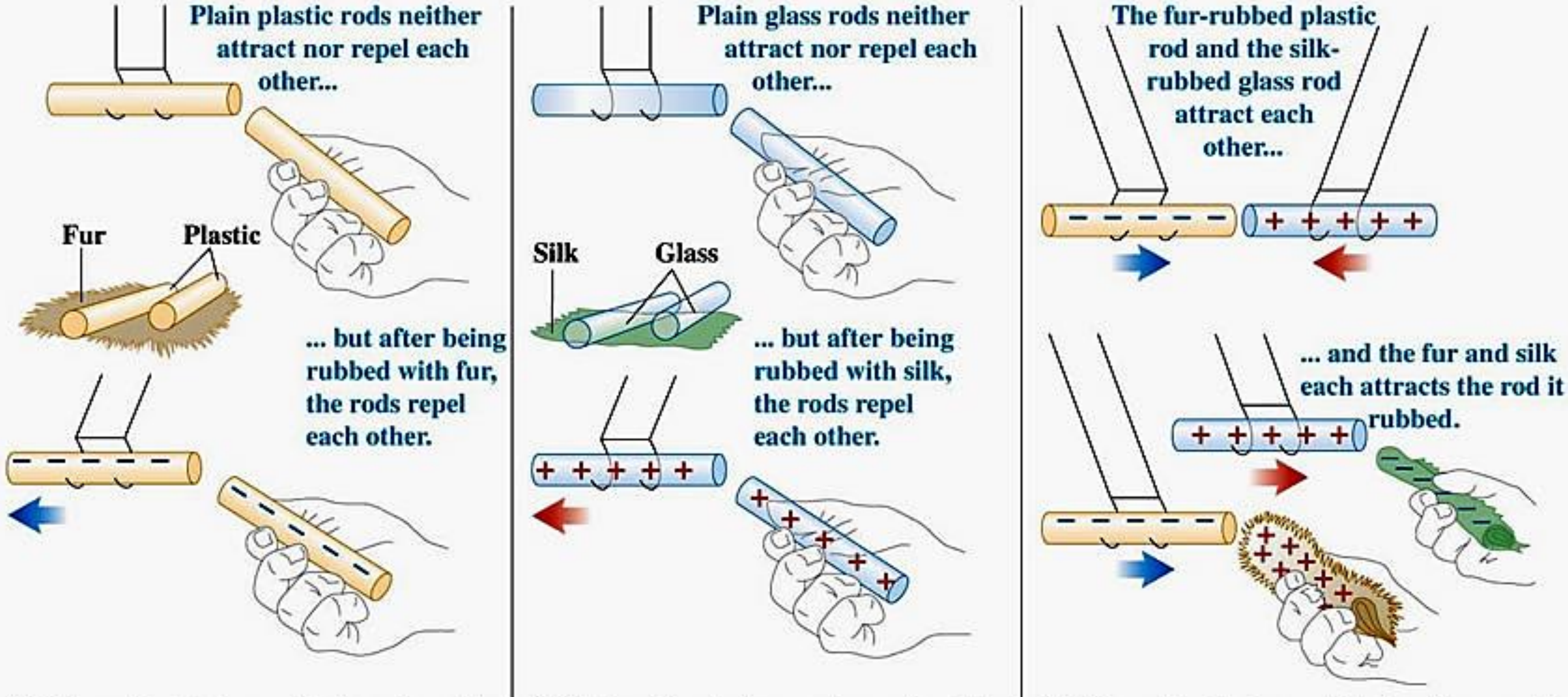
Unit – 2 A

ELECTROSTATICS

SYLLABUS

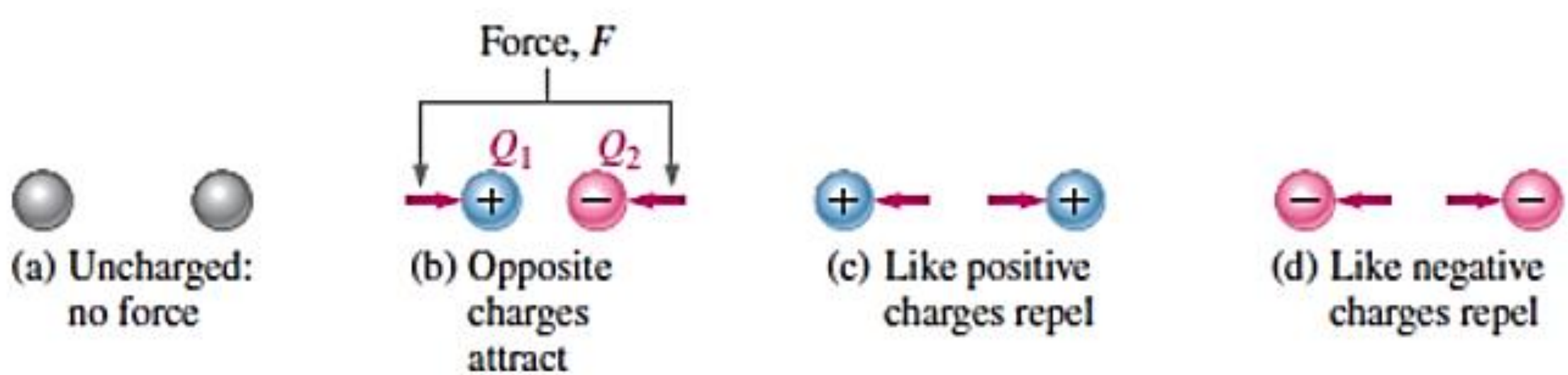
Electrostatics field, Electric flux density, Electric field strength, Absolute permittivity, Relative permittivity, Capacitance and capacitor, Composite dielectric capacitors, Capacitors in series and parallel Energy stored in capacitors, Charging and discharging of capacitors (No derivation) and time constant.



(a) Interaction between plastic rods rubbed on fur

(b) Interaction between glass rods rubbed on silk

(c) Interaction between objects with opposite charges



Laws of Electrostatics

1) Like charges repels and Unlike charges Attract each other.

2) Coulomb's Inverse Square Law: Statement

The mechanical force (either due to attraction or repulsion) between two small charge bodies is:

i) Directly proportional to the product of the charges.

ii) Inversely proportional to the square of distance between them

iii) Depends upon the nature of surrounding medium.

$$\text{Force F} \propto \frac{Q_1 Q_2}{\mathrm{d}^2}$$

$$or \qquad F = K\frac{Q_1 Q_2}{\mathrm{d}^2}$$

$Q_{1,}$ Q_2 are magnitudes of electric charge

K is the constant of proportionality. The value of K depends upon nature of the medium.

$$K = \frac{1}{4\pi \varepsilon} = \frac{1}{4\pi \varepsilon_o \varepsilon_r}$$

ε – *Absolute permitivity of the medium.*

ε_o – *Permitivity of Free space* $= 8.854 \times 10^{-12} F/m$

ε_r – Relative permitivity of the medium.

$$F = \frac{1}{4\pi \varepsilon_o \varepsilon_r} \frac{Q_1 Q_2}{d^2} \quad newtons$$

$If\ \varepsilon_r = 1\ for\ air\ \ and\ \ if\ Q_1 = Q_2 = 1C,\ d = 1m$

$F = 9 \times 10^9\ Newton$

One Coulomb is the total charge possessed by 6.24 ×10^18 electrons.

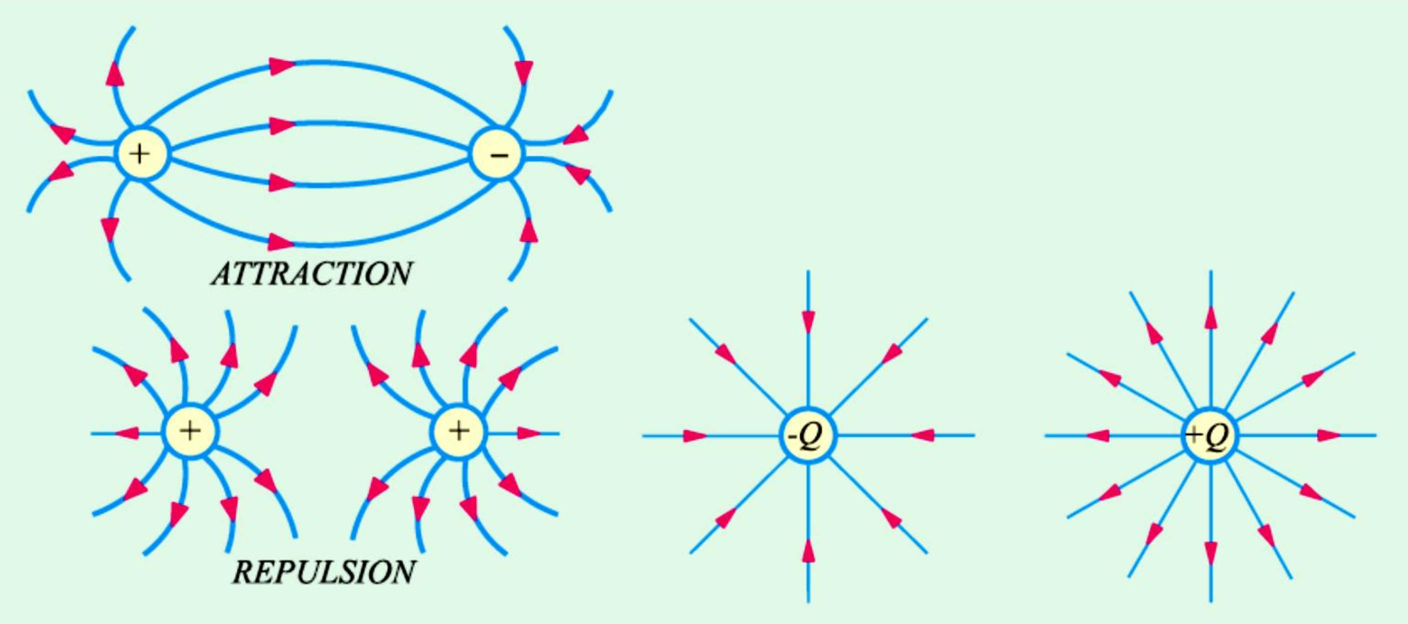
BASIC DEFINATIONS

1) Electrostatic field or Electric field.

The region around a charge in which the influence of the Electrostatic force exists is called electrostatic field or electric field.

2) Electric Lines of Force

The electric field around a charge is imagined in terms of presence of lines of force around it are called electric or Electrostatic Lines of force or field.



Properties of lines of force

The properties of the line of force are

1. In an electric field, each line of force will originate from positive charge and terminate on a negative charge .
2. They never intersect or cross one another .
3. They always enter or leave the conducting surfaces normally.
4. The lines travelling in the same direction repel each other, while traveling in the opposite direction attract one another.
5. They behave like a stretched rubber band, Since they always try to contract in length.
6. They pass only through the insulating medium in between the charges. But do not enter the charged bodies.

3) Electric Flux: Total no of lines of force in any particular electric field is called electric flux.

Electric Flux $\psi = Q \quad coulombs$

4) Electric Flux density: The flux per unit area measured at right angles to the direction of electric flux is known as electric flux density.

$$D = \frac{\psi}{A} coulombs / m^2 = \frac{Q}{A}$$

5) Electric field strength or electric filed intensity : It is defined as the force experienced by a unit positive charge placed at any point in the electric filed

$$E = \frac{F}{Q} N / C$$

6) Electrical Potential & Potential Difference:

$$ElectricPotential\,(V) = \frac{Work\ done\,(W)}{\text{Charge (Q)}}$$

Permittivity

1)Absolute Permittivity:

$$\varepsilon = \frac{D}{E} = \frac{\text{Electric flux density in dielectric medium}}{\text{Electric field strength}} \quad farads/meter$$

$$or\ D = E\varepsilon \qquad couloumbs/meter^2$$

2) Permittivity of free space or Electric space constant:

$$\varepsilon_o = \frac{D_o}{E} = \frac{\text{Electric flux density in vaccum}}{\text{Electric field strength}}$$

$$or\ D_o = E\varepsilon_o$$

3) Relative Permittivity:

$$\text{Relative Permittivity } \varepsilon_r = \frac{D}{D_o} = \frac{\varepsilon}{\varepsilon_o}$$

$$\varepsilon = \varepsilon_r\,\varepsilon_o \qquad farads/meter$$

Material	Relative permittivity
Vacuum	1.0
Air	1.0006
Paper (dry)	2–2.5
Polythene	2–2.5
Insulating oil	3–4
Bakelite	4.5–5.5
Glass	5–10
Rubber	2–3.5
Mica	3–7
Porcelain	6–7
Distilled water	80
Barium titanate	6000+

Breakdown Voltage and Dielectric Strength

Dielectric strength of an insulator or dielectric medium is given by the ***maximum potential difference which a unit thickness of the medium can withstand without breaking down***.

Its unit is volt / metre (V/m) although it is usually expressed in kV/mm.

Sr.No.	Material	Dielectric constant	Dielectric strength in kV/mm
1	Air	1	3
2	Bakelite	5	15 to 25
3	Mica	6	46 to 200
4	Dry paper	2.2	5 to 10
5	Glass	6	6 to 26

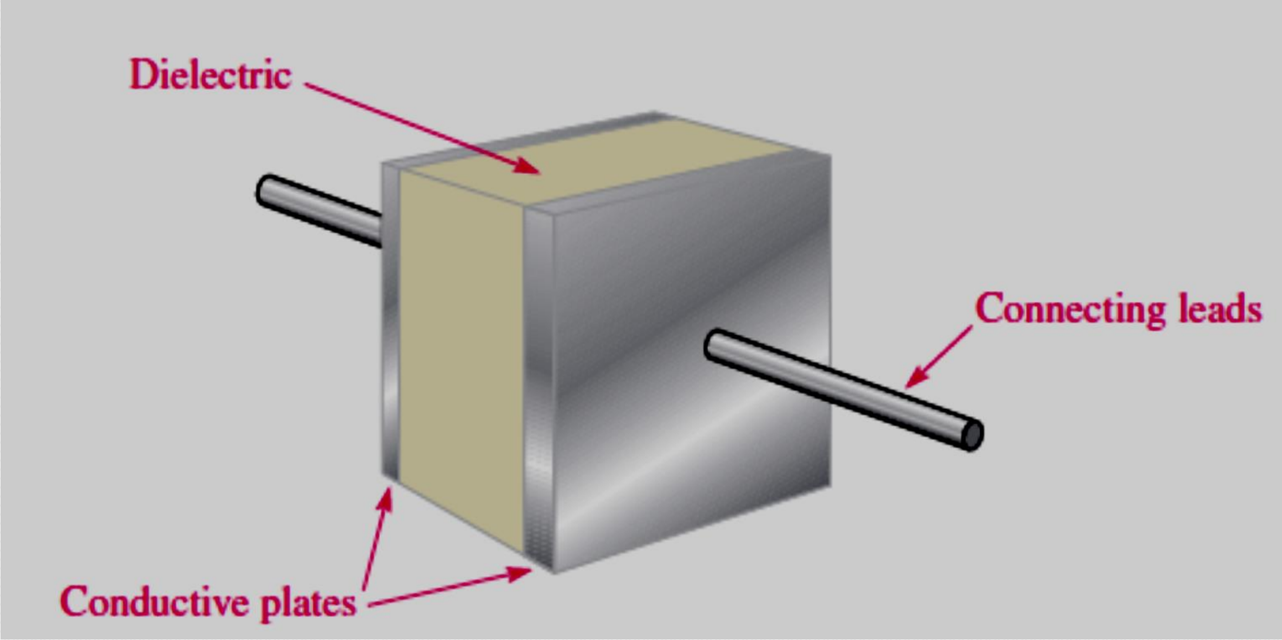
CAPACITOR AND ITS CAPACITANCE

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called ***dielectric***. The dielectric material allows the charge to accumulate on the capacitor plates.

$$Q \;\alpha\; V$$

$$Q = C\,V$$

$$C \;\alpha\; \frac{A}{d}$$

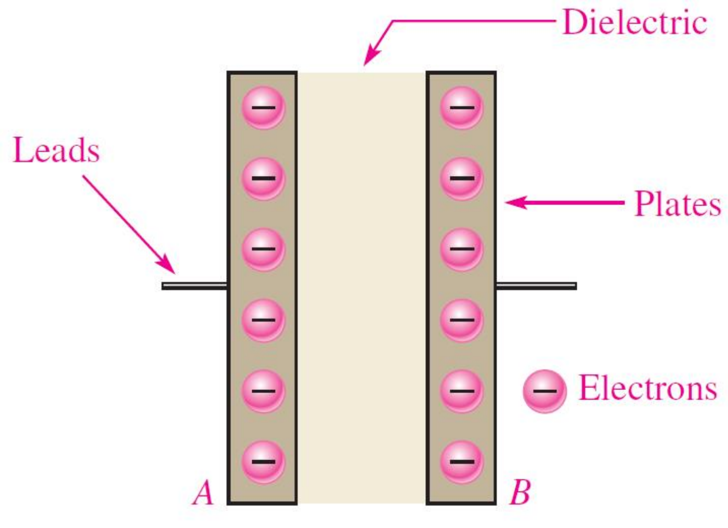


$$C = \varepsilon_0 \varepsilon_r \frac{A}{d} \quad Farad$$

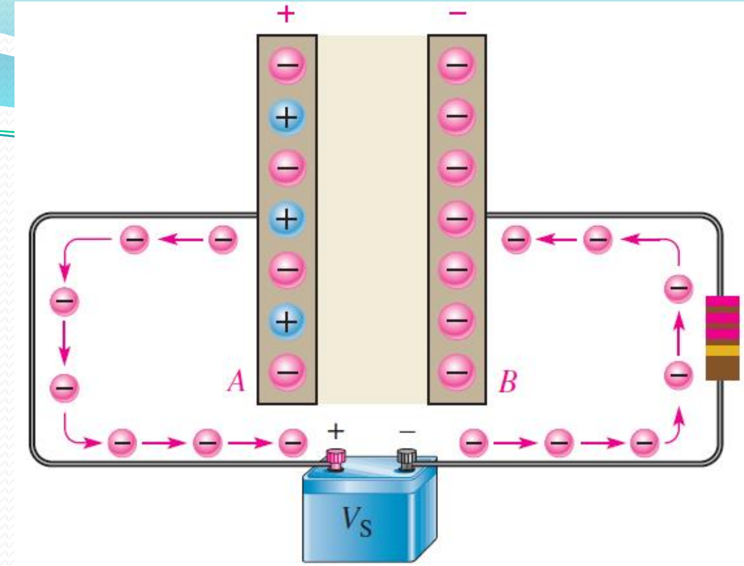
Capacitance is the property of a capacitor to store an electrical energy in the form of electrostatic field.

There are two types of capacitor, **electrolytic** and **non-electrolytic**.

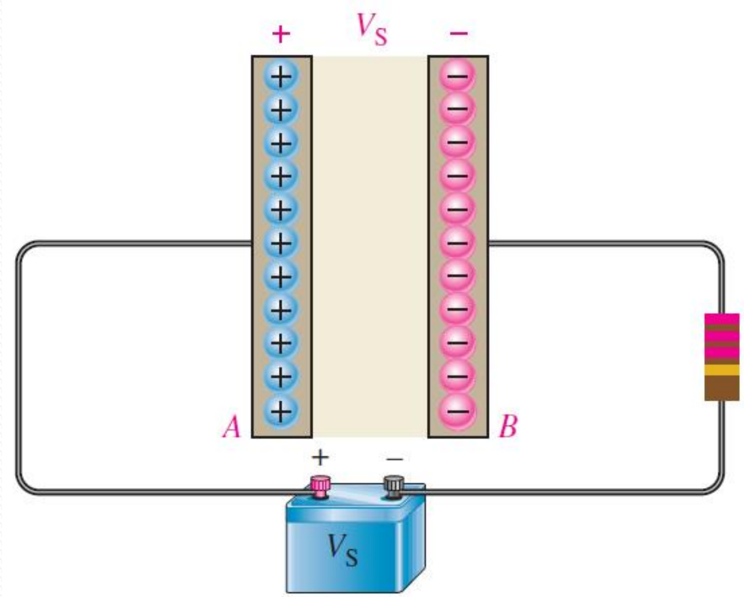
- Electrolytic capacitors hold much more charge;
- Electrolytic capacitors have to be connected with the correct polarity, otherwise they can explode.



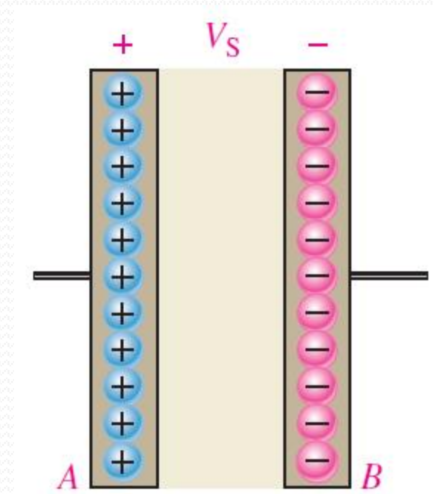
Neutral (uncharged) capacitor (same charge on both plates)



Electrons flow from plate *A* to *B* as the capacitor charges when connected to a voltage source.



After the capacitor charges to *VS*, no electrons flow while connected to the voltage source.



Ideally, the capacitor retains charge when disconnected from the voltage source.

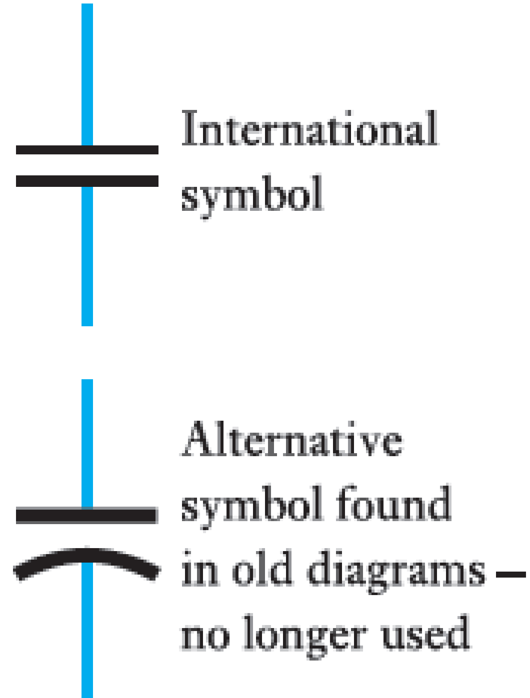
Relation between charge and applied voltage:

$$\frac{\text{Charge [coulombs]}}{\text{Applied p.d. [volts]}} = \text{capacitance [farads]}$$

or in symbols

$$\frac{Q}{V} = C$$

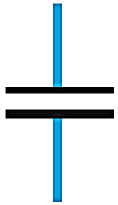
$$\therefore \qquad Q = CV \quad \text{coulombs}$$



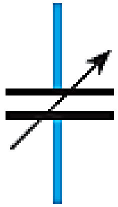
Capacitance Symbol: C Unit: farad (F)

One farad is actually too large for practical purposes. Hence, much smaller units like microfarad (μF), nanofarad (nF) and micro-microfarad (μμF) or picofarad (pF) are generally employed.

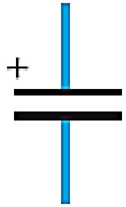
METRIC PREFIX	SYMBOL	POWER OF TEN
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}



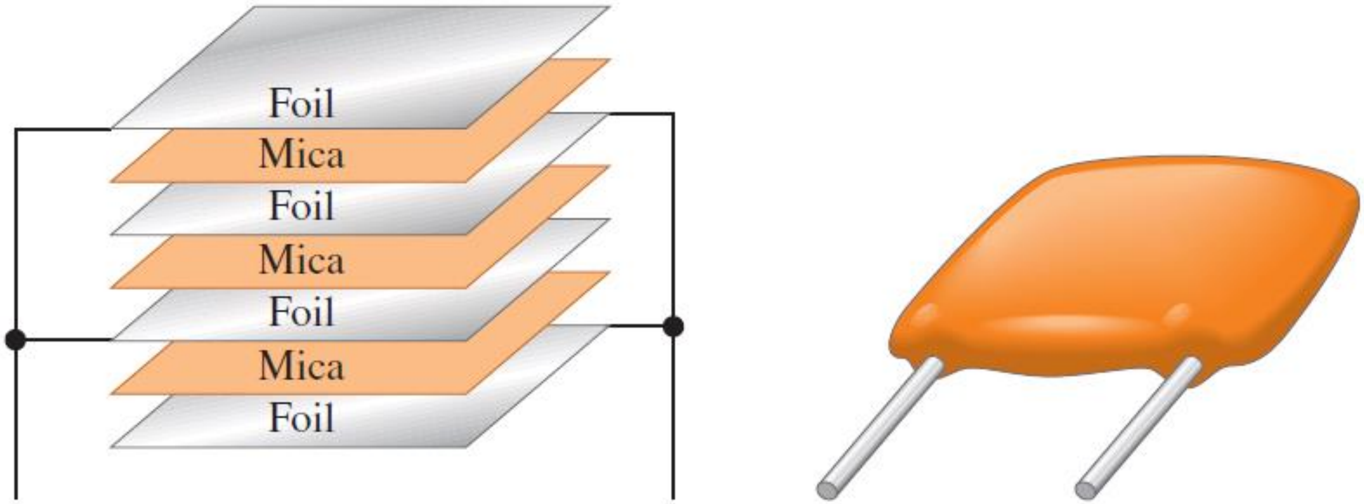
Fixed capacitor



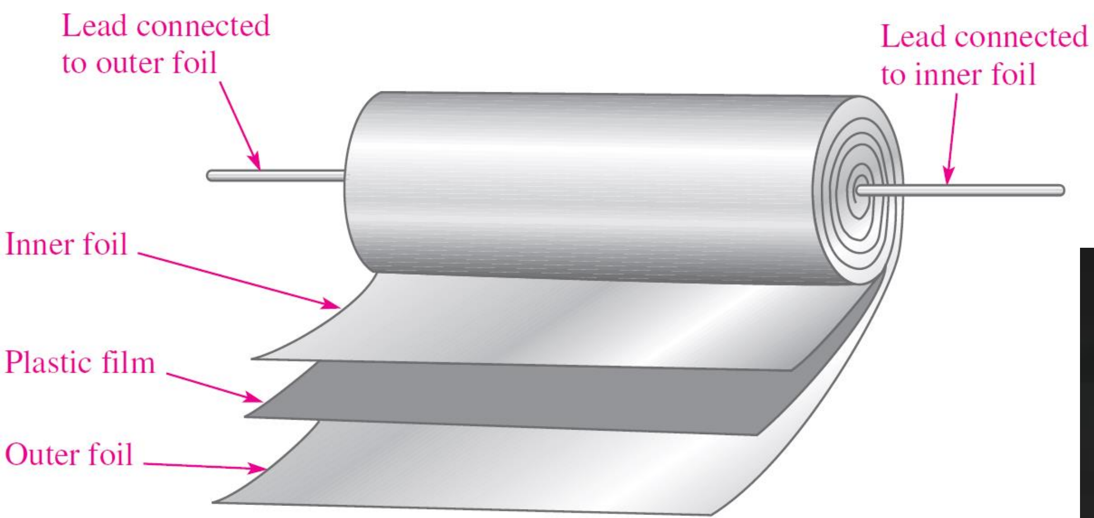
Variable capacitor



Electrolytic capacitor



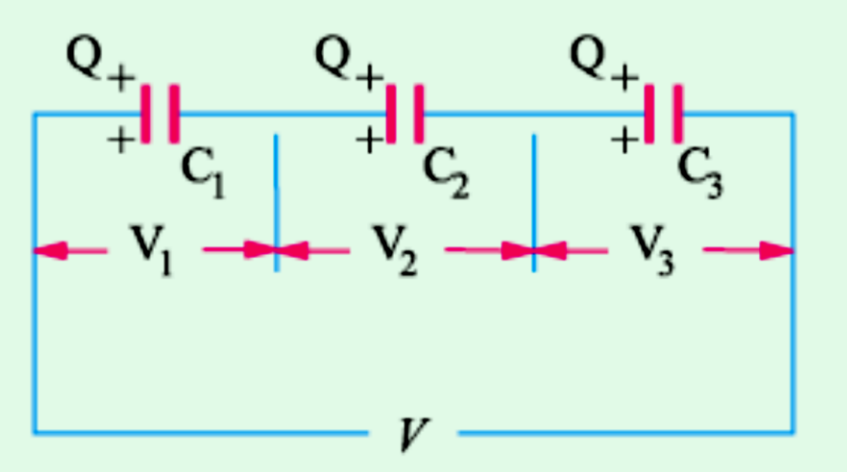
Mica Capacitor :Layers are pressed together and encapsulated.



plastic-film dielectric capacitors.

Capacitors in series and parallel

Capacitors in Series



The potential drops, V_1 ,V_2 and V_3 , across the three capacitors are, in general, different. However, the sum of these drops equals the total potential applied; *i.e.* $V=V_1+V_2+V_3$. The equivalent capacitance of the three capacitors is

$$V = V_1 + V_2 + V_3$$

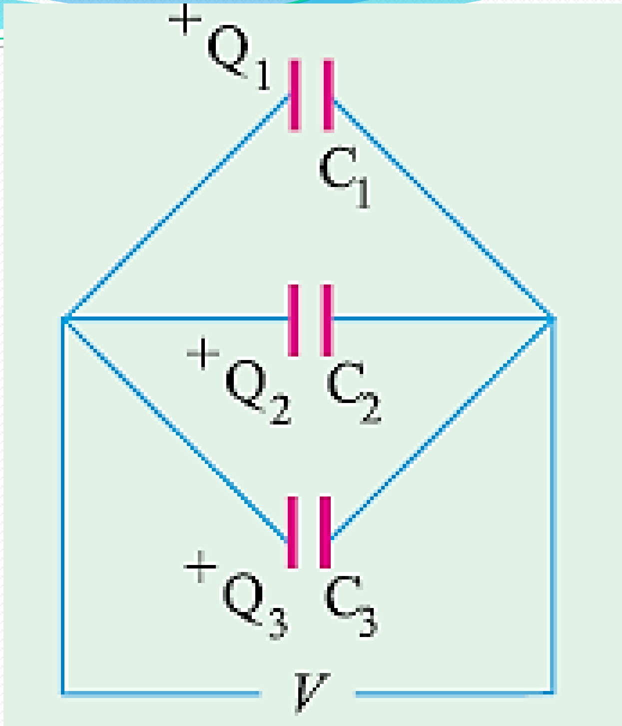
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors connected in Parallel

Consider three capacitors connected in *parallel*; In this case, the potential difference V across the three capacitors is the same, and is equal to the potential difference V.

The total charge Q however, stored in the three capacitors is divided between the capacitors, since it must distribute itself such that the voltage across the three is the same.

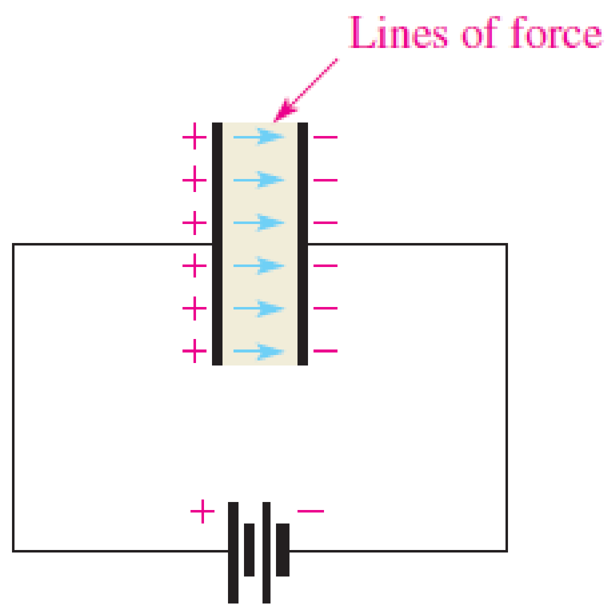


$$Q = Q_1 + Q_2 + Q_3$$

$$C = C_1 + C_2 + C_3$$

Energy Stored in Capacitor

Suppose at any stage of charging, the p.d. across the plates is *v*. By definition, it is equal to the work done in shifting one coulomb from one plate to another. If '*dq*' is charge next transferred, the work done is



$dW = v.dq$

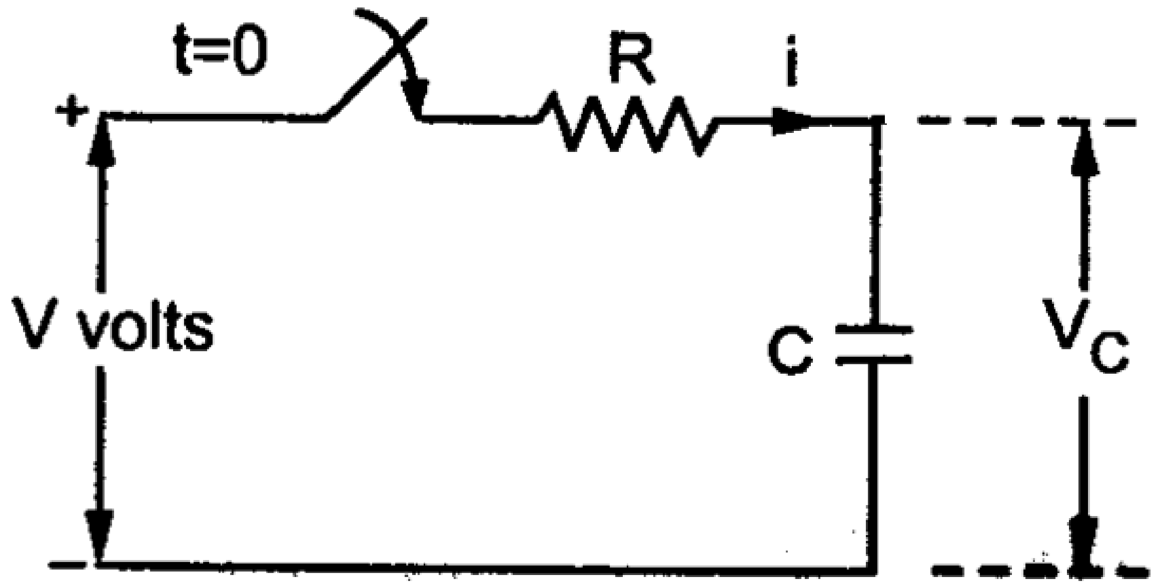
Now $q = Cv$ $dq = C.dv$ $dW = Cv.dv$

Total work done in giving *V* units of potential is

$$W = \int_0^v Cv.dv = C\left|\frac{v^2}{2}\right|_0^v \quad \therefore W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}CV^2 \text{ joules} = \frac{1}{2}QV \text{ joules} = \frac{Q^2}{2C} \text{ joules}$$

Charging of Capacitor through a Resistance

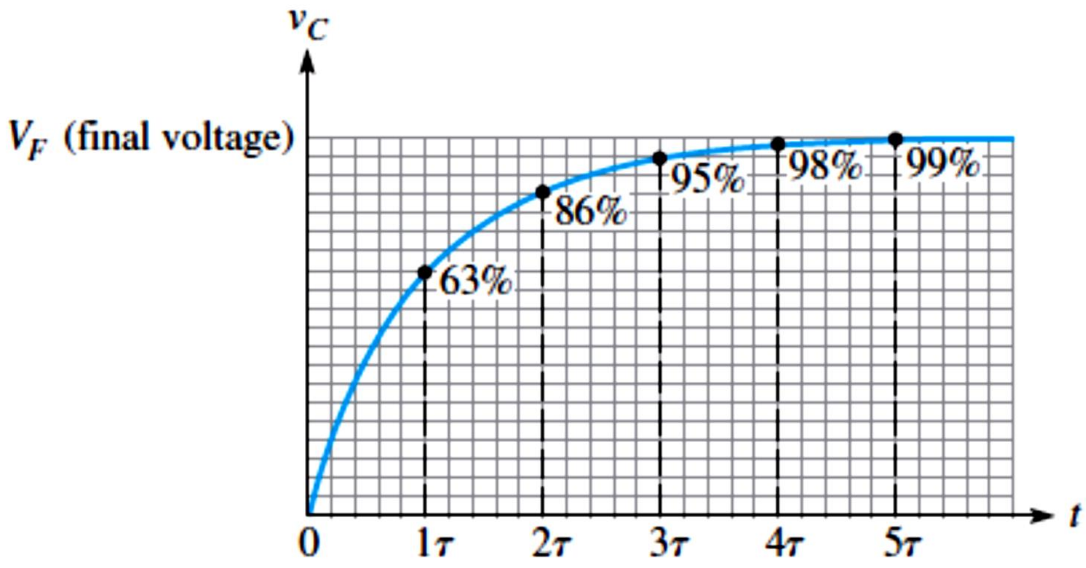


The current at the instant of closing the switch is high and voltage across capacitor V_c at start is zero.

The initial current is

$$i = \frac{V - V_C}{R} = \frac{V}{R} \text{ A}$$

Variation of voltage across capacitor V_C

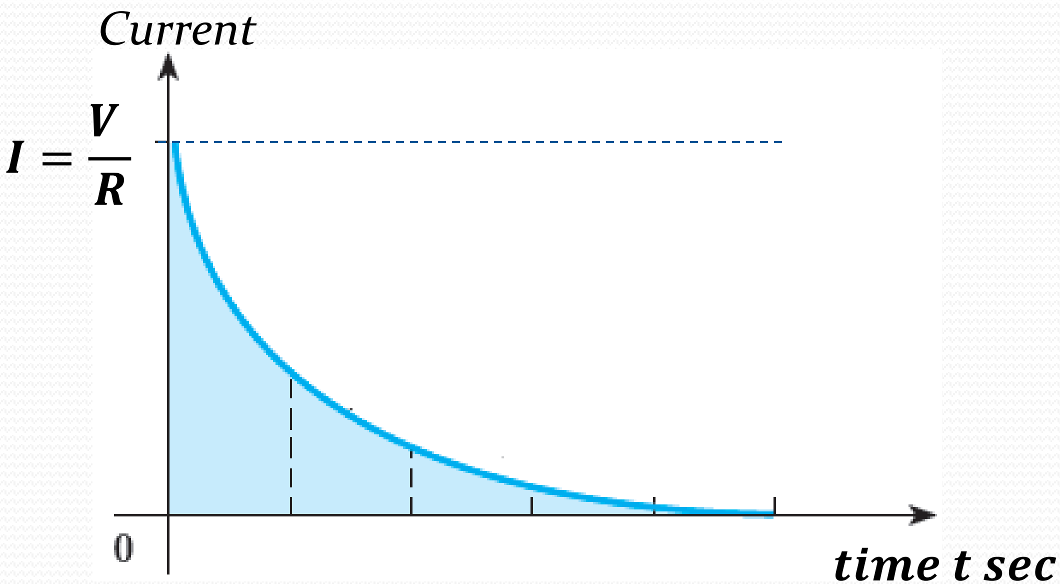


(a) Charging curve with percentages of the final voltage

Initial rate of increase of p.d. $= \dfrac{V}{T}$ volts per second

$$v_c = V\left[1 - e^{-\frac{t}{RC}}\right]$$

Variation of charging current i



$$i = \frac{V}{R}\left[e^{-\frac{t}{RC}}\right]$$

Time Constant

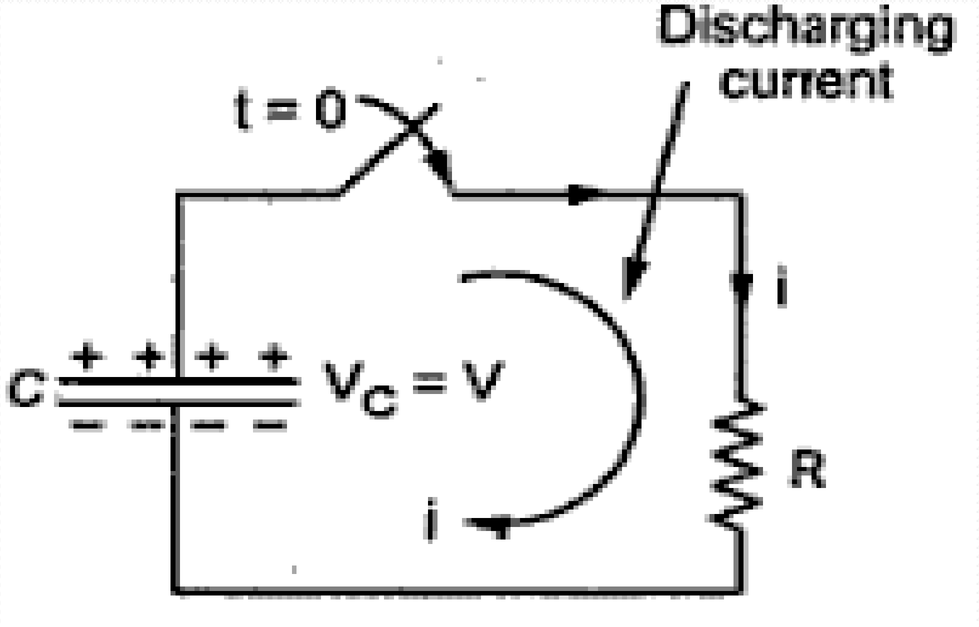
The RC time constant is a fixed time interval that equals the product of the resistance and the capacitance in a series *RC* circuit. The time constant is expressed in seconds when resistance is in ohms and capacitance is in farads. It is symbolized by (Greek letter tau)

$$\tau = RC$$

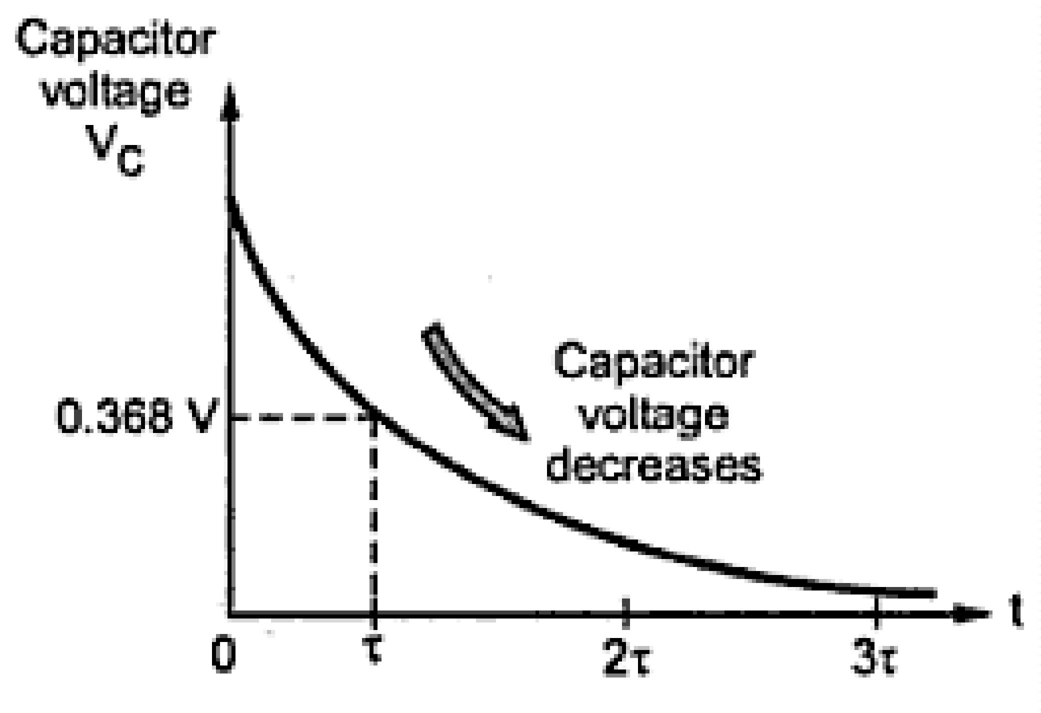
Time constant is defined as it is the time required for capacitor voltage to reach 63.2 percent of its final value.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF FINAL CHARGE
1	63
2	86
3	95
4	98
5	99 (considered 100%)

Discharging of Capacitor through a Resistance

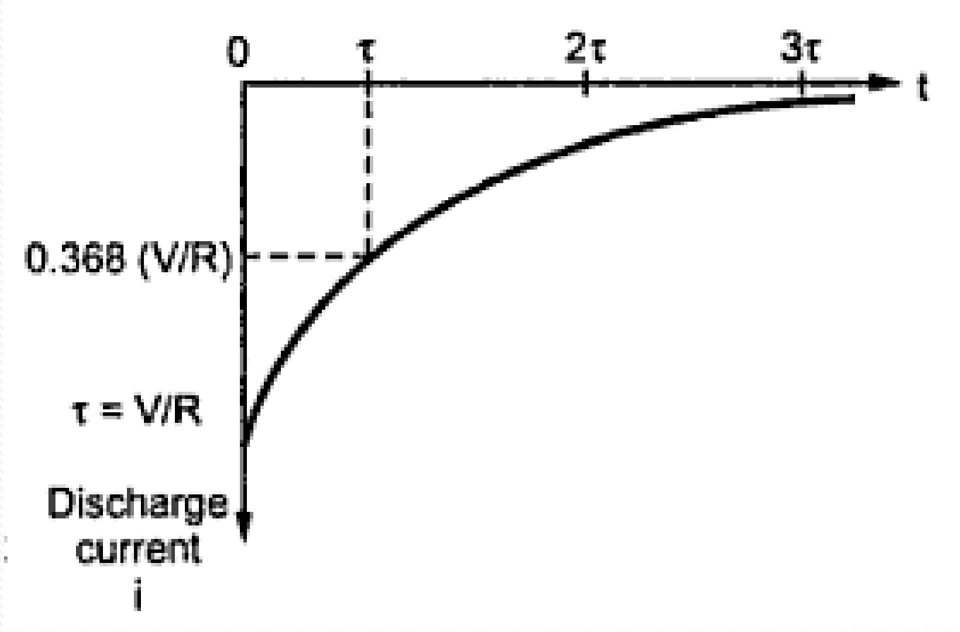


Voltage across Capacitor



$$v = V\left[\, e^{-\frac{t}{RC}} \right]$$

Discharing Capacitor Current



$$i = -\frac{V}{R}\left[e^{-\frac{t}{RC}}\right]$$

Example 1 **A series combination of having R=2MΩand C=1μF is connected across the DC source of 50V. Determine the capacitor voltage and charging current after 2 sec and 6 sec .**

Solution :

$\boldsymbol{R = 2M\Omega \quad C = 1\mu\Omega \quad V = 50V \qquad RC = 2\ sec}$

The voltage across capacitor

$$v_c = V\left[1 - e^{-\frac{t}{RC}}\right]$$

For t=2 sec

$$v_c = 50\left[1 - e^{-\frac{2}{2}}\right] = 31.6V$$

For t=6 sec

$$v_c = 50\left[1 - e^{-\frac{t}{RC}}\right]$$

$$v_c = 50\left[1 - e^{-\frac{6}{2}}\right] = 47.51V$$

The charging current of the capacitor

$$i = \frac{V}{R}\left[e^{-\frac{t}{RC}}\right]$$

For t=2 sec

$$i = \frac{50}{2 \times 10^6}\left[e^{-\frac{2}{2}}\right] = 9.19 \times 10^{-6}A$$

For t=6 sec

$$i = \frac{50}{2 \times 10^6}\left[e^{-\frac{6}{2}}\right] = 1.245 \times 10^{-6}A$$

Example 2. **A parallel plate capacitor has a plate area of 4 cm². The plates are separated by three slabs of different dielectric materials, of thickness 0.3, 0.4 and 0.6mm with relative permittivities of 3 , 5 and 8 respectively. Calculate the capacitance of the capacitor and find the charge on the plates of the capacitor if the supply voltage is 200V. [Oct 2014]**

Solution :

$$plate\ Area\ A = 4\ cm^2 = 4 \times 10^{-4}\ m^2$$

$$d_1 = 0.3mm = 0.3 \times 10^{-3}m$$

$$d_2 = 0.4\ mm = 0.4 \times 10^{-3}m$$

$$d_3 = 0.6mm = 0.6 \times 10^{-3}m$$

$$\varepsilon_1 = 3$$

$$\varepsilon_2 = 5$$

$$\varepsilon_3 = 8$$

$$C_{eq} = \frac{\varepsilon_0\, A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}}$$

$$C_{eq} = \frac{8.854 \times 10^{-12} \times\ 4 \times 10^{-4}}{\frac{0.3 \times 10^{-3}}{3} + \frac{0.4 \times 10^{-3}}{5} + \frac{0.6 \times 10^{-3}}{8}}$$

$$C_{eq} = 1.3888 \times 10^{-11}\ F$$

The charge $Q = C_{eq} V$

$$Q = 1.3888 \times 10^{-11} \times 200 = 2.777 \times 10^{-9}\ \mathrm{C}$$

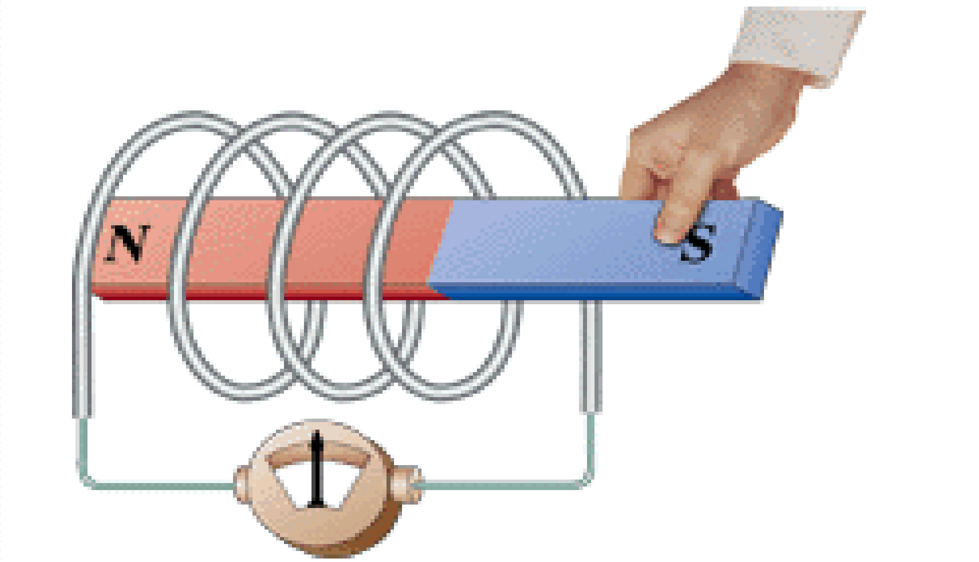
Unit – 2b

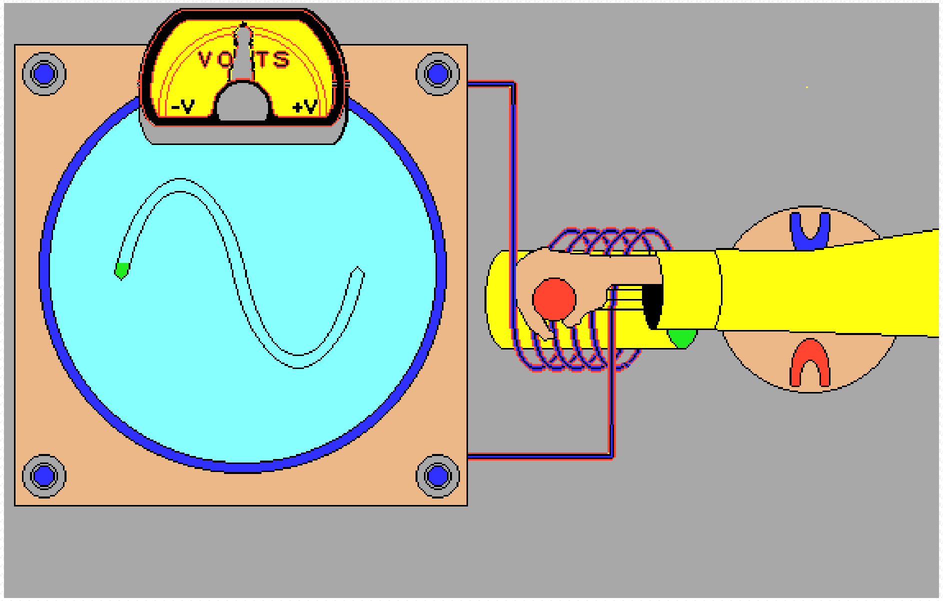
AC FUNDAMENTALS

AC FUNDAMENTALS

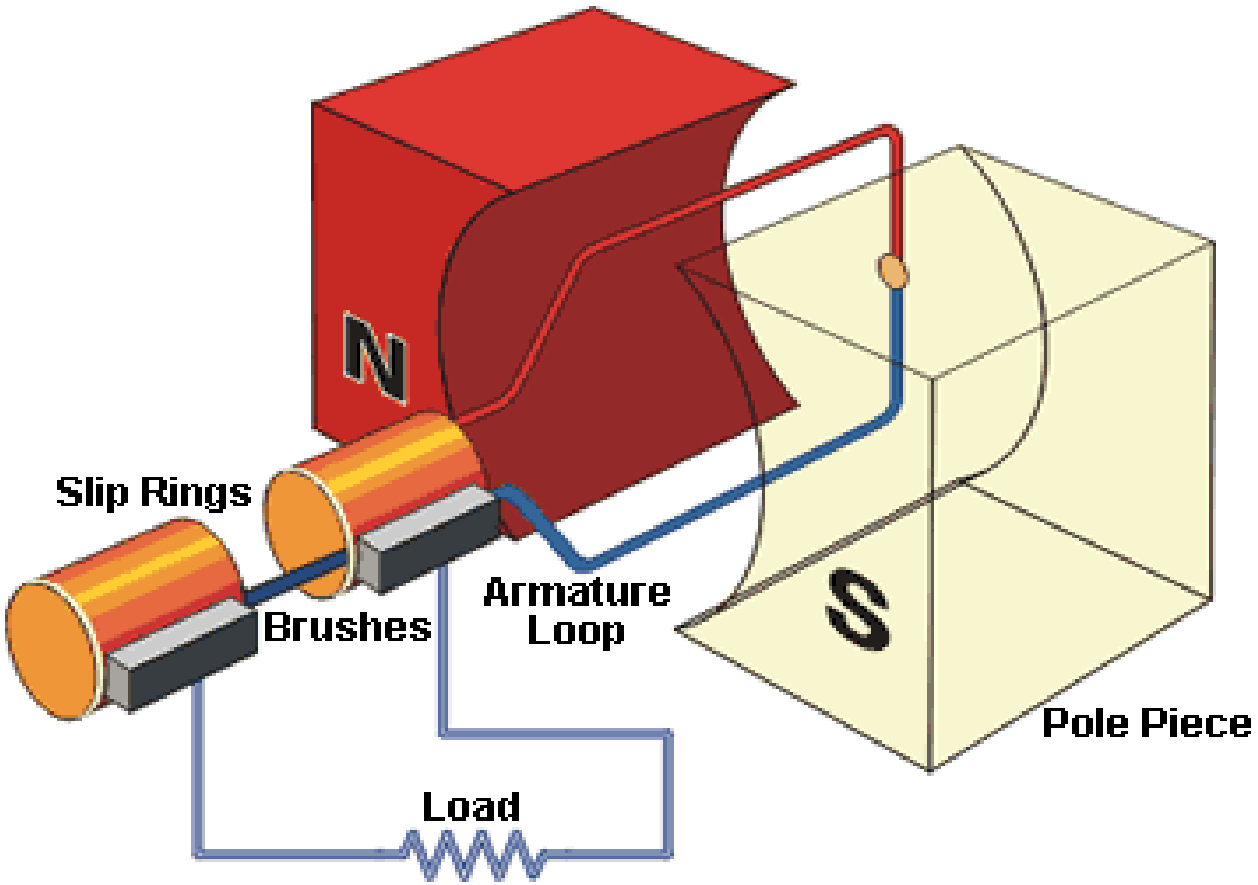
- **Sinusoidal voltages and currents**
- **mathematical and graphical representation of alternating quantities**
- **Concept of instantaneous, peak(maximum), average and r.m.s. values, frequency , cycle, Period, peak factor and form factor**
- **Phase and phase difference, lagging, leading and in phase quantities**
- **Phasor representation, Rectangular and polar representation of phasor.**

Faradays Law of Electromagnetic Induction

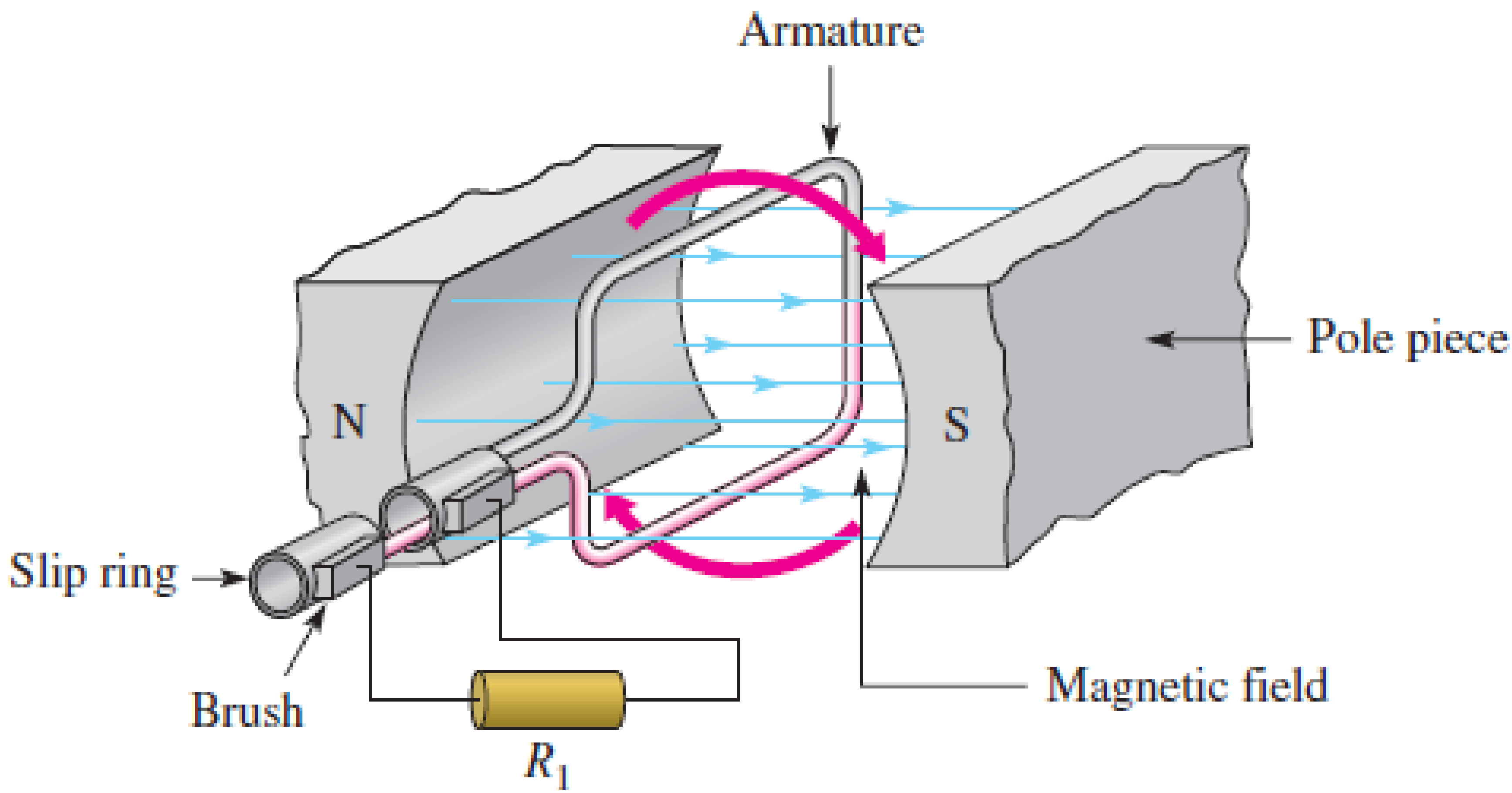




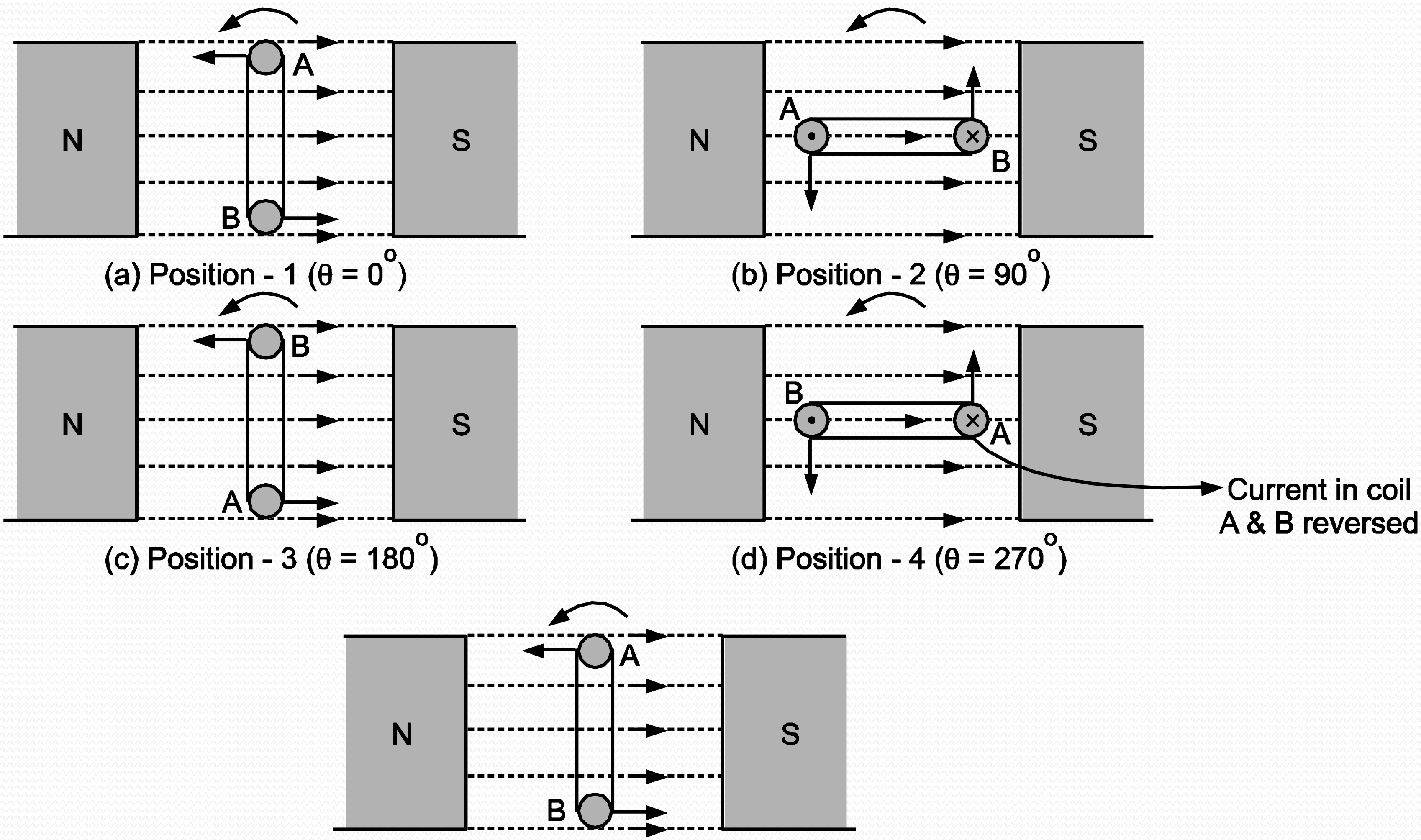
Single Phase Generator



Elementary Generator



Position of the coil to get AC Voltage



(a) Position - 1 ($\theta = 0^{o}$)

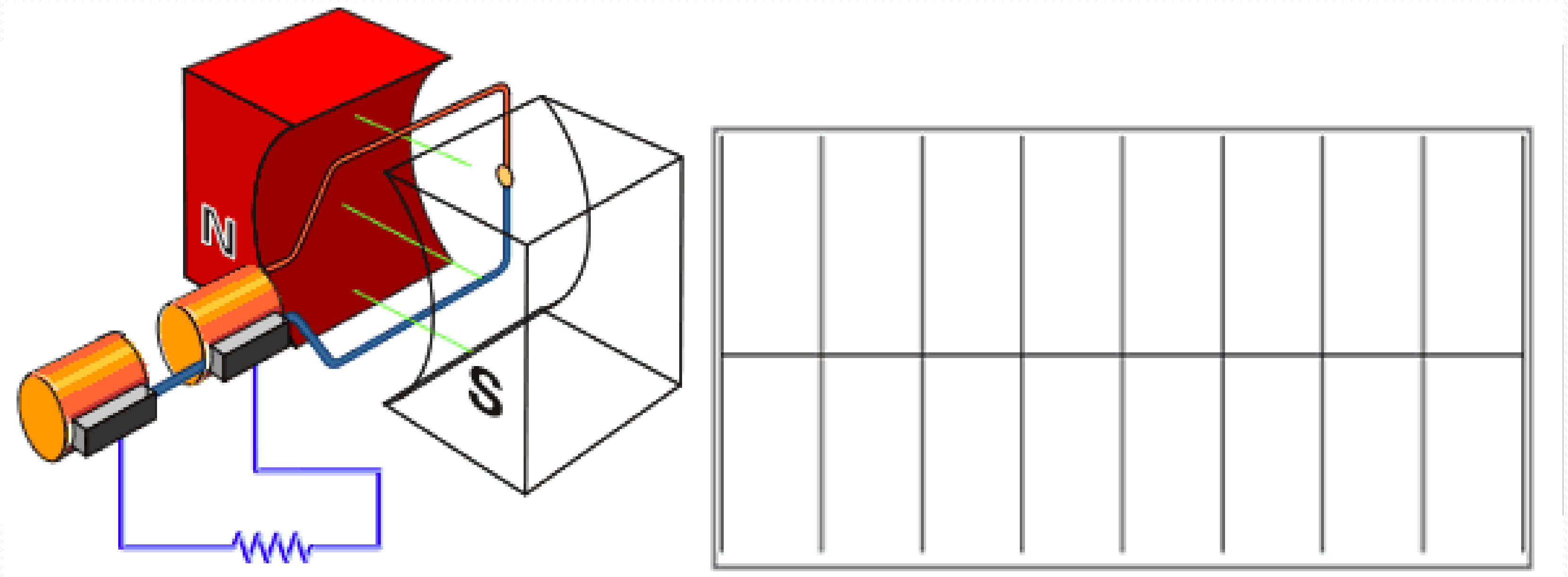
(b) Position - 2 ($\theta = 90^{o}$)

(c) Position - 3 ($\theta = 180^{o}$)

(d) Position - 4 ($\theta = 270^{o}$)

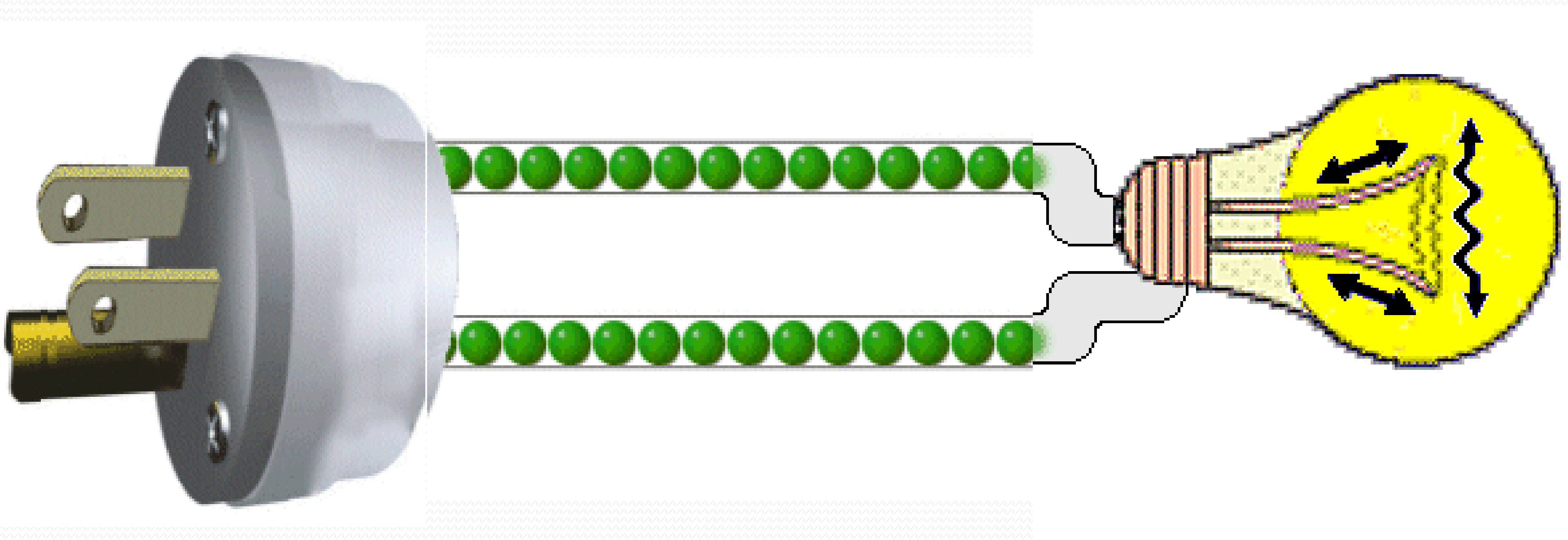
(e) Position - 5 ($\theta = 360^{o}$ or 0^{o})

Formation of sinusoidal waveform

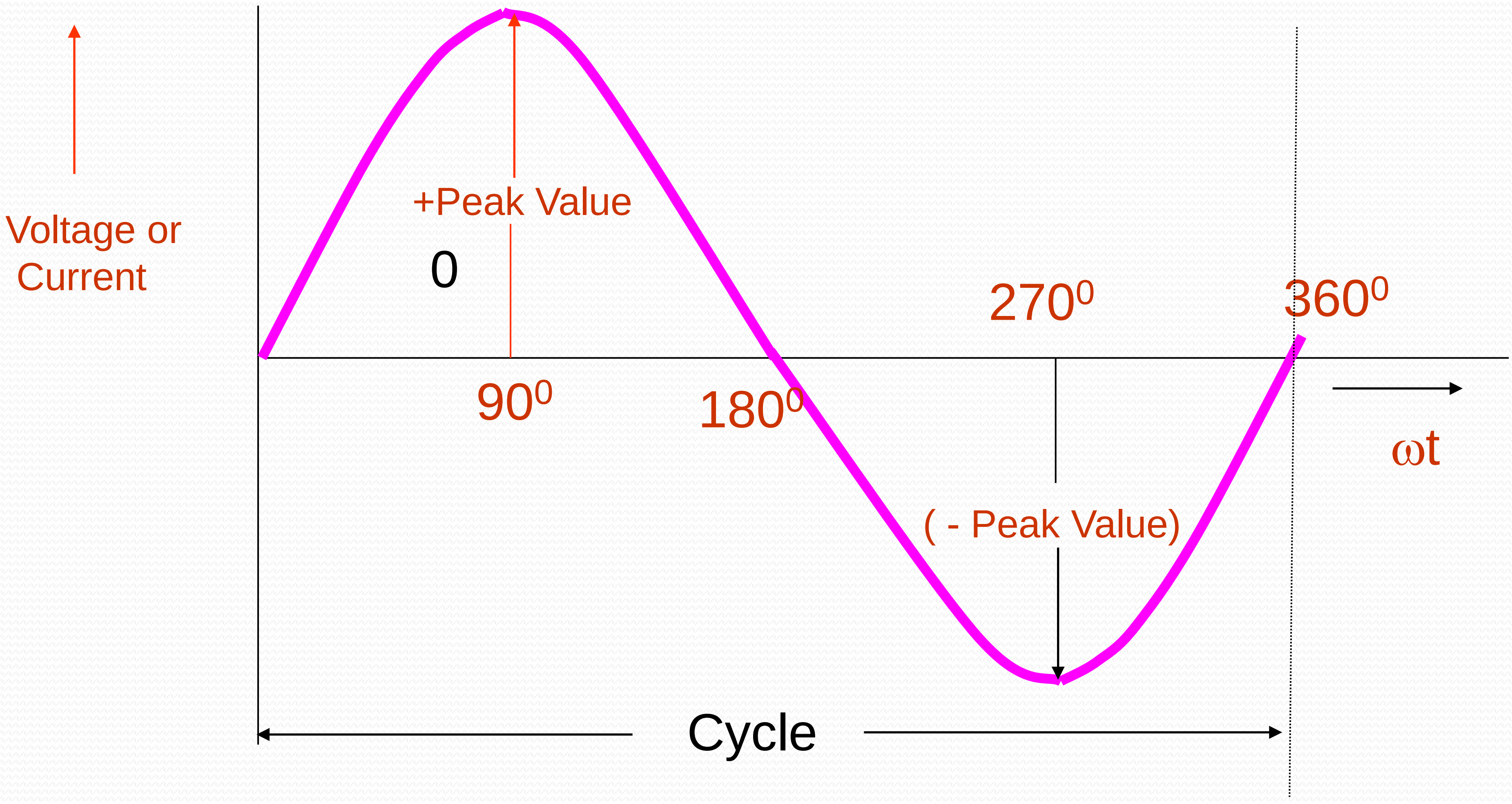


ALTERNATING CURRENT (AC)

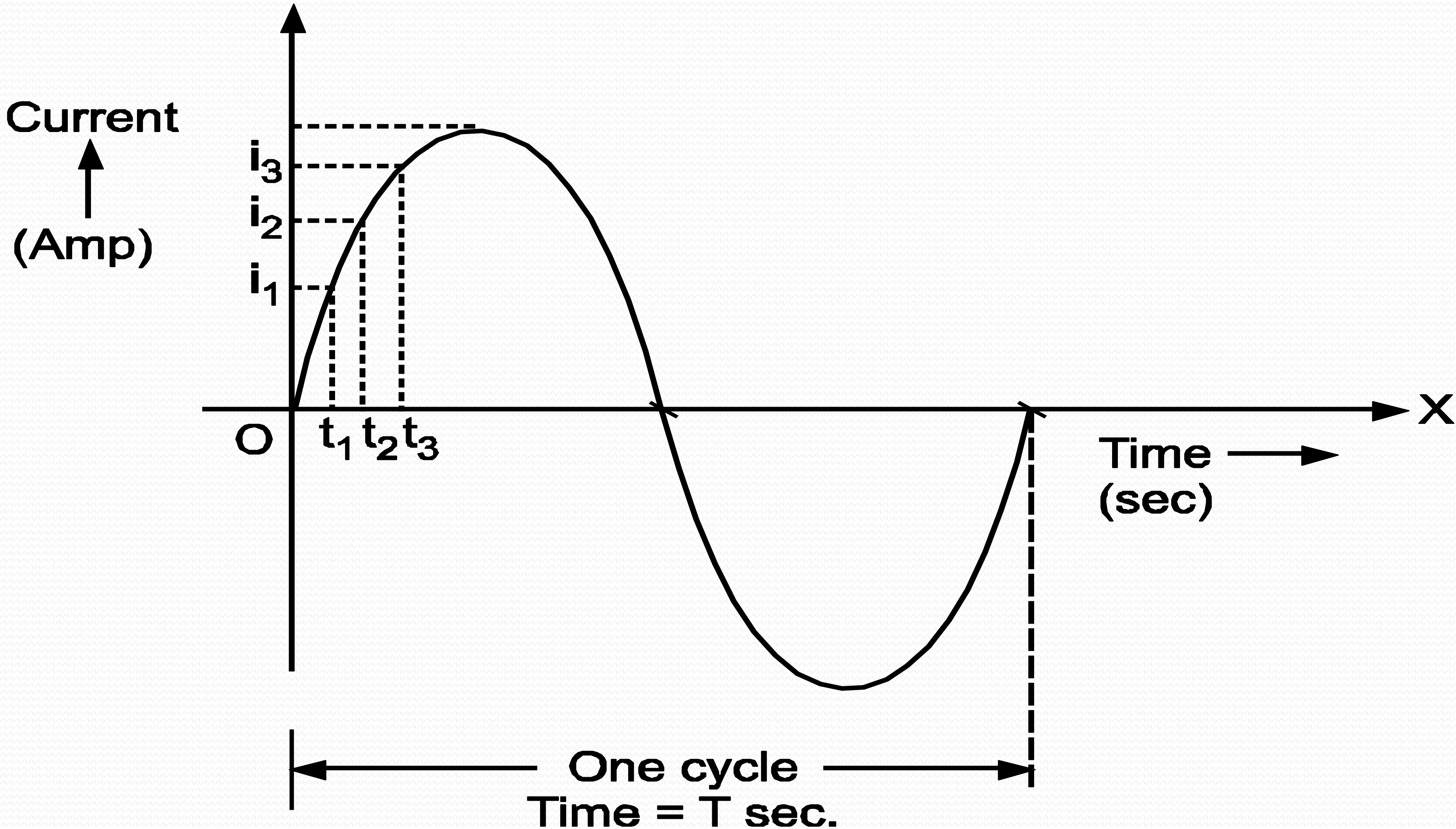
Electricity with electrons flowing back and forth, negative - positive-negative, is called Alternating Current, or AC



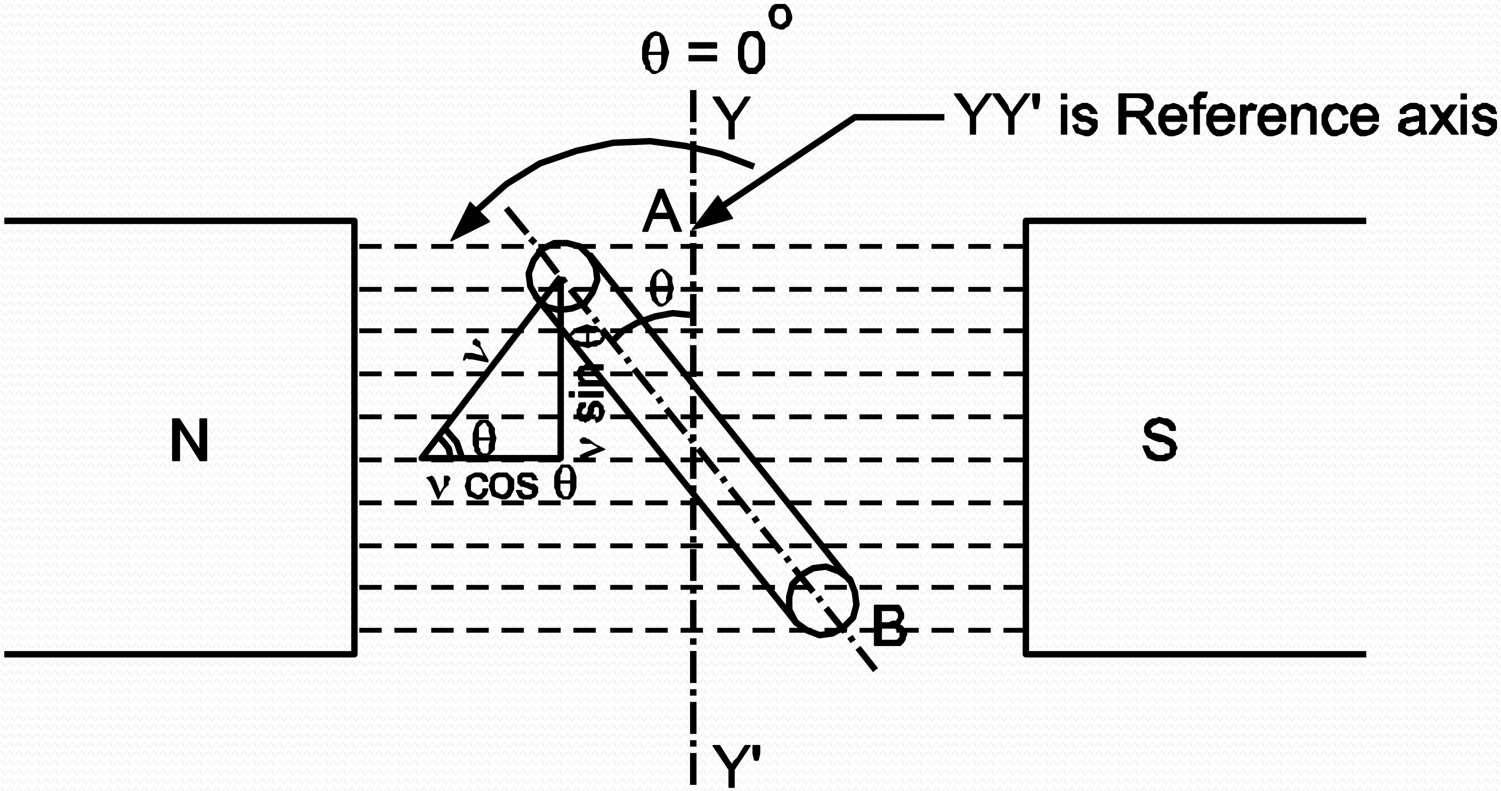
Graphical Representation of induced emf or Current



Sinusoidal waveform of Alternating Current



Equation for Alternating Quantity



v= Vm sin θ volts

i= Im sin θ Amp

Where θ=w t =2πft

$$e = E_m\, sin(\varphi) \quad e = E_m sin(\omega t)$$

$$e = E_m \sin 2\,\pi f t = E_m \sin\left(\frac{2\pi}{T}\right)t$$

$$i = I_m \sin 2\,\pi ft = I_m \sin\left(\frac{2\pi}{T}\right)t$$

$$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin 2\,\pi f t = E_m \sin \frac{2\pi}{T} t$$

Cycle

One complete set of positive and negative values of alternating quantity is known as cycle.

Time Period

The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50-Hz alternating current has a time period of 1/50 second.

Frequency

The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

$$f = 1/T \quad \text{or} \quad T = 1/f$$

Amplitude

The maximum value, attained by an alternating quantity during positive or negative half cycle is known as its *amplitude* or *peak value* or *maximum value.*

Example : *An alternating current of frequency 60 Hz has a maximum value of 12 A :*

i) Write down the equation for instantaneous values. ii) Find the value of the current after 1/360 second. iii) Time taken to reach 9.6 A for the first time.

In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction.

Solution : $f = 60$ Hz and $I_m = 12$ A

$$\omega = 2\pi f = 2\pi \times 60 = 377 \text{ rad/sec}$$

i) Equation of instantaneous value is

$$i = I_m \sin \omega t$$

$$\therefore \quad i = 12 \sin 377\, t$$

ii) $$t = \frac{1}{360} \text{ sec}$$

$$i = 12 \sin 377 \frac{1}{360} = 12 \sin 1.0472 = 10.3924 \text{ A}$$

Note : sin of 1.0472 must be calculated in radian mode.

iii) $$i = 9.6\ A$$

$$\therefore \quad 9.6 = 12 \sin 377\, t$$

$$\therefore \quad \sin 377\, t = 0.8$$

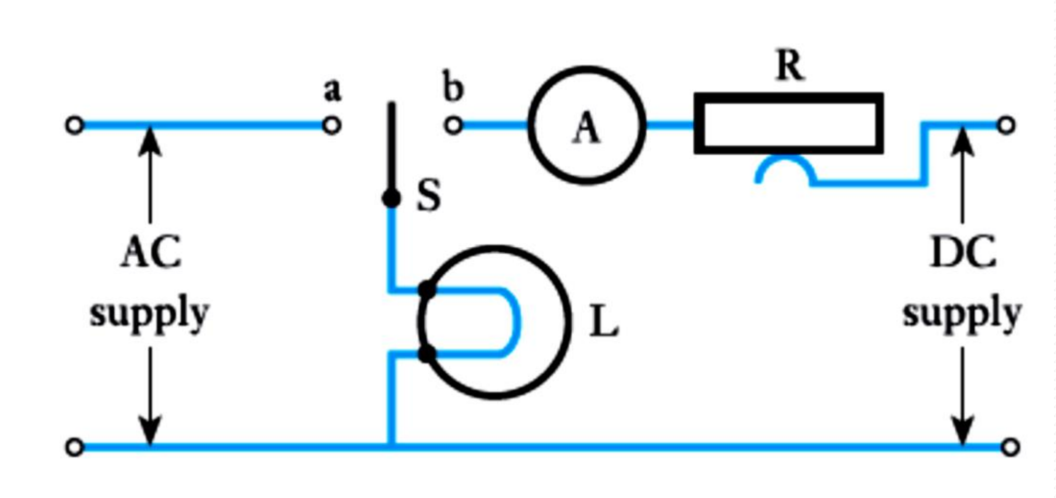
$$\therefore \quad 377\, t = 0.9272$$

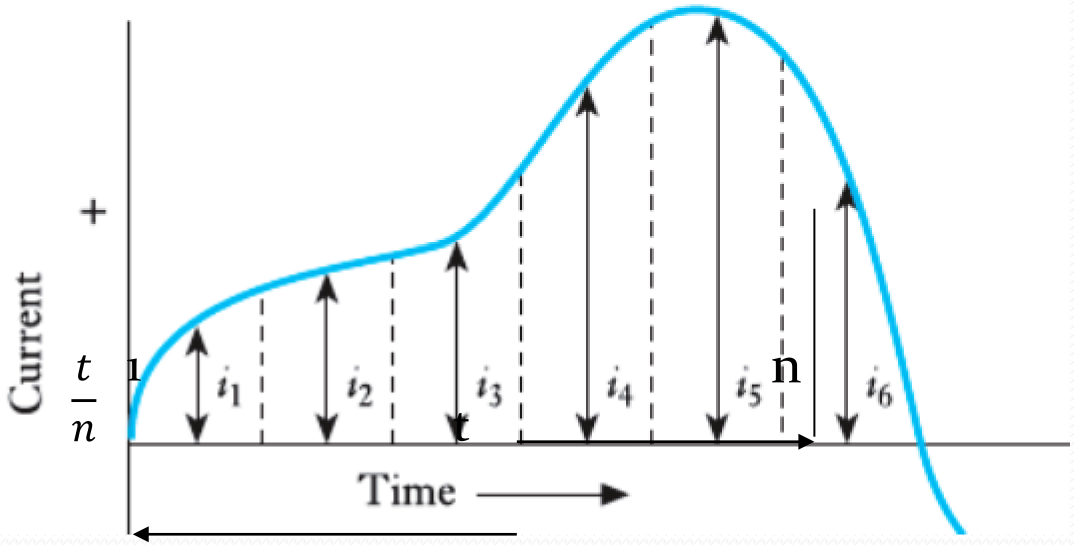
Note : find inverse of sin in radian mode.

$$t = 2.459 \times 10^{-3}\ \text{sec.}$$

Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current ***is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same*** **heat** ***as produced by the alternating current when flowing through the same circuit for the same time*.**





$$\text{Heat produced due to } 1^{st} \text{ interval} = i_1^2 R \frac{t}{n} \quad \text{joules}$$

$$\text{Heat produced due to } 2^{nd} \text{ interval} = i_2^2 R \frac{t}{n} \quad \text{joules}$$

$$\text{Heat produced due to } n^{th} \text{ interval} = i_n^2 R \frac{t}{n} \quad \text{joules}$$

$$\text{Total heat produced in 't' seconds} = R \times t \times \frac{[i_1^2 + i_2^2 + \ldots + i_n^2]}{n}$$

$$\text{Heat produced by DC current} = I^2\,R\,t \quad joules$$

For I to be RMS value then heat produced by dc is equal to heat produced by ac

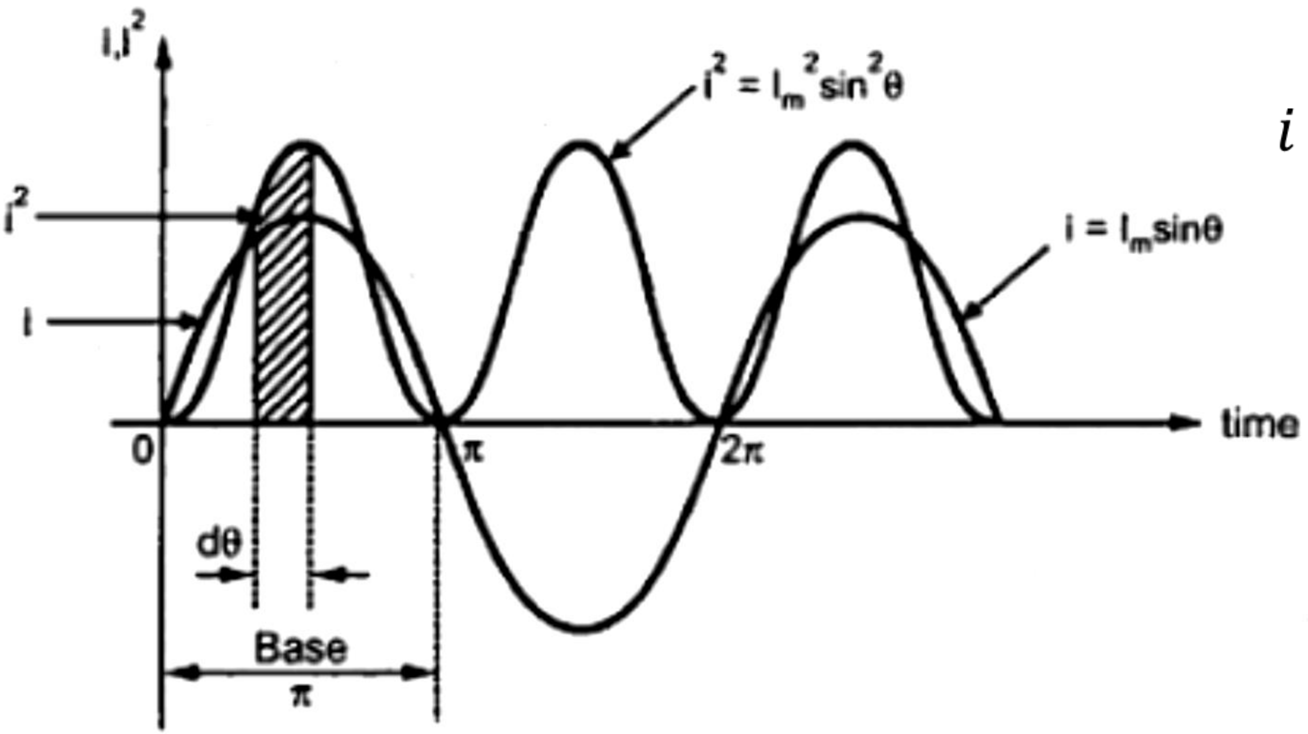
$$I^2\,R\,t = R\,t\,\frac{[\,i_1^2 + i_2^2 + \cdots + i_n^2\,]}{n}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$

$$I = Square\ Root\ of\ Mean\ of\ Squared\ Currents$$

RMS value or Effective Value or Virtual Value

$$V = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}} = \sqrt{\frac{Area\ under\ squared\ curve}{base\ length}}$$



$$i = I_m \sin(\theta)$$

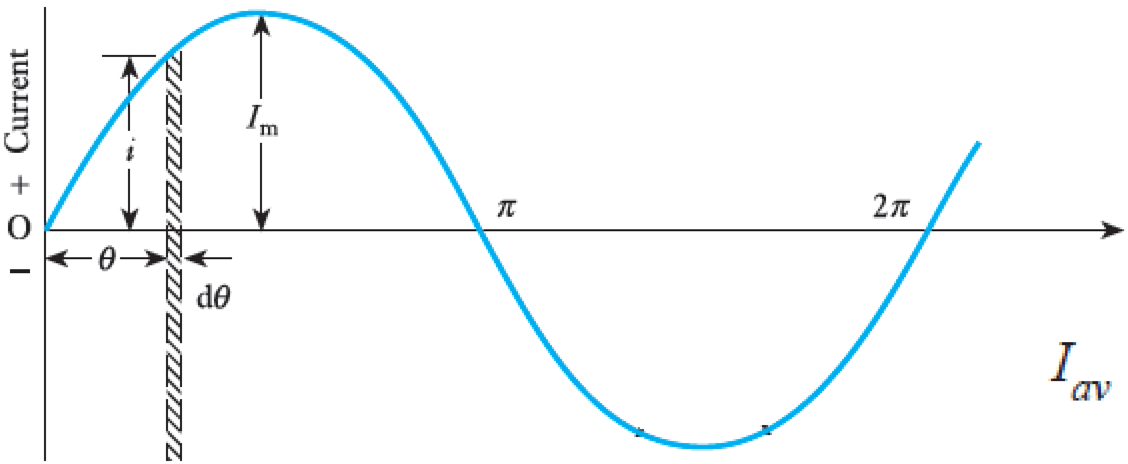
$$I = \sqrt{\frac{\int_0^{\pi} i^2\ d\theta}{\pi}} \sqrt{\frac{\int_0^{\pi} (I_m \sin(\theta)^2\ d\theta}{\pi}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

Average Value :The average value *Ia* of an alternating current is expressed ***by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.***

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

$$\frac{\text{Area enclosed over half-cycle}}{\text{Length of base over half-cycle}}$$

$$i = I_m \sin(\theta)$$



$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi} \int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi} \left| +1 - (-1) \right| = \frac{2I_m}{\pi}$$

$$I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m$$

$\therefore$ average value of current = 0.637 × maximum valu

Form Factor

It is defined as the ratio, $K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.1.$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \frac{0.707\, E_m}{0.637\, E_m} = 1.11$

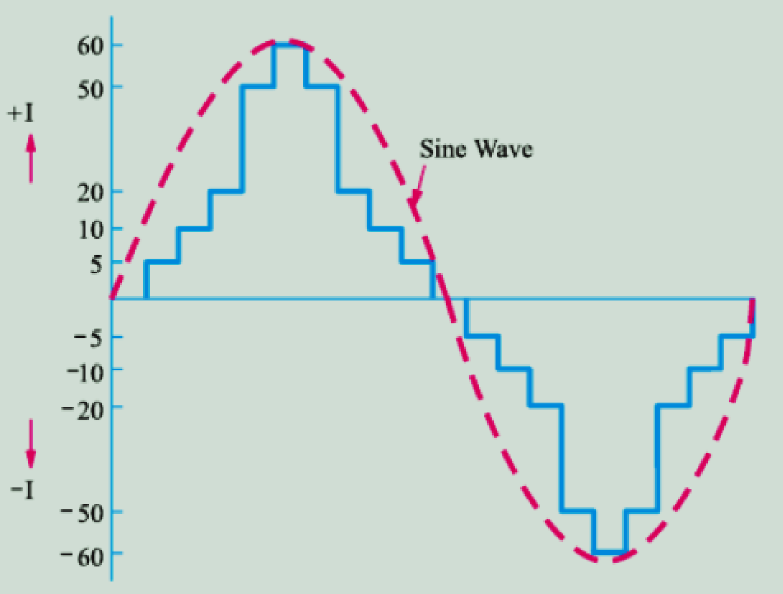
As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice-versa.*

Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$

Calculate the r.m.s. value,the form factor and peak factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, –5, –10 V etc. What would be the r.m.s value of sine wave having the same peak value ?



$$\text{Mean value of } v^2 = \frac{0^2+5^2+10^2+20^2+50^2+60^2+50^2+20^2+10^2+5^2}{10} = \mathbf{965\ V}$$

$$\therefore \quad \text{r.m.s. value} = \sqrt{965} = \mathbf{31\ V\ (approx.)}$$

$$\text{Average value (half-cycle)} = \frac{0+5+10+20+50+60+50+20+10+5}{10} = \mathbf{23\ V}$$

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{31}{23} = \mathbf{1.35.}\quad \text{Peak factor} = 60/31 = \mathbf{2\ (approx.)}$$

Example 11.7. *Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by*

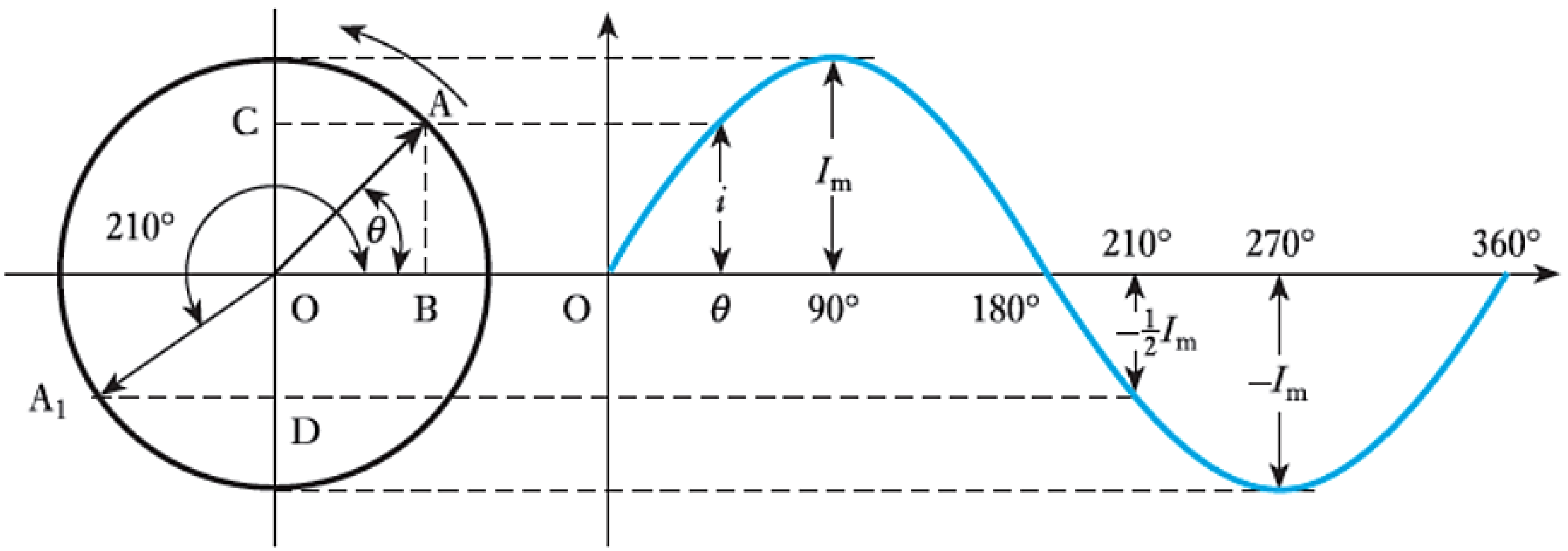
$$v = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$$

The r.m.s. value of individual components are $(200/\sqrt{2})$, $(100/\sqrt{2})$ and $(50/\sqrt{2})$.

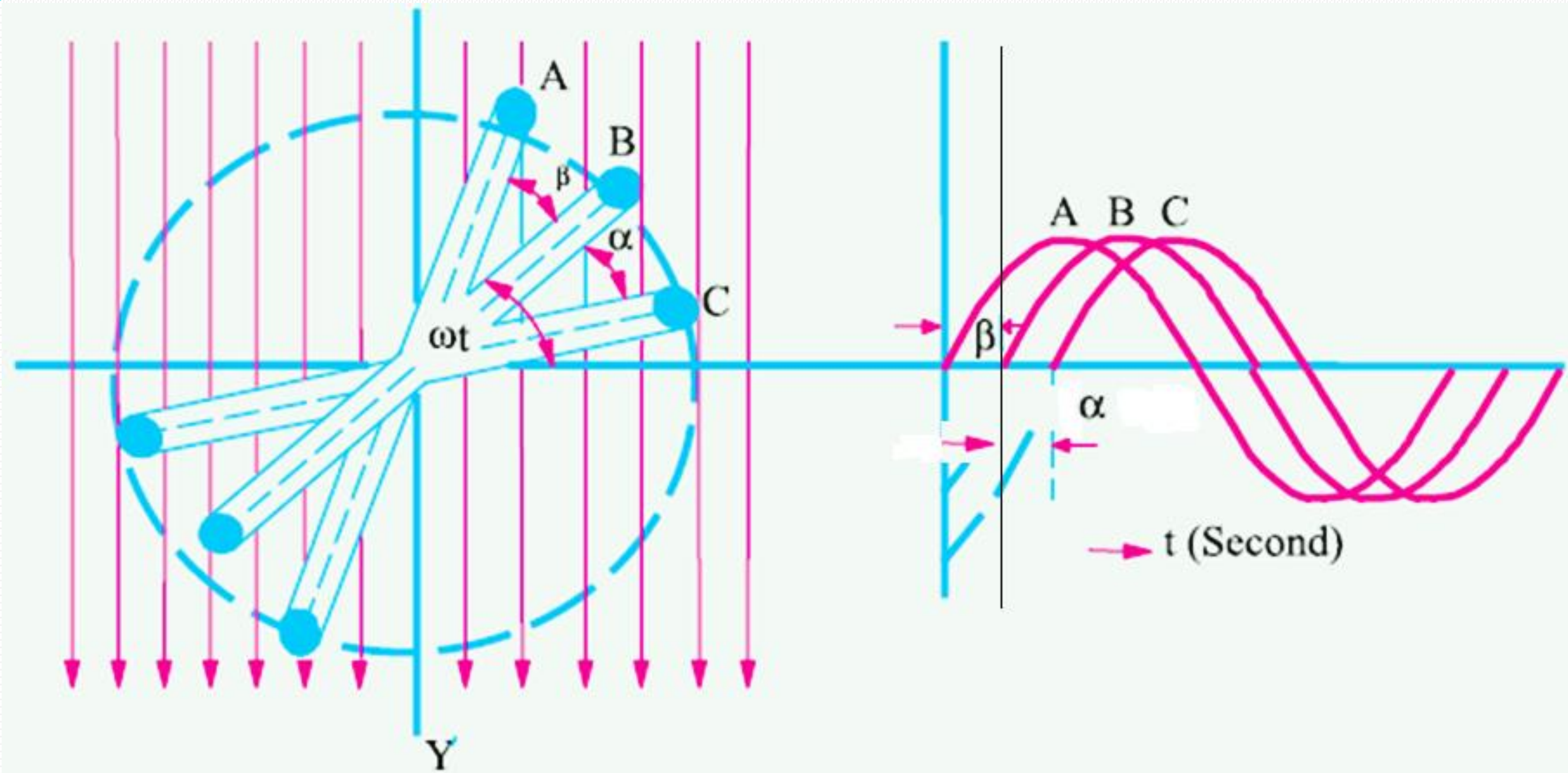
$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(200/\sqrt{2})^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} = \mathbf{162\ V}$$

Phasor Representation of a sine Wave

$$i = I_m \sin \theta = I_m \sin \omega t$$



Lagginig and leading Phase difference



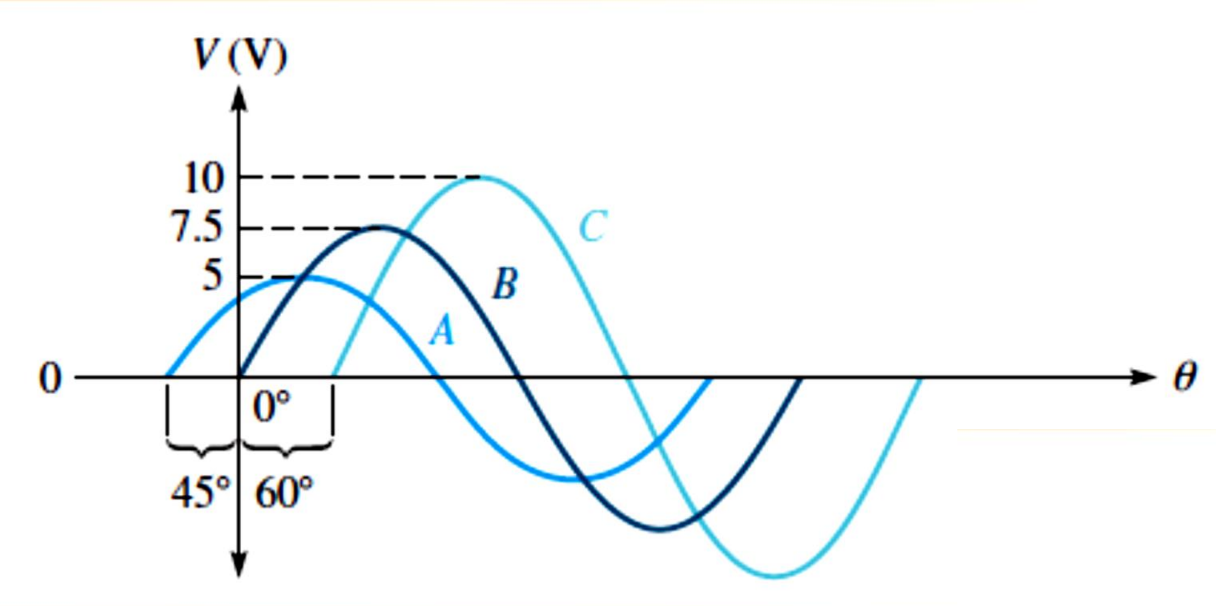
$e_b = E_m \sin(\omega t)$ $e_a = E_m \sin(\omega t + \beta)$ $e_c = E_m \sin(\omega t - \alpha)$

Leading Lagging

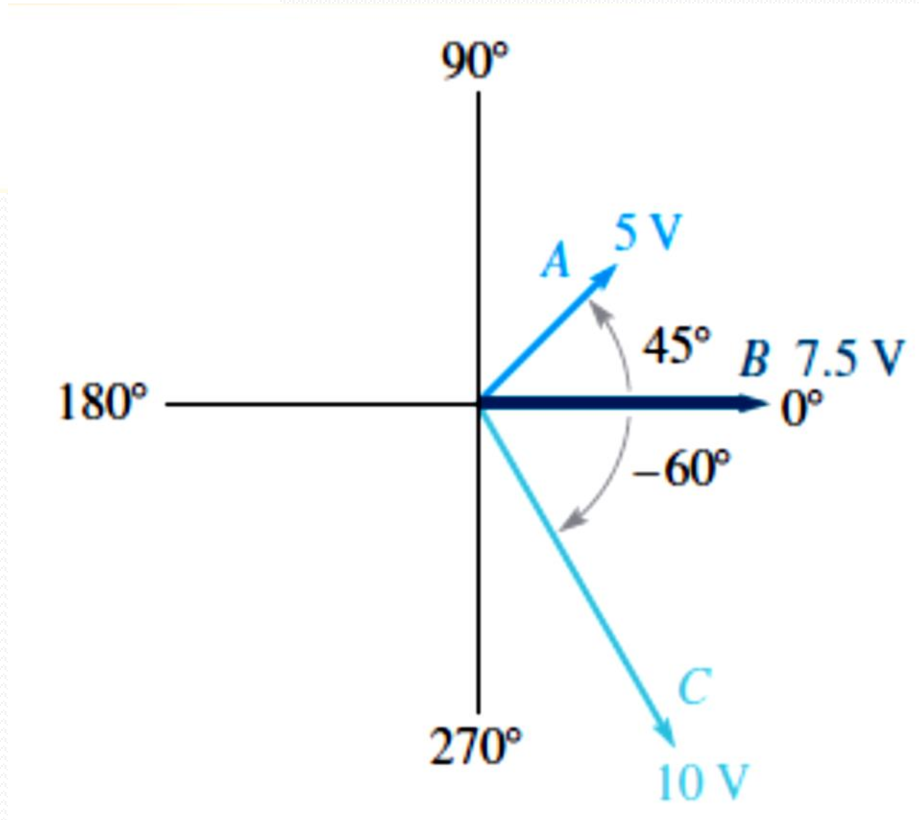
Leading

$$e = E_m \sin(\omega t \pm \varphi)$$

Lagging



V (V)
10
7.5
5
0
θ
C
B
A
0°
45°
60°



90°
180°
270°
0°
A
5 V
45°
B 7.5 V
−60°
C
10 V

Complex Notation of Phasors

(i) Rectangular or Cartesian Form

A vector can be specified in terms of its *X*-component and *Y*-component.

$$\mathbf{E_1} = a_1 + jb_1$$

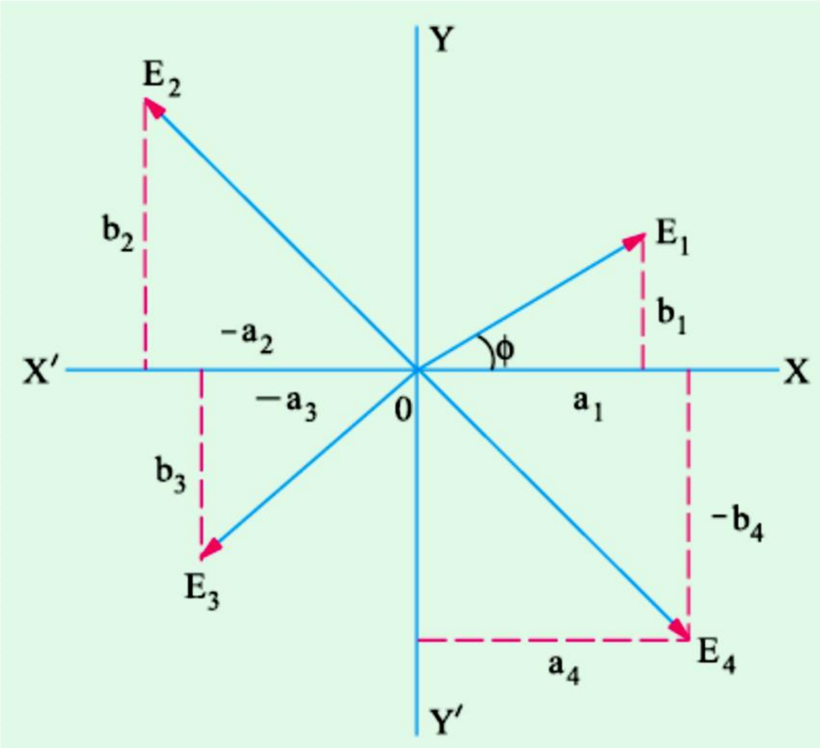
a_1: *Real component*

b_1: *Imaginary component*

j: *Complex no.* $j = \sqrt{-1}$

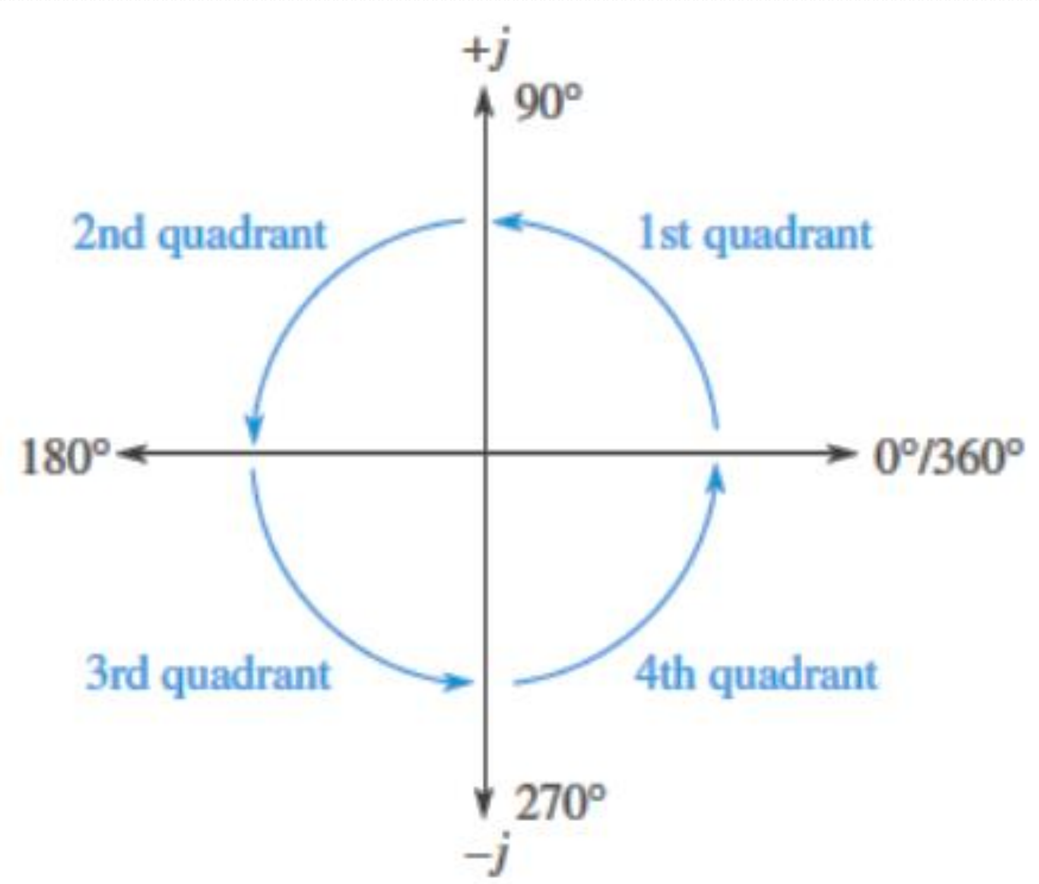
$V_1 = 50 + j\,66 \quad Volts$

$I = 5 + j\,6 \quad Amps$



symbol ***j*** is used to indicate the counter-clockwise rotation of a phasor through 90°.

In Mathematics, ($\sqrt{-1}$) is denoted by ***i*** but in electrical engineering ***j*** is adopted because letter ***i*** is reserved for representing *current*. This helps to avoid confusion.


+j
90°
2nd quadrant
1st quadrant
180°
0°/360°
3rd quadrant
4th quadrant
270°
−j

(ii) Polar Form

$$\bar{E} = E\angle\theta \qquad \bar{I} = 10\angle 30 \text{ Amp}$$

E represents the magnitude of the phasor and θ its inclination (in ccw direction) with the X-axis.

$$E\angle \pm \theta = E_x \pm j\, E_y)$$

(iii) Trigonometrical Form

$$E\angle \pm \theta = E\,(\cos\theta \pm j\sin\theta)$$

(iv) Exponential Form.

$$\bar{V} = V\, e^{\pm j\theta} \quad Volts$$

If $I_1 = |I_1|\angle\theta_1$ $I_2 = |I_2|\angle\theta_2$ then

(i) $I_1 \pm I_2 = (|I_1|\angle\theta_1) \pm (|I_2|\ \angle\theta_2)$

(ii) $I_1 \times\ I_2 = |I_1| \times |I_2|\ \angle(\theta_1+\theta_2)$

(iii) $(I_1 \div\ I_2) = \frac{|I_1|}{|I_2|}\ \angle(\theta_1-\theta_2)$

$V = 16 + j\,12$ $\quad$ $I = -6 + j\,10.4$

$\bar{E} = 220\angle 60$ $\quad$ $I = 12\angle -30$

$I_1 = 10\,e^{j50}$ $\quad$ $I_2 = 5\,e^{-j100}$

$I_1 + I_1 = (10\angle 50°) + (5\angle -100°)$

$I_1 + I_1 = (5.56 + j\,2.74) = (\,6.1965 \angle 26.20°\,)$

$If \quad e_1 = 10\sqrt{2}\sin\left(\omega t + \frac{\pi}{6}\right) \quad and \quad e_2 = 30\sqrt{2}\sin(\omega t - \frac{\pi}{3})$

$Find \;\; e_r = e_1 + e_2$

$\pi \;\; rad = 180\,°$

$E_1 = 10\angle 30°$

$E_2 = 30\angle - 60°$

$$E_r = E_1 + E_2 = (10\angle 30) + (30\angle - 60)$$

$$= (23.66 - \; j\,20.98) = 31.62\angle - 41.56°$$

$$e_r = 31.62\sqrt{2}\sin(\omega t - 41.56°)$$

$$e_r = 31.62\sqrt{2}\sin\left(\omega t - \frac{\pi}{4.33}\right)$$

Example *The following three vectors are given :*

$$A = 20 + j20,\ B = 30\ \angle 120 \text{ and } C = 10 + j0$$

Perform the following indicated operations :

(*i*) $\dfrac{AB}{C}$ and (*ii*) $\dfrac{BC}{A}$.

Solution. Rearranging all three vectors in polar form, we get

$$\mathbf{A} = 28.3 \angle 45°,\ \mathbf{B} = 30 \angle -120°,\ \mathbf{C} = 10 \angle 0°$$

(*i*) $$\frac{\mathbf{AB}}{\mathbf{C}} = \frac{28.3 \angle 45° \times 30 \angle -120°}{10 \angle 0°} = 84.9 \angle -75°$$

(*ii*) $$\frac{\mathbf{BC}}{\mathbf{A}} = \frac{30 \angle -120 \times 10 \angle 0°}{28.3 \angle 45°} = 10.6 \angle -165°$$