Unit 3: Single Phase AC Circuit (Q No. 1 and Q. No. 2)

Pure R, Pure L and Pure C AC Circuit

1. Prove that the voltage and current are in phase in case of purely resistive circuit. Also find power consumed in circuit.

Ans: Consider a circuit which consists of pure resistance R only as shown in Fig.

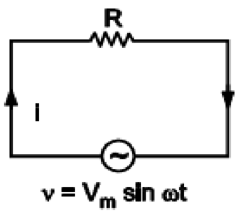


Fig. Purely Resistive Circuit

Let, the applied alternating voltage be given by equation,

$$v = V_m \sin \omega t \qquad \dots (1)$$

As, voltage ***v*** is applied in a close loop so an alternating current will be set up in the circuit. At any instant, the value of current is given by Ohm's law as,

$$i = \frac{v}{R} = \frac{V_m \sin wt}{R} = \frac{V_m}{R} \sin \omega t$$

The current will be maximum when ***sin ωt*** becomes unity, i.e. $I_m = (V_m / R)$,

$$i = (V_m / R) \sin \omega t = I_m \sin \omega t \qquad \dots (2)$$

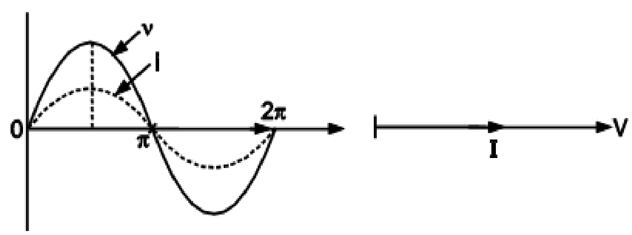


Fig. Voltage and Current Waveforms & Phasor diagram

So in case of Resistive circuit, voltage and current obtains their zero and maximum value at same instant. Hence they are in phase as shown in Fig.

Power

In a.c. circuits, power at any instant is given by the product of instantaneous voltage and instantaneous current.

Instantaneous power, p = v × i

Where, for pure resistance, $v = V_m \sin \omega t$ and $i = I_m \sin \omega t$

$$p = V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Thus, instantaneous power consists of a constant part $(V_m I_m)/2$ and a fluctuating part $[(V_m I_m)/2]Cos2\omega t$. The fluctuating part is a cosine curve of frequency double that of voltage and current. For one complete cycle, average value of $[(V_m I_m)/2]Cos2\omega t$ is zero.

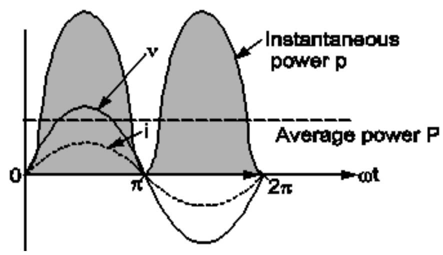


Fig : Voltage, current and power waveforms

Average Power

Average power over a cycle,

$$P = Average\ of \left(\frac{V_m I_m}{2} \right) - Average\ of \left(\frac{V_m I_m}{2} \cos 2(\omega t) \right) = \frac{V_m I_m}{2} - 0$$

$$\boldsymbol{P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V \times I \quad Watt}$$

2. Prove that the current lags behind applied voltage by 90^0 in case of purely inductive circuit. Also find power consumed in circuit.

Ans: Consider a circuit which consists of pure Inductance, L only as shown in Fig.

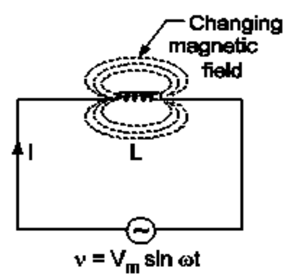


Fig. Purely Inductive Circuit

Wherever, an alternating voltage of ***v*** volt is applied to a purely inductive coil of inductance L Henry, an alternating current starts flowing in the circuit. This current produces an alternating (changing) magnetic field which links with the same coil to produce an emf of self-induction which is given by,

$$e=\left(-L\ \frac{di}{dt}\right)$$

As the circuit contains only L, therefore emf of self-induction will always oppose the applied voltage as per Lenz's law,

$$\therefore v = -e = -\left(-L\ \frac{di}{dt}\right) = L\frac{di}{dt},\quad Where\ v = V_m \sin \omega t\text{----}(1)$$

$$V_m \sin \omega t = L \cdot \frac{di}{dt}$$

$$L \cdot di = V_m \sin \omega t \cdot dt$$

$$di = \frac{V_m}{L} \sin \omega t \cdot dt$$

Integrating on both sides,

$$i=\int\frac{V_m}{L} \sin \omega t \cdot dt$$

$$i= \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right)$$

$$i= \frac{V_m}{\omega L} \sin (\omega t - \pi/2)\ldots (2)$$

When $\boldsymbol{\sin(\omega t - \pi/2)}$ becomes unity, the current attains maximum value which is given by,

$I_m = \dfrac{V_m}{\omega L}$ where $\boldsymbol{\omega L}$ is called as inductive reactance, $\boldsymbol{X_L}$.

$$i = I_m \sin (\omega t - \pi/2)\ or\ \ i= Im \sin (\omega t - 90^o)\ldots(3)$$

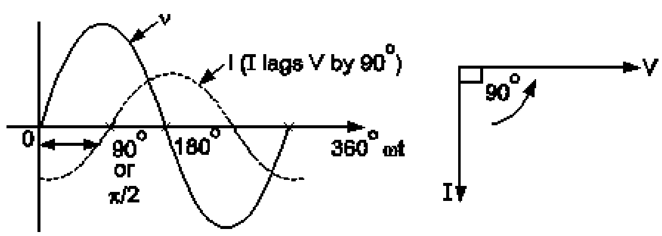


Fig. Voltage and Current Waveforms & Phasor diagram

The current obtains zero value 90° after voltage. Thus, the current always lags behind voltage by 90° as shown in figure.

Power

Instantaneous power, $\boldsymbol{p = v \cdot i}$

$=V_m \sin \omega t \cdot I_m \sin (\omega t - \pi/2)=V_m I_m \sin \omega t \cdot (-\cos\omega t)$

$$= -V_m I_m \cdot \sin \omega t \cdot \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

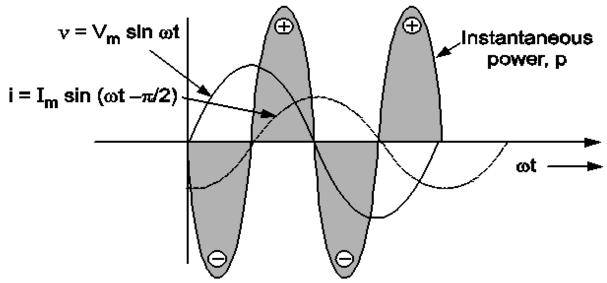


Fig. Voltage, current and power waveforms

Average Power

Average power over one complete cycle,

$$P = Average\ of\left(-\frac{V_m I_m}{2} \sin 2\omega t\right) = \mathbf{0}$$

3 If a sinusoidal voltage of $v = V_m \sin\omega t$ is applied across purely capacitive circuit, derive the expression for current drawn and power consumed.

Ans: Consider a circuit which consists of pure Capacitance, C only as shown in Fig.

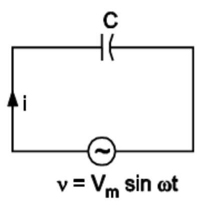


Fig. Circuit Diagram

When a pure capacitor is connected across an alternating voltage of ***v,*** capacitor is charged in one direction then in the opposite direction. Thus instantaneous charge *q* stored on the plates of capacitor depends on that instant (time t).The instantaneous charge *q* on the capacitor of capacitance value ***C*** is given by,

$$q = C \cdot v \quad ,$$

$$where\ v = V_m \sin\omega t \qquad .....(1)$$

$$q = C \cdot V_m \sin\omega t$$

Let *dq* is the small charge which stored on a capacitor plate, in small time interval *dt* second, when the instantaneous value of current is

$$i = \frac{dq}{dt} = \frac{d}{dt}(C \cdot V_m \sin\omega t) = C \cdot V_m \frac{d}{dt}(\sin \omega t)$$

$$= C \cdot V_m\,(\cos\omega t \cdot \omega) = (\omega C) \cdot V_m \cdot \cos\omega t$$

$$= \frac{V_m}{1/(\omega C)} \cdot \cos\,\omega t = \frac{V_m}{1/(\omega C)} \cdot \sin\,(\omega t + \pi/2) \qquad .....(2)$$

Current reaches its maximum value I_m, when $\boldsymbol{\sin\,(\omega t + \pi/2) = 1}$

$$I_m = \frac{V_m}{(1/\omega C)} = V_m \cdot (\omega C)$$

where term $\boldsymbol{(1/\omega C)}$ is called as capacitive reactance $\boldsymbol{X_C}$

Substituting $\frac{V_m}{1/\omega C} = I_m$ in equation (2),

$$\boldsymbol{i = I_m \cdot \sin\,(\omega t + \pi/2)} \qquad .....(3)$$

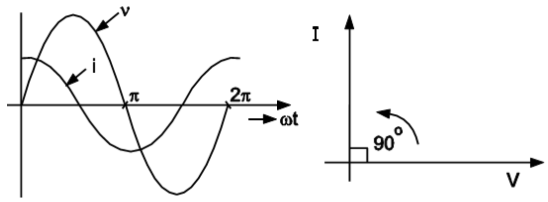


Fig. Voltage and Current Waveforms & Phasor diagram

The current obtains zero value 90° before voltage. Thus the current always leads ahead the applied voltage by 90°.

Power

Instantaneous power, $p = v \cdot i$

$$p = V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2) = V_m\, I_m \sin\omega t \cdot \cos\omega t$$

$$\boldsymbol{p = \frac{V_m\, I_m}{2}\ \sin 2\omega t}$$

Thus instantaneous power wave has double frequency as that of applied voltage and current as shown in Fig.

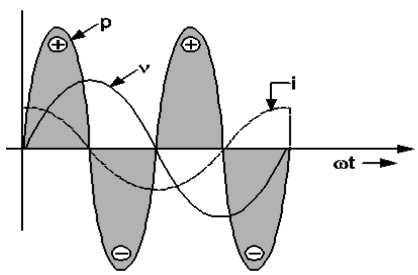


Fig. Voltage, current and power waveforms

Average Power

Average power over a cycle = Average of $\left(\frac{V_m I_m}{2} \sin 2\omega t\right) = 0$

4. Show that the average power taken by pure Inductor or pure Capacitor over a cycle is zero.

Ans: **For Pure Inductive circuit**

Instantaneous power, $p = v \cdot i$

$$p = V_m \sin \omega t \cdot I_m \sin(\omega t - \pi/2) = V_m I_m \sin \omega t \cdot (-\cos \omega t)$$

$$p = -V_m I_m \cdot \sin \omega t \cdot \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

Average power over one complete cycle,

$$P = Average\ of\left(-\frac{V_m I_m}{2} \sin 2\omega t\right) = \int_0^{2\pi} \left(-\frac{V_m I_m}{2} \sin 2\omega t\right) = 0$$

For Pure Capacitive circuit

Instantaneous power, $p = v \cdot i$

$$p = V_m \sin \omega t \cdot I_m \sin(\omega t + \pi/2) = V_m I_m \sin \omega t \cdot (\cos \omega t)$$

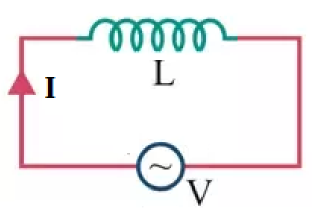
$$p = V_m I_m \cdot \sin \omega t \cdot \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$$

Average power over one complete cycle,

$$P = Average\ of\left(\frac{V_m I_m}{2} \sin 2\omega t\right) = \int_0^{2\pi} \left(\frac{V_m I_m}{2} \sin 2\omega t\right) = 0$$

5. What is Inductive and Capacitive reactance? Explain their dependency on frequency.

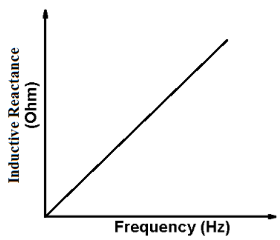
Ans: Inductive Reactance: It is the opposition made by Inductor to flow of an alternating current.



The opposition is expressed as X_L (Inductive reactance) and is given by, $X_L = V/I$

Where, $X_L = 2\pi f L\ Ohm$.

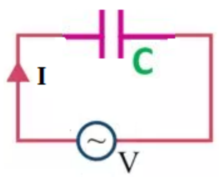
$X_L \alpha f$, When value of L (Self Inductance) is



constant.

The variation of X_L with respect to change in frequency is shown in figure. The higher the frequency, the greater the inductive reactance; the lower the frequency, the less the inductive reactance for a given inductor.

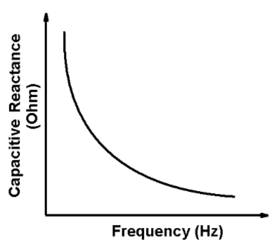
Capacitive Reactance: It is the opposition made by Capacitor to flow of an alternating current.



The opposition is expressed as X_C(Capacitive reactance) and is given by, $X_C = V/I$

Where, $X_C = 1/(2\pi f C)\ Ohm$.

$X_C \alpha 1/f$, When value of C (Capacitance) is constant.



The variation of X_C with respect to change in frequency is shown in figure. The higher the frequency, the less the Capacitive reactance; the lower the frequency, the higher the Capacitive reactance for a given Capacitor.

R-L, R-C and R-L-C series AC Circuit

6. If $v=V_m \sin \omega t$ is applied across single phase circuit and current flowing through the circuit is $i=I_m \sin(\omega t-\Phi)$. Derive the expression for average power consumed in the circuit. Draw voltage, current and power waveforms.

Ans: Let us consider the R-L circuit as shown in fig. Let $v = V_m \sin \omega t$

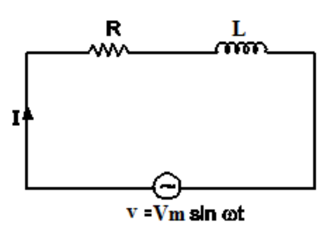


Fig. Circuit diagram

Now the current flowing through the circuit at any instant is $i = I_m (\omega t - \phi)$

The instantaneous power is given as,

$p = v \cdot i$

$= V_m \sin \omega t \cdot I_m \sin(\omega t - \phi) = V_m I_m \sin \omega t \cdot \sin(\omega t - \phi)$

$$= \frac{V_m I_m}{2}\ [\cos\phi - \cos(2\omega t - \phi)]\ = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

In the above expression first term is **constant** and the second term having the double of the supply frequency.

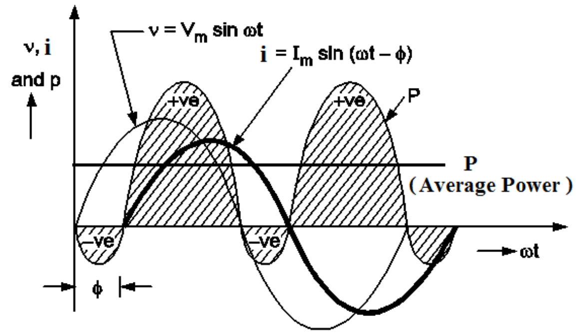


Fig. Voltage, current and power waveforms

Average Power

P = Average of $[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)]$

Hence the ***average power consumed over a cycle by second term is zero.***

$$P = \frac{V_m I_m}{2}\cos\phi = \frac{V_{rms}}{\sqrt{2}} \cdot \frac{I_{rms}}{\sqrt{2}}\cos\phi$$

P = *V. I. cosϕ* Watt

7. **If *v=Vm sin ωt* is applied across single phase circuit and current flowing through the circuit is *i=Im sin (ωt+Φ)*. Derive the expression for average power consumed in the circuit. Draw voltage, current and power waveforms.**

Ans: Let $v = V_m \sin\omega t$

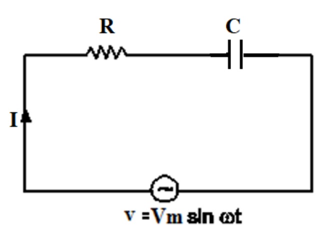


Fig . Circuit diagram

The current flowing through the circuit at any instant is $i = I_m \sin(\omega t + \phi)$

The instantaneous power is given as,

$$p = v \cdot i$$

$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \phi) = V_m I_m \sin\omega t \cdot \sin(\omega t + \phi)$$

$$= \frac{V_m I_m}{2} [\cos\phi - \cos(2\omega t + \phi)] = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

In the above expression first terms is **constant** and the second term having double of the supply frequency, hence the average power consumed by the second term is zero.

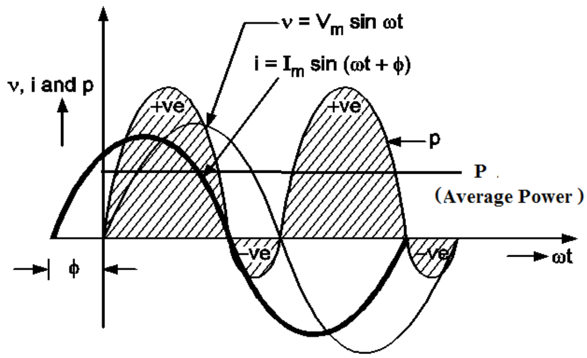


Fig. Voltage, current and power waveforms

Average Power

P = Average of $[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)]$

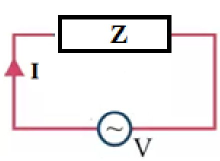
Hence the ***average power consumed over a cycle by second term is zero.***

$$P = \frac{V_m I_m}{2}\cos\phi = \frac{V_{rms}}{\sqrt{2}} \cdot \frac{I_{rms}}{\sqrt{2}} \cos\phi$$

P =*V. I.* $\boldsymbol{\cos\phi}$ **Watt**

8. What is Impedance? Draw Impedance Triangle for R-L and R-C Series circuit

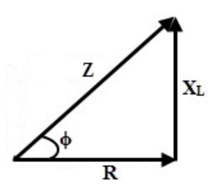
Ans: **Impedance**: It is the opposition made by entire circuit to flow of an alternating current. It is denoted by Z and its unit is ohm.



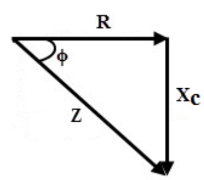
$Z = V/I$, Z may be R-L/ R-C/ R-L-C combination

R-L Series Circuit R-C Series Circuit

Impedance Triangle



$Z=R+jX_L=Z\angle\Phi$



$Z=R-jX_C=Z\angle-\Phi$

9. Explain concept of active, reactive and apparent power. Draw the power triangle for R-L series or R-C series circuit.

Ans:

Active Power or Real Power (P)

Active power is the power which is actually consumed in the circuit.It is the product of RMS value of voltage and current. It is measured in watts or kW and its symbol is P.

$$P = VI \cos \Phi \ \textit{Watt}$$

Reactive Power (Q)

Reactive power is not actually consumed by the circuit rather this power is taken and returned back to the source by components like inductance or capacitance. Reactive power is product of RMS value of voltage and quadrature component of current. Its symbol is Q and unit is VAr or kVAr.

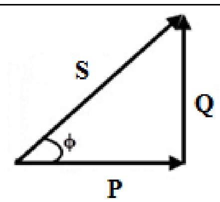
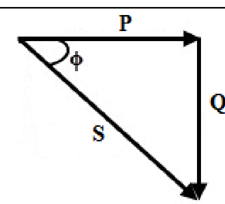
$$Q = V\,I \sin \Phi \ \ \textit{VAr}$$

Apparent Power (S)

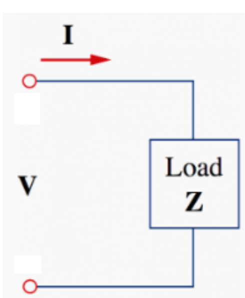
Apparent power is the total power available at source. This power isthe product of RMS value of voltage and current. Its symbol is S and unit is volt ampere or VA.

$$S = V\,I \ \ \textit{VA}$$

$$\text{Apparent power, } S = \sqrt{(\text{Real power})^2 + (\text{Reactive power})^2} = \sqrt{P^2 + Q^2}$$

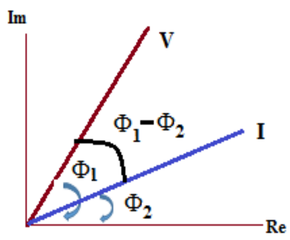
	R-L Series Circuit	R-C Series Circuit
Power Triangle		
Complex Form	S=P+jQ=S∠Φ	S=P-jQ=S∠-Φ

10. What is complex power? Explain its physical significance.



Complex power, *S* (in VA) is the product of the RMS voltage phasor and the complex conjugate of the RMS current phasor. As a complex quantity, its real part is real power *P* and its imaginary part is reactive power *Q*.

$$\overline{S} = \overline{V}\,\overline{I}^* = P + jQ$$



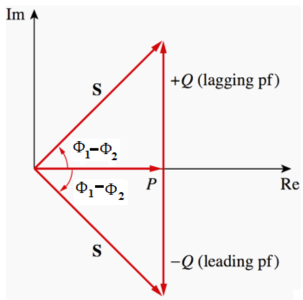
ForRL series circuit, position of voltage and currentphasors are shown in fig. Voltage and current in complex form…

$\overline{V} = V\angle\phi_1$ and $\overline{I} = I\angle\phi_2$

$\overline{S} = \overline{V}\,\overline{I^*} = V\angle\phi_1 \, x \, I\angle-\phi_2 = V\,I\,\angle\phi_1-\phi_2 = P+jQ$

Real Part, $P = VI\cos(\Phi_1-\Phi_2)$

Imaginary Part $Q = VI\sin(\Phi_1-\Phi_2)$



When S lies in the first quadrant, the load is inductive in nature with lagging power factor.

Where, $\Phi_1-\Phi_2>0$

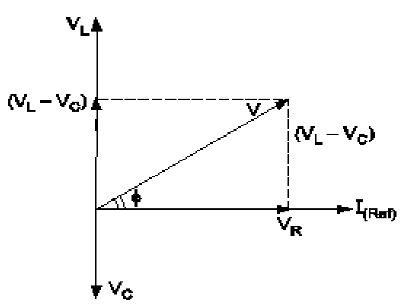
When S lies in the fourth quadrant, the load is capacitive in nature with leading power factor.

Where, $\Phi_1-\Phi_2<0$

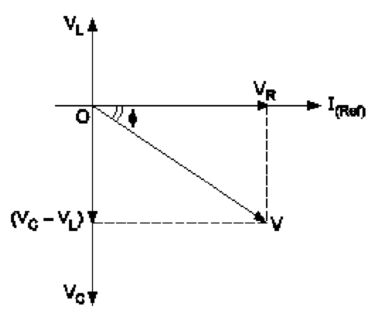
11. Draw the phasor diagram for following condition

(a) $XL > XC$ (b) $XL < XC$ (c) $XL = XC$ for *R-L-C* series circuit.

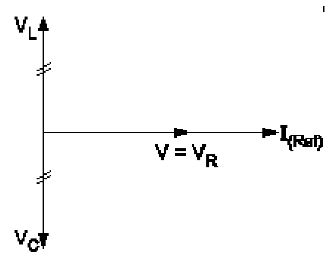
$V_L > V_C$



$V_L < V_C$



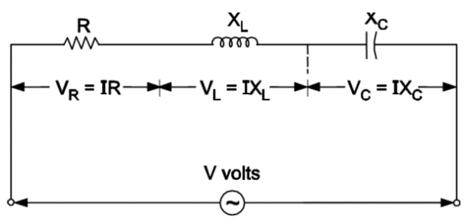
$V_L = V_C$



R-L-C series Resonance Circuit

12. Derive the condition for series resonance in series R-L-C circuit.

Ans: Consider a series circuit of resistance R, inductance L and capacitance



The total impedance of the circuit is given by $Z=\sqrt{R^2 + (X_L - X_C)^2}$

Now, if the supply voltage across the circuit is maintained constant and frequency is gradually increased from zero to a high value then inductive reactance, ($X_L=2\ \pi\ f\ L$), starts increasing from zero while capacitive reactance $\left(X_C = \frac{1}{2\pi fC}\right)$ decreases from its infinitely large value as At a certain frequency f_r, the two reactance's becomes numerically equal i.e. at this frequency, $X_L = X_C$

Let, f_r be the resonant frequency. Then at this frequency,

$$X_L = X_C$$

$$2\ \pi f_r \cdot L = \frac{1}{2\pi f_r \cdot C}$$

$$f_r^2 = \frac{1}{4\pi^2\ LC}$$

$$\boldsymbol{f_r = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

13. State any four characteristics of series resonance. Show the variation of X_L, X_C, Z and I against frequency.

Ans: i) Inductive reactance (X_L) = Capacitive Reactance (X_C)

ii) Impedance, $Z=R + j\ (X_L-X_C)= R+j0=R$. Hence Z reaches to minimum value

iii) Current I=V/Z=V/R. Hence the current reaches to maximum value

iv) As Z=R, the circuit behaves like pure resistive circuit. Hence *pf* (*CosΦ*) becomes **1**

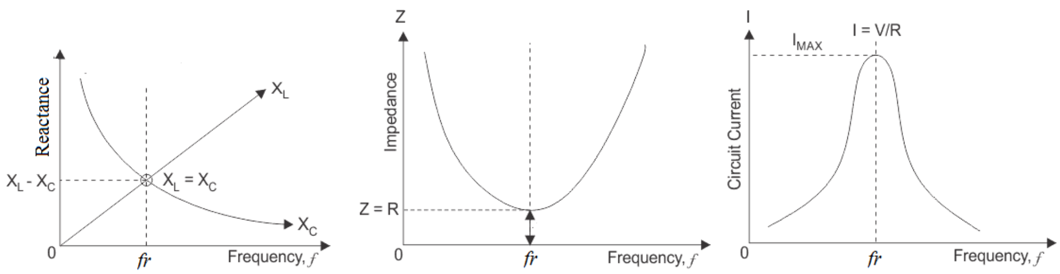


Fig. Variation of X_L, X_C, Z and I with frequency

Parallel AC Circuit

14. Define the terms (i) Admittance (ii) Conductance (iii) Susceptance.

Ans: **(i)** ***Admittance(Y):*** Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in ***unit Siemens or Mho.***

$$Y = \frac{1}{Z} \text{ Siemens}$$

Conductance (G) : It is defined as the ratio of the ***resistance*** to the square of the ***impedance***. It is measured in ***unit Siemens or Mho.***

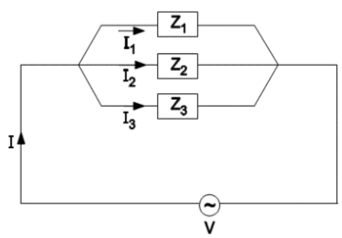
$$G = \frac{R}{Z^2} \text{ Siemens}$$

Susceptance (B) : It is the ratio of the ***reactance*** to the square of the *impedance*. It is measured in the unit **Siemens or Mho.**

$$B = \frac{X}{Z^2} \text{ Siemens}$$

14. What is admittance? What are its two components?

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit Siemens or Mho. Consider a circuit shown in the Fig.The total current is phasor sum individual branch currents.



$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 \text{ and } V = I Z$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_1} + \frac{\bar{V}}{\bar{Z}_2} + \frac{\bar{V}}{\bar{Z}_3}$$

$$\frac{\bar{V}}{\bar{Z}} = \bar{V}\left[\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}\right]$$

$$\frac{1}{\bar{Z}} = \left[\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}\right]$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

Where, Y is the total admittance of the circuit.

Components of Admittance:

Consider an impedance Z is given by,

$Z = R \pm jX$

Where +ve sign is for inductive reactance and – ve sign is for capacitive reactance,

Admittance, $Y = \dfrac{1}{Z} = \dfrac{1}{R \pm jX}$

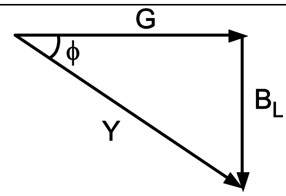
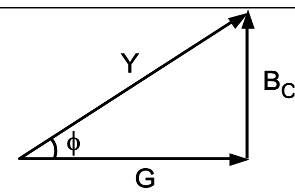
Rationalizing the above expression, we get,

$$Y = \frac{1}{R \pm jX} \; \frac{R \mp jX}{R \mp jX} = \frac{R \mp jX}{R^2 + X^2} = \frac{R}{(R^2 + X^2)} \mp j\frac{X}{R^2 + X^2} \qquad = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$Y = G \mp j,$ where, $G = \dfrac{R}{Z^2}$ is conductance and $B = \dfrac{X}{Z^2}$ is Susceptance

15. Draw the admittance triangle for inductive circuit and capacitive circuit.

The triangle in which sides are representing Conductance, Susceptance and Admittance of the circuit is known as admittance triangle.

	R-L Series Circuit	R-C Series Circuit
Admittance Triangle		
Complex Form	$\bar{Y} = Y \angle -\phi = G - j\,B_L$	$\bar{Y} = Y \angle \phi = G + j\,B_C$

Questions asked in End-Sem Dec 2019 Examination

Q. Define active, reactive and apparent power. State their units. Also draw the power triangle for R-L circuit. [4 M] (Refer Q. No.9)

Q. What is series resonance? Derive the expression for resonant frequency. [6M] (Refer Q. No. 12)

Q. Obtain the expression for current, when voltage v = Vm sinωt is applied across purely inductive circuit. [4M] (Refer Q. No.2)

Q. Derive the expression for power, when voltage v = Vm, sin ωt is applied across R-L series circuit. Draw the phasor diagram [6M] (Refer Q. No. 6)