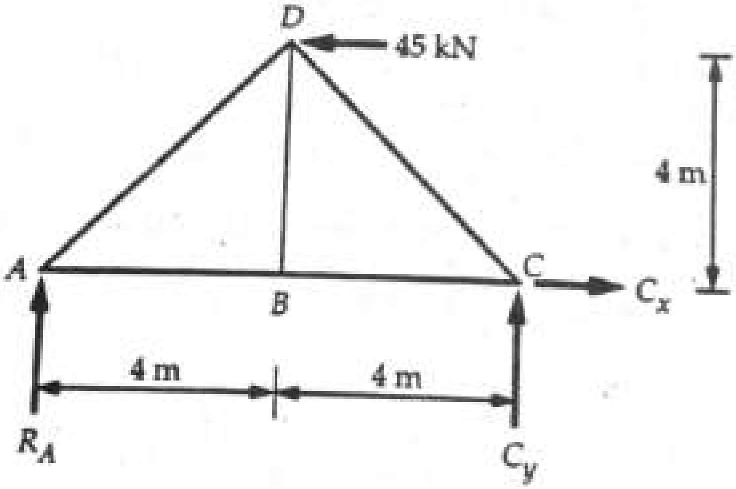

Fig. Q.19.1 (a)

$$\sum M_C = 0 : -(R_A)(8)+(45)(4) = 0$$

$\therefore$ $R_A = 22.5\ KN$

$$\sum F_x = 0 : C_x - 45 = 0$$

$\therefore$ $C_x = 45\ KN$

$$\sum F_y = 0 : \quad R_A + C_y = 0$$

$\therefore$ $C_y = -22.5\ kN = 22.5\ kN \downarrow$

By inspection, $\boxed{F_{BD} = 0}$ ...Ans.

And, $F_{AB} = F_{BC}$

For the FBD of joint A shown in Fig. Q.19.1 (b),

$$\sum F_y = 0 : 22.5 + F_{AD} \sin 45 = 0$$

$$\therefore \quad F_{AD} = -31.82 \text{ kN}$$

$$\therefore \quad \boxed{F_{AD} = 31.82 \text{ kN (C)}} \quad \text{...Ans.}$$

$$\sum F_x = 0 : F_{AD} \cos 45 + F_{AB} = 0$$

$$-31.82 \cos 45 + F_{AB} = 0$$

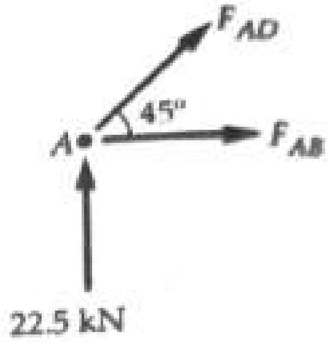

Fig. Q.19.1 (b)

$$\therefore \quad \boxed{F_{AB} = 22.5 \text{ kN (T)}} \quad \text{...Ans.}$$

$$\therefore \quad \boxed{F_{BC} = 22.5 \text{ kN (T)}} \quad \text{...Ans.}$$

For ABD of joint D shown in Fig. Q.19.1 (c),

$$\sum F_y = 0 : 31.82 \sin 45 - F_{CD} \sin 45 = 0$$

$$\therefore \quad \boxed{F_{CD} = 31.82 \text{ KN (T)}} \quad \text{...Ans.}$$

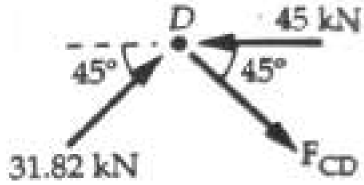

Fig. Q.19.1 (c)

Q.20 The truss supports vertical loads as shown in Fig. Q.20.1. Determine the forces in all the members of the truss and state the nature of the forces in tabular form. **[SPPU : Dec.-18, Marks 7]**

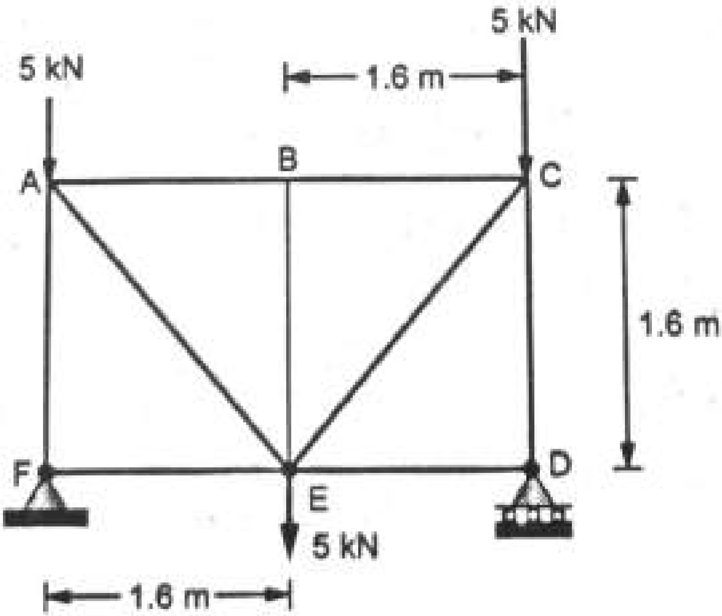

Fig. Q.20.1

Ans. : Applying equilibrium conditions,

$$\sum F_x = 0 \quad \therefore \quad F_x = 0$$

$$\sum F_y = 0 \quad \therefore \quad F_y + R_D - 5 - 5 - 5 = 0$$

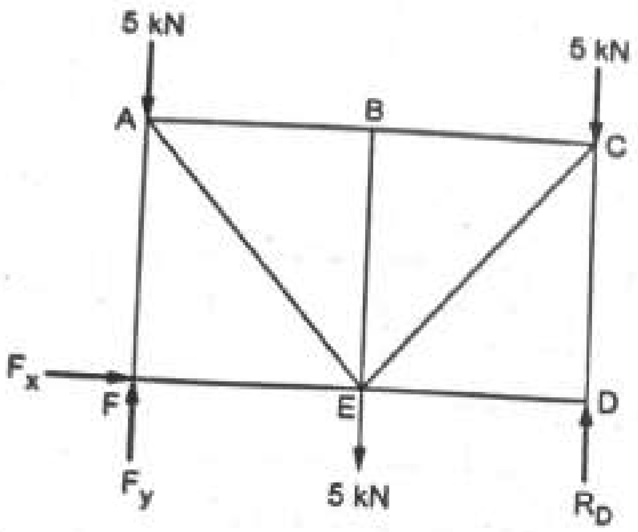

Fig. Q.20.1 (a)

$$\therefore \quad F_y + R_D = 15$$

$$\sum M_F = 0$$

$$= -5 \times 1.6 - 5 \times 3.2 + R_D \times 3.2$$

$$R_D = 7.5 \text{ kN} \quad \therefore \quad F_y = 7.5 \text{ kN}$$

onsider joint F as shown in Fig. Q.20.1 (b)

y observation,

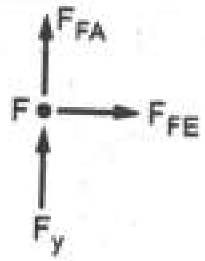

Fig. Q.20.1 (b)

$$F_{FE} = 0 \text{ and } F_{FA} = -F_y = -7.5 \text{ kN}$$

onsider joint A as shown in Fig. Q.20.1 (c)

$$\tan\theta = \frac{1.6}{1.6} \quad \therefore \quad \theta = 45°$$

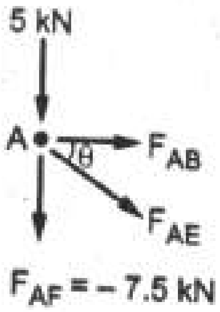

Fig. Q.20.1 (c)

$$\sum F_x = 0 = F_{AB} + F_{AE} \cos 45$$

$$\sum F_y = 0$$

$$= -5 - (-7.5) - F_{AE} \sin 45$$

$$F_{AE} = 3.535 \text{ kN}$$

$$F_{AB} = -2.5 \text{ kN}$$

onsider joint B as shown in Fig. Q.20.1 (d)

y observation,

$$F_{BE} = 0$$

$$F_{BA} = F_{BC} = -2.5 \text{ kN}$$

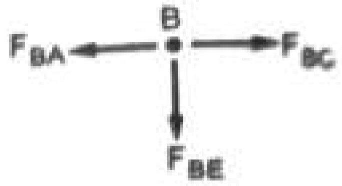

Fig. Q.20.1 (d)

As the given truss is symmetric,

$$F_{CD} = F_{AF} = -7.5 \text{ kN}$$

$$F_{CE} = F_{AE} = 3.535 \text{ kN}$$

$$F_{DE} = F_{FE} = 0$$

All the above values are tabulated as follows :

Member	Magnitude (kN)	Nature
AF, CD	+ 7.5	C
BE	0	-
FE, DE	0	-
AE, CE	3.535	T
AB, BC	+ 2.5	C

Q.21 The truss supports vertical loads as shown in Fig. Q.21.1. Determine the forces in all the members of the truss and state the nature of the forces in tabular form. **[SPPU : May-19, Marks 6]**

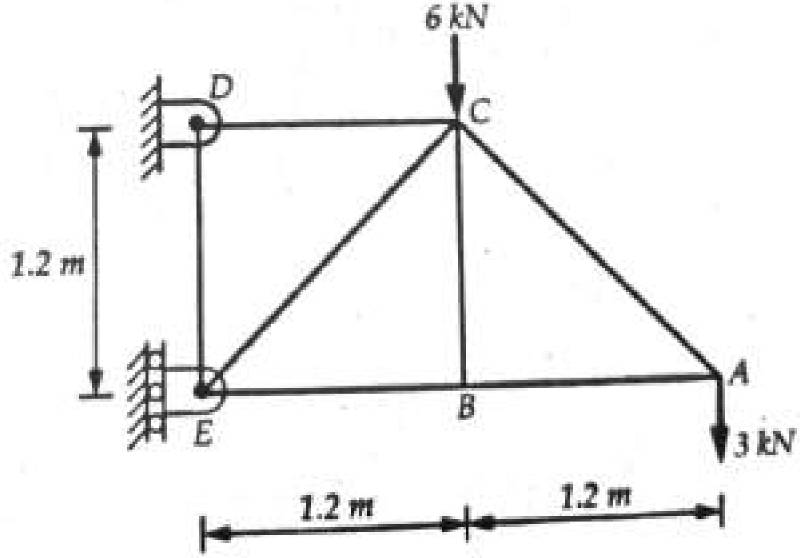

Fig. Q.21.1

Ans. : From FBD of truss as shown in Fig. Q.21.1 (a),

$$\sum M_D = 0 = -6\times1.2 - 3\times2.4 + R_E \times 1.2$$

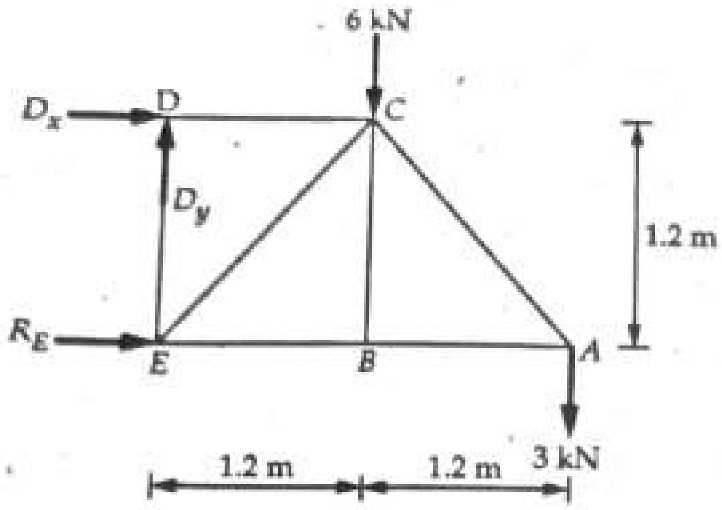

Fig. Q.21.1 (a)

$\therefore$ $\boxed{R_E = 12 \text{ kN} \rightarrow}$

$$\sum F_x = 0 : D_x + R_E = 0$$

$\therefore$ $D_x = -12 \text{ kN} = 12 \text{ kN} \leftarrow$

$$\sum F_y = 0 : D_y - 6 - 3 = 0$$

$\therefore$ $\boxed{D_y = 9 \text{ kN} \uparrow}$

By inspection of B,

$$F_{BC} = 0$$

and $F_{BE} = F_{BA}$

$$\tan\theta_E = \frac{1.2}{1.2}$$

$\therefore$ $\theta_E = 45°$

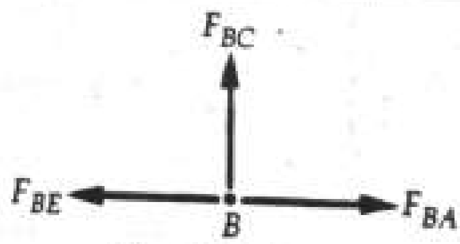

Fig. Q.21.1 (b)

From FBD of joint D,

$\therefore$ $\boxed{\begin{array}{l} F_{DC} = 12 \text{ kN (T)} \\ \text{and } F_{DE} = 9 \text{ kN (T)} \end{array}}$

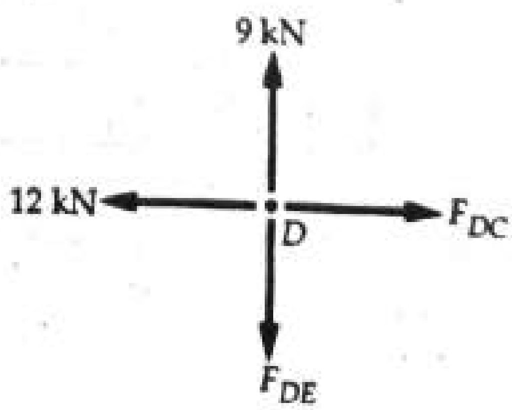

Fig. Q.21.1 (c)

From FBD of joint A,

$$\sum F_x = 0 : -F_{AB} - F_{AC} \cos 45 = 0$$

$$\sum F_y = 0 : -3 + F_{AC} \sin 45 = 0$$

$\therefore$ $\boxed{F_{AC} = 4.24 \text{ kN (T)} \quad \text{and} \quad F_{AB} = -3 \text{ kN} = 3 \text{ kN (C)} = F_{BE}}$ ...Ans.

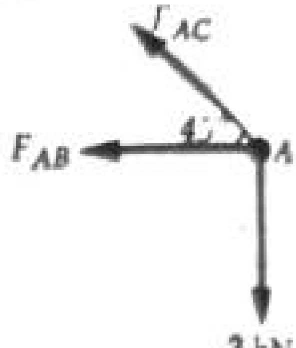

Fig. Q.21.1 (d)

From FBD of joint E,

$$\sum F_x = 0 : 12 - 3 + F_{EC} \cos 45 = 0$$

$\therefore$ $F_{EC} = -12.73$ kN

$\therefore$ $\boxed{F_{EC} = 12.73 \text{ kN (C)}}$ ...Ans.

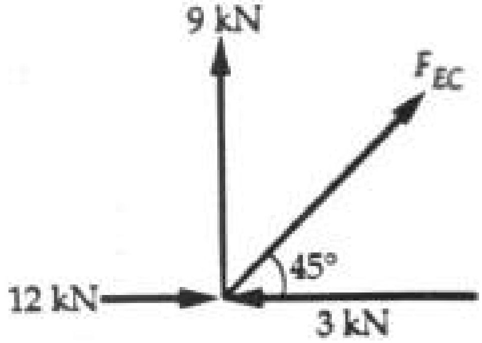

Fig. Q.21.1 (e)

The above results are tabulated as follows :

Member	Magnitude (kN)	Nature
AB	3	C
BE	3	C
AC	4.24	T
BC	0	-
EC	12.73	C
DC	12	T
DE	9	T

Q.22 Determine the forces in all members of truss shown in Fig. Q.22.1. A, B, C, D, E and F are pinned joints. ☞ **[SPPU : May-08]**

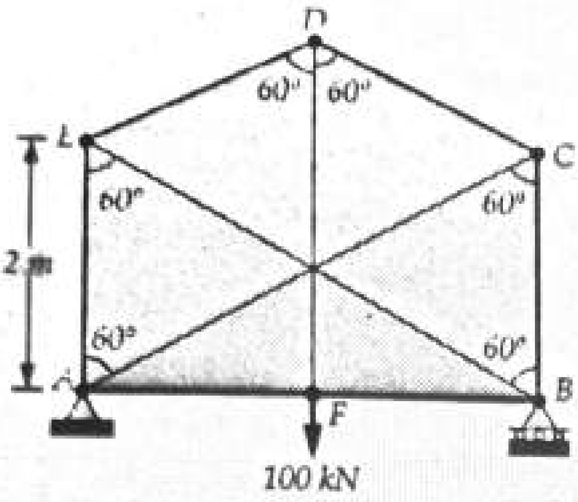

Fig. Q.22.1

Ans. : It is a symmetric truss. By symmetry, $R_A = 50$ kN↑ and $R_B = 50$ kN↑.

($A_x = 0$, since no horizontal force).

Note that there are three unknowns at every joint. But we can find one unknown from F.B.D of joint F and then solve the remaining truss. From **F.B.D of joint** F. [Fig. Q.22.1 (a)]

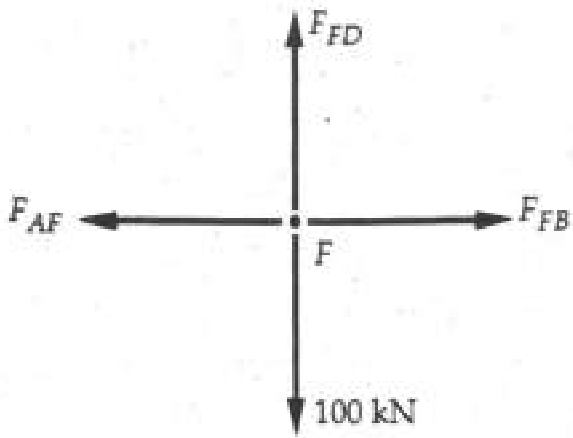

Fig. Q.22.1 (a)

$$\boxed{F_{FD} = 100 \text{ kN (T)}}$$ ...**Ans.**

and $F_{AF} = F_{FB}$

From **F.B.D of joint** D [Fig. Q.22.1 (b)]

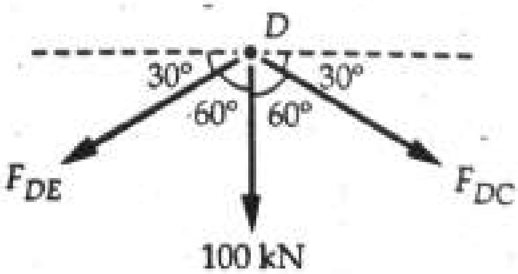

Fig. Q.22.1 (b)

$$\sum F_x = 0 :$$

$$F_{DE} = F_{DC}$$

$$\sum F_y = 0 :$$

$$-F_{DE} \sin 30 - F_{DC} \sin 30 - 100 = 0$$

$$-2 F_{DE} \sin 30 - 100 = 0$$

$$F_{DE} = -100 \text{ kN}$$

$\therefore$ $\boxed{F_{DE} = F_{DC} = 100 \text{ kN (C)}}$ ...Ans.

From F.B.D of joint E [Fig. Q.22.1 (c)]

$\sum F_x = 0$:

$-100 \cos 30 + F_{EB} \cos 30 = 0$

$\therefore$ $\boxed{F_{EB} = 100 \text{ kN (T)}}$ ...Ans.

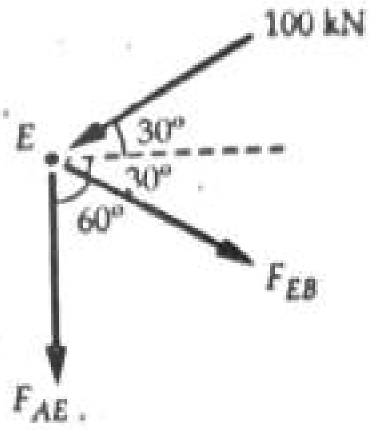

Fig. Q.22.1 (c)

$\sum F_y = 0$:

$-100 \sin 30 - F_{EB} \sin 30 - F_{AE} = 0$

$\therefore$ $F_{AE} = -100 \text{ kN}$

$\therefore$ $\boxed{F_{AE} = 100 \text{ kN (C)}}$

From F.B.D. of joint A [Fig. Q.22.1 (d)]

$\sum F_y = 0$:

$50 - 100 + F_{AC} \sin 30 = 0$

$\therefore$ $\boxed{F_{AC} = 100 \text{ kN (T)}}$ ...Ans.

$\sum F_x = 0$:

$F_{AF} + F_{AC} \cos 30 = 0$

$F_{AF} = -86.6 \text{ kN}$

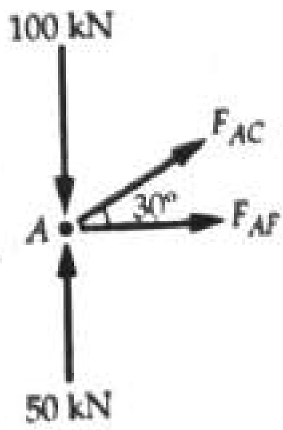

Fig. Q.22.1 (d)

$\therefore$ $\boxed{F_{AF} = 86.6 \text{ kN (C)}}$ ...Ans.

As the truss is symmetric,

$\therefore$ $\boxed{F_{FB} = 86.6 \text{ kN (C)}}$ ...Ans.

$\therefore$ $\boxed{F_{BC} = 100 \text{ kN (C)}}$ ...Ans.

Final check can be obtained from F.B.D of joint B.

Member	Magnitude by force (kN)	Nature
AC	100	T
AE	100	C
AF	86.6	C
BC	100	C
DC	100	C
DE	100	C
EB	100	T
FB	86.6	C
FD	100	T

Q.23 Determine the magnitude and nature of forces in the members BC, HC and HG of the truss loaded and supported as shown in Fig. Q.23.1. [SPPU : May-13, Marks 6]

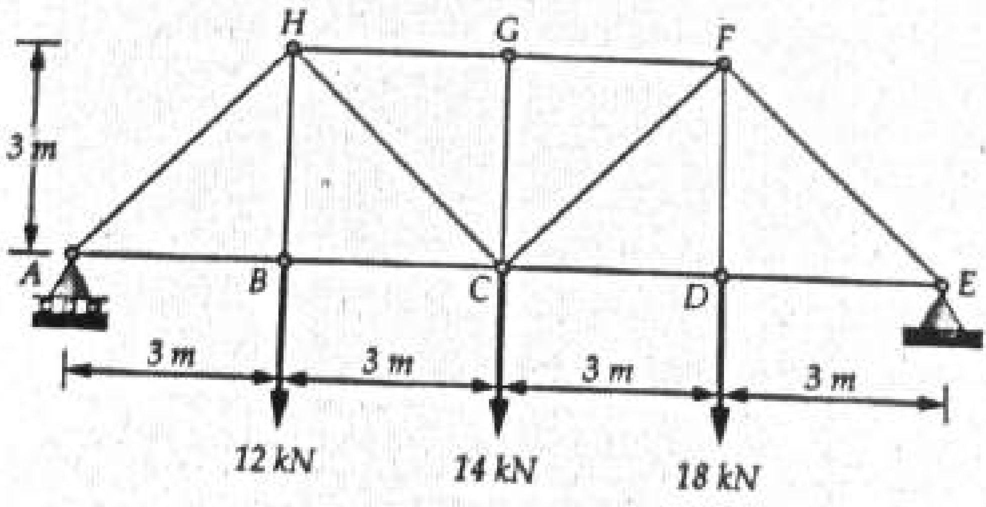

Fig. Q.23.1

Ans. : The vertical reaction at A can be obtained using $\sum M_E = 0$.

$$(18)(3) + (14)(6) + (12)(9) - (R_A)(12) = 0$$

$$R_A = 20.5 \text{ kN} \uparrow$$

Take a vertical section cutting the members BC, HC and HG and draw F.B.D. of portion to the left of the section as shown in Fig. Q.23.1 (a).

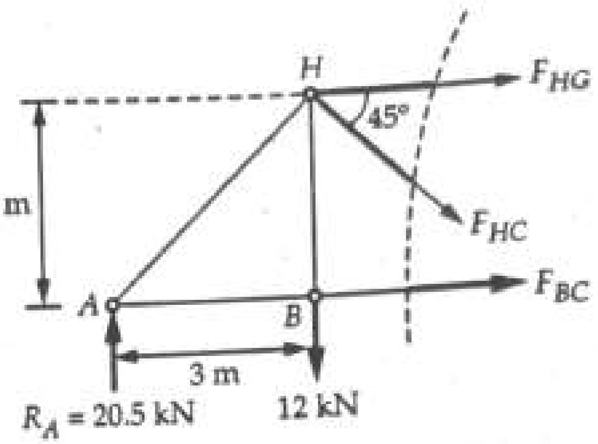

Fig. Q.23.1 (a)

$\sum M_H = 0$:

$(F_{BC})(3) - (20.5)(3) = 0$

$\therefore$ $\boxed{F_{BC} = 20.5 \text{ kN (T)}}$ ... **Ans.**

$\sum F_y = 0$: $-F_{HC} \sin 45 - 12 + 20.5 = 0$

$\therefore$ $\boxed{F_{HC} = 12.02 \text{ kN (T)}}$ ... **Ans.**

$\sum F_x = 0$: $F_{HG} + F_{HC} \cos 45 + F_{BC} = 0$

$\therefore$ $F_{HG} = -29$ kN

$\therefore$ $\boxed{F_{HG} = 29 \text{ kN (C)}}$ ... **Ans.**

Member	Magnitude of force (kN)	Nature
BC	20.5	T
HC	12.02	T
HG	29	C

Q.24 Determine the force in members FE, FC and BC of the truss using method of sections and state if the members are in tension or compression. Given P_1 = 2.22 kN and P_2 = 6.66 kN. Refer Fig. Q.24.1. **[SPPU : Dec.-15, Marks 6]**

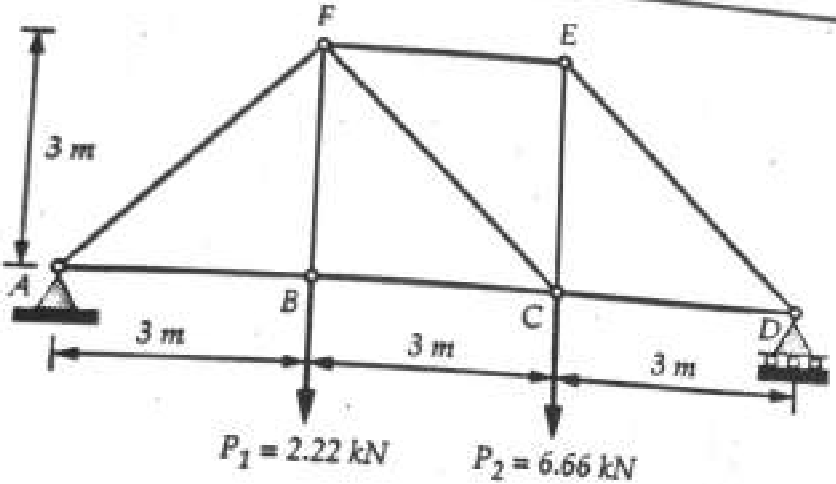

Fig. Q.24.1

Ans. : The FBD of truss is shown in Fig. Q.24.1 (a).

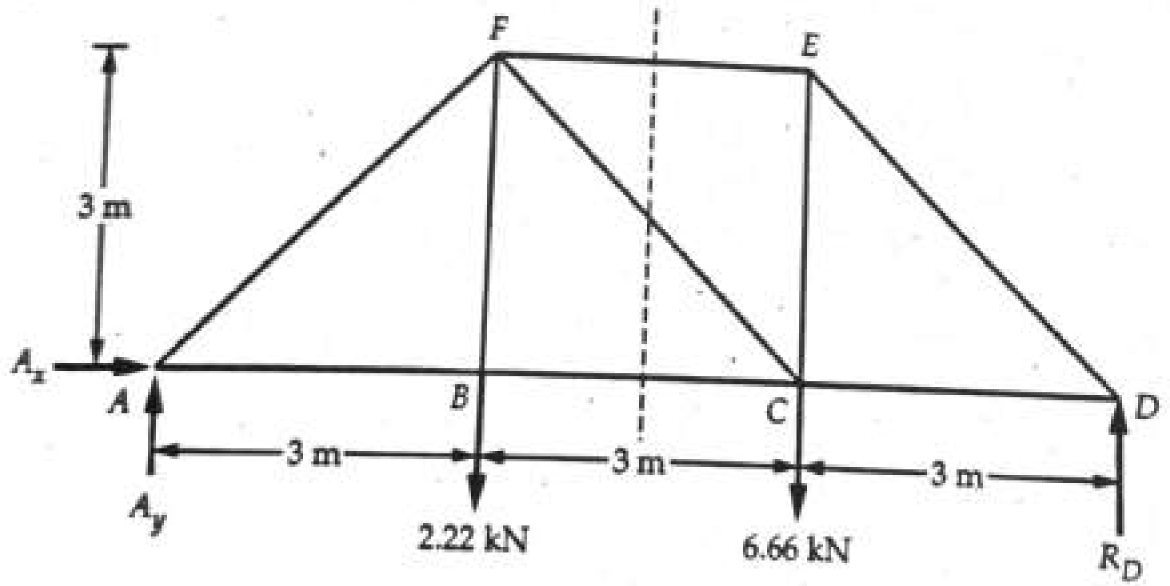

Fig. Q.24.1 (a)

$$\Sigma F_x = 0 : A_x = 0$$

$$\Sigma M_A = 0 : -2.22 \times 3 - 6.66 \times 6 + R_D \times 9 = 0$$

$$\therefore \quad R_D = 5.18 \text{ kN} \uparrow$$

$$\Sigma F_y = 0 : A_y - 2.22 - 6.66 + R_D = 0$$

$$\therefore \quad A_y = 3.7 \text{ kN} \uparrow$$

Take section as shown in Fig. Q.24.1 (a). Draw FBD of part of the truss to the left of the section as shown in Fig. Q.24.1 (b).

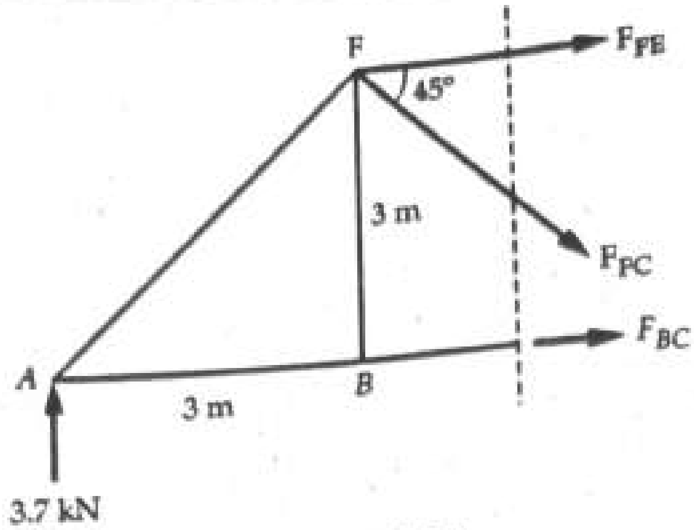

Fig. Q.24.1 (b)

$$\tan \theta_F = \frac{3}{3} \therefore \theta_F = 45°$$

$$\Sigma M_F = 0 : F_{BC} \times 3 - 3.7 \times 3 = 0$$

$\therefore$ $\boxed{F_{BC} = 3.7 \text{ kN (T)}}$ ... Ans.

$$\Sigma F_y = 0 : 3.7 - F_{FC} \sin 2.22 = 0$$

$\therefore$ $\boxed{F_{FC} = 2.09 \text{ kN (T)}}$ ... Ans.

$$\Sigma F_x = 0 : F_{FE} + F_{FC} \cos 45 + F_{BC} = 0$$

$\therefore$ $F_{FE} = -5.17 \text{ kN}$

$\therefore$ $\boxed{F_{FE} = 5.17 \text{ kN (C)}}$ ... Ans.

Member	Magnitude of force (kN)	Nature
BC	3.7	T
FC	2.09	T
FE	5.17	C

Q.25 Determine support reaction and find the forces in magnitude and direction of the members AD, BD and BC of the simply supported truss as shown in Fig. Q.25.1 by method of section.

Tabulate the result with magnitude and nature of force in the members.

☞ [SPPU : May-17, Marks 6]

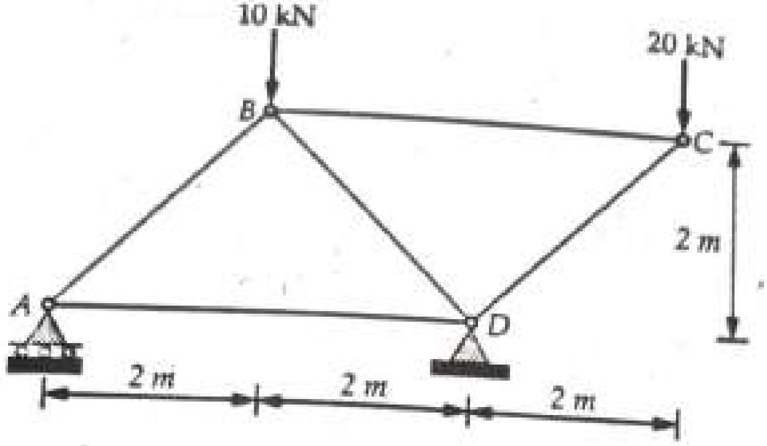

Fig. Q.25.1

Ans. : The F.B.D. of truss is shown in Fig. Q.25.1 (a).

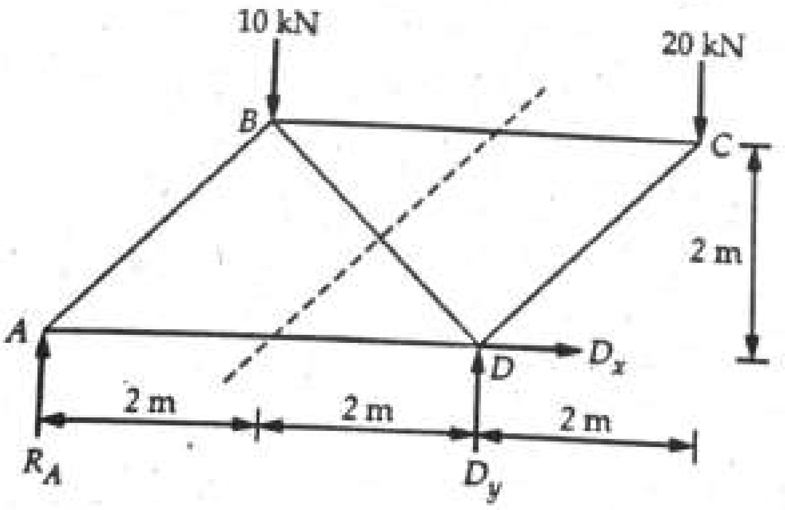

Fig. Q.25.1 (a)

$$\sum M_D = 0 : -R_A \times 4 + 10 \times 2 - 20 \times 2 = 0$$

$$R_A = -5 \text{ kN} = 5 \text{ kN} \downarrow$$

Take section cutting AD, BD and BC as shown in Fig. 10.4.5 (a). Draw FBD of portion of the truss to the left of the section as shown in Fig. Q.25.1 (b).

$$\sum M_B = 0 : (5)\ (2) + F_{AD} \times 2 = 0$$

$$\therefore \quad F_{AD} = -5 \text{ kN}$$

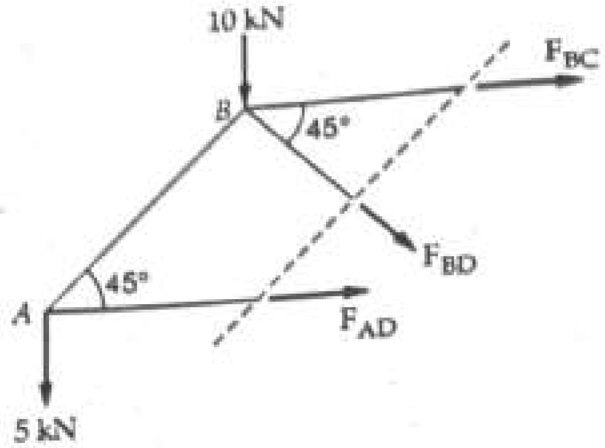

Fig. Q.25.1 (b)

$$\boxed{F_{AD} = 5 \text{ kN (C)}}$$ ... Ans.

$\therefore \quad \sum F_y = 0 : -5 - 10 - F_{BD} \sin 45 = 0$

$\therefore \quad F_{BD} = -21.21 \text{ kN}$

$\therefore \quad \boxed{F_{BD} = 21.21 \text{ kN (C)}}$ ... Ans.

$$\sum F_x = 0 : F_{AD} + F_{BD} \cos 45 + F_{BC} = 0$$

$$-5 - 21.21 \cos 45 + F_{BC} = 0$$

$\therefore \quad \boxed{F_{BC} = 20 \text{ kN (T)}}$ ... Ans.

Member	Magnitude of Force (kN)	Nature
AD	5	C
BD	21.21	C
BC	20	T

Q.26 The roof truss support the vertical loading shown in Fig. Q.26.1. Determine the force in members BC, CK and KJ and state if these members are in tension or compression.

[SPPU : Dec.-17, Marks 7]

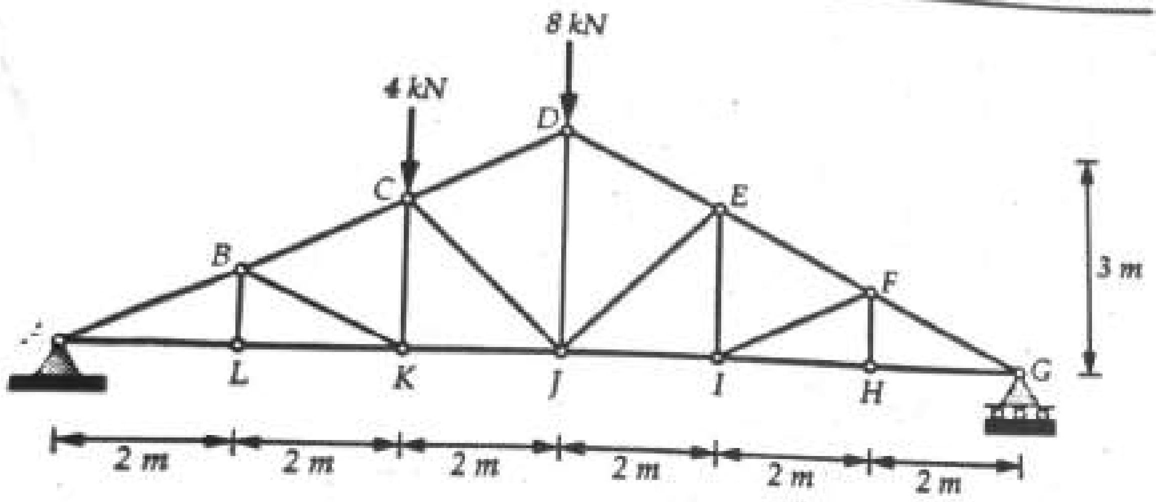

Fig. Q.26.1

Ans. : The FBD of complete truss is shown in Fig. Q.26.1 (a).

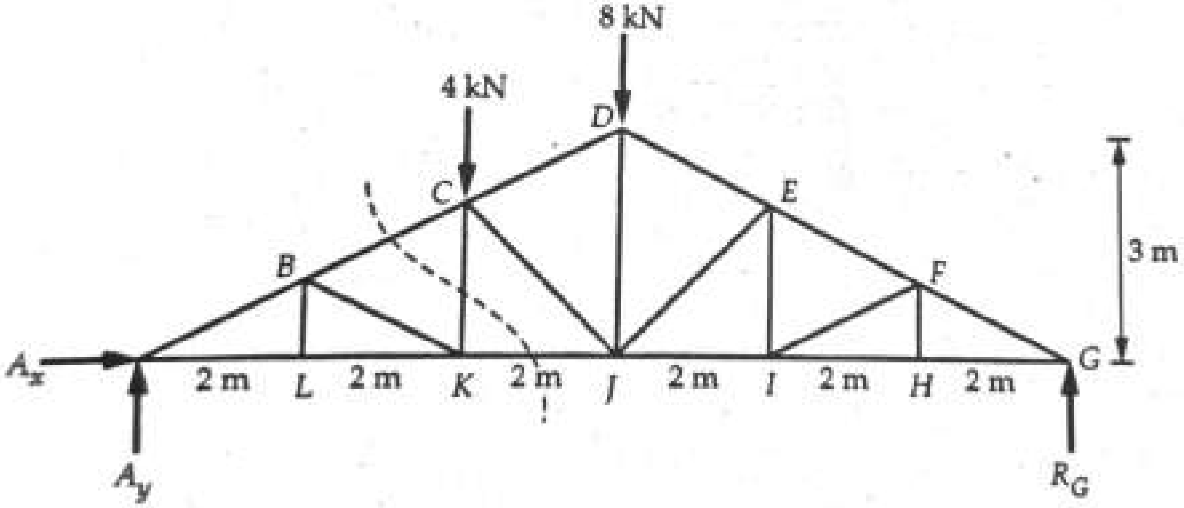

Fig. Q.26.1 (a)

$$\sum F_x = 0 \quad : A_x = 0$$

$$\sum M_G = 0 \quad : (8)(6)+(4)(8)-(A_y)(12) = 0$$

$$\therefore \quad A_x = 6.67 \text{ kN}$$

Take section cutting BC, CK and KJ as shown in Fig. Q.26.1 (a). Draw FBD of part of the truss to the left of the section as shown in Fig. Q.26.1 (b).

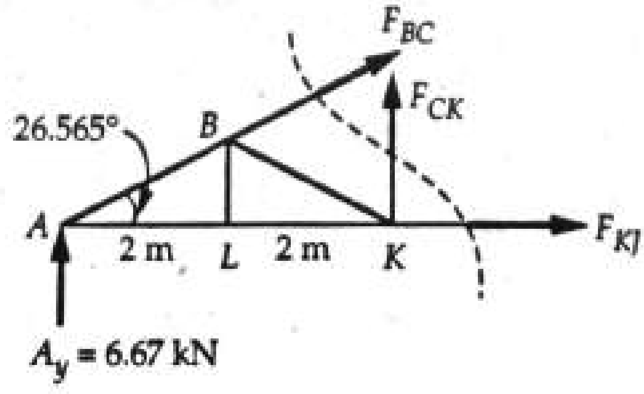

Fig. Q.26.1 (b)

$\angle BAL = \tan^{-1}\left(\frac{3}{6}\right) = 26.565°$

$$\sum M_A = 0 \quad : F_{CK} \times 4 = 0$$

$\therefore$ $\boxed{F_{CK} = 0}$ ... Ans.

$$\sum F_y = 0 \quad : 6.67 + F_{CK} + F_{BC} \sin 26.565 = 0$$

$\therefore$ $F_{BC} = -14.9$ kN

$\therefore$ $\boxed{F_{BC} = 14.9 \text{ kN (C)}}$ ... Ans.

$$\sum F_x = 0 \quad : F_{BC} \cos 26.565 + F_{KJ} = 0$$

$-14.9 \cos 26.565 + F_{KJ} = 0$

$\therefore$ $\boxed{F_{KJ} = 13.33 \text{ kN (T)}}$ ... Ans.

Q.27 The truss supported and loaded as shown in the Fig. Q.27.1. determine the forces in the members AB, AE and EF using section method. Also give the nature of the forces.

[SPPU : Dec.-17, Marks 7]

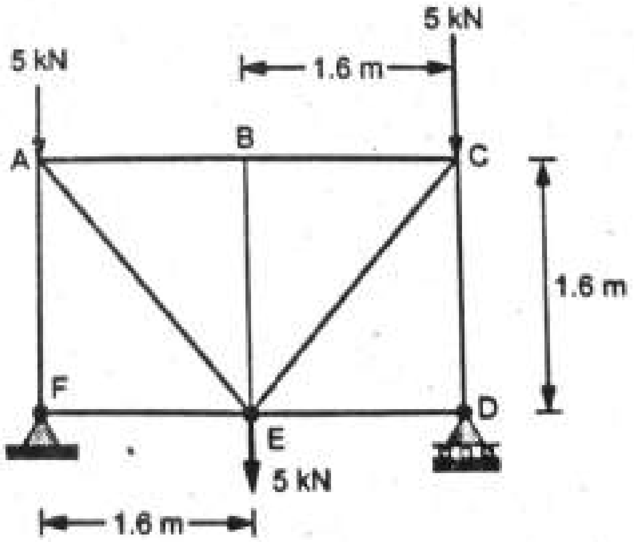

Fig. Q.27.1

Ans. : Draw a section line passing through members AB, EF and AE as shown in Fig. Q.27.1 (a). Consider either part of the truss.

From Q.20.

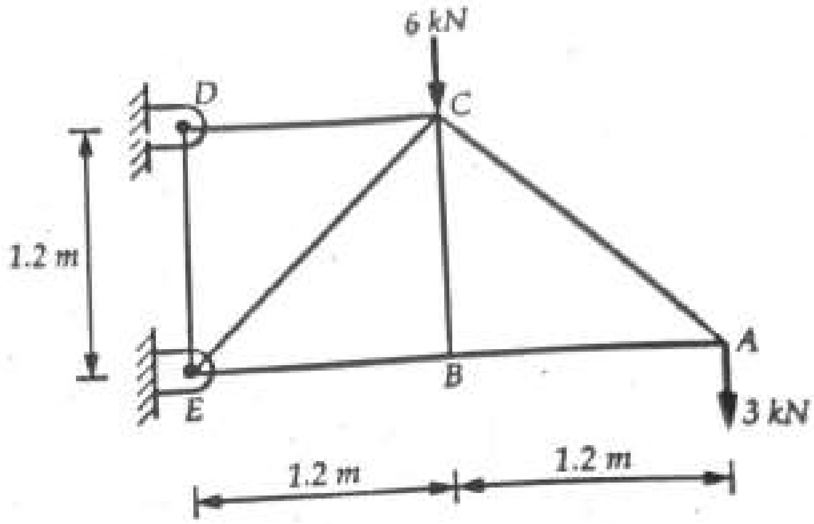

Fig. Q.28.1

Ans. : From FBD of truss as shown in Fig. Q.28.1 (a),

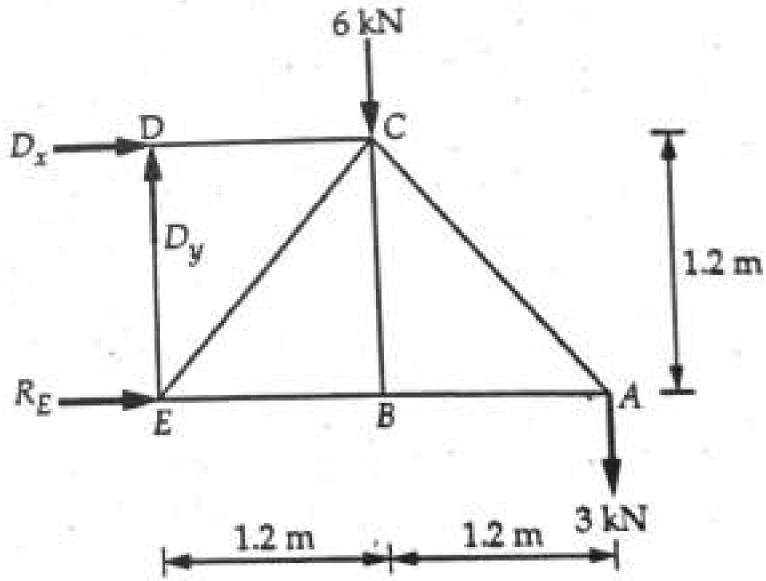

Fig. Q.28.1 (a)

$$\sum M_D = 0 = -6\times1.2-3\times2.4+R_E\times1.2$$

$$\therefore \quad \boxed{R_E = 12 \text{ kN} \rightarrow} \quad \text{... Ans.}$$

$$\sum F_x = 0 : D_x + R_E = 0$$

$$\therefore \quad D_x = -12 \text{ kN} = 12 \text{ kN} \leftarrow$$

$$\sum F_y = 0 : D_y - 6 - 3 = 0$$

$\therefore$ $\boxed{D_y = 9 \text{ kN} \uparrow}$ ... Ans.

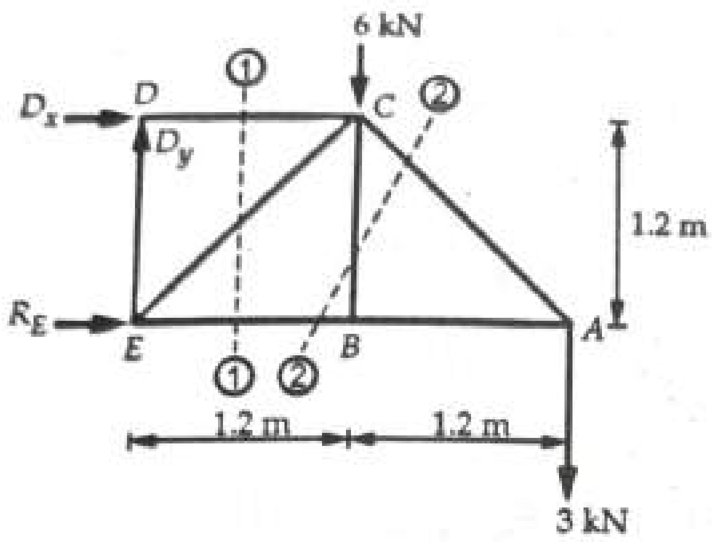

Fig. Q.28.1 (b)

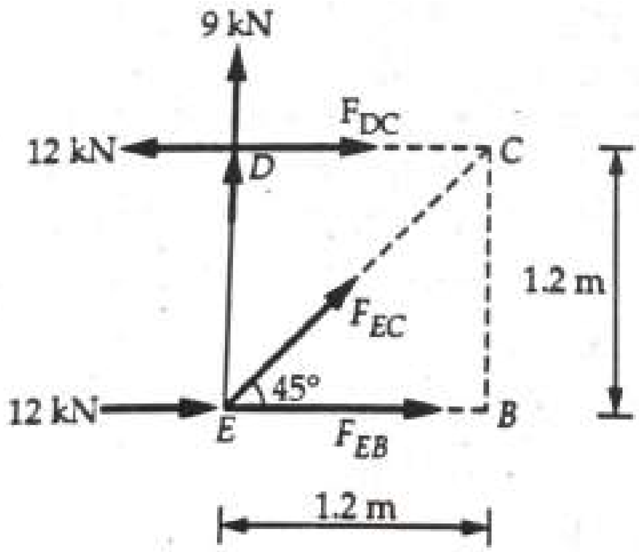

Fig. Q.28.1 (c) Section 1-1

$$\tan\theta_E = \frac{1.2}{1.2}$$

$$\theta_E = 45°$$

For section 1-1,

$$\sum M_E = 0 :$$

$$12 \times 1.2 - F_{DC} \times 1.2 = 0$$

$\therefore$ $\boxed{F_{DC} = 12 \text{ kN (T)}}$ ...Ans.

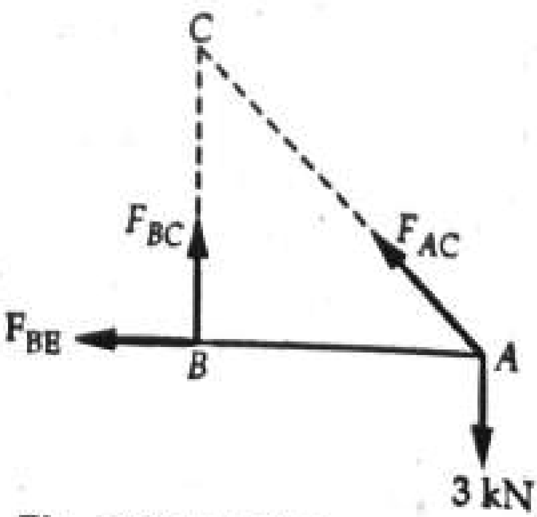

Fig. Q.28.1 (d) Section 2-2

UNIT - IV

10 Frames

Important Points to Remember

Analysis of Frames

i) Use equilibrium of complete frame to find as many support reactions as possible.

ii) Draw F.B.D. of each member of the frame showing equal and opposite reactions at supports common to two members, e.g. common pin connections.

iii) Use $\sum F_x = 0, \sum F_y = 0, \sum M = 0$ for each F.B.D. to find unknown forces.

iv) Calculations can be reduced by identifying two force member, i.e, members which have coaxial force. Such members have forces only at the ends.

Q.1 Determine the horizontal and vertical components of force that pins A and C exert on the frame. Refer Fig. Q.1.1.

[SPPU : May-10]

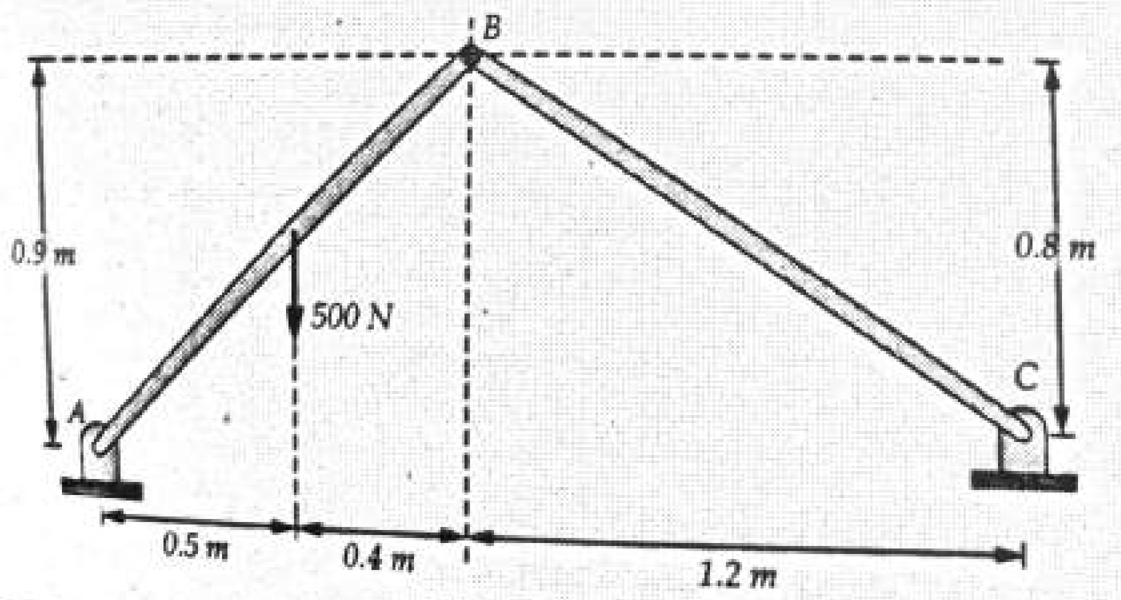

Fig. Q.1.1

Scanned by CamScanner

Ans. : The F.B.D. of frame is shown in Fig. Q.1.1 (a).

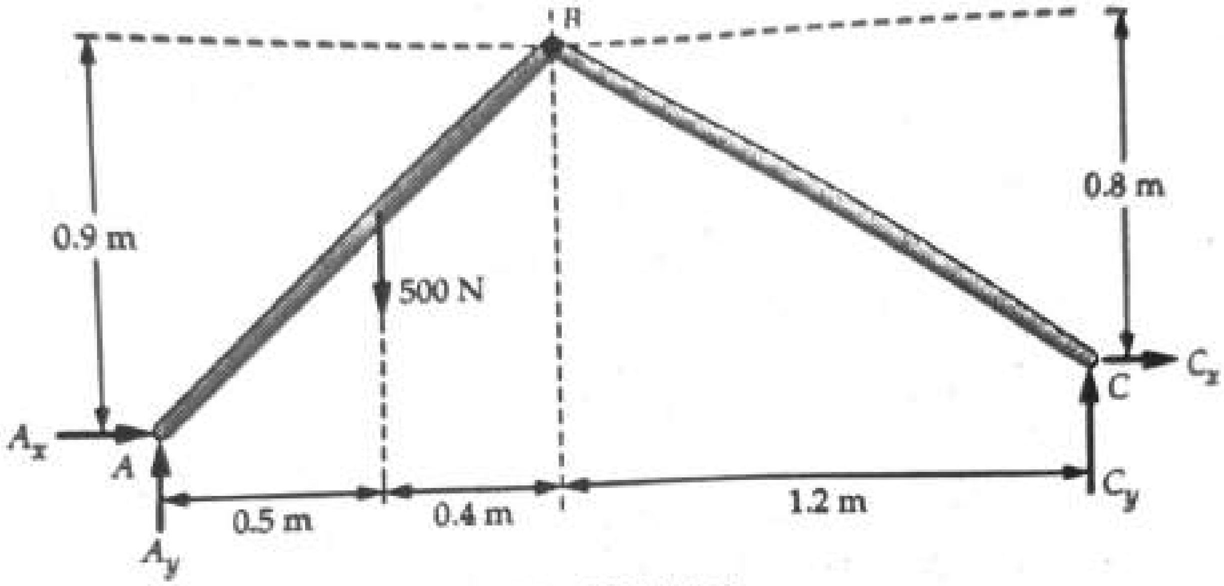

Fig. Q.1.1 (a)

$\sum M_A = 0:$

$$-(C_x)(0.1) + (C_y)(2.1) - (500)(0.5) = 0$$

$$-0.1\ C_x + 2.1\ C_y = 250 \quad \ldots (1)$$

$\sum F_x = 0:$

$$A_x + C_x = 0 \quad \ldots (2)$$

$\sum F_y = 0:$

$$A_y + C_y - 500 = 0 \quad \ldots (3)$$

Consider F.B.D. of BC shown in Fig. Q.1.1 (b).

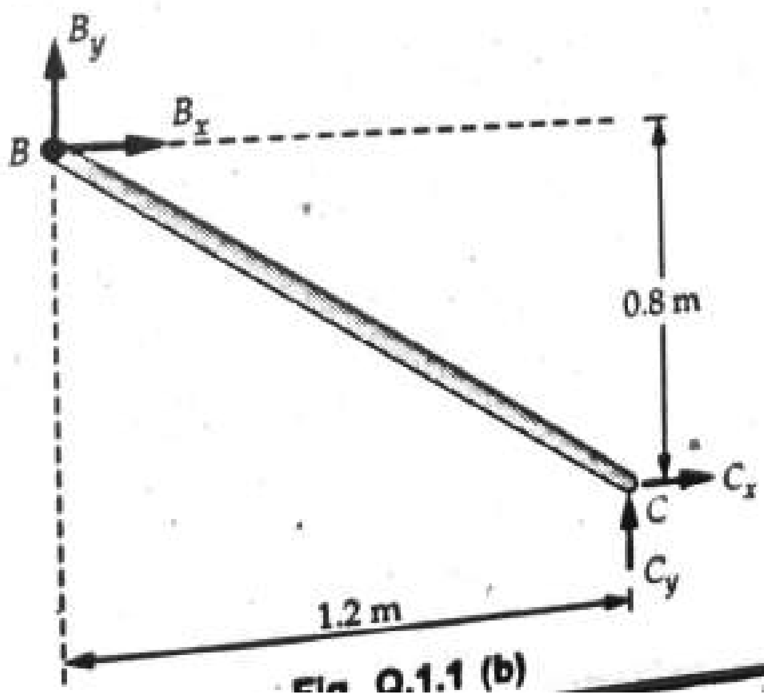

Fig. Q.1.1 (b)

$\sum M_B = 0$:

$$(C_x)(0.8) + (C_y)(1.2) = 0$$

$$0.8\,C_x + 1.2\,C_y = 0 \qquad \text{... (4)}$$

From equations (1) and (4),

$$C_x = -166.67 \text{ N}$$

$\therefore$ $\boxed{C_x = 166.67 \text{ N} \leftarrow}$ ...Ans.

$\boxed{C_y = 111.11 \text{ N} \uparrow}$ ...Ans.

From equation (2),

$$A_x = -C_x = -(-166.67)$$

$\therefore$ $\boxed{A_x = 166.67 \text{ N} \rightarrow}$ ...Ans.

From equation (3),

$$A_y + 111.11 - 500 = 0$$

$\therefore$ $\boxed{A_y = 388.89 \text{ N} \uparrow}$ ...Ans.

Q.2 Determine components of reaction at pin C for the pin jointed frame loaded and supported as shown in Fig. Q.2.1.

[SPPU : May-10]

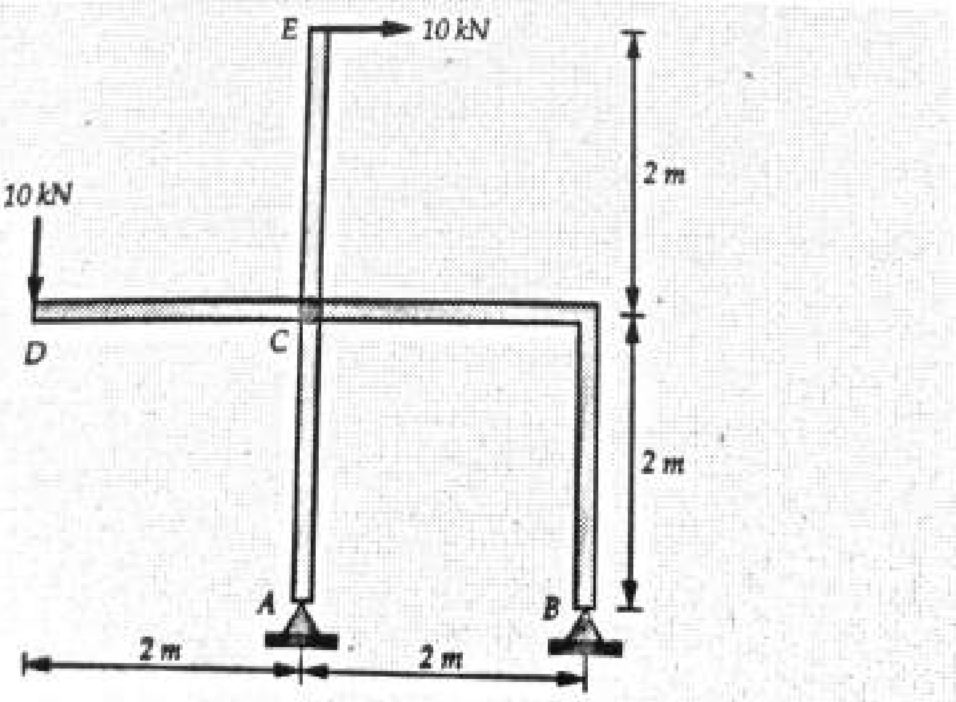

Fig. Q.2.1

Ans. : The F.B.D. by complete frame is shown in Fig. Q.2.1 (a).

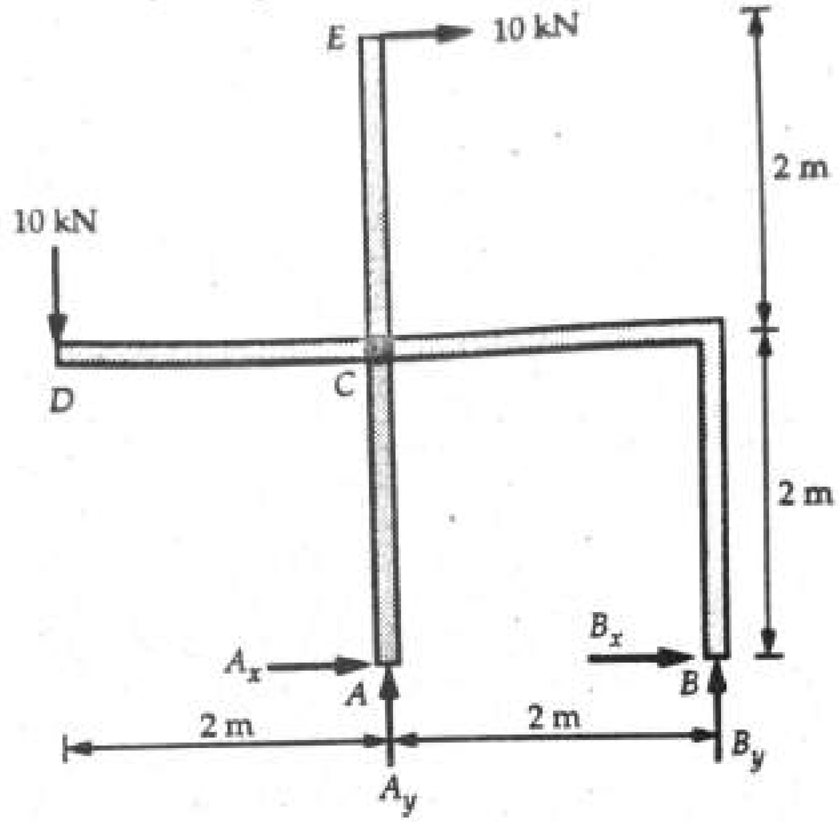

Fig. Q.2.1 (a)

$\sum M_A = 0$:

$$(10)(2) - (10)(4) + (B_y)(2) = 0$$

$$\therefore \quad B_y = 10 \text{ kN} \uparrow$$

$\sum F_y = 0$:

$$-10 + A_y + B_y = 0$$

$$\therefore \quad A_y = 0$$

The free body diagrams for AE and BD are shown in Fig. Q.2.1 (b).

From F.B.D. of AE,

$\sum F_y = 0$:

$$\therefore \quad \boxed{C_y = 0} \quad \text{...Ans.}$$

$\sum M_A = 0$:

$$-(C_x)(2) - (10)(4) = 0$$

$\therefore$ $\boxed{C_x = -20 \text{ kN}}$ ...Ans.

$\therefore$ x-component of reaction at C is 20 kN ← for AE and 20 kN → for BD.

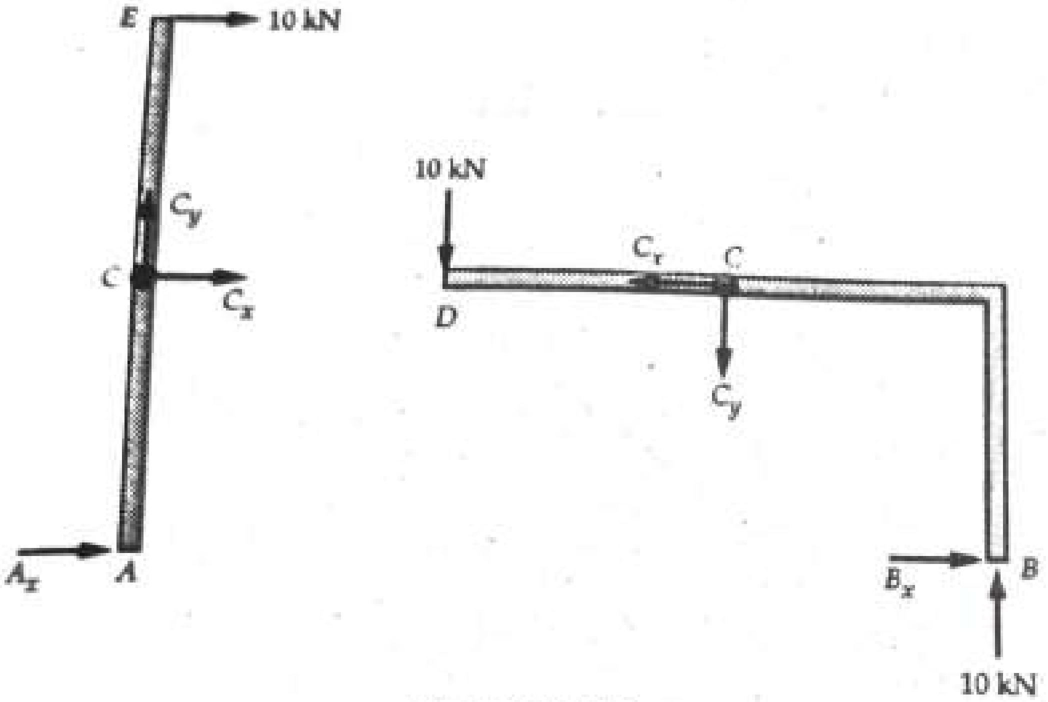

Fig. Q.2.1 (b)

Q.3 Determine the horizontal and vertical components of reaction at pin B and C for the frame shown in Fig. Q.3.1.

[SPPU : May-12, Marks 7]

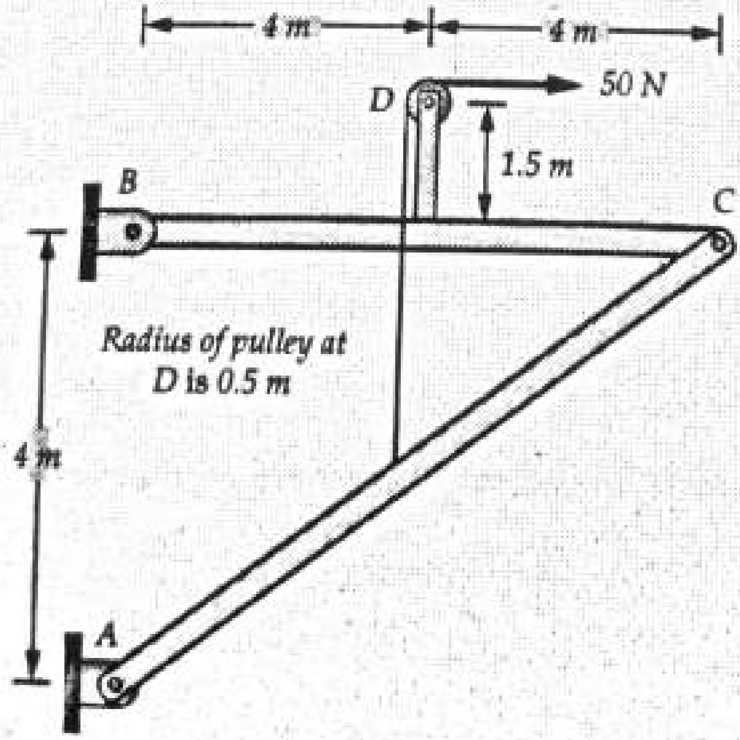

Fig. Q.3.1

Ans. : The F.B.D of complete frame is shown in Fig. Q.3.1 (a).

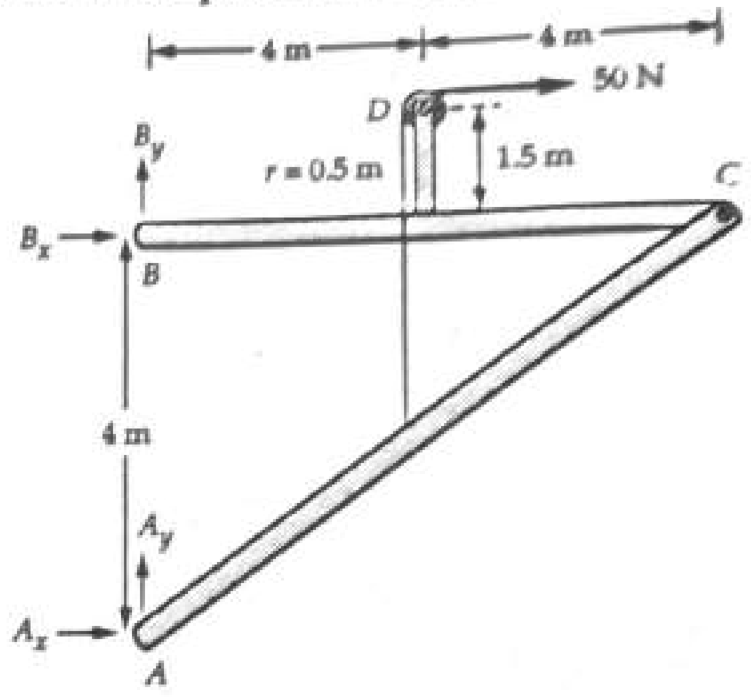

Fig. Q.3.1 (a)

$\sum M_B = 0$:

$$(A_x)(4)-(50)(2) = 0$$

$$\therefore \quad A_x = 25 \text{ N}$$

$\sum F_x = 0$:

$$B_x + A_x + 50 = 0$$

$$\therefore \quad B_x = -75 \text{ N}$$

$$\therefore \quad \boxed{B_x = 75 \text{ N} \leftarrow}$$ ...Ans.

$\sum F_y = 0$:

$$A_y + B_y = 0 \qquad \text{... (1)}$$

Free body diagram of the two members of the frame are shown in Fig. Q.3.1 (b).

For member BC,

$\sum F_x = 0$:

$$-75 + 50 + C_x = 0$$

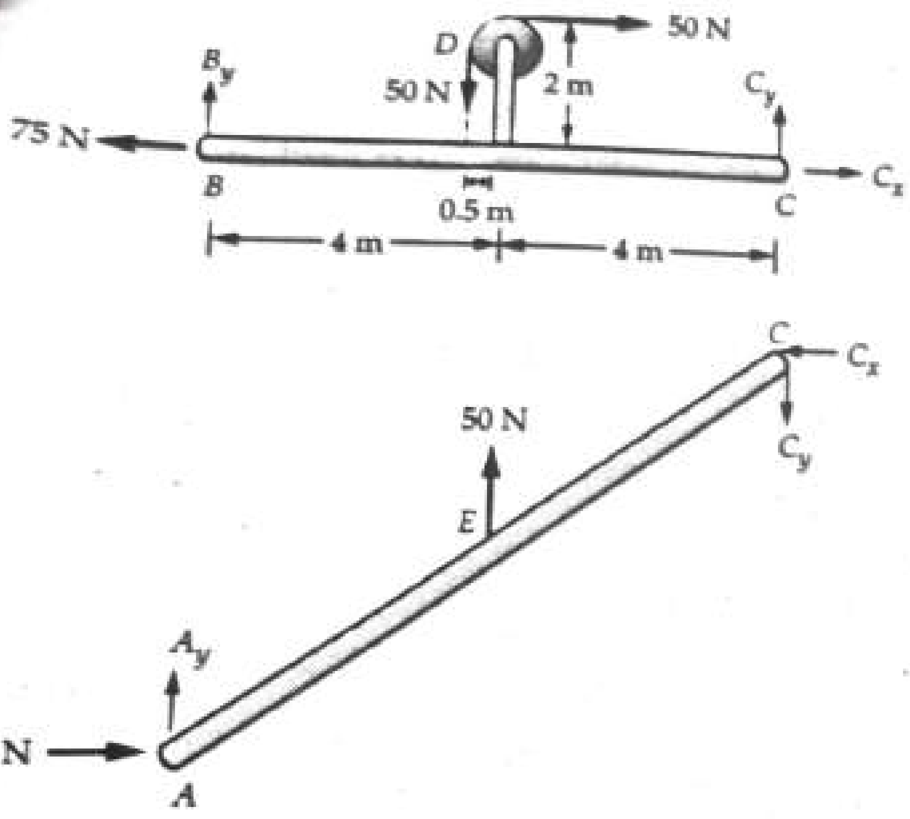

Fig. Q.3.1 (b)

$\therefore \qquad \boxed{C_x = 25 \text{ N}}$ ...Ans.

$\sum M_B = 0$:

$-(50)(2) - (50)(3.5) + (C_y)(8) = 0$

$\therefore \qquad \boxed{C_y = 34.375 \text{ N}}$ ...Ans.

$\sum F_y = 0$:

$$B_y - 50 + C_y = 0$$

$\therefore \qquad \boxed{B_y = 15.625 \text{ N} \uparrow}$ ...Ans.

Q.4 Determine the components of reaction at C for the frame loaded and supported as shown in Fig. Q.4.1.

[SPPU : Dec.-14, Marks 6]

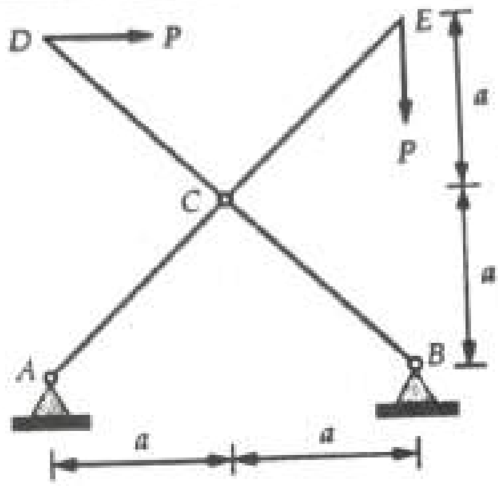

Fig. Q.4.1

Ans. : The FBD of complete frame is shown in Fig. Q.4.1 (a).

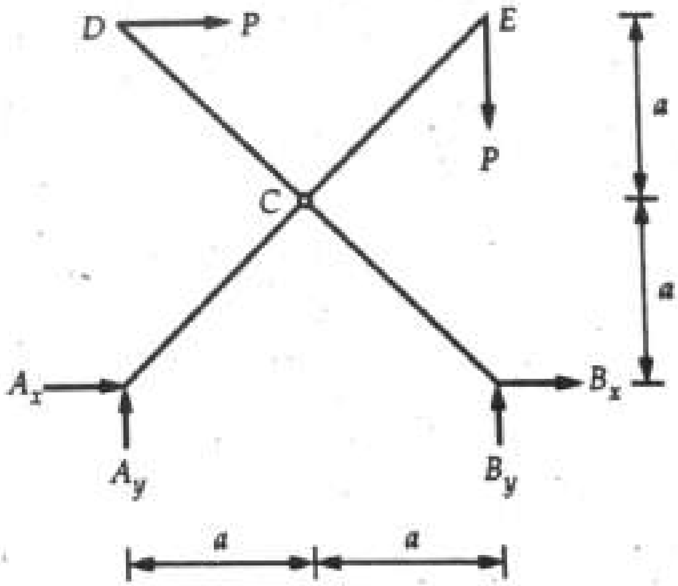

Fig. Q.4.1 (a)

$$\sum M_A = 0 :$$

$$(B_y)(2a) - (P)(2a) - (P)(2a) = 0$$

$$B_y = 2P$$

$$\sum F_y = 0 : A_y + B_y - P = 0$$

$$A_y = -P$$

$$\therefore \quad A_y = P \downarrow \qquad \dots (1)$$

$$\sum F_x = 0 : A_x + B_x + P = 0$$

The separate free body diagrams of ACE and BCD are shown in Fig. Q.4.1 (b)

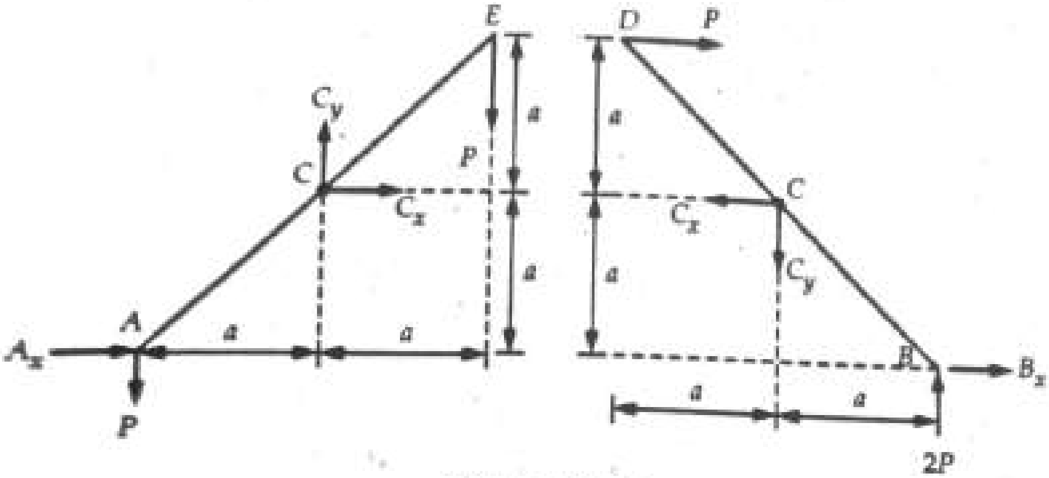

Fig. Q.4.1 (b)

For ACE,

$$\sum M_C = 0:$$

$$(A_x)(a) - (P)(a) + (P)(a) = 0$$

$$\therefore \quad A_x = 0$$

$$\sum F_x = 0 : C_x + A_x = 0$$

$$\therefore \quad \boxed{C_x = 0}$$ ...Ans.

$$\sum F_y = 0 : -P + C_y - P = 0$$

$$\therefore \quad \boxed{C_y = 2P}$$ ...Ans.

Q.5 Determine the reactions at the internal hinge C for the frame loaded and supported as shown in Fig. Q.5.1.

[SPPU : May-15, Marks 6]

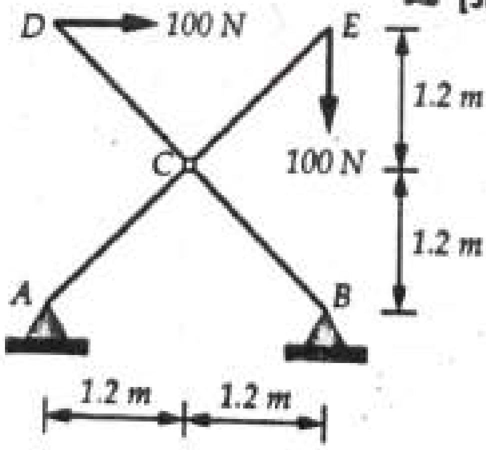

Fig. Q.5.1

Ans. : The FBD of complete frame is shown in Fig. Q.5.1 (a).

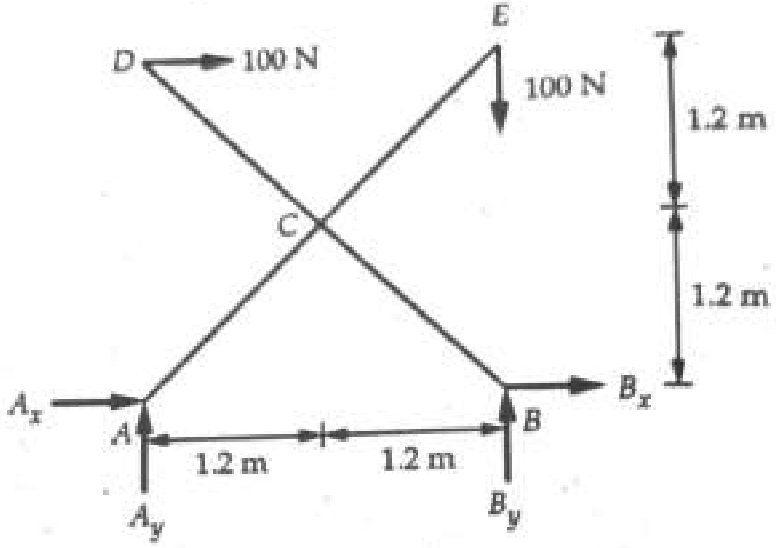

Fig. Q.5.1 (a)

$$\Sigma M_A = 0 : B_y\,(2.4) - (100)\,(2.4) - (100)\,(2.4) = 0$$

$$\therefore \quad B_y = 200 \text{ N}$$

$$\Sigma F_y = 0 : A_y + B_y - 100 = 0$$

$$\therefore \quad A_y = -100 \text{ N} = 100 \text{ N} \downarrow$$

$$\Sigma F_x = 0 : A_x + B_x + 100 = 0 \qquad ...(1)$$

The separate free body diagrams of *ACE* and *BCD* are shown in Fig. Q.5.1 (b).

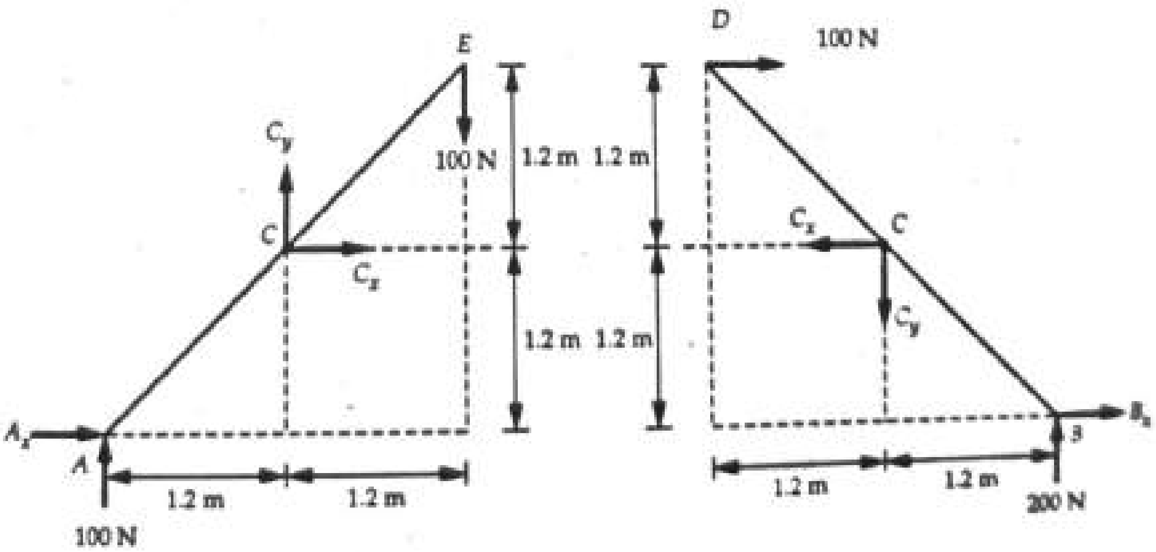

Fig. Q.5.1 (b)

For ACE,

$$\Sigma M_c = 0 :$$

$$(A_x)(1.2) + (100)(1.2) - (100)(1.2) = 0$$

$$\therefore \quad A_x = 0$$

$$\Sigma F_x = 0 : C_x + A_x = 0$$

$$\therefore \quad \boxed{C_x = 0} \quad \text{...Ans.}$$

$$\Sigma F_y = 0 : -100 + C_y - 100 = 0$$

$$\therefore \quad \boxed{C_y = 200\ \text{N}} \quad \text{...Ans.}$$

Q.6 Determine the magnitude of pin reactions at A, B and D for the frame loaded as shown in Fig. Q.6.1. [SPPU : Dec.-15, Marks 6]

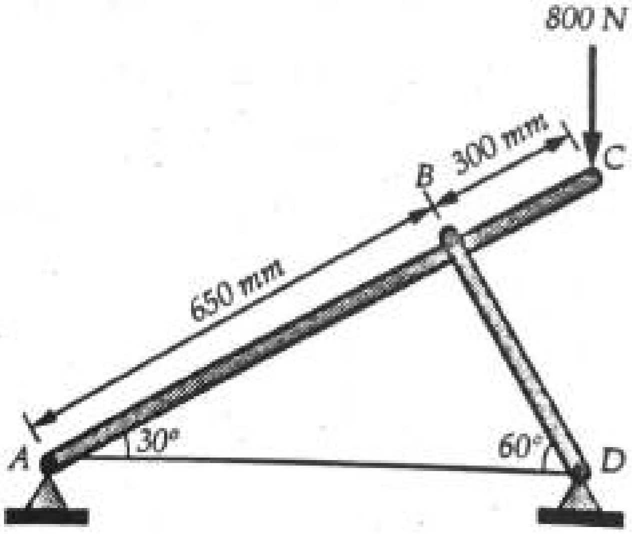

Fig. Q.6.1

Ans. : From given Fig. Q.6.1 (a),

$$AD = \frac{650}{\cos 30} = 750.56 \text{ mm}$$

The FBD of complete frame is shown in Fig. Q.6.1 (a).

$$\sum M_A = 0 : (D_y)(750.56) - (800)(950 \cos 30) = 0$$

$$\therefore \quad D_y = 876.92 \text{ N}$$

$$\sum F_y = 0 : A_y + D_y - 800 = 0$$

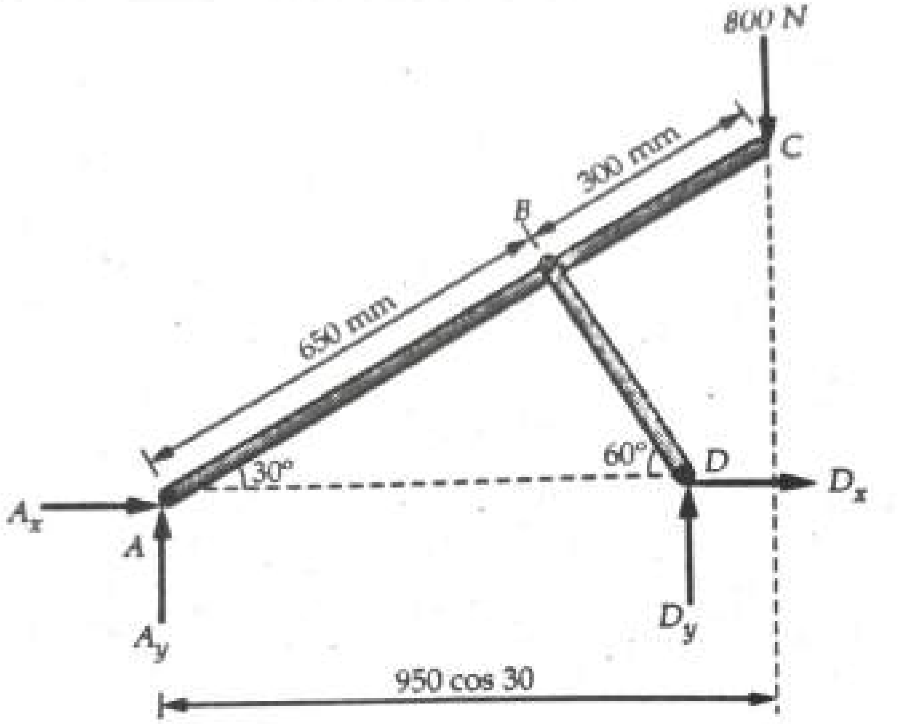

Fig. Q.6.1 (a)

$\therefore \qquad A_y = -76.92 \text{ N} = 76.92 \text{ N} \downarrow$

The free body diagrams of ABC and BD are shown in Fig. Q.6.1 (b).

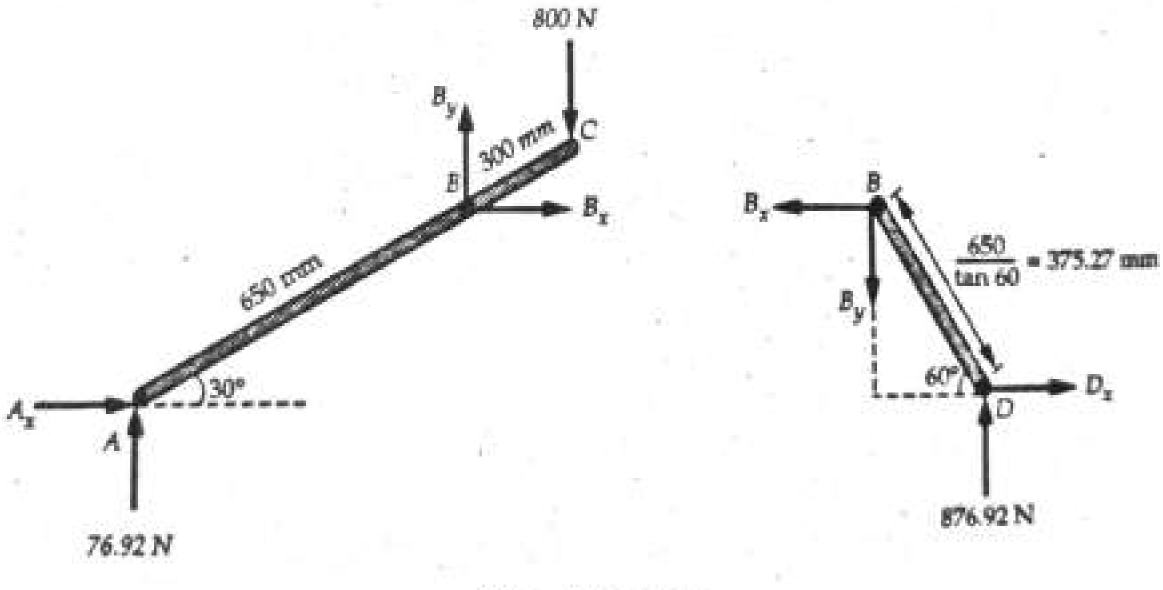

Fig. Q.6.1 (b)

For BD, $\quad \sum F_y = 0:$

$$876.92 - B_y = 0$$

$\therefore \quad B_y = 876.92 \text{ N}$

In Δ BDA, $\tan 60 = \frac{650}{BD}$

$$\therefore \quad BD = \frac{650}{\tan 60} = 375.27 \text{ mm}$$

$$\sum M_D = 0 : (B_y)(375.27 \cos 60) + (B_x)(375.27 \sin 60) = 0$$

$$\therefore \quad B_x = -506.3 \text{ N}$$

$\sum F_x = 0$:

$$-B_x + D_x = 0$$

$$\therefore \quad D_x = -506.3 \text{ N}$$

For ABC, $\sum F_x = 0$:

$$A_x + B_x = 0$$

$$A_x - 506.3 = 0$$

$$\therefore \quad A_x = 506.3 \text{ N}$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{506.3^2 + 76.92^2}$$

$$\therefore \quad \boxed{R_A = 512.1 \text{ N}}$$

...Ans.

$$\theta = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) = \tan^{-1}\left(\frac{76.92}{506.3}\right)$$

$$\therefore \quad \boxed{\theta_A = 8.64^\circ}$$

...Ans.

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{506.3^2 + 876.92^2}$$

$$\therefore \quad \boxed{R_B = 1012.6 \text{ N}}$$

...Ans.

$$R_D = \sqrt{D_x^2 + D_y^2} = \sqrt{506.3^2 + 876.92^2}$$

$$\therefore \quad \boxed{R_D = 1012.6 \text{ N}}$$

...Ans.

$$\theta_D = \tan^{-1}\left(\frac{|D_y|}{|D_x|}\right) = \tan^{-1}\left(\frac{876.92}{506.3}\right)$$

$\therefore$ $\boxed{\theta_D = 60^\circ}$ ...Ans.

Q.7 Determine the horizontal and vertical component of reactions at A and B for the frame loaded and supported as shown in Fig. Q.7.1. [SPPU : May-16, Dec.-16, Marks 5]

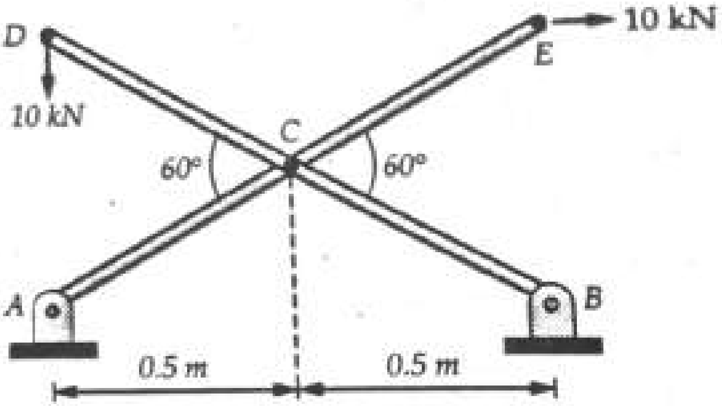

Fig. Q.7.1

Ans. : The FBD of complete frame is shown in Fig. Q.7.1 (a).

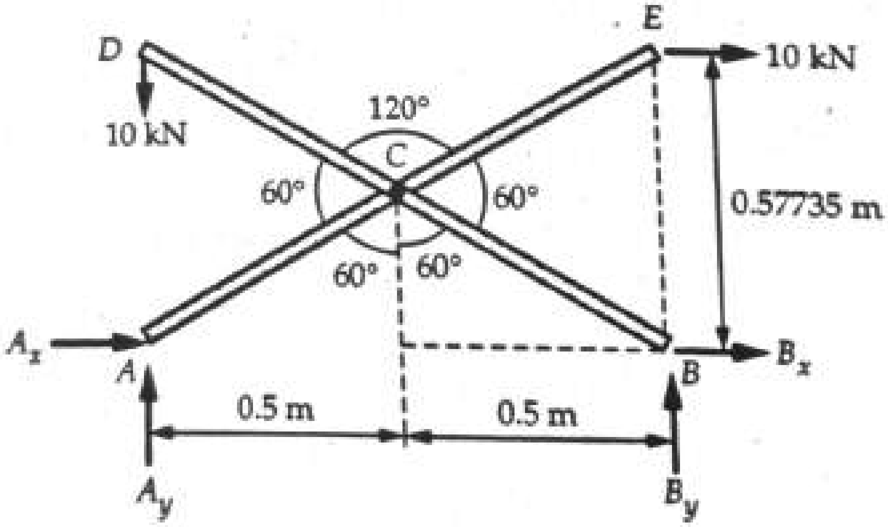

Fig. Q.7.1 (a)

$$BC = \frac{0.5}{\sin 60} = 0.57735 \text{ m}$$

ABCE is equivalent triangle.

$$\therefore \quad BE = 0.57735 \text{ m}$$

$$\sum M_A = 0 : \; B_y \times 1 - 10 \times 0.57735 = 0$$

$\therefore$ $\boxed{B_y = 5.7735 \text{ kN} \uparrow}$ ...Ans.

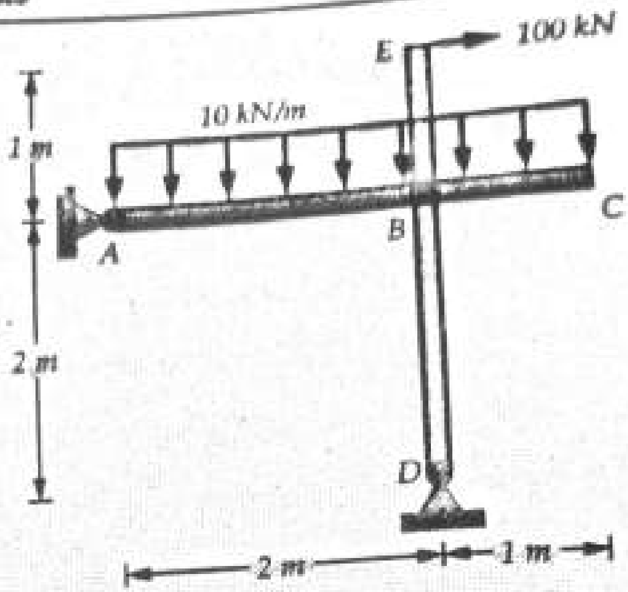

Fig. Q.8.1

Ans. : The F.B.D. of the system of bars is shown in Fig. Q.8.1 (a).

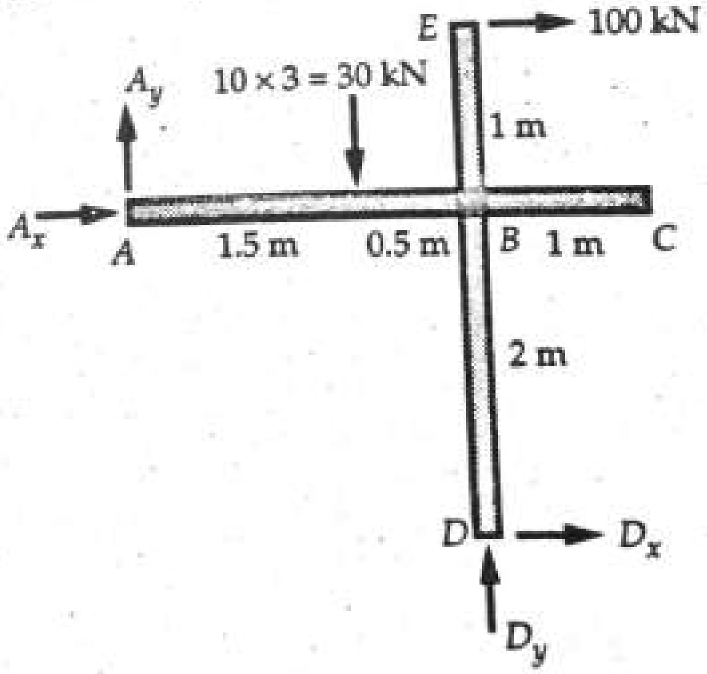

Fig. Q.8.1 (a)

As there are four unknown reactions, we cannot solve for all the unknowns using this F.B.D. we can only get equations relating these unknowns.

$$\sum M_A = 0 :$$

$$-(30)(1.5) - (100)(1) + (D_x)(2) + (D_y)(2) = 0$$

$$\therefore \quad D_x + D_y = 72.5 \qquad \dots (1)$$

$$\sum F_x = 0:$$

$$A_x + D_x + 100 = 0 \quad \ldots (2)$$

$$\sum F_y = 0:$$

$$A_y + D_y - 30 = 0 \quad \ldots (3)$$

The free body diagram of bars *ABC* and *DBE* are shown in Fig. Q.8.1 (b).

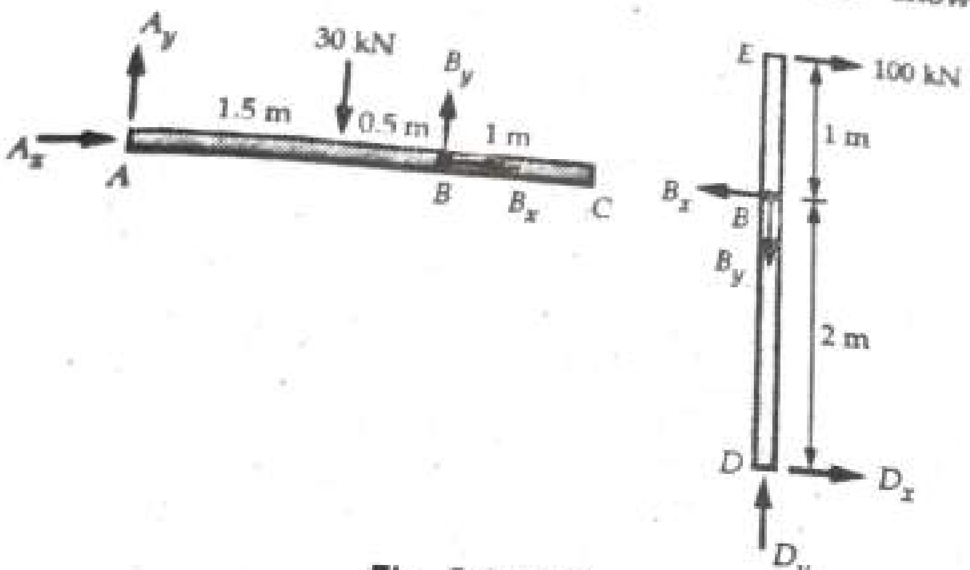

Fig. Q.8.1 (b)

From F.B.D. of *ABC*

$$\sum M_B = 0:$$

$$(30)(0.5) - (A_y)(2) = 0$$

$$A_y = 7.5 \text{ kN} (\uparrow)$$

From equation (3),

$$D_y = 22.5 \text{ kN} \uparrow$$

From equation (1),

$$D_x = 50 \text{ kN} (\rightarrow)$$

From equation (2),

$$A_x = -150 \text{ kN}$$

$$\therefore \quad A_x = 150 \text{ kN} \leftarrow$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{150^2 + 7.5^2}$$

$$\therefore \quad \boxed{R_A = 150.19 \text{ kN}}$$ ...Ans.

$$\theta_A = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) = \tan^{-1}\left(\frac{7.5}{150}\right)$$

$$\therefore \quad \boxed{\theta_A = 2.86^\circ}$$ ...Ans.

$$R_D = \sqrt{D_x^2 + D_y^2} = \sqrt{50^2 + 22.5^2}$$

$$\therefore \quad \boxed{R_D = 54.83 \text{ kN}}$$ ...Ans.

$$\theta_D = \tan^{-1}\left(\frac{|D_y|}{|D_x|}\right) = \tan^{-1}\left(\frac{22.5}{50}\right)$$

$$\therefore \quad \boxed{\theta_D = 24.23^\circ}$$ ...Ans.

END...

UNIT - IV

11 Cables

Important Points to Remember

Analysis of Cables

i) Use equilibrium of complete cable to set up three equations relating the four unknown reactions
$\sum F_x = 0, \sum F_y = 0, \sum M = 0.$

ii) Use method of sections to obtain one more equation.

iii) Solve the four equations simultaneously to obtain reactions at end points.

iv) The reactions at end points are equal to tension in first and last part of the cable. To find tension in other parts of the cable, draw F.B.D. of points where loads are attached and use $\sum F_x = 0$, $\sum F_y = 0$, to find tension and angle made by that part of the cable with horizontal.

v) That part of the cable has maximum tension which makes maximum angle with horizontal. For vertical loads, only the first or last part of the cable can have maximum tension.

Q.1 State the assumptions for the analysis of cable.

☞ [SPPU : Dec.-14, Marks 2]

Ans. : 1) The weight of the cable is neglected due to which cable remains straight between two point loads.

2) Cables have only tensile force.

3) It is assumed that the cables are flexible and inextensible.

Q.2 Two loads are suspended as shown in Fig. Q.2.1 from a cable ABCD. Knowing that d_c = 0.75 m, determine :
a) Distance 'd_B'   b) The reaction components at 'A' and 'D'
c) Maximum tension in cable.

☞ [SPPU : Dec.-11]

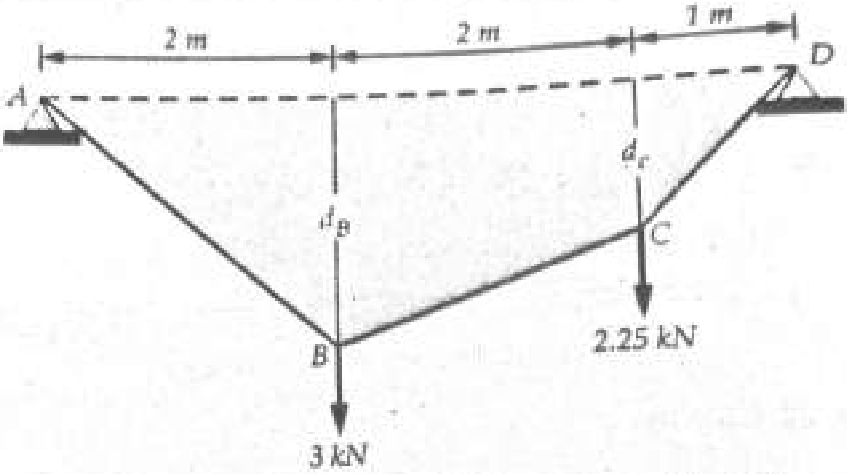

Fig. Q.2.1

Sol. : The F.B.D of elable is shown in Fig. Q.2.1 (a)

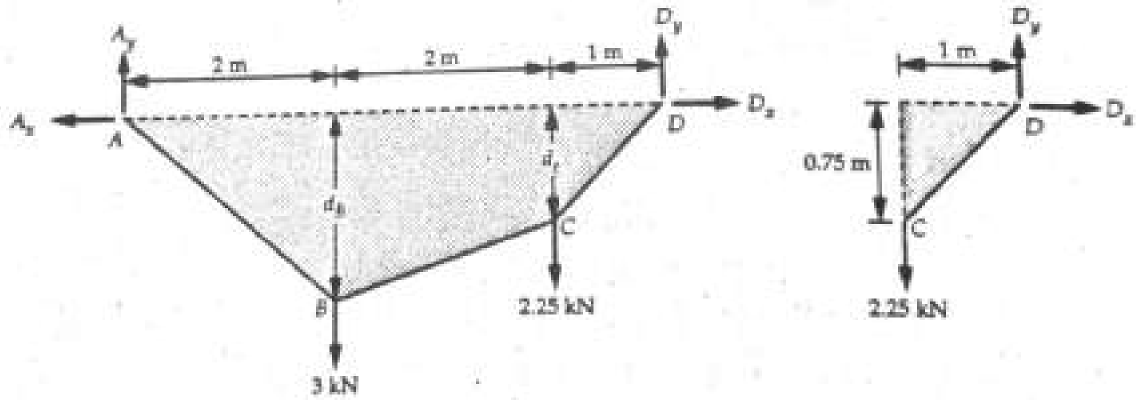

Fig. Q.2.1 (a)

$\sum F_x = 0$:

$$A_x = D_x \qquad \text{... (1)}$$

$\sum F_y = 0$:

$$A_y + D_y - 3 - 2.25 = 0$$

$$\therefore \quad A_y + D_y = 5.25 \qquad \text{... (2)}$$

$\sum M_A = 0$:

$$-(3)(2) - (2.25)(4) + (D_y)(5) = 0$$

$$\therefore \quad \boxed{D_y = 3 \text{ kN} \uparrow} \qquad \text{...Ans.}$$

From equation (2),

$\therefore$ $\boxed{A_y = 2.25 \text{ kN} \uparrow}$ ...Ans.

For section between B and C, $\sum M_C = 0$ for R.H.S.

$$-(D_x)(0.75)+(D_y)(1) = 0$$

$\therefore$ $\boxed{D_x = 4 \text{ kN} \rightarrow}$ ...Ans.

For equation (1),

$\therefore$ $\boxed{A_x = 4 \text{ kN} \leftarrow}$ ...Ans.

At A,

$$\tan\theta_A = \frac{A_y}{A_x} = \frac{d_B}{2}$$

$$\therefore \quad \frac{2.25}{4} = \frac{d_B}{2}$$

$\therefore$ $\boxed{d_B = 1.125 \text{ m}}$ ...Ans.

As $D_y > A_y$, tension is maximum in portion CD

$$\therefore \quad T_{max} = \sqrt{D_x^2 + D_y^2} = \sqrt{4^2+3^2}$$

$\therefore$ $\boxed{T_{max} = 5 \text{ kN}}$ ...Ans.

Q.3 A cable supported at A and D at the same level over a span of 30 m is loaded as shown in Fig. Q.3.1. Determine the maximum tension in the segement of the cable.

[SPPU : May-14, Marks 6]

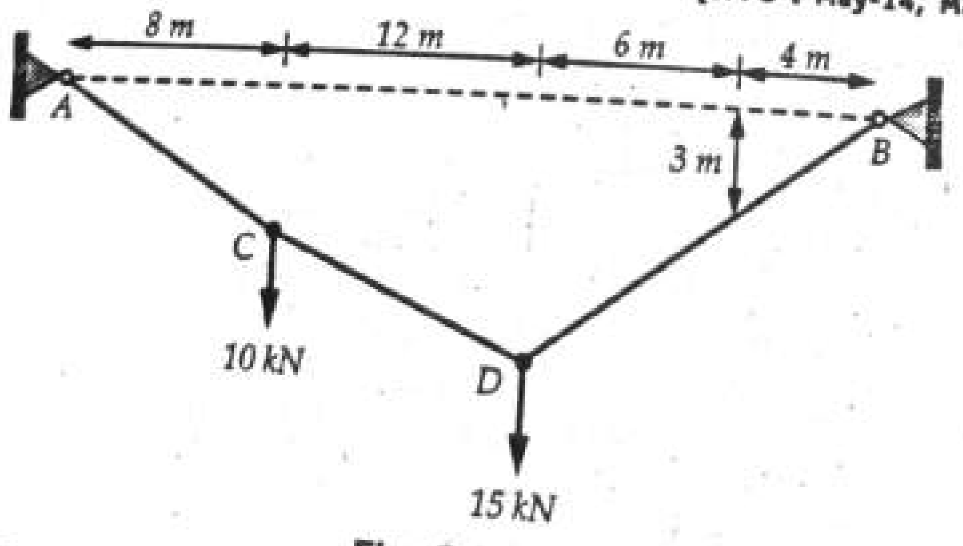

Fig. Q.3.1

Ans. : The FBD of cable is shown in Fig. Q.3.1 (a).

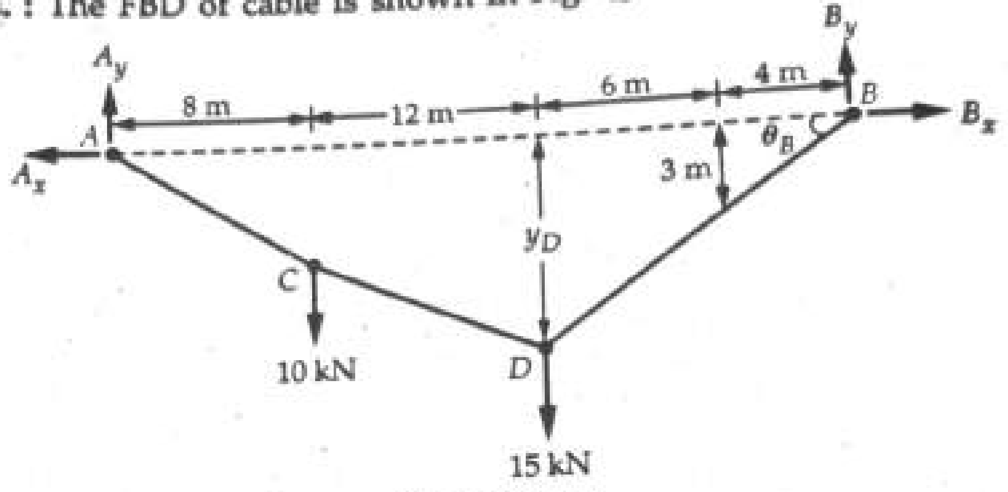

Fig. Q.3.1 (a)

$$\sum M_A = 0 :$$

$$(B_y)(30) - (10)(8) - (15)(20) = 0$$

$$\therefore \quad B_y = 12.667 \text{ kN}$$

$$\sum F_y = 0 : A_y + B_y - 10 - 15 = 0$$

$$\therefore \quad A_y = 12.333 \text{ kN}$$

$$\tan\theta_B = \frac{3}{4} = \frac{B_y}{B_x}$$

$$\therefore \quad B_x = \frac{4}{3} B_y$$

$$B_x = \frac{4}{3} \times 12.667$$

$$\therefore \quad B_x = 16.889 \text{ kN}$$

$$\sum F_x = 0 \therefore A_x = B_x$$

$$\therefore \quad A_x = 16.889 \text{ kN}$$

As $B_y > A_y$,

$$T_{BD} = T_{max} = \sqrt{B_x^2 + B_y^2}$$

$$\therefore \quad T_{max} = \sqrt{16.889^2 + 12.667^2}$$

$$\therefore \quad \boxed{T_{max} = 21.11 \text{ kN}}$$ ...Ans.

Q.4 Cable ABCD supports the 4 kN and 6 kN loads at points B and C as shown in Fig. Q.4.1. Determine the maximum tension in cable and the sag of point B.
[SPPU : May-15, Marks 5]

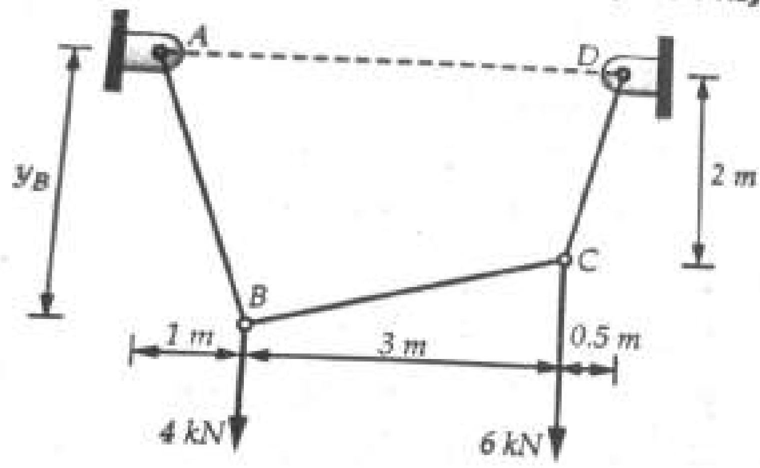

Fig. Q.4.1

Ans. : The FBD of cable is shown in Fig. Q.4.1 (a).

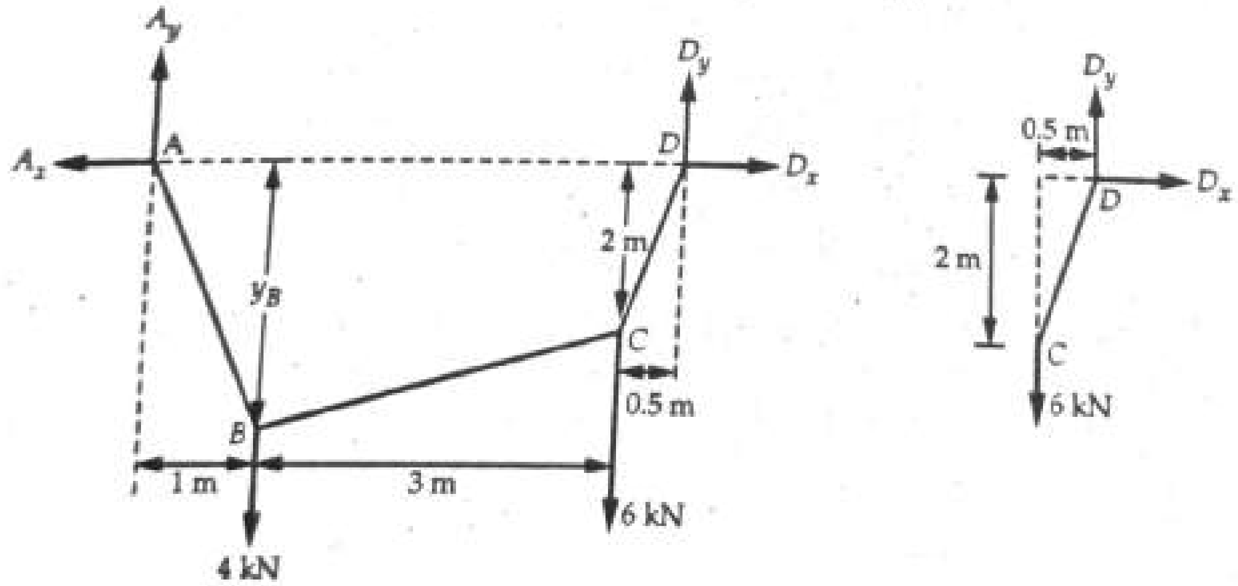

Fig. Q.4.1 (a)

$$\Sigma F_x = 0 : A_x = D_x \quad ...(1)$$

$$\Sigma F_y = 0 : A_y + D_y - 4 - 6 = 0$$

$$\therefore \quad A_y + D_y = 10 \quad ...(2)$$

$$\Sigma M_A = 0 : -4 \times 1 - 6 \times 4 + D_y \times 4.5 = 0$$

$$\therefore \quad D_y = 6.222 \text{ kN}$$

For section between B and C, $\Sigma M_C = 0$ for RHS :

$$-D_x \times 2 + D_y \times 0.5 = 0$$

$$\therefore \quad D_x = 1.556 \text{ kN}$$

From equation (1),

$$A_x = 1.556 \text{ kN}$$

From equation (2),

$$A_y = 3.778 \text{ kN}$$

As $D_y > A_y$,

$$T_{max} = T_{CD} = R_D$$

$$\therefore \quad T_{max} = \sqrt{D_x^2 + D_y^2} = \sqrt{1.556^2 + 6222^2}$$

$$\therefore \quad \boxed{T_{max} = 6.414 \text{ kN}}$$ ...Ans.

$$\tan\theta_A = \frac{y_B}{1} = \frac{A_y}{A_x}$$

$$\therefore \quad y_B = \frac{3.778}{1.556}$$

$$\therefore \quad \boxed{y_B = 2.428 \text{ m}}$$ ...Ans.

Q.5 Two loads are suspended as shown in Fig. Q.5.1 from cable ABCD. Knowing that $d_c = 0.75$ m and $d_b = 1.125$ m, determine the component of reaction at A and maximum tension in the cable. **[SPPU : May-13, Dec.-13, Marks 6]**

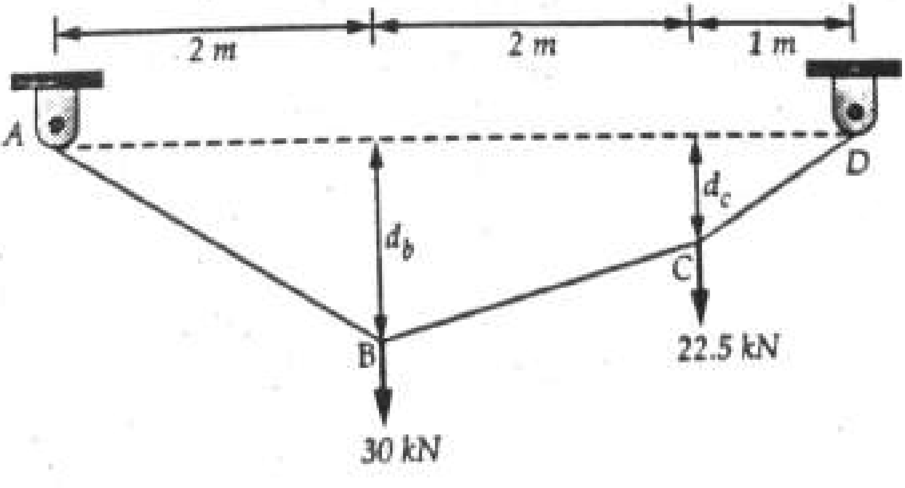

Fig. Q.5.1

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Ans. : The F.B.D. of cable is shown in Fig. Q.5.1 (a).

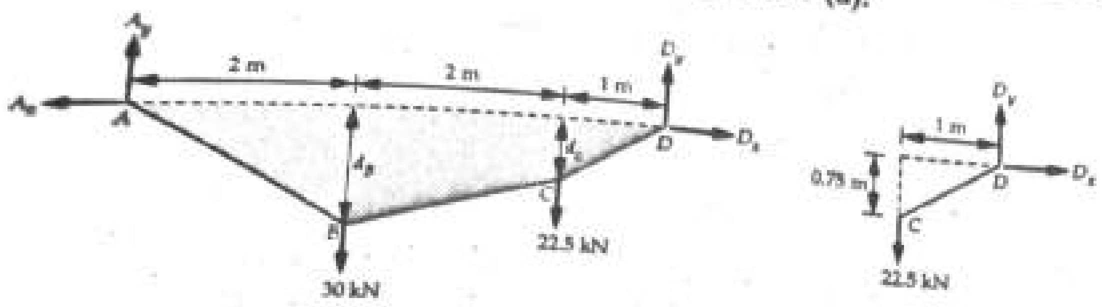

Fig. Q.5.1 (a)

$$\sum F_x = 0 : A_x = D_x \quad \ldots (1)$$

$$\sum F_y = 0 :$$

$$A_y + D_y - 30 - 22.5 = 0$$

$$\therefore \quad A_y + D_y = 52.5 \quad \ldots (2)$$

$$\sum M_A = 0 :$$

$$-(30)(2) - (22.5)(4) + (D_y)(5) = 0$$

$$D_y = 30 \text{ kN} \uparrow$$

From equation (2),

$$\boxed{A_y = 22.5 \text{ kN} \uparrow} \quad \ldots\textbf{Ans.}$$

For section between B and C, $\sum M_C = 0$ for R.H.S.

$$-(D_x)(0.75) + (D_y)(1) = 0$$

$$D_x = 40 \text{ kN} \rightarrow$$

For equation (1),

$$\boxed{A_x = 40 \text{ kN} \leftarrow} \quad \ldots\textbf{Ans.}$$

As $D_y > A_y$, tension is maximum in portion CD

$$\therefore \quad T_{max} = T_{CD} = \sqrt{D_x^2 + D_y^2} = \sqrt{40^2 + 30^2}$$

$$\boxed{T_{max} = 50 \text{ kN}} \quad \ldots\textbf{Ans.}$$

Q.6 Cable ABCD supports the 4 kg block E and 6 kg block F as shown in Fig. Q.6.1. Determine the maximum tension in the cable and the sag of point B. **[SPPU : May-16, Marks 6]**

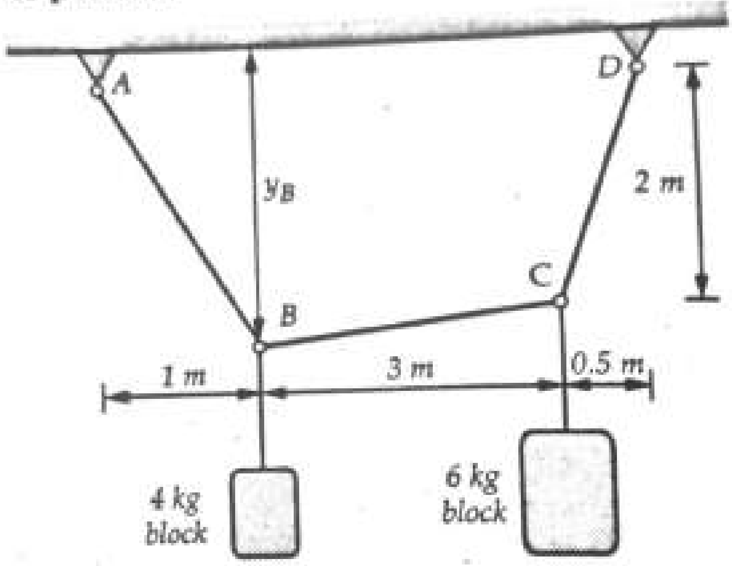

Fig. Q.6.1

Ans. : The FBD of cable is shown in Fig. Q.6.1 (a).

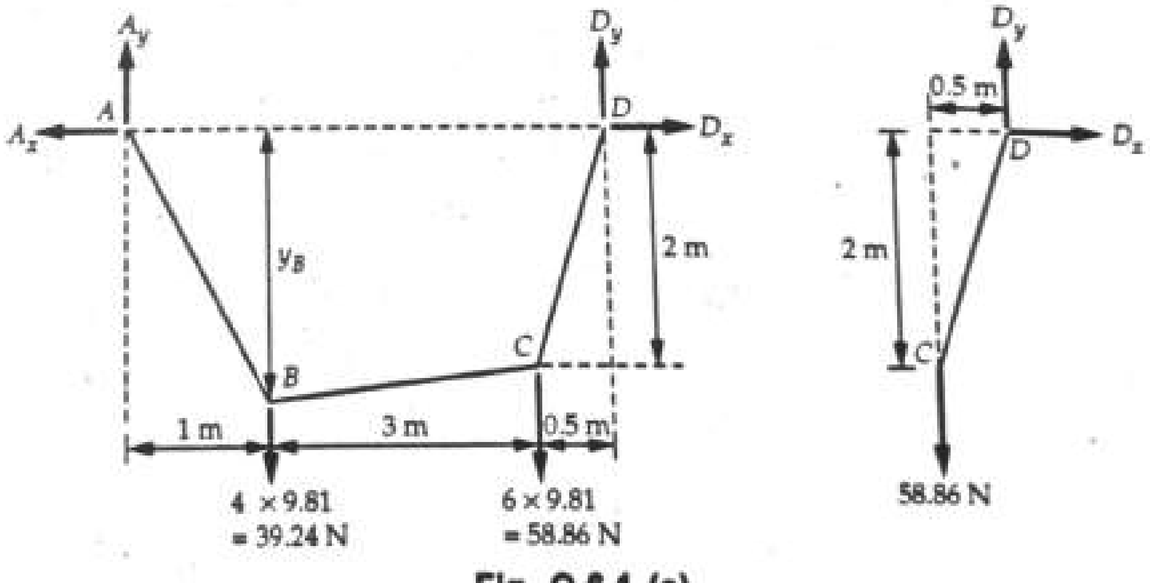

Fig. Q.6.1 (a)

$$\sum F_x = 0 : \quad A_x = D_x \qquad ...(1)$$

$$\sum F_y = 0 : \quad A_y + D_y - 39.24 - 58.86 = 0$$

$$\therefore \quad A_y + D_y = 98.1 \qquad ...(2)$$

$$\sum M_A = 0 : \quad -39.24 \times 1 - 58.86 \times 4 + D_y \times 4.5 = 0$$

$$\therefore \quad D_y = 61.04 \text{ N}$$

For section between B and C, $\sum M_C = 0$ for RHS :

$$-D_x \times 2 + D_y \times 0.5 = 0$$

$$\therefore \quad D_x = 15.26 \text{ N}$$

From equation (1),

$$A_x = 15.26 \text{ N}$$

From equation (2),

$$A_y = 37.06 \text{ N}$$

$$\text{As } D_y > A_y$$

$$T_{max} = T_{CD} = R_D$$

$$\therefore \quad T_{max} = \sqrt{D_x^2 + D_y^2} = \sqrt{15.26^2 + 61.04^2}$$

$$\therefore \quad \boxed{T_{max} = 62.92 \text{ N}} \quad \text{...Ans.}$$

$$\tan\theta_A = \frac{y_B}{1} = \frac{A_y}{A_x}$$

$$\therefore \quad y_B = \frac{37.06}{15.26}$$

$$\therefore \quad \boxed{y_B = 2.43 \text{ m}} \quad \text{...Ans.}$$

Q.7 The cable segment support the loading as shown in Fig. Q.7.1. Determine the support reaction and maximum tension in segment of cable. [SPPU : Dec.-16, 17, May-17, Marks 6]

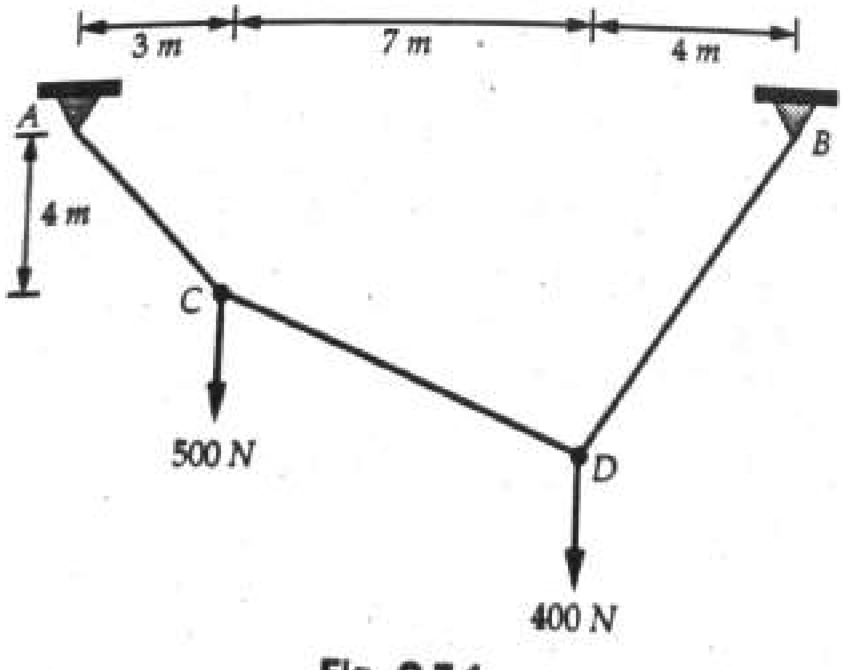

Fig. Q.7.1

Ans. : The FBD of cable segment is shown in Fig. Q.7.1 (a).

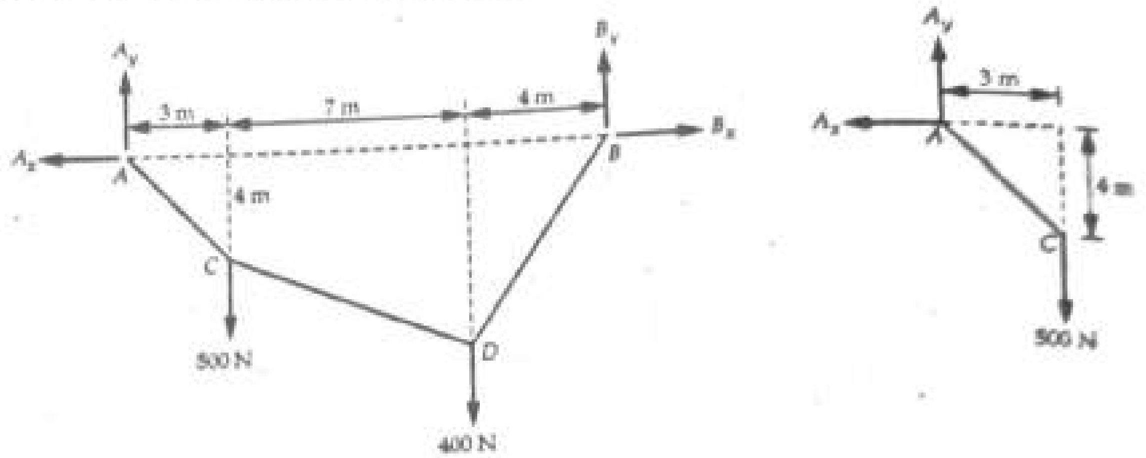

Fig. Q.7.1 (a)

$$\Sigma F_x = 0 : -A_x + B_x = 0$$

$$\therefore \quad A_x = B_x \quad \text{... (1)}$$

$$\Sigma F_y = 0 : A_y + B_y - 500 - 400 = 0$$

$$\therefore \quad A_y + B_y = 900 \quad \text{... (2)}$$

$$\Sigma M_A = 0 : -500 \times 3 - 400 \times 10 + B_y \times 14 = 0$$

$$\therefore \quad B_y = 392.86 \text{ N}$$

From equation (2),

$$A_y = 507.14 \text{ N}$$

For section between C and D, $\Sigma M_C = 0$ for LHS :

$$A_x \times 4 - A_y \times 3 = 0$$

$$\therefore \quad A_x = 380.36 \text{ N} = B_x$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{380.36^2 + 507.14^2}$$

$$\therefore \quad \boxed{R_A = 633.93 \text{ N}} \quad \text{...Ans.}$$

$$\theta_A = \tan^{-}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{507.14}{380.36}\right)$$

$$\therefore \quad \boxed{\theta_A = 53.13^\circ} \quad \text{...Ans.}$$

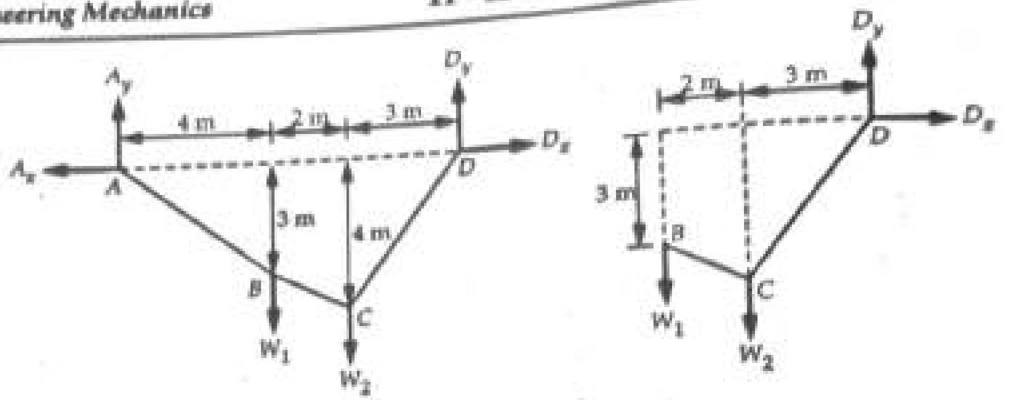

Fig. Q.8.1 (a)

$\therefore$ Tension is maximum in CD which is given 10 kN

$$\therefore \quad T_{max} = T_{CD} = 10 \text{ kN}$$

Reaction at D has same magnitude as T_{CD} but opposite direction.

$$\therefore \quad \boxed{R_D = 10 \text{ kN}, \quad \theta_D = 53.13^\circ} \quad \text{...Ans.}$$

For section between A and B,

$$\sum M_B = 0 \quad \text{for R.H.S.}$$

$$-W_2 \times 2 - 10 \cos 53.13 \times 3 + 10 \sin 53.13 \times 5 = 0$$

$$\therefore \quad \boxed{W_2 = 11 \text{ kN}} \quad \text{...Ans.}$$

For complete cable,

$$\sum M_A = 0$$

$$-W_1 \times 4 - W_2 \times 6 + 10 \sin 53.13 \times 9 = 0$$

$$-W_1 \times 4 - 11 \times 6 + 10 \sin 53.13 \times 9 = 0$$

$$\therefore \quad \boxed{W_1 = 1.5 \text{ kN}} \quad \text{...Ans.}$$

$\sum F_x = 0$ (taking A_x towards left)

$$-A_x + D_x = 0 \quad [\because D_x = R_D \cos\theta_{CD}]$$

$$-A_x + 10 \cos 53.13 = 0$$

$$\therefore \quad A_x = 6 \text{ kN} \leftarrow$$

$\sum F_y = 0$

$$A_y - W_1 - W_2 + 10 \sin 53.13 = 0$$

$$A_y = 1.5 + 11 - 8$$

$$\therefore \quad A_y = 4.5 \text{ kN} \uparrow$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{6^2 + 4.5^2}$$

$$\therefore \quad \boxed{R_A = 7.5 \text{ kN}}$$

...Ans.

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$$\theta_A = \tan^{-1}\left(\frac{4.5}{6}\right)$$

$$\therefore \quad \boxed{\theta_A = 36.87^\circ}$$

...Ans.

Q.9 Cable ABC supports two boxes as shown in Fig. Q.9.1. Knowing that b = 2.7 m, determine the required magnitude of the horizontal force P and the corresponding distance a.

[SPPU : May-11, 19, Dec.-18, Marks 6]

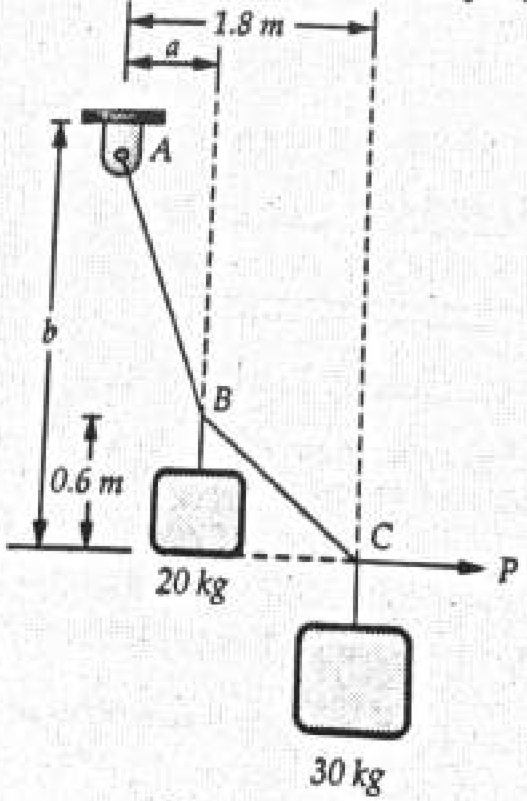

Fig. Q.9.1

Ans. : For section between A and B,

$\sum M_B = 0$ for R.H.S.

$$P \times 0.6 - (30 \times 9.81)(1.8 - a) = 0$$

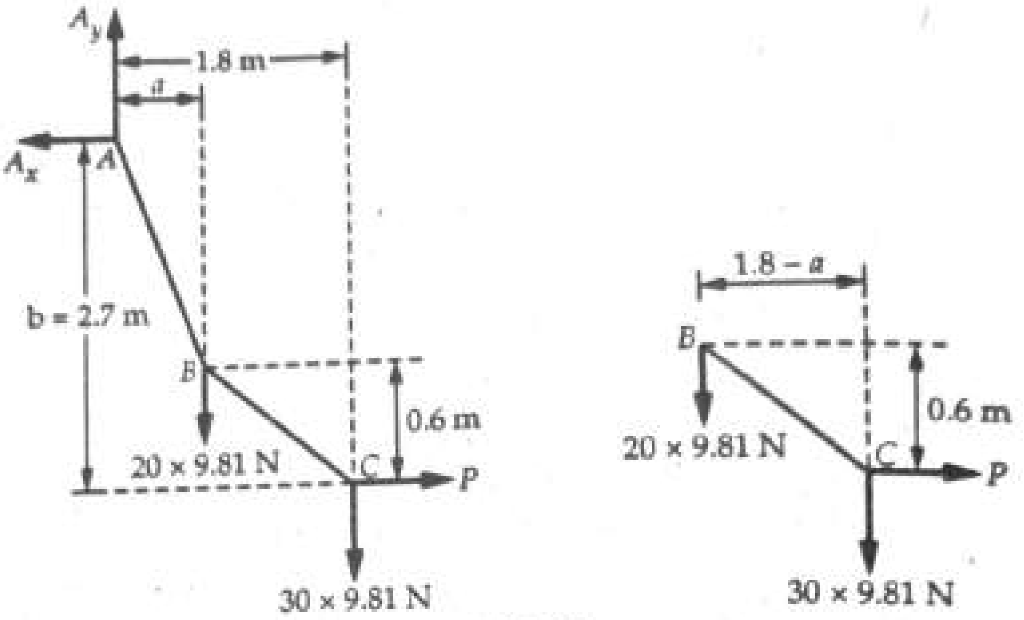

Fig. Q.9.1 (a)

$$P \times 0.6 + 30 \times 9.81 a - 30 \times 9.81 \times 1.8 = 0 \quad \text{... (1)}$$

For complete cable,

$$\sum M_A = 0$$

$$P \times 2.7 - 30 \times 9.81 \times 1.8 - 20 \times 9.81 \times a = 0 \quad \text{... (2)}$$

From equations (1) and (2),

$$\boxed{P = 284.806 \text{ N}} \quad \text{...Ans.}$$

$$\boxed{a = 1.22 \text{ m}} \quad \text{...Ans.}$$

Q.10 Determine the reactions at A, D and tension in BC of the rope ABCD loaded and supported as shown in Fig. Q.10.1.

[SPPU : Dec.-12, Marks 6]

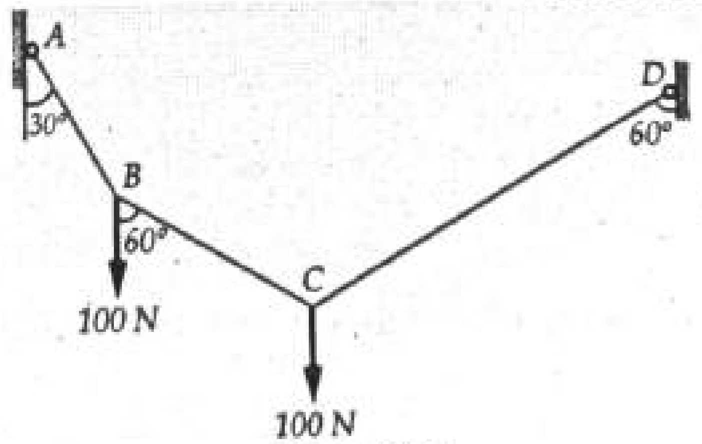

Fig. Q.10.1

Ans. : For the F.B.D. of point B shown in Fig. Q.10.1 (a),

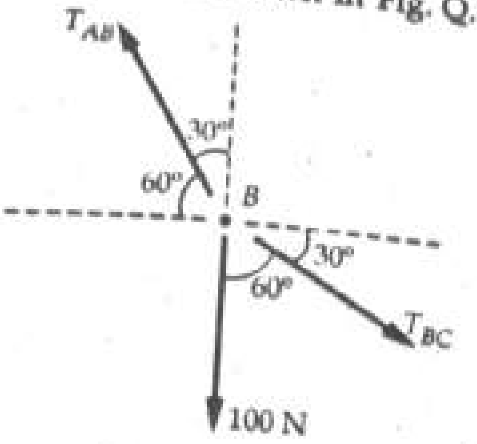

Fig. Q.10.1 (a)

$$\frac{T_{AB}}{\sin 60} = \frac{T_{BC}}{\sin 150} = \frac{100}{\sin 150}$$

$\therefore$ $$\boxed{T_{BC} = 100 \text{ N}}$$ ...Ans.

$$T_{BC} = 173.2 \text{ N}$$

But $$R_A = T_{AB}$$

$\therefore$ $$\boxed{R_A = 173.2 \text{ N} \ 60°}$$ ...Ans.

For the F.B.D. of point C shown in Fig. Q.10.1 (b),

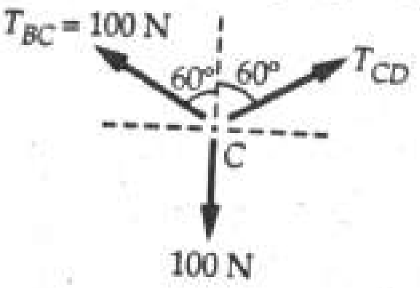

Fig. Q.10.1 (b)

$$\frac{T_{CD}}{\sin 120} = \frac{100}{\sin 120}$$

$\therefore$ $$T_{CD} = 100 \text{ N}$$

But $$R_D = T_{CD}$$

$\therefore$ $$\boxed{R_D = 100 \text{ N}, \ 30°}$$ ...Ans.

Q.11 For the cable AB as shown in Fig. Q.11.1, find the reaction at supports and tension in each segment. [SPPU : May-18, Marks 7]

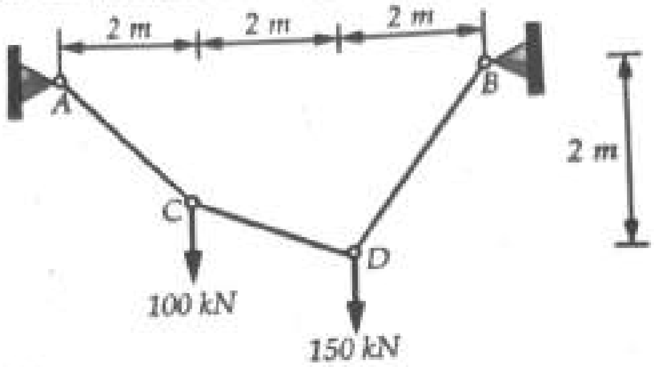

Fig. Q.11.1

Ans. : The FBD of complete cable is shown in Fig. Q.11.1 (a)

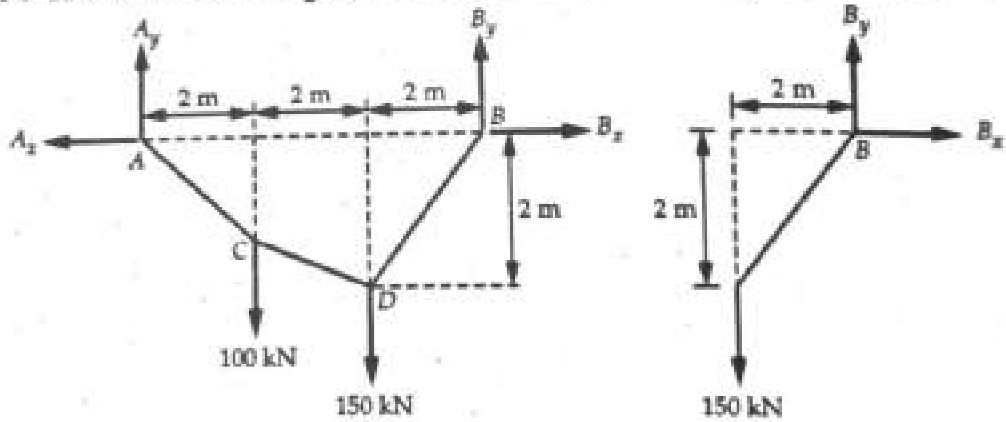

Fig. Q.11.1 (a)

$$\sum F_x = 0 : A_x = B_x \quad \text{...(1)}$$

$$\sum F_y = 0 : A_y + B_y = 250 \quad \text{...(2)}$$

$$\sum M_A = 0 : -(100)(2) - (150)(4) + (B_y)(6) = 0$$

$$\therefore \quad B_y = 133.33 \text{ kN}$$

From equation (2), $A_y = 116.67$ kN

For section between C and D, $\sum M_D = 0$ *for R.H.S.* :

$$(B_y)(2) - (B_x)(2) = 0$$

$$\therefore \quad B_x = B_y = 133.33 \text{ kN}$$

From equation (1), $A_x = 133.33$ kN

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{133.33^2 + 116.67^2}$$

$$\therefore \quad \boxed{R_A = 177.17 \text{ kN}} \quad \text{...Ans.}$$

$$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{116.67}{133.33}\right)$$

$$\therefore \quad \boxed{\theta_A = 41.2°} \quad \text{...Ans.}$$

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{133.33^2 + 133.33^2}$$

$$\therefore \quad \boxed{R_B = 188.56 \text{ kN}} \quad \text{...Ans.}$$

$$\therefore \quad \boxed{\theta_B = 45°} \quad \text{...Ans.}$$

$$T_{AC} = R_A$$

$$\therefore \quad \boxed{T_{AC} = 177.17 \text{ kN}} \quad \text{...Ans.}$$

$$T_{BD} = R_B$$

$$\therefore \quad \boxed{T_{BD} = 188.56 \text{ kN}} \quad \text{...Ans.}$$

For FBD of point C shown in Fig. Q.11.1 (b),

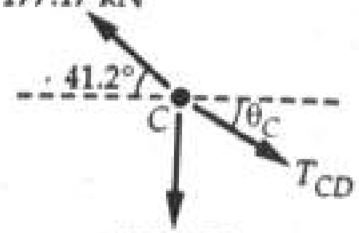

Fig. Q.11.1 (b)

$$\sum F_x = 0 : T_{CD} \cos\theta_C - 177.17 \cos 41.2 = 0$$

$$\therefore \quad T_{CD} \cos\theta_C = 133.3 \quad \text{...(3)}$$

$$\sum F_y = 0 :$$

$$T_{CD} \sin\theta_C - 100 + 177.17 \sin 41.2 = 0$$

$$T_{CD} \sin\theta_C = 16 \quad \text{...(4)}$$

Squaring and adding equations (3) and (4),

$$T_{CD}^2 = 133.3^2 + 16.7^2$$

$$\therefore \quad \boxed{T_{CD} = 134.34 \text{ kN}} \quad \text{...Ans.}$$

END...