Soham

independent

22 Oct

1 Interference

Chapter Outline

- Questions and Answers, Solved Problems
- Formulae at a Glance
- Concept Flowchart

QUESTIONS & ANSWERS

1.1 : Interference in Thin Films of Uniform Thickness

Q.1 Derive the equation of path difference between reflected rays when monochromatic light of wavelength 'λ' falls with angle of incidence 'i' on the uniform thickness film of refractive index 'μ'. Write the conditions of maxima and minima.

☞ **[May-14, 15, Marks 6]**

Ans. :

- Consider a ray of light *AB* incident on a thin film of thickness 't' and refractive index 'μ' surrounded by air as shown in Fig. 1.1.
- The ray *AB* is partially reflected and partially transmitted at *B*. The transmitted ray *BC* is again partially transmitted and partially reflected at *C*.
- The reflected ray *CD* is partially reflected and partially refracted at *D*.
- Although there will be more rays in reflected light, their intensity will be very small. Hence the interference pattern in reflected light will be due to rays BF and *DH* which are coherent as they are both derived from the same ray *AB*.

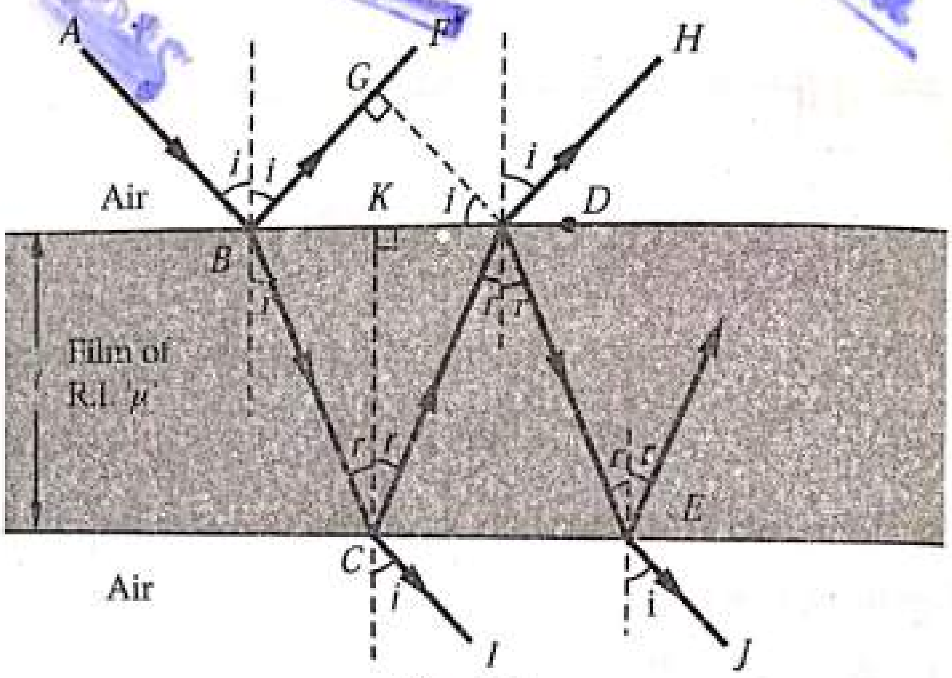


Fig. 1.1

- To find path difference between BF and DH, draw DG perpendicular to BF.
- Path difference between the two rays will be

$$\Delta = \mu (BC + CD) - BG \quad \text{... (1.1)}$$

- ΔBCK and ΔCDK are congruent.

$\therefore$ $$BC = CD = \frac{CK}{\cos r}$$

$\therefore$ $$BC = CD = \frac{t}{\cos r} \quad \text{... (1.2)}$$

- In ΔBDG,

$$\sin i = \frac{BG}{BD} \quad \text{... (1.3)}$$

$$BD = BK + DK = 2\,BK \qquad (\text{as } BK = DK)$$

Also, $$\tan r = \frac{BK}{CK}$$

$\therefore$ $$BK = t \tan r$$

$\therefore$ $$BD = 2t \tan r$$

- From equation (1.3),

$$BG = BD \sin i$$

$$\therefore \quad BG = 2t \tan r \sin i = 2t\left(\frac{\sin r}{\cos r}\right)\sin i$$

- By Snell's law,

$$\boxed{\mu = \frac{\sin i}{\sin r}}$$

$$\therefore \quad \sin i = \mu \sin r$$

$$\therefore \quad BG = 2t\left(\frac{\sin r}{\cos r}\right)\cdot \mu \sin r$$

$$\therefore \quad BG = 2\mu t \frac{\sin^2 r}{\cos r} \qquad \text{... (1.4)}$$

- Substituting equations (1.2) and (1.4) in equation (1.1),

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} = \frac{2\mu t}{\cos r}\left(1-\sin^2 r\right)$$

$$= \frac{2\mu t}{\cos r}\cdot \cos^2 r$$

$$\therefore \quad \Delta = 2\mu t \cos r$$

- Ray BF suffers phase change of π due to reflection from denser medium at B. There is no phase change for ray DH which is transmitted at B, reflected from rarer medium at C and transmitted at D. Therefore, net path difference of $\frac{\lambda}{2}$ is introduced between the two rays due to reflection.

$$\therefore \quad \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} \qquad \text{... (1.5)}$$

- **Constructive interference**

path difference $\Delta = n\lambda$

$$\therefore \quad 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \qquad \text{... (1.6)}$$

- **Destructive interference**

path difference $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \quad 2\mu t \cos r = n\lambda \qquad \dots (1.7)$$

Q.2 Derive an expression for path difference in reflected light for thin parallel film and show that the interference pattern in the reflected and transmitted system is complementary.

[Dec.-11, Marks 7]

Ans. : Refer Q.1 for constructive and destructive interference conditions in reflected light.

Interference in transmitted light

- The rays *CI* and *EJ* in transmitted light have the same path difference as in reflected light. There is no phase change for *CI* due to reflection as it gets transmitted at *C*. The ray *EJ* also does not undergo phase change due to reflection as it is reflected from rarer medium at *C* and *D*.

$$\therefore \quad \Delta = 2\mu t \cos r$$

- For constructive interference,

$$2\mu t \cos r = n\lambda \qquad \dots (1.8)$$

- For destructive interference,

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \qquad \dots (1.9)$$

- As the constructive interference condition in reflected light becomes destructive interference condition in transmitted light, the reflected and transmitted systems are said to be complementary.

Q.3 When seen by reflected light, why does an excessively thin film appear to be perfectly black when illuminated by white light ?

[May-04,06, Marks 3]

Ans. : The path difference in reflected light is

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

If thickness 't' is very small, the first term in the above equation can be neglected.

$\therefore$ the path difference in reflected light is,

$$\Delta \approx \pm \frac{\lambda}{2}$$

As $\Delta \approx \pm\frac{\lambda}{2}$, there is destructive interference.

$\therefore$ **An excessively thin film appears dark in reflected light.**

Q.4 A thin film illuminated by white light appears coloured when observed in reflected light. Explain why ?

☞ **[Dec.-08, Marks 3]**

Ans. : The condition for constructive interference in reflected light is

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

If white light is made to incident on a thin film, constructive interference condition will be satisfied for different wavelength at different angles of refraction 'r' and hence at different angles of incidence.

Hence, different colours will be seen at different angles in reflected light.

Therefore colours will be seen in reflected light.

Important Formulae

1) Condition for constructive interference in reflected light

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

2) Condition for destructive interference in reflected light

$$2\mu t \cos r = n\lambda$$

3) Condition for constructive interference in transmitted light

$$2\mu t \cos r = n\lambda$$

4) Condition for destructive interference in transmitted light

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

Q.5 A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle of 30° from the normal, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46, λ = 5890 A°. ☞ **[May-04]**

Ans. : Given data : $n = 8$, $\lambda = 5890\ \text{A}^{\circ} = 5890 \times 10^{-8}$ cm, $\mu = 1.46$, $i = 30^{\circ}$

Formula : For n^{th} dark band,

$$2\mu t \cos r = n\lambda$$

and, $\frac{\sin i}{\sin r} = \mu$ by Snell's law,

$\therefore \quad \frac{\sin 30}{\sin r} = 1.46$

$\therefore \quad r = 20.072°$

$$t = \frac{n\lambda}{2\mu \cos r} = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \cos 20.072}$$

$\therefore \quad \boxed{t = 1.718 \times 10^{-4} \text{ cm}}$

Q.6 A soap film of refractive index $\frac{4}{3}$ and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference band

[Dec.-05, May-12]

Ans. : Given data : $\mu = \frac{4}{3}$, $t = 1.5 \times 10^{-4}$ cm, $\lambda = 5 \times 10^{-5}$ cm, $i = 45°$

Formula : For n^{th} dark band,

$2\mu t \cos r = n\lambda$

and, $\frac{\sin i}{\sin r} = \mu$ by Snell's law.

$\therefore \quad \frac{\sin 45}{\sin r} = \frac{4}{3}$

$\therefore \quad r = 32.028°$

$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times \frac{4}{3} \times 1.5 \times 10^{-4} \times \cos 32.028}{5 \times 10^{-5}}$$

$\therefore \quad n = 6.78$

Order of interference is a positive integer. The value of n obtained is closer to 7 than 6. Hence order of interference is

$\boxed{n = 7}$

Q.7 **A parallel beam of monochromatic light of wavelength $\lambda = 5890$ A° is incident on a thin film of $\mu = 1.5$ such that the angle of refraction is 60°. Find the minimum thickness of the film so that it appears dark. For normal incidence, what is the thickness required ?** ☞ [May-08, Dec.-10]

Ans. : Given data : $\mu = 1.5$, $r = 60°$, $\lambda = 5890 \text{ A}° = 5890 \times 10^{-8}$ cm

For minimum thickness, $n = 1$

Formula : For n^{th} dark band,

$$2 \mu t \cos r = n \lambda$$

$$t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$

$$\therefore \quad \boxed{t = 3.93 \times 10^{-5} \text{ cm}}$$

For normal incidence, $r = 0$

$$\therefore \quad t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 0}$$

$$\therefore \quad \boxed{t = 1.96 \times 10^{-5} \text{ cm}}$$

Q.8 **An oil drop of volume 0.2 c.c. is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the surface and white light which is incident normally is observed through a spectrometer. The spectrum is seen to contain first dark band whose centre has wavelength 5.5×10^{-5} cm in air. Find the refractive index of oil.** ☞ [May-08]

Ans. : Given data : The volume of oil $V = 0.2 \text{ cm}^3$,
Area $A = 1 \text{ m}^2 = 10^4 \text{ cm}^2$, for normal incidence, $r = 0$, $n = 1$,
$\lambda = 5.5 \times 10^{-5}$ cm

Formula : For n^{th} dark band,

$$2 \mu t \cos r = n \lambda$$

The thickness of oil film $t = \dfrac{V}{A} = \dfrac{0.2}{10^4}$

$$\cos r = \cos 0 = 1$$

$$\therefore \quad \mu = \frac{n\lambda}{2t\cos r} = \frac{1\times5.5\times10^{-5}}{2\times2\times10^{-5}\times1}$$

$$\therefore \quad \boxed{\mu = 1.375}$$

1.2 : Interference in Wedge Shaped Film

Q.9 Define fringe width for wedge shaped film, obtain an expression for it. [Dec.-12, Marks 3]

Ans. : Fringe width is the distance between two consecutive dark fringes.

- If the n^{th} dark fringe is formed at a distance x_n from the edge of the wedge shaped film where the thickness is t_n as shown in Fig. 1.2,

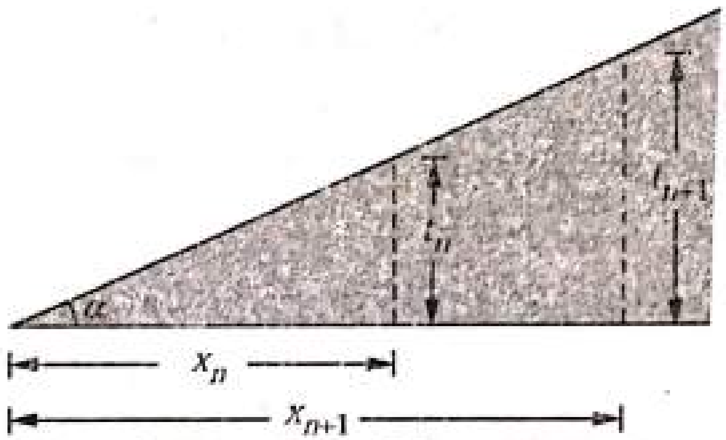


Fig. 1.2

$$t_n = x_n \tan\alpha \qquad \text{... (1.10)}$$

- Similarly, for the $(n+1)^{th}$ bright fringe,

$$t_{n+1} = x_{n+1} \tan\alpha \qquad \text{... (1.11)}$$

- For normal incidence, $i = 0 \Rightarrow r = 0$

  $\therefore$ The condition for n^{th} dark fringe is

$$2\mu t_n \cos\alpha = n\lambda$$

- Substituting for t_n from equation (1.10), we get,

$$2\mu x_n \tan\alpha \cos\alpha = n\lambda$$

$$\therefore \quad 2\mu x_n \sin\alpha = n\lambda \qquad \text{... (1.12)}$$

- For $(n+1)^{th}$ dark fringe,

$$2\mu t_{n+1} \cos\alpha = (n+1)\lambda$$

$\therefore \quad 2\mu t_{n+1} \cos\alpha = (n+1)\lambda$

- Substituting for t_{n+1} from equation (1.11), we get

$2\mu x_{n+1} \tan\alpha \cos\alpha = (n+1)\lambda$

$$\therefore \quad 2\mu x_{n+1} \sin\alpha = (n+1)\lambda \qquad \text{... (1.13)}$$

- Subtracting equation (1.12) from equation (1.13),

$2\mu [x_{n+1} - x_n] \sin\alpha = [n+1-n]\lambda$

$2\mu [x_{n+1} - x_n] \sin\alpha = \lambda$

- The fringe width is $\beta = x_{n+1} - x_n$

$\therefore \quad 2\mu\beta\sin\alpha = \lambda$

$$\therefore \quad \beta = \frac{\lambda}{2\mu\sin\alpha}$$

- For small angle of the wedge, $\sin\alpha \approx \alpha$

$$\therefore \quad \beta = \frac{\lambda}{2\mu\alpha} \qquad \text{... (1.14)}$$

- For air film, $\mu = 1$. Then,

$$\beta_{air} = \frac{\lambda}{2\alpha} \qquad \text{... (1.15)}$$

Important Formula

Fringe width $\quad \beta = \dfrac{\lambda}{2\mu\alpha}$

SOLVED EXAMPLES

Q.10 Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 $A°$. Calculate the angle of wedge in seconds of an arc. ☞ **[Dec.-03, May-07]**

Ans. : Given data : $\lambda = 5893\ A° = 5893 \times 10^{-8}$ cm, $\mu = 1.52$,

$\beta = 1$ mm $= 0.1$ cm

Formula : $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\therefore \quad \alpha = \frac{\lambda}{2\mu\beta} = \frac{5893\times10^{-8}}{2\,(1.52)\,(0.1)}$$

$$\therefore \quad \alpha = 1.9385 \times 10^{-4} \text{ radians}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = \frac{180}{\pi}\times3600 \text{ seconds of arc}$$

$$\alpha = 1.9385\times10^{-4}\times\frac{180}{\pi}\times3600 \text{ seconds}$$

$$\therefore \quad \boxed{\alpha = 39.98 \text{ seconds of an arc}}$$

Q.11 **A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used.**

[May-06, 12]

Ans. : Given data : For air film, $\mu = 1$, $\beta = 0.12$ cm,

$$\alpha = 40 \text{ seconds} = \frac{40\times\pi}{3600\times180} \text{ rad}$$

Formula : $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\therefore \quad \lambda = 2\mu\alpha\beta = 2(1)\left(\frac{40\pi}{3600\times180}\right)(0.12) \text{ cm}$$

$$= 4.654\times10^{-5} \text{ cm}$$

$$= 4654\times10^{-8} \text{ cm}$$

$$\therefore \quad \boxed{\lambda = 4654 \text{ A}^\circ}$$

Q.12 **Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophone whose refractive index is 1.4. The angle of wedge is 20 seconds of an arc and the distance between successive bright fringes is 0.25 cm. Calculate the wavelength of light.**

[Dec.-08]

Ans. : Given data : $\beta = 0.25$ cm, $\mu = 1.4$,

$$\alpha = 20 \text{ seconds of an arc} = \frac{20}{3600} \text{ degree} = \frac{20}{3600}\times\frac{\pi}{180} \text{ radian}$$

$\therefore \quad \alpha = \frac{\pi}{32400}$ radian

Formula : $\beta = \frac{\lambda}{2\mu\alpha}$

$$\lambda = 2\mu\alpha\beta$$

$$= 2\times1.4\times\frac{\pi}{32400}\times0.25$$

$$= 6.7874 \times 10^{-5} \text{ cm}$$

$$\therefore \quad \boxed{\lambda = 6787.4 \text{ A}^{\circ}}$$

Q.13 A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge of wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of interference fringes per cm of the wedge length. ☞ **[Dec.-08]**

Ans. : Given data : $\lambda = 5.82\times 10^{-7}$ m $= 5.82\times 10^{-5}$ cm, $\mu = 1.5$,

$\alpha = 20$ seconds of an arc $= \frac{20}{3600}$ deg $= \frac{20}{3600} \times \frac{\pi}{180}$ radian

$\therefore \quad \alpha = \frac{\pi}{32400}$ radian

Formula : $\beta = \frac{\lambda}{2\mu\alpha}$

$$\beta = \frac{5.82\times10^{-5}}{2\times1.5\times\left(\frac{\pi}{32400}\right)}$$

$$\therefore \quad \beta = 0.2 \text{ cm}$$

Number of fringes per cm $= \frac{1}{\beta} = \frac{1}{0.2} = 5$

$\therefore$ There are 5 interference fringes per cm.

Q.14 Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light

used has wavelength 5900 A°, find the separation between consecutive bright fringes. ☞ [Dec.-09]

Ans. Given data : Wedge shaped film as shown in Fig. 1.3, $\mu = 1$,

$\lambda = 5900\ A^\circ = 5900 \times 10^{-10}$ m

$$\tan\alpha \approx \alpha = \frac{0.01 \times 10^{-1}}{10}$$

$$\therefore \quad \alpha = 10^{-4} \text{ radian}$$

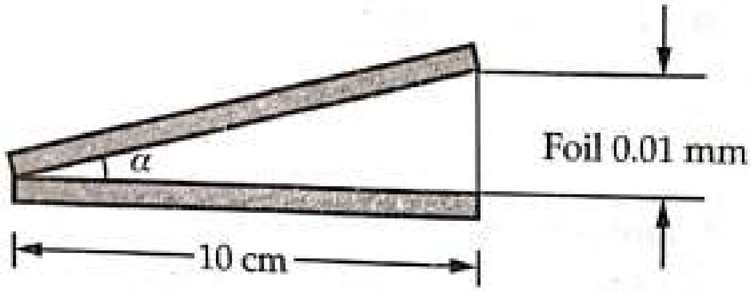


Fig. 1.3

Formula : $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\therefore \quad \beta = \frac{5900 \times 10^{-10}}{2(1)\,(10^{-4})} = 2.95 \times 10^{-3} \text{ m}$$

$$\therefore \quad \boxed{\beta = 2.95 \text{ mm}}$$

Q.15 **Interferences fringes are produced with monochromatic light failing normally on a wedge shaped film of refractive index 1.4. The angle of wedge is 10 sec. of an arc and the distance between successive fringes is 0.5 cm. What is the wavelength of light used ?** ☞ [Dec-13]

Ans. : Given data : $\beta = 0.5$ cm, $\mu = 1.4$,

$$\alpha = 10 \text{ seconds of an arc} = \frac{10}{3600} \text{ degree} = \frac{10}{3600} \times \frac{\pi}{180} \text{ radian}$$

$$\therefore \quad \alpha = \frac{\pi}{64800} \text{ radian}$$

Formula : $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\lambda = 2\mu\alpha\beta = 2\times 1.4\times\frac{\pi}{64800}\times 0.5$$

$$\lambda = 6.7874\times 10^{-5}\text{ cm}$$

$$\therefore \quad \lambda = 6787.4 \text{ A}^\circ$$

1.3 : Newton's Rings Experiment

Q.16 What are Newton's rings ? Draw the experimental set-up to obtain Newton's rings in the laboratory. Show that diameters of Newton's dark rings are proportional to the square root of natural numbers. ☞ **[Dec-13, May-15, Marks 6]**

Ans. : Newton's rings are concentric and alternate bright and dark rings formed in light reflected from a combination of plano - convex lens kept on a plane glass plate.

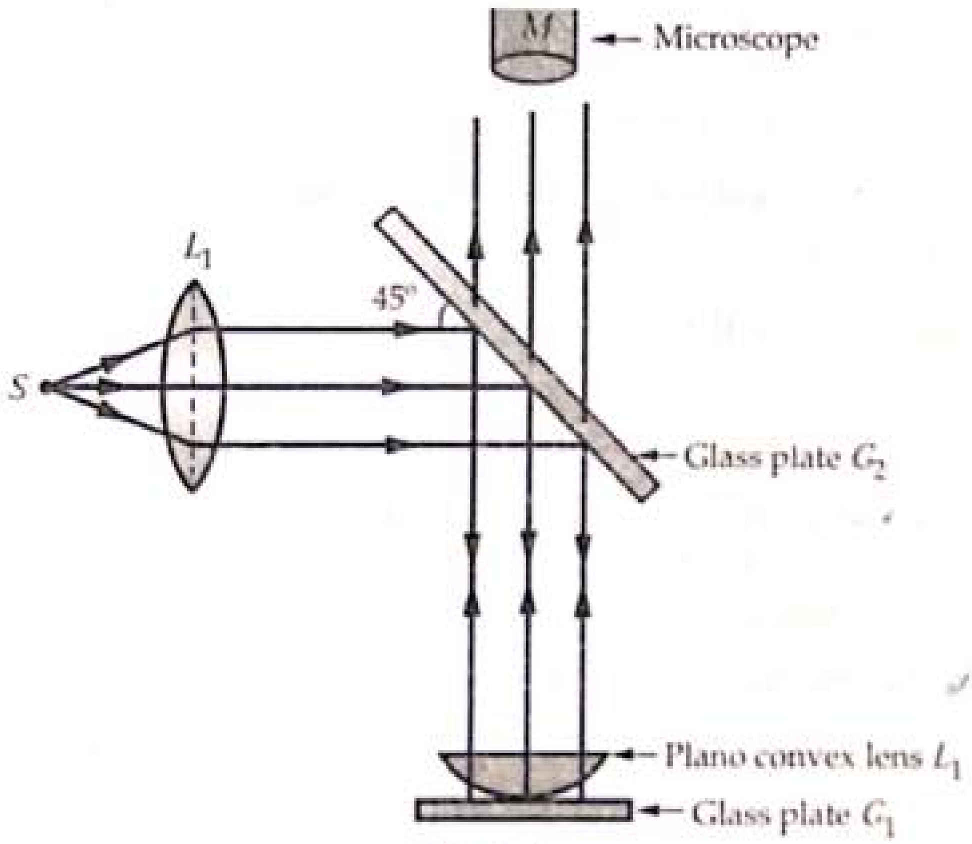


Fig. 1.4

Experimental arrangement

- A plano convex lens 'L' of large radius of curvature is placed on a plane glass plate 'G_1', with the curved surface touching the glass plate as shown in Fig. 1.4. An air film is enclosed between the curved surface of the lens and the glass plate.

- A sodium vapour lamp 'S' is kept at the focus of a biconvex lens 'L_1' which converts the diverging beam of light into a parallel beam. This parallel beam of light is made to fall on a glass plate 'G_2' kept at an angle of 45° with the incident beam.
- A part of incident light is reflected towards the plano convex lens. This light is again reflected back, partially from the top and partially from the bottom of the air film, and transmitted by the glass plate 'G_2'.
- The interference of these rays is observed through a microscope 'M'.

Path difference

- As the radius of curvature of the plano convex lens is large, the air film enclosed between the curved surface of lens and glass plate is approximately wedge shaped. Also, the angle of the wedge will be very small.
- For a wedge shaped film, the path difference is,

$$\Delta = 2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2}$$

- Here, $\mu = 1$ (for air film)

  α is very small and $r \approx 0°$ (due to normal incidence)

$\therefore$ $\cos(r+\alpha) \approx 1.$

- Taking $+\frac{\lambda}{2}$ on R.H.S.,

$$\Delta = 2t + \frac{\lambda}{2} \qquad \text{... (1.16)}$$

- The curved surface of lens is part of sphere of radius 'R' which is the radius of curvature of lens as shown in Fig. 1.5.
- Let 't' be the thickness of air film at a radial distance of 'r' as shown. Then, by Pythagoras theorem,

$$R^2 = (R-t)^2 + r^2$$

$$\therefore \quad R^2 = R^2 - 2Rt + t^2 + r^2$$

$$2Rt = t^2 + r^2$$

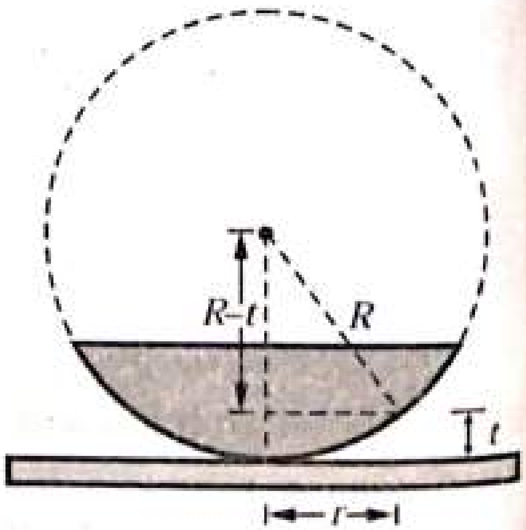


Fig. 1.5

- As 't' is very small, t^2 can be neglected in comparison with r^2.

$$\therefore \quad 2Rt = r^2$$

$$2t = \frac{r^2}{R}$$

- Here, 'r' is the radius of the circle for which thickness is 't'. If 'D' is the diameter of this circle,

$$r = \frac{D}{2} \quad \Rightarrow \quad r^2 = \frac{D^2}{4}$$

$$\therefore \quad 2t = \frac{D^2}{4R}$$

- Substituting in equation (1.16), we get

$$\Delta = \frac{D^2}{4R} + \frac{\lambda}{2} \qquad \text{... (1.17)}$$

Diameters of dark rings : Condition for destructive interference is satisfied for dark rings i.e.,

$$\Delta = (2n+1)\frac{\lambda}{2}$$

If $D = D_n$ is the diameter of n^{th} dark ring,

$$\frac{D_n^2}{4R} + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda$$

$$\therefore \quad D_n = \sqrt{4Rn\lambda} \qquad \text{... (1.18)}$$

where $\quad n = 0, 1, 2, ...$

$$\therefore \quad D_n \propto \sqrt{n}$$

i.e. diameters of dark rings are proportional to square root of natural numbers.

Q.17 Prove that in Newton's Ring by reflected light the diameter of bright ring are proportional to the square root of the odd natural number. ☞ [Dec-12, 14, Marks 6]

Ans. : For derivation of path difference, refer Q.16.

- For bright rings, the condition for constructive interference is satisfied i.e.,

$$\Delta = n\lambda$$

- If $D = D_n$ is the diameter of the n^{th} bright ring,

$$\frac{D_n^2}{4R} + \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad \frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$$

$$\therefore \quad D_n = \sqrt{(2n-1)\,2\lambda R} \qquad \text{... (1.19)}$$

where $\quad n = 1, 2, 3, ...$

$$\therefore \quad D_n \propto \sqrt{2n-1}$$

i.e. diameters of bright rings are proportional to square root of odd natural numbers.

Q.18 Explain the formation of Newton's ring with diagram and drive the diameter of bright ring. ☞ [May-13, Marks 6]

Ans. : Explanation for the formation of Newton's rings

- Division of amplitude takes place at the curved surface of the plano convex lens.
- The incident light is partially reflected and partially transmitted at the curved surface. The transmitted ray is reflected from the glass plate as shown in Fig. 1.6. These two rays interfere in reflected light.
- The path difference between these rays depends on thickness of the air film enclosed between the curved surfaces of lens and glass plate which increases radially outwards from the centre.

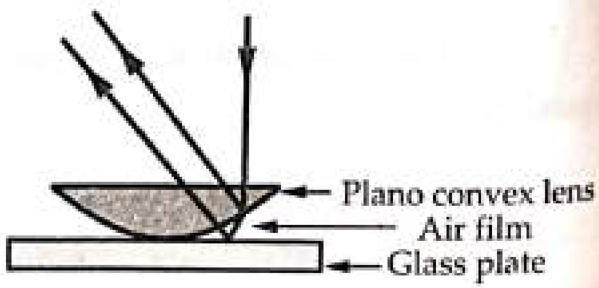


Fig. 1.6

- The thickness of the air film is zero at the centre.

Shape of fringes

- Thickness of air film is zero at the centre and increases radially outwards.
- The locus of points of constant thickness of air film and hence constant path difference is a circle as seen from the top. Hence interference fringes are circular.
- They are concentric alternate bright and dark rings.

Refer Q.17 for diameter of bright rings.

Q.19 Why is the center always dark in Newton's rings for reflected system ?

Ans. : Path difference in reflected light is

$$\Delta = \frac{D^2}{4R} + \frac{\lambda}{2} = 2t + \frac{\lambda}{2}$$

The thickness of air film is zero at the centre.

$$\therefore \quad \Delta = \frac{\lambda}{2}$$

As the path difference at the centre is $\frac{\lambda}{2}$, there is destructive interference and hence the centre of the interference pattern is dark.

Q.20 Explain how rings get closer with increase in order.

OR

Explain why the fringe width decreases with increasing order in Newton's rings experiment.

Ans. : Fringe width is the difference in radii of two adjacent bright or dark rings. The diameter of n^{th} dark ring is

$$D_n = \sqrt{4Rn\lambda}$$

$\therefore$ Its radius will be

$$r_n = \frac{D_n}{2} = \sqrt{Rn\lambda}$$

- The fringe width will be

$$\beta = r_{n+1} - r_n = \sqrt{R(n+1)\lambda} - \sqrt{Rn\lambda}$$

$$\therefore \quad \beta = \left(\sqrt{n+1}-\sqrt{n}\right)\sqrt{R\lambda}$$

Multiply and divide by $\left(\sqrt{n+1}+\sqrt{n}\right)$

$$\therefore \quad \beta = \left(\sqrt{n+1}-\sqrt{n}\right)\sqrt{R\lambda}\times\frac{\left(\sqrt{n+1}+\sqrt{n}\right)}{\left(\sqrt{n+1}+\sqrt{n}\right)}$$

$$\therefore \quad \beta = \frac{1}{\left(\sqrt{n+1}+\sqrt{n}\right)}\sqrt{R\lambda}$$

- As n increases, the fringe width decreases.

$\therefore$ Fringe width decreases and hence fringes come closer and closer as we move away from the centre.

Q.21 Explain how can Newton's rings be obtained in the laboratory and used for the determination of wavelength of monochromatic source.

Ans. : For method to produce Newton's rings in the laboratory, refer Q.16.

- The diameters of n^{th} and $(n+p)^{\text{th}}$ dark rings are measured using a travelling microscope. These diameters are given by

$$D_n^2 = 4Rn\lambda$$

and $$D_{n+p}^2 = 4R(n+p)\lambda$$

$$\therefore \quad D_{n+p}^2 - D_n^2 = 4Rp\lambda$$

$$\therefore \quad \lambda = \frac{D_{n+p}^2 - D_n^2}{4RP}$$

- The above equation can be used for determination of wavelength 'λ' or the radius of curvature 'R' of the plano convex lens.

Q.22 Explain how can Newton's rings be obtained in the laboratory and used for the determination of refractive index of a liquid.

Ans. : For method to produce Newton's rings in the laboratory, refer Q.16.

- First the diameters of n^{th} and $(n+p)^{\text{th}}$ rings are obtained with air film.

$$\therefore \quad \lambda = \frac{D_{n+p}^2 - D_n^2}{4Rp}$$

- Now the diameters of n^{th} and $(n+p)^{\text{th}}$ rings are obtained by introducing the liquid between the plano convex lens and glass plate. If the diameters are D'_n and D'_{n+p} and λ' is the wavelength of light in the liquid.

$$\lambda' = \frac{D'^2_{n+p} - D'^2_n}{4RP}$$

As $$\mu = \frac{\lambda}{\lambda'}$$

$$\therefore \quad \mu = \frac{D^2_{n+p} - D^2_n}{D'^2_{n+p} - D'^2_n}$$

Q.23 What will happen to the diameter of n$^{\text{th}}$ dark ring if air film is replaced by water film ? Explain.

Ans. : If air is replaced by water film of refractive index μ,

$$D_n = \sqrt{\frac{4Rn\lambda}{\mu}}$$

As $\mu > 1$, diameter decreases.

Important Formulae

Diameter of dark rings : $D_n = \sqrt{4Rn\lambda}$

Diameter of bright rings : $D_n = \sqrt{(2n-1)\,2\lambda R}$

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4RP}$$

$$\mu = \frac{D^2_{n+p} - D^2_n}{D'^2_{n+p} - D'^2_n}$$

SOLVED EXAMPLES

Q.24 Newton's rings are formed with monochromatic light source between flat glass plate and a convex lens viewed normally. What will be the order of dark ring which will have double the diameter of that *20th* dark ring ? [Dec-04]

Ans. : Given data : $D_n = 2D_{20}$

Formula : $D_n^2 = 4Rn\lambda$

As $\quad D_n = 2D_{20}$

$$D_n^2 = 4D_{20}^2$$

$\therefore \quad 4Rn\lambda = 4\times4R\times20\lambda$

$\therefore \quad \boxed{n = 80}$

Q.25 **In a Newton's rings experiment, the diameter of the 5^{th} ring was 0.336 cm and that of 15^{th} ring was 0.59 cm. Find the radius of curvature of the plano convex lens, if the wavelength of light used is 5890 A^{o}.** ☞ **[Dec.-04]**

Ans. : Given data : $n = 5$ and $n + p = 15$

$\therefore \quad p = 10$

$D_n = D_5 = 0.336$ cm, $D_{n+p} = D_{15} = 0.59$ cm, $\lambda = 5890\ A^{o} = 5890\times10^{-8}$ cm

Formula : $R = \dfrac{D_{n+p}^2 - D_n^2}{4p\lambda} = \dfrac{0.59^2 - 0.336^2}{4\times10\times5890\times10^{-8}}$

$\therefore \quad \boxed{R = 99.83 \text{ cm}}$

Q.26 **In Newton's rings experiment the diameters of n^{th} and $(n+8)^{th}$ bright rings are 4.2 mm and 7 mm respectively. Radius of curvature of the lower surface of lens is 2.00 m. Determine the wavelength of light.** ☞ **[Dec.-06]**

Ans. : Given data : $D_n = 4.2$ mm $= 0.42$ cm,

$D_{n+p} = D_{n+8} = 7$ mm $= 0.7$ cm, $p = 8$, $R = 2$ m $= 200$ cm.

Formula : $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4Rp}$

$\therefore \quad \lambda = \dfrac{0.7^2 - 0.42^2}{4\times200\times8} = 4.9\times10^{-5}$ cm

$\therefore \quad \boxed{\lambda = 4900\ A^{o}}$

Q.27 **Newton's rings are formed by light reflected normally from a plano convex lens and a plane glass plate with a liquid between them. The diameter of n^{th} ring is 2.18 mm and that of $(n+10)^{th}$ ring is 4.51 mm. Calculate the refractive index of the liquid, given that the radius of curvature of the lens is 90 cm and wavelength of light is 5893 A°.**

☞ **[May-07]**

Ans. : Given data : $D'_n = 2.18 \text{ mm} = 0.218 \text{ cm}$,

$D'_{n+p} = D'_{n+10} = 4.51 \text{ mm} = 0.451 \text{ cm}, \lambda = 5893 \text{ A}^{\circ} = 5893 \times 10^{-8} \text{ cm}$,

$R = 90 \text{ cm}, p = 10$

Formula : $$\mu = \frac{4\,p\,\lambda\,R}{D'^2_{n+p} - D'^2_n} = \frac{4 \times 10 \times 5893 \times 10^{-8} \times 90}{0.451^2 - 0.218^2}$$

$$\therefore \quad \boxed{\mu = 1.361}$$

Q.28 **The diameter of 5^{th} dark ring in Newton's rings experiment was found to be 0.42 cm. Determine the diameter of the 10^{th} dark ring.**

Ans. : Given data : $D_5 = 0.42$ cm

Formula : $D_n^2 = 4\,R\,n\,\lambda$

$$\therefore \quad D_5^2 = 4\,R \times 5\,\lambda$$

$$0.42^2 = 20\,R\,\lambda \qquad ...(1)$$

For 10^{th} dark ring,

$$D_{10}^2 = 4\,R \times 10\,\lambda$$

$$\therefore \quad D_{10}^2 = 40\,R\,\lambda$$

Substituting from equation (1),

$$D_{10}^2 = 40 \times \frac{0.42^2}{20}$$

$$\therefore \quad \boxed{D_{10} = 0.594 \text{ cm}}$$

Q.29 **In Newton's rings experiment, it was found that the n^{th} dark ring due to wavelength 6000 A° coincided with $(n+1)^{th}$ dark ring due to wavelength 5000 A°. If radius of curvature of the plano convex lens used is 2 m, find the diameter of n^{th} dark ring of wavelength 5000 A°.**

Ans. : Given data : n^{th} dark ring of wavelength 6000 A° coincides with $(n+1)^{th}$ dark ring of wavelength 5000 A°.

Formula : $D_n^2 = 4Rn\lambda$

For $\lambda = 6000 \text{ A}^{\circ}$

$$D_n^2 = 4Rn \times 6000$$

For $(n+1)^{th}$ ring of 5000 A°.

$$D_{n+1}^2 = 4R(n+1) \times 5000$$

As the two rings coincide,

$$D_n^2 = D_{n+1}^2$$

$$\therefore \quad 4Rn \times 6000 = 4R(n+1) \times 5000$$

$$6n = 5(n+1) = 5n+5$$

$$\therefore \quad n = 5$$

$$D_n = \sqrt{4Rn\lambda} = \sqrt{4 \times 2 \times 5 \times 6000 \times 10^{-8}} \text{ cm}$$

$$\therefore \quad \boxed{D_n = 0.049 \text{ cm}}$$

Q.30 **The diameter of a bright ring in Newton's rings experiment was observed to decrease from 2.3 cm to 2.0 cm when air was replaced by a liquid in the gap between curved surface of plano convex lens and glass plate. Determine the refractive index of the liquid.**

Ans. : Given data : $D_{air} = 2.3$ cm, $D_{liquid} = 2.0$ cm

Formula : $$\mu = \frac{D_{air}^2}{D_{liquid}^2} = \frac{2.3^2}{2.0^2}$$

$$\therefore \quad \boxed{\mu = 1.3225}$$

Q.31 **In Newton's rings experiment, the diameter of 15th dark ring was found to be 0.590 cm and that of 5th dark ring was 0.336 cm. If the radius of curvature of plano - convex lens is 100 cm, calculate the wavelength of light used.** ☞ **[May-12]**

Ans. : Given data : $n = 5$; $n + p = 15$,

$\therefore \quad p = 10$

$D_{n+p} = 0.590$ cm ; $D_n = 0.336$ cm, $R = 100$ cm

Formula : $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4Rp} = \frac{0.590^2 - 0.336^2}{4 \times 100 \times 10} = 5.880 \times 10^{-5} \text{ cm}$$

$$\therefore \quad \boxed{\lambda = 5880 \text{ A}^\circ}$$

1.4 : Applications of Interference

Q.32 **Explain how interference pattern can be used for testing the optical flatness of surfaces.**

Ans. : A wedge shaped air film is formed between the test piece and a standard optically flat surface by keeping them in contact at one edge and separating them by a spacer at the other edge as shown in Fig. 1.7.

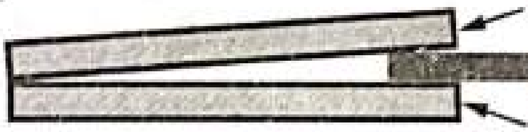


Fig. 1.7

- It is then illuminated by a monochromatic source of light.
- If the test piece is optically flat, the interference pattern will consist of straight fringes of equal width. If the test place is not optically flat, the fringes will have irregular shape.
- When irregular fringes are observed, the test piece is polished and again tested till fringes of equal width are observed.

Q.33 **Explain with diagram how interference principle is used to design anti reflection coating.** ☞ **[May-13, Marks 3]**

Ans. :

- The antireflection coatings reduce intensity of reflected light by destructive interference and hence enhance transmission.

- This is achieved by depositing a thin film of transparent material having refractive index less than that of glass on a glass surface to a thickness of $\frac{\lambda'}{4}$, where λ' in the mean wavelength of light in the film.

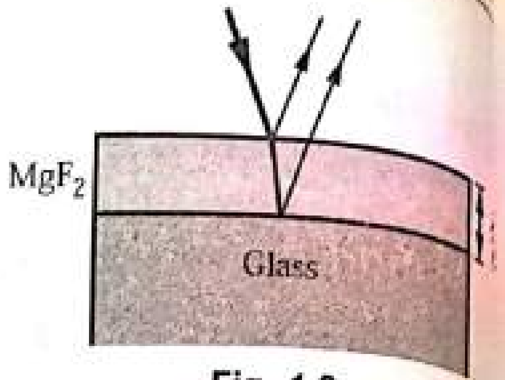


Fig. 1.8

- Magnesium fluoride ($Mg F_2$) of refractive index 1.38 is commonly used.
- For near normal incidence as shown in Fig. 1.8 the path difference between the two rays in reflected light is

$$\Delta = \mu\left(2\frac{\lambda'}{4}\right) = \frac{\mu\lambda'}{2}$$

as $$\mu = \frac{\lambda}{\lambda'}, \quad \lambda = \mu\lambda'$$

$\therefore$ $$\Delta = \frac{\lambda}{2}$$

- There is phase change of π for both rays as they are reflected from denser media. Hence there is no net phase change due to reflection and the path difference remains $\Delta = \frac{\lambda}{2}$. This causes destructive interference in reflected light due to which intensity of reflected light decreases and intensity of transmitted light increases.
- Such coatings are used on camera lenses to enhance the brightness of images.
- As the destructive interference condition is satisfied exactly only for mean wavelength, there will be some reflected light near the red and violet ends of the spectrum. As a result the camera lenses appear purple in reflected light. Also, there is some reflection for angles of incidence more than 0°.
- To reduce intensity of reflected light further, multilayer coatings are used.

Important Formulae

$$\lambda' = \frac{\lambda}{\mu}$$

$$t = \frac{\lambda'}{4}$$

Q.34 A monochromatic beam of light of wavelength 5893 A° is incident normally on the top of a glass which is coated by transparent material MgF_2 having R.I. 1.38. Calculate smallest thickness of the MgF_2 layer which will act as a non reflecting surface. ☞ **[Dec-14, Marks 3]**

Ans. : Given data : $\lambda = 5893$ A°, $\mu = 1.38$

Formula : $\lambda' = \dfrac{\lambda}{\mu}$ and $t = \dfrac{\lambda'}{4}$

$$\therefore \quad \lambda' = \frac{5893}{1.38} = 4270.3 \text{ A°}$$

$$t = \frac{4270.3}{4} = 1067.6 \text{ A°}$$

$$\therefore \quad \boxed{t = 1.068 \times 10^{-5} \text{ cm}}$$

FORMULAE AT A GLANCE

1) For uniform thickness film,

Condition for constructive interference in reflected light

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

Condition for destructive interference in reflected light

$$2\mu t \cos r = n\lambda$$

Condition for constructive interference in transmitted light

$$2\mu t \cos r = n\lambda$$

Condition for destructive interference in transmitted light

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

2) For wedge shaped film,

Fringe width $\beta = \dfrac{\lambda}{2\mu\alpha}$

3) For Newton's rings experiment

Diameter of dark rings : $D_n = \sqrt{4Rn\lambda}$

Diameter of bright rings : $D_n = \sqrt{(2n-1)\,2\lambda R}$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4RP}$$

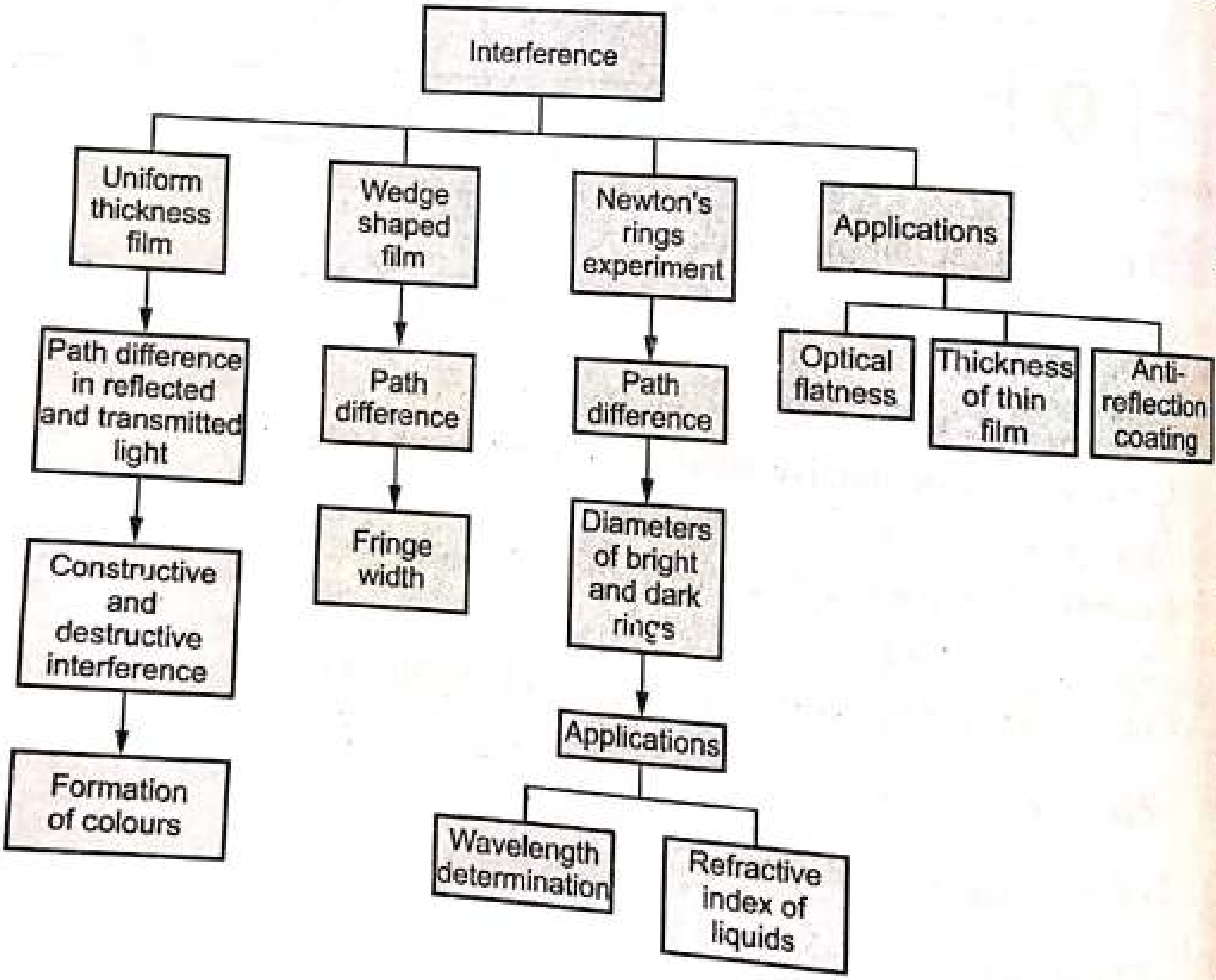
$$\mu = \frac{D_{n+p}^2 - D_n^2}{D'^2_{n+p} - D'^2_n}$$

4) For antireflection coating,

$$\lambda' = \frac{\lambda}{\mu}$$

$$t = \frac{\lambda'}{4}$$

CF CONCEPT FLOWCHART



END...

2 Diffraction

Chapter Outline

- Questions and Answers, Solved Problems
- Formulae at a Glance
- Concept Flowchart

QUESTIONS & ANSWERS

2.1 : Diffraction and Types of Diffraction

Q.1 What is diffraction ? Distinguish between Fresnel and Fraunhofer diffraction (2 points). ☞ [Dec.-13, Marks 3]

OR

What is diffraction ? What are the types of diffraction ? Distinguish between them (any two point).

☞ [Dec - 14, Marks 3]

Ans. : Bending of light around sharp edges of objects into the geometrical shadow is called diffraction.

- The two types of diffraction are Fresnel diffraction and Fraunhofer diffraction.

Sr. No.	Fresnel diffraction	Fraunhofer diffraction
1.	Distance of slit from source and screen is finite.	Distance of slit from source and screen is infinite.
2.	Wavefront incident on the slit is spherical or cylindrical.	Wavefront incident on the slit is plane.

3.	Wavefront incident on the screen is spherical or cylindrical.	Wavefront incident in the screen is plane.
4.	There is path difference between the rays before entering the slit which depends on the distance between the source and slit.	There is no path difference between the rays before entering the slit.
5.	Path difference between the rays forming the diffraction pattern depends on distance of slit from source as well as the screen and the angle of diffraction. Hence mathematical treatment is complicated.	Path difference depends only on angle of diffraction. Hence mathematical treatment is comparatively easier.
6.	Lenses are not required to observe Fresnel diffraction in the laboratory.	Lenses are required to observe Fraunhofer diffraction in the laboratory.

2.2 : Fraunhofer Diffraction at a Single Slit

Q.2 Discuss the Fraunhofer diffraction at a single slit and obtain th conditions for principal maximum and minima. Draw th intensity distribution curve. ☞ **[May-04,07,10]**

Ans. : Consider a parallel beam of light incident normally on a slit of width 'a', diffracted at angle 'θ' as shown in Fig. 2.1.

- The path difference between extreme rays from the slit is

$$\Delta = BC = AB \sin \theta$$

$$\therefore \Delta = a \sin \theta$$

- The path difference between these rays is

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta$$

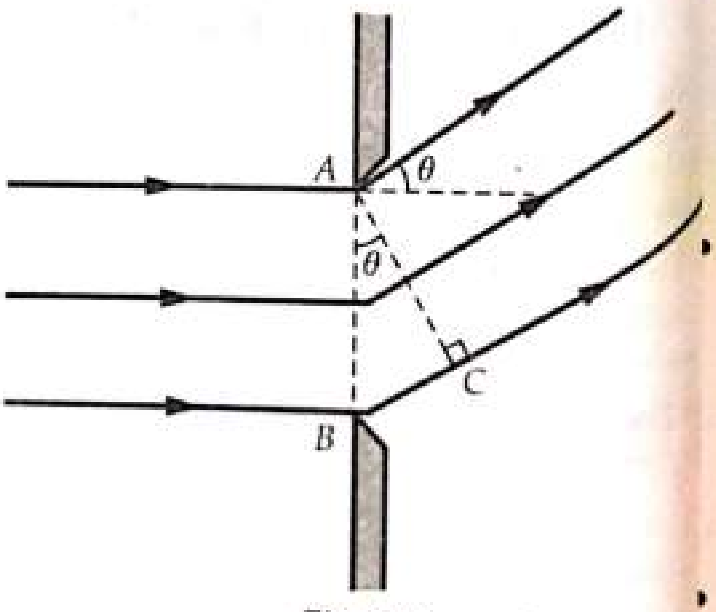


Fig. 2.1

$$\therefore \quad \phi = \frac{2\pi}{\lambda} a \sin\theta \qquad \text{... (2.1)}$$

- The slit is now divided into 'N' parts of equal width Δx, as shown in Fig. 2.2. The width of each part is so small that it behaves like a point source.

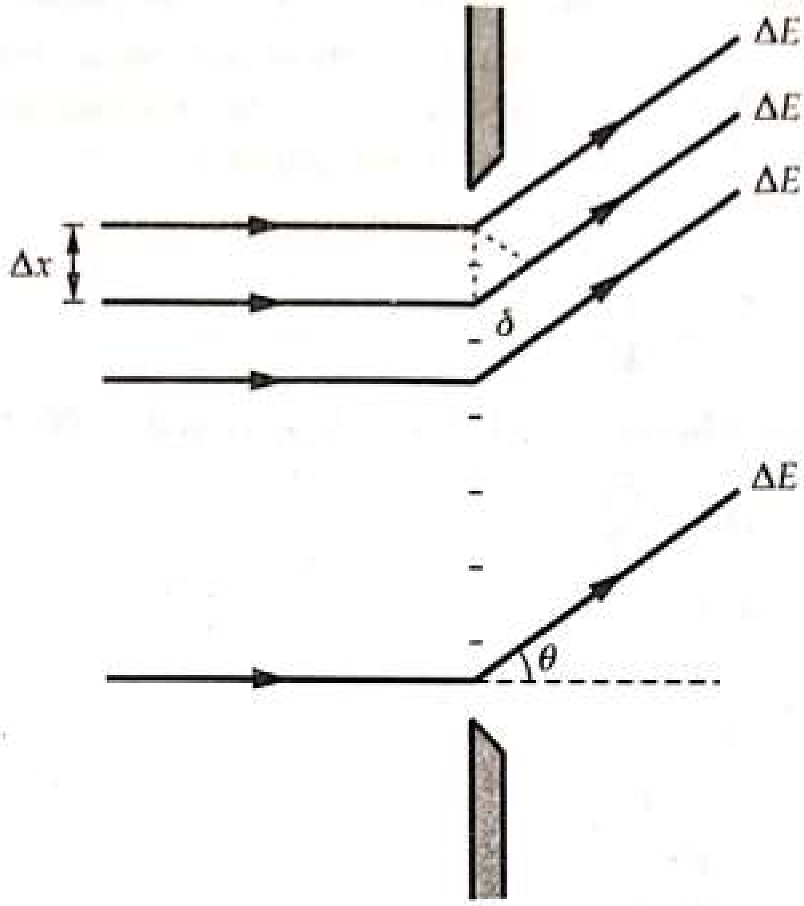


Fig. 2.2

- As all parts are of equal width, the amplitude ΔE of waves transmitted by them will be same. The path difference between waves transmitted by two adjacent parts is $\delta = \Delta x \sin\theta$ and hence the phase difference is

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x \sin\theta$$

- Thus there will be 'N' amplitude phasors of the same amplitude ΔE and the same phase difference $\Delta\phi$ between adjacent phasors. The resultant of these phasors can be obtained geometrically using the phasor diagram shown in Fig. 2.3.

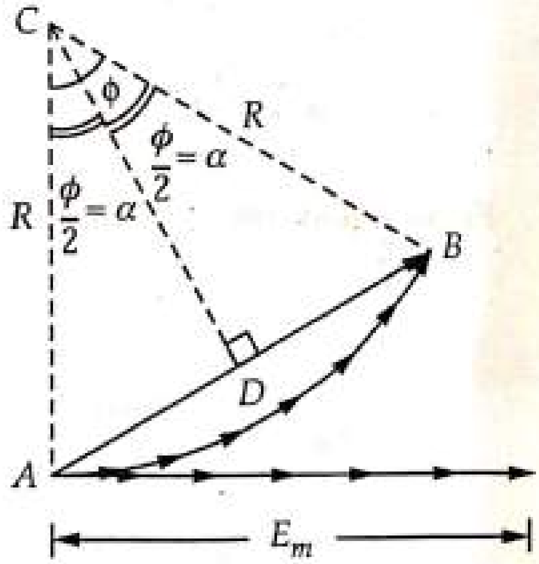


Fig. 2.3

- The first phasor is chosen as the reference phasor and can be drawn in any arbitrary direction. The other phasors are drawn one after the other, each at angle $\Delta\phi$ from the tip of the previous phasor. The '*N*' phasors of equal amplitude and same phase difference between adjacent phasors form an arc of a circle '*AB*' as shown in Fig. 2.3 with centre '*C*' and radius '*R*'.

- The length of arc AB represents the maximum amplitude 'E_m' which can be obtained by adding the phasors without any phase difference between them. The length of chord *AB* represents the resultant amplitude (E_θ) of the diffracted wave at an angle of diffraction 'θ'.

$$AC = BC = R \quad \text{(radius of the circle)}$$

Let
$$\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda} a \sin\theta \qquad \text{... (2.2)}$$

- Draw *CD* perpendicular to *AB*. Then, $\Delta\, ACD$ and $\Delta\, BCD$ are congruent.

$$\therefore \quad AD = DB = \frac{E_\theta}{2}$$

- In $\Delta\, ACD$,

$$\sin\alpha = \frac{AD}{AC}$$

$$\therefore \quad \sin\alpha = \frac{E_\theta / 2}{R}$$

$$\therefore \quad R = \frac{E_\theta}{2\sin\alpha} \qquad \text{... (2.3)}$$

- Using $S = R\,\theta$,

$$\text{arc length } AB = R\,\phi$$

$$\therefore \quad E_m = R \cdot 2\alpha$$

$$R = \frac{E_m}{2\alpha} \qquad \text{... (2.4)}$$

- From equations (2.3) and (2.4),

$$\frac{E_\theta}{2\sin\alpha} = \frac{E_m}{2\alpha}$$

$$\therefore \quad E_\theta = E_m \frac{\sin\alpha}{\alpha} \qquad \text{... (2.5)}$$

$$E_\theta^2 = E_m^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{... (2.6)}$$

- As intensity is proportional to square of the amplitude,

$$I_\theta = K E_\theta^2 \quad \text{and} \quad I_m = K E_m^2$$

- Substituting in equation (2.6),

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{... (2.7)}$$

Principal maximum

- The resultant amplitude for diffraction from a single slit is

$$E_\theta = E_m \frac{\sin\alpha}{\alpha} = \frac{E_m}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$\therefore \quad E_\theta = E_m \left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

$\therefore$ E_θ will be maximum if $\alpha = 0$.

i.e., if $\frac{\pi}{\lambda} a \sin\theta = 0$

$$\therefore \quad \theta = 0$$

- Thus the principal maximum is formed along the incident direction and hence is also called the **central maximum**.

Minima

- For minimum intensity, $I_\theta = 0$

$\therefore$ From equation (2.7),

$$I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$\therefore \quad \sin\alpha = 0 \quad \text{but} \quad \alpha \neq 0$$

$$\therefore \quad \alpha = n\pi \; ; \quad n = \pm 1, \pm 2, \pm 3, ...$$

- From equation (2.2),

$$\frac{\pi}{\lambda} a \sin\theta = n\pi$$

$$\therefore \quad a \sin\theta = n\lambda \qquad \text{... (2.8)}$$

- The minimum intensities are formed at angles given by

$$\theta = \sin^{-1}\left(\frac{n\lambda}{a}\right) \; ; \quad n = \pm 1, \pm 2, \pm 3, ...$$

Here 'n' is called the order of the minimum intensity.

Secondary maxima

- Between any two adjacent minimum intensities, there will be a maximum intensity called secondary maximum. These secondary maxima are formed for $\alpha = \pm\left(n+\frac{1}{2}\right)\pi$, where $n = 1, 2, 3, ...$

$$\therefore \qquad \frac{\pi}{\lambda} a \sin\theta = \left(n+\frac{1}{2}\right)\pi$$

$$a \sin\theta = \left(n+\frac{1}{2}\right)\pi$$

As
$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_m\left[\frac{\sin\left(n+\frac{1}{2}\right)\pi}{\left(n+\frac{1}{2}\right)\pi}\right]^2$$

$$\sin\left(n+\frac{1}{2}\right)\pi = \pm 1$$

$$\therefore \qquad \left[\sin\left(n+\frac{1}{2}\right)\pi\right]^2 = 1$$

$$\therefore \qquad I_\theta = I_m \times \frac{1}{\left(n+\frac{1}{2}\right)^2\pi^2}$$

$$\frac{I_\theta}{I_m} = \frac{1}{\left(n+\frac{1}{2}\right)^2\pi^2}$$

for
$$n = 1, 2, 3, ...$$

$$\frac{I_\theta}{I_m} = 0.0450, 0.0162, 0.0083, ...$$

- Thus the intensity of secondary maxima decreases rapidly.

Intensity pattern due to single slit

- The graph of $\frac{I_\theta}{I_m}$ vs α is shown in Fig. 2.4. The intensity is maximum along the incident direction. The intensity then decreases to zero at $\alpha = \pm\pi$ on either sides of the central maxima.

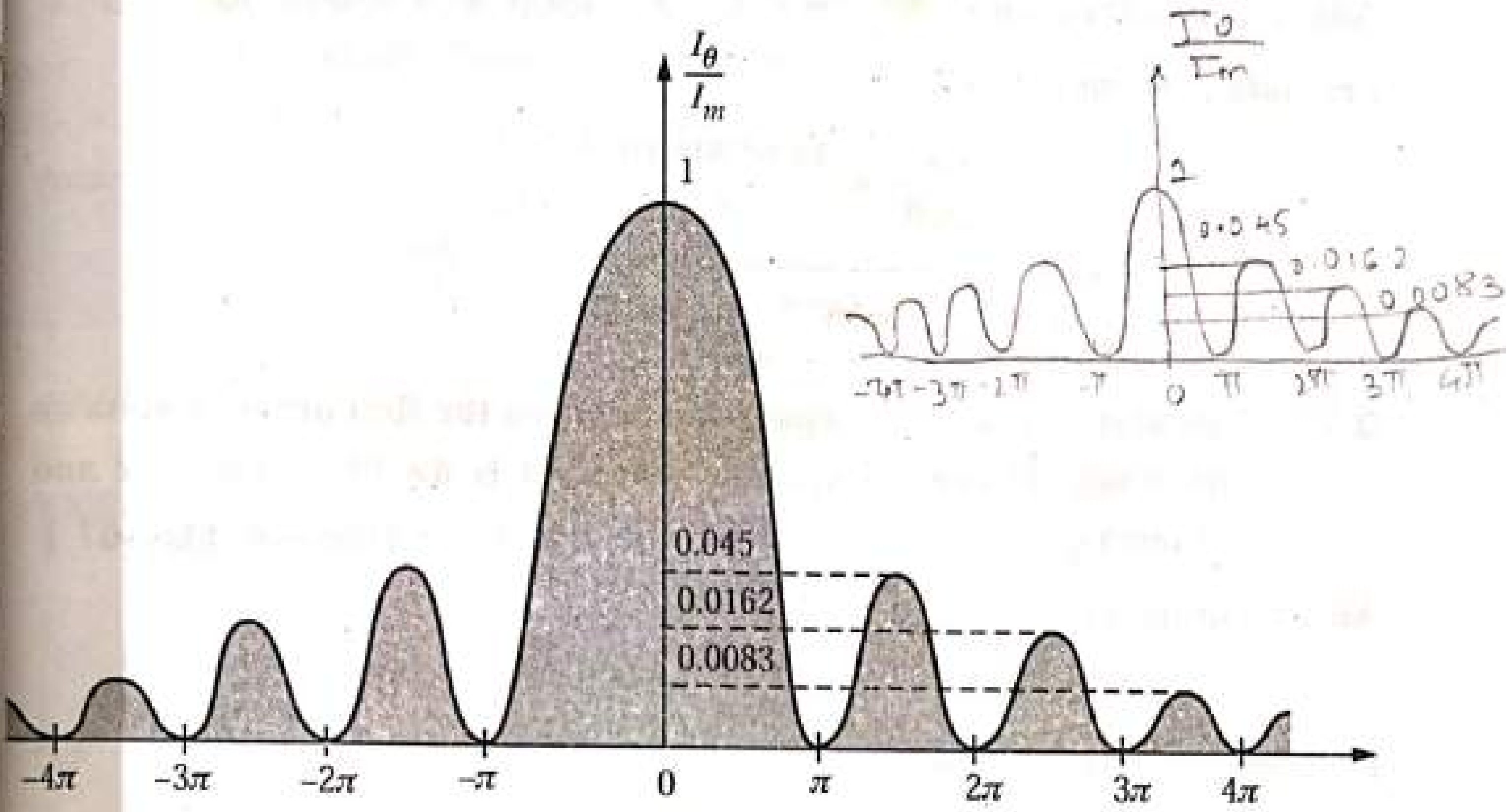


Fig. 2.4

- Secondary maxima and minima are alternately formed after the first minimum intensity on both sides of the central maximum. The intensity of secondary maxima decreases rapidly away from the centre.

Q.3 **Define diffraction of light. Draw intensity distribution pattern obtained because of diffraction of light at a single slit and label the significant points in the same.** ☞ **[May-14 Marks 3]**

Ans. : Refer Q.1 and Q.2.

Important Formulae

Condition for minimum intensity : $a \sin \theta = n \lambda$

Width of central maximum : $W = 2d = \frac{2D\lambda}{a}$

SOLVED EXAMPLES

Q.4 **A slit of width 'a' is illuminated by white light. For what value of 'a' will the first minimum for red light fall at an angle of 30° ? Wavelength of red light is 6500 A°.** ☞ **[May-06]**

Ans. : Given Data : $\theta = 30°$, $n = 1$, $\lambda = 6500$ A° = 6500×10^{-8} cm

Formula : $a \sin \theta = n \lambda$

$$\therefore \quad a = \frac{n\lambda}{\sin\theta} = \frac{1 \times 6500 \times 10^{-8}}{\sin 30}$$

$$\therefore \quad \boxed{a = 1.3 \times 10^{-4} \text{ cm}}$$

Q.5 **Calculate the angular separation between the first order minima either side of central maxima when slit is 6×10^{-4} cm wide and $\lambda = 6000$ A°.**

☞ **[Dec.-04, May-07]**

Ans. : Given Data : $a = 6 \times 10^{-4}$ cm

$n = 1$, $\lambda = 6000$ A° = 6000×10^{-8} cm

Formula : $a \sin \theta = n \lambda$

$$\therefore \quad \sin \theta = \frac{n\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{6 \times 10^{-4}}$$

$$\sin \theta = 0.1$$

$$\therefore \quad \theta = 5° 44' 21''$$

Angular separation between the first order minima

$$= 2\theta = 2 \times 5° 44' 21'' = 11° 28' 42''$$

Q.6 **A single slit diffraction pattern is formed using white light. For what wavelength of light does the second minimum coincide with the third minimum for the wavelength 4000 A° ?**

☞ **[Dec.-08, May-11]**

Ans. : Given Data : Third minimum for the wavelength 4000 A° coincides with second minimum of wavelength λ.

Formula : $a \sin \theta = n \lambda$

For second order minimum of wavelength 'λ' formed at angle θ,

$$a \sin \theta = 2 \lambda \quad \ldots (1)$$

For third minimum of wavelength 4000 A°,

$$a \sin \theta = 3 \times 4000 \quad \ldots (2)$$

From equations (1) and (2),

$$2\lambda = 3 \times 4000$$

$$\therefore \quad \boxed{\lambda = 6000 \text{ A°}}$$

Q.7 **A slit of width 0.16 mm is illuminated by a monochromatic light of wavelength 5600 A°. Find half angular width of a principal maximum.** ☞ **[Dec.-10]**

Ans. : Given Data : $a = 0.16 \text{ mm} = 0.16 \times 10^{-3}$ m

$n = 1$ for first minimum

$\lambda = 5600 \text{ A}^\circ = 5600 \times 10^{-10}$ m

Formula : $a \sin\theta = n\lambda$

The angle θ, at which the first minimum is formed, is the half angular width of central maximum.

$$0.16 \times 10^{-3} \sin\theta = 1 \times 5600 \times 10^{-10}$$

$$\therefore \quad \boxed{\theta = 0.2^\circ = 0^\circ\, 12'\, 1.93''}$$

Q.8 **Find the half angular width of the central maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm, when illuminated by light of wavelength 6000 A°.** ☞ **[Dec.-11]**

Ans. : Given Data : $a = 12 \times 10^{-5}$ cm

$n = 1$ for first minimum

$\lambda = 6000 \text{ A}^\circ = 6000 \times 10^{-8}$ cm

Formula : $a \sin\theta = n\lambda$

$$12 \times 10^{-5} \sin\theta = 1 \times 6000 \times 10^{-8}$$

$$\therefore \quad \boxed{\theta = 30^\circ}$$

Q.9 **A slit of width 2 μm is illuminated by light of wavelength 6500 A°. Calculate the angle at which the first minimum will be observed.** ☞ **[May-12]**

Ans. : Given Data : $a = 2 \text{ μm} = 2 \times 10^{-6}$ m

$n = 1$ for first minimum

$\lambda = 6500 \text{ A}^\circ = 6500 \times 10^{-10}$ m

Formula $a \sin\theta = n\lambda$

$$2 \times 10^{-6} \sin\theta = 1 \times 6500 \times 10^{-10}$$

$$\therefore \quad \boxed{\theta = 18.97^\circ}$$

Q.10 **Find the half angular width of the central maxima in Fraunhofer diffraction pattern of slit having width 10×10^{-5} cm. When illuminated by light having wave length 5000 A°.**

☞ [Dec.-12

Ans. : Given Data : $a = 10 \times 10^{-5}$ cm

$$n = 1$$

$$\lambda = 5000 \text{ A}^{\circ} = 5000 \times 10^{-8} \text{ cm}$$

Formula : $a \sin \theta = n\lambda$

$$10 \times 10^{-5} \sin \theta = 1 \times 5000 \times 10^{-8}$$

$$\therefore \quad \boxed{\theta = 30^{\circ}}$$

2.3 : Plane Diffraction Grating

Q.11 **What is diffraction grating ? How it is obtained ?**

☞ [Dec.-04, May-07, Marks 2

Ans. :

- A plane diffraction grating consists of large number of parallel slits of equal width separated by opaque spaces.
- All opaque spaces have equal width.
- It is constructed by drawing lines on a flat glass plate using a diamond point.
- The line becomes opaque and the region between two lines, which is transparent, acts as the slit.

Q.12 **Give the theory of plane diffraction grating. Obtain the conditions for maxima and minima.**

Ans.: Let $a =$ Width of each slit.

$b =$ Width of each opaque space.

$N =$ Total number of lines on the grating.

then $d = a + b =$ Width of a line (grating element)

$$\frac{1}{a+b} = \text{Number of lines per unit length.}$$

- Consider a parallel beam of light incident normally on the grating diffracted through an angle 'θ' as shown in Fig. 2.5.

①

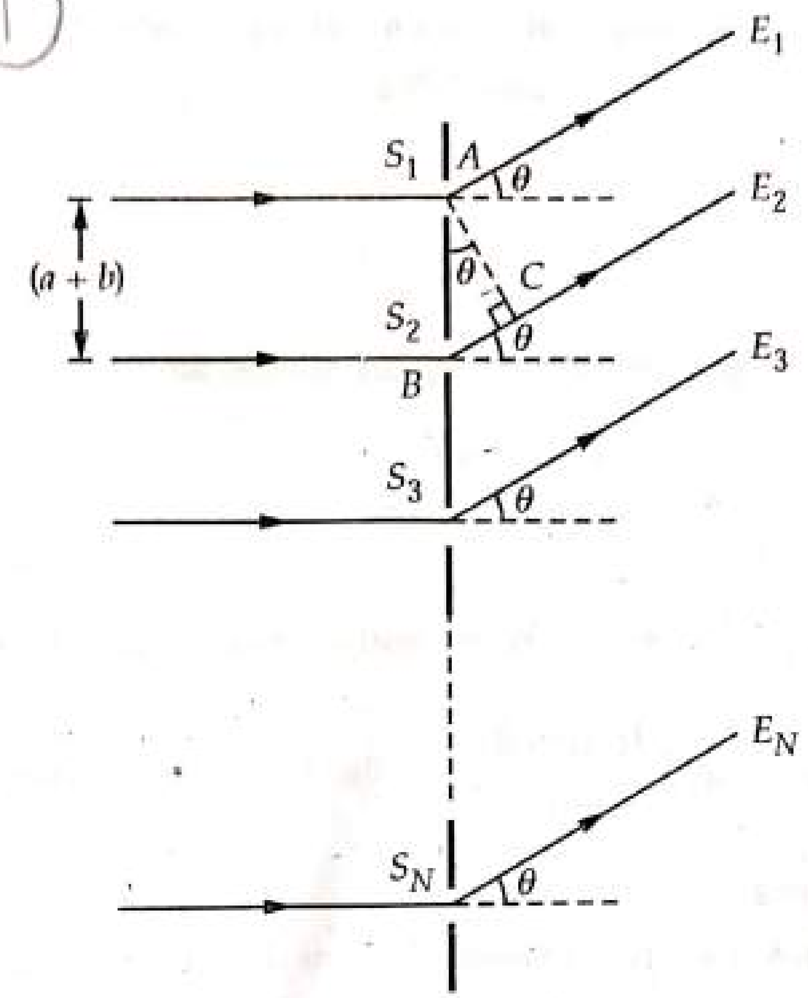


Fig. 2.5

- As all slits are of equal width, resultant amplitude of light transmitted by them will be same at angle θ. From theory of single slit diffraction, this amplitude is given by,

$$\therefore \quad E_1 = E_2 = ...= E_N = \frac{E_m \sin\alpha}{\alpha}$$

where $\alpha = \frac{\pi}{\lambda} a \sin\theta$ and E_m is the maximum amplitude.

- The path difference between adjacent amplitude vectors is

$$BC = (a+b)\sin\theta$$

- $\therefore$ Phase difference (2β) between them is

$$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$$

$$\therefore \quad \beta = \frac{\pi}{\lambda}(a+b)\sin\theta \qquad ...(2.10)$$

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- Thus there are 'N' amplitude vectors of equal amplitude E_θ with cons phase difference 2β between adjacent vectors. The resultant ampli E_θ is given by

$$E_\theta = \left(E_m \frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right) \quad \text{... (2.}$$

- As intensity is proportional to square of the amplitude,

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)^2 \quad \text{... (2.}$$

- Here $I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$, which is intensity from single slit diffraction, is diffraction term and $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ is the interference term due to 'N' slit

Principal maxima

- Principal maxima are obtained for $\sin\beta = 0$, i.e., for $\beta = m\pi$ whe $m = 0, \pm 1, \pm 2, \ldots$

$$\therefore \quad \frac{\pi}{\lambda}(a+b)\sin\theta = m\pi$$

$$\therefore \quad (a+b)\sin\theta = m\lambda \quad \text{... (2.13)}$$

- Equation (2.13) gives the angles 'θ' for which principal maxima formed. 'm' is called the order of the spectrum.

- For $\beta = m\pi$, the term $\frac{\sin N\beta}{\sin\beta}$ becomes indeterminate. Hence, tak limits as $\beta \to m\pi$ in equation (2.13), we get,

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \left[\lim_{\beta\to m\pi} \frac{\sin N\beta}{\sin\beta}\right]^2$$

- Using L'Hospital's rule,

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \left[\lim_{\beta\to m\pi} \frac{d(\sin N\beta)}{d(\sin\beta)}\right]^2$$

$$= I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \left[\lim_{\beta\to m\pi} \frac{N\cos N\beta}{\cos\beta}\right]^2$$

For $\beta = m\pi$, $\cos N\beta = \cos N m\pi = \pm 1$ and $\cos\beta = \pm 1$

$$\therefore \quad I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 [\pm N]^2$$

$$\therefore \quad I_\theta = N^2 I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{... (2.14)}$$

$$\therefore \quad I_\theta = N^2 \times \text{ Intensity due to single slit.}$$

Minima

- For minimum intensity, $I_\theta = 0$

$$\therefore \quad \frac{\sin N\beta}{\sin\beta} = 0$$

$\therefore$ $\sin N\beta = 0$ but $\sin\beta \neq 0$

$\therefore$ $N\beta = n\pi$ but $\beta \neq l\pi$ where n and l are integers.

$\therefore$ $\beta = \frac{n\pi}{N}$ where $\frac{n}{N}$ must not be an integer.

$\therefore$ $n = 1, 2, 3, \ldots (N-1), (N+1), (N+2), \ldots (2N-1), (2N+1), \ldots$

or, $n \neq 0, N, 2N, \ldots$ as these values of 'n' give principal maxima.

as $\quad \beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = \frac{n}{N}\pi$$

$$\therefore (a+b)\sin\theta = \frac{n}{N}\lambda \qquad \text{... (2.15)}$$

- where $n = 1, 2, \ldots (N-1), (N+1), (N+2), \ldots (2N-1), (2N+1) \ldots$
- From the above values of 'n', we can see that there are $(N-1)$ minimum intensities between any two adjacent principal maxima.

Q.13 What is diffraction grating ? Give the equation of resultant intensity of light with the meaning of each symbol, when monochromatic light is diffracted from grating. Obtain the equation of maxima and minima.

☞ [May-11, Dec.-11, Marks 6]

Ans. : Refer Q.11 and Q.12.

Q.14 State the factors on which resolving power of grating depends.

[Dec.-08, Marks 2]

Ans. : $\frac{d\theta}{d\lambda}$ is called the dispersive power of the grating. It is given by

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta}$$

It depends upon, the order of diffraction, grating element and the angle of diffraction.

Important Formulae

Condition for principal maxima : $(a + b) \sin \theta = m \lambda$

Condition for minimum intensity : $(a + b) \sin \theta = \frac{n}{N} \lambda$

Number of lines per unit length $= \frac{1}{a+b}$

Maximum order $m_{max} = \frac{a+b}{\lambda}$

where $n = 1, 2, \ldots (N-1), (N+1), (N+2), \ldots\ldots (2N-1), (2N+1) \ldots$

Dispersive power of the grating : $\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta}$

SOLVED EXAMPLES

Q.15 In a plane transmission grating, the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines / cm of the grating surface.

[Dec.-08, May -14]

Ans. : Given Data : $m = 2$, $\lambda = 5 \times 10^{-5}$ cm

$\theta = 30°$

Formula : $(a + b) \sin \theta = m \lambda$

$$\therefore \quad a + b = \frac{m\lambda}{\sin\theta} = \frac{2 \times 5 \times 10^{-5}}{\sin 30} = 2 \times 10^{-4} \text{ cm}$$

$$\text{Number of lines/cm} = \frac{1}{a+b} = \frac{1}{2\times10^{-4}} = \mathbf{5000}$$

$\therefore$ There are 5000 lines/cm.

Q.16 What is the longest wavelength that can be observed in the third order for a transmission grating having 7000 lines per cm ? Assume normal incidence. ☞ **[May-07]**

Ans. : Given Data : For maximum wavelength, $\sin\theta = 1$

$$(a+b) = \frac{1}{7000}\text{ cm}$$

$$m = 3$$

Formula : $(a+b)\sin\theta = m\lambda$

$$\therefore \quad \frac{1}{7000}\times 1 = 3\lambda$$

$$\therefore \quad \lambda = 4.762\times10^{-5}\text{ cm}$$

$$\therefore \quad \boxed{\lambda = \mathbf{4762\ A^\circ}}$$

Q.17 Monochromatic light of wavelength 6560 A° falls normally on a grating 2 cm wide. The first order spectrum is produced at an angle of 16° 12′ from the normal. Calculate total number of lines on the grating. ☞ **[Dec.-04]**

Ans. : Given Data : $m = 1, \quad \lambda = 6560\text{ A}^\circ = 6560\times10^{-8}$ cm

$$\theta = 16^\circ\, 12'$$

Formula : $(a+b)\sin\theta = m\lambda$

$$(a+b) = \frac{m\lambda}{\sin\theta} = \frac{1\times6560\times10^{-8}}{\sin 16^\circ 12'} = 2.3513\times10^{-4}\text{ cm}$$

$$\text{Number of lines / cm} = \frac{1}{a+b} = \frac{1}{2.3513\times10^{-4}} = 4253$$

As width of grating is 2 cm, total number of lines

$$= 4253\times2 = \mathbf{8506}$$

Q.18 Monochromatic light of wavelength 6.56×10^{-5} cm falls normally on a grating 2 cm wide. The first order spectrum is produced at

an angle of 18° 14′ from the normal. What is the total number lines on the grating ? ☞ [Dec.-06

Ans. : Given Data : $m = 1$, $\lambda = 6.56 \times 10^{-5}$ cm

$$\theta = 18^\circ 14'$$

Formula : $(a + b) \sin \theta = m \lambda$

$$(a + b) = \frac{m\lambda}{\sin\theta} = \frac{1 \times 6.56 \times 10^{-5}}{\sin 18^\circ 14'} = 2.0966 \times 10^{-4} \text{ cm}$$

$$\text{Number of lines/cm} = \frac{1}{a+b} = \frac{1}{2.0966 \times 10^{-4}} = 4770$$

As width of grating is 2 cm, total number of lines = $4770 \times 2 = 9540$

Q.19 What is the highest order spectrum that is visible with light wavelength 6000 A° by means of a grating having 5000 lines per cm ? ☞ [Dec.-03,05, May-10

Ans. : Given Data : $a + b = \frac{1}{5000}$ cm

$$\lambda = 6000 \text{ A}^\circ = 6 \times 10^{-5} \text{ cm}$$

Formula : $m_{max} = \frac{a+b}{\lambda}$

$$m_{max} = \frac{1}{5000 \times 6 \times 10^{-5}} = 3.333$$

As m_{max} is an integer,

$$\boxed{m_{max} = 3}$$

Q.20 What is the highest order spectrum which may be seen with light of wavelength 6328 A° by means of a grating with 3000 lines/cm ? ☞ [May-08

Ans. : Given Data : $(a + b) = \frac{1}{3000}$ cm

$$\lambda = 6328 \text{ A}^\circ = 6328 \times 10^{-8} \text{ cm}$$

Formula : $m_{max} = \frac{a+b}{\lambda}$

$$\therefore \quad m_{max} = \frac{1}{3000 \times 6328 \times 10^{-8}} = 5.267$$

As m_{max} is a whole number,

$$\boxed{m_{max} = 5}$$

Q.21 A grating has 6000 lines per cm. How many orders of light of wavelength 4500 A° can be seen ? ☞ [Dec. 2008]

Ans. : Given Data : $a + b = \frac{1}{6000}$ cm

$$\lambda = 4500 \text{ A}^\circ = 4500 \times 10^{-8} \text{ cm}$$

Formula : $m_{max} = \frac{a+b}{\lambda}$

$$\therefore \quad m_{max} = \frac{1}{6000 \times 4500 \times 10^{-8}} = 3.704$$

$$\therefore \quad \boxed{m_{max} = 3}$$

Note : As $m_{max} = 3.704$, the 4^{th} order will not be visible.

Q.22 When the parallel waves of monochromatic light of wavelength 5790 A° fall normally on a grating 2.54 cm wide. The first order spectrum is produced at an angle of 19.994° from the normal. Calculate total number of lines of the grating. ☞ [Dec. 10]

Ans. : Given Data : $m = 1$, $\lambda = 5790$ A° $= 5790 \times 10^{-8}$ cm

$$\theta = 19.994^\circ$$

Formula : $(a + b) \sin\theta = m\lambda$

$$(a + b) = \frac{m\lambda}{\sin\theta} = \frac{1 \times 5790 \times 10^{-8}}{\sin 19.994}$$

$$\therefore \quad (a + b) = 1.6934 \times 10^{-4} \text{ cm}$$

$$\text{Number of lines/cm} = \frac{1}{a+b} = \frac{1}{1.6934 \times 10^{-4}} = 5905$$

As width of grating is 2.54 cm, total number of lines

$$= 5905 \times 2.54 = 14998.7 \simeq \mathbf{15000 \text{ lines}}$$

Q.23 **Monochromatic light from laser of wavelength 6238 A° incident normally on a diffraction grating containing 60[00] lines / cm. Find the angles at which the first and second ord[er] maximum are obtained.** [May-12, 13]

Ans. : Given Data : $(a+b) = \frac{1}{6000}$ cm

For first order, $m = 1$.

$$\lambda = 6238\ A^{\circ} = 6238 \times 10^{-8}\ \text{cm}$$

Formula : $(a+b)\sin\theta = m\lambda$

$$\therefore \left(\frac{1}{6000}\right)\sin\theta = 1 \times 6238 \times 10^{-8}$$

$$\therefore \boxed{\theta = 21.98^{\circ}}$$

For second order, $m = 2$.

$$\therefore \left(\frac{1}{6000}\right)\sin\theta = 2 \times 6238 \times 10^{-8}$$

$$\therefore \boxed{\theta = 48.47^{\circ}}$$

21.98
× 2
.96

FORMULAE AT A GLANCE

1) For Fraunhofer diffraction at a single slit,

Condition for minimum intensity : $a \sin\theta = n\lambda$

Width of central maximum : $W = 2d = \frac{2D\lambda}{a}$

2) For Diffraction grating,

Condition for principal maxima : $(a+b)\sin\theta = m\lambda$

Condition for minimum intensity : $(a+b)\sin\theta = \frac{n}{N}\lambda$

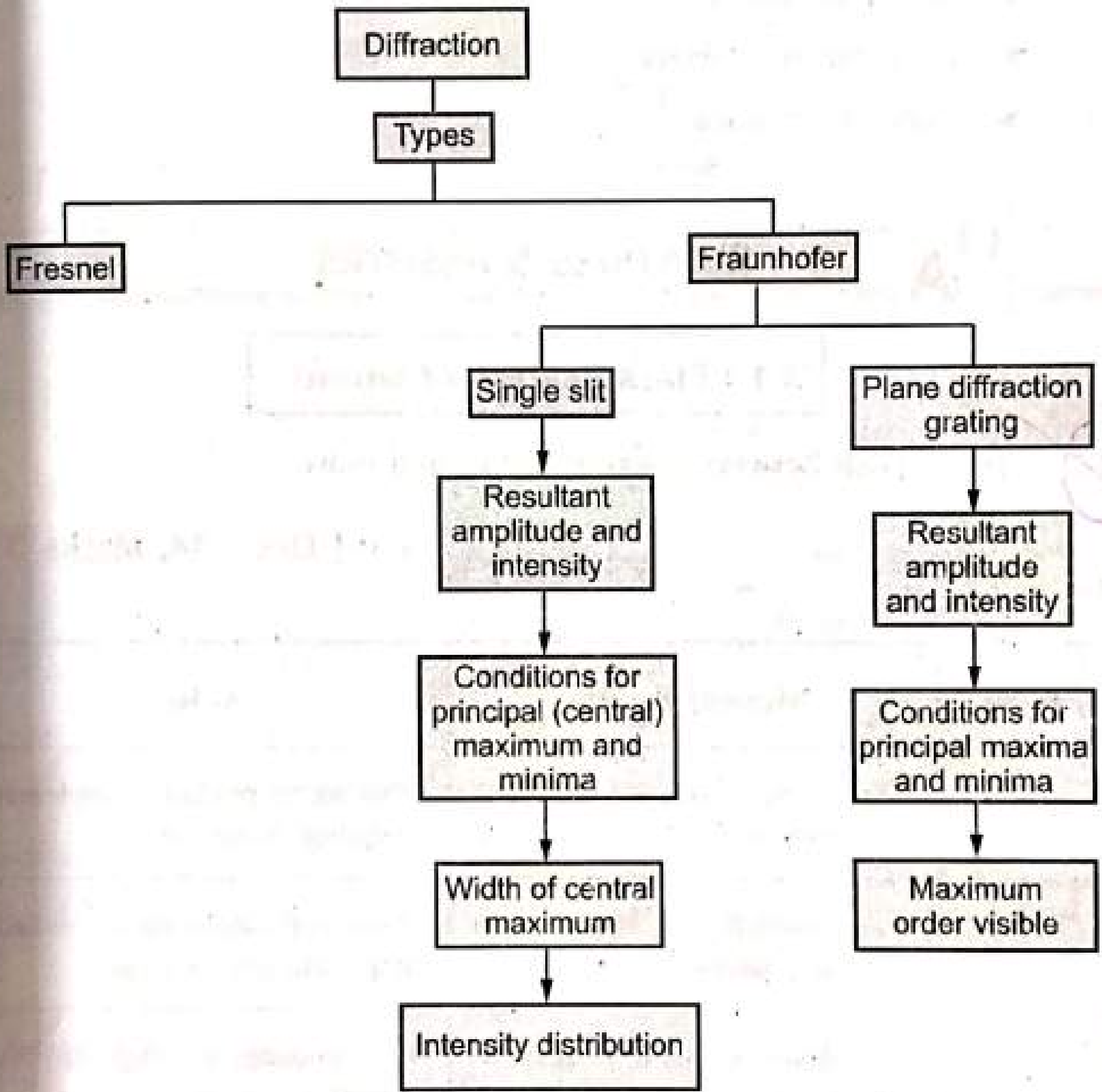
Number of lines per unit length $= \frac{1}{a+b}$

Maximum order $m_{max} = \dfrac{a+b}{\lambda}$

where $n = 1, 2, \ldots (N-1), (N+1), (N+2), \ldots\ldots (2N-1), (2N+1) \ldots$

Dispersive power of the grating : $\dfrac{d\theta}{d\lambda} = \dfrac{m}{(a+b)\cos\theta}$

CF CONCEPT FLOWCHART



END...

5 Polarization

Chapter Outline

- Questions and Answers, Solved Problems
- Formulae at a Glance
- Concept Flowchart

QUESTIONS & ANSWERS

5.1 : Basic Definitions

Q.1 Define the following terms : i) Unpolarized light ii) plane polarized light iii) plane of vibration iv) plane of polarization.

Ans. :

Unpolarized light : It is light which has vibrations in all possible directions perpendicular to the direction of propagation.

Plane polarized light : Light having vibrations only in one plane and in a direction perpendicular to the direction of propagation is called plane polarized light.

Plane of vibration : It is the plane in which the electric field vector of plane polarized light vibrates.

Plane of polarization : It is a plane perpendicular to the plane of vibration.

5.2 : Law of Malus

Q.2 State and prove law of Malus. ☞ **[May-15, Marks 3]**

Ans. : Statement : When unpolarized light is incident on the polarizer, the ratio of intensity of light transmitted by analyzer to the intensity transmitte by polarizer is equal to square of the cosine of the angle between the axes polarizer and analyzer. i.e.,

$$\frac{I}{I_0} = \cos^2 \theta$$

where I = Intensity transmitted by analyzer

I_0 = Intensity transmitted by polarizer

θ = Angle between axes of polarizer and analyzer

Proof : Consider the axis of analyzer to be at angle 'θ' with the axis polarizer as shown in Fig. 5.1 (a).

- The amplitude of light (E_0) transmitted by polarizer is parallel to axis polarizer and hence will make angle 'θ' with axis of analyzer.
- E_0 can be resolved into two mutually perpendicular components shown in Fig. 5.1 (b).

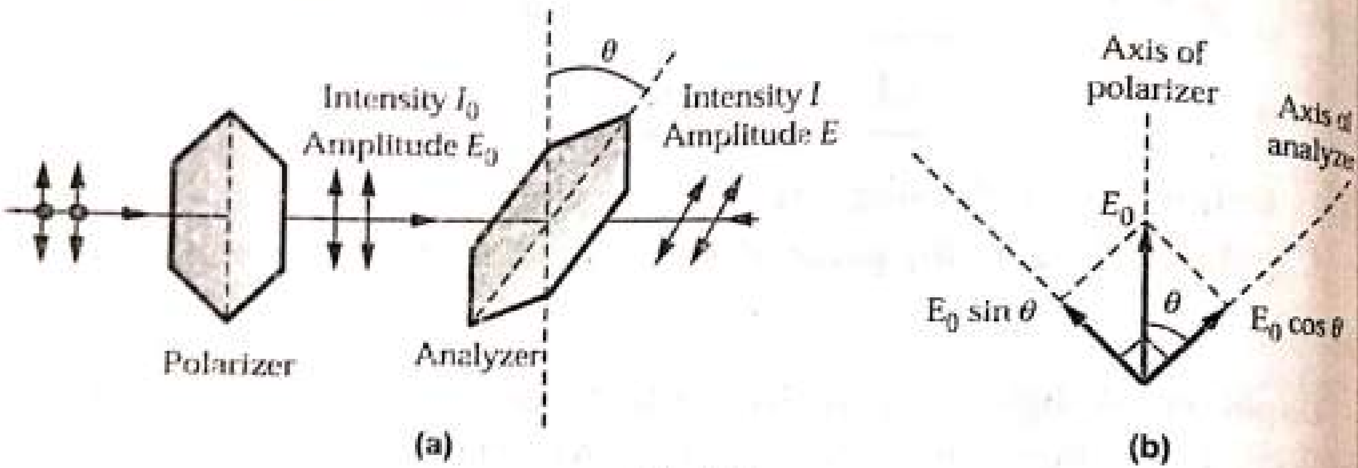


Fig. 5.1

i) $E_0 \cos\theta$ parallel to axis of analyzer, which is allowed to pass through.

ii) $E_0 \sin\theta$ perpendicular to axis of analyzer, which is blocked.

$\therefore$ Amplitude 'E' of light transmitted by analyzer is

$$E = E_0 \cos\theta$$

- Squaring, we get

$$E^2 = E_0^2 \cos^2\theta$$

$$I = I_0 \cos^2\theta$$

$$\frac{I}{I_0} = \cos^2 \theta$$

- The above equation gives two values of 'θ' for a given ratio of $\frac{I}{I_0}$. One value is 'θ' and the other $(180-\theta)$.

Important Formula

$$\frac{I}{I_0} = \cos^2 \theta$$

SOLVED EXAMPLES

Q.3 **A polarizer and analyzer are oriented so that the amount of transmitted light is maximum. Through what angle should either be turned so that the intensity of transmitted light is reduced to i) 0.75 and ii) 0.25 times the maximum intensity ?**

[Dec.-06]

Ans. : Given data : i) $\frac{I}{I_0} = 0.75$ ii) $\frac{I}{I_0} = 0.25$

Formula : $\frac{I}{I_0} = \cos^2 \theta$

i) $0.75 = \cos^2 \theta$

$\therefore$ $\theta = 30°$ or $\theta = 180 - 30 = 150°$

$\therefore$ $\boxed{\theta = 30° \text{ or } 150°}$

ii) $0.25 = \cos^2 \theta$

$\therefore$ $\theta = 60°$ or $\theta = 180 - 60 = 120°$

$\therefore$ $\boxed{\theta = 60° \text{ or } \theta = 120°}$

Q.4 **Polarizer and analyzer are set with their polarizing directions parallel so that the intensity of transmitted light is maximum. Through what angle should either be turned so that the intensity be reduced to i) $\frac{1}{2}$ and ii) 25% of the maximum intensity ?**

[Dec.-03,11, 12, 14, Marks 3]

Ans. : Given data : i) $\frac{I}{I_0} = \frac{1}{2}$ ii) $\frac{I}{I_0} = \frac{25}{100} = 0.25$

Formula : $\frac{I}{I_0} = \cos^2 \theta$

i) $\frac{1}{2} = \cos^2 \theta$

$\therefore$ $\theta = 45°$ or $\theta = 180 - 45 = 135°$

$\therefore$ $\boxed{\theta = 45° \text{ or } 135°}$

ii) $0.25 = \cos^2 \theta$

$\therefore$ $\theta = 60°$ or $\theta = 180 - 60 = 120°$

$\therefore$ $\boxed{\theta = 60° \text{ or } 120°}$

Q.5 **Two polarizing plates have polarizing directions parallel so as transmit maximum intensity of light. Through what angle m... either plates be turned if the intensity of the transmitted beam one third the intensity of the incident beam.** ☞ **[Dec.-09**

Ans. : Given data : $\frac{I}{I_0} = \frac{1}{3}$

Formula : $\frac{I}{I_0} = \cos^2 \theta$

$\therefore$ $\frac{1}{3} = \cos^2 \theta$

$\therefore$ $\theta = \cos^{-1}\left(\pm \frac{1}{\sqrt{3}}\right)$

$\therefore$ $\boxed{\theta = 54.74° \text{ or } 125.26°}$

5.3 : Polarization by Reflection

Q.6 **What is polarization by reflection? State Brewster's law. He... prove that the refractive index of a material is equal to t... tangent of its polarizing angle.**

Ans. : When unpolarized light is incident on any transparent medium, the reflected light is found to be completely plane polarized for a particular angle of incidence known as the polarizing angle.

- The electric field vector of the reflected plane polarized light is parallel to the reflecting surface.
- Brewster found that if light is made incident on any transparent material at the polarizing angle, the reflected ray and the refracted. Ray are perpendicular to each other. This is known as Brewster's law.

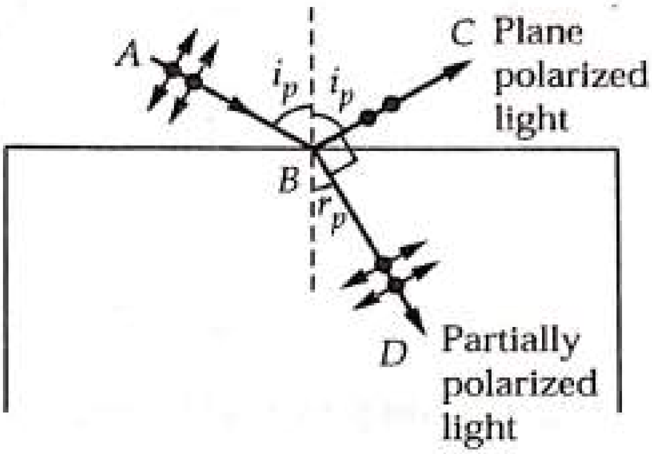


Fig. 5.2

- Consider unpolarized light incident on a transparent medium at the polarizing angle i_P as shown in Fig. 5.2.
- By Brewster's law, the reflected ray and the refracted ray are perpendicular to each other.

$$\therefore \quad \angle CBD = 90^\circ$$

$$\therefore \quad i_P + 90^\circ + r_P = 180^\circ$$

$$\therefore \quad i_P + r_P = 90^\circ$$

$$\therefore \quad r_P = 90^\circ - i_P$$

- By Snell's law, the refractive index 'μ' of the transparent medium is given by

$$\mu = \frac{\sin i_P}{\sin r_P}$$

$$\therefore \quad \mu = \frac{\sin i_P}{\sin(90 - i_P)}$$

$$\mu = \frac{\sin i_P}{\cos i_P}$$

$$\mu = \tan i_P \quad \text{... (5.1)}$$

- Hence the alternative statement of Brewster's law is as follows :

  The refractive index of any transparent medium is equal to the tangent of its polarizing angle.

Q.7 Prove that when light is incident on a transparent medium at polarizing angle, the reflected ray and the refracted ray perpendicular to each other.

Ans. : According to Brewster's law, the refractive index of any transparent medium is equal to the tangent of its polarizing angle.

$$\therefore \quad \mu = \tan i_p$$

$$= \frac{\sin i_p}{\cos i_p}$$

$$\mu = \frac{\sin i_p}{\sin (90 - i_p)}$$

- But according to Snell's law,

$$\mu = \frac{\sin i_p}{\sin r_p}$$

$$\therefore \quad 90° - i_p = r_p$$

$$\therefore \quad i_p + r_p = 90°$$

- From Fig. 5.7.1,

$$i_p + r_p + \angle CBD = 180°$$

$$\therefore \quad 90° + \angle CBD = 180°$$

$$\therefore \quad \angle CBD = 90°$$

$\therefore$ The refracted ray and the reflected ray are perpendicular to each other.

Important Formulae

$$i_p + r_p = 90°$$

$$\mu = \tan i_p$$

SOLVED EXAMPLES

Q.8 For an ordinary glass plate μ = 1.54. Find the angle polarization and angle r_L the corresponding angle of refraction.

Ans. : Given data : $\mu = 1.54$

Formulae : $\mu = \tan i_p$ and $i_p + r_p = 90°$

$$\therefore \quad i_p = \tan^{-1} \mu$$

$$\mu = 1.54$$

$\therefore$ $$i_p = \tan^{-1} 1.54$$

$\therefore$ $$i_p = 57^\circ$$

$$r_L = 90 - 57^\circ$$

$\therefore$ $$\boxed{r_L = 33^\circ}$$

Q.9 **A ray of light is incident on a surface of benzene of refractive index 1.50. If the reflected light is linearly polarized, calculate the angle of refraction.**

Ans. : Given data : $\mu = 1.50$

Formulae : $\mu = \tan i_p$ and $i_p + r_p = 90^\circ$

$\therefore$ $$i_p = \tan^{-1} 1.5$$

$\therefore$ $$i_p = 56.31^\circ$$

$$r_p = 90 - i_p$$

$$= 90 - 56.31$$

$\therefore$ $$\boxed{r_p = 33.69^\circ}$$

Q.10 **At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light**
(Data given : Critical angle for diamond = 24.5°).

☞ [May - 13, Marks 3]

Ans. : Given data : $\mu_2 = 1$, $\theta_c = 24.5^\circ$, $\mu_1 = \mu$

Formulae : $\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$, $i_p = \tan^{-1}(\mu)$

$\therefore$ $$24.5 = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\therefore$ $$\mu = 2.41$$

The polarizing angle is given by,

$$i_p = \tan^{-1}(\mu)$$

$\therefore \qquad i_p = \tan^{-1} 241$

$\therefore \qquad \boxed{i_p = 67.46°}$

5.4 : Polarization by Refraction

Q.11 Explain polarization by refraction.

Ans. :

- If unpolarized light is incident on a transparent medium like glass, tho waves having vibrations in the plane of incidence are partly reflected a partly refracted for all angles of incidence except the polarizing ang (given by $i_p = \tan^{-1} \mu$) for which these waves are completely refracte
- Those waves having vibrations perpendicular to the plane of inciden are partly reflected and partly refracted at all angles of incidence.
- When light is incident on glass ($\mu = 1.5$) at the polarizing angle, 10 % of light having vibrations in the plane of incidence is transmitte whereas about 85 % of light having vibrations perpendicular to the plan of incidence is transmitted and 15 % is reflected.
- If many transparent plates are kept parallel to each other, at each surfac 15 % of the light having vibrations perpendicular to the plane o incidence will be reflected.
- If sufficient number of plates are used, the transmitted light will main consist of the vibrations parallel to plane of incidence as shown i Fig. 5.3.
- Thus plane polarized light is obtained by refraction using a pile of plate

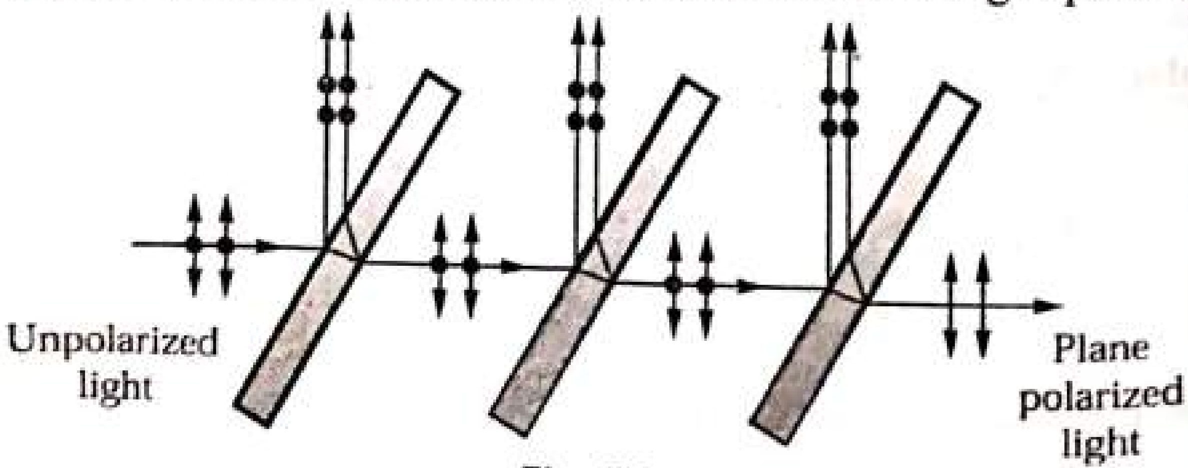


Fig. 5.3

5.5 : Polarization by Double Refraction

Q.12 Explain the term double refraction.

Ans. :

- When objects are seen through crystals like calcite, quartz or tourmaline, two images of an object are observed for certain orientations of these crystals. This phenomenon is called double refraction or birefringence.
- On rotating the crystal, it is observed that one of the images remains stationary and is known as ordinary image.
- The other image rotates with the crystal and is known as the extraordinary image.
- The ordinary and the extraordinary waves forming these images are observed to be plane polarized with mutually perpendicular planes of vibration.

Q.13 Define plane of polarization and plane of vibration. Explain the phenomenon of double refraction in calcite. ☞ [Dec.-09]

Ans. :

- Calcite is a crystal of calcium carbonate ($CaCO_3$).

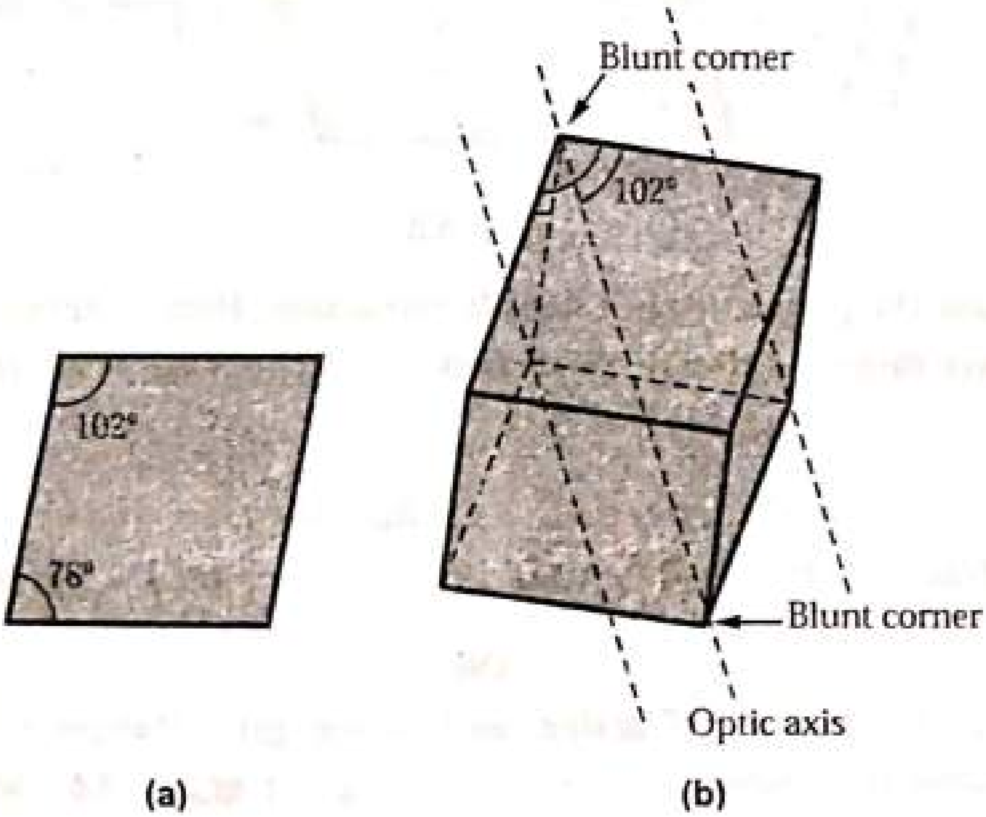


Fig. 5.4

- It has rhombohedral structure in which each face of the crystal is a parallelogram as shown in Fig. 5.4 (a) with an acute angle of 78° and obtuse angle of 102°.
- There are two diagonally opposite corners in the crystal where all the three angles are obtuse as shown in Fig. 5.4 (b) which are called blunt corners.
- At all other corners, there are two acute angles and one obtuse angle.

Optic axis : An imaginary line inside the crystal from one of the blunt corners making equal angles with all the three edges or any other line parallel to it is defined as the optic axis. If light is incident parallel or perpendicular to optic axis, double refraction is not observed.

Principal plane : A plane perpendicular to that face of the crystal on which light is incident and which contains the optic axis is called the principal plane. The principal plane does not intersect the optic axis. All principal planes are parallelograms with acute angle of 71° and obtuse angle of 109°. Vibrations of the ordinary ray (O-ray) are perpendicular to principal plane while those of the extraordinary ray (E-ray) are parallel to principal plane as shown in Fig. 5.5.

Fig. 5.5

Q.14 State the phenomena of double refraction. Hence explain Huygen's wave theory of double refraction. ☞ **[Dec - 12, Marks 6]**

OR

Define double refraction. Explain Huygen's theory of double refracting crystal with diagram. ☞ **[May - 13, Marks 6]**

OR

Explain double refraction and hence give Huygen's theory of double refraction. ☞ **[May - 14, Marks 6]**

Ans. : Refer Q.11 for double refraction.

Huygen's wave theory : Huygen explained the phenomenon of double refraction based on the following assumptions.

i) Every point in a double refracting medium is a source of two types of wavefronts :

a) Ordinary wavefront which is spherical as ordinary waves travel with same velocity in all directions. The refractive index of the crystal is same for these waves in all directions.

b) Extraordinary wavefront which is ellipsoidal as these waves travel with different velocity in different directions. The refractive index of the crystal for these waves is different in different directions.

ii) The *O*-waves and *E*-waves travel with same velocity along the optic axis. Hence the two wavefronts meet at the optic axis.

iii) In some crystals, the velocity (v_0) of the ordinary waves is greater than the velocity (v_e) of the extraordinary waves in all directions except along the optic axis. Such crystals are known as positive crystals.

As $v_0 > v_e$ and $\mu = \frac{c}{v}$,

$\mu_0 < \mu_e$ in all directions except the optic axis.

Quartz and ice are positive crystals. The wavefronts are shown in Fig. 5.6 (a). The spherical *O*-wavefront lies outside the ellipsoidal E-wavefront.

iv) In some crystals like calcite, $v_0 < v_e$ in all directions except along optic axis. For these crystals, $\mu_0 > \mu_e$. These crystals are known as negative crystals. The wavefronts are shown in Fig. 5.6 (b). The *O*-wavefront, which is spherical, lies inside the *E*-wavefront which is ellipsoidal.

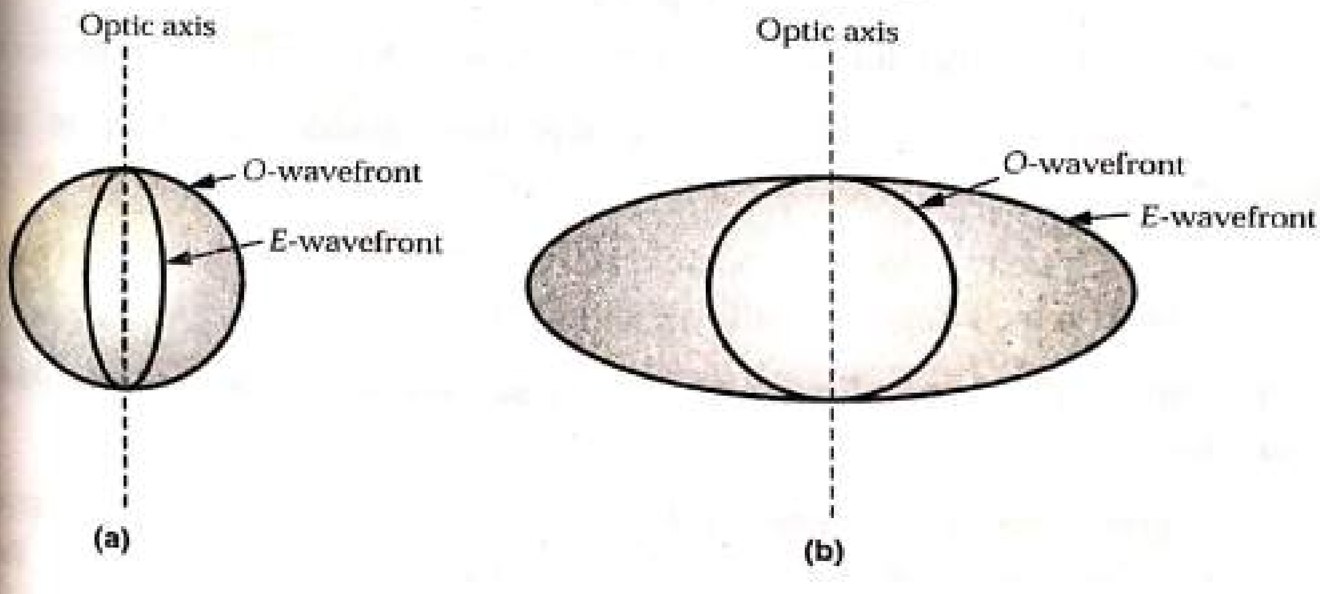


Fig. 5.6

Q.15 Explain the propogation of light through a quartz crystal plate for normal incidence. When

i) Optic axis is parallel to the crystal surface and lying in the plane of incidence.

ii) Optic axis is perpendicular to the crystal surface and lying in the plane of incidence.

iii) Optic axis is inclined to the crystal surface and lying in the plane of incidence. ☞ **[Dec.-13, Marks 6]**

Ans. : As quartz is a positive crystal, $V_o < V_e$ and the *E*-wavefront lies inside the *O*-wavefront. The two wavefronts coincide along the optic axis. We consider light to be incident normally on the crystal surface.

i) Optic axis parallel to crystal surface and lying in the plane of incidence :

The wavefronts are as shown in Fig. 5.7.

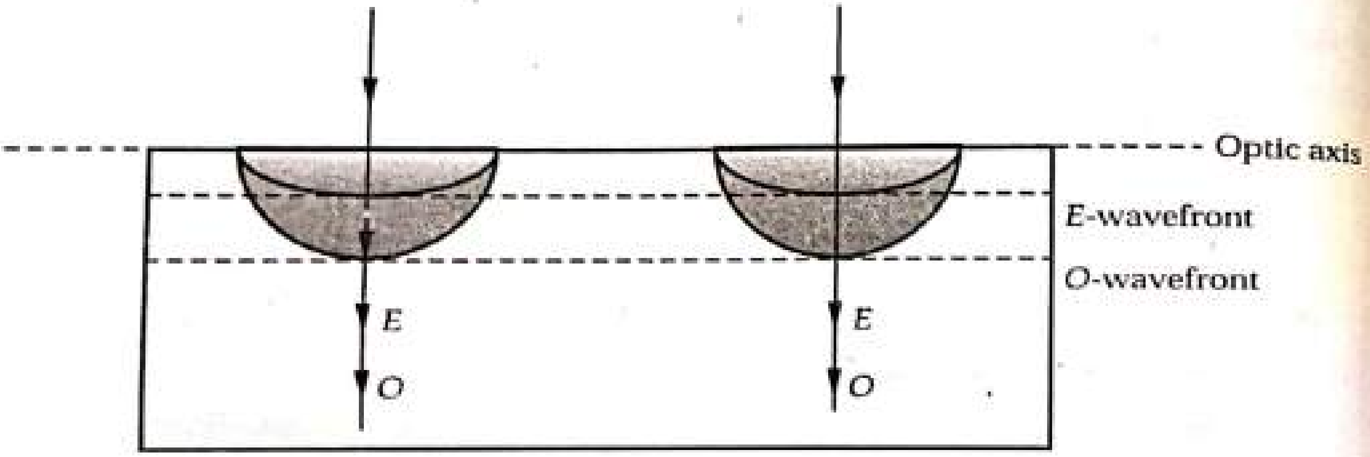


Fig. 5.7

The *O* and *E* waves travel in the same direction with different velocities.

As the waves travel in the same direction, double refraction is not observed.

Due to different velocities, some path difference is introduced between the two waves which depends on thickness of the crystal.

ii) Optic axis perpendicular to crystal surface and lying in the plane of incidence :

The wavefronts are as shown in Fig. 5.8. The *O* and *E* wavefronts travel in the same direction with the same velocity. Hence double refraction is not observed.

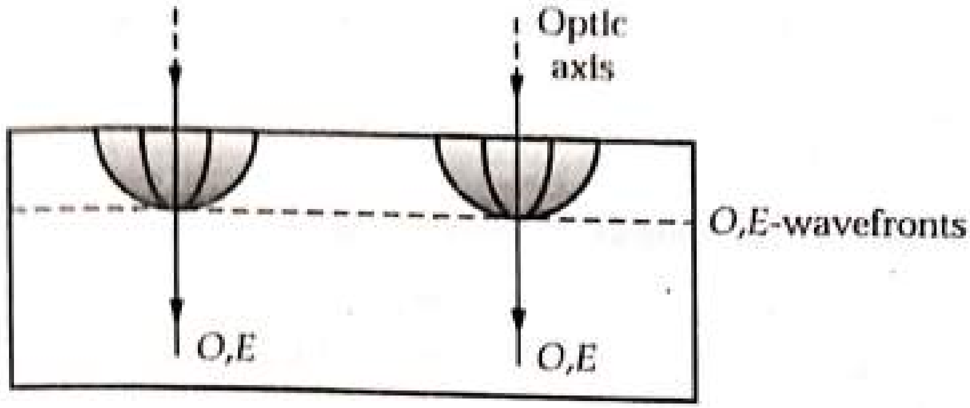


Fig. 5.8

iii) Optic axis inclined to the crystal surface and lying in the plane of incidence :

The wavefronts are as shown in Fig. 5.9. The O and E waves travel in different directions with different velocities. Hence double refraction is observed.

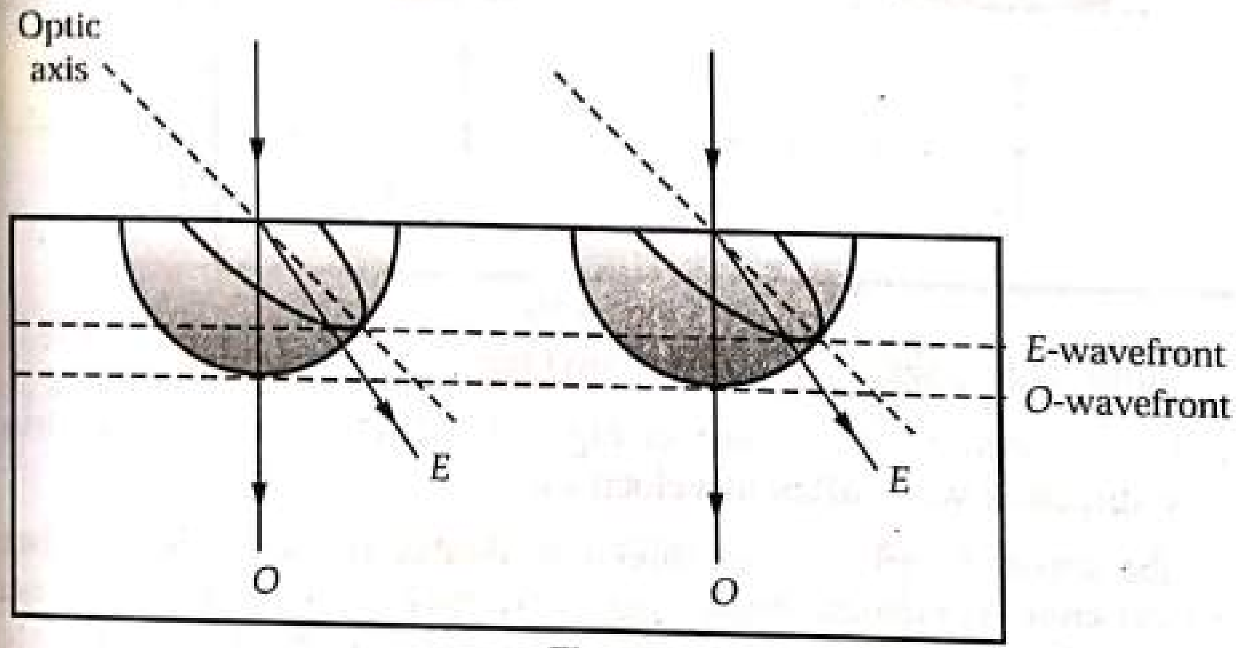


Fig. 5.9

Q.16 Explain the propogation of light through a calcite crystal plate for normal incidence. When

i) Optic axis is parallel to the crystal surface and lying in the plane of incidence.

ii) Optic axis is perpendicular to the crystal surface and lying in the plane of incidence.

iii) Optic axis is inclined to the crystal surface and lying in the plane of incidence.

Ans. : As calcite is a negative crystal, $v_0 < v_e$ and the ellipsoidal E-wavefront lies outside the spherical O-wavefront. The two wavefronts coincide along the optic axis. We consider unpolarized parallel beam of light to be incident normally on the crystal surface for the three cases discussed below. In all cases the optic axis is considered to be in the plane of incidence.

i) Optic axis perpendicular to crystal surface

The wavefronts are as shown in Fig. 5.10. The *O* and *E*-wavefronts travel in the same direction with the same velocity. Hence double refraction is not observed.

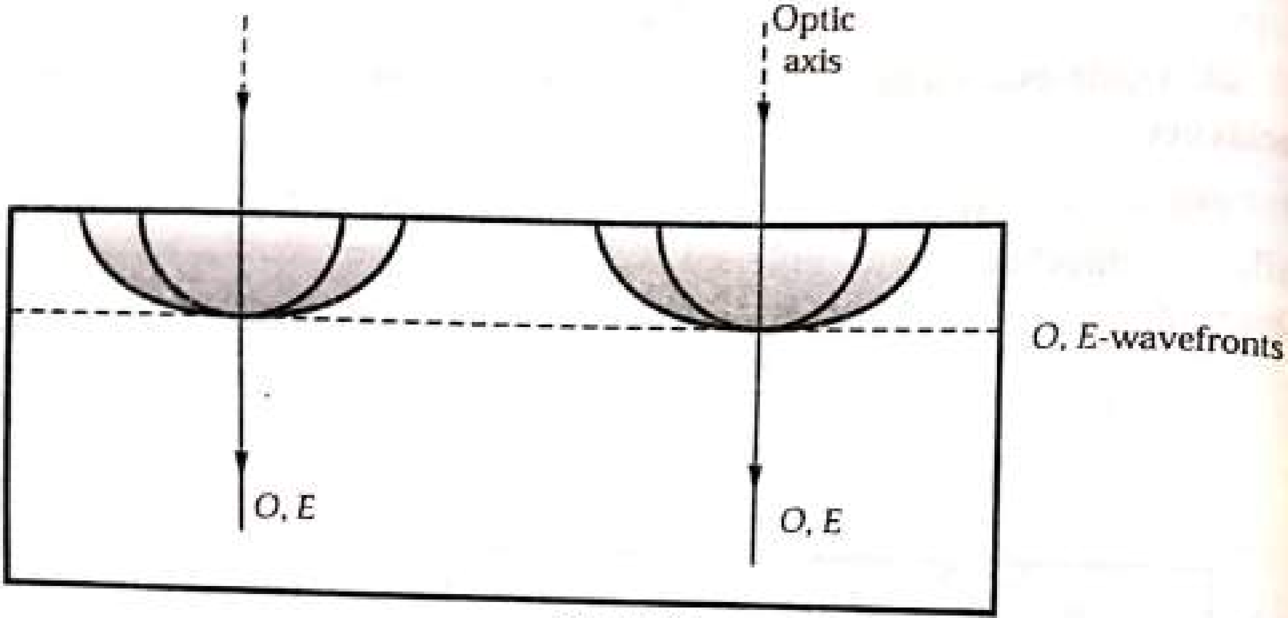


Fig. 5.10

ii) Optic axis parallel to crystal surface

The wavefronts are as shown in Fig. 5.11. The *O* and *E*-waves travel in the same direction with different velocities.

As the waves travel in same direction, double refraction is not observed. Due to different velocities, some path difference is introduced between the two waves which depends on thickness of the crystal. Such crystal plates are called retardation plates.

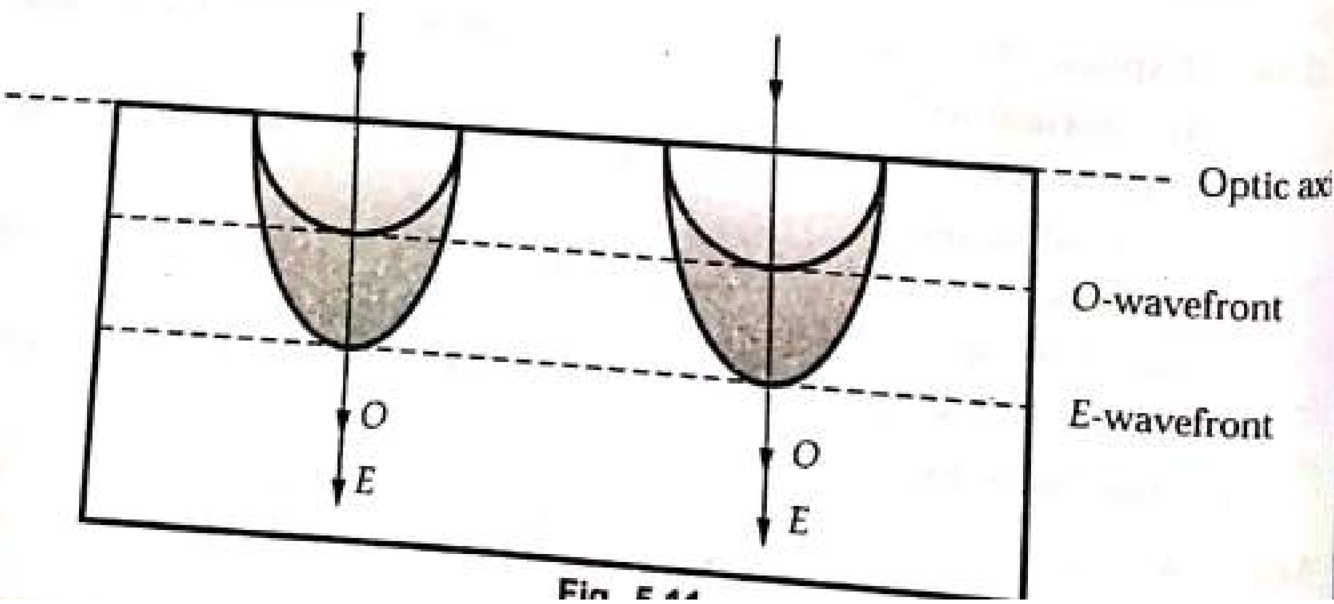


Fig. 5.11

iii) Optic axis inclined to the crystal surface

The wavefronts are as shown in Fig. 5.12. The O and E-waves travel in different directions with different velocities. Hence double refraction is observed.

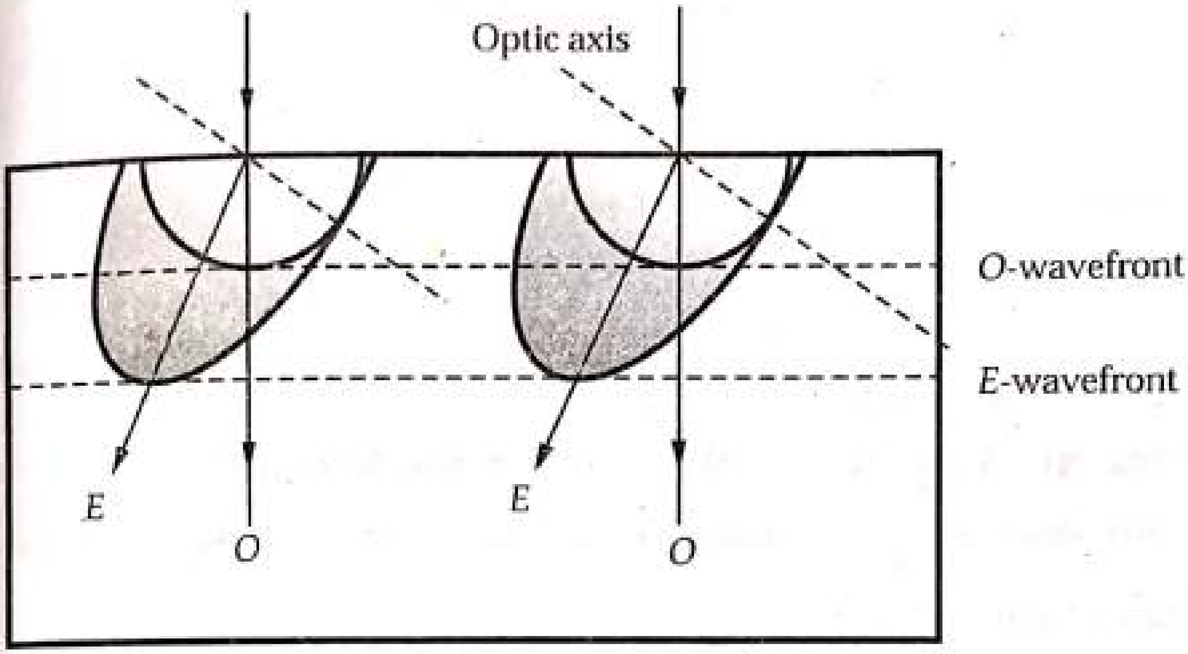


Fig. 5.12

Q.17 Explain propagation of light in a doubly refracting crystal when the optic axis is parallel to the crystal surface, with the help of neat diagram. ☞ **[Dec - 14, Marks 3]**

Ans. : Refer Q.14.

5.6 : Retardation Plates

Q.18 What are retardation plates ? What are their types ? Obtain expression for their thickness. ☞ **[May-12, Marks 6]**

Ans. :

- Retardation plates are double refracting crystals cut in such a way that the optic axis becomes parallel to the surface.
- When light is incident normally on such crystals, some path difference is introduced between O and E-waves which depends on the thickness of the crystal.
- If μ_0 and μ_e are the refractive indices for O and E-waves respectively and 't' is the thickness of retardation plates then the path difference introduced by the plate is

$$\Delta = \mu_e t - \mu_0 t = (\mu_e - \mu_0)t \quad \text{for positive crystals.}$$

and $$\Delta = \mu_0 t - \mu_e t = (\mu_0 - \mu_e)t \quad \text{for negative crystals.}$$

- The phase difference is

$$\phi = \frac{2\pi}{\lambda}(\mu_e - \mu_0)t \quad \text{for positive crystal.}$$

and $$\phi = \frac{2\pi}{\lambda}(\mu_0 - \mu_e)t \quad \text{for negative crystal.}$$

- Depending upon the amount of path difference introduced between O a *E*-waves, there are two types of retardation plates.

  i) Quarter wave plate and

  ii) Half wave plate.

Quarter Wave Plate (QWP) : It is a retardation plate which produc path difference of $\frac{\lambda}{4}$ i.e., quarter of a wavelength or phase difference of between O and E-waves.

- For positive crystal,

$$(\mu_e - \mu_o)t = \frac{\lambda}{4}$$

$$\therefore \quad t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

- For negative crystals,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Half Wave Plate (HWP) : It is a retardation plate which produces pa difference of $\frac{\lambda}{2}$. i.e., half of a wavelength, or phase difference of π betwe O and E-waves.

- For positive crystal,

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

For negative crystals,

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

Important Formulae

The phase difference $\phi = \frac{2\pi}{\lambda}(\mu_e - \mu_0)t$ for positive crystal.

$\phi = \frac{2\pi}{\lambda}(\mu_0 - \mu_e)t$ for negative crystal.

Thickness of QWP $t = \frac{\lambda}{4(\mu_e - \mu_o)}$ for positive crystal.

$t = \frac{\lambda}{4(\mu_o - \mu_e)}$ for negative crystal.

Thickness of HWP $t = \frac{\lambda}{2(\mu_e - \mu_o)}$ for positive crystal.

$t = \frac{\lambda}{2(\mu_o - \mu_e)}$ for negative crystal.

SOLVED EXAMPLES

Q.19 Calculate the thickness of :
i) Quarter wave plate ii) Half wave plate.
Given $\mu_e = 1.553$, $\mu_0 = 1.544$, $\lambda = 5000$ A° ☞ **[May-09]**

Ans. : Given data : $\mu_e = 1.553$, $\mu_0 = 1.544$, $\lambda = 5000$ A°

i) For QWP

Formula : $$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

$$\therefore \quad t = \frac{5000 \times 10^{-10}}{4\,(1.553 - 1.544)}$$

$$\therefore \quad \boxed{t = 1.39 \times 10^{-5} \text{ m}}$$

ii) For HWP

Formula : $t = \dfrac{\lambda}{2(\mu_e - \mu_0)}$

$\therefore \quad t = \dfrac{5000 \times 10^{-10}}{2(1.553 - 1.544)}$

$\therefore \quad \boxed{t = 2.78 \times 10^{-5} \text{ m}}$

Q.20 Calculate thickness of a mica plate required to make a quarter wave plate and a half wave plate for light of wavelength 5890 A°. (Given : $\mu_0 = 1.586$ and $\mu_e = 1.592$) ☞ [Dec.-10]

Ans. : Given data : $\mu_0 = 1.586$, $\mu_e = 1.592$, $\lambda = 5890$ A°

i) For QWP

Formula : $t = \dfrac{\lambda}{4(\mu_e - \mu_0)}$

$\therefore \quad t = \dfrac{5890 \times 10^{-8}}{4(1.592 - 1.586)}$

$\therefore \quad \boxed{t = 2454 \times 10^{-3} \text{ cm}}$

ii) For HWP

Formula : $t = \dfrac{\lambda}{2(\mu_e - \mu_0)} = \dfrac{5890 \times 10^{-8}}{2(1.592 - 1.586)}$

$\therefore \quad \boxed{t = 4.908 \times 10^{-3} \text{ cm}}$

Q.21 Calculate the thickness of doubly refracting crystal required to introduce a phase difference of π radians between O and E rays. Given that λ = 6000 A°, $\mu_0 = 1.55$, $\mu_e = 1.54$. ☞ [May-12]

Ans. : Given data : λ = 6000 A°, $\mu_0 = 1.55$, $\mu_e = 1.54$

Formula : $t = \dfrac{\lambda}{2(\mu_0 - \mu_e)} = \dfrac{6000 \times 10^{-8}}{2(1.55 - 1.54)}$

$\therefore \quad \boxed{t = 3 \times 10^{-3} \text{ cm}}$

5.7 : Dichroism

Q.22 Explain the term Dichroism. What are polaroids and how are they produced ? ☞ **[Dec.-11, Marks 6]**

Ans. :

- Certain double refracting crystals have the property of absorbing either the *O* or the *E*-wave to a larger extent than the other. This property is known as dichroism.
- For example, tourmaline absorbs the *O*-waves to a larger extent than the *E*-waves.
- By cutting the crystal to a sufficient thickness, the *O*-waves can be completely eliminated. The transmitted light then will be only the *E*-wave which is plane polarized.
- Thus plane polarized light can be obtained using selective absorption.
- Polaroids are polarizing sheets used to produce plane polarized light.
- They are constructed from dichroic crystals like iodosulphate of quinine.
- A paste of these tiny crystals is prepared in nitro cellulose and then squeezed through a set of fine, parallel slits.
- A thin sheet is obtained in which crystals are arranged with their axis parallel to each other. This sheet is then sandwiched between two glass or plastic plates.
- Polaroids are also constructed by stretching a sheet of polyvenyl alcohol which orients the molecules in the direction of the stress and makes them double refracting.
- They are then saturated with iodine which makes them dichroic. These polaroids are known as *H*-polaroids.
- When stretched polyvenyl films are heated with *HCl*, the film darkens and shows dichroic properties. These are known as *K*-polaroids.

5.8 : Optical Activity

Q.23 Explain the term : Optical activity. ☞ [May-09, Marks 3]

Ans. :

- When plane polarized light is passed through certain substances lik sugar crystals, sugar solutions, quartz crystals etc. the plane of vibratio is rotated.
- The amount of rotation depends upon the distance travelled in th medium.
- This phenomenon of rotation of the plane of vibration of plane polarize light is called optical activity.
- Some crystals rotate the plane of vibration to the right (i.e., clockwise when looking against the direction of light. Such crystals are called righ handed or dextrorotatory.
- Those crystals which rotate the plane of vibration to the left (i.e anticlockwise) are called left handed or levorotatory.

Q.24 Define specific rotation and give expression for it.

Ans. : The specific rotation of a solution at a given temperature an wavelength is defined as the rotation of plane of vibration of plane polarize light produced by one decimeter (i.e., 10 cm) length of solution when th concentration of the solution is one gram per c.c. i.e.,

$$S = \frac{\theta}{lc}$$

where θ = Angle of rotation

l = Length of solution in decimeter

c = Concentration in g/c.c.

If length l is measured in cm,

$$S = \frac{10\theta}{lc}$$

Important Formula

$$S = \frac{10\theta}{lc}$$

SOLVED EXAMPLES

Q.25 **A 20 cm long tube containing 48 c.c. of sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the mass of sugar in the solution.** ☞ **[Dec.-08]**

Ans. : **Given data :** $S = 66°$, $\theta = 11°$, $l = 20$ cm

Formula : $$S = \frac{10\theta}{lc}$$

$$\therefore \quad c = \frac{10\theta}{lS}$$

$$= \frac{10 \times 11}{20 \times 66}$$

$$\therefore \quad c = \frac{1}{12} \text{ g/c.c}$$

i.e. 1 c.c of solution contains $\frac{1}{12}$ gm of sugar

$\therefore$ 48 c.c of solution contains $48 \times \frac{1}{12} =$ **4 gram**

Q.26 **Calculate the specific rotation of sugar solution using the following data : length of the tube containing solution = 25 cm, volume of the solution = 100 c.c., amount of sugar in the solution = 10 gm and angle of rotation = 16° 15′.**

Ans. : **Given data :** $\theta = 16° \, 15' = 16.25°$

$$l = 25 \text{ cm}, \quad c = \frac{10}{100} = \frac{1}{10} \text{ gm/c.c.}$$

Formula : $$S = \frac{10\theta}{lc}$$

$$S = \frac{10 \times 16.25}{25 \times \left(\frac{1}{10}\right)}$$

$$\therefore \quad \boxed{S = 65°}$$

Q.27 **Calculate the specific rotation of the sugar solution of 4.5 % concentration, if the plane of polarization is rotated through 6.8° in passing through a length of 1.8 decimeter of the solution.**

☞ [Dec.-13, Marks 3]

Ans. : Given data : $\theta = 6.8°$, $l = 1.8$ decimeter $= 18$ cm

$$c = 4.5\ \% = \frac{4.5}{100}\ \text{gm/cm}^3$$

Formula : $$S = \frac{10\theta}{lc} = \frac{10 \times 6.8}{18 \times \frac{4.5}{100}} = 83.95°$$

FORMULAE AT A GLANCE

Law of Malus : $$\frac{I}{I_0} = \cos^2\theta$$

Polarization by reflection : $i_P + r_P = 90°$

$$\mu = \tan i_P$$

The phase difference $$\phi = \frac{2\pi}{\lambda}(\mu_e - \mu_0)t$$ for positive crystal.

$$\phi = \frac{2\pi}{\lambda}(\mu_0 - \mu_e)t$$ for negative crystal.

Thickness of QWP $$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$ for positive crystal.

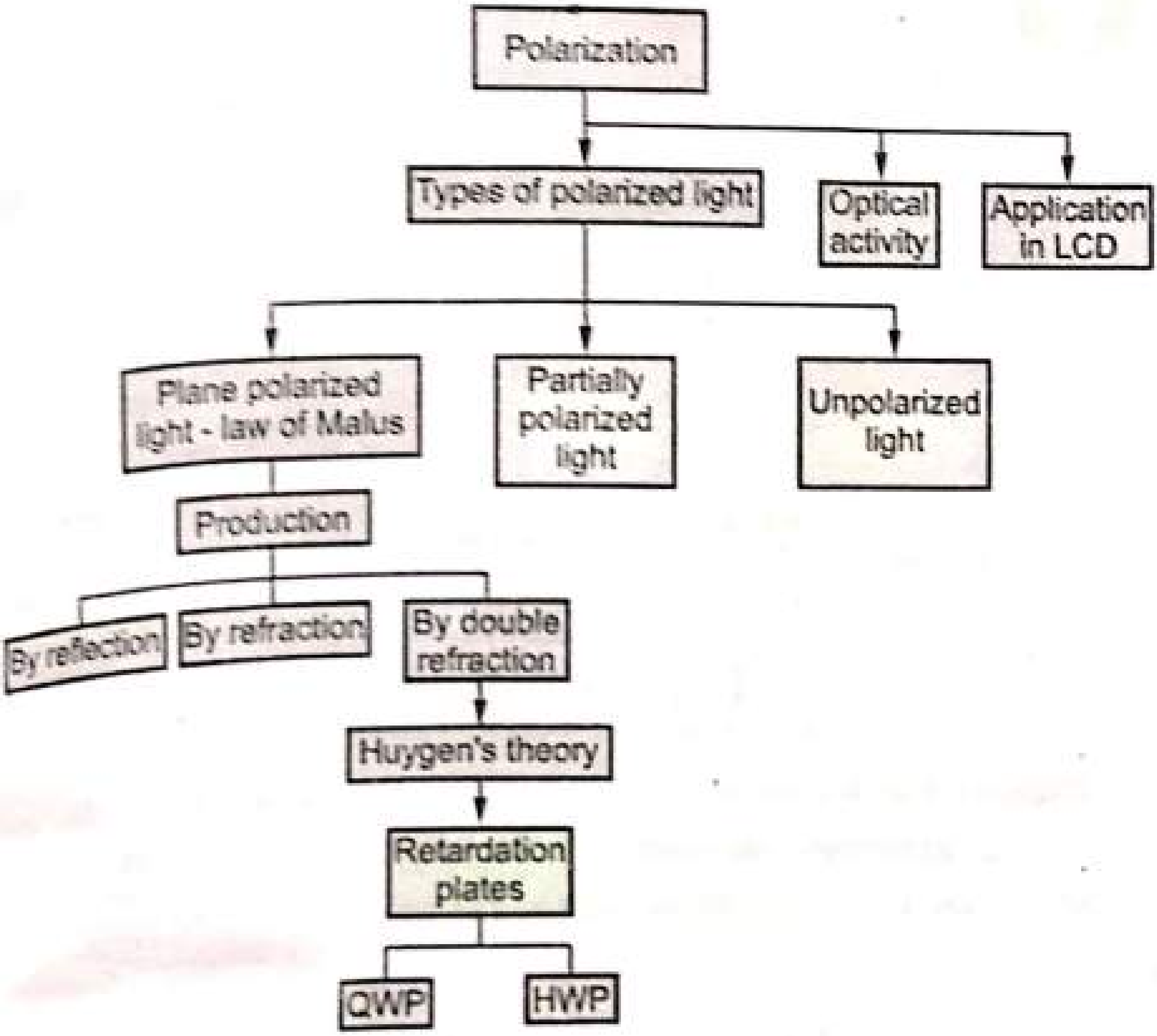
$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$ for negative crystal.

Thickness of HWP $$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$ for positive crystal.

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$ for negative crystal.

$$S = \frac{10\theta}{lc}$$

CF CONCEPT FLOWCHART



END...

Sol. : Given data : As total internal reflection takes place for light travelling within 5° of the fibre axis, the critical angle is

$$\theta_c = 90 - 5 = 85°$$

$$n_1 = 1.50$$

Formula :

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\therefore \quad \sin 85° = \frac{n_2}{1.50}$$

$$\therefore \quad \boxed{n_2 = 1.4943}$$

Q.9 The angle of acceptance of an optical fibre is 30° when kept in air. Find the angle of acceptance when it is in a medium of refractive index 1.33.

Sol. : **Given data :** $\theta_0 = 30°$, $n_0 = 1.33$

Formula : In air,

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2} = (\sin \theta_0)_{air}$$

In medium of refractive index n_0,

$$n_0 \sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad \sqrt{n_1^2 - n_2^2} = \sin 30$$

$$\therefore \quad \sqrt{n_1^2 - n_2^2} = 0.5$$

$$\therefore \quad 1.33 \sin \theta_0 = 0.5$$

$$\therefore \quad \boxed{\theta_0 = 22.08°}$$

Q.10 The numerical aperture of an optical fibre is 0.2 when surrounded by air. Determine the refractive index of its core given the refractive index of cladding as 1.59. Also find the acceptance angle when it is in a medium of refractive index 1.33.

Ans. : Formula : In air,

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2} = (\sin \theta_0)_{air}$$

In medium of refractive index n_0,

- The core diameter is typically about 20 μm to 100 μm with cladding thickness of about 25 μm.
- They are commonly used for medium distance communication.

5.3 : Attenuation

Q.13 Explain attenuation mechanisms in optic fibres.

Ans. : • The power loss of optical signals when they travel through optical fibres is known as attenuation.

- The power loss P_L in decibel (dB) is given by,

$$P_L = -10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad \text{... (1)}$$

where P_{out} is the output power and P_{in} is the input power.

- The **attenuation constant** (α) for optic fibres is defined as the power loss per unit length and is expressed in dB/km. It is given by

$$\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad \text{... (2)}$$

- where L is the length of the cable in km.
- Attenuation in optic fibres is due to absorption, scattering and radiation losses.

Absorption :

- Light can be absorbed either by the pure fibre material which is known as **intrinsic absorption** or it can be absorbed by impurities present in the core of the fibre.

Scattering :

- Light traveling through the core can get scattered by impurities or small regions with sudden change in refractive index.
- Rayleigh scattering varies as λ^{-4} and leads to significant power loss at smaller wavelengths. Rayleigh scattering is responsible for maximum losses in optic fibres.

$$\theta_0 = \sin^{-1}\left(\sqrt{1.6^2 - 1.3^2}\right)$$

$\therefore$ $$\boxed{\theta_0 = 68.866°}$$

Angle of acceptance cone $= 2\theta_0 = 2 \times 68.866$

$\therefore$ $$\boxed{2\theta_0 = 137.732°}$$

Q.6 A fibre cable has an acceptance angle of 30° and a core index of refraction of 1.4. Calculate the refractive index of the cladding.

Sol. : Given data : $n_1 = 1.4,\ \theta_0 = 30°$

Formula : $$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$\therefore$ $$\sin 30 = \sqrt{1.4^2 - n_2^2}$$

$\therefore$ $$\sin^2 30 = 1.4^2 - n_2^2$$

$\therefore$ $$n_2^2 = 1.4^2 - \sin^2 30$$

$\therefore$ $$\boxed{n_2 = 1.3077}$$

Q.7 Calculate the fractional index change for a given optical fibre if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Sol. : Given data : $n_1 = 1.563,\ n_2 = 1.498$

Formula : Fractional index change $\Delta = \dfrac{n_1 - n_2}{n_1}$

$\therefore$ $$\Delta = \frac{1.563 - 1.498}{1.563}$$

$\therefore$ $$\boxed{\Delta = 0.0416}$$

Q.8 An optical glass fibre of refractive index 1.50 is to be clad with another glass to ensure internal reflection that will contain light travelling within 5° of the fibre axis. What maximum index of refraction is allowed for the cladding ?

- The critical angle can be obtained using Snell's law.

$$n_1 \sin i = n_2 \sin r$$

- where i is the angle of incidence in the medium of refractive index n_1 and r is the angle of refraction in the medium of refractive index n_2. Here, $i = \theta_c$ and $r = 90°$.

$$\therefore \quad n_1 \sin \theta_c = n_2 \sin 90°$$

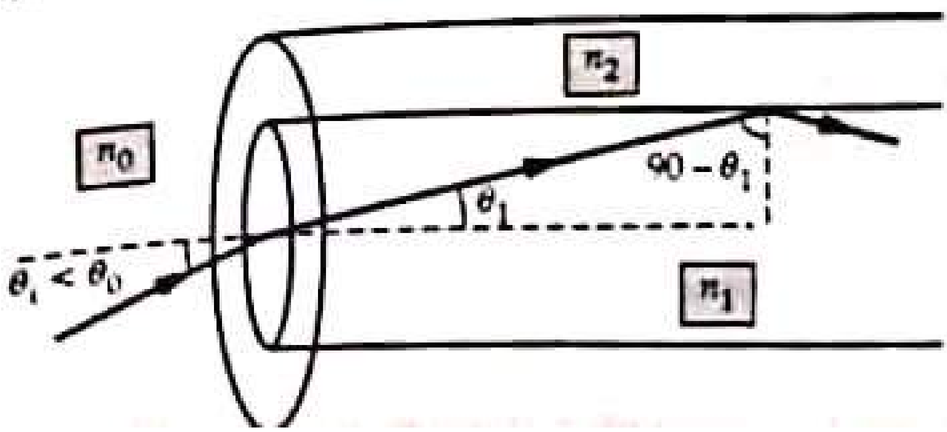
$$\therefore \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad \text{... (1)}$$

- When light is introduced into the core and is incident on the core-cladding interface, total internal reflection takes place for angles of incidence greater than the critical angle.
- Due to multiple total internal reflections, light is confined to the core. If sharp bends are avoided, there will be negligible loss of energy.

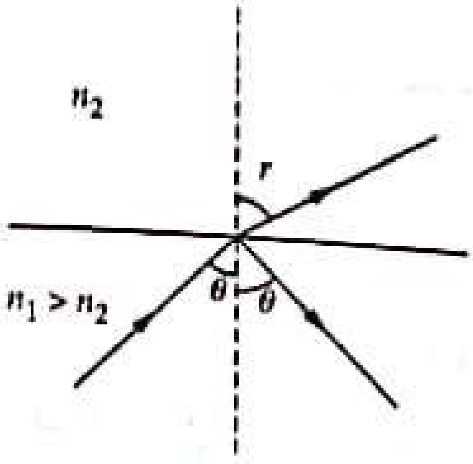
Q.3 Derive expressions for acceptance angle, numerical aperture and fractional index change.

Ans. : • If light is transmitted into the core of the optic fibre from a surrounding medium of refractive index n_0 at angle θ_i which is less than the refractive index n_1 of the core, it bends towards the normal.

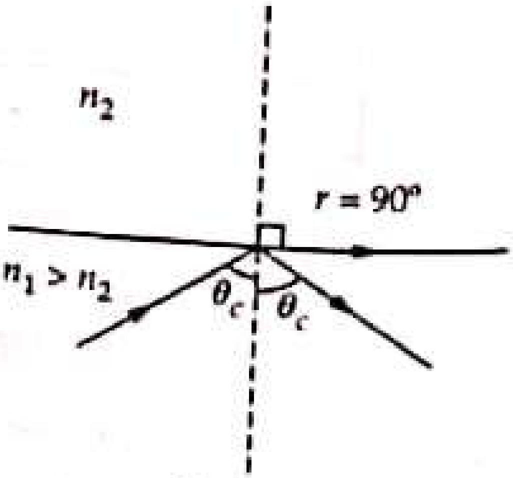
- This bending increases the angle of incidence on the core-cladding interface. For very small θ_1, the angle of incidence on the core-cladding interface is more than the critical angle $(90 - \theta_1)$ as shown in Fig. Q.3.1 (a).



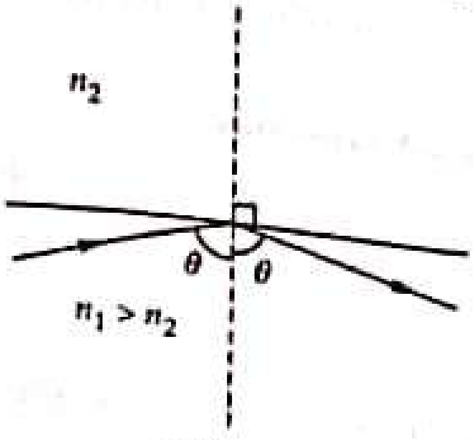
- On increasing the angle of incidence, the angle of refraction of the transmitted ray increases according to Snell's law.
- When angle of incidence becomes equal to the critical angle, the angle of refraction becomes 90° as shown in Fig. Q.2.1 (b). i.e., the transmitted ray does not emerge out in the medium of smaller refractive index.
- Thus light is confined to the denser medium only. For angles of incidence greater than the critical angle, there is no transmitted light in the rarer medium as shown in Fig. Q.2.1 (c).
- This phenomenon in which light is confined only to the denser medium while travelling from denser medium to rarer when the angle of incidence is greater than the critical angle is known as **total internal reflection.**



(a) $\theta < \theta_c$



(b) $\theta = \theta_c$



(c) $\theta > \theta_c$

Fig. Q.2.1

Step index fibres :

- Such fibres have a core of homogeneous transparent material of refractive index n_1 and cladding of another homogeneous transparent material of refractive index n_2 ($< n_1$).

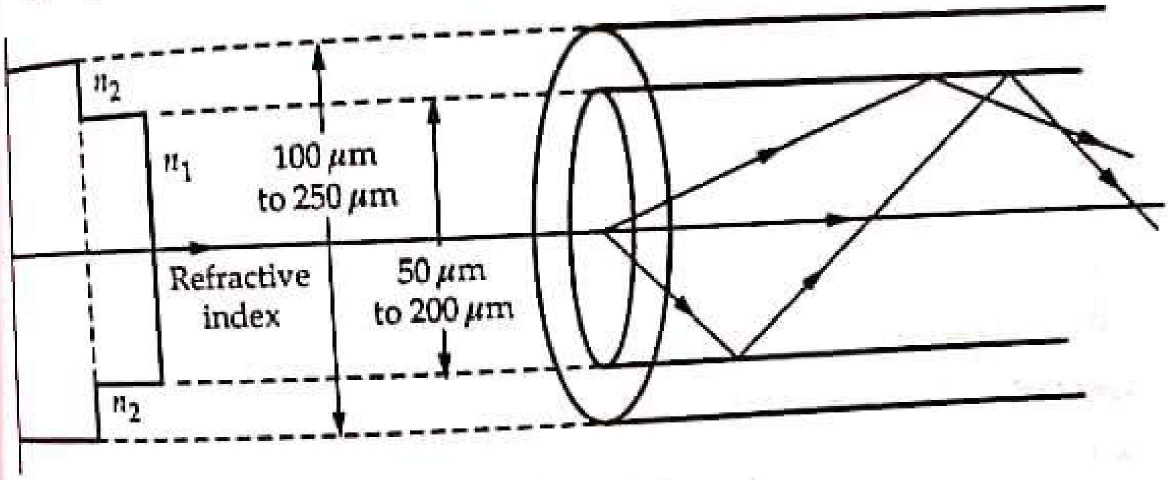


Fig. Q.12.1

- There is an abrupt change in refractive index at the core-cladding interface due to which they are known as **step index fibres.**
- If the core diameter of the step index fibre is of the order of 50 mm to 200 μm, it can transmit a large number of modes as shown in Fig. Q.12.1. The cladding thickness is typically 20 to 25 μm.
- These fibres are known as **multimode step index fibres.**
- These fibres are cheaper compared to other fibres but not suitable for long distance communication as intermodal dispersion causes significant signal distortion.
- They are used mainly for short distance communication for which signal distortion due to intermodal dispersion is not significant.
- Step index fibres which have core diameters in the range of 2 μm to 10 μm are known as **single mode step index fibres.**
- These fibres essentially transmit only the axial mode due to very small core diameter as shown in Fig. Q.12.2.

Radiation loss :

- These losses are due to changes in angle of incidence on the core-cladding interface due to bending of cable (macroscopic bends) or due to microscopic bends at the core-cladding interface.
- Light is partly lost by refraction due to microscopic bends (as shown in Fig. Q.13.1) as angle of incidence becomes smaller than the critical angle.
- These losses can be minimized by bending the cable in larger radius of curvature.
- Similarly, light is lost due to decrease in angle of incidence when the core-cladding interface has microscopic irregularities as shown in Fig. Q.13.2.

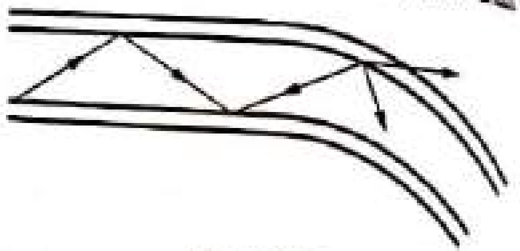


Fig. Q.13.1

Fig. Q.13.2

5.4 : Optic Fibre Communication System

Q.14 Describe the optical fibre communication system with a neat block diagram. State its advantages.

Ans. : • A general block diagram of a communication system based on optic fibres is shown in Fig. Q.14.1.

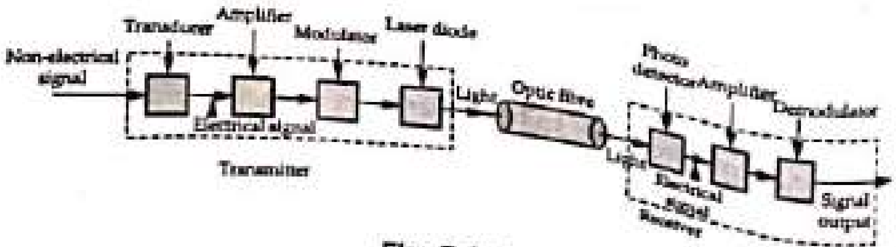


Fig. Q.14.1

- A non-electrical signal like sound, in telecommunication, is converted into electrical signal by a transducer.
- The signal is amplified, modulated and then fed to a laser diode or light emitting diode which converts the electrical signal into a light signal.
- Thus the transmitter transmits a light signal. The light signal is transmitted through the optic fibre.
- In the receiver, the light signal is converted into corresponding electrical signal by a photo detector which is amplified by an amplifier.
- The amplified electrical signal is then demodulated and in telecommunication is fed to a speaker which converts it into sound signal.
- In data transfer systems, which transfer digital data from one computer to another, there is no transducer in the transmitter.
- The input of transmitter and the output of receiver are both electrical signals.

Advantages of optical fibres

1) As the frequency of light is large ($\sim 10^{14}$ Hz), large bandwidth is available. Hence more channels become available due to which data transfer speeds are larger.
2) As optic fibres are made up of dielectric materials, the communication system is not affected by electromagnetic disturbances, specially during thunderstorms.
3) There is no cross talk as signals traveling in one cable do not give rise to induced signals in a nearby cable. Also there is more security as the signals are confined to the core and cannot be tapped.
4) Quality of communication is better as the transmission losses are low.

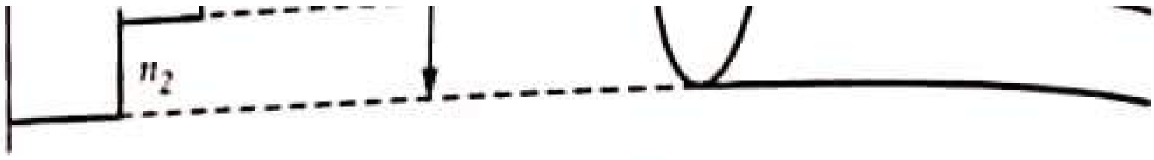
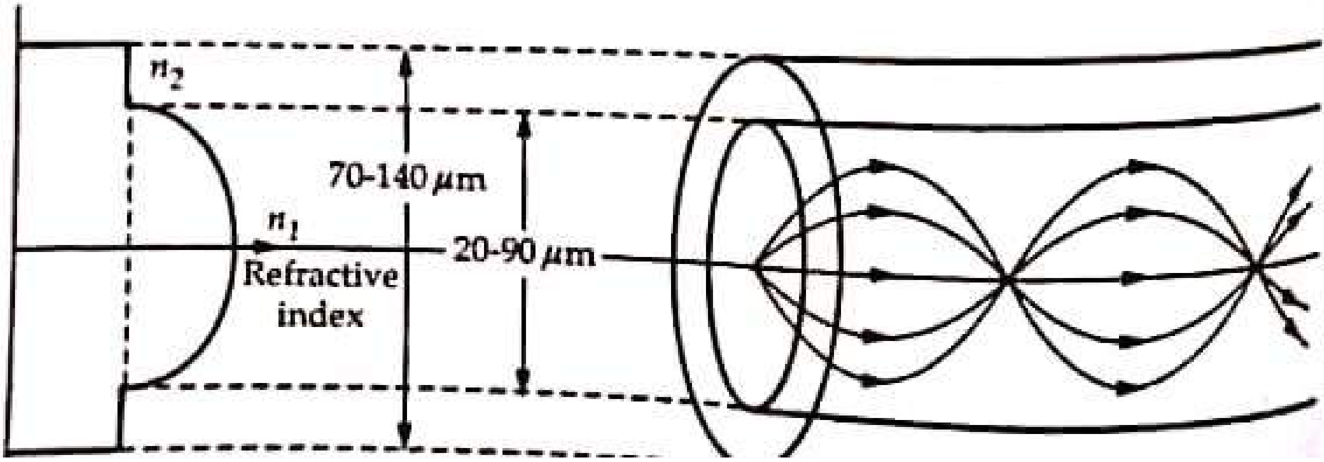


Fig. Q.12.2

- The thickness of the core is typically 25 μm to 30 μm. These fibres are used for long distance communication as intermodal dispersion is eliminated.

Graded index (GRIN) multimode fibres :

- In graded index fibres, the refractive index is maximum at the axis of the core, decreases gradually upto the cladding and then remains constant throughout the cladding as shown in Fig. Q.12.3



Numerical aperture : N.A. $= \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

Numerical aperture (Fibre in air) :

$$\text{N.A.} = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

Fractional index change : $\Delta = \frac{n_1 - n_2}{n_1}$

Q.4 Calculate the numerical aperture and acceptance angle of an optical fibre having $n_1 = 1.49$ and $n_2 = 1.44$.

Ans. :

Formula : $\text{N.A.} = \sqrt{n_1^2 - n_2^2}$

Angle of acceptance $\theta_0 = \sin^{-1}(\text{N.A})$

Given data : $n_1 = 1.49$,

$n_2 = 1.44$

$$\text{N.A} = \sqrt{1.49^2 - 1.44^2}$$

$$\therefore \quad \boxed{\text{N.A} = 0.38275}$$

Angle of acceptance $\theta_0 = \sin^{-1}(\text{N.A}) = \sin^{-1} 0.38275$

$$\therefore \quad \boxed{\theta_0 = 22.5^\circ}$$

Q.5 In an optical fibre, the core material has refractive index 1.6 and refractive index of clad material is 1.3. What is the value of critical angle ? Also calculate the value of angle of acceptance cone.

Ans.. : Given data : $n_2 = 1.3$, $n_1 = 1.6$

Formula : Critical angle $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Acceptance angle $\theta_0 = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

Angle of acceptance cone $= 2\theta_0$

$$\therefore \quad \theta_c = \sin^{-1}\left(\frac{1.3}{1.6}\right) = 54.34^\circ$$

- Light is transmitted through the fibre when

$$\theta_i < \theta_0$$

i.e.,
$$\sin\theta_i < \sin\theta_0$$

$\therefore$
$$\sin\theta_i < \sqrt{n_1^2 - n_2^2}$$

i.e.
$$\sin\theta_i < \text{N.A.}$$

- Light will be transmitted through the fibre with multiple total internal reflections when the above condition is satisfied.

Fractional index change :

- The fractional index change 'Δ' is defined as the ratio of the difference in refractive indices of core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1} \qquad \text{... (5)}$$

- From equation (4), the numerical aperture in air is,

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2}$$

$\therefore$
$$\text{N.A.} = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

- For small difference between n_1 and n_2,

$$n_1 + n_2 \approx 2\,n_1$$

and from equation (5),

$$n_1 - n_2 = n_1\Delta$$

$\therefore$
$$\text{N.A.} = \sqrt{(2n_1)(n_1\Delta)}$$

$\therefore$
$$\text{N.A.} = n_1\sqrt{2\Delta} \qquad \text{... (6)}$$

- Thus the numerical aperture can be increased by increasing the fractional index change.

- **Acceptance angle** is the maximum angle of incidence on the core for which total internal reflection takes place inside the core.
- The light entering the core in a cone of semivertical angle θ_0 is transmitted in the core through total internal reflections. This cone is known as the **acceptance cone.**
- Using Snell's law at A,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\therefore \quad \sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \qquad \text{... (1)}$$

- At B, the angle of refraction is 90 °.

$$\therefore \quad n_1 \sin(90-\theta_1) = n_2 \sin 90°$$

$$\therefore \quad \cos\theta_1 = \frac{n_2}{n_1}$$

As

$$\sin\theta_1 = \sqrt{1-\cos^2\theta_1}$$

$$\therefore \quad \sin\theta_1 = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\therefore \quad \sin\theta_1 = \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

- Substituting in equation (1),

$$\sin\theta_0 = \frac{n_1}{n_0}\times\frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad \text{... (2)}$$

- As numerical aperture is $\sin\theta_0$,

$$\text{N.A.} = \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad \text{... (3)}$$

- If the fibre is in air, $n_0 = 1$.

$$\text{N.A.} = \sin\theta_0 = \sqrt{n_1^2 - n_2^2} \qquad \text{... (4)}$$

Then.

5) As the fibres are very thin and can accommodate more channels, they occupy smaller space compared to copper cables for the same number of communication channels.

6) There is good electrical isolation between transmitter and receiver.

7) They are not affected by corrosion and atmospheric conditions. Hence optic fibres have longer life than copper cables.

END...

$$n_0 \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

Given data : N.A = 0.2, n_2 = 1.59, n_0 = 1.33

In air,

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2} = 0.2, \qquad n_2 = 1.59$$

$$\therefore \quad 0.2 = \sqrt{n_1^2 - 1.59^2}$$

$$\therefore \quad \boxed{n_1 = 1.6}$$

In medium of refractive index n_0,

$$n_0 \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad 1.33 \sin\theta_0 = 0.2$$

$$\therefore \quad \boxed{\theta_0 = 8.65°}$$

Q.11 Calculate the numerical aperture and hence the acceptance angle for an optical fibre whose core and cladding has refractive index of 1.59 and 1.40 respectively. [SPPU : Dec.-12, Marks 2]

Ans. :

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2}$$

$$n_1 = 1.59,$$

$$n_2 = 1.40$$

$$\text{N.A} = \sqrt{1.59^2 - 1.40^2}$$

$$\therefore \quad \boxed{\text{N.A} = 0.7537}$$

Angle of acceptance $\theta_0 = \sin^{-1}(\text{N.A}) = \sin^{-1} 0.38275$

$$\therefore \quad \boxed{\theta_0 = 48.91°}$$

5 Optic Fibre

5.1 : Propagation Mechanism in Optic Fibres

Q.1 What is an optic fibre ? How does light propagate through it ?

Ans. : • Optic fibres consist of a central core of a transparent material surrounded by a cladding of another transparent material as shown in Fig. Q.1.1.

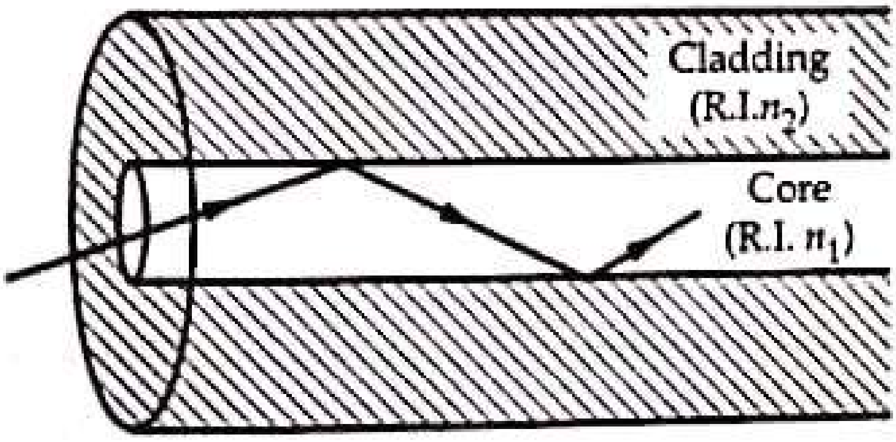


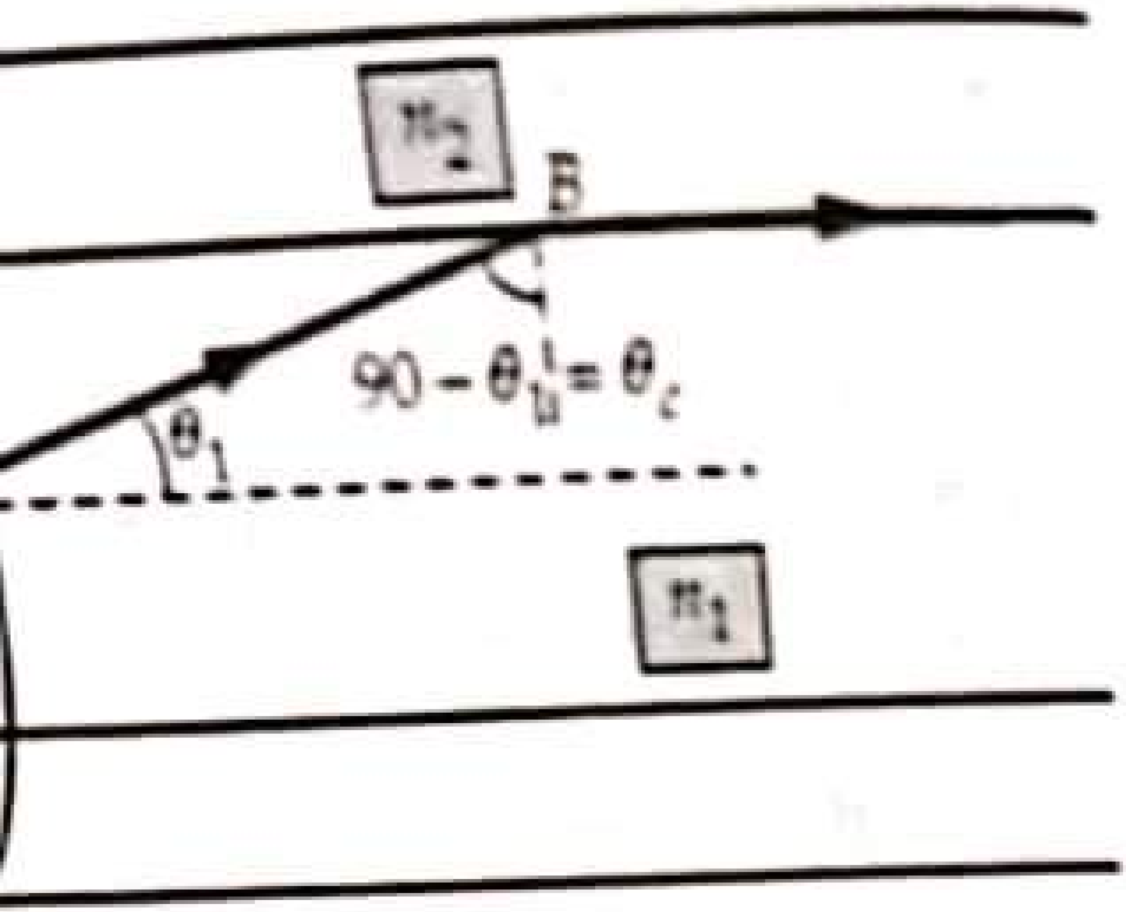
Fig. Q.1.1

• The refractive index of the core is greater than that of the cladding. Light introduced into the core suffers multiple total internal reflections due to which it travels long distances without significant loss of energy.

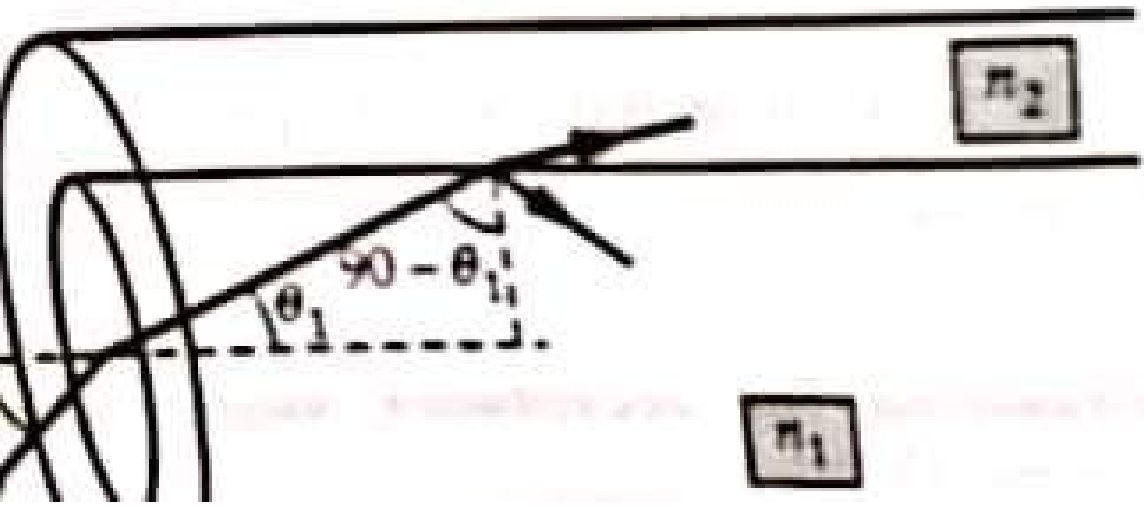
Q.2 Explain total internal reflection.

Ans. : • When light travels from denser medium to rarer medium, it is partially reflected and partially transmitted for angles of incidence 'θ' less than a particular angle 'θ_c' known as the critical angle.

• The transmitted ray bends away from the normal as shown in Fig. Q.2.1 (a).



(b)



6 Lasers

Chapter Outline

- Questions and Answers
- Concept Flowchart

QUESTIONS & ANSWERS

6.1 : Basic Definitions

Q.1 Explain the following terms : i) Coherence ii) Metastable state iii) Spontaneous emission iv) Stimulated emission v) Population inversion vi) Pumping vi) Resonant cavity.

Ans. :

i) Coherence : Two waves are said to be coherent if the phase difference between them remains constant.

- If the phase difference between two points along any ray remains constant, the coherence is called temporal coherence.
- The distance along the ray over which the phase difference remains constant is called coherence length and the corresponding time is called coherence time.
- If the phase difference between two points in a plane normal to the ray direction remains constant the coherence is called spatial or lateral coherence.

ii) Metastable state : These are excited states of an atom with relatively larger life times of the order of 10^{-3}s. As these energy states are neither as stable as ground state nor as unstable as the other excited states, they are known as metastable states.

iii) Spontaneous emission : When an atom in its excited energy state E_2 makes a transition to the ground state E_1 on its own, without any external stimulus, it emits a photon of energy $E_2 - E_1$ as shown in Fig. 6.1. This is known as spontaneous emission of radiation.

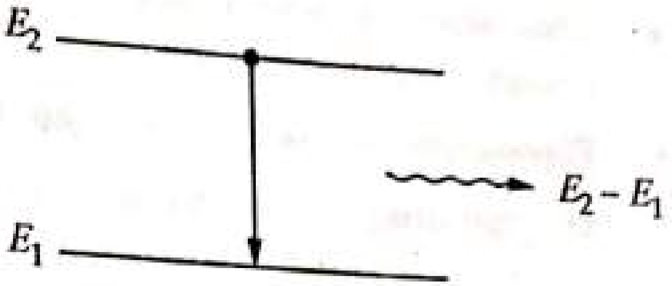


Fig. 6.1

- When a large group of atoms emit radiation spontaneously, the emitted radiation contains photons which have randomly distributed phases, directions and polarization states, even though they have the same energy and wavelength.

iv) Stimulated emission : If a photon having energy $E_2 - E_1$ interacts with an atom in the energy state E_2, the photon forces the atom to undergo transition to the ground state E_1 giving rise to another photon of the same energy $E_2 - E_1$ as shown in Fig. 6.2.

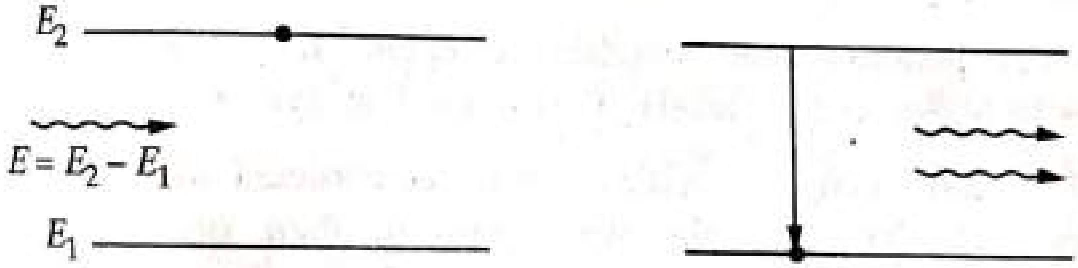


Fig. 6.2

- The emitted photon has the same phase wavelength, direction and polarization as that of the stimulating photon. Hence the light produced by stimulated emission is coherent.
- Stimulated emission of radiation is possible from atoms in metastable states. Spontaneous emission of radiation takes place from excited states with very small life times.

v) Population inversion : The number of atoms is more in the lower energy state than that in the higher energy state.

- If a photon having energy $E_2 - E_1$ is incident on such a gas, the probability of induced absorption is more than the probability of stimulated emission.
- To increase the probability of stimulated emission, the number of atoms in the higher energy state must be made greater than the number of atoms in the lower energy state.

- This state, in which there is a larger number of atoms in the higher energy state than the lower, is called population inversion.
- Population inversion ensures amplification of light.

vi) Pumping : To achieve a state of population inversion, the atoms in the lower energy state have to be 'pumped' to the higher energy state by providing energy.

- The process of raising the atoms from a lower energy state to higher, to create population inversion, is called pumping.
- Energy can be supplied to atoms in different forms.
- In optical pumping, light energy from gas discharge is used to raise atoms to higher energy states. This is used in ruby laser.
- In electrical pumping, which is suitable for gases, the gas is ionized by applying a suitable potential difference across it. The atoms in the ionized gas collide with free electrons causing transfer of energy. Other atoms and molecules can also be introduced into the gas to transfer energy through inelastic collisions, as in He – Ne laser.
- Chemical pumping uses exothermic chemical reactions to pump the atoms to higher energy levels. This is used in dye lasers.

vi) Resonant cavity : Cavities can be constructed using different types of mirrors such that the light rays return to their original location and orientation after travelling through the cavity for a certain number of times. Such cavities are known as resonant cavities.

- For example, in a cavity formed by two parallel mirrors as shown in Fig. 6.3 (a), photons travelling along the axis bounce back and forth whereas those photons that do not travel along the axis escape from the cavity after a few reflections.
- When the active system is placed in a resonant cavity, the intensity of light builds up along the axis. As a result, the laser beam has a very high degree of uni-directionality.
- One of the mirrors of the resonant cavity is completely silvered and the other partially silvered. The laser beam emerges from the resonant cavity through the partially silvered mirror.
- Further, stationary waves are formed and the cavity resonates when the distance 'L' between the mirrors is an integer multiple of $\frac{\lambda}{2}$ where λ is the wavelength of the radiation in the active system.

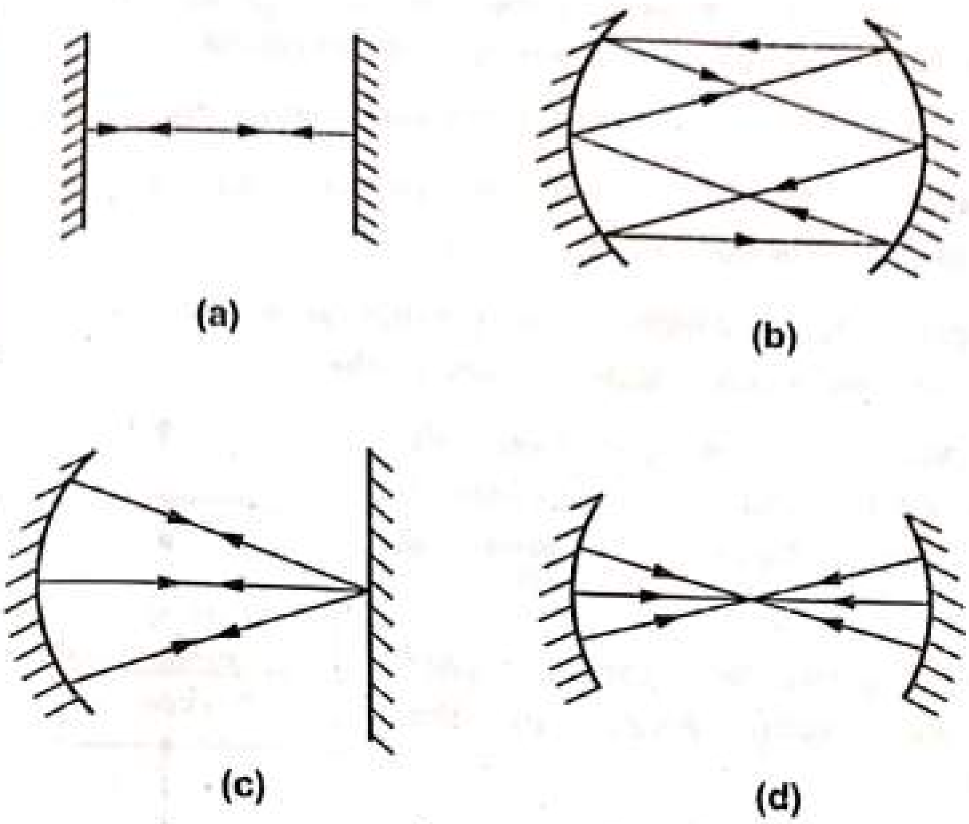


Fig. 6.3

Q.2 Explain the following :
i) Stimulated emission ii) Metastable state iii) Population inversion.
☞ **[Dec - 13, Marks 3]**

Ans. : Refer Q.1.

Q.3 Explain the terms - Optical pumping, Population inversion.
☞ **[Dec.-10, May-11, Marks 4]**

Ans. : Refer Q.1.

6.2 : Semiconductor Laser

Q.4 Explain the construction and working of semiconductor laser with the help of energy band diagram. ☞ **[May-12, Marks 6]**

Ans. : Principle : When a P-N junction is forward biased, the conduction band electrons from N-type combine with the valence band holes in the depletion region and radiation is emitted.

- For silicon and germanium junctions, the radiation is in infrared whereas for materials like Gallium Arsenide (GaAs) and Gallium Arsenic Phosphide (GaAsP) the radiation is in the visible region.

- The P-N junction is heavily doped and a large current is made to fl... through the junction to create population inversion.
- Laser can be obtained by using a resonant cavity for such a junction.

Construction : Metallic contacts are provided to the P and N types i... heavily doped P-N junction diode.

- Two opposite faces which are perpendicular to the plane of the junct... are polished and made parallel to each other.
- These parallel faces constitute the resonant cavity and laser is obtained through these faces as shown in Fig. 6.4.
- The remaining two faces are roughened to prevent lasing action in that direction.

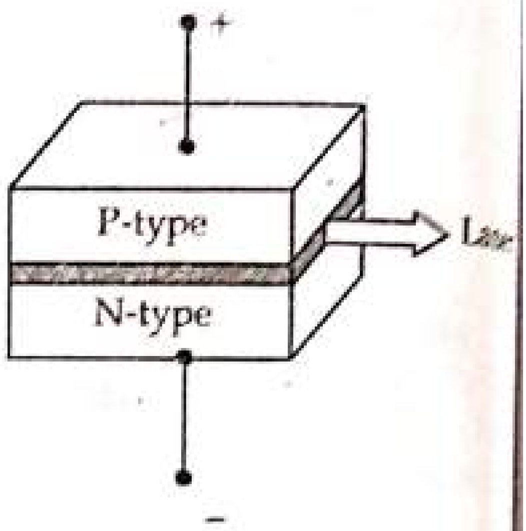


Fig. 6.4

Working : As the P and N types are heavily doped, the Fermi level (E_F) in N type lies in the conduction band and in P-type it lies in the valence band.

- The Fermi level is uniform throughout the unbiased diode as shown in Fig. 6.5 (a).
- When the junction is forward biased, the energy levels shift as shown in Fig. 6.5 (b).
- The width of depletion region decreases due to injection of electrons and holes.

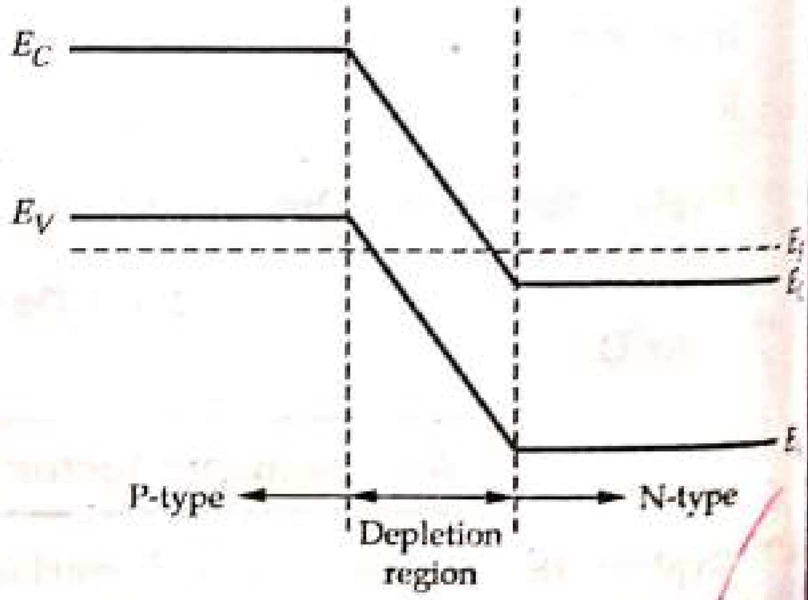


Fig. 6.5 (a)

- At low forward currents, the electron-hole recombination caus... spontaneous emission of radiation and the diode acts as a LED.
- When current is increased and reaches a threshold value, populat... inversion is achieved in the depletion region due to large concentrat...

of electrons in conduction band and holes (i.e. vacancies) in valence band.

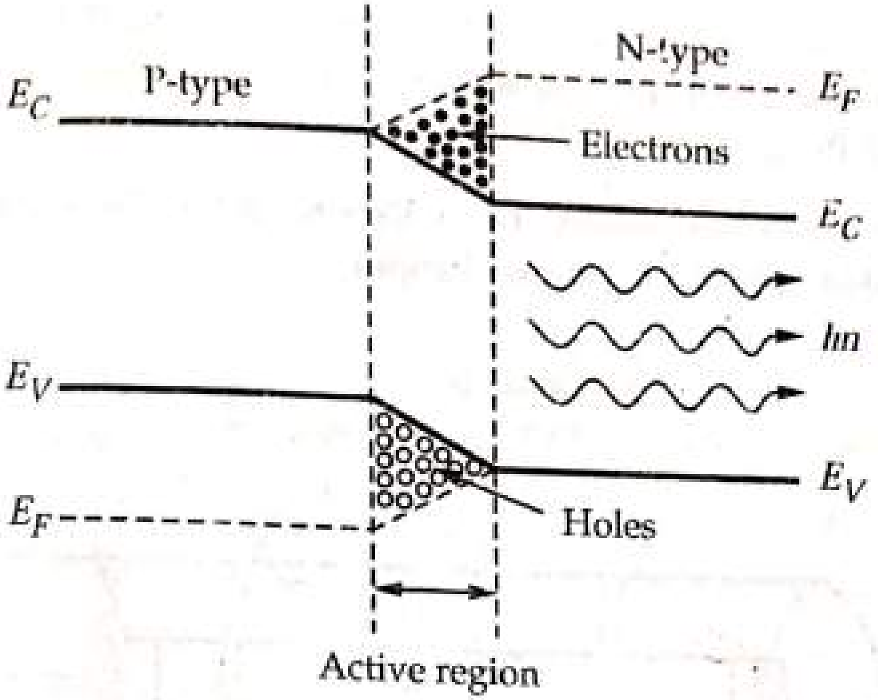


Fig. 6.5 (b)

- The narrow region where population inversion is achieved becomes the active region where lasing action takes place. The forward bias applied to the junction is thus the pumping mechanism which produces population inversion.
- The photons travelling in the junction along the resonant cavity stimulate recombination of electron-hole pairs due to which the intensity of coherent light builds up along the axis of the cavity.
- The semiconductor lasers have low power consumption are compact and highly efficient. But the laser output is less monochromatic and more divergent compared to other lasers.

6.3 : Ruby Laser

Q.5 **Explain the operation of solid state ruby laser with a neat labelled diagram.**

☞ **[May-04,09,10,11; Dec.-05,06,07,09,10, Marks 6]**

OR

Explain the construction and working of ruby laser with the help of energy level diagram. ☞ **[Dec-14, May-15, Marks 6]**

Ans. : Ruby is a crystal of Al_2O_3 in which a few aluminium atoms are replaced by chromium atoms. The chromium atoms have metastable states which are used for lasing action.

Construction : The end faces of a ruby rod in the form of a cylinder are polished flat and parallel. One of the end faces is completely silvered and the other partially silvered.

- The ruby rod is surrounded by a helical xenon flash tube as shown in Fig. 6.6 which provides optical pumping.

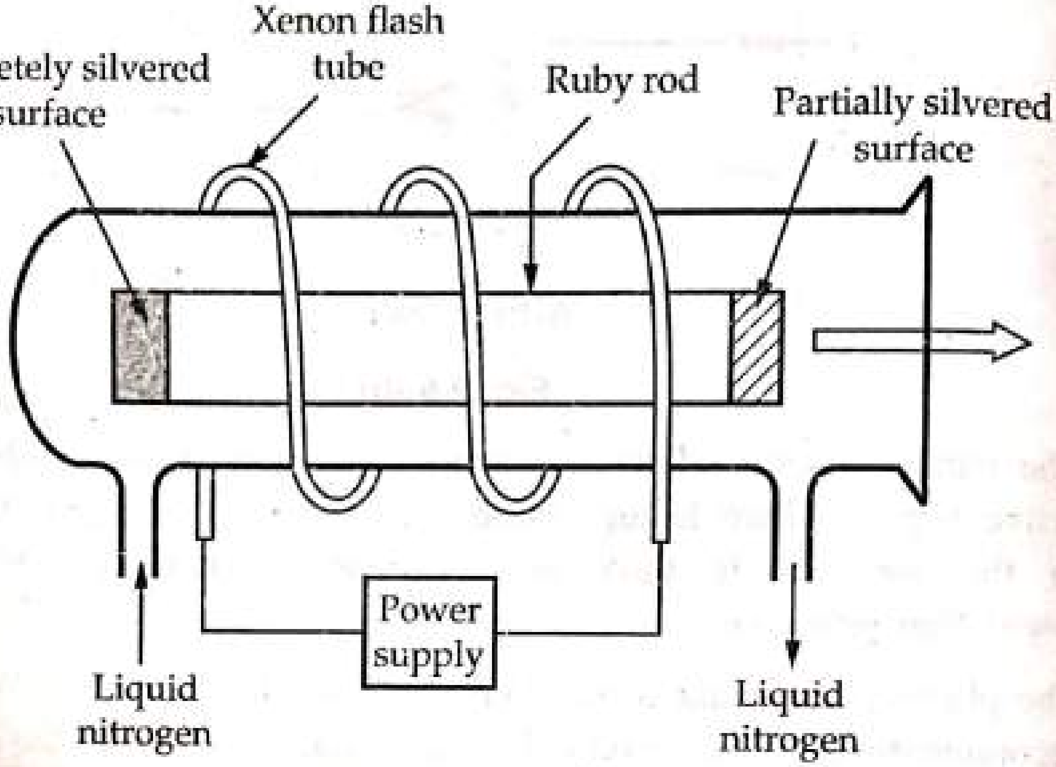


Fig. 6.6

- The ruby rod is placed in a glass tube through which liquid nitrogen is circulated for cooling as very large amount of heat is produced.

Working : An intense flash of light is produced by the xenon flash tube.

- The chromium atoms in ground state absorb this light in the wavelength region of 5500 A° and are raised to a wide band of energy levels as shown in Fig. 6.7.
- The mean life time of atoms in these states is very small (of the order of 10^{-8} s).
- Some atoms from this energy band jump back to the ground state whereas some drop to intermediate metastable states E_3 and E_4.

- A very intense flash from the xenon tube produces population inversion in these states where the mean life time is a few milliseconds.

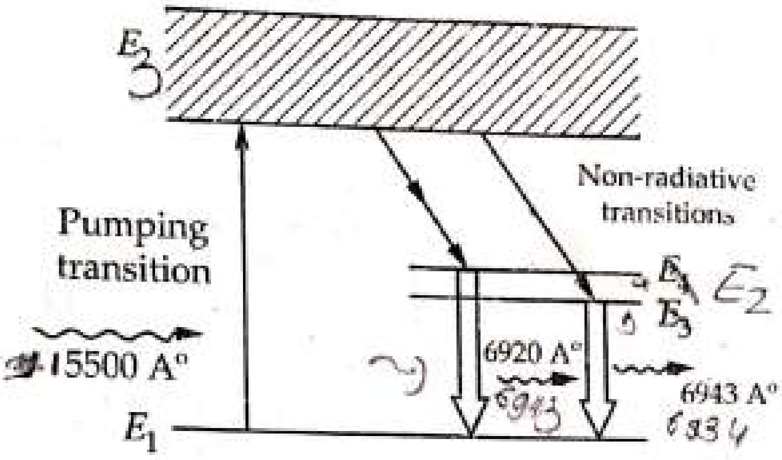


Fig. 6.7

- The transition of atoms from E_2 to E_3 and E_4 is non-radiative as this energy is converted into vibrational energy of atoms in the crystal.
- Some atoms in state E_3 emit photons spontaneously by falling to the ground state. Some of these photons which travel along the axis of resonant cavity produce stimulated emission due to which intensity of light builds up along the axis of the cavity.
- The laser is transmitted through the partially silvered surface.
- Each flash of light from the xenon lamp is used to produce a pulse of laser. Hence the ruby laser is a pulsed laser.
- The ruby laser is a high power pulsed laser in the visible region but the efficiency of the laser is very low.

6.4 : Helium - Neon Laser

Q.6 Explain the construction and working of He-Ne low power gas laser with neat labelled diagram.

☞ [May-04,05,06,07,09,11,12, Marks 6]

Ans. : A mixture of Helium (*He*) and Neon (*Ne*) is used. Lasing transition in Ne is utilized to produce laser and He atoms are used to transfer energy to Ne atoms.

Construction : The *He-Ne* laser consists of a glass tube of length 10 cm to 100 cm and few millimetre in diameter.

- Two electrodes are inserted in the tube as shown in Fig. 6.8.
- One end of the tube is completely silvered and the other partially silvered.

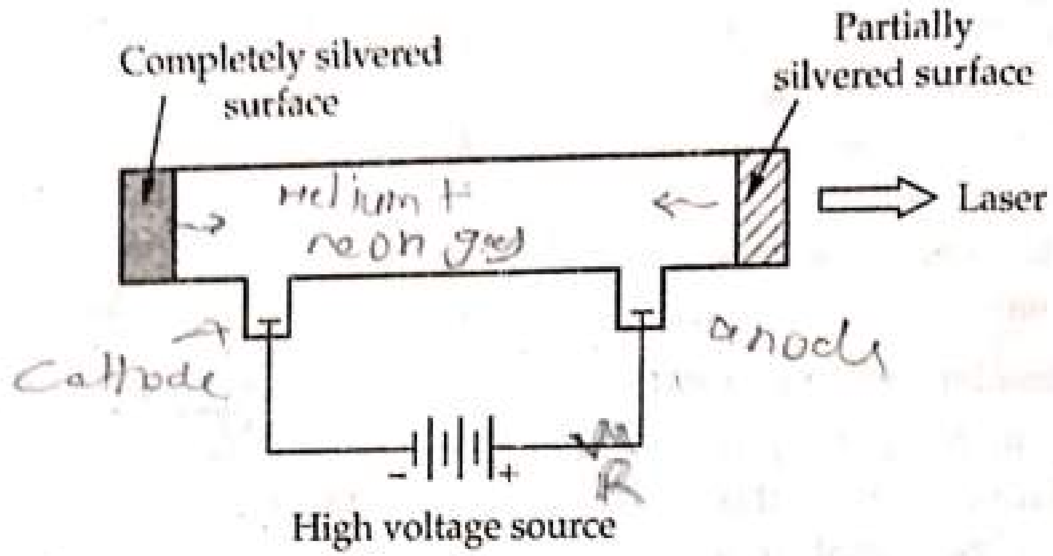


Fig. 6.8

- The tube is filled with *He* to a pressure of 1 Torr and *Ne* to a pressure of 0.1 Torr. This mixture contains about 85 % of He atoms and 15 % Ne atoms.
- A high voltage D.C. source is connected across the electrodes to strike discharge in the gas.

Working : The high voltage applied across the electrodes ionizes the gas. The electrons gain energy more easily than the ions due to smaller mass and transfer this energy to *He* and *Ne* atoms by collision. Transfer of energy to *He* is more due to its smaller mass.

- The He atoms accumulate in He_2 and He_3 states which are metastable. These *He* atoms transfer energy to *Ne* atoms through inelastic collisions to raise them to Ne_4 and Ne_6 levels which have same energy as He_2 and He_3 states respectively as shown in Fig. 6.9.
- Due to large number of *He* atoms in the mixture, *Ne* atoms are continuously excite to Ne_4 and Ne_6 levels and population inversion in these levels can be maintained at all times.
- Ne_2, Ne_4 and Ne_6 are the metastable states in *Ne* atoms. The transitions from Ne_6 to Ne_5 and Ne_3 give rise to 3.39 μm and 6328 A° wavelengths respectively whereas transition from Ne_4 to Ne_3 gives radiation of wavelength 1.15 μm.
- The laser can be operated at any of these wavelengths but is more commonly operated at 6328 A° which is in the visible region.
- Stimulated emission at other wavelengths can be suppressed by using highly reflecting coatings at 6328 A° on the mirrors that absorb the other

wavelengths. Other methods of suppressing certain wavelengths include use of etalons and variation of the length of resonant cavity.

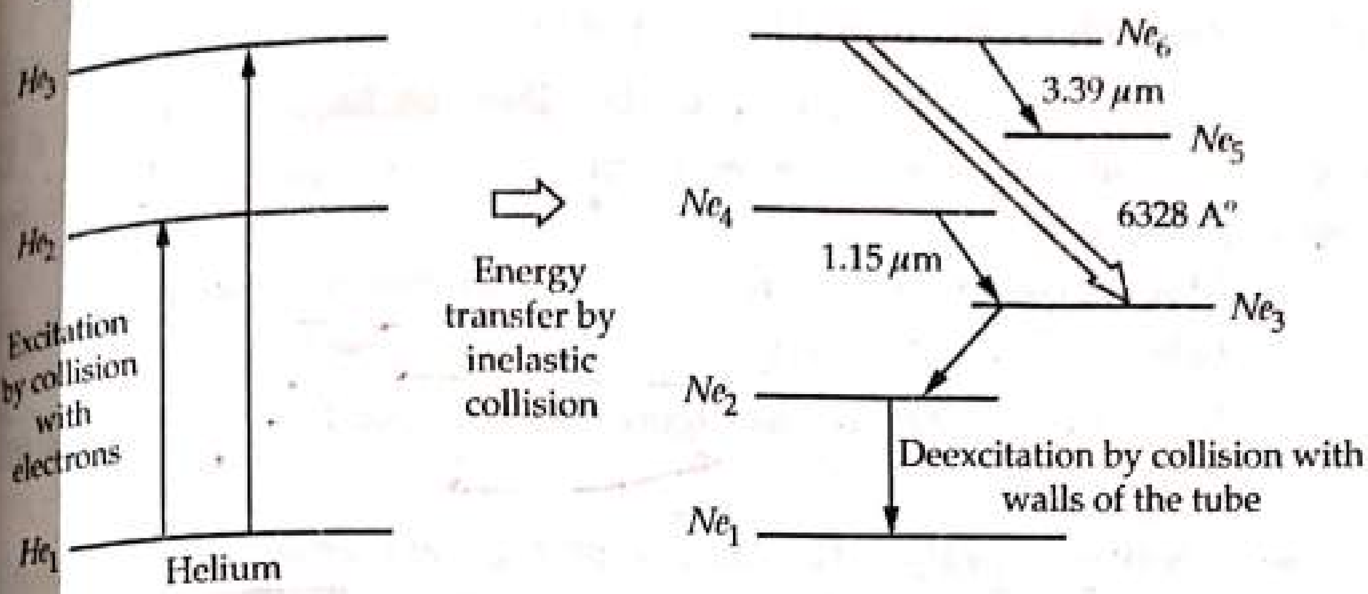


Fig. 6.9

- Neon atoms in Ne_2 level can get excited to Ne_3 level disturbing the population inversion between Ne_3 and Ne_6 levels. This can be avoided by decreasing the diameter of the tube so that the atoms in Ne_2 level lose energy and come down to Ne_1 level through collisions with walls of the tube.
- When population inversion is achieved between the Ne_3 and Ne_6 levels a few spontaneously emitted photons travelling along the axis of the resonant cavity stimulate emission of the 6328 A° wavelength.
- The intensity of coherent radiation builds up along the axis of the resonant cavity and the laser is transmitted through the partially silvered surface.
- *He-Ne* laser is a low power continuous laser. It is highly monochromatic as the energy levels in gaseous state have very small spread in energy due to negligible interaction between different atoms.

Q.7 Explain only the pumping process in ruby laser and He - Ne laser. ☞ **[May-14, Marks 3]**

Ans. : Refer Q.6.

6.5 : Properties of Lasers

Q.8 State important properties of lasers.

☞ [May-05,06; Dec.-05,06,07,09, Marks 4]

Ans. : The following important properties of laser make it different fro[m] other ordinary sources of light :

i) **Monochromaticity :** The laser beam is highly monochromatic. Th[e] light is emitted in a very narrow frequency band.

ii) **Coherence :** The laser is highly coherent due to stimulated emissio[n] of radiation.

iii) **Unidirectionality :** The laser beam has very small divergence due t[o] the resonant cavity. Hence light intensity does not decrease as fa[st] with distance as it does in ordinary sources of light.

iv) **Brightness :** Light from laser is much more brighter than othe[r] ordinary sources of light.

6.6 : Applications of Laser

Q.9 Explain the industrial applications of laser.

Ans. : i) Welding : The two metal plates to be welded are held in contact at their edges as shown in Fig. 6.10 and a high power laser is focussed on the line of contact.

- The metal at the line of contact melts and solidifies on cooling which causes the two plates to stick together.

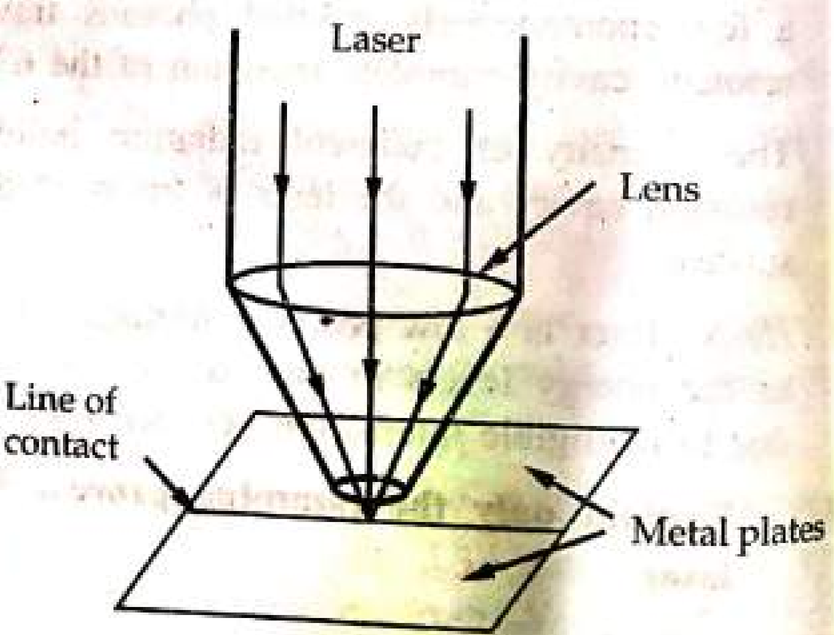


Fig. 6.10

- As the laser can be focussed to a very sharp point, the heat affected area is very small. Hence laser welding does not cause distortion of the plates.

Also, it is a contact - less procedure. Hence there is no possibility of introducing impurities.

- Laser welding is commonly used in automobile, ship building and aircraft manufacturing industries. CO_2 and $NdYAG$ lasers are used for this purpose.

ii) **Cutting** : Cutting of metal sheets is achieved using high power lasers like $NdYAG$ or CO_2 lasers.

- The laser is focussed on the metal sheet and a jet of oxygen is blown on the spot.
- A significant part of the energy required for cutting is supplied by burning of the metal in oxygen.
- The oxygen jet also blows away the vapourized metal and also cools the adjacent edges. An arrangement of gas laser cutting is shown in Fig. 6.11.

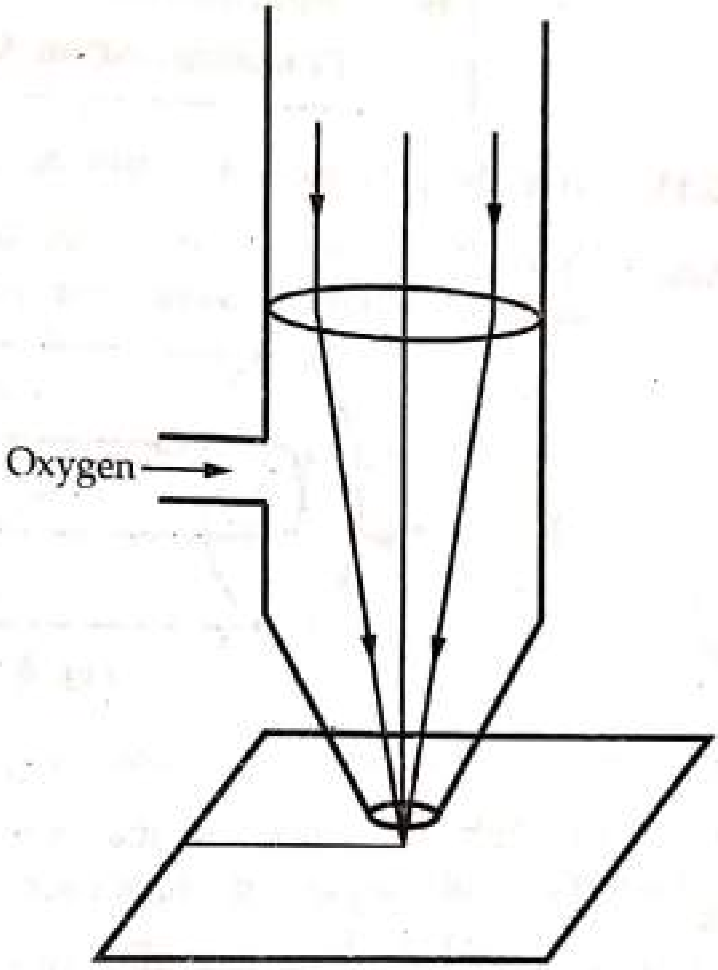


Fig. 6.11

- Higher cutting speeds have been achieved with lasers compared to other conventional methods. Laser cutting produces higher quality of the cut edges.

iii) **Drilling** : Holes can be drilled into materials using high power pulsed lasers of 10^{-4} to 10^{-3} s duration. The laser pulse evaporates the material which leaves a hole in its place.

- $NdYAG$ laser is commonly used for this purpose.
- Laser drilling has a very high degree of precision and holes can be drilled in any direction.
- As the laser can be focussed to a very fine spot, very small holes can be drilled having diameters of the order of a few microns.

- As there are no mechanical vibrations, holes can be drilled very close to the edges without damaging the metal plates.

Q.10 Explain any one application of laser. ☞ **[Dec - 12, Marks 3]**

Ans. : Refer Q.8.

6.7 : Application of Laser in Optic Fibre Communication System

Q.11 Give the principle of communication system based on laser.

Ans. : Optic fibers consist of a central core of a transparent material surrounded by cladding of another transparent material as shown in Fig. 6.12

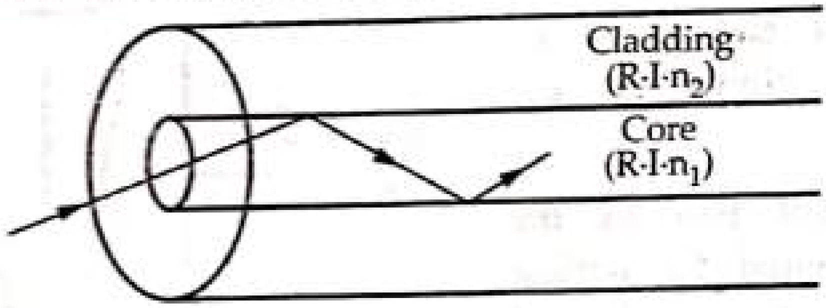


Fig. 6.12

- The refractive index of the core is greater than that of the cladding.
- When light travelling in the core is incident on the core-cladding interface at angle of incidence greater than the critical angle, $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$, there is total internal reflection.
- Due to multiple total internal reflections, light is confined to the core and can travel long distances without significant attenuation.

Q.12 Give the advantages of using fibre optic media for communication. ☞ **[May-04,12 Dec.-04, Marks 4]**

Ans. :

1) As the frequency of light is large ($\sim 10^{14}$ Hz), large bandwidth is available. Hence more channels become available due to which data transfer speeds are larger.

2) As optic fibres are made up of dielectric materials, the communication system is not affected by electromagnetic disturbances, specially during thunderstorms.

3) There is no cross talk. Signals travelling in one cable do not give rise to induced signals in a nearby cable.

4) Quality of communication is better as the transmission losses are low.

5) As the fibres are very thin and can accomodate more channels, they occupy smaller space compared to copper cables for the same number of communication channels.

6) There is good electrical isolation between transmitter and receiver.

7) They are not affected by corrosion.

Q.13 What is holography ? Explain the process of holography recording and reconstruction.

☞ **[Dec.-04,07, May-05,08; Marks 6]**

Ans. : Holography is a technique of producing three dimensional images.

- In holography, the amplitude as well as the phase of light waves coming from the object are stored in the form of an interference pattern. The technique involves two steps-recording and reconstruction.
- Best interference patterns are produced using lasers which are the most coherent sources.

Recording : Coherent light from a laser is partly reflected from a mirror (known as reference beam) and partly reflected from the object (known as object beam).

- The interference pattern of these two beams is recorded on a photographic plate as shown in Fig. 6.13.
- The developed photographic plate is known as hologram. It consists of alternate bright and dark fringes.

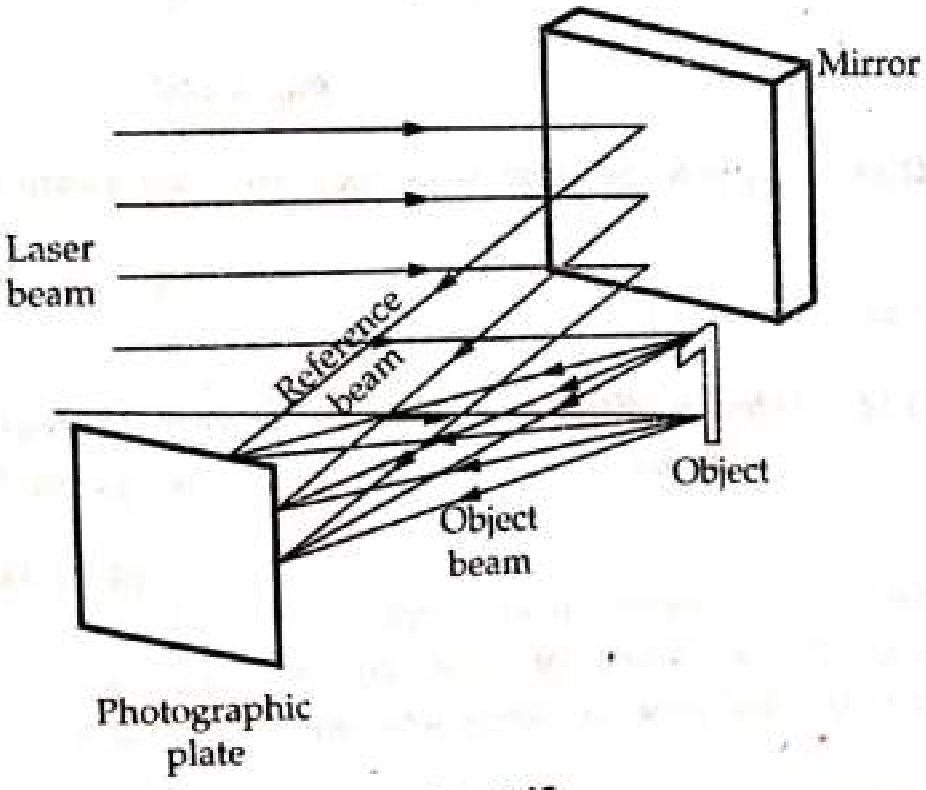


Fig. 6.13

Reconstruction : To see the three dimensional image of the object, the hologram is illuminated by the reference beam only or a laser beam with the orientation of the reference beam with respect to the hologram.

- The hologram acts as a diffraction grating.
- One of the first order diffracted beam produces a real three dimensional and inverted image and the other first order diffracted beam produces a virtual three dimensional image which is formed at the same place where the object was kept during recording as shown in Fig. 6.14.

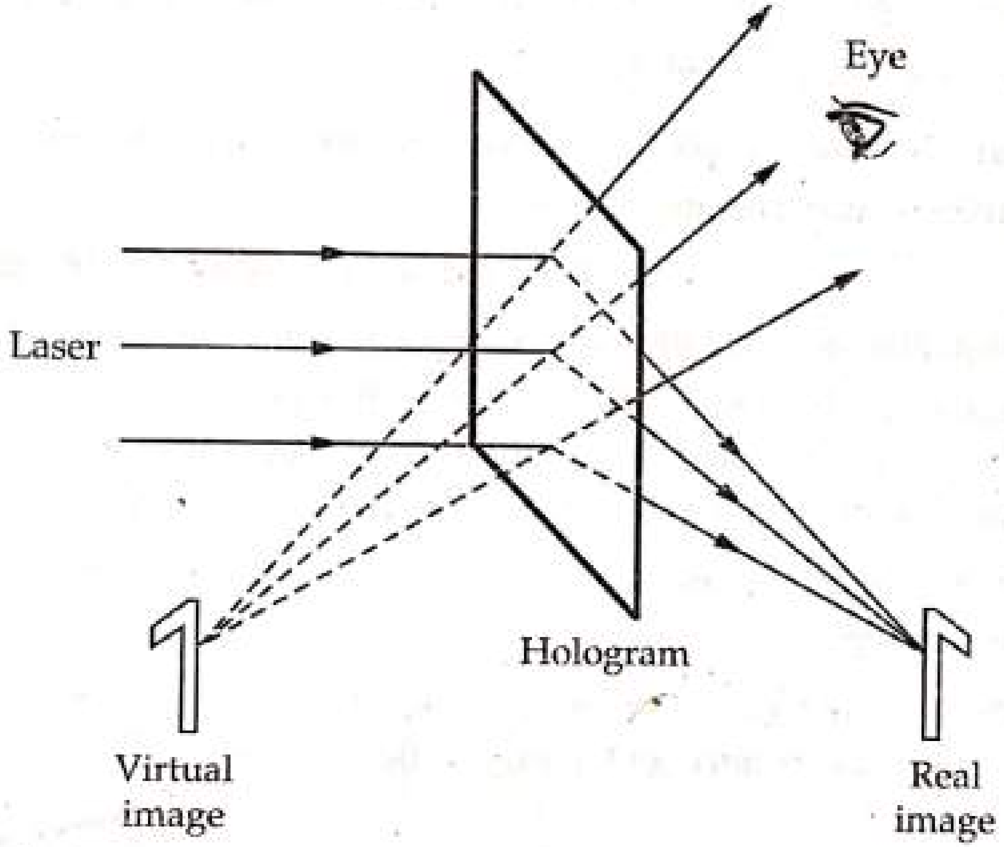


Fig. 6.14

Q.14 Explain the process of recording hologram with the help of laser.

☞ **[May-13, Marks 3]**

Ans. : Refer Q.12.

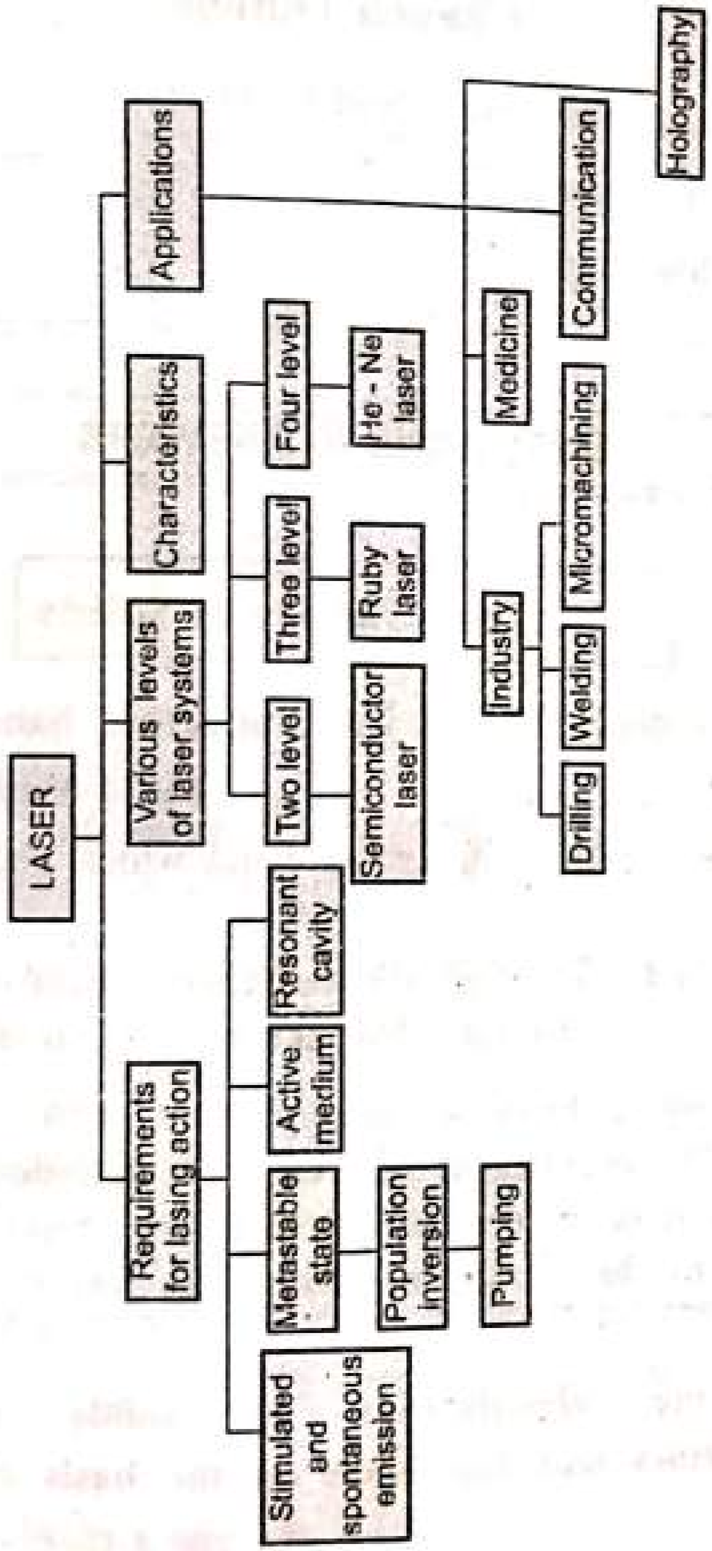
Q.15 What is difference between normal photography and holography ? Why lasers are used to record hologram ?

☞ **[May - 14, Marks 3]**

Ans. : In conventional photography, two dimensional images are formed as only the amplitude of light coming from an object is recorded on a photographic plate. In holography, the amplitude as well as the phase of light waves coming from the object are stored in the form of an interference pattern.

As holograms are interference patterns, best holograms are produced using lasers which are the most coherent sources.

CF CONCEPT FLOWCHART



END...

UNIT - III

6 Quantum Mechanics

6.1 : de Broglie Hypothesis

Q.1 State de Broglie's hypothesis and derive the equation for de Broglie's wavelength in terms of 1) Energy 2) For an electron

☞ [SPPU : May-12, Marks 6]

Ans. : de Broglie hypothesis : According to de Broglie's hypothesis, every moving particle is associated with a wave, called the matter wave or de Broglie wave, whose wavelength (λ) is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where, h = Planck's constant,

m = Mass of the particle

and v = Particle velocity.

de Broglie Wavelength for a Free Particle in terms of its Kinetic Energy

- For a free particle the total energy is same as its kinetic energy which is given by

$$E = \frac{1}{2}mv^2$$

- Multiplying and dividing by 'm',

$$E = \frac{m^2v^2}{2m}$$

But $p = mv$

$$\therefore \quad E = \frac{p^2}{2m}$$

$$\therefore \quad p = \sqrt{2mE}$$

By de Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}}$$

de Broglie Wavelength of an Electron Accelerated through Some Potential Difference

- If an electron is accelerated through a potential difference of 'V' volts, the work done on the electron is eV which is converted into kinetic energy.

$$\therefore \quad \frac{1}{2}mv^2 = eV$$

$$\therefore \quad \frac{p^2}{2m} = eV$$

$$\therefore \quad p = \sqrt{2meV}$$

- The de Broglie wavelength is given by,

$$\lambda = \frac{h}{p}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2meV}}$$

As $h = 6.63 \times 10^{-34}$ J-s,

$m = 9.1 \times 10^{-31}$ kg

(Neglecting relativistic correction)

$e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}\ V}}$$

$$= \frac{12.27 \times 10^{-10}}{\sqrt{V}}\ m$$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{V}}\ A^{o}$$

Q.2 State de Broglie's hypothesis. Hence obtain the relation for de Broglie's wave length in terms of energy.

Ans. : Refer Q.1.

Q.3 Starting from $\lambda = \frac{h}{mv}$, obtain $\lambda = \frac{h}{\sqrt{2mE}}$, where E is KE of the particle. [SPPU : May-14, Marks 3]

Ans. : Refer Q.1.

Q.4 What is De-Broglie's hypothesis of matter waves. Show that the De-Broglie's wavelength of a charged particle is inversely proportional to the square root of the accelerating potential.

Ans. : Refer Q.1.

Q.5 What is de-Broglie hypothesis. Derive an expression for de-Broglie wavelength for an electron when it is accelerated by potential difference 'V'.

Ans. : Refer Q.1.

Q.6 State de-Broglie hypothesis of matter waves. Derive the expression for matter waves for an accelerating particle in terms of its kinetic energy.

Ans. : Refer Q.1.

Q.7 What is De-Broglie hypothesis ? Derive an expression for de-Broglie wavelength for an electron when it is accelerated by potential difference V.

Ans. : Refer Q.1.

Important Formulae

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{12.27}{\sqrt{V}}\ A^{o}$$

Q.8 Determine the velocity and kinetic energy of a neutron having de Broglie wavelength 1 A°. Mass of neutron = 1.67×10^{-27} kg. [SPPU : Dec.-09,09, May-04, Marks 4]

Ans. : Given data : $h = 6.63\times 10^{-34}$ J-s

$$m = 1.67\times 10^{-27} \text{ kg}$$

$$\lambda = 1 \text{ A}° = 1\times 10^{-10} \text{ m}$$

Formulae : $\lambda = \frac{h}{mv}$ and $\lambda = \frac{h}{\sqrt{2mE}}$

$$\therefore \quad v = \frac{h}{m\lambda}$$

$$\therefore \quad v = \frac{6.63\times 10^{-34}}{1.67\times 10^{-27}\times 1\times 10^{-10}}$$

$$\therefore \quad \boxed{v = 3970 \text{ m/s}}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.63\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (1\times 10^{-10})^2}$$

$$\therefore \quad E = 1.316\times 10^{-20} \text{ J} = \frac{1.316\times 10^{-20}}{1.6\times 10^{-19}} \text{ eV}$$

$$\therefore \quad \boxed{E = 8.225\times 10^{-2} \text{ eV}}$$

Q.9 Find the de Broglie wavelength of
i) An electron accelerated through a potential difference of 182 volts and
ii) 1 kg object moving with a speed of 1 m/s.
Comparing the results, explain why the wave nature of matter is not more apparent in daily observations. [SPPU : Dec.-01]

Ans. : Given data : $V = 182$ V , $h = 6.63\times 10^{-34}$ J-s , $m = 1$ kg, $v = 1$ m/s.

Formulae : i) $\lambda = \frac{12.27}{\sqrt{V}}$ A° ii) $\lambda = \frac{h}{mv}$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{182}}$$

$$\therefore \quad \boxed{\lambda = 0.91 \text{ A}°}$$

$$\lambda = \frac{6.63\times 10^{-34}}{1\times 1} = 6.63\times 10^{-34} \text{ m}$$

$$\therefore \quad \boxed{\lambda = 6.63\times 10^{-24} \text{ A}°}$$

This wavelength is too small and hence is not measurable whereas the wavelength of electron is measurable. Hence wave nature of matter is not apparent in daily observations.

Q.10 If electron had existed inside the nucleus, then its de Broglie wavelength would be roughly of the order of nuclear distance, i.e. 10^{-14} m. How much momentum corresponds to this wavelength ? How much energy corresponds to this momentum ? Express this energy in MeV and explain how this result proves that the electron cannot exist inside the nucleus. (The maximum nuclear binding energy is 8.8 MeV per nuclear particle.)
Given : Planck's constant = 6.63×10^{-34} J-s. [SPPU : May-02]

Ans. : Given data : $h = 6.63\times 10^{-34}$ J-s

$$\lambda = 10^{-14} \text{ m}$$

Formula : $\lambda = \frac{h}{p}$

$$\therefore \quad p = \frac{h}{\lambda} = \frac{6.63\times 10^{-34}}{10^{-14}}$$

$$\therefore \quad \boxed{p = 6.63\times 10^{-20} \text{ kg-m/s}}$$

The corresponding energy is,

$$E = \frac{p^2}{2m} = \frac{(6.63\times 10^{-20})^2}{2\times 9.1\times 10^{-31}}$$

$$\therefore \quad E = 2.415\times 10^{-9} \text{ J}$$

As $\quad 1 \text{ MeV} = 1.6\times 10^{-13}$ J,

$$E = \frac{2.415\times 10^{-9}}{1.6\times 10^{-13}} \text{ MeV}$$

$\therefore$ $\boxed{E = 15093.75 \text{ MeV}}$

This energy of the electron is much larger than the 8.8 MeV energy required to keep the electron inside the nucleus. Hence, the electron cannot exist inside the nucleus.

Q.11 An electron is accelerated through a potential difference of 10 kV. Calculate the de Broglie wavelength and momentum of electron. [SPPU : Dec.-04]

Ans. : Given data : $V = 10 \text{ kV} = 10 \times 10^3 \text{ V}$

Formula : $\lambda = \frac{12.27}{\sqrt{V}} \text{ A}^\circ$ and $p = \frac{h}{\lambda}$

$$\lambda = \frac{12.27}{\sqrt{10 \times 10^3}}$$

$$\therefore \quad \boxed{\lambda = 0.1227 \text{ A}^\circ}$$

$$p = \frac{6.63 \times 10^{-34}}{0.1227 \times 10^{-10}}$$

$$\therefore \quad \boxed{p = 5.403 \times 10^{-23} \text{ kg-m/s}}$$

Q.12 A proton and an α-particle are accelerated by the same potential difference. Show that the ratio of the de Broglie wavelengths associated with them is $2\sqrt{2}$. Assume the mass of α-particle to be 4 times the mass of proton. [SPPU : May-05]

Ans. : Given data : The charge of proton is 'e' and that of an α-particle is $2e$, $m_\alpha = 4\, m_p$

Formula : $\lambda = \frac{h}{\sqrt{2meV}}$

For proton,

$$\lambda_p = \frac{h}{\sqrt{2m_p eV}} \quad \text{... (1)}$$

For α-particle,

$$\lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(2e)V}} \quad \text{... (2)}$$

From equations (1) and (2),

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p eV}} \times \frac{\sqrt{2(4m_p)(2e)V}}{h} = \sqrt{8}$$

$$\therefore \quad \boxed{\frac{\lambda_p}{\lambda_\alpha} = 2\sqrt{2}}$$

Q.13 Calculate the velocity and de Broglie wavelength of an α-particle of particle of energy 1 keV. Given : Mass of α - particle $= 6.68 \times 10^{-27}$ kg. [SPPU : May-07]

Ans. : Given data : $h = 6.63 \times 10^{-34}$ J-s

$$m = 6.68 \times 10^{-27} \text{ kg}$$

$$E = 1 \text{ keV} = 1.6 \times 10^{-16} \text{ J}$$

Formula : $\lambda = \frac{h}{\sqrt{2mE}}$, $\lambda = \frac{h}{mv}$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 1.6 \times 10^{-16}}}$$

$$\therefore \quad \lambda = 4.535 \times 10^{-13} \text{ m}$$

$$\therefore \quad \boxed{\lambda = 4.535 \times 10^{-3} \text{ A}^\circ}$$

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{6.68 \times 10^{-27} \times 4.535 \times 10^{-13}}$$

$$\therefore \quad \boxed{v = 2.19 \times 10^5 \text{ m/s}}$$

Q.14 Which has a shorter wavelength - 1 eV photon or 1 eV electron ? Calculate the value and explain.

Ans.: Given data : $h = 6.63 \times 10^{-34}$ J-s

$$c = 3 \times 10^8 \text{ m/s}$$

$$E = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Formula : The energy of photon is given by,

$$E = h\nu = \frac{hc}{\lambda}$$

For an electron with energy 'E'

$$\lambda_e = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \lambda_{ph} = \frac{hc}{E}$$

$$h = 6.63\times10^{-34} \text{ J-s}$$

$$c = 3\times10^{8} \text{ m/s}$$

$$E = 1 \text{ eV} = 1.6\times10^{-19} \text{ J}$$

$$\therefore \quad \lambda_{ph} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}}$$

$$= 1.2431\times10^{-6} \text{ m}$$

$$\therefore \quad \boxed{\lambda_{ph} = 12431 \text{ A}^\circ}$$

For an electron with energy 'E'

$$\lambda_e = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}}}$$

$$= 1.229\times10^{-9} \text{ m}$$

$$\therefore \quad \boxed{\lambda_e = 12.29 \text{ A}^\circ}$$

The wavelength of electron is shorter than the photon. This is because of the larger momentum of electron (mv or $\sqrt{2mE}$) compared to photon $\left(\frac{h\nu}{c} \text{ or } \frac{E}{c}\right)$

Q.15 An electron has kinetic energy equal to its rest mass energy. Calculate de Broglie's wavelength associated with it.
[SPPU : Dec.-07]

Ans.: Given data : $E = m_0c^2$

where,

$$m_0 = 9.1\times10^{-31} \text{ kg}$$

$$c = 3\times10^{8} \text{ m/s}$$

Formula :

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad E = 9.1\times10^{-31}\times(3\times10^{8})^2$$

$$\therefore \quad E = 8.19\times10^{-14} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times8.19\times10^{-14}}}$$

$$= 1.717\times10^{-12} \text{ m}$$

$$\therefore \quad \boxed{\lambda = 1.717\times10^{-2} \text{ A}^\circ}$$

Q.16 In a T.V. set, electrons are accelerated by a potential difference of 10 kV. What is the wavelength associated with these electrons ?
[SPPU : May-08, Dec.-18, Marks 3]

$$m = 1.67\times10^{-27} \text{ kg}$$

$$E = 1 \text{ MeV} = 1.6\times10^{-13} \text{ J}$$

Formula :

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-13}}}$$

$$\therefore \quad \boxed{\lambda = 2.87\times10^{-14} \text{ m}}$$

Q.17 Find the de Broglie wavelength associated with monoenergetic electron beam having momentum 10^{-23} kg m/s. **[SPPU : May-09]**

Ans. : Given data : $h = 6.63\times10^{-34}$ J-s

$$p = 10^{-23} \text{ kg-m/s}$$

Formula :

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{10^{-23}}$$

$$\therefore \quad \lambda = 6.63\times10^{-11} \text{ m}$$

$$\therefore \quad \boxed{\lambda = 0.663 \text{ A}^\circ}$$

Q.18 Calculate de Broglie wavelength of 10 keV protons in A°.
[SPPU : May-10,11, Dec.-17, Marks 3]

Ans. : Given data : $h = 6.63 \times 10^{-34}$ J-s.

$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$E = 10 \text{ keV} = 10 \times 1.6 \times 10^{-16} \text{ J}$$

$\therefore$ $$E = 1.6 \times 10^{-15} \text{ J}$$

Formula : $$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore$ $$E = 1.6 \times 10^{-15} \text{ J}$$

$\therefore$ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-15}}}$$

$$= 2.868 \times 10^{-13}$$

$\therefore$ $$\boxed{\lambda = 2.868 \times 10^{-3} \text{ A}^\circ}$$

Q.19 At what kinetic energy an electron will have a wavelength of 5000 A° ? [SPPU : Dec.-10]

Ans. : Given data : $\lambda = 5000 \text{ A}^\circ = 5000 \times 10^{-10}$ m

Formula : $$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore$ $$E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2(9.1 \times 10^{-31})(5000 \times 10^{-10})^2}$$

$$= 9.66 \times 10^{-25} \text{ J} = \frac{9.66 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV}$$

$\therefore$ $$\boxed{E = 6.038 \times 10^{-6} \text{ eV}}$$

Q.20 Calculate the de Broglie wavelength associated with 1 MeV proton ($m_p = 1.67 \times 10^{-27}$ kg). [SPPU : May-13, Marks 3]

Ans.: Given data : $h = 6.63 \times 10^{-34}$ Js

$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$E = 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

Formula : $$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore$ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}}}$$

$\therefore$ $$\boxed{\lambda = 2.87 \times 10^{-14} \text{ m}}$$

Q.21 What accelerating potential would be required for a proton with zero initial velocity to acquire a velocity corresponding to its de Broglie wavelength of 10^{-10}m. [Given : m = 1.67×10^{-27} kg] [SPPU : May-14, Marks 3]

Ans. : Given data : $\lambda = 10^{-10}$ m , $m = 1.67 \times 10^{-27}$ kg

$$h = 6.63 \times 10^{-34} \text{ Js}, \quad e = 1.6 \times 10^{-19} \text{ C}$$

Formula : $$\lambda = \frac{h}{\sqrt{2meV}}$$

$\therefore$ $$V = \frac{h^2}{2me\lambda^2}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times (10^{-10})^2}$$

$\therefore$ $$\boxed{V = 0.08225 \text{ V}}$$

Q.22 Calculate the de Broglie wavelength of electron having kinetic energy 1 keV. [SPPU : Dec - 14, 15, Marks 3]

Ans. : Given data : $h = 6.63 \times 10^{-34}$ J s, $m = 9.1 \times 10^{-31}$ kg

$$E = 1 \text{ keV} = 1.6 \times 10^{-16} \text{ J}$$

Formula : $$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore$ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}}}$$

$$= 3.89 \times 10^{-11} \text{ m}$$

$\therefore$ $$\boxed{\lambda = 0.39 \text{ A}^\circ}$$

Q.23 Calculate de Broglie wavelength for a proton moving with velocity 1 percent of velocity of light. [SPPU : May-16, Marks 3]

Ans. : The de Broglie wavelength is given by

$$\lambda = \frac{h}{mV}$$

$$h = 6.63\times10^{-34}\ Js,\ m = 1.673\times10^{-27}\ kg$$

$$V = \frac{1}{100}\times3\times10^{8} = 3\times10^{6}\ m/s$$

$$\therefore \quad \lambda = \frac{6.67\times10^{-34}}{1.673\times10^{-27}\times3\times10^{6}}$$

$$\therefore \quad \boxed{\lambda = 1.32\times10^{-13}\ m}$$

Q.24 An electron beam is accelerated from rest through a potential difference of 200 V. Calculate the associated wavelength.

[SPPU : Dec.-16, Marks 3]

Ans. : For electrons accelerated through potential difference of V volts,

$$\lambda = \frac{12.27}{\sqrt{V}}\ \mathring{A}$$

$$V = 200\ V$$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{200}}$$

$$\therefore \quad \boxed{\lambda = 0.868\ A^{\circ}}$$

Q.25 Calculate the energy (in eV) with which a proton has to acquire de-Broglie wavelength of 0.1 $\mathring{A}$.

Ans. : By de-Broglie hypothesis,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad E = \frac{h^2}{2m\lambda^2}$$

$$h = 6.63\times10^{-34}\ Js,\quad m = 1.673\times10^{-27}\ kg$$

$$\lambda = 0.1\ \mathring{A} = 0.1\times10^{-10}\ m$$

$$\therefore \quad E = \frac{(6.63\times10^{-34})^2}{2\times1.673\times10^{-27}\times(0.1\times10^{-10})^2}$$

$$= 1.314\times10^{-18}\ J = \frac{1.314\times10^{-18}}{1.6\times10^{-19}}\ eV$$

$$\therefore \quad \boxed{E = 8.2\ eV}$$

Q.26 A proton and an alpha particle are accelerated by the same potential difference. Find the ratio of their de-Broglie wavelength. (m_α = 6.68 0^{-27}kg, charge on alpha particle = 2 × Charge on electron, m_p = 1.673× 10^{-27}kg)

[SPPU : May-18, Marks 3]

Ans. : Given data : The charge of proton is '*e*' and that of an α-particle is $2e$, $m_\alpha = 4\ m_p$

Formula : $$\lambda = \frac{h}{\sqrt{2meV}}$$

For proton, $$\lambda_p = \frac{h}{\sqrt{2m_p eV}} \quad \dots (1)$$

For α-particle, $$\lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(2e)V}} \quad \dots (2)$$

From equations (1) and (2),

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p eV}}\times\frac{\sqrt{2(4m_p)(2e)V}}{h} = \sqrt{8}$$

$$\therefore \quad \boxed{\frac{\lambda_p}{\lambda_\alpha} = 2\sqrt{2}}$$

Q.27 An electron initially at rest is accelerated through a potential difference of 3000 V. Calculate for the electron wave the following parameters :

i) The de-Broglie wavelength and

ii) The momentum (h : 6.63 × 10^{-34}Js)

Ans. : Given data : $V = 3000\ V$

Formula : $$\lambda = \frac{12.27}{\sqrt{V}}\ A^{\circ},\quad \lambda = \frac{h}{p}$$

$$\therefore \quad \lambda = \frac{12.27}{\sqrt{3000}}$$

$$\therefore \quad \boxed{\lambda = 0.224\ A^\circ}$$

$$\lambda = \frac{h}{p}$$

$$\therefore \quad p = \frac{h}{\lambda} = \frac{6.63\times 10^{-34}}{0.224\times 10^{-10}}$$

$$\therefore \quad \boxed{p = 2.96\times 10^{-23}\ kg\text{-}m/s}$$

6.2 : Phase Velocity and Group Velocity

Q.28 Define phase velocity and group velocity. Hence obtain the relation between v_p and v_g for de Broglie wave.
[SPPU : Dec - 12, 16, Marks 6]

OR Define phase velocity, group velocity and derive their expressions.
[SPPU : May - 13, Marks 4]

Ans. : • Phase velocity (also known as wave velocity) is the velocity with which a particular phase of the wave propagates in a medium.

If k = Propagation constant

and ω = Angular frequency.

The phase velocity v_p is,

$$\therefore \quad v_p = \frac{\omega}{k} \quad \ldots (1)$$

• Mathematically, a wave group can be described in terms of a superposition of individual waves of different wavelengths. The interference of these waves gives variation in amplitude defining the shape of the wave group.

• If velocities of the waves are same, the wave group travels with this same velocity. But, if the wave velocity changes with wavelength the wave group has different velocity from those of the individual waves.

• If the two waves have angular velocities differing by $d\omega$ and propagation constants differing by dk (due to difference $d\lambda$ in their wavelengths), the group velocity is given by

$$v_g = \frac{d\omega}{dk} \quad \ldots (2)$$

Relation between Phase Velocity and Group Velocity

$$k = \frac{2\pi}{\lambda}$$

• From equation (1) $\omega = v_p k$

• Substituting in equation (2),

$$v_g = \frac{d}{dk}(v_p k)$$

$$\therefore \quad v_g = v_p + k\frac{dv_p}{dk} = v_p + k\frac{dv_p}{d\lambda}\cdot\frac{d\lambda}{dk}$$

$$\therefore \quad v_g = v_p + k\frac{dv_p}{d\lambda}\times\frac{1}{\left(\frac{dk}{d\lambda}\right)} \quad \ldots (3)$$

As $\quad k = \frac{2\pi}{\lambda}$,

$$\frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$$

• Substituting in equation (3),

$$v_g = v_p + \frac{2\pi}{\lambda}\frac{dv_p}{d\lambda}\times\frac{1}{\left(\frac{-2\pi}{\lambda^2}\right)}$$

$$\therefore \quad v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

Q.29 Define phase velocity of a matter wave. Show that phase velocity of matter wave is greater than velocity of light.
[SPPU : May - 14, 15, Dec.-15, Marks 4]

Ans. : Refer Q.28 for definition of phase velocity.

• The phase velocity v_p is,

$$v_p = \frac{\omega}{k} \quad \ldots (1)$$

• For de Broglie waves,

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} \quad \ldots (2)$$

- To find the frequency ν, we equate the energy '$h\nu$' with relativistic total energy 'mc^2',

$$h\nu = mc^2$$

$\therefore$ $$\nu = \frac{mc^2}{h}$$

As $$\omega = 2\pi\nu$$

$\therefore$ $$\omega = \frac{2\pi mc^2}{h} \quad \dots (3)$$

- Substituting equation (2) and (3) in equation (1),

$$v_p = \frac{\left(\frac{2\pi mc^2}{h}\right)}{\left(\frac{2\pi mv}{h}\right)}$$

$\therefore$ $$v_p = \frac{c^2}{v} \quad \dots (4)$$

- The particle velocity 'v' is always less than the velocity of light 'c'.

$$v_p = \frac{c}{v} \cdot c$$

As $v < c$, $\frac{c}{v} > 1$

$\therefore$ $$v_p > c$$

i.e., velocity of de Broglie waves is always greater than the velocity of light.

Q.30 Define group velocity. Show that the group velocity of matter wave is equal to particle velocity. **[SPPU : Dec - 14, Marks 4]**

Ans. : Refer Q.28 for definition of group velocity.

- The group velocity is given by

$$v_g = \frac{d\omega/dv}{dk/dv} \quad \dots (1)$$

- To find the frequency ν, we equate the energy '$h\nu$' with relativistic total energy 'mc^2'.

$$h\nu = mc^2$$

$\therefore$ $$\nu = \frac{mc^2}{h}$$

As $$\omega = 2\pi\nu$$

$\therefore$ $$\omega = \frac{2\pi mc^2}{h}$$

As $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\omega = \frac{2\pi m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0}{h} v\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad \dots (2)$$

- For de Broglie waves,

$$\lambda = \frac{h}{mv}$$

$\therefore$ $$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h}$$

$\therefore$ $$k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad \dots (3)$$

- Substituting equation (2) and (3) in equation (1),

$$v_g = v \quad \dots (4)$$

i.e., the de Broglie wave group associated with moving particle travels with the same velocity as the particle.

Q31 Define phase velocity and group velocity. Show that group velocity is equal to particle velocity.

Ans. : Refer Q.28 & Q.30

6.3 : Properties of Matter Waves

Q.32 State and explain properties of matter waves.

[SPPU : May-11, Marks 6]

Ans. : Matter particles moving with velocity 'v' exhibit wave nature for which the wavelength is given by,

$$\lambda = \frac{h}{mv}$$

The properties of these waves are as follows :

1. If m = Constant, $\lambda \propto \frac{1}{v}$ i.e., for particles having same mass, the particles moving with smaller velocities will have longer wavelengths associated with them.
2. If v = Constant, $\lambda \propto \frac{1}{m}$, i.e., for particles of different masses moving with the same velocities the lighter particles will have longer wavelengths associated with them.
3. For particles at rest, $v = 0$

   $\therefore \lambda = \infty$

   i.e., particles at rest do not exhibit wave nature.
4. Matter waves are associated with charged as well as uncharged particles. Their velocity is not constant as in the case of electromagnetic waves. Hence they are not electromagnetic in nature. In fact, they cannot be classified into any of the types which we have encountered so far, i.e., transverse or longitudinal, electromagnetic or mechanical etc.
5. The phase velocity of matter waves is,

   $$v_p = \frac{c^2}{v}$$

   which is greater than 'c', the velocity of light as the particle velocity 'v' is always less than 'c'. Also, this velocity depends on the particle velocity 'v' and hence, is not constant.
6. For waves in a string, the position of particles varies with time. In sound waves pressure varies with time. In electromagnetic waves the electric and magnetic fields vary with time. Similarly, the quantity whose variations consitute matter waves is called the wave function (represented by ψ). However, the wave

function ψ has no direct physical significance. The value of the wavefunction at any point in space at any time is related to the probability of finding the particle there at that time.

7. A particle is localized in space whereas wave is spread out. Hence wave nature of matter introduces uncertainty in position of the particle.
8. The wave and particle properties are not exhibited simultaneously.

Q.33 Are the matter waves electromagnetic waves ? Explain.

[SPPU : May-13, Marks 3]

Ans. : Refer Q.32.

Q.34 State and explain de Broglie's hypothesis of matter waves. State any two properties of matter waves.

[SPPU : Dec.-13, 17, Marks 4]

Ans. : Refer Q.32.

6.4 : Heisenberg's Uncertainty Principle

Q.35 State and explain Heisenberg's uncertainty principle.

[SPPU : May-16, Marks 4]

Ans. : • Describing a moving particle as a wave group introduces uncertainty in the measurement of particle properties like position and momentum.

- This uncertainty is not due to any inadequacy in the measuring instruments but is inherent in wave nature.
- The position of particle becomes uncertain when it is described as a wave group. The particle may be located any where within the wave group but its exact location within the wave group is not known.
- Narrower the wave group, smaller is the uncertainty in its position as illustrated in Fig. Q.35.1 (a) and (b).

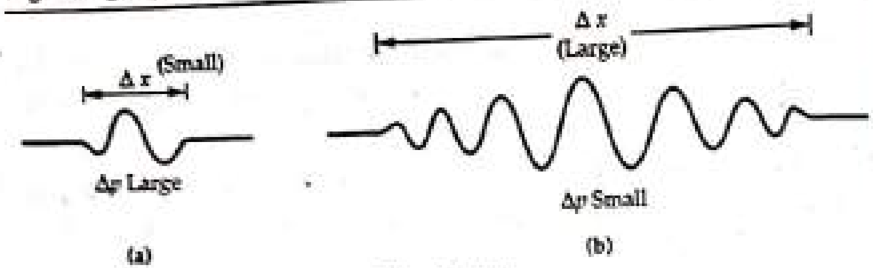


Fig. Q.35.1

- If the wave group is narrower, it consists of larger number of frequencies and hence wavelengths due to which the wavelength becomes more uncertain.
- As $\lambda = \frac{h}{p}$, an uncertainty in wavelength leads to uncertainty in momentum.
- Thus for broader wave groups, there is smaller uncertainty in wavelength and momentum, whereas for narrower wavegroups there is larger uncertainty in wavelength and momentum.
- Hence, we can conclude that for a narrow wave group there is small uncertainty in position but large uncertainty in momentum, whereas for a broader wave group there is larger uncertainty in position and smaller uncertainty in momentum.
- The Heisenberg's uncertainty principle states that it is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to the Planck's constant 'h'.

$$\Delta x \, \Delta p_x \geq h \quad \dots (1)$$

Q.36 State and explain Heisenberg's uncertainty principle. Prove the same for pair of variables energy and time.

[SPPU : May - 14, Dec.-17, 18, Marks 6]

Ans. : For statement and explanation of Heisenberg's uncertainty principle, refer Q.35.

Uncertainty Principle Applied to Energy and Time

The kinetic energy of particles can be written as

$$E = \frac{1}{2} m v^2$$

The uncertainty in energy ΔE can be obtained by differentiating the above equation.

$$\Delta E = \frac{1}{2} m \cdot 2v \, \Delta v = v \cdot (m \Delta v)$$

As $\quad p = mv$

$$\Delta p = m \Delta v$$

$\therefore \quad \Delta E = v \Delta p$

As $\quad v = \frac{\Delta x}{\Delta t}$,

$$\Delta E = \frac{\Delta x}{\Delta t} \Delta p$$

$\therefore \quad \Delta E \, \Delta t = \Delta x \, \Delta p$

By Heisenberg's uncertainty principle,

$$\Delta x \, \Delta s \geq h$$

$\therefore \quad \Delta E \, \Delta t \geq h \quad \dots (1)$

Q.37 Obtain an expression for Heisenberg's uncertainty principle for energy and time.

Ans. : Refer Q.36

Important Formulae

$$\Delta x \, \Delta p_x \geq h$$

$$\Delta E \, \Delta t \geq h$$

Q.38 Compute the minimum uncertainty in the location of a 2 gm mass moving with a speed of 1.5 m/s and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/s, given that the uncertainty in the momentum is $\Delta p = 10^{-3} p$ for both.

[SPPU : May-1999]

Ans. : Given data : $h = 6.63 \times 10^{-34}$ J-s

$$\Delta p = 10^{-3} p = 10^{-3} mv, \quad m = 2 \text{ gm} = 2 \times 10^{-3} \text{ kg}$$

$$v = 1.5 \text{ m/s}$$

Formula : $\Delta x \, \Delta p = h$

$\therefore$ $$\Delta x = \frac{h}{\Delta p}$$

$\therefore$ $$\Delta x = \frac{h}{10^{-3} mv}$$

$\therefore$ $$\Delta x = \frac{6.63 \times 10^{-34}}{10^{-3} \times 2 \times 10^{-3} \times 1.5}$$

$$\boxed{\Delta x = 2.21 \times 10^{-28} \text{ m}}$$

For the electron,

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$v = 0.5 \times 10^{8} \text{ m/s}$$

$\therefore$ $$\Delta x = \frac{6.63 \times 10^{-34}}{10^{-3} \times 9.1 \times 10^{-31} \times 0.5 \times 10^{8}}$$

$\therefore$ $$\boxed{\Delta x = 1.46 \times 10^{-8} \text{ m}}$$

Q.39 An electron has a speed of 600 m/s with an accuracy of 0.005 %. Calculate the uncertainty with which we can locate the position of the electron. [SPPU : Dec.-99]

Ans. : Given data : $\Delta v = \frac{0.005}{100} \times 600 = 0.03$ m/s

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

Formula : $\Delta x \, \Delta p = h$

$\therefore$ $$\Delta x = \frac{h}{\Delta p} = \frac{h}{m \Delta v}$$

$\therefore$ $$\Delta x = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.03}$$

$\therefore$ $$\boxed{\Delta x = 0.0243 \text{ m}}$$

Q.40 A bullet of mass 25 grams is moving with a speed of 400 m/s. The speed is measured accurate upto 0.02 %. Calculate the certainty with which the position of the bullet can be located. [SPPU : May-2000]

Ans. : Given data : $h = 6.63 \times 10^{-34}$ J-s

$$m = 25 \text{ gm} = 25 \times 10^{-3} \text{ kg}$$

$$\Delta v = \frac{0.02}{100} \times 400 = 0.08 \text{ m/s}$$

Formula : $\Delta x \, \Delta p = h$

$\therefore$ $$\Delta x = \frac{h}{\Delta p} = \frac{h}{m \Delta v}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{25 \times 10^{-3} \times 0.08}$$

$\therefore$ $$\boxed{\Delta x = 3.315 \times 10^{-31} \text{ m}}$$

Q.41 Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A°. [SPPU : Dec.-2000]

Ans. : Given data : $(\Delta x)_{max} = 10 \text{ A}^{\circ} = 10 \times 10^{-10}$ m

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

Formula : $\Delta x \, \Delta p = h$

$\therefore$ $$\Delta x \cdot m \Delta v = h$$

$$\Delta v = \frac{h}{m \Delta x}$$

$\therefore$ $$(\Delta v)_{min} = \frac{h}{m (\Delta x)_{max}}$$

$\therefore$ $$(\Delta v)_{min} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10 \times 10^{-10}}$$

$\therefore$ $$\boxed{(\Delta v)_{min} = 7.286 \times 10^{5} \text{ m/s}}$$

Q.42 If the uncertainty in location of a particle is equal to its de Broglie wavelength, show that the uncertainty in its velocity is equal to its velocity. [SPPU : May-12, Dec-13, Marks 3]

Ans. : Given data : $\Delta x = \lambda$

Formula : $\Delta x \Delta p = h$

$$\Delta x = \lambda = \frac{h}{mv}$$

$$\Delta p = m\Delta v$$

$$\therefore \quad \frac{h}{mv} \cdot m\Delta v = h$$

$$\therefore \quad \boxed{\Delta v = v}$$

Q.43 An electron is confined to a box of length 2A°. Calculate the minimum uncertainty in its velocity. [SPPU : Dec.-06]

Ans. : Given data :

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$(\Delta x)_{max} = 2 \text{ A}^\circ = 2 \times 10^{-10} \text{ m}$$

Formula : $\Delta x \Delta p = h$

$$\Delta p = m\Delta v$$

$$\therefore \quad \Delta x \cdot m\Delta v = h$$

$$\Delta v = \frac{h}{m\Delta x}$$

For minimum uncertainty in velocity, the uncertainty in position is maximum.

$$\therefore \quad (\Delta v)_{min} = \frac{h}{m(\Delta x)_{max}}$$

$$\therefore \quad (\Delta v)_{min} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^{-10}}$$

$$\therefore \quad \boxed{(\Delta v)_{min} = 3.643 \times 10^{6} \text{ m/s}}$$

Q.44 The position and momentum of 1 keV electron are simultaneously measured. If its position is located with 1 A°. Find the percentage of uncertainty in its momentum.
(Given : h = 6.64 × 10^{-34} J-sec., m = 9.1 × 10^{-31} kg) [SPPU : Dec.-12, Marks 3]

Ans. : Given data : $\Delta x = 1 \text{ A}^\circ = 1 \times 10^{-10}$ m

$$h = 6.64 \times 10^{-34} \text{ Js}$$

$$E = 1 \text{ keV} = 1.6 \times 10^{-16} \text{ J}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

Formula : $\Delta x \, \Delta p = h$

$$\therefore \quad 1 \times 10^{-10} \, \Delta p = 6.64 \times 10^{-34}$$

$$\therefore \quad \Delta p = 6.64 \times 10^{-24} \text{ kg-m/s}$$

$$E = \frac{p^2}{2m}$$

$$\therefore \quad p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-16}}$$

$$\therefore \quad p = 1.706 \times 10^{-23} \text{ kg-m/s}$$

$$\text{Percentage uncertainty} = \frac{\Delta p}{p} \times 100 = \frac{6.64 \times 10^{-24}}{1.706 \times 10^{-23}} \times 100$$

$$\therefore \quad \boxed{\text{Percentage uncertainty} = 38.92 \ \%}$$

6.5 : Wave Function and its Physical Significance

Q.45 What is wave function ψ ? Write down the conditions satisfied by wave function ψ. [SPPU : Dec - 13, May 15, Marks 4]

Ans. : • Every wave is characterized by periodic variation in some physical quantity. For example, pressure varies periodically in sound waves whereas electric and magnetic fields vary periodically in an electromagnetic wave.

• Similarly, the quantity whose periodic variations make up the matter wave is called wave function ψ. The value of ψ at a particular point (x, y, z) in space at time 't' is related to the probability of finding the particle there at that time.

- ψ itself does not have any direct physical significance and is not an experimentally measurable quantity.

Conditions satisfied by wave function ψ

i) ψ should be normalized wave function so that $|\psi|^2$ will represent probability.

ii) As probability is a single valued function, ψ must also be a single valued function at every point in space.

iii) ψ must be finite at each and every point in space.

iv) ψ must be continuous in the region where it is defined.

v) The first order derivatives of ψ, i.e., $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}$ and $\frac{\partial \psi}{\partial z}$ must be continuous in the region where ψ is defined.

Q.46 Explain the physical significance of ψ and $|\psi|^2$.
[SPPU : Dec.-12, May-15, Marks 4]

Ans. : • According to Max Born's interpretation, the probability of finding the particle described by the wave function ψ at a point (x, y, z) in space at time 't' proportional to the value of $|\psi|^2$ at that point at time 't'.

- A large value of $|\psi|^2$ represents larger probability of finding the particle.
- The probability of finding the particle is zero at a point only if $|\psi|^2 = 0$ at that point. Even if there is a small value of $|\psi|^2$ at a point, there will be some probability of detecting the particle there.
- The probability of finding the particle in a certain volume element $dv = dx\ dy\ dz$ is $|\psi|^2\ dv$ which is called the probability density if $|\psi|^2$ represents probability.
- As the particle has to exist somewhere in space, the total probability of finding the particle is 1. i.e,

$$\int_{-\infty}^{\infty} |\psi|^2\ dv = 1 \qquad \ldots (1)$$

- The above condition is satisfied only if $|\psi|^2$ represents probability. But, in general, the wave functions obtained as solutions of differential equations do not satisfy equation (1). Instead, on integrating over the whole volume, yield a finite positive and real quantity, say N^2 i.e.,

$$\int_{-\infty}^{\infty} |\psi|^2\ dv = N^2$$

- Normalized wave function ψ_N is then constructed from ψ as,

$$\psi_N = \frac{\psi}{N}$$

and
$$\psi_N^* = \frac{\psi^*}{N}$$

- The normalized wave function satisfies equation (1) so that $|\psi_N|^2$ represents probability. The process of constructing ψ_N from ψ is called normalization of the wave function.

Q.47 What is wave function ? Explain what is normalization of wave function.
[SPPU : Dec.-14, 15, Marks 4]

Ans. : Refer Q.45 and Q.46.

Q.48 Explain why probability of finding of a particle cannot be predicted by the interpretation of wave function Ψ. Explain physical significance of $|\Psi|^2$.
[SPPU : May-16, Marks 4]

Ans. : Refer Q.45 and Q.46.

Q.49 Write down the conditions which are to be satisfied by well behaved wave function.
[SPPU : Dec.-16, May-19, Marks 4]

Ans. : Refer Q.45 and Q.46.

Q.50 Explain physical significance of wave function Ψ and $(\Psi)^2$. State the mathematical conditions that wave function Ψ should satisfy.
[SPPU : May-17, Marks 4]

Ans. : Refer Q.45 and Q.46.

Q.51 Explain wave-function ψ. Give the physical significance of $|\psi^2|$
[SPPU : May-18, Dec.-18, Marks 4]

Ans. : Refer Q.45 and Q.46.

6.6 : Schrödinger's Time Independent Wave Equation

Q.52 Derive Schrodinger's time independent wave equation.

[SPPU : Dec-12,14,15, May-13,14,15,16,17,19, Marks 6]

Ans. : • The general differential equation for a wave with wave function 'ψ', travelling with velocity 'u' in three dimensions is

$$\frac{\partial^2 \psi}{\partial t^2} = u^2\left[\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+\frac{\partial^2 \psi}{\partial z^2}\right]$$

i.e.,

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \nabla^2 \psi \quad \ldots (1)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

is called the Laplace operator.

• The general solution of equation (1) is of the form

$$\psi(x, y, z, t) = \psi_0 (x, y, z) e^{-i\omega t} \quad \ldots (2)$$

where $\psi_0(x, y, z)$ is the amplitude of the wave at point (x, y, z).

• The above equation can also be written as

$$\psi(\vec{r}, t) = \psi_0(\vec{r}) e^{-i\omega t} \quad \ldots (3)$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of the point (x, y, z).

• Differentiating equation (3) partially with respect to 't' twice,

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0 e^{-i\omega t}$$

But

$$\psi = \psi_0 e^{-i\omega t}$$

∴

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

• Substituting in equation (1),

$$-\omega^2 \psi = u^2 \nabla^2 \psi$$

$$\nabla^2 \psi + \frac{\omega^2}{u^2}\psi = 0 \quad \ldots (4)$$

$$\omega = 2\pi\nu = 2\pi \frac{u}{\lambda}$$

∴

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2} \quad \ldots (5)$$

• From de Broglie hypothesis for matter waves,

$$\lambda = \frac{h}{p}$$

∴

$$\lambda^2 = \frac{h^2}{p^2} \quad \ldots (6)$$

• The total energy (E) of particle is a sum of kinetic energy $\left(\frac{1}{2} mv^2\right)$ and potential energy V.

$$E = \frac{1}{2} mv^2 + V = \frac{m^2 v^2}{2m} + V$$

∴

$$E = \frac{p^2}{2m} + V$$

∴

$$p^2 = 2m(E - V)$$

• Substituting in equation (6),

$$\lambda^2 = \frac{h^2}{2m(E-V)}$$

• Substituting in equation (5),

$$\frac{\omega^2}{u^2} = 4\pi^2 \times \frac{2m(E-V)}{h^2}$$

∴

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2}(E - V)$$

6.6 : Schrodinger's Time Independent Wave Equation

Q.82 Derive Schrodinger's time independent wave equation.

[SPPU : Dec-12,14,15, May-13,14,15,16,17,19, Marks 6]

Ans. : • The general differential equation for a wave with wave function 'ψ', travelling with velocity 'u' in three dimensions is

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right]$$

i.e.,

$$\frac{\partial^2\psi}{\partial t^2} = u^2\nabla^2\psi \quad \text{... (1)}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

is called the Laplace operator.

• The general solution of equation (1) is of the form

$$\psi(x, y, z, t) = \psi_0 (x, y, z) e^{-i\omega t} \quad \text{... (2)}$$

where $\psi_0(x, y, z)$ is the amplitude of the wave at point (x, y, z).

• The above equation can also be written as

$$\psi(\vec{r}, t) = \psi_0(\vec{r}) e^{-i\omega t} \quad \text{... (3)}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of the point (x, y, z).

• Differentiating equation (3) partially with respect to 't' twice,

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi_0 e^{-i\omega t}$$

But

$$\psi = \psi_0 e^{-i\omega t}$$

$\therefore$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi$$

• Substituting in equation (1),

$$-\omega^2\psi = u^2\nabla^2\psi$$

$$\nabla^2\psi+\frac{\omega^2}{u^2}\psi = 0 \quad \text{... (4)}$$

$$\omega = 2\pi\nu = 2\pi\frac{u}{\lambda}$$

$\therefore$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2} \quad \text{... (5)}$$

• From de Broglie hypothesis for matter waves,

$$\lambda = \frac{h}{p}$$

$\therefore$

$$\lambda^2 = \frac{h^2}{p^2} \quad \text{... (6)}$$

• The total energy (E) of particle is a sum of kinetic energy $\left(\frac{1}{2}mv^2\right)$ and potential energy V.

$$E = \frac{1}{2}mv^2 + V = \frac{m^2v^2}{2m}+V$$

$\therefore$

$$E = \frac{p^2}{2m}+V$$

$\therefore$

$$p^2 = 2m(E-V)$$

• Substituting in equation (6),

$$\lambda^2 = \frac{h^2}{2m(E-V)}$$

• Substituting in equation (5),

$$\frac{\omega^2}{u^2} = 4\pi^2\times\frac{2m(E-V)}{h^2}$$

$\therefore$

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2}(E-V)$$

- Substituting in equation (4),

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \quad \dots (7)$$

- The quantity $\frac{h}{2\pi}$ appears very frequently in quantum mechanics and hence is substituted as $\hbar$.

$$\therefore \nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \dots (8)$$

Equation (7) or (8) is the Schrödinger's time independent wave equation.

Q.53 Starting from de-Broglie hypothesis, derive Schrodinger's time independent wave equation.

Ans. : Refer Q.52

6.7 : Schrödinger's Time Dependent Wave Equation

Q.54 Derive Schrodinger's time dependent wave equation.

[SPPU : May-11, Marks 7]

Ans. : • The general differential equation for a wave with wave function ψ, travelling with velocity 'u' in three dimensions is,

$$\frac{\partial^2\psi}{\partial t^2} = u^2\nabla^2\psi \quad \dots (1)$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

- The general solution of equation (2) is of the form,

$$\psi(x, y, z, t) = \psi_0(x, y, z)e^{-i\omega t} \quad \dots (2)$$

where $\psi_0(x, y, z)$ is the amplitude of the wave at point (x, y, z).

- The above equation can also be written as,

$$\psi(\vec{r}, t) = \psi_0(\vec{r})e^{-i\omega t} \quad \dots (3)$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of the point (x, y, z).

- Differentiating equation (3) partially with respect to 't',

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$\therefore \quad \frac{\partial\psi}{\partial t} = -i\omega\psi \quad \dots (4)$$

As $\omega = 2\pi\nu$ and $E = h\nu$,

$$\omega = 2\pi\frac{E}{h}$$

- From equation (4),

$$\frac{\partial\psi}{\partial t} = \frac{-i\,2\pi E}{h}\psi$$

$$\therefore \quad E\psi = \frac{-h}{i2\pi}\frac{\partial\psi}{\partial t}$$

$$\therefore \quad E\psi = \frac{ih}{2\pi}\frac{\partial\psi}{\partial t} \quad \dots (5)$$

- Schrödinger's time independent equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$$\therefore \quad \frac{h^2}{8\pi^2 m}\nabla^2\psi + (E-V)\psi = 0$$

$$\frac{-h^2}{8\pi^2 m}\nabla^2\psi + V\psi = E\psi$$

- Substituting for $E\psi$ from equation (5),

$$\frac{-h^2}{8\pi^2 m}\nabla^2\psi + V\psi = \frac{ih}{2\pi}\frac{\partial\psi}{\partial t} \quad \dots (6)$$

- The above equation is known as Schrödinger's time dependent wave equation.

6.8 : Particle in a Rigid Box (Infinite Potential Well)

Q.55 Derive equation of energy and wave function when a free particle is trapped in an infinite potential well.

[SPPU : Dec.-09, 11, May-10,12, Marks 7]

Ans. :

- The physical problem of particle confined between two rigid walls can be converted into a problem of potential distribution by specifying the potential energy of the particle to be infinite at and beyond the walls as shown in Fig. Q.55.1 i.e.,

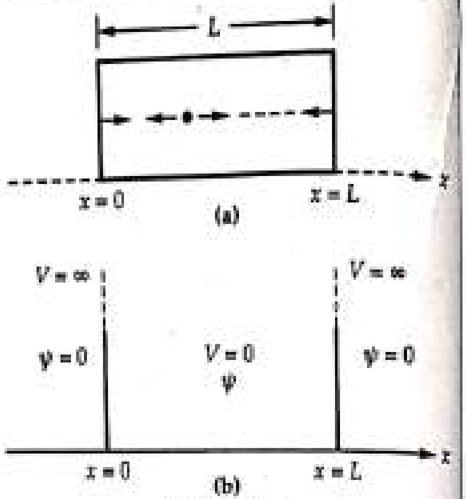


Fig. Q.55.1

$V = \infty$ for $x \leq 0$ and for $x \geq L$

- The potential energy of the particle is constant within the box which can be taken to be zero for convenience i.e.,

$$V = 0 \quad \text{for } 0 < x < L$$

- The potential energy distribution is shown in Fig. Q.55.1 (b). It is as if the particle is inside an infinite potential well. As the particle does not exist at the walls and beyond them,

$$\psi = 0 \quad \text{for} \quad x \leq 0 \quad \text{and} \quad x \geq L$$

- The wave function ψ exists only for $0 < x < L$. Schrödinger's time independent wave equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \text{...(1)}$$

- As it is a problem in one dimension along x-axis, $\nabla^2\psi$ can be replaced by $\frac{\partial^2\psi}{\partial x^2}$ which in turn can be replaced by $\frac{d^2\psi}{dx^2}$ as ψ is a function of x only. Also, substituting $V = 0$ for $0 < x < L$ in equation (1),

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \qquad \text{... (2)}$$

Let

$$K^2 = \frac{8\pi^2 mE}{h^2} \qquad \text{... (3)}$$

$$\therefore \quad \frac{d^2\psi}{dx^2} + K^2\psi = 0 \qquad \text{... (4)}$$

- The general solution of equation (4) is

$$\psi = A \sin Kx + B \cos Kx \qquad \text{... (5)}$$

where A and B are arbitrary constants which are to be determined using boundary conditions.

- The first boundary condition is

$\psi = 0$ at $x = 0$

$\therefore$ From equation (5),

$$0 = A \sin 0 + B \cos 0$$

$$B = 0$$

$\therefore$ From equation (5)

$$\psi = A \sin Kx \qquad \text{... (6)}$$

- The second boundary condition is

$\psi = 0$ at $x = L$

$\therefore$ From equation (6),

$$0 = A \sin KL$$

$A \neq 0$ as for $A = 0$, $\psi = 0$ for all values of x which will mean that the particle does not exist inside the box.

$$\therefore \quad \sin KL = 0$$

$$\therefore \quad KL = n\pi \; ; \; n = 1, 2, 3, \ldots$$

$n \neq 0$ as $n = 0 \Rightarrow \psi = 0$ for all values of x which is not possible.

$$\therefore \quad K = \frac{n\pi}{L} \qquad \text{... (7)}$$

- From equations (3) and (6),

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

- As energy 'E' depends on 'n', we use suffix 'n' for 'E'.

$$\therefore \qquad E_n = \frac{n^2h^2}{8mL^2} \qquad \dots (8)$$

where, $\quad n = 1, 2, 3, \dots$

- From the above equation, the smallest value of energy that the particle can have is

$$E_1 = \frac{h^2}{8mL^2}$$

which is non-zero. This contradicts classical mechanics, according to which the particle can have zero energy.

- The other possible values of energy are $E_2 = \frac{4h^2}{8mL^2}$, $E_3 = \frac{9h^2}{8mL^2}$, etc.
- These energy values are discrete. They are not continuous as expected from classical mechanics.
- Thus, according to quantum mechanics, the particle inside a rigid box cannot have all values of energy, but only those discrete values which are given by equation (8).
- These discrete energy values are known as energy eigen values. These energy levels are shown in Fig. Q.55.2

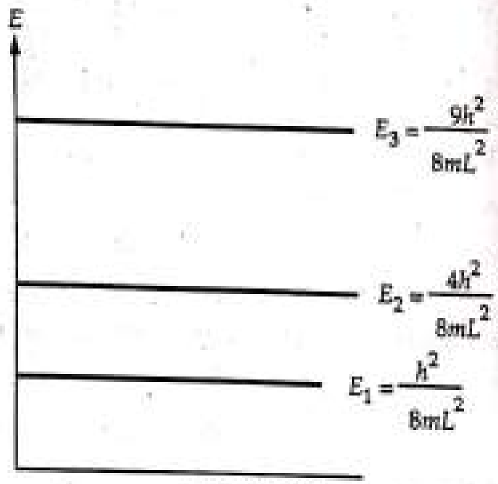


Fig. Q.55.2

Wave function :

- Substituting equation (7) in equation (6), we get,

$$\psi = A \sin\left(\frac{n\pi x}{L}\right) \qquad \dots (9)$$

- The complex conjugate of ψ is

$$\psi^* = A \sin\left(\frac{n\pi x}{L}\right)$$

To normalize the wave function, we find $\int_0^L \psi\psi^* dx$

$$\int_0^L \psi\psi^* dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= A^2 \int_0^L \left[\frac{1-\cos\left(\frac{n\pi x}{L}\right)}{2}\right] dx$$

$$= \frac{A^2}{2}\left[x - \frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)}\right]_0^L$$

$$= \frac{A^2}{2}[L-0]$$

$$\therefore \qquad \int_0^L \psi\psi^* dx = \frac{A^2 L}{2}$$

- Let the R.H.S. be N^2 i.e.,

$$N^2 = \frac{A^2 L}{2}$$

or

$$N = A\sqrt{\frac{L}{2}}$$

- The normalized wave function ψ_n is obtained using

$$\psi_n = \frac{\psi}{N}$$

$$\therefore \qquad \psi_n = \frac{A \sin\left(\frac{n\pi x}{L}\right)}{\left(A\sqrt{\frac{L}{2}}\right)}$$

$$\therefore \quad \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad \dots (10)$$

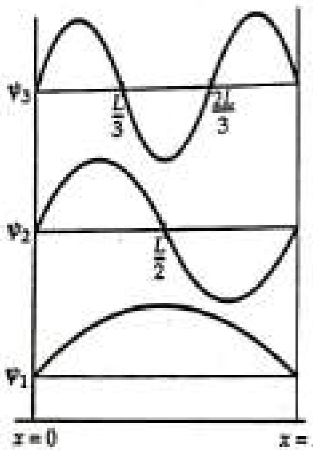
- These normalized wave functions are called eigen functions.

$$\psi_n^* = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

- The probability function is

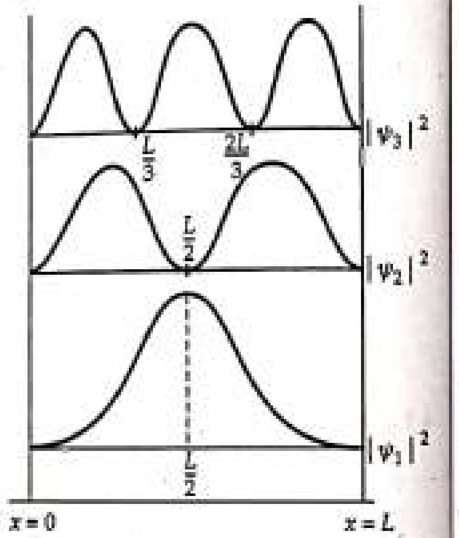
$$P(x) = |\psi_n|^2 = \psi_n \psi_n^* = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right) \quad \dots (11)$$

- The wave function and probability functions for $n = 1, 2, 3$ are shown in Fig. Q.55.3



(a) Wave function

Fig. Q.55.3 (a)



(b) Probability function

Fig. Q.55.3 (b)

Q.56 Derive an expression for energy of a particle trapped in an infinite potential well. **[SPPU : Dec.-13, 16 Marks 6]**

Ans. : Refer Q.55

Q.57 Derive equation of energy when a particle is confined to an infinite potential well. Draw first three energy levels for an electron confined in it. **[SPPU : May-18, Marks 6]**

Ans. : Refer Q.55

Q.58 Derive expression for the energy and wave function of a particle enclosed in an infinite potential well (rigid box). **[SPPU : Dec.-18, Marks 6]**

Ans. : Refer Q.55.

Important Formula

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Q.59 Compare the lowest three energy states for (i) An electron confined in a potential well of width 10 A° and (ii) A grain of dust with mass 10^{-6} gm in an infinite potential well of width 0.1 mm. What can you conclude from this comparison ? **[SPPU : Dec.-03]**

Ans. : For a particle of mass 'm' in an infinite potential well, the energy levels are given by,

Formula : $E_n = \dfrac{n^2 h^2}{8mL^2}$

i) Given :

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$L = 10 \text{ A°} = 10 \times 10^{-10} \text{ m}$$

$$\therefore \quad L = 10^{-9} \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$\therefore \quad E_n = \frac{n^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} \text{ J}$$

$$= n^2 \times 6.038 \times 10^{-20} \text{ J}$$

$$= \frac{n^2 \times 6.038 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E_n = 0.3774 \, n^2 \text{ eV}$$

The lowest three energy states are obtained for $n = 1, 2$ and 3.

$$E_1 = 0.3774 \text{ eV}$$

$$E_2 = 0.3774 \times 2^2 = 1.5096 \text{ eV}$$

$$E_3 = 0.3774 \times 3^2 = 3.3966 \text{ eV}$$

ii) **Given :**

$$m = 10^{-6}\ \text{gm} = 10^{-9}\ \text{kg}$$

$$L = 0.1\ \text{mm} = 10^{-4}\ \text{m}$$

$$E_n = \frac{n^2 \times (6.63\times 10^{-34})^2}{8\times 10^{-9} \times (10^{-4})^2}\ \text{J}$$

$$= n^2 \times 5.4946\times 10^{-51}\ \text{J}$$

$$= \frac{n^2 \times 5.4946\times 10^{-51}}{1.6\times 10^{-19}}\ \text{eV}$$

$\therefore$ $$E_n = 3.434\times 10^{-32}\ n^2\ \text{eV}$$

The lowest three energy states are,

$$E_1 = 3.434\times 10^{-32}\ \text{eV}$$

$$E_2 = 3.434\times 10^{-32} \times 2^2 = 1.37365\times 10^{-31}\ \text{eV}$$

$$E_3 = 3.434\times 10^{-32} \times 3^2 = 3.091\times 10^{-31}\ \text{eV}$$

The energy levels of dust are very close to each other and hence will appear continuous whereas the energy difference is larger for electrons due to which the energy levels will appear discrete. Hence the quantization of energy will be observed more easily for the smaller particle, i.e., electron, compared to the larger dust particles.

Q.60 Lowest energy of an electron trapped in a potential well is 38 eV. Calculate the width of the well.

[SPPU : Dec.-04,17, May-05,19]

Ans. : Given :

$$E_1 = 38\ \text{eV} = 38\times 1.6\times 10^{-19}\ \text{J}$$

$$h = 6.63\times 10^{-34}\ \text{J-s}$$

$$m = 9.1\times 10^{-31}\ \text{kg}$$

Formula : $$E_1 = \frac{h^2}{8mL^2}$$

$$L^2 = \frac{h^2}{8mE_1}$$

$$= \frac{(6.63\times 10^{-34})^2}{8\times 9.1\times 10^{-31} \times 38\times 1.6\times 10^{-19}}$$

$$L^2 = 9.93\times 10^{-21}$$

$\therefore$ $$L = 9.965\times 10^{-11}\ \text{m}$$

$\therefore$ $$\boxed{L = 0.9965\ \text{A}^\circ}$$

Q.61 An electron is bound by a potential which closely approaches an infinite square well of width 1 A°. Calculate the lowest three permissible energies (in electron volts) the electron can have.

[SPPU : Dec.-05, May-10,11,18]

Ans. : Given data :

$$h = 6.63\times 10^{-34}\ \text{J-s}$$

$$m = 9.1\times 10^{-31}\ \text{kg}$$

$$L = 1\ \text{A}^\circ = 10^{-10}\ \text{m}$$

Formula : $$E_n = \frac{n^2h^2}{8mL^2}$$

$\therefore$ $$E_n = \frac{n^2 \times (6.63\times 10^{-34})^2}{8\times 9.1\times 10^{-31} \times (10^{-10})^2}\ \text{J}$$

$$= n^2 \times 6.038\times 10^{-18}\ \text{J}$$

$$= \frac{n^2 \times 6.038\times 10^{-18}}{1.6\times 10^{-19}}\ \text{eV}$$

$\therefore$ $$E_n = 37.74\ n^2\ \text{eV}$$

The lowest three permissible energies are

$$E_1 = 37.74\ \text{eV}$$

$$E_2 = 37.74\times 2^2 = 150.96\ \text{eV}$$

$$E_3 = 37.74 \times 3^2 = 339.66 \text{ eV}$$

Q.62 An electron is trapped in a rigid box of width 2 A°. Find its lowest energy level and momentum. Hence find energy of the third energy level. **[SPPU : May-06,12, Dec.-18]**

Ans. : Given :

For lowest energy, $n = 1$

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$L = 2 \text{ A}° = 2 \times 10^{-10} \text{ m}$$

Formula :

$$E_n = \frac{n^2 h^2}{8mL^2}, \; p = \sqrt{2mE}$$

$$E_1 = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} \text{ J}$$

$$= 1.5095 \times 10^{-18} \text{ J}$$

$$= \frac{1.5095 \times 10^{-18}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore \quad \boxed{E_1 = 9.434 \text{ eV}}$$

The corresponding momentum is,

$$p_1 = \sqrt{2m E_1}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 1.5095 \times 10^{-18}}$$

$$\therefore \quad \boxed{p_1 = 1.66 \times 10^{-24} \text{ kg-m/s}}$$

The third energy level is

$$E_3 = E_1 \times 3^2$$

$$= 9.434 \times 3^2$$

$$\therefore \quad \boxed{E_3 = 84.906 \text{ eV}}$$

Q.63 An electron is confined in an infinite potential well of width 5 A°. Calculate the energy and wavelength of the emitted photon if the electron makes a transition from its n = 2 energy level to n = 1.

Ans. : Given data :

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$L = 5 \text{ A}° = 5 \times 10^{-10} \text{ m}$$

Formula :

$$E_n = \frac{n^2 h^2}{8mL^2}$$

For $n = 1$,

$$E_1 = \frac{h^2}{8mL^2}$$

For $n = 2$,

$$E_2 = \frac{4h^2}{8mL^2}$$

The energy of emitted photon is $E_2 - E_1$, i.e.,

$$E_2 - E_1 = \frac{3h^2}{8mL^2}$$

$$\therefore \quad E_2 - E_1 = \frac{3 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (5 \times 10^{-10})^2}$$

$$= 7.246 \times 10^{-19} \text{ J}$$

$$= \frac{7.246 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore \quad \boxed{E_2 - E_1 = 4.53 \text{ eV}}$$

The energy of photon in terms of wavelength is $\frac{hc}{\lambda}$

$$\therefore \quad \frac{hc}{\lambda} = 7.246\times 10^{-19}\ J$$

$$c = 3\times 10^{8}\ m/s$$

$$\therefore \quad \lambda = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{7.246\times 10^{-19}}$$

$$\therefore \quad \boxed{\lambda = 2.745\times 10^{-7}\ m}$$

Q.64 Compute energy diffrence between the ground state and first excited state for an electron in a one-dimensional rigid box of length 10^{-8} cm. [SPPU : Dec.-10, Marks 4]

Ans. : Given data :

$$h = 6.63\times 10^{-34}\ J\text{-}s$$

$$m = 9.1\times 10^{-31}\ kg$$

$$L = 10^{-8}\ cm = 10^{-10}\ m$$

Formula :

$$E_1 = \frac{h^2}{8\,mL^2}$$

and in first excited state,

$$E_2 = \frac{4\,h^2}{8\,mL^2}$$

The difference in energy is,

$$E_2 - E_1 = \frac{3h^2}{8\,mL^2}$$

$$= \frac{3\times(6.63\times 10^{-34})^2}{8(9.1\times 10^{-31})\,(10^{-10})^2}$$

$$= 1.8114\times 10^{-17}\ J$$

$$= \frac{1.8114\times 10^{-17}}{1.6\times 10^{-19}}\ eV$$

$$\therefore \quad \boxed{E_2 - E_1 = 113.21\ eV}$$

Q.65 The lowest energy of an electron trapped in a rigid box is 4.19 eV. Find the width of the box in A.U. [SPPU : Dec.-15, Marks 3]

Ans. : Given data :

$$E_1 = 4.19\ eV = 4.19\times 1.6\times 10^{-19}\ J$$

$$h = 6.63\times 10^{-34}\ Js$$

$$m = 9.1\times 10^{-31}\ kg$$

Formula :

$$E_1 = \frac{h^2}{8\,m\,L^2}$$

$$\therefore \quad 4.19\times 1.6\times 10^{-19} = \frac{(6.63\times 10^{-34})^2}{8\times 9.1\times 10^{-31}\times L^2}$$

$$\therefore \quad \boxed{L = 3\times 10^{-10}\ m}$$

Q.66 A neutron is trapped in an infinite potential well of width 10^{-14} m. Calculate its first energy eigen value in eV. [SPPU : May-16, Marks 3]

Ans. : Given data :

$$h = 6.63\times 10^{-34}\ Js$$

$$m = 1.675\times 10^{-27}\ kg$$

$$L = 10^{-14}\ m$$

Formula :

$$E_1 = \frac{h^2}{8\,mL^2}$$

$$E_1 = \frac{(6.63\times 10^{-34})^2}{8\times 1.675\times 10^{-27}\times(10^{-14})^2}$$

$$= 3.28\times 10^{-13}\ J$$

$$= \frac{3.28\times 10^{-13}}{1.6\times 10^{-19}}\ eV$$

$$\therefore \quad \boxed{E_1 = 2.05\times 10^{6}\ eV}$$

Q.67 Calculate the energy required to excite the electron from its ground state to fourth excited state in rigid box of length 0.1 nm. [SPPU : Dec.-16, Marks 3]

Ans. : Given data :

$$h = 6.63\times10^{-34}\ \text{J s}$$
$$m = 9.1\times10^{-31}\ \text{kg}$$
$$L = 0.1\ \text{nm} = 0.1\times10^{-9}\ \text{m}$$

Formula : $$E_n = \frac{n^2h^2}{8mL^2}$$

For ground state, $$E_1 = \frac{h^2}{8mL^2}$$

For fourth excited state,

$$E_5 = \frac{5^2h^2}{8mL^2} = \frac{25h^2}{8mL^2}$$

Energy required to excite the electron is

$$E_5 - E_1 = \frac{25h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2}$$

$$E_5 - E_1 = \frac{3\times(6.63\times10^{-34})^2}{8\times9.1\times10^{-31}\times(0.1\times10^{-9})^2}$$

$$= 181\times10^{-17}\ \text{J}$$

$$= \frac{181\times10^{-17}}{1.6\times10^{-19}}\ \text{eV}$$

$$\therefore \boxed{E_5 - E_1 = 113.125\ \text{eV}}$$

Q.68 A neutron is trapped in an infinite potential well of width 1 Å. Calculate the values of energy and momentum in its ground state. [SPPU : May-17, Marks 3]

Ans. : Given data :

$$h = 6.63\times10^{-34}\ \text{Js},\quad m = 1.675\times10^{-27}\ \text{kg}$$

$$L = 1\ \text{Å} = 1\times10^{-10}\ \text{m}$$

Formula : $$E = \frac{h^2}{8mL^2},\ p = \sqrt{2mE}$$

$$E = \frac{(6.63\times10^{-34})^2}{8\times1.675\times10^{-27}\times(1\times10^{-10})^2}$$

$$\therefore \boxed{E = 3.28\times10^{-21}\ \text{J}}$$

$$p = \sqrt{2mE}$$
$$= \sqrt{2\times1.675\times10^{-27}\times3.28\times10^{-21}}$$

$$\therefore \boxed{p = 3.315\times10^{-24}\ \text{kg-m/s}}$$

6.9 : Particle in a Non-Rigid Box (Finite Potential Well)

Q.69 Discuss the problem of particle in a non - rigid box.

Ans. :
- Consider another one dimensional problem of a particle of mass 'm' confined to a non-rigid box of length 'L'.
- As the walls are not rigid, the potential energy of the particle would be finite, say V_0 outside the box. Inside the box, the potential energy is constant, taken to be zero for convenience

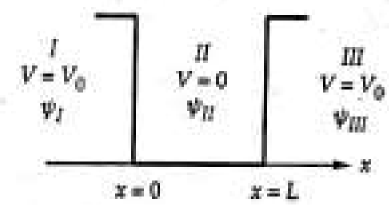


Fig. Q.69.1

- The potential energy distribution is shown in Fig. Q.69.1
- We consider a particular case of interest where $E < V_0$, i.e. the particle energy is less than the energy required to overcome the barrier represented by the walls.
- Schrödinger's time independent wave equation in one dimension is

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \quad \text{... (1)}$$

- In region I, $V = V_0$ and $\psi = \psi_I$

$$\therefore \quad \frac{d^2\psi_I}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V_0)\psi_I = 0$$

As $E < V_0$, $E - V_0 < 0$

$\therefore$ Let $\quad \frac{8\pi^2 m}{h^2}(E-V_0) = -K'^2$

$$\therefore \quad \frac{d^2\psi_I}{dx^2} - K'^2\psi_I = 0$$

- The general solution of this equation is,

$$\psi_I = Ae^{K'x}+Be^{-K'x} \quad \text{... (2)}$$

- In region II, $V = 0$ and $\psi = \psi_{II}$

$$\therefore \quad \frac{d^2\psi_{II}}{dx^2}+\frac{8\pi^2 m}{h^2}E\psi_{II} = 0$$

Let $\quad K^2 = \frac{8\pi^2 mE}{h^2}$

$$\therefore \quad \frac{d^2\psi_{II}}{dx^2}+K^2\psi_{II} = 0$$

- The general solution of this equation is,

$$\psi_{II} = Ce^{iKx}+De^{-iKx} \quad \text{... (3)}$$

- In region III, $V = V_0$, $\psi = \psi_{III}$

$$\therefore \quad \frac{d^2\psi_{III}}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V_0)\psi_{III} = 0$$

$$\therefore \quad \frac{d^2\psi_{III}}{dx^2} - K'^2\psi_{III} = 0$$

- The general solution of this equation is

$$\psi_{III} = Fe^{K'x}+Ge^{-K'x} \quad \text{... (4)}$$

- The condition in region I is

$$\psi_I \to 0 \text{ as } x \to -\infty$$

$\therefore$ From equation (2),

$$B = 0$$

$$\therefore \quad \psi_I = Ae^{K'x} \quad \text{... (5)}$$

- Similarly in region III,

$$\psi_{III} \to 0 \text{ as } x \to \infty$$

$$\therefore \quad F = 0$$

$$\therefore \quad \psi_{III} = Ge^{-K'x} \quad \text{... (6)}$$

- The boundary conditions to be imposed on the wave function are :

i) Continuity of wave function at $x = 0$:

$$[\psi_I]_{x=0} = [\psi_{II}]_{x=0}$$

ii) Continuity of wave function at $x = L$:

$$[\psi_{II}]_{x=L} = [\psi_{III}]_{x=L}$$

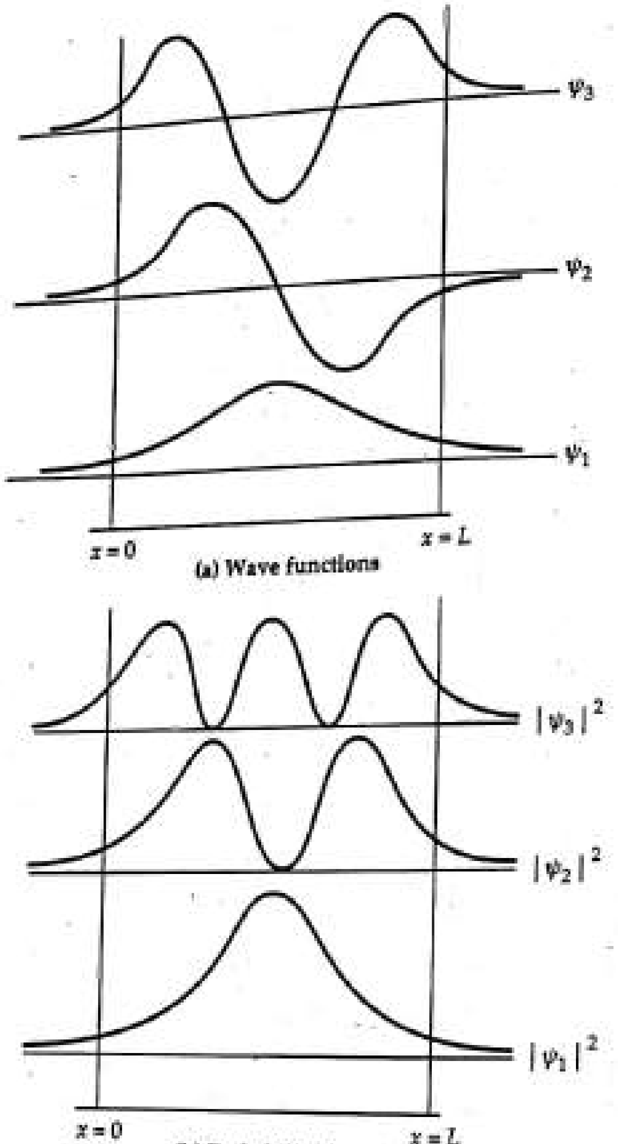
iii) Continuity of first order derivative of wave function at $x = 0$:

$$\left[\frac{d\psi_I}{dx}\right]_{x=0} = \left[\frac{d\psi_{II}}{dx}\right]_{x=0}$$

iv) Continuity of first order derivative of wave function at $x = L$

$$\left[\frac{d\psi_{II}}{dx}\right]_{x=L} = \left[\frac{d\psi_{III}}{dx}\right]_{x=L}$$

- Using these four boundary conditions, the four arbitrary constants A, C, D and G in equations (3), (5) and (6) can be evaluated to obtain solution for ψ_I, ψ_{II} and ψ_{III}.
- The nature of ψ_{II} is similar to the wave function in a rigid box, except that the values at $x = 0$ and $x = L$ are not zero. ψ_I has a non-zero value at $x = 0$ and decreases exponentially to zero as $x \to -\infty$. ψ_{III} has a non-zero value at $x = L$ and decreases exponentially to zero as $x \to +\infty$. The energy eigen values are discrete. The eigen functions and the corresponding probability functions are shown in Fig. Q.69.2



(a) Wave functions

(b) Probability functions

Fig. Q.69.2

- In addition to the discrete energy values and prediction of certain forbidden regions, like in the case of a rigid box, another significant result is the finite probability of finding the particle outside the box (as shown in Fig. Q.69.2 (b)).
- Even though the particle does not have sufficient energy to overcome the barrier, still there is some finite probability of the particle escaping from the box.
- This explains the phenomenon of α - decay where α-particles do not have sufficient energy to come out of the nucleus but still do escape from the nucleus. The tunnel diode and scanneling tunneling microscope work on the same principle of particles being able to 'tunnel' through a barrier even though they do not have sufficient energy.

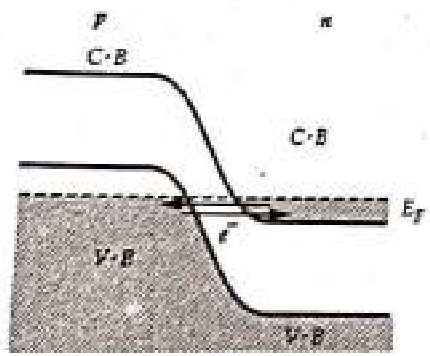
6.10 : Tunnel Diode

Q.70 Explain tunneling effect. How is this principle used in a tunnel diode. [SPPU : Dec.-17, Marks 4]

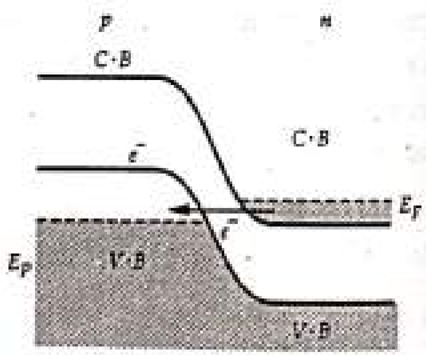
Ans. : • When impurity concentration in an n-type semiconductor is small. The donor levels are discrete as there is negligible interaction amongst the donor atoms.

- As the donor impurity concentration is increased the interaction amongst donor atoms increases. Due to this the donor levels which are just below the conduction band spread to form an energy band and overlap with the bottom of the conduction band.
- As a result the fermilevel for n-type semiconductors with high doping concentration lies in the conduction band.
- Similarly, the fermi level in a heavily doped p-type semiconductor lies in the valence band.
- The tunnel diode is formed by heavily doped p and n-type semiconductors. The depletion region in these diodes is very thin.

- The conduction band of n-type overlaps with the valence band of p-type so as to equalize the fermi levels as shown in Fig. Q.70.1
- Under equilibrium conditions when no voltage is applied across the diode, the electrons from conduction band of n-type tunnel across the potential barrier in the depletion layer into valence band of p-type and the electrons from valence band of p-type tunnel into conduction band of n-type in equal numbers. Hence there is no current through the diode.
- When a small forward biased voltage is applied to the diode, the conduction band and fermi level in n-type is slightly raised with respect to valence band in p-type as shown in Fig. Q.70.1.
- The lower part of the n-type conduction band containing electrons is just next to the upper part of the valence band in p-type which is empty.
- As a result the electrons from the higher energy levels in conduction band of n-type tunnel into the lower energy levels in valence band of p-type.
- Tunnelling of electrons is not possible from p-type to n-type as the filled energy levels of p-type are at lower level compared to



(a)



(b)

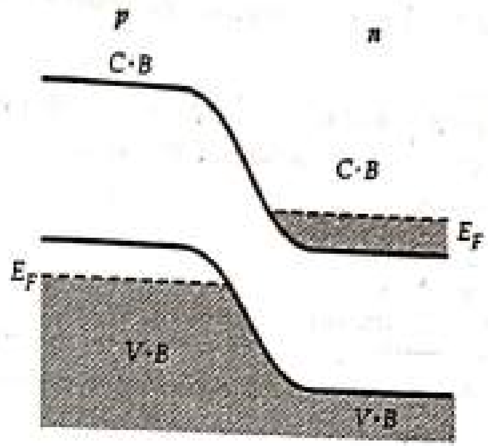


Fig. Q.70.1 (c)

empty levels of n-type. As the electrons tunnel from n-type to p-type, it constitutes a current from p-type to n-type.

- If the forward baised voltage is increased the conduction band of n-type is raised further with respect to valence band and the two bands no longer overlap as shown in Fig. Q.70.1 (c) as a result the tunnelling current decreases.

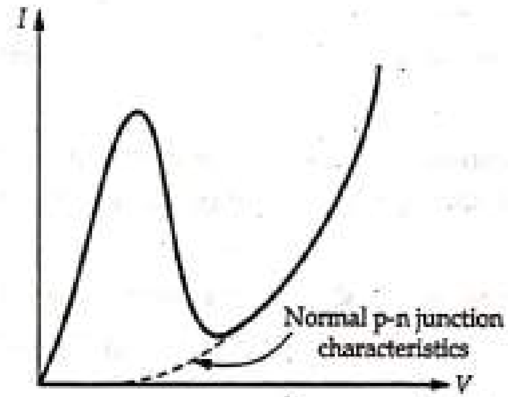


Fig. Q.70.2

- For further increase in voltage, the diffussion current starts due to injected charge carriers and the current increases exponentially as in any other p-n junction diode.

- The tunnel diode voltage-current characteristics are as shown in Fig. Q.70.2.
- The response time of tunnel diodes to small changes in voltage is very small. Hence they are used in high frequency oscillators and fast switching circuits.

6.11 : Scanning Tunnelling Microscope

Q.71 Explain in brief, working of Scanning Tunnelling Microscope (STM). [SPPU : May - 14, Marks 4]

Ans. : • The scanning tunneling microscope consists of a probe with a fine tip that is made to move on a surface which is to be studied.

- The probe is kept at a positive potential with respect to that surface and kept at a distance of about 1 nm above the surface.
- The microscope can be used in either constant height mode or the constant current mode.

Constant height mode :

- In this mode, the probe is maintained at a constant height above the surface.
- Due to the positive potential of the probe tip, electrons on the surface tunnel through to the probe tip giving rise to a small current.
- As the tip moves on the surface at a constant height, the distance of the tip from the surface changes due to the irregularities on the surface.
- As a result, the probe current changes. It increases when the distance of the probe tip from the surface decreases.
- Thus the probe current gives an indication of the surface topography when the probe tip is scanned on the surface as shown in Fig. Q.71.1 (see fig.Q.71.1 on next page)

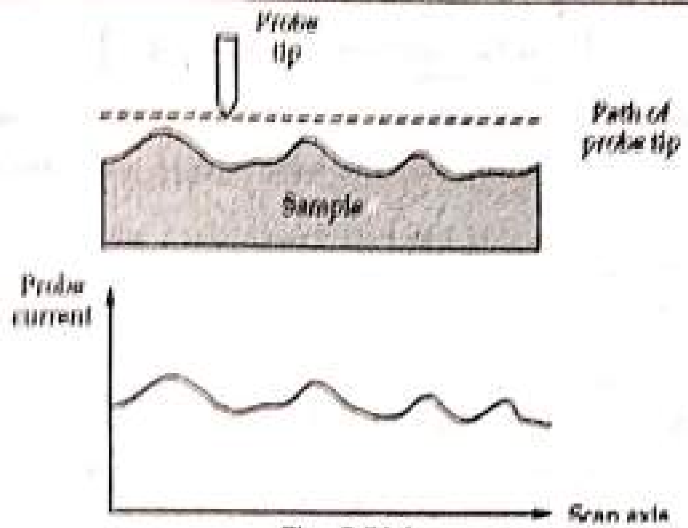


Fig. Q.71.1

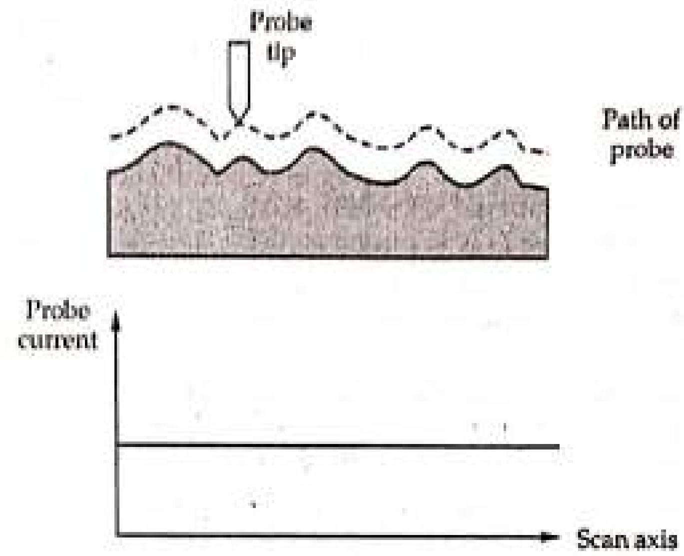


Fig. Q.71.2

Constant current mode :

- In this mode, the distance of the probe tip from the surface is varied so as to get a constant current.
- The movement of the probe tip perpendicular to the surface is recorded which is directly the surface topography as shown in above Fig. Q.71.2.

6.12 : Quantum Computing

Q.72 Explain the basic principle of working of quantum computers.

Ans. : • In digital computers, the basic unit of information is the 'bit'.

- A bit is a system that has two states, which we call '0' and '1'. For example, it can be a swith with two states 'ON' representing '1' and 'OFF' representing '0'
- The bits are classical states and hence a system will be either in its' '0' state or '1' state.
- The information is processed by digital logic gates.
- Quantum computing uses a basic unit of information known as qubit . It is acronyn for quantum bit.
- Quantum mechanics predicts that if a system can exist in two distinguishable states say 0 and 1, it can also exist in states which are coherent superposition of the two distinguishable states.
- A qubit can exist in state 0, state 1 or both 0 and 1 simultaneously in some proportion.
- A classical 3-bit register can store 8 numbers in on of its 8 possible configurations. 000, 001, 010, ... 111, which represent numbers from 0 to 7. It is in one of these states at an instant.
- A quantum 3-bit register consisting of 3 qubits stores these 8 numbers simultaneously. The 8 different numbers are physically present in the register simultaneously.
- A register made of n qubits is capable of storing 2^n numbers simultaneously. Hence it is possible to perform methematical operations on all 2^n numbers simultaneously.
- This will greatly enhance the computing speed.
- A qubit can be an atom. A register of 250 atom is capable of storing more numbers simultaneously than the estimated number of atoms in the universe.
- The quantum computer is made of qubits to store information which is processed by quantum logic gates.
- Quantum computers have the potential to increase computing power exponentially.

END...

UNIT - IV

7 Semiconductor Physics

7.1 : Free Electron Theory

Q.1 State the assumption in classical free electron theory.

Ans. :

Assumptions in classical free electron theory

1) The metal atoms have a core, which consists of a nucleus surrounded by the inner electrons and the valence electrons. The positive ion core along with the valence electrons is electrically neutral. The valence electrons are free to move throughout the volume of the sample.
2) The electrostatic forces of attraction between the free electrons and the ion cores are negligible.
3) The electrostatic forces of repulsion amongst the free electrons are negligible.
4) The electrons cannot escape from the metal due to potential barrier at the surface. Inside the metal the potential is constant.
5) The free electrons are equivalent to gas molecules and hence the kinetic theory of gases is applicable to them. In absence of electric field, they have random thermal motion. Their energies are distributed according to the Maxwell-Boltzmann distribution.

Q.2 Define (i) drift velocity, (ii) mean tree path, (iii) relaxation time.

Ans. : (i) drift velocity : The electrons do not move in a straight line opposite to the electric field due to collisions with ion cores. They move in a zig-zag path with a small average velocity opposite to the electric field which is known as the **drift velocity**.

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(ii) mean free path : The average time between two collisions is known as **mean collision time** and the average distance covered by the electrons between two collisions is known as **mean free path.**

(iii) relaxation time : When applied electric field is turned off, the drift velocity decreases and falls off to zero exponentially. The time required by the drift velocity to decrease to $\left(\frac{1}{e}\right)$ times the initial value is known as the **relaxation time.**

Q.3 List the successes and failures of classical free electron theory.

Ans. : Success of Classical Free Electron Theory

1) The classical free electron theory is able to explain the electrical and thermal conductivity of metals.
2) Ohm's law can be derived using classical free electron theory.
3) Optical properties of metals can be explained on the basis of this theory.

Failure of Classical Free Electron Theory

1) The classical free electron theory predicts that the resistivity is directly proportional to square root of the absolute temperature. However, experimental evidence shows that the resistivity is directly proportional to the absolute temperature.
2) According to classical theory, the free electron gas should contribute to the molar specific heat at constant volume of metal an amount equal to

$$C_V = \frac{3}{2}R$$

Experiments show that the contribution of free electrons to the molar specific heat at constant volume of any metal is much smaller ($C_V = 10^{-4}\ RT$). Also, the contribution of free electrons is found to be temperature dependent which is not predicted by the classical free electron theory.

3) According to classical theory, the mean free path must be of the order of the interatomic spacing in the crystal. However, calculations from various experiments show that the mean free

path is many interatomic spacings and it also *depends on* temperature. As the temperature decreases, the *mean free path* increases.

4) The classical free electron theory predicts larger conductivity for divalent and trivalent atoms like zinc and aluminium compared to monovalent atoms like copper and silver as it is based on the assumption that the valence electrons take part in conduction. This is contrary to observations that copper and silver are better conductors than zinc and aluminium.
5) The classical free electron theory predicts that the magnetic susceptibility of metals to be inversely proportional to temperature whereas experimentally it is established that the susceptibility is independent of temperature.

7.2 : Kronig-Penney Model

Q.4 Explain the Kronig-Penney model. Describe the concept of allowed and forbidden energy bands in a single crystal qualitatively from the result of Kronig-Penney model.

Ans. : • Kronig and Penney considered the electrons to be moving in a variable potential region in the crystal instead of being free.

• The potential was approximated by a square well periodic potential as shown in Fig. Q.4.1.

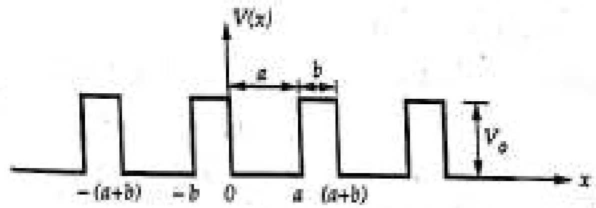


Fig. Q.4.1

• The time independent Schroedinger wave equation in one dimension is,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

or $$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+(E-V)\psi = 0$$

- In region $0 < x < a$, where $v = 0$, the general solution for wave equation is,

$$\psi_1 = Ae^{iKx}+Be^{-iKx} \quad \dots (1)$$

and the energy is,

$$E = \frac{\hbar^2 K^2}{2m} \quad \dots (2)$$

- In the region $-b < x < 0$, $V = V_0$. *The solution of wave equation is,*

$$\psi_2 = Ce^{Qx}+De^{-Qx} \quad \dots (3)$$

- The continuity of wave function at $x = 0$ requires that the values of ψ_1 and ψ_2 be equal at this point.

$$\psi_1\big|_{x=0} = \psi_2\big|_{x=0}$$

$\therefore$ $$A+B = C+D \quad \dots (4)$$

- $\frac{d\psi}{dx}$ is also continuous at $x = 0$

$\therefore$ $$\frac{d\psi_1}{dx}\bigg|_{x=0} = \frac{d\psi_2}{dx}\bigg|_{x=0}$$

$\therefore$ $$iK(A-B) = Q(C-D) \quad \dots (5)$$

- By Bloch theorem,

$$\psi(a<x<a+b) = \psi(-b<x<0)e^{ik(a+b)}$$

- where k is the wave vector.
- The condition for continuity of ψ at $x = a$ with Bloch theorem is

$$\psi_1\big|_{x=a} = \psi_2\big|_{x=-b}\, e^{ik(a+b)}$$

$\therefore$ $$Ae^{iKa}+Be^{-iKa} = (Ce^{-Qb}+De^{Qb})e^{ik(a+b)} \quad \dots (6)$$

- For continuity of $\frac{d\psi}{dx}$ at $x = a$,

$$\frac{d\psi_1}{dx}\bigg|_{x=a} = \frac{d\psi_2}{dx}\bigg|_{x=-b} \cdot e^{ik(a+b)}$$

$\therefore$ $$iK(Ae^{iKa}-Be^{iKa}) = Q(Ce^{-Qb}-De^{Qb})e^{ik(a+b)} \quad \dots (7)$$

- Equation (4), (5), (6) and (7) have solutions only if the determinant of coefficients of A, B, C and D vanishes. This leads to the condition.

$$\frac{P}{Ka}\sin Ka+\cos Ka = \cos ka \quad \dots (8)$$

where $$P = \frac{Q^2 ba}{2}$$

- The R.H.S. of equation (8) is $\cos ka$ which lies between -1 and $+1$ Hence solutions are obtained only when the L.H.S lies between -1 and $+1$.
- The graph of $\frac{P}{Ka}\sin Ka+\cos Ka$ plotted for different values of Ka is shown in Fig. Q.4.2.
- In regions of the graph where the value of the function is either greater than 1 or less than 1, solutions for ψ do not exist.
- The corresponding energies which are forbidden can be obtained using equation (2).

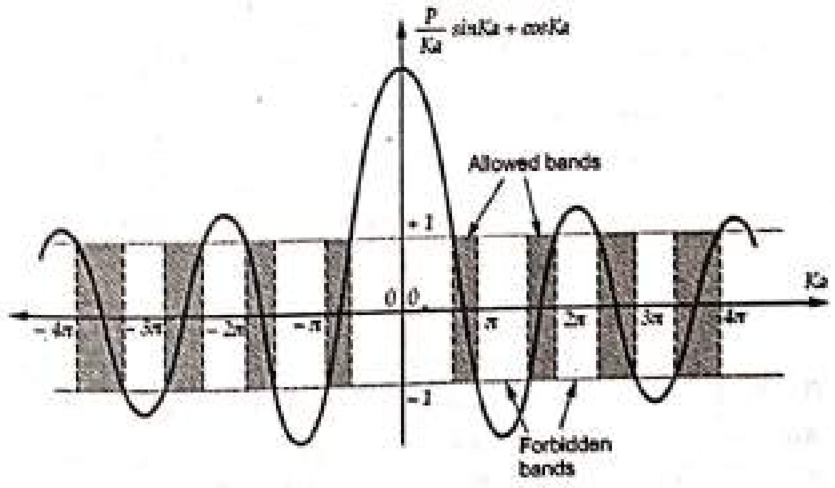


Fig. Q.4.2

- Thus, there are bands of allowed energies a separated by energies which are not allowed and hence known as forbidden bands.
- The graph of energy E for different wave vectors k is shown in Fig. Q.4.3. There are discontinuities at $k = \pm\frac{\pi}{a}, \pm\frac{2\pi}{a}$, ... which correspond to the condition that $\left|\frac{P}{Ka} \sin Ka + \cos Ka\right| > 1$.
- Thus according to Kronig-Penney model, the motion of electrons in a periodic potential in crystals gives rise to certain allowed energy bands separated by forbidden energy bands.

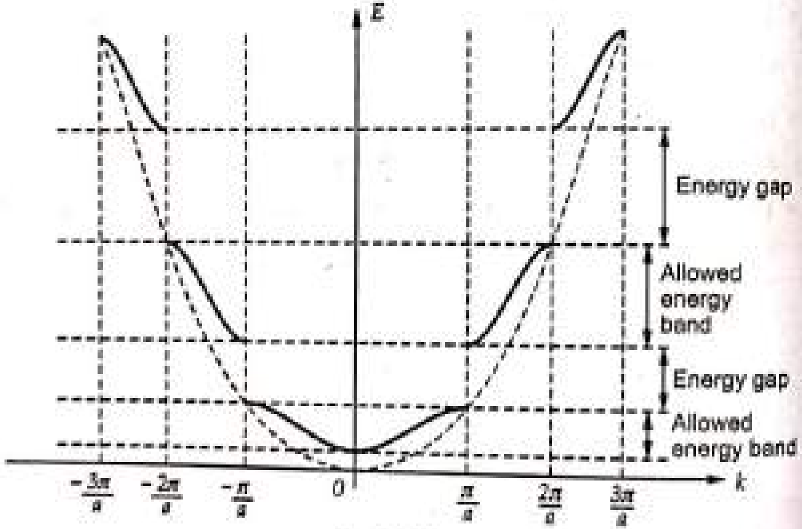


Fig. Q.4.3

7.3 : Band Theory of Solids

Q.5 Explain Classification of solids on the basic of band theory.

Ans. : Solids can be classified into conductors, insulators and semiconductors based on their energy band structure.

1) **Conductors :** • In conductors, the valence band and the conduction band overlap. There is no forbidden band.

- The electrons can be made to move and constitute a current by applying a small potential difference.
- The resistivity of conductors is very low and increases with temperature. Hence the conductors are said to have a positive temperature coefficient of resistance.
- The energy band structure is shown in Fig. Q.5.1.
- Metals like copper, silver, gold, aluminium etc. are good conductors of electricity.

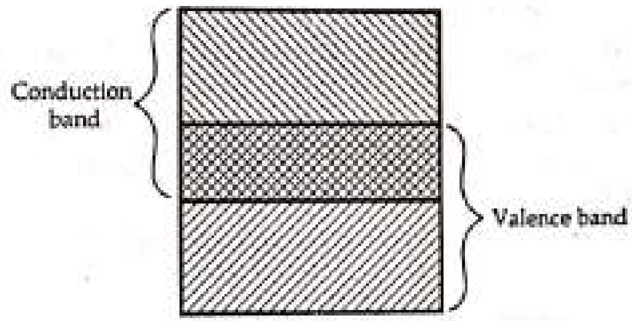


Fig. Q.5.1

2) **Insulators :** • Insulators have a completely filled valence band and an empty conduction band which are separated by a large forbidden band.

- The band gap energy is large (of the order of 5 eV). Hence large amount of energy is required to transfer electrons from valence band to conduction band.
- The insulators have very low conductivity and high resistivity. Diamond, wood, glass etc., are insulators.

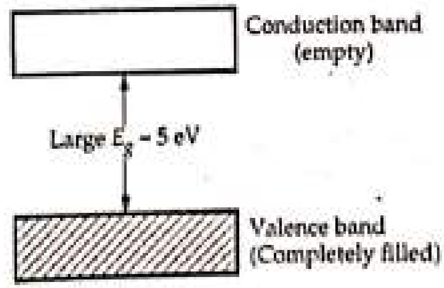


Fig. Q.5.2

- The energy band structure is shown in Fig. Q.5.2.

3) Semiconductors : • In semiconductors, the valence band is completely filled and the conduction band is empty at absolute zero temperature.

- The valence band and conduction band are separated by a small forbidden band of the order of 1 eV.
- Hence, compared to insulators, smaller energy is required to transfer the electrons from valence band to conduction band. Hence the conductivity is better than insulators but not as good as the conductors.
- Silicon and germanium are semicondutors having band gap energies of 1.1 eV and 0.7 eV respectively.
- Some compounds formed between group III and group V elements like gallium arsenide (GaAs) are also semiconductors.
- The energy band structure is shown in Fig. Q.5.3.

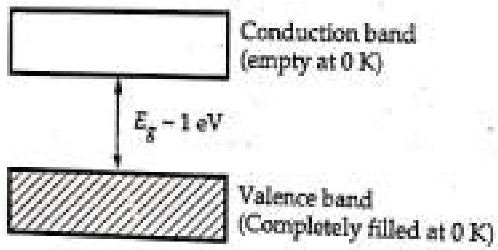


Fig. Q.5.3

- As temperature is increased, electrons from valence band jump to conduction band leaving a vacancy in valence band which is known as hole.
- The free electrons in conduction band and the holes in valence band take part in conduction.
- Hence conductivity increases and resistivity decreases with increase in temperature. The semiconductors are said to have **a negative temperature coefficient.**

Q.6 Calculate the band gap energy in silicon given that it is transparent to radiation of wavelength greater than 11000 A°

[SPPU : May-07, Dec.-17, 18, Marks 3]

Ans. : Given data :

$$h = 6.63 \times 10^{-34} \text{ J - s}$$
$$c = 3 \times 10^8 \text{ m/s}$$
$$\lambda = 11000 \text{ A}° = 11000 \times 10^{-10} \text{ m}$$

Formula : $E_g = \dfrac{hc}{\lambda}$

$$\therefore \quad E_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{11000 \times 10^{-10}}$$
$$= 1.8082 \times 10^{-19} \text{ J} = \frac{1.8082 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore \quad \boxed{E_g = 1.13 \text{ eV}}$$

Q.7 Calculate the wavelength at which germanium starts to absorb light. The energy gap in germanium is 0.7 eV.

Ans. : Given data :

$$h = 6.63 \times 10^{-34} \text{ J - s}$$
$$c = 3 \times 10^8 \text{ m/s}$$
$$E_g = 0.7 \text{ eV} = 0.7 \times 1.6 \times 10^{-19} \text{ J}$$

Formula : $E_g = \dfrac{hc}{\lambda}$

$$\therefore \quad \lambda = \frac{hc}{E_g}$$
$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.7 \times 1.6 \times 10^{-19}} = 1.776 \times 10^{-6} \text{ m}$$
$$\therefore \quad \boxed{\lambda = 17760 \text{ A}°}$$

Q.8 Calculate the band gap energy in Germanium. Given that it is transparent to radiation of wavelength greater than 17760 A°.

[SPPU : May-10]

Ans. :

Given data :

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$c = 3 \times 10^{8} \text{ m/s}$$

$$\lambda = 17760 \text{ A}^{\circ} = 17760 \times 10^{-10} \text{ m}$$

Formula :

$$E_g = \frac{hc}{\lambda}$$

$$E_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{17760 \times 10^{-10}}$$

$$= 1.12 \times 10^{-19} \text{ J} = \frac{1.12 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore \quad \boxed{E_g = 0.7 \text{ eV}}$$

7.4 : Effective Mass of an Electron

Q.9 Derive an expression for effective mass of an electron. How is it used to describe motion of a hole ? Also find effective mass of a free electron.

Ans. : • An electron in a crystal is not free. When an external field is applied, the electron in a crystal behaves as if it had a mass different from its actual mass.

- Consider an electron described as a wave packet having wave function in the region of wave vector k.
- Let the electron be in a crystal to which an electric field $\mathcal{E}$ is applied. The group velocity of the wave packet will be

$$v_g = \frac{d\omega}{dk}$$

- If E is the energy of the wave packet,

$$E = h\nu = \frac{h}{2\pi} \cdot 2\pi\nu$$

$$\therefore \quad E = \hbar\omega$$

$$\therefore \quad \omega = \frac{E}{\hbar}$$

$$v_g = \frac{d\omega}{dk}$$

$$\therefore \quad v_g = \frac{1}{\hbar}\frac{dE}{dk} \quad \ldots (1)$$

$$\frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk\,dt}$$

$$\therefore \quad \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} \quad \ldots (2)$$

- The work done dE by the electric field $\mathcal{E}$ in a time interval dt is

$$dE = -e\mathcal{E}\, v_g\, dt \quad \ldots (3)$$

- (The force on the electron is $-e\,\mathcal{E}$ and the displacement is $v_g dt$)
- As $\quad dE = \frac{dE}{dk} \cdot dk$

and $\quad E = \hbar v_g \quad$ from equation (1)

$$dE = \hbar v_g\, dk \quad \ldots (4)$$

- From equations (3) and (4),

$$\hbar\, dk = -e\mathcal{E}\, dt$$

$$\therefore \quad -e\mathcal{E} = \hbar\frac{dk}{dt} \quad \ldots (5)$$

- The above equation describes the force $-e\mathcal{E}$ due to an external field $\mathcal{E}$ on an electron interms of the rate of change of wave vector k. Hence we can write

$$F = \hbar\frac{dk}{dt} \quad \ldots (6)$$

$$\therefore \quad \frac{dk}{dt} = \frac{F}{\hbar}$$

- Substituting in equation (2)

$$\frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk^2} \cdot \frac{F}{\hbar}$$

$$\therefore \quad F = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}\frac{dv_g}{dt}$$

- As $\frac{dv_g}{dt}$ has dimensions of acceleration, the quantity $\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$ is defined as the effective mass (m^*) of an electron.

$$\therefore \quad m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \quad \dots (7)$$

- Thus the curvature $\left(\frac{d^2E}{dk^2}\right)$ of the energy band decides the effective mass of an electron in a crystal.
- The curvature $\frac{d^2E}{dk^2}$ is negative for an electron at the top of the valence band and positive at the bottom of conduction band as shown in Fig. Q.9.1.
- Hence the effective mass of an electron is negative near the top of valence band and positive near the bottom of conduction band.

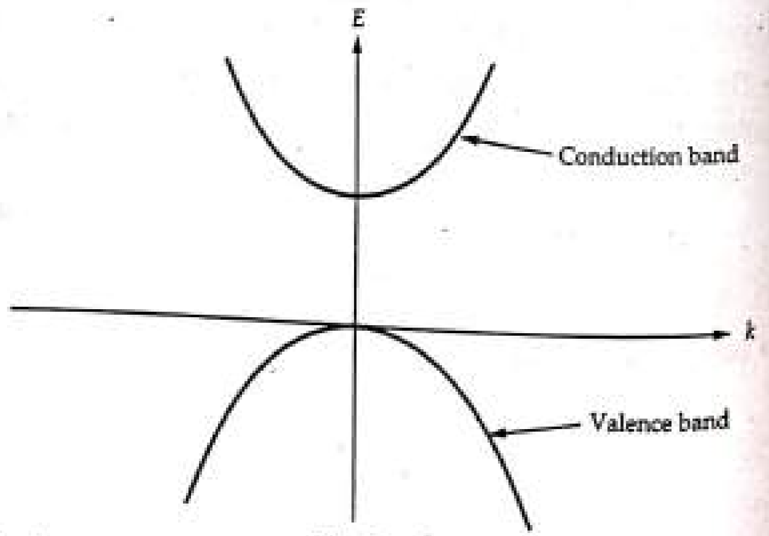


Fig. Q.9.1

- The motion of valence band electrons with negative charge and negative mass can be equivalently described by motion of holes having positive charge and positive effective mass in same direction.
- For a free electron,

$$E = \frac{p^2}{2m}$$

$$P = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda}$$

$$\therefore \quad P = \hbar k$$

$$\therefore \quad E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m} \quad \text{From equation (7),}$$

$$m^* = m$$

i.e. the effective mass is same as its mass for a free electron.

7.5 : Density of States

Q.10 Derive an expression for density of states for free electrons in metals.

Ans. : • The conduction band electrons in metals are treated as free electrons in the free electron gas model which more in three dimensions within the boundary of the specimen.

- As the electrons are considered as free electrons within the specimen, their potential energy can be taken as zero inside the specimen which we will consider to be a cube of side L.
- The time independent Schroedinger wave equation in three dimensions with $V = 0$ becomes,

$$\nabla^2\psi + \frac{8\pi^2 m}{4^2} E\psi = 0 \quad \dots (1)$$

$$-\frac{h^2}{8\pi^2 m}\nabla^2\psi = E\psi$$

$$\therefore \quad -\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = E\psi \qquad \dots (2)$$

- The solution of above equation i.e. of the form

$$\psi = A\, e^{i\vec{k}\cdot\vec{r}} \qquad \dots (3)$$

- The wavefunction ψ has to be periodic in x, y and z with period L i.e,

$$\psi(x+L, y, z) = \psi(x, y, z)$$
$$\psi(x, y+L, z) = \psi(x, y, z) \text{ and}$$
$$\psi(x, y, z+L) = \psi(x, y, z)$$

- These conditions are satisfied only if the components of wave vector k_x, k_y, k_z take values $0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \dots$ etc.

- Hence the components of k must be of the form $\frac{2n\pi}{L}$.

$$\therefore \quad k_x = \frac{2n_x\pi}{L},\ k_y = \frac{2n_y\pi}{L} \text{ and } k_z = \frac{2n_z\pi}{L}$$

where n_x, n_y and n_z are integers

- From equation (3)

$$\psi = A e^{i(k_x x + k_y y + k_z z)} \qquad \dots (4)$$

$$[\text{as } \vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k} \text{ and } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}]$$

$$\frac{\partial^2\psi}{\partial x^2} = -Ak_x^2\, e^{i(k_x x + k_y y + k_z z)}$$

$$\frac{\partial^2\psi}{\partial y^2} = -Ak_y^2\, e^{i(k_x x + k_y y + k_z z)}$$

$$\frac{\partial^2\psi}{\partial z^2} = -Ak_z^2\, e^{i(k_x x + k_y y + k_z z)}$$

Substituting in equation (2),

$$\left(\frac{-h^2}{8\pi^2 m}\right)\left[-A\left(k_x^2 + k_y^2 + k_z^2\right) e^{i(k_x x + k_y y + k_z z)}\right] = EAe^{i(k_x x + k_y y + k_z z)}$$

$$\therefore \quad \frac{h^2}{8\pi^2 m}(k_x^2 + k_y^2 + k_z^2) = E$$

$$\therefore \quad E = \frac{h^2}{8\pi^2 m}\left[\left(\frac{2\pi n_x}{L}\right)^2 + \left(\frac{2\pi n_y}{L}\right)^2 + \left(\frac{2\pi n_z}{L}\right)^2\right]$$

$$\therefore \quad E = \frac{h^2}{8\pi^2 m} \times \frac{4\pi^2}{L^2}(n_x^2 + n_y^2 + n_z^2)$$

- Let $n_x^2 + n_y^2 + n_z^2 = n^2$ then,

$$E = \frac{h^2 n^2}{2mL^2} \qquad \dots (5)$$

- The equation $n_x^2 + n_y^2 + n_z^2 = n^2$ represents a sphere in n-space with radius n. Hence the number of allowed energy states per unit volume can be obtained by using the volume of this sphere in n-space.

- As each energy level corresponding to a unique set of values of n_x, n_y and n_z can accommodate two electrons according to Pauli's exclusion principle, the total number of allowed states will be $2\times\left(\frac{4}{3}\pi n^3\right)$.

- If N is the number of energy states per unit volume,

$$N \cdot L^3 = 2\times\left(\frac{4}{3}\pi n^3\right)$$

$$\therefore \quad n^2 = \left(\frac{3NL^3}{8\pi}\right)^{2/3} \qquad \dots (6)$$

Substituting in equation (5) we get,

$$E = \frac{h^2}{2mL^2}\left(\frac{3NL^3}{8\pi}\right)^{2/3}$$

$$\therefore \quad E = \frac{h^2}{2m}\left(\frac{3N}{8\pi}\right)^{2/3}$$

$$\therefore \quad N = \left(\frac{2mE}{h^2}\right)^{3/2} \times \frac{8\pi}{3} = \frac{8\pi}{3} \times \frac{2\sqrt{2}\, m^{3/2}\, E^{3/2}}{h^2}$$

$$\therefore \quad N = \frac{16\sqrt{2}\,\pi\, m^{3/2}\, E^{3/2}}{3h^3} \quad \dots (7)$$

- The density of states, $g(E)dE$, which is the number of allowed energy states in the energy range E and $E+dE$, is obtained by differentiating equation (7) w.r.t. E.

$$g(E)\,dE = \frac{dN}{dE}\,dE = \frac{16\sqrt{2}\,\pi\, m^{3/2}}{3h^3} \times \frac{3}{2}E^{\frac{1}{2}}\,dE$$

$$\therefore \quad g(E)\,dE = \frac{8\sqrt{2}\,\pi\, m^{3/2}}{h^3}E^{\frac{1}{2}}\,dE \quad \dots (8)$$

7.6 : Fermi-Function and Fermi-Level

Q.11 Explain Fermi-Dirac distribution function specifying the meaning of each term in it. [SPPU : May - 14, Dec - 14, Marks 3]

Ans. : The Fermi-Dirac probability distribution function, also known as Fermi function, is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

where $P(E)$ = Probability of an electron occupying the energy state E

E_F = Fermi energy

k = Boltzmann constant

and T = Absolute temperature

Q.12 Explain Fermi energy and Fermi level. Show the variation of Fermi function with energy at 0 K and at a higher temperature. [SPPU : May - 15, Marks 3]

Ans. : The Fermi-Dirac probability distribution function, also known as Fermi function, is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

where $P(E)$ = Probability of an electron occupying the energy state E

E_F = Fermi energy

k = Boltzmann constant

and T = Absolute temperature

for $T = 0$ K, if $E > E_F$

$$P(E) = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty}$$

$$\therefore \quad P(E) = 0$$

i.e. no electron can have energy greater than the Fermi energy at 0 K. It means that all energy states above the Fermi energy are empty at 0 K.

for $T = 0$ K, if $E < E_F$,

$$P(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0}$$

$$\therefore \quad P(E) = 1$$

i.e, all electrons occupy energy states below the Fermi energy at 0 K.

- Thus, all energy states below Fermi energy are filled and energy states above Fermi energy are empty at 0 K. Hence Fermi energy as the highest occupied energy state at 0 K.

for $T > 0$ K, if $E = E_F$,

$$P(E_F) = \frac{1}{1+e^{0}} = \frac{1}{1+1}$$

$$P(E_F) = \frac{1}{2}$$

∴ i.e, the Fermi energy level represents the energy state with a 50 % probability of being filled if forbidden gap does not exist as in the case of good conductors.

- The Fermi function is shown for different temperatures in Fig. Q.12.1.

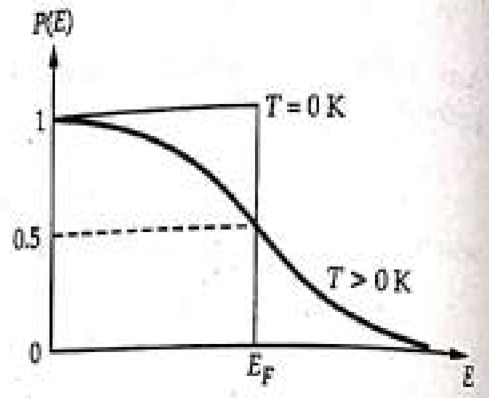


Fig. Q.12.1

Important Formula

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

Q.13 Find the temperature at which there is 1.0 % probability that a state with an energy 0.5 eV above Fermi energy will be occupied.

Ans. : The Fermi distribution function is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$P(E) = 1.0\ \% = 0.01$$

$$E-E_F = 0.5\ \text{eV} = 0.5 \times 1.6 \times 10^{-19}\ \text{J}$$

∴ $$E-E_F = 8 \times 10^{-20}\ \text{J}$$

$$k = 1.38 \times 10^{-23}\ \text{J/K}$$

∴ $$0.01 = \frac{1}{1+e^{\left(\frac{8\times10^{-20}}{1.38\times10^{-23}\,T}\right)}}$$

$$1+e^{\left(\frac{5797.1}{T}\right)} = 100$$

$$e^{\left(\frac{5797.1}{T}\right)} = 99$$

$$\frac{5797.1}{T} = 4.595$$

∴ $$\boxed{T = 1261.6\ \text{K}}$$

Q.14 Show that if the probability of occupancy is x at an energy level ΔE below the Fermi level, then x is also probability of non-occupancy at an energy level ΔE above the Fermi level.

Ans. : The Fermi distribution function is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

For $E-E_F = -\Delta E$, $P(E) = x$

∴ $$x = \frac{1}{1+e^{-\Delta E/kT}} \quad \text{... (1)}$$

For $E-E_F = \Delta E$,

$$P(E+\Delta E) = \frac{1}{1+e^{\Delta E/kT}}$$

- $P(E+\Delta E)$ is the probability of occupancy of energy level ΔE above the Fermi level.
- ∴ The probability of non-occupancy of energy level ΔE above the Fermi level is

$$1-P(E+\Delta E) = 1-\frac{1}{1+e^{\Delta E/kT}}$$

$$= \frac{1+e^{\Delta E/kT}-1}{1+e^{\Delta E/kT}} = \frac{e^{\Delta E/kT}}{1+e^{\Delta E/kT}}$$

$$= \frac{1}{e^{-\Delta E/kT}+1}$$ (dividing numerator and denominator by $e^{\Delta E/kT}$)

$$\boxed{1-P(E+\Delta E) = x}$$

Q.15 Find the probability with which an energy level 0.02 eV below Fermi level will be occupied at room temperature of 300 K and at 1000 K.

Ans. : The Fermi distribution function is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$E-E_F = -0.02 \text{ eV}$$

$$k = 1.38\times10^{-23} \text{ J/K} = 8.625\times10^{-5} \text{ eV/K}$$

For $T = 300$ K

$$P(E) = \frac{1}{1+e^{\left(\frac{-0.02}{8.625\times10^{-5}\times300}\right)}}$$

$\therefore$ $P(E) = 0.684$

For $T = 1000$ K

$$P(E) = \frac{1}{1+e^{\left(\frac{-0.02}{8.625\times10^{-5}\times1000}\right)}}$$

$\therefore$ $P(E) = 0.56$

> Note : It is clear from these values that the probability of occupancy of energy levels below Fermi level decreases on increasing the temperature.

Q.16 Calculate the probability of an electron occupying an energy level 0.02 eV above the Fermi level at 200 K and 400 K in a material.

Ans. : The Fermi distribution function is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$E-E_F = 0.02 \text{ eV}$$

$$k = 1.38\times10^{-23} \text{ J/K}$$

$$= 8.625\times10^{-5} \text{ eV/K}$$

For $T = 200$ K,

$$P(E) = \frac{1}{1+e^{\left(\frac{0.02}{8.625\times10^{-5}\times200}\right)}}$$

$\therefore$ $\boxed{P(E) = 0.239}$

For $T = 400$ K

$$P(E) = \frac{1}{1+e^{\left(\frac{0.02}{8.625\times10^{-5}\times400}\right)}}$$

$\therefore$ $\boxed{P(E) = 0.359}$

> Note : The probability of occupancy of energy levels above Fermi level increases on increasing the tempereture.

Q.17 The Fermi level in potassium is 2.1 eV. What are the energies for which the probabilities of occupancy at 300 K are 0.99, 0.01 and 0.5 ?

Ans. : The Fermi distribution function is

$$P(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$1+e^{(E-E_F)/kT} = \frac{1}{P(E)}$$

$$e^{(E-E_F)/kT} = \frac{1}{P(E)}-1$$

$$\frac{E-E_F}{kT} = ln\left[\frac{1}{P(E)}-1\right]$$

$\therefore$
$$E = E_F + kT\,ln\left[\frac{1}{P(E)}-1\right]$$

$$E_F = 2.1 \text{ eV}$$

$$k = 1.38\times10^{-23} \text{ J/K} = 8.625\times10^{-5} \text{ eV/K}$$

$$T = 300 \text{ K}$$

For $P(E) = 0.99$

$$E = 2.1 + 8.625\times10^{-5}\times300\,ln\left[\frac{1}{0.99}-1\right]$$

$$\therefore \quad \boxed{E = 1.981 \text{ eV}}$$

For $\quad P(E) = 0.01$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300 \, ln \left[\frac{1}{0.01} - 1\right]$$

$$\therefore \quad \boxed{E = 2.219 \text{ eV}}$$

For $\quad P(E) = 0.5$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300 \, ln \left[\frac{1}{0.5} - 1\right]$$

$$\therefore \quad \boxed{E = 2.1 \text{ eV}}$$

7.7 : Conductivity of Conductor

Q.18 Derive expressions for conductivity and current in a conductor.

Ans. : • Resistivity or specific resistance is defined as the resistance of a conductor of unit length and unit cross-section area. It is related to the resistance 'R' by the following expression.

$$R = \rho \frac{l}{a} \qquad \dots (1)$$

where

ρ = Resistivity

l = Length of conductor and

a = Area of cross-section

• Conductivity is defined as the reciprocal of resistivity. i.e.,

$$\text{Conductivity } \sigma = \frac{1}{\rho}$$

• By ohm's law,

$$V = IR$$

$$\therefore \quad I = \frac{V}{R}$$

• Substituting from equation (1),

$$I = \left(\frac{V}{\rho \frac{l}{a}}\right) = \frac{V}{l} \times \frac{a}{\rho}$$

• The current density 'J' is defined as the current flowing per unit cross section area.

$$\therefore \quad J = \frac{I}{a} = \frac{V}{l} \cdot \frac{1}{\rho}$$

• As $\frac{V}{l} = E$, the electric field and

$$\frac{1}{\rho} = \sigma, \text{ the conductivity,}$$

$$J = \sigma E \qquad \dots (2)$$

• Current density is also defined as the amount of charge that flows through unit cross section area per unit time.

• Consider a conductor of unit cross section area and length 'v_d' where 'v_d' is the drift velocity of the electrons.

Let n = Number of electrons per unit volume

and e = Charge of an electron

Then the distance travelled by electrons in 1 s = $v_d \times 1 = v_d$

• As electrons travel distance v_d in 1 s, all electrons within a distance 'v_d' from the unit cross-section area shown in Fig. Q.18.1 will cross the area.

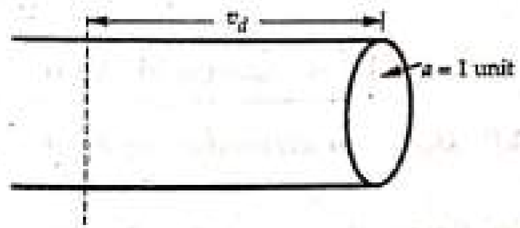


Fig. Q.18.1

$\therefore$ The volume of the conductor from which all electrons will cross the unit area = $v_d \times a = v_d$ (as a = 1 unit)

$\therefore$ Number of electrons that cross unit area in 1 s = $n v_d$

$\therefore$ Amount of charge that flows through the unit area in 1 s = $J = n e v_d$ ... (3)

- From equations (2) and (3),

$$\sigma E = n e v_d$$

$$\therefore \quad \sigma = n e \frac{v_d}{E} \quad \text{... (4)}$$

- For the same electric field the charges which acquire larger drift velocity are said to be more mobile. Hence the quantity $\frac{v_d}{E}$ is a measure of mobility of charge carriers.

$$\therefore \quad \text{Mobility } \mu = \frac{v_d}{E} \quad \text{... (5)}$$

$\therefore$ From equation (4)

$$\sigma = n e \mu \quad \text{... (6)}$$

- An equation for current can be obtained from equation (3). As $J = \frac{I}{a}$,

$$I = J \cdot a$$

$$\therefore \quad I = n e a v_d \quad \text{... (7)}$$

Important Formulae

$$I = n e a v_d$$

$$\sigma = n e \mu$$

7.8 : Conductivity in a Semiconductor

Q.19 Derive an expression for conductivity in semiconductor. [SPPU : Dec - 12, May - 11, Marks 6]

OR Derive the expression for the conductivity of intrinsic and extrinsic semiconductor. [SPPU : May - 14, Marks 6]

Ans. : • In a semiconductor, the current is due to free electrons as well as holes.

- The current due to electrons can be written as,

$$I_e = n_e e a v_e \quad \text{... (1)}$$

where v_e = Drift velocity of electrons

n_e = Number density of electrons

- Similarly the current due to holes is

$$I_h = n_h e a v_h \quad \text{... (2)}$$

where v_h = Drift velocity of holes

and n_h = Number density of holes

- The total current is

$$I = I_e + I_h$$

$$\therefore \quad I = e a (n_e v_e + n_h v_h)$$

- The current density $J = \frac{I}{a}$

$$\therefore \quad J = e (n_e v_e + n_h v_h) \quad \text{... (3)}$$

- Also,

$$J = \sigma E$$

$$\therefore \quad \sigma E = e (n_e v_e + n_h v_h)$$

$$\sigma = e\left(n_e \frac{v_e}{E} + n_h \frac{v_h}{E}\right)$$

$$\frac{v_e}{E} = \mu_e, \text{ the mobility of electrons}$$

and $$\frac{v_h}{E} = \mu_h, \text{ the mobility of holes.}$$

$$\therefore \quad \sigma = e (n_e \mu_e + n_h \mu_h) \quad \text{... (4)}$$

- For intrinsic semiconductors, $n_e = n_h = n_i$ where n_i is called the density of intrinsic charge carriers.

$$\therefore \quad \sigma = n_i e (\mu_e + \mu_h) \quad \text{... (5)}$$

- For n-type semiconductors, $n_e >> n_h$

$\therefore$ From equation (4),

$$\sigma \approx n_e e \mu_e \quad \text{... (6)}$$

- As each donor atom contributes one free electron, n_e is also the density of donor impurity atoms.

- For p-type semiconductors, $n_h >> n_e$

$\therefore$
$$\sigma = n_h e \mu_h$$

where n_h is the density of holes which is same as the density of acceptor impurity atoms.

- The current due to motion of electrons and holes in semiconductor under the action of an externally applied electric field is known as drift current.
- In semiconductors, another type of current can flow when there is difference in density of charge carriers. This current which is due to density difference of charge carriers is known as diffusion current.

Important Formulae

For intrinsic semiconductors, conductivity $\sigma = n_i e(\mu_e + \mu_h)$

For n-type semiconductors, conductivity $\sigma = n_e e \mu_e$

For p-type semiconductors, conductivity $\sigma = n_h e \mu_h$

Q.20 Calculate the conductivity of a germanium sample if a donor impurity is added to the extent of one part in 10^7 Ge atoms at room temperature.

Given : Avagadro number $N_a = 6.02 \times 10^{23}$ atoms/gm-mole
Atomic weight of Ge = 72.6, Mobility μ_e = 3800 cm^2/V-sec,
Density of Ge = 5.32 gm/cc

Ans. : Given data :

As density of Ge = 5.32 gm/cc, mass of 1 cc of Ge is 5.32 gm. 72.6 gm of Ge contains 6.02×10^{23} atoms.

$\therefore$ Number of Ge atoms in 1 cc $= \frac{6.02 \times 10^{23}}{72.6} \times 5.32 = 4.41 \times 10^{22}$

i.e., there are 4.41×10^{22} Ge atoms per cc. As there is one donor impurity per 10^7 Ge atoms, the density of donor impurity atoms which is same as density of free electrons is

$$n_e = \frac{4.41 \times 10^{22}}{10^7} = 4.41 \times 10^{15}/cc$$

Formula : $\sigma_n = n_e e \mu_e$

$e = 1.6 \times 10^{-19}$ C, $\mu_e = 3800\ cm^2/V\text{-}s$

$\therefore$ $\sigma_n = 4.41 \times 10^{15} \times 1.6 \times 10^{-19} \times 3800$

$\therefore$ $\boxed{\sigma_n = 2.6813 \text{ mho/cm}}$

Q.21 : Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is 1.6×10^{10} per cm^3. Take $\mu_e = 1500\ cm^2/V\text{-}s$ and $\mu_h = 500\ cm^2/V\text{-}s$ at room temperature. [SPPU : May-04, 05]

Ans. : Given data : $n_i = 1.6 \times 10^{10}$ per cm^3

$e = 1.6 \times 10^{-19}$ C

$\mu_e = 1500\ cm^2/V\text{-}s$

$\mu_h = 500\ cm^2/V\text{-}s$

Formula : $\sigma_i = n_i e(\mu_e + \mu_h)$

$\therefore$ $\sigma_i = 1.6 \times 10^{10} \times 1.6 \times 10^{-19}(1500+500)$

$\therefore$ $\boxed{\sigma_i = 5.12 \times 10^{-6} \text{ mho/cm}}$

Q.22 : Calculate the number of donor atoms which must be added to an intrinsic semiconductor to obtain the resistivity as $10^{-6}\ \Omega$ cm.

Given : $\mu_e = 1000\ cm^2/V$ - sec [SPPU : Dec.-07]

Ans. : Given data : $\rho = 10^{-6}\ \Omega$-cm

$e = 1.6 \times 10^{-19}$ C

$\mu_e = 1000\ cm^2/V\text{-sec}$

Formula : $\rho = \frac{1}{\sigma}$

$\therefore$ $\rho = \frac{1}{n_e e \mu_e}$

$$\therefore \quad n_e = \frac{1}{\rho e \mu_e}$$

$$\therefore \quad n_e = \frac{1}{10^{-6} \times 1.6 \times 10^{-19} \times 1000}$$

$$\therefore \quad \boxed{n_e = 6.25 \times 10^{21} \ per\ cm^3}$$

Q.23 Calculate the number of acceptors to be added to a Germanium sample to obtain the resistivity of 10 Ω-cm.
Given : $\mu = 1700\ cm^2/V$ - sec. **[SPPU : May-08, 12, 19, Marks 3]**

Ans. : Given data : $\rho = 10\ \Omega\text{-cm}$

$$e = 1.6 \times 10^{-19}\ C$$

$$\mu_h = 1700\ cm^2/V\text{-}s$$

Formula : $$\rho = \frac{1}{n_h e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{\rho e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{10 \times 1.6 \times 10^{-19} \times 1700}$$

$$\therefore \quad \boxed{n_h = 3.676 \times 10^{14} \ per\ cm^3}$$

Q.24 Calculate the current produced in a small Germanium plate of area 1 cm^2 and of thickness 0.3 mm when a P.D. of 2 V is applied across the faces.
Given : $n_i = 2 \times 10^{19}/m^3$, $e = 1.6 \times 10^{-19} C$, $\mu_e = 0.36\ m^2/V\text{-}s$, $\mu_h = 0.17\ m^2/V\text{-}s$

Ans. : Given data : $V = 2\ V, l = 0.3\ mm = 0.3 \times 10^{-3}\ m$

$$A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$$

Formula : $$J = \frac{I}{A} = \sigma E$$

$$\therefore \quad I = \sigma E A$$

As $$E = \frac{V}{l}, \text{ and } \sigma = n_i e (\mu_e + \mu_h)$$

$$I = n_i e (\mu_e + \mu_h) \frac{V}{l} A$$

$$\therefore \quad I = 2 \times 10^{19} \times 1.6 \times 10^{-19} (0.36 + 0.17) \times \frac{2}{0.3 \times 10^{-3}} \times 1 \times 10^{-4}$$

$$\therefore \quad \boxed{I = 1.13\ A}$$

Q.25 Calculate the conductivity of Ge specimen if donor impurity is added to the extent of one part in 10^8 Ge atoms at room temperature. (Given : Atomic weight of Ge = 72.6, Density of Ge : 5.32 gm/cm^3, mobility of electrons = 3800 cm^2/v - sec. Avagadro number = 6.02×10^{23} atoms/mole). **[SPPU : Dec.-09, 18, May-17, Marks 3]**

Ans. : Given : As density of Ge = 5.32 gm/cm^3, mass of 1 cm^3 of Ge is 5.32 gm.

72.6 gm of Ge contains 6.02×10^{23} atoms.

$$\therefore \text{ Number of Ge atoms in } 1\ cm^3 = \frac{6.02 \times 10^{23}}{72.6} \times 5.32 = 4.41 \times 10^{22}$$

i.e, there are 4.41×10^{22} Ge atoms per cm^3.

As there is one donor impurity per 10^8 Ge atoms, the density of donor impurity atoms and hence density of free electrons is,

$$n_e = \frac{4.41 \times 10^{22}}{10^8} = 4.41 \times 10^{14} \text{ per } cm^3$$

$$e = 1.6 \times 10^{-19}\ C$$

$$\mu_e = 3800\ cm^2/v\text{-sec}$$

Formula : $$\sigma_n = n_e e \mu_e$$

$$\therefore \quad \sigma_n = 4.41 \times 10^{14} \times 1.6 \times 10^{-19} \times 3800$$

$$\therefore \quad \boxed{\sigma_n = 0.268\ \text{mho/cm}}$$

Q.26 Intrinsic silicon is doped with phosphorus, with the atomic ratio of 10^8(Si) : 1 (P). Calculate the conductivity of N type of silicon thus formed. Given mobility of electrons in silicon $m_e = 1400\ cm^2V^{-1}s^{-1}$. Atomic weight of intrinsic silicon = 28.085, Avogadro's number = 6.022×10^{23} atoms per mole, Density of silicon = 2.33 gm/cm^3. [SPPU : Dec.-11]

Ans. : Given : As density of silicon is 2.33 gm/cm^3, mass of 1 cm^3 silicon is 2.33 gm.

28.085 gm of silicon contains 6.022×10^{23} atoms,

$\therefore$ Number of silicon atoms in 1 cm^3 $= \frac{6.022\times10^{23}}{28.085}\times2.33 = 5\times10^{22}$

As there is 1 phosphorus atom 10^8 atoms of silicon, the density of free electrons is

$$n_e = \frac{5\times10^{22}}{10^8} = 5\times10^{14} \text{ per cm}^3$$

Formula :

$$\sigma_n = n_e e \mu_e$$

$$\sigma_n = n_e e \mu_e = 5\times10^{14}\times1.6\times10^{-19}\times1400$$

$$\therefore \quad \sigma_n = 0.112 \text{ mho/cm}$$

Q.27 The resistivity of copper wire of diameter 1.03 mm is 6.51 ohm per 300 m. The concentration of free electrons in copper is $8.4\times10^{28}/m^3$. If the current is 2 A, find the mobility of free electrons. [SPPU : May-12]

Ans. : Given data : $R = 6.51$ ohm ; $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(1.03\times10^{-3})^2$

$$l = 300 \text{ m}$$

Formula :

$$\rho = R\times\frac{A}{l}$$

$$\therefore \quad \rho = 6.51\times\frac{\frac{\pi}{4}\times(1.03\times10^{-3})^2}{200}$$

$$= 1808\times10^{-8} \text{ ohm-m}$$

$$\sigma = \frac{1}{\rho} = n e \mu$$

$$\therefore \quad \mu = \frac{1}{n e \rho} = \frac{1}{8.4\times10^{28}\times1.6\times10^{-19}\times1808\times10^{-8}}$$

$$\therefore \quad \boxed{\mu = 4.115\times10^{-3}\ m^2/V\text{-}s}$$

Q.28 Calculate the number of acceptors to be added to a germanium sample to obtain the resistivity of 20 Ω cm. Given : μ = 1700 $cm^2/V.sec$ [SPPU : Dec.-15, May-18, Marks 3]

Ans. : Given data : $\rho = 20\ \Omega$ cm , $e = 1.6\times10^{-19}$ C,

$$\mu_h = 1700\ cm^2/V\ sec.$$

Formula :

$$\rho = \frac{1}{n_h e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{\rho e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{20\times1.6\times10^{-19}\times1700}$$

$$\therefore \quad \boxed{n_h = 1.838\times10^{14} \text{ per cm}^3}$$

Q.29 Calculate the number of acceptor atoms that need to be doped in germanium sample to obtain the resistivity of 8 Ω cm. [Given : mobility μ = 1600 $cm^2/V\cdot s$] [SPPU : May-16, Marks 3]

Ans. : Given : $\rho = 8\ \Omega$ cm, $e = 1.6\times10^{-19}$ C

$$\mu_h = 1600\ cm^2/V\cdot s$$

Formula :

$$\rho = \frac{1}{n_h e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{\rho e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{8\times1.6\times10^{-19}\times1600}$$

$$\therefore \quad \boxed{n_h = 4.883\times10^{14} \text{ per cm}^3}$$

7.9 : Fermi-Level in Semiconductors

Q.30 Using expression for concentration of free electrons and holes, derive an expression for fermi level in intrinsic semiconductors. Describe its variation with temperature.

Ans. : In an intrinsic semiconductor,

$$n = p$$

$$\therefore 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_c - E_F)/kT} = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{-(E_F - E_v)/kT}$$

$$\therefore \quad (m_e^*)^{3/2} e^{-(E_c-E_F)/kT} = (m_h^*)^{3/2} e^{-(E_F-E_v)/kT}$$

$$e^{-(E_c-E_F)/kT} e^{(E_F-E_v)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$e^{-(E_c+E_v)/kT} e^{2E_F/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$e^{2E_F/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{(E_c+E_v)/kT}$$

Taking logs on both sides,

$$\frac{2E_F}{kT} = \frac{3}{2}\log_e\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c+E_v}{kT}$$

$$E_F = \frac{3}{4}kT\log_e\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c+E_v}{2} \quad \ldots(1)$$

If $m_h^* = m_e^*$,

$$\log_e\left(\frac{m_h^*}{m_e^*}\right) = \log_e 1 = 0$$

$$\therefore \quad E_F = \frac{E_c+E_v}{2} \quad \ldots(2)$$

- Hence the Fermi level lies at the centre of the forbidden band as shown in Fig. Q.30.1.
- Actually, $m_h^* > m_e^*$. Hence E_F is slightly raised above the centre of the forbidden band as temperature increases.

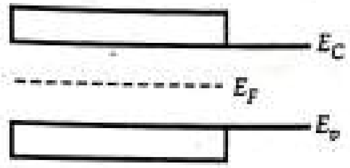


Fig. Q.30.1

Q.31 Derive expression for fermi level in an n-type semiconductor and describe its variation with temperature.

Ans. : The number of electrons in conduction band per unit volume is given by,

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_c - E_F)/kT} \quad \ldots(1)$$

- Let N_d be the density of donor atoms added to an intrinsic semiconductor and E_d be the energy of the donor impurities (which are just below the bottom of conduction band).
- The density of free electrons in conduction band will be equal to the density of ionized donor atoms as for every donor atom that is ionized, there will be one free electron.
- The probability of an electron having energy E_d is $f(E_d)$. The electron will either be in the donor level or in the conduction band. Hence the probability of finding the electron in conduction band is same as the probability that it is not in donor energy level, i.e. $1 - f(E_d)$.

$\therefore$ Density of ionized donor atoms $= N_d\,[1 - f(E_d)]$

$$= N_d\left[1 - \frac{1}{1+e^{(E_d-E_F)/kT}}\right] = N_d\left[\frac{1+e^{(E_d-E_F)/kT} - 1}{1+e^{(E_d-E_F)/kT}}\right]$$

$$= N_d \left[\frac{e^{(E_d-E_F)/kT}}{1+e^{(E_d-E_F)/kT}}\right] = N_d \left[\frac{1}{e^{-(E_d-E_F)/kT}+1}\right]$$

As E_F lies more than a few kT above E_d,

$$e^{-(E_d-E_F)/kT} >> 1$$

Hence, 1 can be neglected in the denominator.

$$\therefore \text{Density of ionized donor atoms} = N_d \left[\frac{1}{e^{-(E_d-E_F)/kT}}\right]$$

$$= N_d\, e^{(E_d-E_F)/kT}$$

Density of electrons in conduction band = Density of ionized donor atoms

$$\therefore 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_c-E_F)/kT} = N_d\, e^{(E_d-E_F)/kT}$$

$$2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} = N_d\, e^{(E_c+E_d-2E_F)/kT}$$

$$2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} = N_d\, e^{(E_c+E_d)/kT} e^{-2E_F/kT}$$

$$e^{2E_F/kT} = \frac{N_d e^{(E_c+E_d)/kT}}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}$$

$$e^{E_F/kT} = \frac{N_d^{1/2} e^{(E_c+E_d)/2kT}}{2^{1/2}\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4}} \quad \dots (2)$$

Substituting in equation (1) for $e^{E_F/kT}$,

$$n = 2^{1/2}\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4} N_d^{1/2} e^{-E_c/kT} e^{(E_c+E_d)/2kT}$$

$$\therefore \quad n = (2N_d)^{1/2}\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4} e^{(E_d-E_c)/2kT} \quad \dots (3)$$

Fermi Level in n - Type

Taking log of equation (2),

$$\frac{E_F}{kT} = \frac{E_c+E_d}{2kT} + \frac{1}{2}\log\left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

$$E_F = \frac{E_c+E_d}{2} + \frac{kT}{2}\log\left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

At 0 K,

$$E_F = \frac{E_c+E_d}{2}$$

- i.e, the Fermi level lies mid way between donor impurity levels and bottom of conduction band.
- As temperature increases, E_F shifts downwards. The Fermi level shifts upwards on increasing the doping concentration N_d.

Q.32 Derive expression for fermi level in p-type semiconductor and describe its variation with temperature.

Ans. : • The number of holes in valence band per unit volume is given by,

$$p = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{-(E_F-E_v)/kT} \quad \dots (1)$$

- Let N_a be the density of acceptor atoms added to an intrinsic semiconductor and E_a be the energy of the acceptor impurities (which are just above the top of valence band).

- When an acceptor atom gains an electron, it becomes a negative ion and the electron is in the energy level E_a.
- The probability of finding an electron in acceptor energy level is $f(E_a)$.

$\therefore$ Density of ionized acceptor atoms $= N_a f(E_a)$

$$= N_a \times \frac{1}{1+e^{(E_a-E_F)/kT}}$$

$E_a - E_F$ is large compared to kT.

$\therefore \quad e^{(E_a-E_F)/kT} >> 1$

Hence, 1 can be neglected in the denominator.

$\therefore$ Density of ionized acceptor atoms $= N_a \times \dfrac{1}{e^{(E_a-E_F)/kT}}$

Density of holes in valence band = Density of ionized acceptor atoms

$$\therefore 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{-(E_F-E_v)/kT} = N_a \times \frac{1}{e^{(E_a-E_F)/kT}}$$

$$2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} = N_a \times e^{(E_F-E_v-E_a+E_F)/kT}$$

$$2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} = N_a \times e^{2E_F/kT} e^{-(E_a+E_v)/kT}$$

$$e^{-2E_F/kT} = \frac{N_a e^{-(E_a+E_v)/kT}}{2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}}$$

$$e^{-E_F/kT} = \frac{N_a^{1/2} e^{-(E_a+E_v)/2kT}}{2^{1/2}\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/4}} \quad ...(2)$$

Substitute in equation (1).

$$\therefore \quad p = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{-(E_F-E_v)/kT}$$

$$= 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{-E_F/kT} e^{E_v/kT}$$

$$= 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} e^{E_v/kT} \times \frac{N_a^{1/2} e^{-(E_a+E_v)/2kT}}{2^{1/2}\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/4}}$$

$$\therefore \quad p = (2N_a)^{1/2}\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/4} e^{-(E_a-E_v)/2kT} \quad ...(3)$$

Fermi Level in p - Type

Taking log of equation (2),

$$-\frac{E_F}{kT} = \frac{1}{2}\log\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right] - \frac{E_a+E_v}{2kT}$$

$$\therefore \quad E_F = \frac{E_a+E_v}{2} - \frac{kT}{2}\log\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right] \quad ...(4)$$

At T = 0 K,

$$E_F = \frac{E_C+E_v}{2}$$

- i.e, the Fermi level lies mid way between acceptor impurity levels and top of valence band.
- As temperature increases, E_F shifts upwards. The Fermi level shifts downwards on increasing the doping concentration N_a.

7.10 : p-n Junction Diode

Q.33 Explain the working of p-n junction diode in i) Zero bias ii) Forward Bias iii) Reverse Bias On the basis of energy level diagram.

Ans. : • **i) Open circuited *p-n* junction (zero bias) :** • The Fermi level is close to valence band in *p*-type and close to conduction band in *n*-type as shown in Fig. Q.33.1.

- When the *p* and *n*-type are brought in contact, the electrons in the *n*-type have a higher average energy than the *p*-type.
- The lower position of fermi level in *p*-type compared to *n*-type means that there are lower vacant energy states for the electrons in *p*-type as compared to *n*-type. Hence electrons start diffusing from *n*-type to *p*-type.

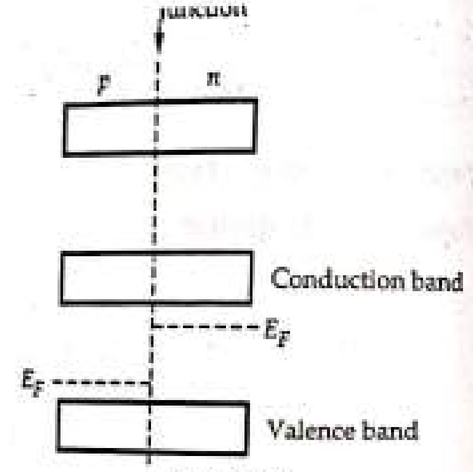


Fig. Q.33.1

- Diffusion of electrons continues till the fermi levels are equalized across the junction.
- Under equilibrium conditions, there is a single Fermi level throughout the diode which is close to valence band in *p*-type and close to conduction band in *n*-type.

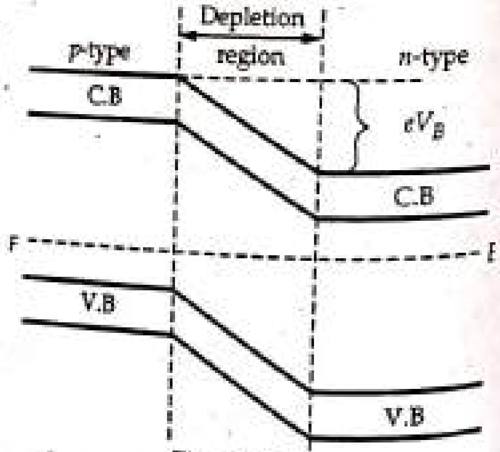


Fig. Q.33.2

- The energy bands of the *n*-type shift downward with respect to energy bands of *p*-type by eV_B where V_B is the barrier potential for alignment of the fermi levels as shown in Fig. Q.33.2.

- **ii) Forward bias :** • Diode is said to be forward biased when the *p*-type is connected to positive terminal and *n*-type to negative terminal of battery.
- As electrons are removed from *p*-type by the positive terminal of battery, holes are created which lowers the Fermi level.
- The negative terminal of battery connected to *n*-type supplies free electrons due to which the fermi level is raised.
- Hence the energy bands of *n*-type are raised compared to those of *p*-type. If 'V' is the applied voltage, the energy difference between the conduction bands reduces to $e(V_B - V)$.
- Due to this reduced barrier the electrons can cross the junction and conduction starts when $V > V_B$, i.e. when conduction band in *n*-type is at higher level than conduction band of *p*-type.

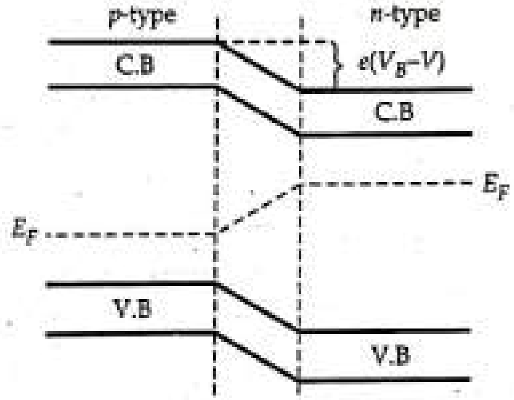


Fig. Q.33.3

- The energy band structure of forward biased diode is shown in Fig. Q.33.3.

iii) Reverse bias : • Diode is said to be reverse biased when *p*-type is connected to negative terminal and *n*-type to positive terminal of battery.

- The negative terminal supplies free electrons into p-type which raises the Fermi level.
- The positive terminal takes away free electrons from n-type which lowers the Fermi level.
- Therefore the energy bands of n-type are shifted downwards with respect to p-type.
- The energy difference between the conduction bands of n and p-type increases to $e(V_B + V)$ where 'V' is the applied voltage.
- The free electrons in n-type now face a larger potential barrier and the diode does not conduct.

The energy band structure of reverse biased diode is shown in Fig.Q.33.4.

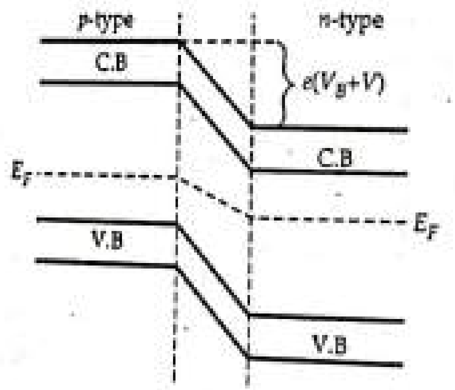


Fig. Q.33.4

Q.34 What is Fermi energy in semiconductor ? With the help of labeled diagram show the position of Fermi level in the case of a diode that is connected in forward bias. **[SPPU : May-16, Marks 3]**

Ans. : Refer Q.12 and Q.33.

Q.35 Derive an expression for barrier potential in diodes.

Ans. : • Consider the band diagram of an open circuited p-n junction as shown in Fig. Q.35.1

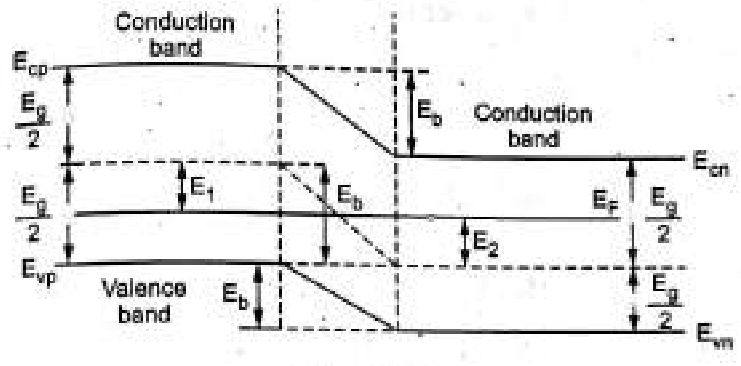


Fig. Q.35.1

- Let all energies be in electron volt (eV) so that the potential differences will be numerically equal to the corresponding energy difference.
- Let E_{vp} and E_{vn} be valence band energies in p and n types. E_{cp} and E_{cn} the conduction band energies in p and n types and E_b be the numerical value of the barrier potential. E_g is the energy gap.
- From Fig. Q.35.1,

$$E_b = E_{cp} - E_{cn} = E_{vp} - E_{vn}$$

$$= E_1 + E_2 \quad \ldots(1)$$

$$E_F - E_{vp} = \frac{E_g}{2} - E_1 \quad \ldots(2)$$

$$E_{cn} - E_F = \frac{E_g}{2} - E_2 \quad \ldots(3)$$

- Adding equations (2) and (3),

$$(E_F - E_{vp}) + (E_{cn} - E_F) = E_g - (E_1 + E_2)$$

$$\therefore \quad E_1 + E_2 = E_g - (E_{cn} - E_F) - (E_F - E_{vp})$$

- From equation (1),

$$E_b = E_g - (E_{cn} - E_F) - (E_F - E_{vp})$$

- But,

$$E_g = KT \ln \frac{N_c N_v}{n_i^2}$$

$$E_{cn} - E_F = KT \ln \frac{N_c}{N_D}$$

And,

$$E_F - E_{vp} = KT \ln \frac{N_v}{N_A}$$

$\therefore$

$$E_b = KT\left[\ln \frac{N_c N_v}{n_i^2} - \ln \frac{N_c}{N_D} - \ln \frac{N_v}{N_A}\right]$$

$$= KT \ln \left[\frac{N_c N_v}{n_i^2} \times \frac{N_D}{N_c} \times \frac{N_A}{N_v}\right]$$

$$E_b = KT \ln \left(\frac{N_D N_A}{n_i^2}\right) \quad \ldots(4)$$

Q.36 State the ideal diode equation. Explain diode characteristics predicted by this equation.

Ans. : • The ideal diode equation relates the current (I) through the diode with the voltage (V) across it.

The ideal diode equation, also known as Shockley ideal equation or diode law is given by

$$I = I_0 \left(e^{V/\eta V_T} - 1\right) \quad \ldots(1)$$

Where, I_0 is the reverse saturation current,

$V_T = \frac{T}{11,600}$ is known as volt equivalent of temperature,

T = Absolute temperature

• η is a constant which is 1 for germanium and approximately 2 for silicon.

At room temperature i.e. T = 300 K, $\quad V_T = 26 \times 10^{-3}$ V

• For reverse bias, v is negative. As V_T is very small, the exponential term in equation (Q.36.1) is nearly zero.

$\therefore$

$$I \approx -I_0$$

• Hence, in reverse bias, the current is constant which is the reverse saturation current and the negative sign indicates that it flows from n to p type.

• In forward bias the exponential term is large compared to 1.

$\therefore$

$$I = I_0 e^{v/\eta V_T}$$

• Hence, current rises exponentially and positive sign indicates that it flows from p to n type.

• The diode characteristics predicted by ideal diode equation are shown in Fig. Q.36.1.

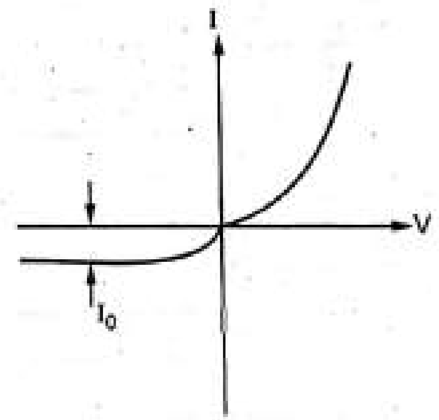


Fig. Q.36.1

7.11 : Solar Cell

Q.37 Explain the construction and working of solar cell. Also draw the IV characteristics of solar cell and define fill factor.

[SPPU : May-18, Marks 4]

Ans. : • Solar cell is a photovoltaic device used to convert solar energy into electrical energy. It is a p-n junction with a large surface area as compared to p-n junction diodes and a very thin p-layer.

- When light falls on the upper *p*-type layer, electron - hole pairs are created. As the *p*-layer is very thin, the electron hole pairs are formed close to the depletion region.
- The electric field in the depletion region (due to barrier potential), separates the electron-hole pairs.
- The electrons diffuse into *n*-type and holes from *n*-type to *p*-type.
- The electrons in *p*-type are the minority charge carriers and diffuse towards *n*-type as lower energy states are available for these electrons in *n*-type as shown in the energy band diagram in Fig. Q37.1.

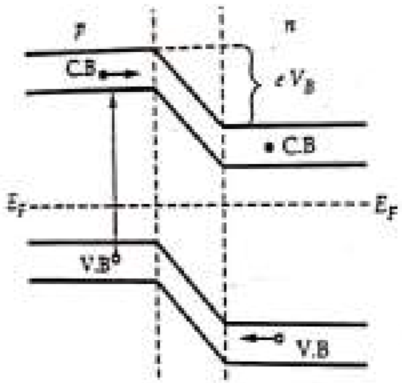


Fig. Q.37.1

- As a result of this diffusion a potential difference develops across the solar cell which is proportional to the intensity of light but has a maximum value of about 0.5 V for silicon cells and 0.1 V for germanium cells.
- If an external load resistance is connected across the cell, current flows through the load and power is delivered to it.
- The circuit used to study characteristics of solar cell is shown in Fig. Q37.2.

- When load resistance is infinite, no current flows through it and maximum voltage appears across the solar cell which is known as **open circuit voltage** V_{oc}.
- If $R_L = 0$, maximum current flows through it which is called the short circuit current I_{sc}.

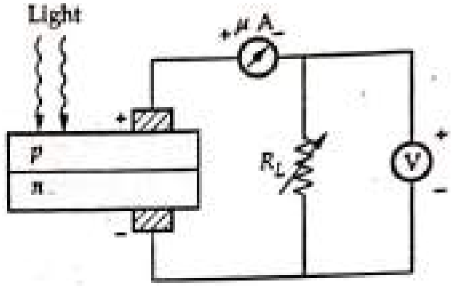


Fig. Q.37.2

- The typical *I-V* characteristics of a solar cell are shown in Fig.Q.37.3. $V_{oc} I_{sc}$ is theoretically the maximum power that can be obtained from the solar cell. Experimentally the maximum power is $V_m I_m$.

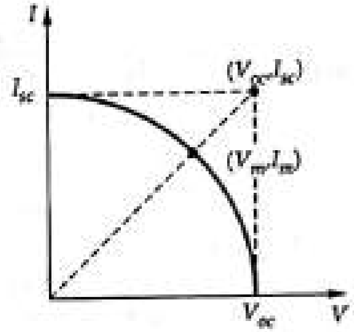


Fig. Q.37.3

- The fill factor of the solar cell is defined as the ratio of the experimentally maximum power to the ideal power.

$$\therefore \quad \text{Fill factor} = \frac{V_m I_m}{V_{oc} I_{sc}}$$

Q.38 List any three applications of solar cell. Explain any one of them in brief. **[SPPU : May-19, Marks 3]**

Ans. : Applications of solar cells

- Solar cells are used in satellites, space vehicles and in remote places as a source of energy.

- They are used in battery charging system, outdoor lighting system and in telecommunication system.
- A few countries like USA and Germany have photovoltaic power plants.
- The solar cells are widely used in electronic equipments with low power consumption like calculators.
- As calculators require only about 2V at very low current, a series combination of 4 solar cells is sufficient to provide this power.
- As these cells are very small, they do not make the equipment bulky.

7.12 : Hall Effect

Q.39 What is hall effect ? Derive the expression for hall voltage and hall coefficient. State applications of hall effect.

[SPPU : May-17, Marks 6]

Ans. : Hall voltage and Hall coefficient

- Consider a conductor of rectangular cross section of dimensions $w \times d$ in which current I flows along x-axis, magnetic field is applied along z-axis and Hall voltage develops along y-axis which is measured across terminals 1 and 2 as shown in Fig. Q.39.1.

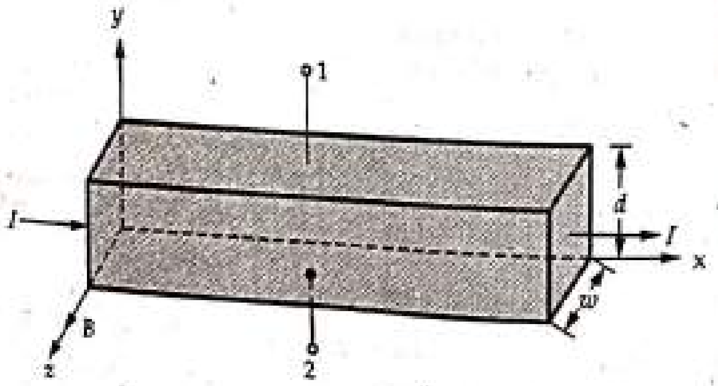


Fig. Q.39.1

- The dimension 'w' (the width) is parallel to the direction of magnetic field and ('d' the thickness) is parallel to the axis along which Hall voltage develops.
- Let V_H = Hall voltage and E_H, the corresponding electric field

and v = Drift velocity of charges

- Under equilibrium conditions, force on charge carriers due to magnetic field will be balanced by the force on them due to E_H.

$$\therefore \quad q E_H = q v B$$

$$\therefore \quad E_H = v B$$

- Also
$$E_H = \frac{V_H}{d}$$

$$\therefore \quad \frac{V_H}{d} = v B$$

$$\therefore \quad V_H = v B d \qquad \ldots (1)$$

- From equation (1),

$$I = n q a v$$

$$\therefore \quad v = \frac{I}{n q a}$$

- Substituting in equation (1),

$$V_H = \frac{I B d}{n q a} \qquad \ldots (2)$$

As
$$a = w \times d$$

$$V_H = \frac{I B}{n q w} \qquad \ldots (3)$$

- The quantity $\frac{1}{n q}$ is the reciprocal of charge density and is defined as the Hall coefficient 'R_H'.

$$\therefore \quad R_H = \frac{1}{n q} \qquad \ldots (4)$$

From equation (2),

$$V_H = R_H \frac{I\,B\,d}{a} \quad \text{... (5)}$$

- As V_H, B, d and a are measurable quantities, R_H and hence charge density nq can be determined using equation (7.19.7).
- If E is the electric field to produce current, V is the applied voltage along x-axis, and l is the length of the conductor along x - axis,

$$E = \frac{V}{l}$$

Also,
$$E_H = vB = \frac{BI}{nqa}$$

$\therefore$
$$\frac{E_H}{E} = \frac{BI}{nqa} \times \frac{l}{V}$$

$$R = \frac{V}{I}$$

$\therefore$
$$\frac{E_H}{E} = \frac{B}{nqa} \times \frac{l}{R}$$

But,
$$\rho = \frac{Ra}{l}$$

$\therefore$
$$\frac{E_H}{E} = \frac{B}{nq\rho}$$

- Charges in the conductor will be under the influence of two mutually perpendicular electric fields - E along x - axis and E_H along y - axis.
- The angle made by the resultant electric field with the x - axis is known as the **Hall angle** (θ_H).

$\therefore$
$$\tan\theta_H = \frac{E_H}{E} = \frac{B}{nq\rho} \quad \text{... (6)}$$

- Once charge density is known, we can determine mobility of charge carriers using

$$\sigma = n\,q\,\mu$$

- Also,
$$\sigma = \frac{\mu}{R_H} \quad \text{...(7)}$$

...(8)

Hence,
$$\rho = \frac{R_H}{\mu} \quad \text{... (9)}$$

- Equation (6) can be written as

$$\tan\theta_H = \frac{B\sigma}{nq}$$

$\therefore$
$$\tan\theta_H = B\mu \quad \text{...(10)}$$

- Conductivity σ can be determined using

$$\sigma = \frac{1}{\rho} = \frac{l}{R\,a}$$

- Thus the Hall effect can be used to determine

i) Whether charge carriers are positive or negative which in turn determines whether semiconductor is n-type or p-type.

ii) Density of charge carriers.

iii) Mobility of charge carriers.

Q.40 Explain Hall effect. Derive the equations of Hall voltage and Hall coefficient. **[SPPU : Dec.-15, Marks 3]**

Ans. : Refer Q.39.

Important Formulae

$$V_H = \frac{I\,B\,d}{n\,q\,a} \quad \text{... (1)}$$

$$V_H = \frac{I\,B}{n\,q\,w} \quad \text{... (2)}$$

$\therefore$
$$R_H = \frac{1}{n\,q} \quad \text{... (3)}$$

$$V_H = R_H \frac{I\,B\,d}{a} \quad \text{... (4)}$$

Q.41 A copper specimen having length 1 meter, width 1 cm and thickness 1 mm is conducting 1 ampere current along its length and is applied with a magnetic field of 1 tesla along its thickness. It experience Hall effect and a Hall voltage of 0.074 microvolts appear along its width. Calculate the Hall coefficient and the mobility of electrons in copper. Conductivity of copper is $\sigma = 5.8 \times 10^7 \,(\text{Wm})^{-1}$. **[SPPU : Dec.-05]**

Ans. : Given data : $V_H = 0.074\ \mu V = 0.074 \times 10^{-6}\ V$

$a = 1 \times 10^{-2} \times 1 \times 10^{-3} = 10^{-5}\ m^2$

$B = 1\ T,\ I = 1\ A,\ d = 1\ cm = 10^{-2}\ m$

Formula : $V_H = R_H \dfrac{B I d}{a}$

$\therefore \quad R_H = \dfrac{V_H a}{B I d}$

$\therefore \quad R_H = \dfrac{0.074 \times 10^{-6} \times 10^{-5}}{1 \times 1 \times 10^{-2}}$

$\therefore \quad \boxed{R_H = 7.4 \times 10^{-11}\ m^3/C}$

Mobility $\mu = \sigma R_H$

$\sigma = 5.8 \times 10^7\ (\Omega m)^{-1}$

$\therefore \quad \mu = 5.8 \times 10^7 \times 7.4 \times 10^{-11}$

$\therefore \quad \boxed{\mu = 4.292 \times 10^{-3}\ m^2/V\text{-}s}$

Q.42 A slab of copper 2.0 mm in thickness and 1.5 cm wide is placed in a uniform magnetic field with magnitude 0.40 T. When a current of 75 amp flows along the length, the voltage measured across the width is 0.81 μV. Determine the concentration of mobile electrons in copper. [SPPU : May-06]

Ans. : Given data : $B = 0.4\ T$

$I = 75\ A$

$d = 1.5\ cm = 1.5 \times 10^{-2}\ m$

$V_H = 0.81\ \mu V = 0.81 \times 10^{-6}\ V$

$q = 1.6 \times 10^{-19}\ C$

$a = 2 \times 10^{-3} \times 1.5 \times 10^{-2} = 3 \times 10^{-5}\ m^2$

Formula : $V_H = \dfrac{B I d}{n q a}$

$\therefore \quad n = \dfrac{B I d}{V_H q a}$

$\therefore \quad n = \dfrac{0.4 \times 75 \times 1.5 \times 10^{-2}}{0.81 \times 10^{-6} \times 1.6 \times 10^{-19} \times 3 \times 10^{-5}}$

$\therefore \quad \boxed{n = 1.157 \times 10^{29}\ per\ m^3}$

Q.43 A silver wire is in the form of a ribbon 0.50 cm wide and 0.10 mm thick . When a current of 2 amp passes through the ribbon, perpendicular to 0.80 tesla magnetic field, calculate the Hall voltage produced. The density of silver is 10.5 gm/cc and atomic weight of Ag = 108. [SPPU : May-07]

Ans. : Given data : Mass of 1 cc of silver is 10.5 gm

108 gm of silver contains 6.025×10^{23} atoms

$\therefore$ Density of electrons $n = \dfrac{6.025 \times 10^{23}}{108} \times 10.5 = 5.8576 \times 10^{22}$ per cm^3

$\therefore \quad n = 5.8576 \times 10^{28}$ per m^3

$B = 0.8\ T,\ I = 2\ A$

$w = 0.1\ mm = 0.1 \times 10^{-3}\ m$ (Assuming that magnetic field is applied along this length)

$q = 1.6 \times 10^{-19}\ C$

$a = 0.1 \times 10^{-3} \times 0.5 \times 10^{-2}$

Formula : $V_H = \dfrac{BI}{n q w}$

$\therefore \quad V_H = \dfrac{0.8 \times 2}{5.8576 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.1 \times 10^{-3}}$

$\therefore \quad V_H = 1.71 \times 10^{-6}\ V$

$\therefore \quad \boxed{V_H = 1.71\ \mu V}$

Q.44 Calculate the mobility of charge carriers in a doped silicon whose conductivity is 100 per $\Omega \cdot m$ and the Hall coefficient is $3.6 \times 10^{-4}\ m^3/coulomb$. [SPPU : May-09,11, Dec.-16, 17, Marks 3]

Ans. : Given data : $\sigma = 100\ (\Omega\text{-}m)^{-1}$

$$R_H = 3.6\times10^{-4}\ m^3/C$$

Formula : $\mu = \sigma R_H$

$\therefore$ $\mu = 100\times 3.6\times10^{-4}$

$\therefore$ $\boxed{\mu = 3.6\times10^{-2}\ m^3/V\text{-}s}$

Q.45 A specimen having length 1.00 cm, width 1.00 mm and thickness 0.1 mm is made to conduct with 1.00 mA current and is placed in a magnetic field of 1.0 Wb/m^2, acting along the thickness. Calculate the hall voltage in case of i) N type semiconductor with hall coefficient of $-3.44\times10^{-8}\ m^3/C$ and ii) Aluminum with hall coefficient of $-0.3\times10^{-10}\ m^3/C$. Which of these materials is more sensitive to hall effect ? Why ?

[SPPU : Dec.11, Marks 5]

Ans. : Given data :

$$I = 1\ mA = 1\times 10^{-3}\ A$$

$$B = 1.0\ Wb/m^2$$

$$W = 1.00\ mm = 1\times 10^{-3}\ m$$

$$d = 0.1\ mm = 1\times 10^{-4}\ m$$

$$a = 1\times 10^{-3}\times 1\times 10^{-4} = 1\times 10^{-7}\ m^2$$

Formula : $V_A = R_H \dfrac{B\,Id}{a}$

i) $R_H = 3.44\times 10^{-8}\ m^3/C$

(The negative sign only indicates that the charge carriers are negative)

$\therefore$ $V_H = \dfrac{3.44\times 10^{-8}\times 1.0\times 1\times 10^{-3}\times 1\times 10^{-4}}{1\times 10^{-7}}$

$\therefore$ $V_H = 3.44\times 10^{-8}\ V$

ii) $R_H = 0.3\times 10^{-10}\ m^3/C$

$\therefore$ $V_H = \dfrac{0.3\times 10^{-10}\times 1.0\times 1\times 10^{-3}\times 1\times 10^{-4}}{1\times 10^{-7}}$

$\therefore$ $V_H = 0.3\times 10^{-10}\ V$

- The N-type semiconductor is more sensitive to hall effect as more hall voltage is produced in it for the same current and magnetic field.

END...

UNIT - V

8 Magnetism

8.1 : Origin of Magnetism

Q.1 Explain the origin of magnetic properties of materials.

Ans. : • Magnetic properties of materials are due to electrons in the atoms.

- In atoms, the electrons have orbital angular momentum and spin angular momentum.
- The orbital as well as spin angular momentum are quantized and give rise to orbital magnetic dipole moment and spin magnetic dipole moment respectively.
- The vector addition of these two magnetic dipole moments gives the resultant magnetic dipole moment for an electron.
- The resultant of magnetic dipole moments of all electrons in an atom gives the magnetic dipole moment of an atom and the dipole moments of all atoms in a specimen of the material will give the resultant magnetic dipole moment of that specimen.
- If the resultant magnetic dipole moment produces a magnetic field, the material behaves like a magnet.
- Nuclei also have a magnetic moment known as nuclear magnetic moment. But this moment is about 10^3 times smaller than the magnetic moment due to electrons and hence is neglected.
- When materials are placed in external magnetic fields, the magnetic dipole moments of the electrons are modified.

Q.2 Derive an expression for magnetic moment due to orbital angular momentum of electron. Hence define Bohr magneton.

Ans. : • The orbital motion of an electron around the nucleus is equivalent of a current loop. Every current loop is associated with a magnetic moment.

• The magnetic moment associated with a loop carrying current I and having area A is

$$\mu_L = IA. \quad \text{...(1)}$$

Let e = charge of an electron and

T = Time period of revolution of electron motion around the nucleus.

• Current is defined as the charge per unit time. Hence the current I associated with this electron motion will be

$$I = \frac{e}{T}$$

• If the electron moves with velocity v in a circular path of radius r around the nucleus,

$$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{ev}{2\pi r} \quad \text{...(2)}$$

• The area of current loop is

$$A = \pi r^2 \quad \text{...(3)}$$

• Substitute equations (2) and (3) in equation (1)

$$\mu_L = \left(\frac{ev}{2\pi r}\right)\times\left(\pi r^2\right)$$

$$\therefore \quad \mu_L = \frac{evr}{2}$$

• Multiply and divide by the mass m of the electron.

$$\therefore \quad \mu_L = \left(\frac{evr}{2}\right)\times\frac{m}{m} = \frac{e}{2m}\times mvr$$

$L = mvr$ is the orbital angular momentum of the electron.

$$\therefore \quad \mu_L = \frac{e}{2m}\times L \quad \text{...(4)}$$

• According to quantum theory, the orbital angular momentum is an integer multiple of $\frac{h}{2\pi}$ where h is the Planck's constant. i.e.

$$L = \frac{nh}{2\pi}$$

$$\therefore \quad \mu_L = \frac{e}{2m}\times\frac{nh}{2\pi}$$

$$\therefore \quad \mu_L = n\left(\frac{eh}{4\pi m}\right) \quad \text{...(5)}$$

where n = 1, 2, 3, ... is the principal quantum number.

• The above equation gives the magnetic moment associated with the orbital motion of electron.

Bohr Magneton

• Bohr magneton is defined as the angular momentum of an electron in ground state i.e, with principal quantum number 1. It is denoted by μ_B.

$$\therefore \quad \mu_B = \frac{eh}{4\pi m} \quad \text{...(6)}$$

8.2 : Types of Magnetic Materials

Q.3 What is diamagnetism ? Explain using Langevin's theory.

Ans. : • The atoms of diamagnetic materials do not possess any net magnetic dipole moment.

• When diamagnetic materials are placed in an external magnetic field, magnetic dipole moments are developed which are directed opposite to the external field. Hence the magnetic flux density inside the material decreases. i.e. the material has a tendency to expel the magnetic flux.

• All materials show diamagnetic property. Ordinary materials are only weakly diamagnetic. Superconductors show very strong diamagnetic property.

• The magnetic susceptibility is negative for diamagnetic materials. When diamagnetic materials are placed in non uniform magnetic fields, they move from region of larger magnetic field to region of smaller magnetic field. This phenomenon is used in magnetic levitation.

- The induced magnetic dipole moments disappear on switching off the external field. Copper, gold and zinc show diamagnetic properties.

Langevin's Theory of Diamagnetism

- According to Langevin's theory of diamagnetism, the orbital motion of electrons gives rise to diamagnetic properties.
- The orbital motion of electrons gives rise to magnetic moment.
- In an atom of a diamagnetic material, the magnetic moments of all electrons cancel each other as they are in random directions.
- When an external magnetic field is applied, the orbital motion of the electrons in atoms are altered. It causes current loops to develop.
- The magnetic moments produced by these induced current loops are opposite to the applied field in accordance with Lenz's law.
- Based on the above considerations, Langevin derived the equation for susceptibility of a diamagnetic material as,

$$\chi = -\frac{NZe^2\mu_0}{6m}\langle r^2\rangle$$

where μ_0 = Permeability of free space,

N = Number of atoms per unit volume,

Z = Number of electrons in the atom = atomic number,

e = Charge of an electron,

m = Mass of an electron and

$\langle r^2\rangle$ = Mean square distance of the electrons from the nucleus.

- From the above equation, it is clear that the diamagnetic susceptibility of a diamagnetic substance is always negative. Also,

it is independent of the temperature and the strength of applied field.

Q.4 What is paramagnetism ? Explain.

Ans. : • In paramagnetic materials, each atom possesses a net magnetic dipole moment.

- In absence of external magnetic field, these dipole moments are randomly oriented due to which the net magnetic dipole moment of the material is zero.
- When placed in a magnetic field, the dipoles tend to get aligned with the field which leads to an increase in magnetic flux density inside it.
- When paramagnetic materials are placed in non uniform magnetic fields, they move towards region of stronger magnetic field.
- The net magnetic dipole moment of the material is lost when the external field is switched off.
- The susceptibility of paramagnetic materials is a small positive value. Salts of iron and nickel show paramagnetism.

Langevin's Theory of Paramagnetism

- According to Langevin's theory of paramagnetism, the magnetic dipole moments get aligned in the direction of the magnetic field. The random thermal motion tends to disturb this alignment making this effect temperature dependent.
- The paramagnetic susceptibility is given by,

$$\chi = \frac{\mu_0\mu_m^2 N}{3kT} = \frac{C}{T}$$

where, $C = \frac{\mu_0\mu_m^2 N}{3k}$ is a constant known as Curie constant,

μ_0 = Permeability of free space,

μ_m = Magnetic dipole moment,

N = Number of atoms per unit volume,

k = Boltzmann constant,

T = Absolute temperature.

- Hence the susceptibility of a paramagnetic material is inversely proportional to the temperature. The above equation is known as Curie's law.
- Langevin's theory failed to explain the exact temperature dependence of susceptibility of certain paramagnetic materials like crystals and salts.

Weiss Theory of Paramagnetism

- Weiss assumed that in addition to the externally applied field, the internal molecular field produced by neighboring molecules also acts on the magnetic dipole moment of any molecule.
- According to Weiss theory, the expression for susceptibility of a paramagnetic material is,

$$\chi = \frac{\mu_0 \mu_m^2 N}{3k(T-\theta_c)} = \frac{C}{T-\theta_c}$$

where,

μ_0 = Permeability of free space,

μ_m = Magnetic dipole moment,

$C = \frac{\mu_0 \mu_m^2 N}{3k}$ is the Curie constant,

k = Boltzmann constant,

T = Absolute temperature.

N = Number of atoms per unit volume,

θ_c = Paramagnetic Curie point.

- The above equation is called Curie - Weiss law.

Q.5 Explain ferromagnetism.

Ans. : • The atoms of ferromagnetic materials have a large magnetic dipole moment. In unmagnetized specimen, these magnetic dipole moments are randomly oriented due to which the net magnetic dipole moment of the sample is zero.

- When the specimen is kept in an external magnetic field, these magnetic dipole moments get aligned with the external field to give the specimen a large net magnetic dipole moment.
- On removing the external magnetic field, the material retains the magnetic properties and the material shows permanent magnetism.
- The material loses its magnetic properties on increasing the temperature upto the Curie temperature beyond which it becomes paramagnetic.
- The ferromagnetic materials have a very large positive value of magnetic susceptibility. The magnetic susceptibility changes with temperature above the Curie temperature according to the Curie-Weiss law given by,

$$\chi = \frac{C}{T-\theta} \qquad for\ T > \theta$$

where C is the Curie constant and θ is the Curie temperature.

- Iron, nickel and cobalt show ferromagnetic properties.

8.3 : Applications of Magnetic Devices

Q.6 Explain low magnetic properties of materials are utilized for recording and reading data.

Ans. : Magnetic recording : • Magnetic recording is based on the concept that data (in the form of sound or optical signal, or in digital form) can be converted to electrical signal which in turn is used to produce a magnetic field that orients magnetic moments in a ferromagnetic material.

- To achieve this, a coil is wound on a ferrite ring with an air gap as shown in Fig. Q.6.1.
- A magnetic tape which consists of a coating of ferromagnetic material on a plastic tape is made to move below the air gap.
- The input signal converted to electrical form is fed to the coil which sets up a magnetic field in the ferrite ring.

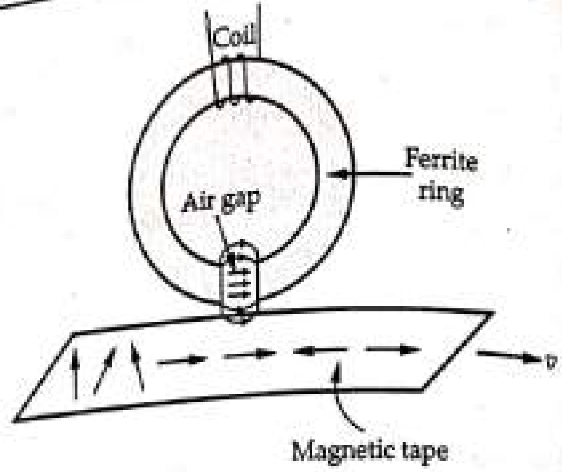


Fig. Q.6.1

- As the air gap has less permeability than the magnetic material just below it, most of the magnetic flux completes its path through the magnetic material.
- This flux aligns the randomly oriented magnetic moments according to the applied signal in the horizontal direction. This recording method is known as longitudinal recording as the bits are magnetized along the length of the tape.
- For small magnetic fields, the number of dipoles getting aligned with the applied magnetic field increases with the magnitude of the field. This is used in analog recording as in the case of audio recording.
- For sufficiently large magnetic fields, most of the magnetic dipoles get aligned with the applied magnetic field due to saturation. As a result, the number of magnetic dipole moments getting aligned with the applied magnetic field does not change with the field. This condition is used for digital recording.
- To read the data, the tape is made to move under a read head which is similar to the write head. The flux associated with the ring changes as it moves over the aligned magnetic moments and induces current in the coil.

Q.7 Explain the construction and working of a magnetic tape. State its advantages and disadvantages.

Ans. : • Magnetic tapes used for data storage are made up of a plastic strip with a thin layer of ferric oxide. The tape width is typically about half an inch. These are rolled into large reels or smaller cassettes.

- In this type of recording, the magnetic moments are aligned in a vertical direction perpendicular to the plane of the tape. This type of recording increases the data density on the tape.
- The recording head is made up of a ferrite with a coil wound around it as shown in Fig. Q.7.1. There is an air gap in the ferrite with one end sharp and the other flat.

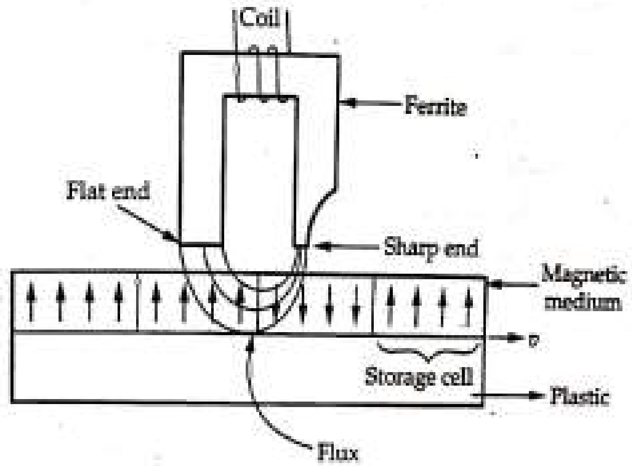


Fig. Q.7.1

- The magnetic flux emerging out from the sharp end is nearly perpendicular to the plane of the tape whereas the flux at the flat end has lower density due to larger area and is not perpendicular to the tape.
- The tape is moved from flat end towards the sharp end so that the flux at the sharp end will be utilized to align the magnetic moments. The data is stored on multiple tracks.

- The data is stored in cells in the form of '0' and '1'. A '0' is represented by one direction of magnetic moments and the opposite direction represents a '1'.
- While reading, voltages of opposite polarity are induced in the coil connected to the ferrite for '0' and '1' when the tape passes below the read head which is similar to the writing head.

Advantages

1) Magnetic tapes have very high data storage capacity. They can have capacity of upto 5 tera bytes.
2) Low cost.
3) Easy to handle and portable.

Disadvantage

- The access to data is sequential. Random access to a particular data on the tape takes longer time as the tape head has to be moved to that position which requires the tape to be either wound or unwound.

Q.8 Explain construction and working of a floppy disk.

Ans. : • Floppy disks are made of thin plastic disk with a coating of magnetic material on it. The disk is enclosed in a thick plastic covering with an opening for the read / write head.

- The opening remains closed until it is inserted into the floppy disk drive.
- The floppy disk is divided into circular concentric tracks as shown in Fig.Q.8.1 where the data is recorded. The disk is divided into sectors as shown.
- The disk is rotated and the write head aligns the

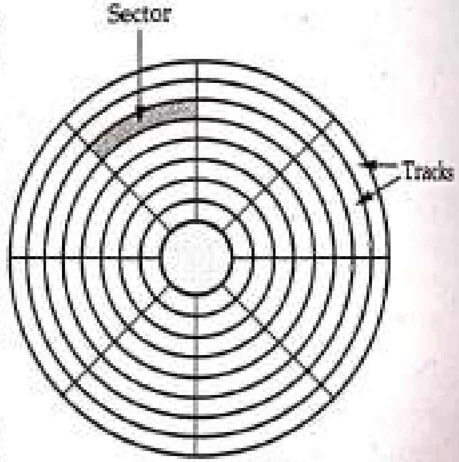


Fig. Q.8.1

magnetic moments on the tracks as it passes over them. The movement of the head is controlled by a servo mechanism.

- The floppy disks have small storage capacity of 1.44 MB. These are now phased out and new computer systems do not have floppy drives. They have been replaced by the optical devices like compact disk (CD) and DVD.

Q.9 Explain construction and working of a hard disk.

Ans. : • Hard disks are magnetic disk drives like the floppy disks but with a larger number of disks.

- A large number of disks mounted on a common central shaft are coated on both sides with a magnetic material except the two outermost disks which are coated only on the inner side.
- One read / write head is provided per surface as shown in Fig. Q.9.1.

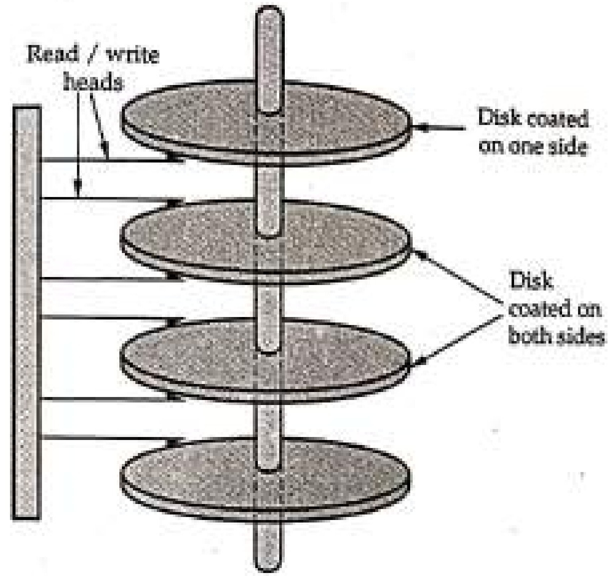


Fig. Q.9.1

- The read / write operation is similar to that in floppy disks. Each disk is divided into tracks and sectors. The write head aligns the magnetic moments on the tracks as it passes over the track.

- Vertical alignment of magnetic moments is used for higher storage density.
- Data can be stored on both sides of the disks. Hence these hard disks have a very large storage capacity. Hard disks with capacity upto 5 tera bytes (TB) have become available in the market.
- These are available as both internal hard disks which are used inside computer systems or as external disks.

END...

UNIT - V

9 Superconductivity

9.1 : Properties of Superconductors

Q.1 What is superconductivity ?

Ans. : For certain materials, like mercury, the resistivity suddenly drops to zero at very low temperature typically near the boiling point of liquid helium. This phenomenon is known as superconductivity.

Q.2 Explain zero electrical resistance property of superconductor.

[SPPU : Dec.-16, Marks 2]

Ans. : Zero electrical resistance property :

- The electrical resistivity of superconductors is zero.
- The experimental testing of zero resistance was tried by connecting the speciman at room temperature to a battery. A voltmeter was connected across the speciman as shown in Fig. Q.2.1 to measure the potential difference across the specimen.

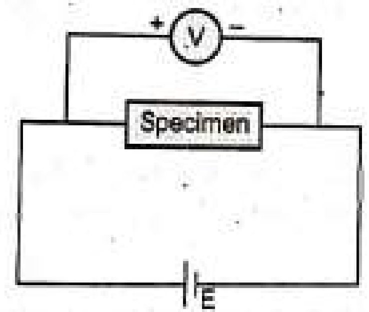


Fig. Q.2.1

- The temperature of the speciman is then lowered.
- It was observed that below a particular temperature, known as the critical temperature, the potential difference across the speciman suddenly dropped to zero.

Isotope effect : Refer Q.5

Q.3 State and explain :
a) Meissner effect b) Persistent current [SPPU : Dec-12, May-15, Marks 3]

Ans. :

a) Meissner effect :

- When a specimen is placed in a weak magnetic field and cooled below the critical temperature, the magnetic flux originally present in the specimen is ejected from the specimen as shown in Fig. Q.3.1. This effect is known as Meissner effect.

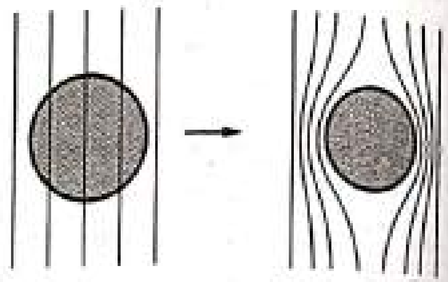
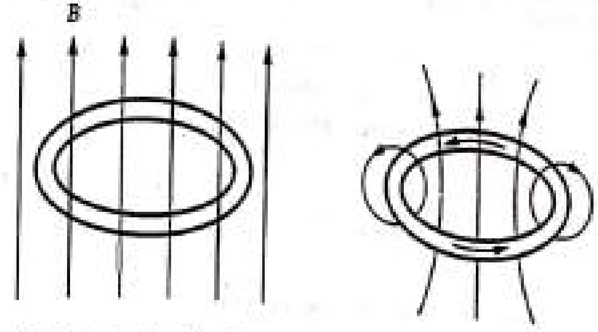


Fig. Q.3.1

- This effect is reversible and the specimen returns to its normal state when its temperature is raised above its critical temperature. Thus superconductors act as perfect diamagnets with zero magnetic induction in the interior when placed in weak magnetic fields.
- This indicates perfect diamagnetism.
- Meissner effect cannot be explained by assuming that the superconductor is a perfect conductor with zero electrical conductivity.
- Due to Meissner effect, superconductors strongly repel external magnets which leads to magnetic levitation.

b) Persistent current :

- When a superconducting ring is placed in a magnetic field and the field is switched off, a current is induced in the ring as shown in Fig Q.3.2.



(a) Superconducting ring in magnetic field B

(b) Persistent current and associated magnetic field with external field B is switched off

Fig. Q.3.2

- The magnitude of current remains constant even though there is no source of e.m.f. as the resistance of the superconducting ring is zero.
- Such steady currents flowing in a superconductor are called persistent currents. These currents flowing in the ring produce a magnetic field.
- Thus superconducting rings can be used to produce magnetic fields which do not require any power supply to maintain a constant field.

Q.4 Explain Meissner effect and critical magnetic field for superconductivity. [SPPU : May-13, Marks 6]

Ans. : For Meissner effect, refer Q.3.

Critical magnetic field :

- Superconductivity is destroyed by sufficiently strong magnetic fields.

- The minimum value of the applied magnetic field required to destroy superconductivity is called the critical field H_c and is a function of temperature.
- Above the critical temperature, $H_c = 0$.
- Below critical temperature the variation of H_c with temperature 'T' is as shown in Fig. Q.4.1 and can be represented by the equation

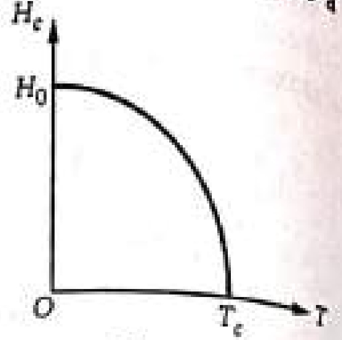


Fig. Q.4.1

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

where H_0 is the critical field at absolute zero.

Q.5 Explain the isotope effect and its significance.
[SPPU : Dec.-05, Marks 4]

Ans. : • According to observations, the critical temperature of superconductors varies with isotopic mass. The critical temperature is smaller for larger isotopic mass.

- This phenomenon is known as isotope effect. The relation between critical temperature 'T_c' and isotopic mass 'M' is given by,

$$T_c M^{1/2} = \text{Constant}$$

or

$$T_c \propto M^{-1/2}$$

- As lattice vibrations are reduced for larger isotopic masses, the isotope effect indicates that superconductivity is due to interaction between electrons and lattice vibrations.

Q.6 Show that superconductors are perfectly diamagnetic.

Ans. : • The magnetic induction inside the specimen in the normal state is given by,

$$B = \mu_0(H+M) \quad \text{... (1)}$$

where 'H' is the externally applied magnetic field and 'M' is the magnetization inside the specimen.

- When the temperature 'T' of the specimen is lowered below its critical temperature 'T_c',

$$B = 0$$

$$\therefore \quad \mu_0(H+M) = 0$$

$$H = -M$$

- The susceptibility χ is given by,

$$\chi = \frac{M}{H}$$

$$\therefore \quad \chi = -1 \quad \text{... (2)}$$

- As the relative permeability $\mu_r = 1+\chi$

$$\therefore \quad \mu_r = 0$$

This indicates perfect diamagnetism.

- Meissner effect cannot be explained by assuming that the superconductor is a perfect conductor with zero electrical conductivity. The electric field is given by,

$$E = \frac{V}{l} = \frac{IR}{l} = \frac{IR}{l} \times \frac{A}{A} = \frac{RA}{l} \cdot \frac{I}{A}$$

$$\therefore \quad E = \rho J \quad \text{... (3)}$$

where ρ = Resistivity

and J = Current density

- If ρ becomes zero for a finite current density J, then $E = 0$.

From Maxwell's equations,

$$\nabla \times \vec{E} = \frac{-d\vec{B}}{dt}$$

$$\text{As } E = 0, \frac{d\vec{B}}{dt} = 0$$

$$\therefore \quad \overline{B} = \text{Constant}$$

$\therefore$ In a conductor the flux cannot change on cooling below the critical temperature. This contradicts Meissner effects according to which the flux must reduce to zero and hence superconductor is not just a perfect conductor.

- Due to Meissner effect, superconductors strongly repel external magnets which leads to magnetic levitation.

Q.7 Explain : I) Critical field II) Meissner effect. [SPPU : Dec.-15, Marks 6]

Ans. : Refer Q.4.

Q 8 What is superconductivity ? Explain the Meissner effect in superconductors. [SPPU : Dec.-16, Marks 6]

Ans. : Refer Q.1 and Q.3.

Q.9 Explain zero electrical resistance property and isotope effect in superconductor. [SPPU : Dec.-16, Marks 4]

Ans. : Refer Q.2 and Q.5.

Q.10 Explain Meissner effect and show that superconductors exhibit perfect diamagnetism. [SPPU : May-17, Marks 4]

Ans. : Refer Q.3 and Q.6.

Q.11 Explain the following terms of superconductivity with the help of necessary figure. Give formula and graph wherever necessary. I) Meissner effect II) Critical magnetic field. [SPPU : Dec.-17, Marks 6]

Ans. : Refer Q.4.

Q.12 Give the statement of Meissner effect and show that super conductors are perfectly diamagnetic. [SPPU : Dec.-17, May-19, Marks 4]

Ans. : Refer Q.3 and Q.6.

Q.13 What is superconductivity ? Explain Meissner effect and show that superconductors are perfectly diamagnetic. [SPPU : Dec.-18, Marks 6]

Ans. : Refer Q.3 and Q.6.

Q.14 Explain the following terms of superconductivity, i) Critical Magnetic Field ii) Persistent Current. [SPPU : Dec.-18, Marks 4]

Ans. : Refer Q.3. and Q.4.

Q.15 Superconducting tin (Sn) has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 tesla at 0 K. Find the critical field at 2 K.

Ans. :

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

As

$$H_c = \frac{B_c}{\mu_0} \text{ and } H_0 = \frac{B_0}{\mu_0},$$

$$B_c = B_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$B_0 = 0.0306 \text{ T}$$

$$T_c = 3.7 \text{ K}$$

$$T = 2 \text{ K}$$

$$B_c = 0.0306\left[1-\left(\frac{2}{3.7}\right)^2\right]$$

$$\therefore \quad \boxed{B_c = 0.02166 \text{ T}}$$

Q.16 Calculate the critical current for a wire of lead having a diameter of 1 mm at 4.2 K. Critical temperature of lead is 7.18 K and H_c at 0 K is 6.5×10^4 A/m.

Ans. :

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$H_0 = 6.5 \times 10^4 \text{ A/m}$$

$$T_c = 7.18 \text{ K}$$

$$T = 4.2 \text{ K}$$

$$H_c = 6.5 \times 10^4\left[1-\left(\frac{4.2}{7.18}\right)^2\right] = 4.276 \times 10^4 \text{ A/m}$$

$$I_c = 2\pi r H_c$$

$$r = 0.5 \text{ mm} = 0.5\times10^{-3} \text{ mm}$$

$$\therefore \quad I_c = 2\pi\times0.5\times10^{-3}\times4.27\times10^{4}$$

$$\therefore \quad \boxed{I_c = 134.33 \text{ A}}$$

Q.17 The critical temperature for a metal with isotopic mass 199.5 is 4.185 K. Calculate the isotopic mass if the critical temperature falls to 4.133 K. Assume α = 0.5.

Ans. :

$$T_c M^{\alpha} = \text{Constant}$$

$$\alpha = 0.5$$

$$\therefore \quad T_c M^{0.5} = \text{Constant}$$

$$T_{c1} M_1^{0.5} = T_{c2} M_2^{0.5}$$

$$T_{c1} = 4.185 \text{ K} ; M_1 = 199.5 ; T_{c2} = 4.133 \text{ K}$$

$$4.185\times199.5^{0.5} = 4.133\times M_2^{0.5}$$

$$\therefore \quad \boxed{M_2 = 204.55}$$

Q.18 The critical temperature for lead is 7.2 K. However at 5 K it loses its superconductivity when subjected to a magnetic field of 3.3 × 10⁴ A/m. Find the maximum value of critical magnetic field which will allow the metal to retain its super conductivity at 0 K.

[SPPU : May-18, Marks 3]

Ans. : Given data :

$$T_c = 7.2 \text{ K}, T = 5 \text{ K},$$

$$H_c = 3.3\times10^4 \text{ A/m}$$

Formula :

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$3.3\times10^4 = H_0\left[1-\left(\frac{5}{7.2}\right)^2\right]$$

$$\therefore \quad \boxed{H_0 = 6.37\times10^4 \text{ A/m}}$$

9.2 : BCS Theory

Q.19 Explain BCS theory of super-conductivity.

[SPPU : Dec-12, Marks 4]

Ans. : • Superconductivity was explained by Bardeen, Cooper and Schrieffer on the basis of quantum theory. This theory which is named after the three scientists is known as the BCS theory.

- When a free electron passes near an ion in a lattice, there is coulomb attraction between them. This causes deformation of the lattice.
- A second free electron passing nearby, takes advantage of the deformation to lower its energy.
- Thus the second electron interacts attractively with the first through the lattice deformation.
- This interaction is called electron - lattice - electron interaction which can be considered to take place through phonons.
- The quantum of lattice vibrational energy is called phonon, similar to quantum of electromagnetic which energy which is called photon.
- The first electron emits a phonon which is absorbed by the second electron.
- Electrons in such an interaction have same energy and equal and opposite spins. Such pairs of electrons coupled together through a phonon are called Cooper pairs.
- The attraction between electrons in a Cooper pair is very weak and the pair can be separated by a small increase in temperature which causes thermal vibrations.
- At very low temperature, density of Cooper pairs is large and they move collectively through the lattice which minimizes collisions.
- Also, due to large density of Cooper pairs, their velocity is small even for large currents.
- Due to small velocity and the collective movement of the Cooper pairs, collisions are reduced and resistivity decreases.

- The BCS theory was able to explain the critical field, the isotope effect and the Meissner effect.
- It could also explain the quantization of magnetic field in a superconducting ring with effective charge '2e' instead of 'e' as the ground state in BCS theory consists of pairs of electrons.

Q.20 Explain in brief the BCS theory of superconductivity. [SPPU : Dec.-15, Marks 4]

Ans. : Refer Q.19.

Q.21 Explain BCS theory of superconductivity. Mention why superconductivity is observed below critical temperature. [SPPU : May-16, Marks 6]

Ans. : Refer Q.19.

Q.22 Explain the formation of cooper pairs in superconductors with the help of electron phonon interaction. [SPPU : Dec.-17, 18, Marks 3]

Ans. : Refer Q.19.

Q.23 What is superconductivity ? Explain BCS theory of superconductors. [SPPU : May-18, Marks 6]

Ans. : Refer Q.1 and Q.19.

9.3 : Types of Superconductors

Q.24 Explain the phenomena of super-conductivity. Explain type-I and type-II super-conductors. [SPPU : Dec-12, Marks 6]

Ans. : For explanation of phenomenon of super-conductivity, refer Q.1.

Type-I Superconductors (Soft superconductors)

- The magnetization curve for a Type-I superconductor is shown in Fig. Q.24.1.
- Type-I superconductors show complete Meissner effect when the applied magnetic field H is less than the critical field H_c.

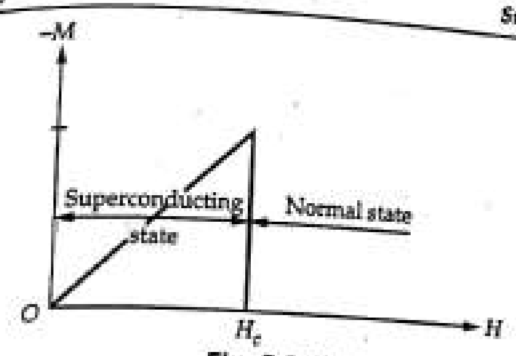


Fig. Q.24.1

- The magnetization in the specimen is proportional to the applied field and the specimen acts like a perfect diamagnet.
- For $H > H_c$, the magnetization is negligible and the specimen becomes a normal conductor.
- Pure elements show such magnetization curves. The values of H_c are very low.

Type-II Superconductors (Hard superconductors)

- The magnetization curve for a type-II superconductor is shown in Fig. Q.24.2.
- Type-II superconductors are characterized by two critical fields H_{c1} and H_{c2}.

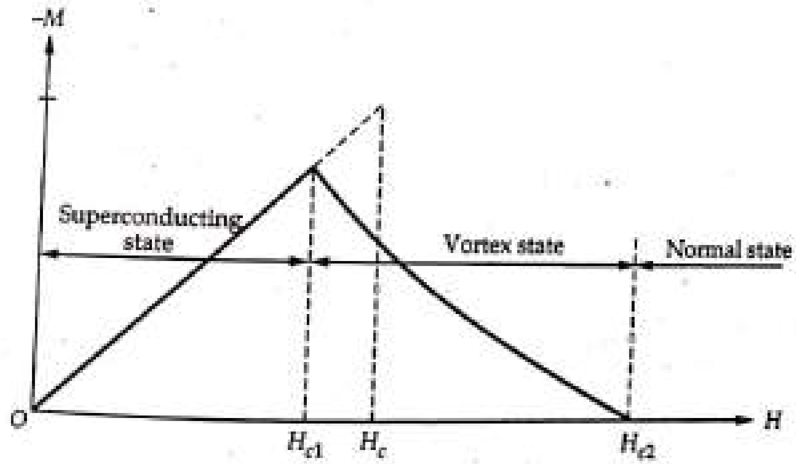


Fig. Q.24.2

- Upto the lower critical field H_{c1} the specimen is perfectly diamagnetic and flux is completely ejected out from the specimen.
- The magnetization in the specimen is proportional to the applied field. Between H_{c1} and H_{c2}, Meissner effect is incomplete but electrically the specimen is a superconductor.
- This state is called the mixed state or the vortex state as the specimen is a electrically a superconductor but not magnetically.
- The values of H_{c2} may be 100 times or more higher than the critical field H_c for type-I superconductors.
- Above the higher critical field H_{c2}, the specimen becomes a normal conductor.
- Type-II superconductors are usually alloys or transition metals with high values of electrical resistivity in the normal state.

Q.25 Differentiate between type-I and type-II superconductor with diagram. **☞ [SPPU : May-13, 15 Marks 4]**

OR Differentiate between type-I and type-II superconductor with diagram. (Any three points) **☞ [SPPU : Dec-13, Marks 3]**

Ans. : For diagrams, refer Q.24.

Sr.No.	Type - I superconductor	Type - II superconductor
1	There is only one critical field.	There are two critical fields.
2	They show complete Meissner effect when the applied magnetic field H is less than the critical field H_c.	Upto the lower critical field H_{c1} the specimen is perfectly diamagnetic. Between H_{c1} and H_{c2}, Meissner effect is incomplete but electrically the specimen is a superconductor.
3	There is no vortex state.	There is a vortex state.
4	These are normally pure elements.	These are usually alloys or transition metals with high values of electrical resistivity in the normal state.

Q.26 Explain critical magnetic field of superconductor. Differentiate between type-I and type-II superconductors (four points). **☞ [SPPU : May-17, Marks 6]**

Ans. : Refer Q.24 and Q.25.

Q.27 Give any four points to distinguish between type I and type II superconductors. **☞ [SPPU : May-18, 19, Marks 4]**

Ans. : Refer Q.25.

9.4 : High Temperature Superconductors

Q.28 What are high temperature superconductors ? Explain.

Ans. : • Kamerlingh Onnes discovered superconductivity in mercury in 1911 with a critical temperature of 4.2 K.

- Research was then focussed on materials with larger critical temperatures so that the phenomenon could become commercially viable.
- But, till about 1986, the highest critical temperatures were observed to be nearly 20 K.
- In 1986, an oxide of lanthanum, barium and copper ($La_{1.85}Ba_{0.15}CuO_4$) was observed to have a critical temperature of 36 K.
- In 1987, an oxide of yttrium, barium and copper ($YBa_2Cu_3O_7$) was found to have a critical temperature of 90 K and then an oxide of thallium, barium, calcium and copper($Tl_2Ba_2Ca_2Cu_3O_{10}$) was found to have a critical temperature of 120 K.
- The main feature of these high temperature superconductors is the presence of parallel sheets of CuO_2.

- Efforts are going on to find superconductors with critical temperatures near room temperature.

9.5 : Josephson Effect

Q.29 Explain D.C. and A.C. Josephson effect.

[SPPU : Dec-13, Marks 4]

Ans. : • Two superconductors separated by a thin layer of insulator is called a weak link or Josephson junction.

- The superconducting electron pairs (Cooper pairs) tunnel through the insulating layer from one superconductor to another.
- The following two effects are observed due to this tunneling of electron pairs :

D.C. Josephson effect : In absence of any electric or magnetic field, a d.c. current flows across the junction.

A.C. Josephson effect : Radio frequency current oscillations are produced across the junction when a d.c. voltage is applied across it. Also, when a radio frequency voltage is applied along with the d.c. voltage, it can cause d.c. current across the junction.

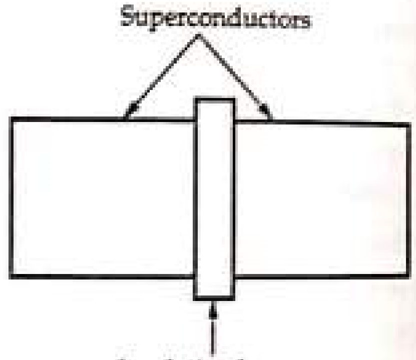


Fig. Q.29.1 Josephson junction

- If a d.c. voltage 'V' is applied across the junction, the current oscillates with frequency

$$\nu = \frac{2eV}{h}$$

- Josephson junctions are used in superconducting quantum interference devices (SQUIDS) which are used as diagnostic tools in detecting brain signals.

9.6 : Applications of Superconductors

Q.30 Explain two applications of superconductivity.

[SPPU : May-13, Dec-14, Marks 3]

Ans. : 1) Magnetic levitation :

- When a magnet is brought near a superconducting coil, current is induced in the superconductor which in turn produces a magnetic field which repels the magnet.
- The magnet can thus be made to float in air (i.e., levitate) on the superconducting coil.
- This phenomenon is called magnetic levitation.
- This concept has been used in the development of Maglev trains (i.e., magnetic levitation trains) which can attain speeds of upto 500 km/h as there is no friction between the rails and the wheels.

2) Superconducting magnets :

- Solenoids made of superconducting wires can generate strong magnetic fields without consuming large amount of power.
- As superconductors can carry much larger current densities without energy loss, the superconducting magnets will be light weight and compact.

Q.31 Explain the applications of superconductors in the field of electronics.

[SPPU : May-16, Marks 3]

Ans. : • In integrated circuits, there is generation of heat in interconnecting metal films due to their resistance. Also, their finite resistance makes the charging and discharging of capacitors slow. The use of superconductors will decrease heat loss and make the charging and discharging of capacitors very fast.

- This will make the circuits work faster and will make it possible to increase the density of components very fast switching times have been obtained using superconductors in digital electronic circuits. This will increase the speed of digital communication. Very sensitive microwave detectors, magnetometers and SQUIDS have been developed using Josephson junctions.

Q.32 Give any six applications of superconductivity. [SPPU : Dec.-15, Marks 3]

OR State any six applications of superconductors. [SPPU : May-14, 19, Marks 3]

Ans. :

1) Superconductors are used in magnetic levitation trains.

2) Superconductors are used in superconducting magnets.

3) They can be used in superconducting transmission lines.

4) They can be used in superconducting bearings.

5) Superconductors are used in IC fabrication.

6) They are used in supercomputers.

Q.33 Explain any one application of superconductors in brief. [SPPU : Dec.-16, Marks 3]

OR State applications of superconductors. Explain any one application. [SPPU : May-17, Marks 3]

Ans. : Refer Q.30, Q.31 and Q.32.

END...

UNIT - VI

10 Non-Destructive Testing

10.1 : Testing Methods

Q.1 Compare destructive and non-destructive testing.

Ans. :

Sr. No	Destructive Testing	Non-Destructive Testing
1.	The component is destroyed	The component is not destroyed.
2.	All components cannot be tested	All components can be tested.
3.	Tests are more time consuming.	Testing is rapid.
4.	Testing during operation is not possible.	Testing during operation is possible.
5.	Test results are quantitative	Test results are generally qualitative.

10.2 : Types of Defects

Q.2 List the various types of defects.

Ans. : • The following types of defects are required to be detected as they affect the performance of components :

1) Surface cracks
2) Surface irregularities
3) Internal cracks
4) Presence of impurities or air bubbles

5) Variation in dimensions

6) Creep : It is the permanent deformation of a component when subjected to constant load for a long time.

7) Fatigue : It is the failure of a material due to repeated cycles of loading and unloading.

10.3 : Objectives of Non-Destructive Testing

Q.3 List the objectives of non-destructive testing.

Ans. : • The objectives of non-destructive testing are as follows :

1) To detect flaws during the manufacturing of components so that defective components can be detected at an early stage of production. This saves time and money.
2) To detect flaws of finished components which will enhance their performance and reliability.
3) To detect flaws which may develop during operation which will ensure that there is no reduction in the rate of production
4) To increase the safety by ensuring that the defective components are replaced during periodic checks.

10.4 : Methods of Non-Destructive Testing

Q.4 List the methods of non-destructive testing.

Ans. : • The following methods are used for non-destructive testing :

1) Liquid/dye penetrant test
2) Radiography methods :
   a) *x-ray radiography* and b) *γ-ray radiography.*
3) Ultrasonic inspection
4) Magnetic particle inspection.
5) Visual inspection
6) Eddy current inspection.
7) Thermography.

10.5 : Acoustic Emission Testing

Q.5 Describe acoustic emission testing. State its advantages and disadvantages.

Ans. : • Acoustic emission is defined as the generation of an elastic wave due to rapid release of energy within the material.

• If a structure is subjected to change in pressure, temperature or load, some sites within the structure develop more deformation than its surrounding regions. This leads to release of energy in the form of stress waves.

• The stress waves propagate to the surface of the structure and can be detected by sensors.

• Such acoustic emission is also observed when iron, tin or ceramics are cooled from high temperatures.

• Acoustic emission also takes place due to development of cracks and breaking of fibres in a stressed material. Hence, these can also be detected by sensors.

• The sensors are piezo-electric crystals which are placed in arrays. They can defect the presence of defects and also locate their position.

• For example, if two sensors are placed at some distance from each other and acoustic emission takes place at the centre of the line joining them, the time interval between each sensor receiving the signal will be zero. If the acoustic emission takes place towards one of the detector, there will be some time interval between the each sensor receiving the signal. The position of the site of acoustic emission can be obtained from this time interval.

• This method gives real time data on development of cracks and breaking of fibres within the structure.

• This method is used in bridges, dams and aerospace structures.

Advantages :

1) Gives real time data during operation of the structure.
2) This method is highly sensitive.
3) Leads to early detection of flaws.
4) Cost is less.
5) There is no need to shut down the structure as the sensors are mounted permanently on the structure which continuously monitor it.

Disadvantages :

1) Requires the loading to be sufficiently high to produce a significant signal.
2) Requires sensors to be permanently mounted on the structure.

10.6 : X-ray Radiography

Q.6 Describe X-ray radiography method for non-destructive testing of engineering components. State its advantages and disadvantages.

Ans. : • X-ray have high penetration power. The smaller the wavelength, larger is the penetration power.

- When x-rays are passed through metals, the intensity decreases due to absorption.
- Larger the distance travelled by x-rays in the metal, larger is the decrease in intensity and smaller is the transmitted intensity.
- The decrease in intensity also depends on the density of the material.
- If x-rays are passed through a metal piece containing a flaw (e.g an air bubble) as shown in Fig. Q.6.1, larger intensity of x-rays will be transmitted through that region due to small thickness of metal in that region compared to surrounding region.
- The x-ray film kept behind the object will become darker in the region of the flaw as shown in Fig. Q.6.1.

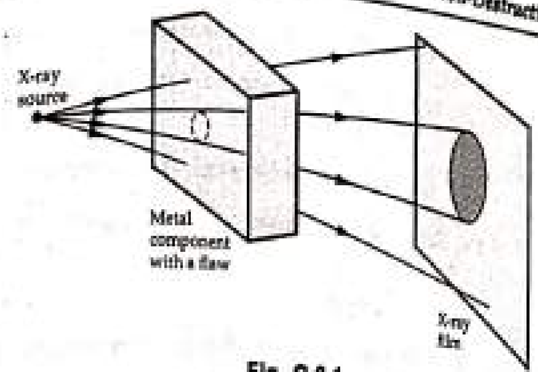


Fig. Q.6.1

- This is similar to the x-rays of human body in which the bones appear transparent on the film as they block the x-rays and remaining regions are darker.
- This method is able to reveal the flaws hidden in the interior of the component.
- If x-ray image is taken for only one position of the source, the flaw can be detected but its position cannot be located.
- To locate position of the flaw, the image is taken for two positions of the source as shown in Fig. Q.6.2

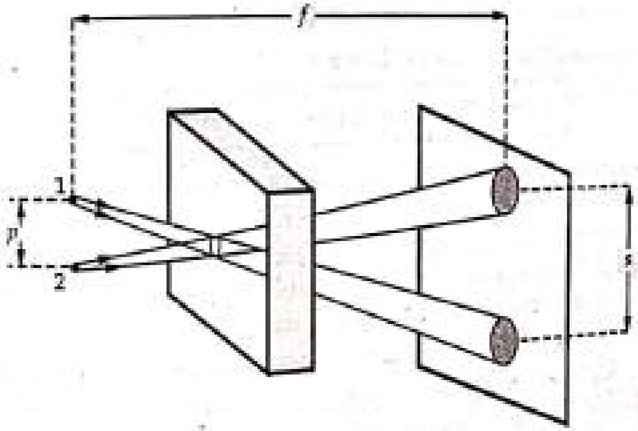


Fig. Q.6.2

If f = Distance between source and film,

p = Distance between two positions of source

and s = Distance between the two image,

The depth d of the flaw from the upper surface is given by

$$d = \frac{f s}{s + p}$$

- A similar approach is used in x-ray fluoroscopy except that the x-ray film is replaced by a fluorescent screen.
- This method gives live images of the components.
- Instead of x-rays, γ-rays can also be used to form images.

Advantages :

1) Hidden flaws can be detected.
2) Inspection takes very little time.
3) A wide variety of materials can be tested.

Disadvantages :

1) It is expensive compared to other methods.
2) Inspection has to be carried out in an isolated place as long exposure to x-rays is harmful to human beings.
3) Very small flaws cannot be detected.

10.7 : Ultrasonic Inspection Method

Q.7 Describe the ultrasonic testing method of flow detection. State its advantages and disadvantages.

Ans. : • Ultrasonic flaw detection technique uses the property of ultrasonic waves that they get partially reflected from any boundary separating two media having different densities.

- The time interval between the transmission of an ultrasonic pulse and the arrival of an echo is proportional to the distance of the boundary separating the two media from the transducer.

- An ultrasonic pulse is transmitted into the specimen from one end by an ultrasonic transducer like quartz crystal which also performs the function of a receiver.
- In absence of any flaw, the pulse is reflected from the other end of the specimen and the two pulses are displayed on a C.R.O. as shown in Fig. Q.7.1.

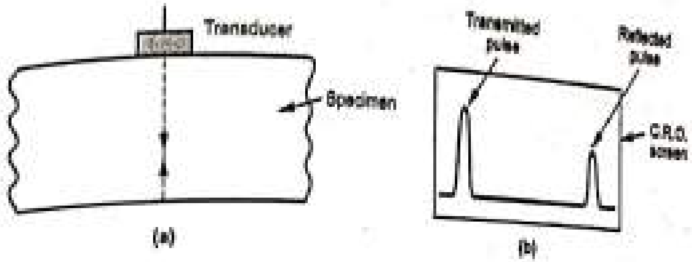


Fig. Q.7.1

- The distance between the two pulses on the C.R.O. screen can be calibrated in terms of the distance travelled by the ultrasonic waves in the specimen.
- If there is a flaw, an additional peak will be seen between the transmitted pulse and the pulse reflected from the other end as shown in Fig. Q.7.2.

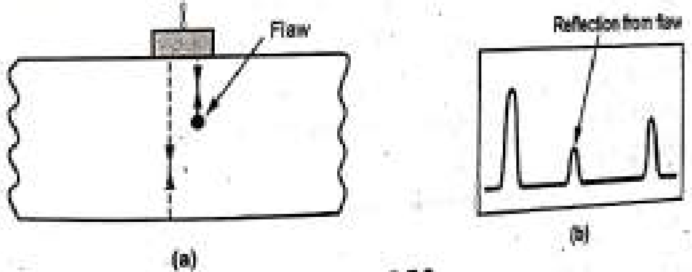


Fig. Q.7.2

- From the distance between transmitted pulse and the pulse corresponding to the flaw on the C.R.O. screen, the distance of the flaw from the surface through which ultrasonic wave was transmitted can be obtained.

- The position and dimensions of the flaw can be obtained by moving the transducer on the specimen surface and finding the distance of the flaw from different locations on the specimen surface.
- The ultrasonic flaw detection method is used in automobile, aircraft, railway and shipping industries to test machine parts and rails for flaws.
- Three types of visual display units are used in ultrasonic inspection. They are known as A-scan display, B-scan display and C-scan display.
- The A-scan display shows peaks corresponding to transmitted and reflected pulses as shown in Fig. Q.7.2 (b).
- The B-scan display gives a two dimensional cross sectional view. It gives two horizontal lines corresponding to reflection from top and bottom surfaces and additional lines in between if there are flaws as shown in Fig. Q.7.3.

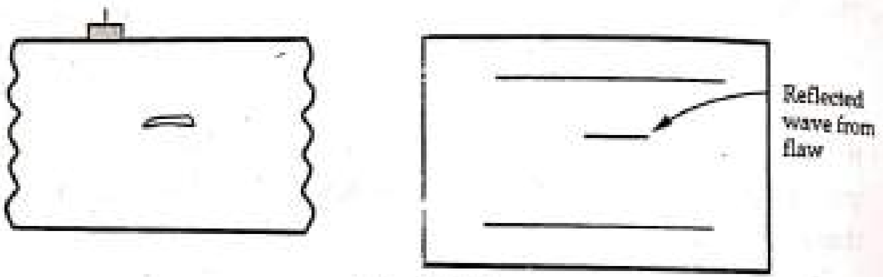


Fig. Q.7.3

- The C-scan display gives image similar to an x-ray in which the shape of the flaw is visible in two dimensions as shown in Fig. Q.7.4.

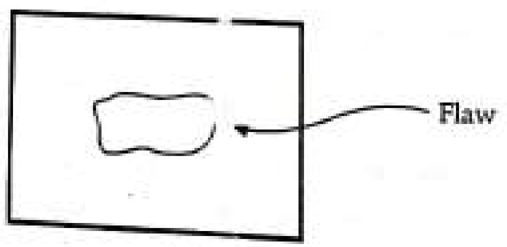


Fig. Q.7.4

Advantages :

1) Components with larger thicknesses can be tested compared to x-ray radiography.
2) This method is more accurate.
3) Low cost
4) High speed
5) Small flaws can also be detected.

Disadvantages :

1) Surface of the component needs to be smooth.
2) Very thin specimens cannot be tasted.
3) Specimen must have homogeneous composition.
4) Defects very close to the surface are not detected.

END...

UNIT - VI

11 Nanotechnology

11.1 : Nano-particles

Q.1 What are nano-particles ? Explain.

Ans. : • The radii of atoms and most of the molecules are less than a nanometer.

- Nanoparticles are generally considered to have radius in the range of *1 nm* to *100 nm* which can have *25* to 10^6 atoms. A cluster of *1 nm* radius has approximately *25* atoms.
- This definition of nanoparticles based on size does not distinguish between molecules and nanoparticles as many organic molecules contain more than *25* atoms.
- Nanoparticles can be more appropriately defined as an aggregate of atoms between *1 nm* and *100 nm* with dimensions less than the characteristic length of some physical phenomena.
- When particle size is less than the characteristic length of some physical phenomena, the particles show different properties.
- The nanoparticles show unique properties that change with their size.
- Classical mechanics is able to explain properties of bulk materials but is unable to explain properties of nanoparticles.
- Quantum mechanical principles have to be used to explain properties of the nanoparticles.
- The state of matter around the nanosize is known as **mesoscopic state.**

11.2 : Quantum Confinement

Q.2 Explain quantum confinement and its effect on allowed energies for electrons in various types of quantum structures.

Ans. : • Density of states is the possible electron quantum energy states between energies *E* and *E+dE* per unit volume. The density of states for metals in three dimensions in a bulk material is given by

$$g(E) = \frac{8\sqrt{2}\,\pi\, m^{3/2}}{h^3}\, E^{1/2}$$

- Hence, the density of states increases with energy as shown in Fig. Q.2.1 (a). From the above equation, we can conclude that the density of states in a bulk material is proportional to $E^{1/2}$ and hence increases with energy.
- If the electrons are confined in one or more directions by reducing the dimensions of the material in those directions, the density of states changes due to quantization of energy.
- For a quantum well, which is a material reduced in one dimension to nano - scale, there is quantization of energy due to confinement of electrons in one direction. The density of states for a quantum well is given by,

$$g(E) = \frac{4\pi m}{h^2}$$

- Hence, the density of states for a given quantum state will be constant as shown in Fig. Q.2.1(b)(ii). For the different quantum states, it has different constant values as shown in Fig. Q.2.1(b)(iii).
- For a quantum wire, which is a material reduced in two dimensions to nano - scale, there is quantization of energy due to confinement of electrons in two directions. The density of states for a quantum wire is given by

$$g(E) = \frac{2\sqrt{2}\, m^{1/2}}{h^3}\, E^{-1/2}$$

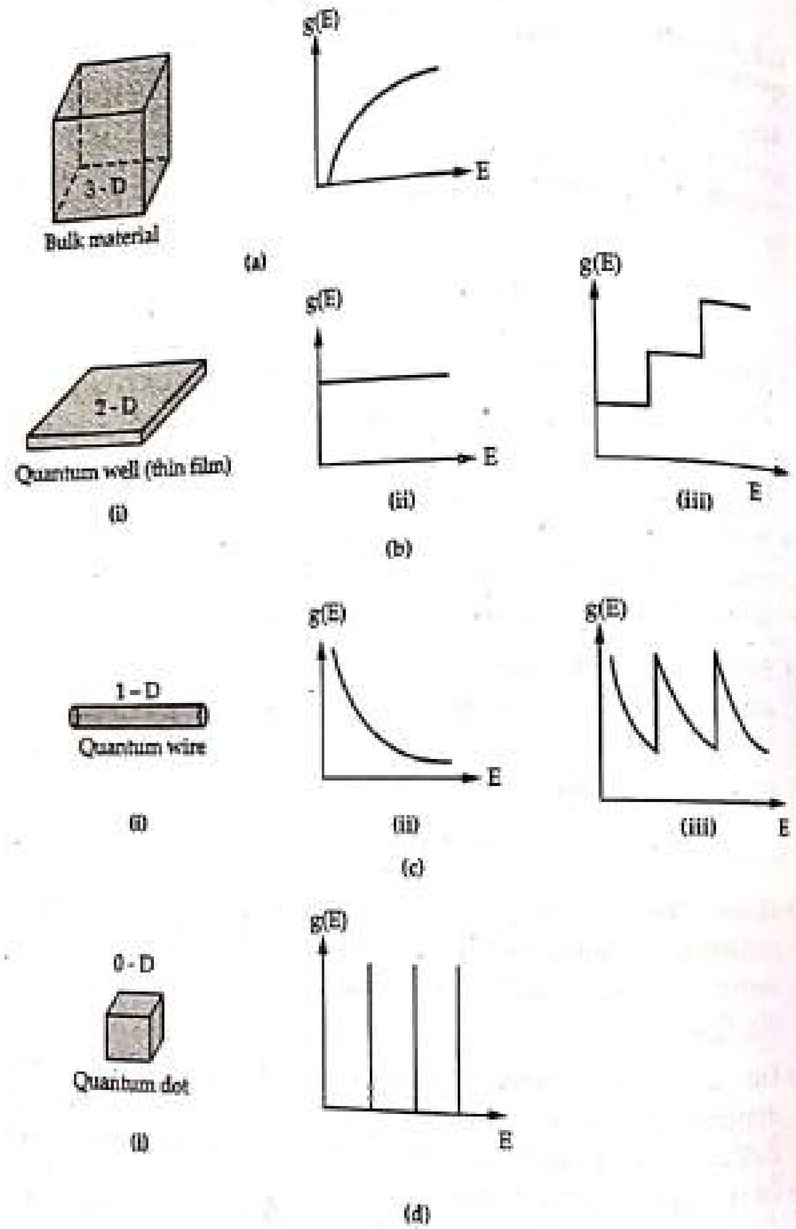


Fig. Q.2.1

- Hence, the density of states for a given quantum state will decrease with increasing energy as shown in Fig. Q.2.1 (c)(ii). For the different quantum states, it has variation as shown in Fig. Q.2.1 (c)(iii).
- For a quantum dot, which is a material reduced in all three dimensions to nano - scale, there is quantization of energy due to confinement of electrons in all the directions. Hence, for a quantum dot, only some discrete energy states are allowed for the electrons. As only certain energy states are allowed, the density of states has a discrete structure as shown in Fig. Q.2.1 (d).

11.3 : Surface to Volume Ratio

Q.3 Describe the effect of surface to volume ratio on properties of nanomaterials.

Ans. : • When particle size decreases, the fraction of atoms on the surface to the total number of atoms increases.

- Hence, nanomaterials have larger fraction of atoms at the surface compared to bulk materials.
- Also, the surface energy per unit mass is larger for nanomaterials.
- The surface to volume ratio, which represents the fraction of the atoms on the surface, is larger for nanomaterials.
- The large surface to volume ratio is responsible for higher reactivity of nanomaterials and also their energy band structure.

11.4 : Properties of Nanoparticles

Q.4 Explain optical, electrical and mechanical properties of nanoparticles.

Ans. : **1) Optical properties :**

- The colour of nanoparticles is different from the bulk material.

- When a bulk material is reduced in size to a few hundred atoms, the energy band structure of the bulk material changes to a set of discrete energy levels.
- Atomic clusters of different sizes will have different energy level separations.
- As clusters of different sizes have different energy level separations, the colour of the clusters (which are due to transitions between the energy levels) will depend on their size.
- Hence the size of the cluster can be altered to change the colour of a material.
- For example, gold in bulk form appears yellow but gold nanoparticles appear bright red in colour.
- The medieval glass makers produced tinted glass with beautiful variety of colours by dissolving metal particles like gold, silver, cobalt, iron etc.
- Due to these metal nanoparticles, the glasses appear coloured.
- In semiconductor nanoparticles (which are used in quantum dots) there is significant shift in the optical absorption spectra towards blue as the particle size is reduced.

2) Electrical properties :

- The resistivity in bulk matter is mainly due to scattering of electrons by ions and crystal defects.
- In nanostructures, the resistivity mainly depends on scattering from boundaries of nanoparticles when particle size becomes less than the mean free path between collisions.
- Thus smaller particle size increases the resistivity.
- Various types of defects in the lattice also increase the resistivity by limiting the mean free path. But many nanostructures are too small to have internal defects.

- Another effect of reduced size is the confinement of conduction electrons. In bulk conductors, the electrons move freely throughout the entire conductor.
- The situation changes when one or more dimensions of the conductor are made very small.
- In a **quantum well**, the electron will be confined along one dimension but will move freely along the remaining two dimensions.
- If a conducting wire has a long length but very small diameter, the electrons can move freely along the length but will be confined in two mutually perpendicular transverse directions. This configuration is known as a **quantum wire**.
- If all the three dimensions of the conductor are in nanometer range, the configuration is called a **quantum dot** and the electron is confined in all the three dimensions.
- Confinement of electrons to small dimensions leads to quantization of energy.
- The level of doping in semiconductors gives rise to another important phenomenon.
- For typical doping levels of 1 donor impurity atom in 10^8 atoms of semiconductor atoms, a quantum dot of 10^7 semiconductor atoms would have an average of 10^{-1} electrons.
- In other words, on an average, one quantum dot in 10 will have a free electron.
- These result in the phenomena of single-electron tunneling and coulomb blockade.
- The conduction is due to tunneling of electrons through the quantum dot. The electrons are blocked from tunneling except at discrete voltage change positions.
- This phenomenon is called coulomb blockade.

- The *I-V* characteristic shown in Fig. Q.4.1 is called coulomb staircase.

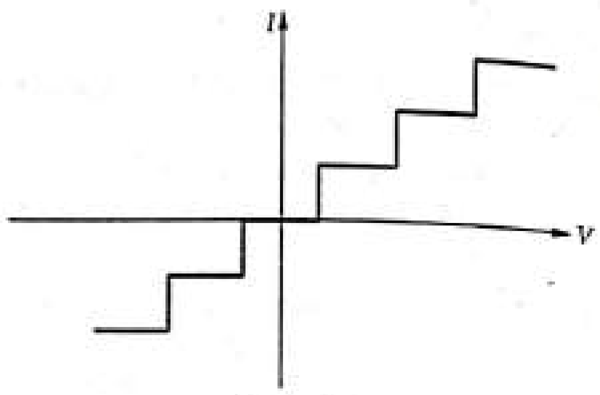


Fig. Q.4.1

3) Mechanical properties :

- Mechanical properties like hardness, elasticity and ductility depend upon the bonds between atoms.
- Imperfections in the crystal structure and impurities result in changes in these properties.
- As the nanoparticles are highly pure and free from imperfections, they show different mechanical properties than the bulk material.
- It has been observed that the Young's modulus decreases in metallic nanocrystals with decrease in particle size.
- The yield stress has been observed to increase with the decrease in grain size in bulk materials with nanosized grains.
- Hence stronger materials can be produced by making materials with nanosized grains.
- The carbon nanotubes are estimated to be about 20 times stronger than steel.

Q.5 Explain any two properties of nano-particles in brief.
☞ [SPPU : Dec.-15, Marks 4]

Ans. : Refer Q.4.

Q.6 Why are the properties of nano-particles different from that of the bulk materials ? Explain any two properties of nano-particles.
☞ [SPPU : May-16, Marks 6]

Ans. : Refer Q.4.

Q.7 Explain the mechanical property of nano-particles.
☞ [SPPU : Dec.-16, Marks 2]

Ans. : Refer Q.4.

Q.8 What is nano technology ? Explain optical and electrical properties of nano-particles.
☞ [SPPU : May-17, Marks 6]

Ans. : Refer Q.1 and Q.4.

Q.9 Explain the mechanical properties of nanoparticles.
☞ [SPPU : Dec.-17, Marks 3]

Ans. : Refer Q.4.

Q.10 Explain the optical and electrical properties of nanoparticles.
☞ [SPPU : May-18,19, Marks 6]

Ans. : Refer Q.4.

11.5 : Applications of Nanoparticles

Q.11 Explain applications of nanoparticles.

Ans. : Nanotechnology has found wide ranging applications in many fields. Some of these applications are discussed below.

1) **Electronics :** Nanosized electronic components show unique properties which are different from the larger semiconductor components. The semiconductor devices are based on the concept of charge transport only whereas the nanosized components work on the concept of charge as well as spin transport of electrons. This has been used in devices like spin FET, spin LED etc. These devices have increased the data

storage capacities of hard disks and have led to small and faster microprocessors.

2) **Energy :** Attempts are being made to increase the efficiency of solar cells by using nanotechnology. Another important area of research is the use of hydrogen as a fuel. The main problem with hydrogen is that it is highly combustible and hence cannot be stored easily. Efforts are being made to use carbon nanotubes to trap and store hydrogen.

Nanoparticles are also being used to increase the energy density of rechargeable batteries. Which are used in laptops and mobile phones.

3) **Automobiles :** Nanotube composites have better mechanical strength compared to steel but are costly at present. Efforts are being made to develop cheaper nanotube composites that can replace steel which is used to construct the body structure of automobiles. Use of nanoparticles in paints provides thin and smooth coatings.

Nanoparticles are being used to develop light weight and less rubber consuming tyres for automobiles which will increase the mileage of the automobiles. The use of carbon nanotubes for storing hydrogen is being explored so that the automobiles can be run on hydrogen as a fuel.

Nanonmaterial catalysts can be used as catalysts to convert the harmful emissions from automobiles to less harmful gases.

4) **Space and defence :** Aerogels are porous materials with nanosized pores. They have very low density and are poor conductors of heat. They can be used in spacecrafts, light weight suits and jackets.

Polymer composites using silica fibers and nanoparticles have larger mechanical strength and low temperature coefficient of expansion. They can be used in spacecrafts which have to withstand high temperature and stress conditions during launching and re-entry into the earth's atmosphere.

Satellites and spacecrafts use solar energy.

The efficiency of solar cells can be increased using nanoparticles. The use of nanoparticles will also make the solar cells smaller in size and light weight.

5) **Medical :** Nanoparticles can be used for detection and treatment of cancers and tumours. The nanoparticles are injected into the body and guided towards a specific part. Drugs can be encapsulated in nanocapsules and guided towards any specific part where the drug can be delivered in a controlled manner by opening the capsule at a desired rate using magnetic fields or infrared light. This targeted drug delivery does not affect the healthy organs. Nanotechnology based tests are being developed for fast detection of viruses and antibodies.

6) **Environmental :** Nanoparticle based sensors are capable of detecting water and air pollution due to toxic ions and pesticides with a very high sensitivity. Nanomaterial catalysts can be used as catalysts to convert the harmful emissions from industries and automobiles to less harmful gases.

Nanotubes can be used to store hydrogen fuel which, when used in automobiles, will reduce harmful emissions.

7) **Textiles :** The use of nanotechnology in textile industry has led to the development of water repellent and wrinkle free clothes.

8) **Cosmetics :** Zinc oxide and titanium oxide nanoparticles are used in sunscreen lotions which protect the skin from the ultraviolet radiations. These nanoparticles absorb ultraviolet radiations. Nanoparticle based dyes and colours are harmless to the skin and hence are used in hair creams, gels and hair dyes.

Q.12 Explain the applications of nano-particles in electronic industry.

[SPPU : Dec.-15, Marks 6]

Ans. : Refer Q.11.

Q.13 Explain the applications of nano-particles in the field of automobiles.

[SPPU : May-16, Marks 3]

Ans. : Refer Q.11.

Q.14 Explain the application of nano particles in medical and automobile field.
☞ [SPPU : Dec.-16, Marks 3]

Ans. : Refer Q.11.

Q.15 State applications of nano-particles. Explain any one application.
☞ [SPPU : May-17,19, Marks 3]

Ans. : Refer Q.11.

Q.16 List the applications of nanotechnology in the field of automobile. Explain any one application in brief.
☞ [SPPU : May-18, Marks 3]

Ans. : Refer Q.11.

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