Unit 6

Work, Power, Energy and Batteries

Factors Governing the value of Resistance

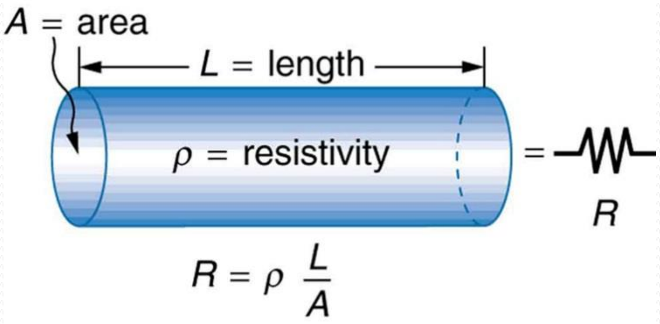
The resistance R offered by a conductor depends on the following factors :

(i) It varies directly as its length, l.

(ii) It varies inversely as the cross-section A of the conductor.

(iii) It depends on the nature of the material.

(iv) It also depends on the temperature of the conductor.

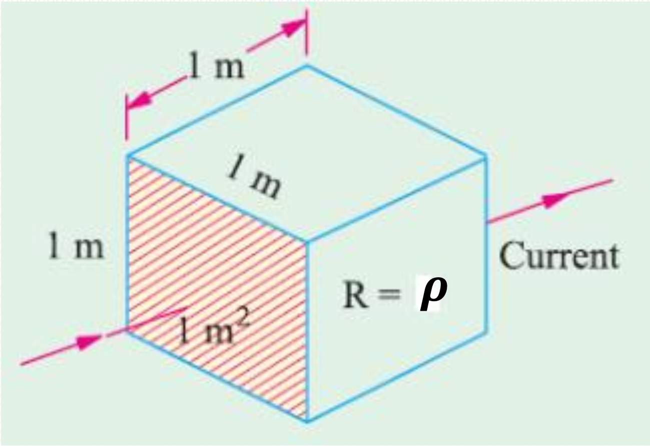


$$R = \frac{\rho l}{A} \quad \Omega$$

where ρ is a constant depending on the nature of the material of the conductor and is known as its *specific resistance or resistivity*.

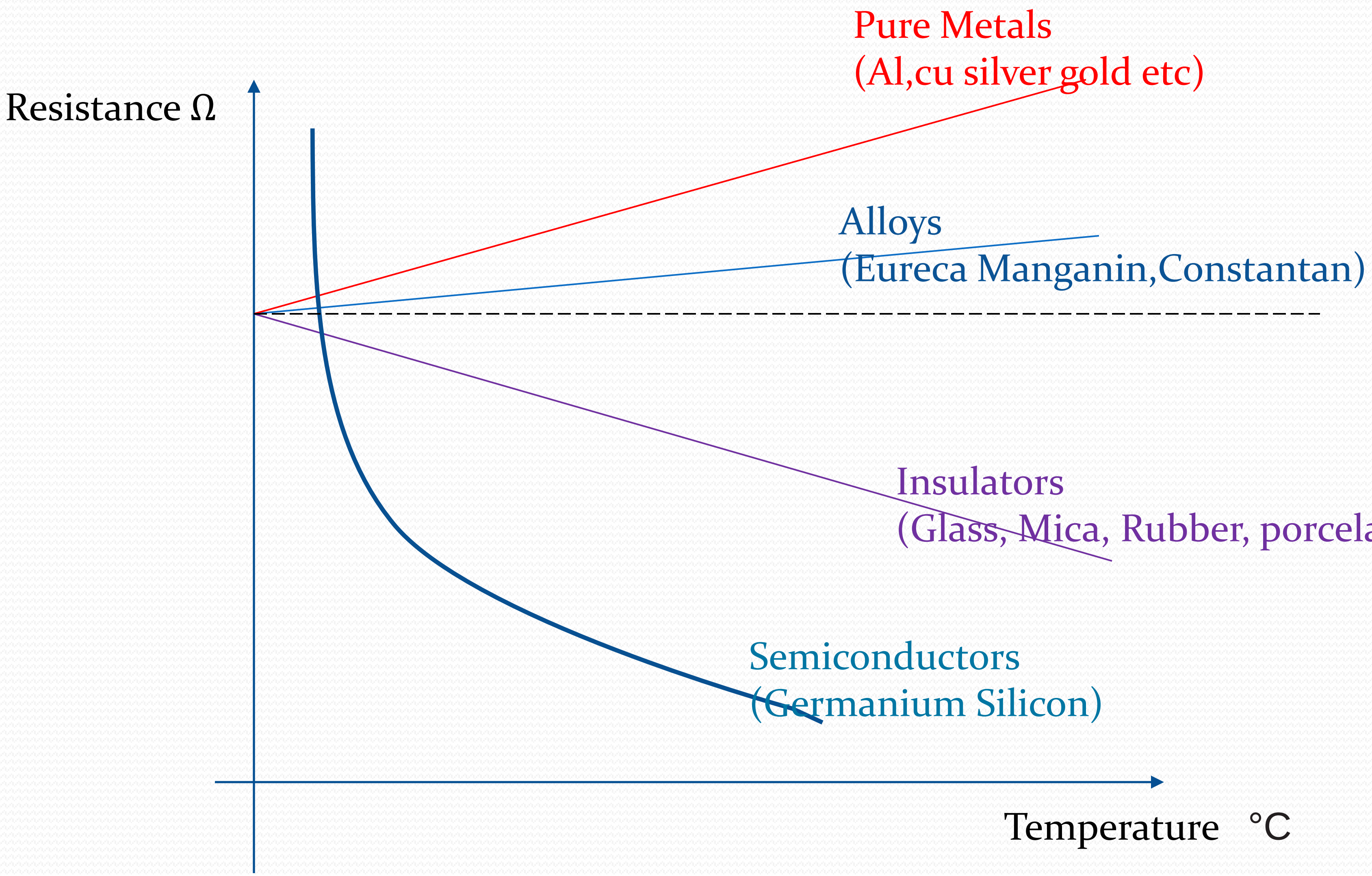
If $l = 1$ m and $A = 1\ m^2$, then $R = \rho$

Hence, specific resistance of a material may be defined as the resistance between the opposite faces of a metre cube of that material.



Resistivity of the material is measured *in* $\Omega - \text{m}$

Effect of Temperature on Resistance



(i) With increase in the temperature of pure metals Resistance will *increases*. The increase is large and fairly regular for normal ranges of temperature.

(ii) With increase in the temperature of Alloys Resistance will *increases*. But increase is relatively small and irregular. For some high-resistance alloys like Eureka (60%Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.

(iii) With increase in the temperature of electrolytes, carbon and insulators (such as paper, rubber, glass, mica etc.) Resistance will *decreases* linearly.

(iv) With increase in the temperature of Semiconductors (such as Silicon,Germanium etc.) Resistance will *decreases* Nonlinearly.

Temperature Coefficient of Resistance / Resistance Temperature Coefficient (RTC)

Let a metallic conductor having a resistance of R_0 at 0°C be heated of t°C and let its resistance at this temperature be R_t. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

(i) directly on its initial resistance

(ii) directly on the rise in temperature

(iii) on the nature of the material of the conductor.

$$(R_t - R_0) \propto R_0(t - 0)$$

$$(R_t - R_0) = \alpha_0 \; R_0 \,(t)$$

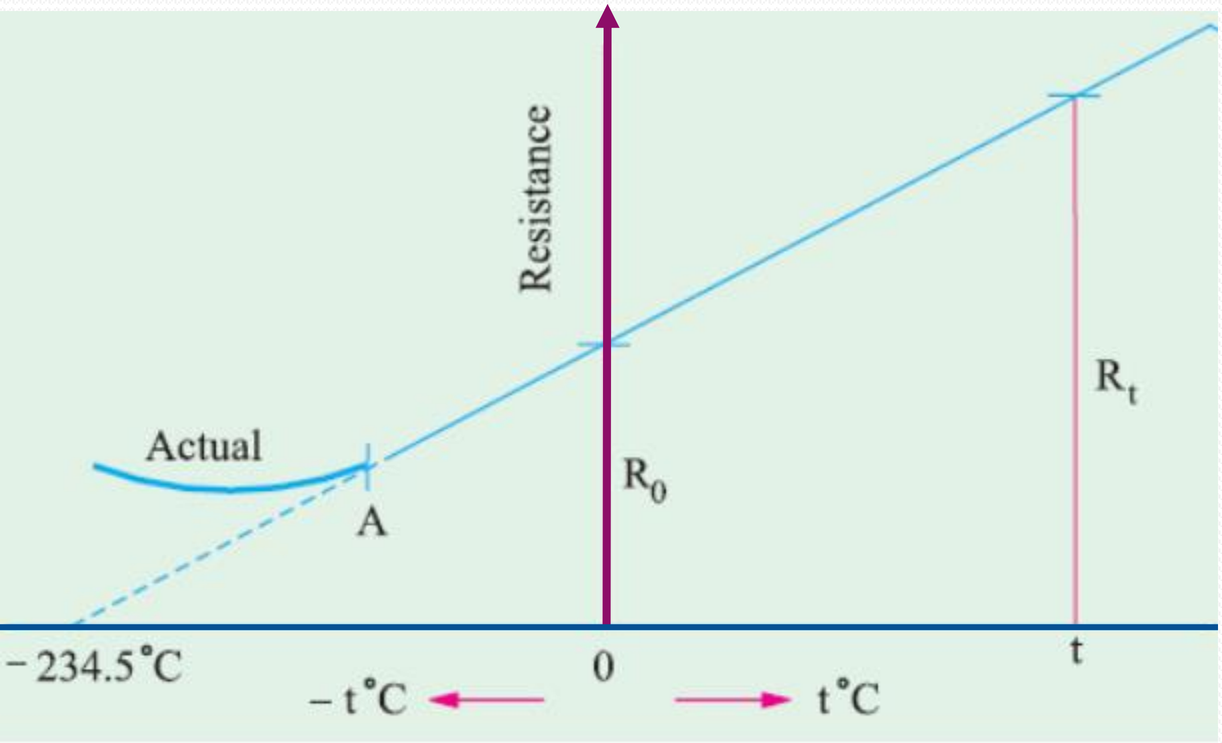
where α_0 (alpha) is a constant and is known as the *temperature coefficient of resistance* of the conductor.

$$\alpha_0 = \frac{R_1 - R_0}{R_0(t)} \quad / \ °C$$

Reciprocal(per) degree centigrade (/°C)

RTC is the change in resistance per ohm original resistance per °C change in temperature.

$$R_t = R_0(1 + \alpha_0 t)$$



For copper

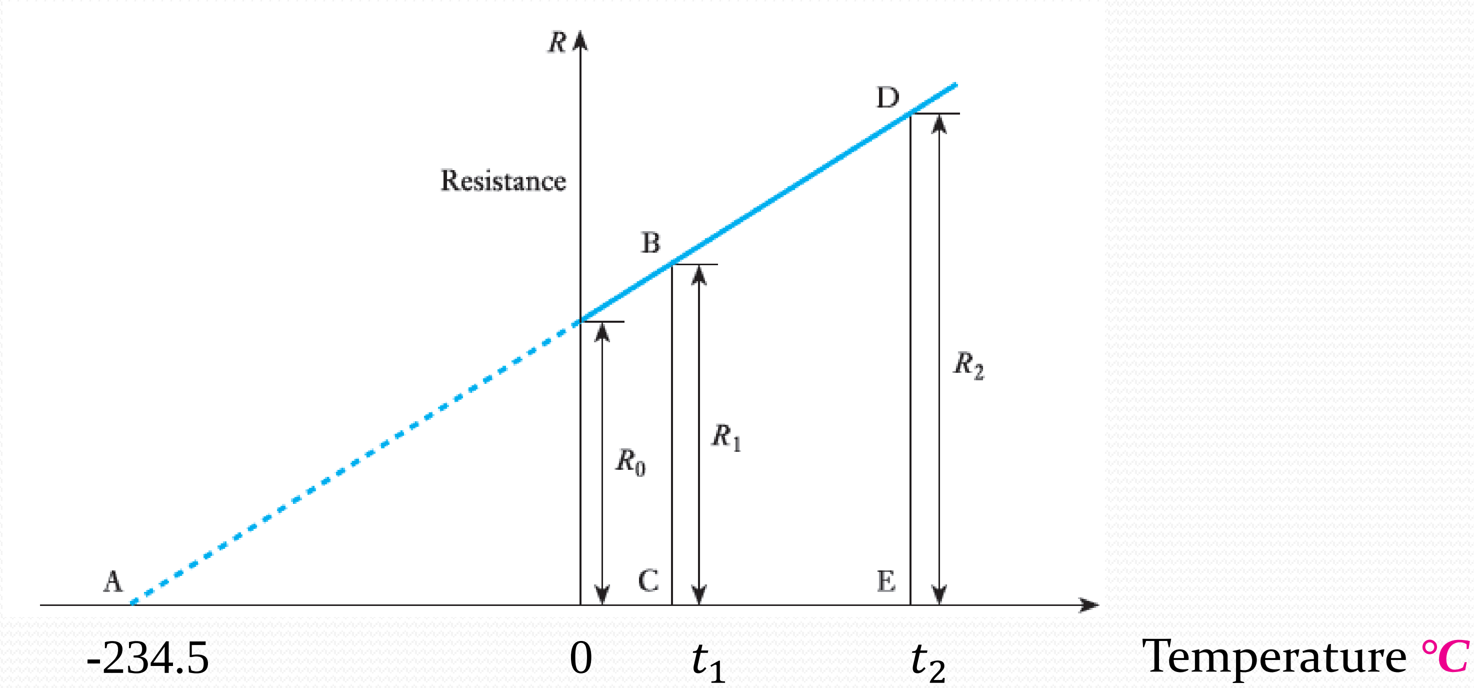
t = - 234.5 °C

$$0 = R_0(1 + \alpha_0(-234.5))$$

$$\alpha_0 = \frac{1}{234.5} = 0.0042 \text{ per } °C$$

Typical temperature coefficients of resistance at 0 °C

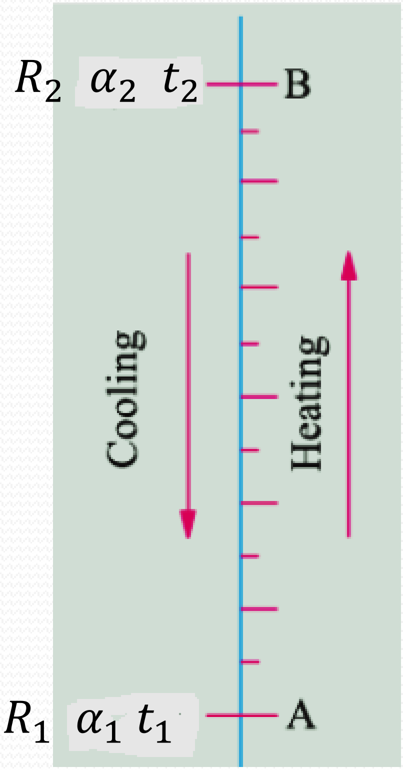
Material	α_0(/°C) at 0 °C
Aluminium	0.003 81
Copper	0.004 28
Silver	0.004 08
Nickel	0.006 18
Tin	0.004 4
Zinc	0.003 85
Carbon	–0.000 48
Manganin	0.000 02
Constantan	0
Eureka	0.000 01
Brass	0.001

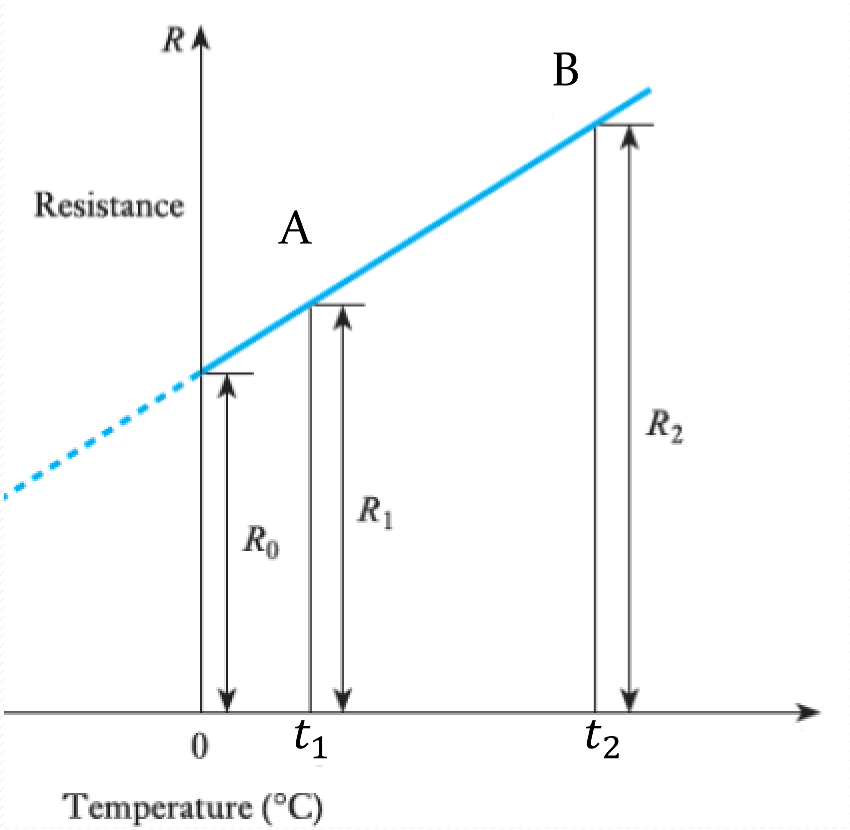


$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

Effect of temperature on Resistance Temperature Coefficien

The value of RTC (α)is not constant it will vary with temperature. As the temperatu increases the value of RTC Decreases.





Consider a specimen of material initially it operating at point A where temperature is t_1 and resistance is R_1 if the specimen is Heated to temperature t_2 then operating point will shift to B where resistance is R_2
This change in resistance is given by

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)] \qquad (1)$$

Now suppose the specimen is Cooled to temperature t_1 then operating point will shift to A where resistance is R_1 This change in resistance is given by

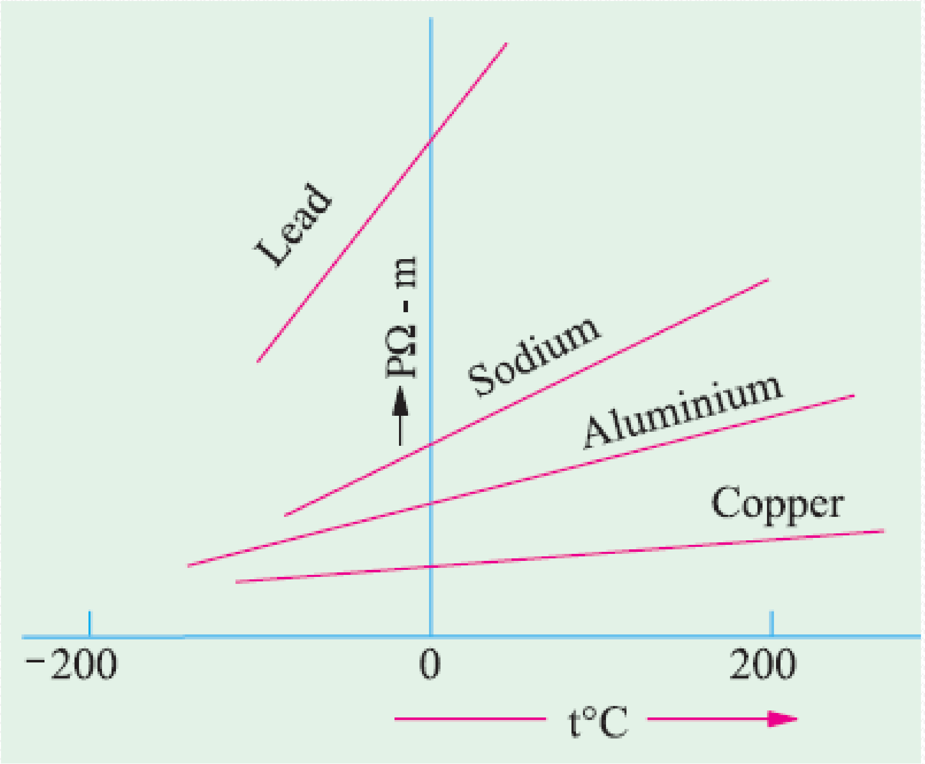
$$R_1 = R_2[1 + \alpha_2(t_1 - t_2)] \qquad (2)$$

$$\boldsymbol{\alpha_2 = \frac{\alpha_1}{1+\alpha_1(t_2 - t_1)}}$$

If $\boldsymbol{t_1}$= 0 and $\boldsymbol{t_2 = t}$ ***then***

$$\boldsymbol{\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}}$$

Variations of Resistivity with Temperature



$$R = \frac{\rho \;\; l}{A} \quad \Omega$$

$$\rho_2 = \rho_1[1 + \alpha_1(t_2 - t_1)]$$

$$\rho_t = \rho_0[1 + \alpha_0(t)]$$

Example 1.

At 0°C, a specimen of copper wire has its resistance equal to 4-mΩ and its temperature coefficient of resistance equal to (1/234.5) per °C. Find the values of its resistance and temperature coefficient of resistance at 70°C.

Solution : Given : Resistance at 0°C = R_0 = 4 mΩ, Temperature co-efficient of resistance at 0°C = $\alpha_0 = \frac{1}{234.5}$ / °C.

$$\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$$

$$\therefore \quad \alpha_{70} = \frac{\frac{1}{234.5}}{1 + \frac{1}{234.5} \times 70}$$

$$\alpha_{70} = 0.003284/°C$$

Now,

$$R_t = R_0 (1 + \alpha_0 t)$$

$$R_{70} = 4\left[1 + \frac{1}{234.5} \times 70\right]$$

$$R_{70} = 5.194 \text{ m}\Omega$$

Example 2

Determine the current flowing at the instant of switching a 100 watt lamp on 230 V supply. The ambient temperature is 25°C. The filament temperature is 2000°C and the resistance temperature coefficient is 0.005/°C at 0°C.

Solution : **Given :** $P = 100$ W, $V = 230$ V, $t_1 = 25°C$, $t_2 = 2000$ °C, $\alpha_0 = 0.005/°C$

R_2 = Resistance of filament in ON condition

$$= \frac{V^2}{P} = \frac{(230)^2}{100} = 529\ \Omega$$

α_1 = R.T.C. at t_1

$$= \frac{\alpha_0}{1 + \alpha_0 t_1} = \frac{0.005}{1 + 0.005 \times 25}$$

$$\alpha_1 = 4.44 \times 10^{-3}/°C$$

Now, $$R_2 = R_1 (1 + \alpha_1 \Delta t)$$

$\therefore$ $$529 = R_1 [1 + 4.44 \times 10^{-3} \times (2000 - 25)]$$

$\therefore$ $$R_1 = 54.15\ \Omega$$

I = Current at switching time

$$I_{25} = \frac{V}{R_1} = \frac{230}{54.15} = 4.25\ A$$

Example 3

A D.C. shunt motor, after running several hours on constant voltage of 400 V, takes field current of 1.6 A. If temperature rise is 40°C, what value of extra resistance is required in field circuit to maintain field current equal to 1.6 A ? Assume motor started from cold at 20°C and $\alpha_{20} = 0.0043/°C$

Solution : Given : $V = 400$ V, $I_f = 1.6$ A, $\Delta t = 40°C$, $t_1 = 20$ °C, $\alpha_{20} = 0.0043$ / °C.

$$R_2 = \frac{V}{I_f} = \frac{400}{1.6} = 250\ \Omega$$

...field circuit resistance after 4 hours at t_2°C

$$R_2 = R_1\left[1 + \alpha_1\, \Delta t\right],\ \text{where, } \alpha_1 = \alpha_{20}$$

$$250 = R_1\,[1 + 0.0043 \times 40]$$

$$\therefore \quad R_1 = 213.3105\ \Omega \qquad \text{...field circuit resistance at } t_1\text{°C}$$

But I_f is to be maintained constant.

$$\therefore \quad I_f = \frac{V}{R_1 + R_x} = 1.6$$

$$\therefore \quad R_1 + R_x = \frac{400}{1.6} = 250$$

$$\therefore \quad R_x = 250 - 213.3105$$

(Extra resistance required) $\quad R_x = 36.6894\ \Omega$

Example 4

Two coils connected in series have resistances of 600 Ω and 400 Ω with temperature coefficients of 0.1% and 0.4% respectively at 20°C. Find the effective temperature coefficient of series combination at 20°C. When the combination is heated to 50°C, find the resistance of series combination.

Solution : Given : $R_1 = 600\ \Omega$, $R_2 = 400\ \Omega$, $\alpha_1 = 0.1\% = \frac{0.1}{100}$, $\alpha_2 = 0.4\% = \frac{0.4}{100}$

All values are given at $t_1 = 20\ ^\circ C$

Let R_1' and R_2' be resistance values at t_2°C

$\therefore \quad R'_1 = R_1 [1 + \alpha_1 (t_2 - t_1)]$

$R'_2 = R_2 [1 + \alpha_2 (t_2 - t_1)]$

At t_1°C, $\quad R_{12} = R_1 + R_2$

and at t_2°C $\quad R'_{12} = R'_1 + R'_2$

while, $\quad R'_{12} = R_{12} [1 + \alpha_{12} (t_2 - t_1)]$

where, $\quad \alpha_{12}$ = R.T.C. of series combination at t_1°C

$\therefore \quad R'_1 + R'_2 = (R_1 + R_2) [1 + \alpha_{12} (t_2 - t_1)]$

$\therefore \quad R_1 [1 + \alpha_1 (t_2 - t_1)] + R_2 [1 + \alpha_2 (t_2 - t_1)] = (R_1 + R_2) [1 + \alpha_{12} (t_2 - t_1)]$

Simplifying, $\alpha_{12} = \dfrac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$

$$= \frac{\left[600 \times \left(\frac{0.1}{100}\right)\right] + \left[400 \times \left(\frac{0.4}{100}\right)\right]}{600 + 400}$$

$$\alpha_{12} = 2.2 \times 10^{-3} / ^\circ C$$

Now, $R_{12} = R_1 + R_2 = 600 + 400 = 1000\ \Omega$, at $t_1 = 20^\circ C$

$\therefore$ R'_{12} = Resistance of series combination at $t_2 = 50^\circ C$

$$= R_{12}\left[1 + \alpha_{12}(t_2 - t_1)\right]$$

$$= 1000\left[1 + 2.2 \times 10^{-3}(50 - 20)\right]$$

$$R'_{12} = 1066\ \Omega$$

(**Note :** Alternatively, find R'_1 and R'_2 at 50°C and $R'_{12} = R'_1 + R'_2$)

Example 5

A coil has resistances of 18 Ω when its temperature is 20°C and 22 Ω when its temperature is 50°C. Find the rise in temperature, when the resistance becomes 24 Ω. The room temperature is 18°C.

Solution : Given : $R_{20} = 18\ \Omega$, $R_{50} = 22\ \Omega$. Find temperature when R becomes 24 Ω, Room temperature = 18°C.

Using

$$R_t = R_0 (1 + \alpha_0 \cdot t)$$

$$R_{20} = R_0 (1 + \alpha_0 \times 20)$$

$$18 = R_0 (1 + \alpha_0 \times 20) \quad \ldots(I)$$

$$R_{50} = R_0 (1 + \alpha_0 \times 50)$$

$$22 = R_0 (1 + \alpha_0 \times 50) \quad \ldots(II)$$

Dividing equation (I) by (II),

$$\frac{18}{22} = \frac{R_0 (1 + \alpha_0 \times 20)}{R_0 (1 + \alpha_0 \times 50)}$$

$$18 + 900\,\alpha_0 = 22 + 440\,\alpha_0$$

$$\alpha_0 (900 - 440) = 22 - 18$$

$$460\,\alpha_0 = 4$$

$$\therefore \quad \alpha_0 = 0.00869\ /^\circ C$$

Substituting in equation (I),

$$18 = R_0 (1 + 0.00869 \times 20)$$

$$18 = R_0 (1 + 0.1738)$$

$$\therefore \quad R_0 = \frac{18}{1.1738}$$

$$\therefore \quad R_0 = 15.33\ \Omega$$

$$R_t = R_0 (1 + \alpha_0 \cdot t)$$

$$24 = 15.33 (1 + 0.00869 \times t)$$

$$\frac{24}{15.33} = (1 + 0.00869 \times t)$$

$$1.56 - 1 = 0.00869\, t$$

$$\therefore \quad t = \frac{0.56}{0.00869} = 64.44°C$$

$$\therefore \quad \text{Temperature rise} = 64.44 - 18$$

$$= 46.44°C$$

- Example 6

Determine the current flowing as the instant of switching a 60 W lamp on a 240 V supply. The ambient temperature is 24° C. The filament temperature is 2000 °C and R.T.C. of filament material is 0.005 per °C at 0°C.

Solution : Given : $P = 60$ W, $V = 240$ volt, $t_1 = 24°C$, $t_2 = 2000$ °C, $\alpha_o = 0.005/°C$.

When the lamp is on, filament temperature is 2000° C

$$R \text{ at } 2000° C = R_{2000} = \frac{V^2}{P} = \frac{(240)^2}{60} = 960\ \Omega$$

R at 24° C = R_{24} (at the instant of switching)

As
$$R_2 = R_1 (1 + \alpha_1 (t_2 - t_1))$$

$$R_{2000} = R_{24} (1 + \alpha_{24} (2000 - 24)) \quad \ldots(1)$$

$$\alpha_{24} = \frac{\alpha_o}{1 + \alpha_o (24)} \qquad \text{as,} \quad \alpha_t = \frac{\alpha_o}{1 + \alpha_o t}$$

$$= \frac{0.005}{1 + 0.005 \times 24} = 0.004464/°C$$

by putting value of α_{24} in equation (1)

$$960 = R_{24} (1 + 0.004464 (1976))$$

$$R_{24} = \frac{960}{1 + (0.004264 \times 1976)} = \frac{960}{9.820} = 97.75\ \Omega$$

current at the instant of switching

$$I = \frac{V}{R_{24}} = \frac{240}{97.75} = 2.45 \text{ Amp.}$$

Example 7

A heater coil, after running for several hours from switched on, draws a current of 2 amp from constant voltage 400 V. If the temperature rise is 70 °C. Find the change in voltage required to maintain same current at the time of starting the heater coil. Assume the heater is started at 20 °C and RTC for the heater coil at 0 °C is 0.0038/°C.

Solution : Given : $\alpha_o = 0.0038/°C$, $t_1 = 20°C$, $t_2 = 70° + 20°\ C = 90°C$

After running for several hours, the temperature of heater coil is 90°C.

$$R \text{ at } 90°C = R_{90} = \frac{V}{I_{90}} = \frac{400}{2} = 200\ \Omega$$

as
$$R_2 = R_1\,(1 + \alpha_1\,(t_2 - t_1))$$

$$R_{90} = R_{20}\,(1 + \alpha_{20}\,(90 - 20)) \qquad \ldots(1)$$

(where R_{20} is R at 20°C)

$$\alpha_{20} = \frac{\alpha_o}{1 + \alpha_o\,(20)}$$

$$= \frac{0.0038}{1 + (0.0038 \text{ x } 20)} = 3.53 \times 10^{-3}\ /°C$$

Putting value of α_{20} in (1)
$$200 = R_{20}\,(1 + 0.00353 \times 70)$$

$$R_{20} = \frac{200}{(1 + (0.00353 \times 70))} = 160.37\ \Omega$$

To maintain the same current of 2 Amp, the voltage required

$$V_2 = 160.37 \times 2 = 320.74 \text{ volt}$$

$$\text{change in voltage} = 400 - 320.74 = 79.26 \text{ volt}$$

Example 8

The filament of 240 V of metal filament lamp is to be constructed from a wire having a diameter of 0.03 mm and a resitivity of 4.3 μΩcm. If the RTC of the filament material is 0.005/°C at 20°C, what length of the filament is necessary for the lamp to dissipitate 60 watt at a filament temperature of 2420°C. Assume room temperature as 20°C.

Solution : $V = 240$ V, $d = 0.03$ m, $\rho = 4.3\ \mu\Omega$ cm, $\alpha_{20} = 0.005/°C$, $t_1 = 20°C$, $t_2 = 2420\ °C$

$$R \text{ at } 2420°\ C = R_{2420}$$

$$R_{2420} = \frac{V^2}{P} = \frac{240^2}{60} = 960\ \Omega$$

$$R_2 = R_1\,(1 + \alpha_1\,(t_2 - t_1))$$

$$R_{2420} = R_{20}\,(1 + \alpha_{20}\,(2420 - 20))$$

$$960 = R_{20}\,(1 + 0.05\,(2400))$$

$$R_{20} = \frac{960}{1 + 0.005 \text{ x } 2000} = 73.84\ \Omega$$

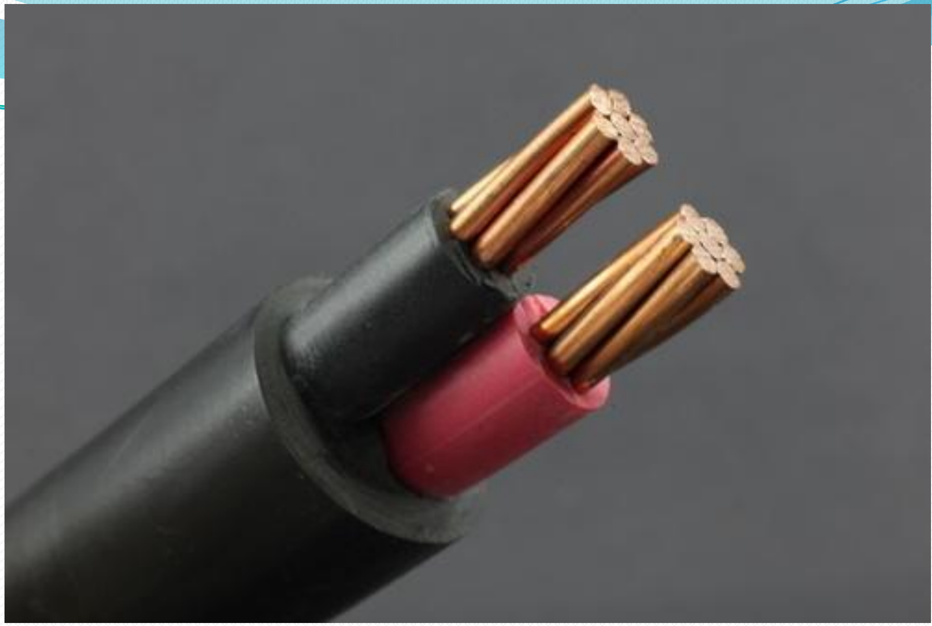
$$R_{20} = \rho\,\frac{l}{a} = \frac{4.3 \text{ x } 10^{-6} \times 10^{-2} \times l}{\pi\, d^2/4}$$

$$l = 73.84 \times \pi \times \frac{(0.03 \times 10^{-3})^2}{4} \;/\; 4.3 \times 10^{-8}$$

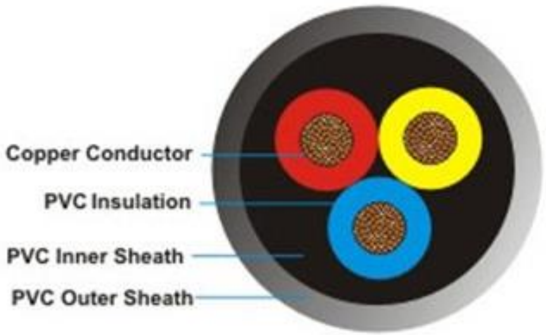
$$= 1.21 \text{ meter or } 121.25 \text{ cm.}$$

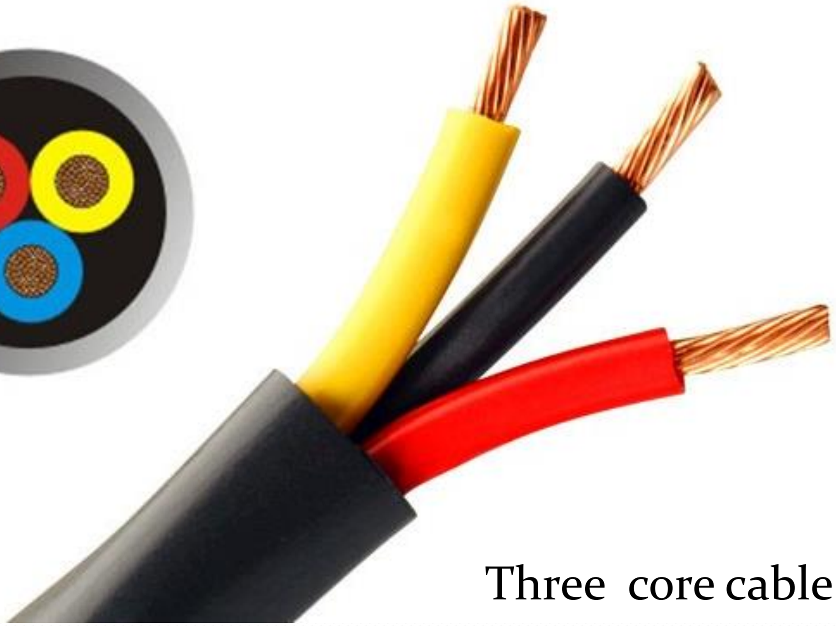
CABLES

Single core cable

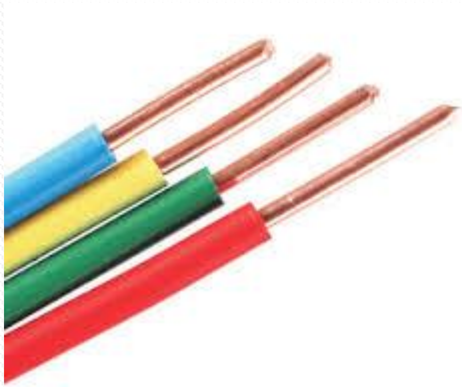


two core cable





Three core cable



INSULATION RESISTANCE

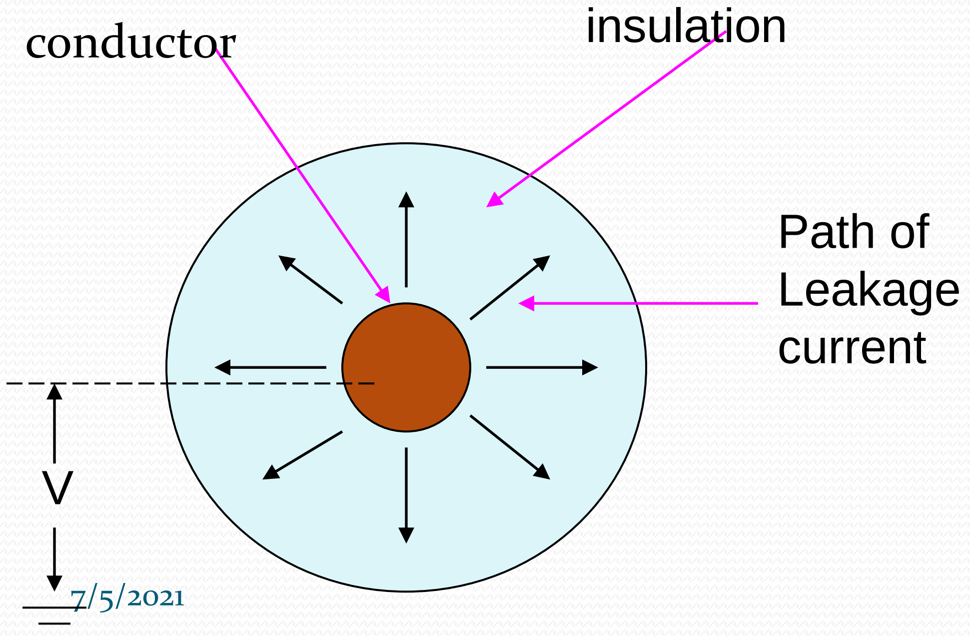
The insulation resistance is defined as the resistance offered by an insulator to the flow of leakage current.

$$R_i = \frac{V}{I_l}$$

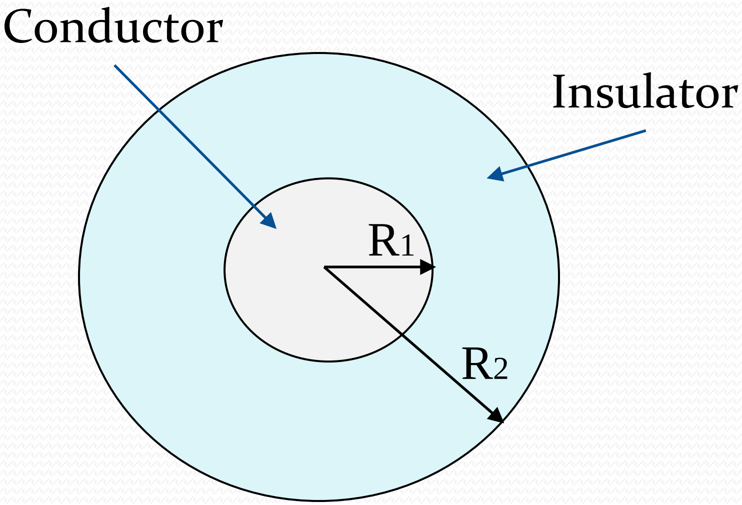
V = Voltage measured between conductor and earth

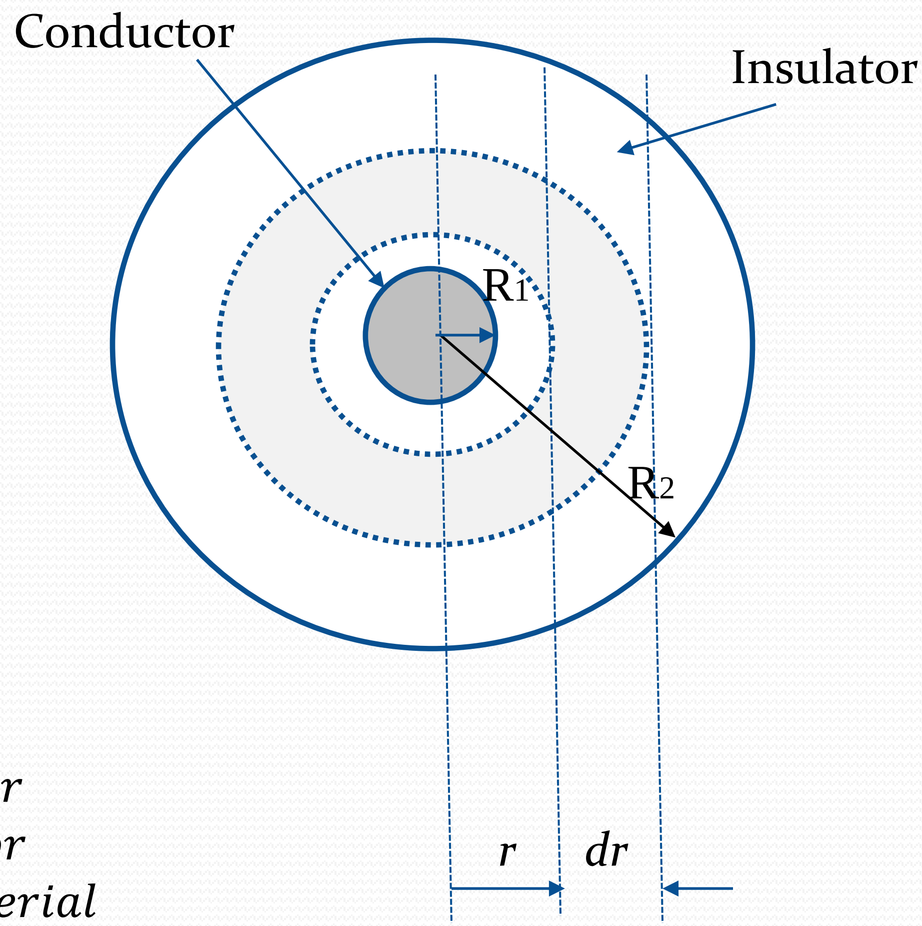
I_l = Leakage current

R_i = Insulation resistance



Insulation Resistance of a Single core cable





R_1 $-$ *Inner Radius of Insulator*

R_2 $-$ *Outer Radius of Insulator*

$\boldsymbol{\rho}$ $-$ *Resistivity of Insulating Material*

l $-$ *Length of the cable*

Some leakage current flows from the conductor in radially outward direction.

to obtain the expression for *Ri*, consider a small section of insulator at radius *r* with thickness *dr* of the cable.

$$dRi = \rho \, \frac{length\ of\ path\ followed\ by\ leakage\ current}{Area\ of\ cross\ section}$$

Resistance of this ring $\mathrm{dRi} = \rho \frac{l}{A} = \rho \, \frac{dr}{2\,\pi\, r\, l}$

The total insulation resistance *Ri* can be obtained by integrating "*dRi*" over the entire radius of insulating material i.e. from R_1 and R_2.

$$R_i = \int_{R_1}^{R_2} dR_i = \int_{R_1}^{R_2} \frac{\rho\, dr}{2\pi r l}$$

$$= \frac{\rho}{2\pi l} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\rho}{2\pi l} \left[\log_e r\right]_{R_1}^{R_2}$$

$$R_i = \frac{\rho}{2\pi l} \left[\log_e R_2 - \log_e R_1\right]$$

$$= \frac{\rho}{2\pi l}\, l_n \left(\frac{R_2}{R_1}\right) \Omega$$

$$\boxed{R_i = \frac{\rho}{2\pi l} \log_e \left[\frac{R_2}{R_1}\right]}$$

$$R_i = \frac{\rho}{2\pi l}\, ln \frac{R_2}{R_1}$$

Effect of Temperature and Moisture on Insulation Resistance

Example 1

A single-core cable has 1.5 cm diameter conductor and thickness of insulation is 2.2 cm. The resistivity of the insulating material is 9.2×10^{12} Ω-m. Determine the insulation resistance per km length of cable. If the working voltage of conductor is 1100 V, what is the leakage current per km length of cable ?

Solution :

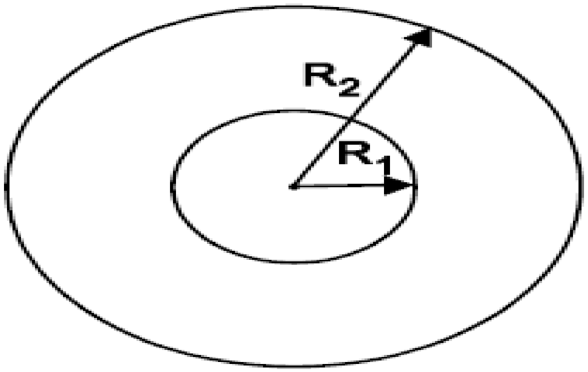


Fig. 1.4

Given : Conductor diameter = 1.5 cm, Radius = $R_1 = 0.75$ cm $= \frac{1.5}{2}$

$V = 1100$ volt, $\rho = 9.2 \times 10^{12}$ Ω-m

$$\begin{aligned}
\text{Leakage current/km} &= ? \\
R_2 &= R_1 + 2.2 = 2.2 + 0.75 \\
R_i &= \frac{\rho}{2\pi l} \log_e\left[\frac{R_2}{R_1}\right] \\
&= \frac{9.2 \times 10^{12}}{2 \times \pi \times 1000} \log_e\left[\frac{2.2 + 0.75}{0.75}\right] \\
&= 2 \times 10^9 \ \Omega \\
\text{Leakage current} &= \frac{V}{R_i} = \frac{1100}{2 \times 10^9} = 0.55 \times 10^{-6} \text{ A.}
\end{aligned}$$

Example 2

A single-core cable has a conductor of diameter 1.2 cm and its insulation thickness of 1.6 cm. The specific resistance of insulating material is 7.5×10^8 MΩ-cm.

(a) Calculate the insulation resistance per km of cable.

(b) If resistance to be increased by 20%, calculate the thickness of additional layer required.

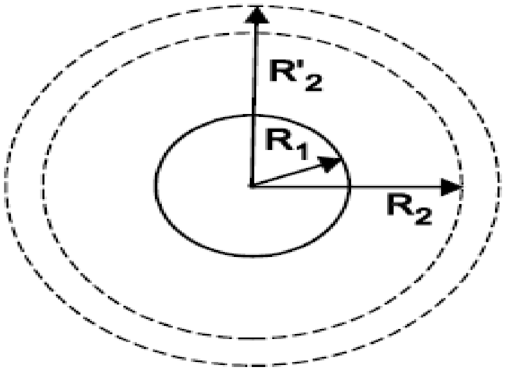


Fig. 1.5

Given : $\rho = 7.5 \times 10^8$ MΩ-cm $= 7.5 \times 10^8 \times 10^6$ Ω-cm $= 7.5 \times 10^{14} \times 10^{-2}m= 7.5 \times 10^{12}$ m,

$R_1 = \frac{D}{2} = \frac{1.2}{2} = 0.6$ cm, $R_2 = R_1 + t = 1.6 + 0.6 = 2.2$ cm

$$R_i = \frac{\rho}{2\pi l} \log_e\left[\frac{R_2}{R_1}\right] = \frac{7.5 \times 10^{12}}{2 \times \pi \times 1000} \log_e\left[\frac{2.2}{0.6}\right] = 1550.9 \text{ M}\Omega$$

New $\quad R_i = R_i' = 1550.9 + (0.2 \times 1550.9) = 1861.08$ MΩ

$$R_i' = 1861.08 = \frac{\rho}{2\pi l} \log_e\left[\frac{R_2'}{R_1}\right] = \frac{7.5 \times 10^{12}}{2 \times \pi \times 1000} \times \log_e\left[\frac{R_2'}{0.6}\right]$$

$$R_2' = 2.8528 \text{ cm}$$

Additional thickness required $= 2.8528 - R_1 -$ (original thickness)

$= 2.8528 - 0.6 - 1.6 = 0.6528$ cm

Example 3

The insulation resistance per km of a single core cable having a conductor diameter of 1.2 cm and insulation thickness of 1.5 cm is 550 MΩ. Find the % age increase in insulation resistance if the insulation thickness is increased by 50%.

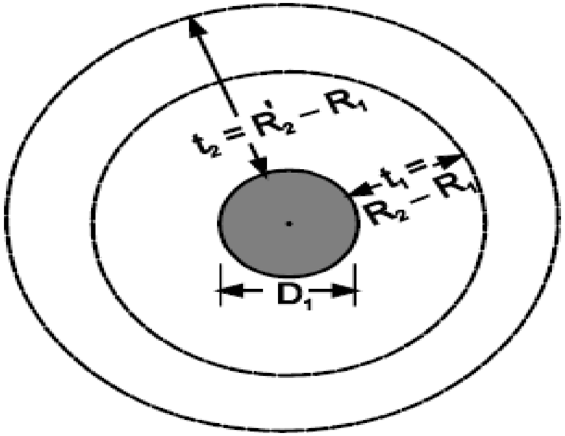


Fig. Ex. 4

Solution : Given : $D_1 = 1.2$ cm, $R_1 = D_1/2 = 1.2/2 = 0.6$ cm, $t_1 = R_2 - R_1 = 1.5$ cm

$R_2 = R_1 + t_1 = 0.6 + 1.5 = 2.1$ cm., $t_2 = t_1 + 0.5\, t_1 = 1.5 + 0.5 \times 1.5 = 2.25$ cm

$R_2' = R_1 + t_2 = 0.6 + 2.25 = 2.85$ cm

$$R_i = \frac{\rho_i}{2\pi l} \ln\left(\frac{R_2}{R_1}\right)$$

$$550 \times 10^6 = \ln\left(\frac{2.1}{0.6}\right) \qquad \ldots(1)$$

Insulation Resistance with additional thickness

$$R_i' = \frac{\rho_i}{2\pi l} \ln\left(\frac{2.85}{0.6}\right) \qquad \ldots(2)$$

By taking ratio of 2 and 1

$$\frac{R_i'}{550 \times 10^6} = \frac{\rho/2\,\pi l}{\rho/2\,\pi l} \times \frac{l_n\,(2.85/0.6)}{l_n\,(2.1/0.6)}$$

$$R_i' = (550 \times 10^6) \times \left(\frac{1.558}{1.252}\right) = 1.24 \times 550 \times 10^6$$

$$= 682 \times 10^6\,\Omega = 682\text{ M}\Omega$$

$$\%\text{ age increase} = \frac{682 - 550}{550} \times 100 = 24\%$$

ELECTRICAL SYSTEM

WORK : *Work is said to be done when a charge is transferred from one point to the other through a potential difference.*

$$\text{Work done} = \text{charge} \times \text{potential difference}$$

$$1\ \text{joule} = 1\ \text{coulomb} \times 1\ \text{volt}$$

$$W = Q.V = I.t.V = I^2.R.t = \frac{V^2}{R}t$$

POWER: *Power is the rate of doing work.*

$$P = \frac{\text{Work done}}{\text{Time}} = V.I = I^2R = \frac{V^2}{R}$$

power is measured in Watt.

1 watt =1 joule /sec

Electric motors, Cranes
Water pumps,lifts, Gyeser
Electric locomotive

ENERGY: *work done is always taken as a measure of the energy used.*

Energy expended = Work done electrically

$$= V\,I\,t = I^2.R.t \quad = \frac{V^2}{R}\text{t} \quad \text{Watt-sec or (joules)}$$

Energy consumed by an electrical circuit is said to be 1 watt-sec when it Utilizes the power of 1 watt for 1 second.

1 Watt-sec = 1 Joule

Board Of Trade (BOT) Unit of energy is kWh.

$$1\ kWh = 1000\ Wh = 1000 \times 3600 = 36 \times 10^5\ Watt\text{-}seconds\ \ or\ Joules$$

1 kWh = 1 Unit

Mechanical Systems

WORK : *Work is said to be done on a body when a force acts on it and the body moves through some distance.*

$$\text{Work done} = \text{Force} \times \text{distance}$$

$$W \quad = F \; d \quad \text{joules}$$

The work done is one joule if a force of 1 Newton moves the body through one meter in the direction of force.

POWER : rate of doing work

$$\text{Power} = \frac{\text{Work Done}}{\text{Time}} = \frac{W}{t} \; \text{joules/sec or watt}$$

ENERGY : Energy *is defined as the capacity to do the work.*

The work done is stored in the body in the form of energy.

Hence work and energy are measured in the same unit i.e. joules.

There are two forms of energy

1. *KINETIC ENERGY : Energy possessed by a body due to its motion.*

$$\text{KE} = \frac{1}{2}\, mass \times (velocity)^2$$

$$= \frac{1}{2} m\, v^2$$

2. *POTENTIAL ENERGY : Energy possessed by a body due to its position.*

$$\text{PE} = mass \times gravitational\ force \times height$$

PE=mgh $= W.h$ *joules*

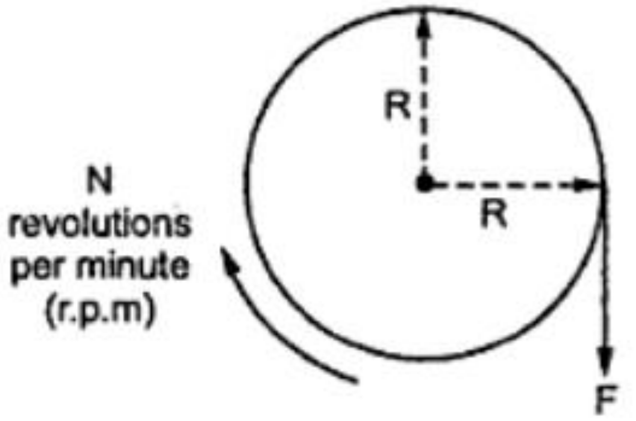
m= mass kg

h= meter

g= $9.81\ m/s^2$

W= weight =mg *1kg=9.81 N*

Relation between Torque and Power



Consider a pulley of radius R and F is the force applied as shown in the Fig.

$$T = F \times R \quad Nm$$

Let speed of pulley is N revolutions per minute. Now work done in one revolution is force into distance travelled in one revolution.

$$d = \text{distance travelled in 1 revolution}$$

$$= 2\pi R$$

$$\therefore \quad W = \text{work done in 1 revolution} = F \times d = 2\pi R F \quad J$$

The time required for a revolution can be obtained from speed N r.p.m.

$$t = \text{time for a revolution} = \frac{60}{N} \quad \text{sec}$$

$$P = \text{power} = \frac{W}{t} = \frac{2\pi R F}{\frac{60}{N}} = \left(\frac{2\pi N}{60}\right) \times (F \times R)$$

$$\boxed{P = T \times \omega}$$

$$T = F \times R = \text{torque in Nm}$$

$$\boxed{\omega = \frac{2\pi N}{60} = \text{angular velocity in rad/sec}}$$

THERMAL SYSTEM

When the body is heated from temperature t_1 to t_2 heat is gained by the body.
When the body is cooled from temperature t_2 to t_1 heat is lost by the body.

The amount of heat gained / lost by the body is

1. directly proportional to its mass
2. directly proportional to change in the temperature
3. depend upon the nature of the material of the body

$Q = s\, m\, (t_2 - t_1)$ *joules* *(sensible heat)*

s- Specific Heat Capacity (Amount of heat energy required to rise the temperature of 1kg of mass of a substance through 1 °C.

J / kg- °C or J / kg-K

for water c= 4186 J/kg- °C

The unit of heat is calories. 1 calorie is the amount of heat required to rais
The temperature of 1g of water by 1°C.
Since heat is a form of energy it can also be measured in joules.
The relation between joule and other units of heat is as given below

1 calorie = 4.186 joules
1 joule =0.2389 calories
1kWh = 860 kcal

Specific latent heat L: when a body is heated beyond its melting point Its solid state gets converted in to liquid state. During this conversion Temperature of the body remains at its melting point.
Heat required to convert a body from solid to liquid state at a constant Temperature is called *specific latent heat of fusion.*

-----liquid state to gaseous state ----*specific latent heat of vaporization*

Let m –mass of the body

L-specific latent heat of the material (J/kg)

Then heat required to change the state of the material without change in its temperature is given by

$$Q = m\,L \quad joules$$

FURNACE
HEATER
BOILER
DISEIL ENGINE

Calorific value : Heat energy can be produced by burning the fuels. The calorific value of a fuel is defined as the amount of heat produced by completely burning unit mass of that fuel.

It is measured in kJ/gram, kJ/kg etc.

$$\boxed{\text{Heat produced in joules} = \text{Mass in kg} \times \text{calorific value in J/kg}}$$

Solid state to liquid state latent heat of fusion or liquefacation

Liquid state to solid state latent heat of freezing or solidification

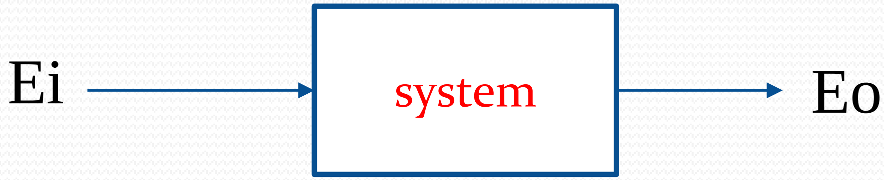
liquid state to vapore state latent heat of vaporization

Gaseas state to liquid state latent heat of condensation

CONVERSION OF ENERGY FROM ONE FORM TO ANOTHER

Energy neither be created nor be destroyed but it can be converted from one form to another while converting from one form to another some of the energy is lost as heat.

Electric motor, water pump, lifts, traction, water heater, Generator, Diesel Engine are the examples of systems



The efficiency can be defined as the ratio of energy output to energy input. It can also expressed as ratio of power output to power input.

Its value is always less than 1. Higher its value, more efficient is the system of equipment. Generally expressed in percentage. Its symbol is η.

$$\boxed{\begin{aligned} \%\,\eta &= \frac{\text{Energy output}}{\text{Energy input}} \times 100 \\ &= \frac{\text{Power output}}{\text{Power input}} \times 100 \end{aligned}}$$

Example 1

In a house, various electrical appliances are put on to the supply as per the following schedule during a day :

(i) 4 fluorescent tubes, each of 40 W, for 6 hours.

(ii) 3 kW electric geyser for 1 hr.

(iii) 750 W electric iron for 40 minutes.

(iv) Other miscellaneous electrical load of 400 W for 3.5 hours.

Estimate the bill for consumption of electrical energy in this house, for a month of 30 days, at a rate of Rs. 2.50 per unit.

Solution : The daily energy consumption is as follows :

Sr. No.	Appliance (1)	Power in kW (2)	Hours/day (3)	Energy consumed kWh/day $(1 \times 2 \times 3)$
1.	4 fluorescent tubes	0.04 kW	06 hrs.	0.96 kWh
2.	1 electric geyser	3 kW	1 hr.	3.00 kWh
3.	1 electric iron	0.75 kW	0.67 hrs.	0.50 kWh
4.	Miscellaneous load	0.4 kW	3.5 hrs.	1.40 kWh
			Total	5.86 kWh

$$\text{Energy consumed per day} = 5.86 \text{ kWh}$$

$$\text{Energy consumed per month} = 5.86 \times 30 = 175.8 \text{ kWh}$$

$\therefore$ Monthly bill for energy consumption at a rate of Rs. 2.50 /unit

$$\text{So for } 175.8 \text{ units} = 175.8 \times 2.50 = \text{Rs. } 439.50/–$$

A belt driven pulley of 0.4 m in diameter rotates at a speed of 4 revolutions per second. The tension in the tight side of the belt is 420N and in the slack side 80N. Calculate

i) The torque on pulley

ii) The power developed

Example 1 : *An electric furnace is used in order to melt 50 kg of tin per hour. Melting temperature of tin is 235 °C and room temperature is 15 °C. Latent heat of fusion for tin is 13.31 kcal/kg. Specific heat of tin is 0.055 kcal/kg°K. If input to furnace is 5 kW, find the efficiency of the furnace.*

Solution : $m = 50$ kg, $t_1 = 15$ °C, $t_2 = 235$ °C, $L = 13.31$ kcal/kg

$C = 0.055$ kcal/kg°K, $P_{in} = 5$ kW

The melting takes place in two steps :

1. Heat required to raise temperature from 15 °C to 235 °C = $m\,C\,\Delta t$
2. Latent heat required to convert solid state to liquid state = mL

$$\therefore \quad H = \text{heat output required} = m\,C\,\Delta t + m\,L$$

$$= 50 \times 0.055 \times (235 - 15) + 50 \times 13.31 = 1270.5 \text{ kcal}$$

Now $\quad 1 \text{ cal} = 4.2 \text{ J}$

$$\therefore \quad H = 1270.5 \times 10^3 \times 4.2 \text{ J} = 5336.1 \text{ kJ}$$

$$\text{time} = 1 \text{ hour} = 3600 \text{ sec}$$

$$\therefore \quad P_{out} = \frac{H\,(\text{output})}{\text{time}} = \frac{5336.1 \times 10^3}{3600} = 1482.25 \text{ W}$$

While $\quad P_{in} = 5$ kW

$$\therefore \quad \%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{1482.25}{5 \times 10^3} \times 100 = 29.645\ \%$$

Example 2 : *An electric pump lifts 12 m^3 of water per minute to a height of 15-m. If its overall efficiency is 60%, find the input power. If the pump is used for 4-hours a day, find the daily cost of energy at Rs. 2.25 per unit.*

$m = 12\ m^3 = 12 \times 1000$ kg as $1\ m^3$ of water = 1000 kg, h = 15 m, $\eta = 60\%$,

time = 1 minute = 60 sec

Energy output = $mgh = 12 \times 1000 \times 9.81 \times 15 = 1.7658 \times 10^6$ J

Energy input = $\dfrac{\text{output}}{\eta} = \dfrac{1.7658 \times 10^6}{0.6} = 2.943 \times 10^6$ J i.e. watt-sec

time for lifting = 1 minute = 60 sec

$\therefore$ $P_{in} = \dfrac{\text{input}}{\text{time}} = \dfrac{2.943 \times 10^6}{60} = 49.05$ kW

For 4 hours, pump consumes $P_{in} \times 4$ = 196.2 kWh

Thus total units per day = 196.2

$\therefore$ Daily cost = 196.2×2.25 = Rs. 441.45

Example 3 : *The effective head of 100 MW power station is 220 m. Station supplies full load for 12 hours a day. The overall efficiency of power station is 86.4 %. Find the volume of water used.*

Solution : h = 220 m , Full load output $P_{out} = 100\ \text{MW} = 100 \times 10^6\ \text{W}$

$$t = 12\ \text{hours},\ \eta = 86.4\ \%,\ P_{out} = 100 \times 10^6\ \text{W i.e. J/sec}$$

$\therefore$ Energy output for full load $= P_{out} \times$ hours per day $= 100 \times 10^6 \times 12 = 1200 \times 10^6$ Wh

$$= 1200 \times 10^6 \times 3600\ \text{J} = 4.32 \times 10^{12}\ \text{J}$$

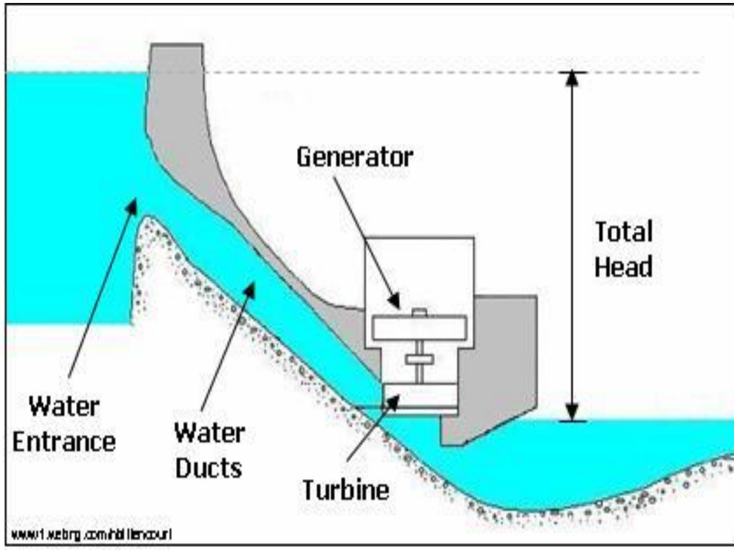
$$\therefore \quad \text{Input energy} = \frac{\text{Energy output}}{\eta} = \frac{4.32\times10^{12}}{0.864} = 5 \times 10^{12}\ \text{J}$$

This is potential energy of water mgh.

$\therefore$ P.E. of water = Input energy supplied

$$\therefore \quad m \times g \times h = 5 \times 10^{12}$$

$$\therefore \quad m = \frac{5\times10^{12}}{9.81\times220} = 2.3167 \times 10^9\ \text{kg}$$



But 1 m^3 of water = 1000 kg

$$\therefore \quad \text{volume of water used} = \frac{2.3167\times10^9}{1000}\ m^3 = 2.3167 \times 10^6\ m^3$$

Example 4 : *An electric furnace is used to melt aluminium. Initial temperature of the solid aluminium is 32 °C and its melting point is 680 °C. Specific heat capacity of aluminium is 0.95 kJ/kg°K, and the heat required to melt 1 kg of aluminium at its melting point is 450 kJ. If the input power drawn by the furnace is 20 kW and its overall efficiency is 60%, find the mass of aluminium melted per hour.*

Solution : $t_1 = 32\ °C$, $t_2 = 680\ °C$, $C = 0.95$ kJ/kg°K, $P_{in} = 20$ kW, $\eta = 60\% = 0.6$

Heat required to melt 1 kg of Al at melting point is 450 kJ is the information related to **Latent Heat.**

Total heat = sensible heat + Latent heat = $m\ C\ \Delta t + mL$

Where $L = 450$ kJ/kg as given

$\therefore$ Total output energy $= m\ [C\ \Delta\ t + L] = m\ [0.95 \times (680 - 32) + 450]$

$= m \times 1.0656 \times 10^3$ kJ ... Note both C and L in kJ

$P_{in} = 20$ kW

and time = 1 hour as mass of Al per hour to be obtained

$\therefore$ Input energy $= P_{in} \times \text{time} = 20 \times 10^3 \times 3600 = 72 \times 10^6$ J

$\therefore$ Output energy = Input $\times\ \eta = 72 \times 10^6 \times 0.6 = 43.2 \times 10^6\ J = 43.2 \times 10^3$ kJ

Equating with total output energy required,

$m \times 1.0656 \times 10^3 = 43.2 \times 10^3$

$\therefore$ m = 40.5405 kg aluminium per hour will be melted

Example 5 : *In a thermal generating station the heat energy obtained by burning 1 kg of coal is 16,000 kJ. Find the mass of coal required to get an output electrical energy of 1 kWh from the station, if its overall efficiency is 18 %.*

Solution : m = 1 kg, Heat energy = 16000 kJ, output = 1 kWh, η = 18 %

$$\text{Calorific value of coal} = \text{heat energy/kg} = 16000 \text{ kJ/kg}$$

$$\therefore \quad \text{Total input energy} = m \times 16000 \times 10^3 \text{ J} \qquad \ldots (1)$$

$$m = \text{mass of coal burned}$$

$$\text{Output required} = 1 \text{ kWh} = 1 \times 10^3 \text{ Wh}$$

$$= 1 \times 10^3 \times 3600 \text{ W-sec i.e. J}$$

$$\therefore \quad \text{Input required} = \frac{\text{Output energy in J}}{\eta} = \frac{1 \times 10^3 \times 3600}{0.18}$$

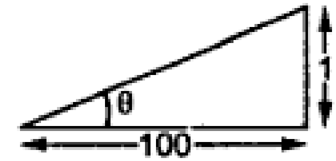
$$= 20 \times 10^6 \text{ J} \qquad \ldots (2)$$

Equating (1) and (2),

$$m \times 16000 \times 10^3 = 20 \times 10^6$$

$$\therefore \quad m = 1.25 \text{ kg} \qquad \ldots \text{mass of coal required}$$

Example 6 : *Calculate the current required by a 1500 V D.C. locomotive when driving a total load of 100×10^3 kg at 25 km per hour up on incline of 1 in 100. Assume tractive resistance of 0.069 N/kg and efficiency of motor's gearing as 70 %.*



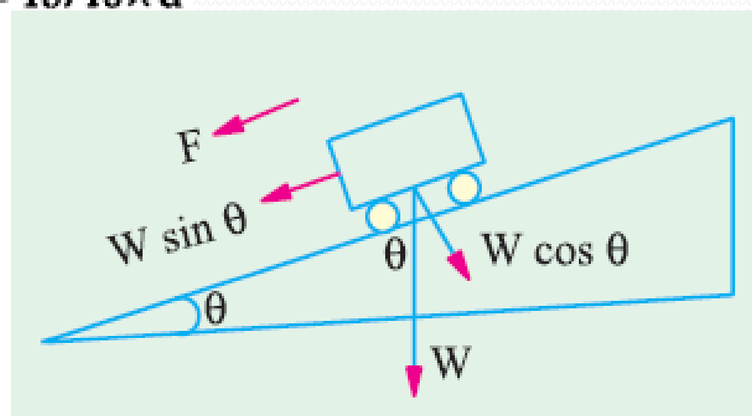
$$\sin\theta = \tan\theta = \frac{1}{100} = 0.01$$

$$W\sin\theta = 100\times10^3\times0.01 = 1000\text{ kg} = 9810\text{ N}$$

$$\text{Track resistance} = 0.069\text{ N/kg} = 0.069\times100\times10^3 = 6900\text{ N}$$

$$\text{Total resistance} = 9810 + 6900 = 16710\text{ N}$$

$$\text{Work done} = \text{Force}\times\text{distance travelled in 1 sec} = 16710\times d$$



Now $$\text{speed} = 25\text{ km/hr}$$

$$\therefore \quad d = \frac{25\times10^3}{3600} = 6.944\text{ m in 1 sec}$$

$$\therefore \quad \text{work done} = 16710\times6.944 = 116040.924\text{ J}$$

$$\therefore \quad P_{out} = \frac{W}{\text{time}} = \frac{116040.924}{1\text{ sec}} = 116040.924\text{ W}$$

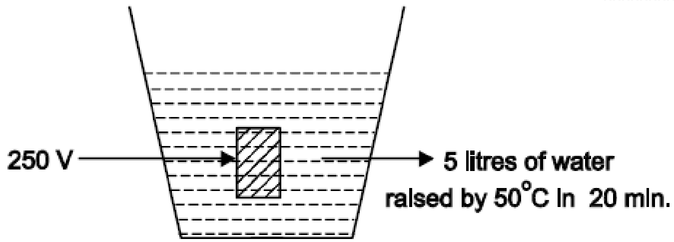
$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_{gear}} = \frac{116040.924}{0.7} = 165.77274\times10^3\text{ W}$$

But $$P_{in} = V\times I$$

$$\therefore \quad I = \frac{P_{in}}{V} = \frac{165.77274\times10^3}{1500} = 110.5151\text{ A}$$

An immersion heater supplied at 250 V D.C. is placed in a bucket containing 5 litres of water. The temperature of water goes up by 50°C in 20 min. The water equivalent of the bucket is 500 g. Calculate the heater resistance. Specific heat of water is 4180 J/kg K and heat lost due to radiation is negligible.

Solution : Given : V = 250 V, Specific heat of water = 4180 J/kg K, Water equivalent of the bucket = 500 g.



$$\text{Sensible heat (H)} = m \cdot s \cdot (t_2 - t_1)$$

$$\text{Output energy} = (5 + 0.5) \times 4180 \times 50$$

$$= 1149500 \text{ J}$$

$$= \text{Input energy (as radiation losses are absent)}$$

$$\text{Input power} = \frac{\text{Sensible heat}}{\text{Time}} = \frac{1149500}{20 \times 60} = 957.916 \text{ W}$$

If R is the heater resistance, $\frac{250^2}{R} = 957.916$

$$\therefore \quad R = 65.246\ \Omega$$

Example 8

An electric furnace is used to melt 1000 kg of tin per hour. Input to the furnace is 50 kW. Melting point of the tin is 235°C. Latent heat of liquification for tin is 13.3 kcal/kg. Specific heat is 231 J/kg-K and the initial temperature is 15°C. Calculate the efficiency of the furnace.

Solution : Given : Input = 50 kW, m = 1000 kg, Specific heat = 231 J/kg-K, t_1 = 15°C,

L = 13.3 kcal/kg

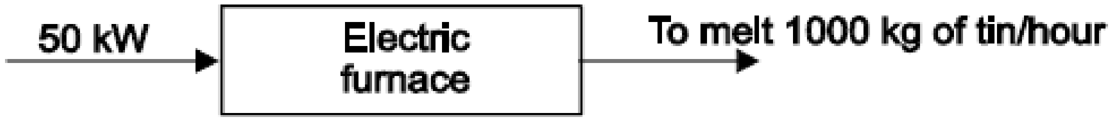


Fig. 1.10

$$\text{Total output energy in one hour} = \text{Sensible heat} + \text{Latent heat of liquification}$$

$$= ms\Delta t + mL$$

$$= 1000 \times 231 \times (235 - 15) + 1000 \times 13.3 \times 4186$$

$$= 106.49 \times 10^6 \text{ J/hr}$$

$$\text{Output power} = \frac{\text{Output energy}}{\text{time}}$$

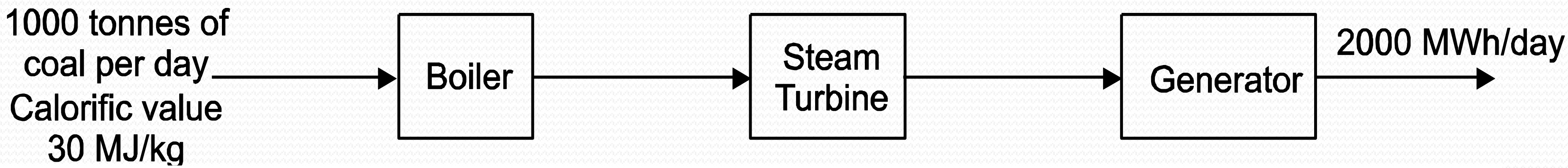
$$= \frac{106.49 \times 10^6}{3600} \text{ J/s or watt}$$

$$= 29580.55 \text{ watt} = 29.58 \text{ kW}$$

$$\% \text{ efficiency} = \frac{\text{output}}{\text{input}} \times 100 = \frac{29.58}{50} \times 100 = 59.16\%$$

Example 3

A boiler steam turbine generator system consumes 1000 tonnes of coal and generates 2000 MWh energy in a day. The calorific value of the coal is 30 MJ/kg. Find overall efficiency of the system.



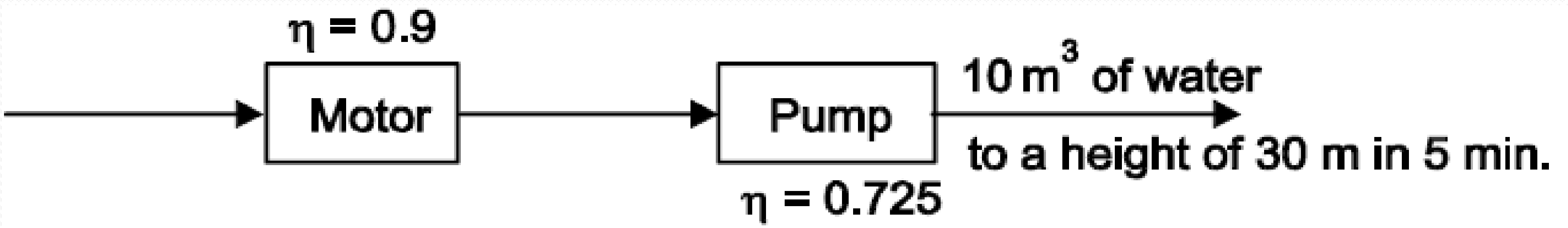
Solution : Given : Output = 2000 MWh, calorific value of coal = 30 MJ/kg, m = 1000 tonnes.

$$
\begin{aligned}
\text{Output of the system} &= 2000 \text{ MWh} \\
\text{Input to the system} &= \text{mass} \times \text{calarotic value} \\
&= (1000 \times 1000) \times 30 \text{ MJ} \\
&= \frac{1000 \times 1000 \times 30}{3600} \text{ MWh} \\
&= 8333.33 \text{ MWh}
\end{aligned}
$$

$$\text{Overall efficiency of the system} = \frac{2000}{8333.33} = 0.24 \text{ x } 100 = 24\%$$

Example 1

A pump driven by an electric motor lifts 10 m^3 of water to a height of 30 m, in 5 min. Efficiency of the pump and the motor are 72.5% and 90% respectively. Calculate power input to the motor and the energy consumed by motor if it works for 4 hours.



Solution : Given : $h = 30$ m, $t = 5$ min, $\eta_p = 72.5\%$, $\eta_m = 90\%$.

$$\text{Output energy in 5 min.} = mgh = 10 \times 1000 \times 9.81 \times 30 \text{ N-m}$$

$$= 2.943 \times 10^6 \text{ N-m}$$

(Note that 1 m^3 of water has a mass of 1000 kg)

$$\text{Output power} = \frac{\text{Output power}}{t} = \frac{2.943 \times 10^6}{5 \times 60} = 9810 \text{ W}$$

$$\text{Input power} = \frac{\text{Output Power}}{t} = \frac{9810}{0.725 \times 0.9} = \frac{1}{1000}$$

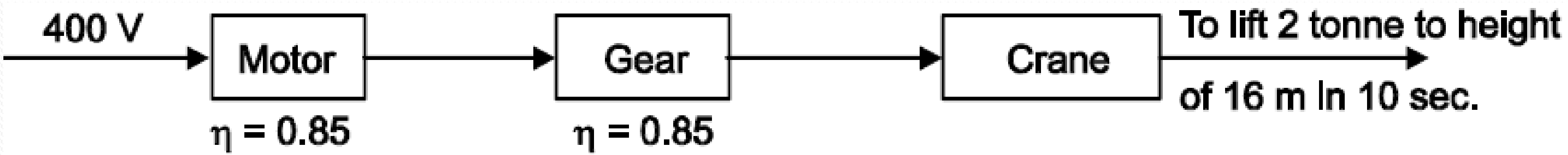
$$= 15.034 \text{ kW}$$

$$\text{Energy consumed in 4 hours} = 15.034 \times 4$$

$$= 60.137 \text{ kWh}$$

Example 2

Find the current and energy input to a crane driving motor when raising a load of 1 tonne through a height of 16 m in 10 sec. Motor efficiency and gear efficiency are 85% and 60% respectively. Supply voltage is 400 V D.C.



Solution : Given : Motor efficiency = 85%, Gear efficiency = 60%, V = 400 V, h = 16 m, t = 10 sec.

$$\text{Output energy} = mgh = 1000 \times 9.81 \times 16 \text{ N-m}$$

$$\begin{aligned}
\text{Input energy} &= \frac{\text{Output energy}}{\eta_{gear} \times \eta_{motor}} = \frac{1000 \times 9.81 \times 16}{0.85 \times 0.6} \\
&= 307764.7 \text{ W.sec.} \\
&= 0.0855 \text{ kWh}
\end{aligned}$$

$$\begin{aligned}
\text{Input current} &= \frac{\text{Input energy}}{t \times \text{voltage}} = \frac{307764.7}{10 \times 400} \\
&= 76.94 \text{ A}
\end{aligned}$$

Example 3

An electric motor is driving a train weighing 100 thousand kilograms upon an inclined track of 1 in 100 at a speed of 60 km/h. The frictional force of tracks is 10 kg per 1000 kg of its weight. If the motor operates on 11 kV, find the current taken by the motor assuming the overall efficiency of the system as 70%.

Solution : Given : $\eta = 70\%$, $V = 11$ kV.

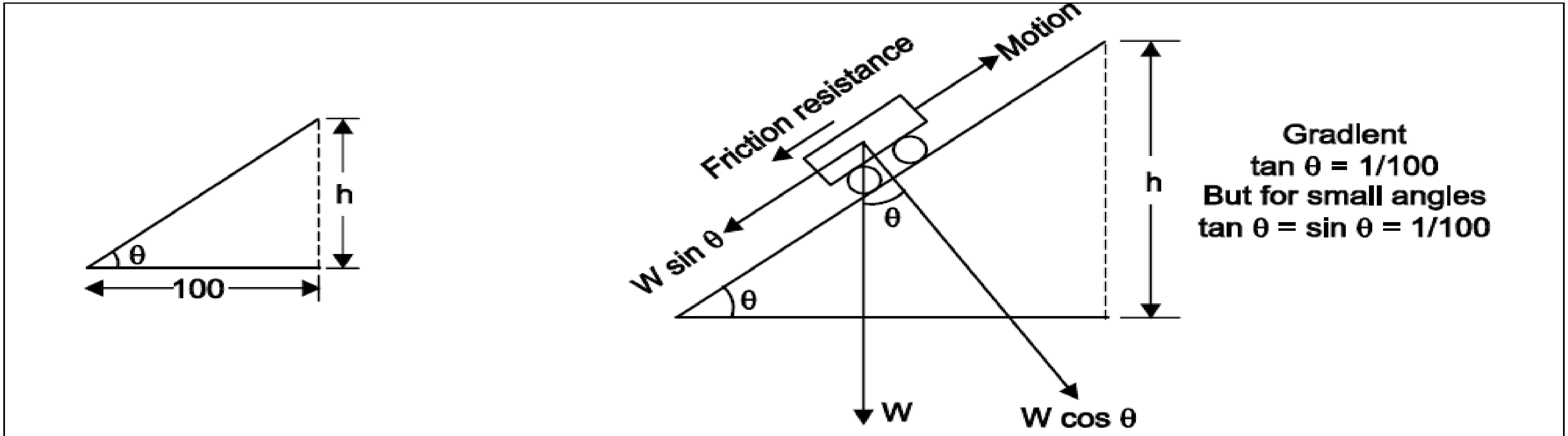


Fig.

Force trying to pull the train downward is W sin θ

$$= 100 \times 1000 \times 9.81 \times \frac{1}{100}$$

$$= 9810 \text{ N}$$

$$\text{Frictional force} = 10 \times 9.81 \times 100$$

$$= 9810 \text{ N}$$

$$\text{Total opposing force} = 9810 + 9810$$

$$= 19620 \text{ N}$$

Work done against this force at a speed of 60 km/hr.

$$= \frac{60 \times 1000}{3600} \text{ m/sec. is}$$

$$\text{o/p power} = \text{force} \times \text{distance/sec} = 19620 \times 60 \times \frac{1000}{3600} = 327000 \text{ watts} = 327 \text{ kW}$$

$$\text{Input power} = \frac{\text{O/p Power}}{\eta} = \frac{327}{0.7}$$

$$= 467.1428 \text{ kW}$$

But, $\quad \text{Input} = V \times I$

$$\therefore \quad I = \frac{467.1428}{11} = 42.467 \text{ A}$$

- Example 1

A bucket contains 15 litres of water at 20°C. A 2 kW immersion heater is used to raise the temperature of water to 95°C. The overall efficiency of the process is 90% and the specific heat capacity of water is 4187 J/kg K. Find the time required for the process.

Solution : Given : m = 15 kg (1 litre = 1 kg),

Specific heat capacity of water (s) = 4187 J/kg K,

$t_1 = 20°C$, $t_2 = 95°C$, Input power = P_{in} = 2 kW, efficiency = η = 90%.

Energy required to heat the water is the output energy.

$$\therefore \quad \text{Output energy} = m\ s\ \Delta t$$
$$= 15 \times 4187 \times (95 - 20)$$
$$= 4.7103 \times 10^6\ \text{J}$$
$$\text{Input energy} = \frac{\text{Output}}{\eta}$$
$$= \frac{4.7103 \times 10^6}{0.9} = 5.2337 \times 10^6\ \text{J}$$
$$P_{in} = \frac{\text{Input in J}}{\text{Time in sec.}}$$
$$\therefore \quad 2 \times 10^3 = \frac{5.2337 \times 10^6}{\text{Time}}$$
$$\therefore \quad \text{Time} = 2616.875\ \text{sec.}$$
$$= 43.614\ \text{minutes}$$

Example 2

An electric furnace is used to melt aluminium. Initial temperature of the solid aluminium is 32°C and its melting point is 680°C. Specific heat capacity of aluminium is 0.95 kJ/kg°K and the heat required to melt 1 kg of aluminium at its melting point is 450 kJ. If the input power drawn by the furnace is 20 kW and its overall efficiency is 60%, find the mass of aluminium melted per hour.

Solution : Given : $t_1 = 32°C$, $t_2 = 680°C$,

Specific heat capacity of aluminium (s) = 0.95 kJ/kg K, $P_{in} = 20$ kW,

efficiency = 60% = 0.6, Latent heat (L) = 450 kJ/kg

Heat required to melt 1 kg of aluminium at melting point is 450 kJ is the information related to latent heat.

$$\text{Total heat} = \text{Sensible heat} + \text{Latent heat}$$

$$= m\, s\, \Delta t + mL$$

where, $\quad \text{Latent heat} = 450 \text{ kJ/kg} \quad$ (given)

$$\therefore \quad \text{Total output energy} = m\,[s\,\Delta t + L]$$

$$= m\,[0.95 \times (680 - 32) + 450]$$

$$= m \times 1.0656 \times 10^3 \text{ kJ}$$

$$P_{in} = 20 \text{ kW}$$

and time = 1 hour as mass of Al per hour is to be obtained.

$\therefore$ Input energy = $P_{in} \times$ time = $20 \times 10^3 \times 3600 = 72 \times 10^6$ J

$\therefore$ Output energy = input $\times \eta = 72 \times 10^6 \times 0.6 = 43.2 \times 10^6$ J = 43.2×10^3 kJ

Equating with total output energy required,

$$\begin{aligned} m \times 1.0656 \times 10^3 &= 43.2 \times 10^3 \\ m &= 40.5405 \text{ kg} \end{aligned}$$

Example 3

In a thermal generating station, the heat energy obtained by burning 1 kg of coal is 16,000 kJ. Find the mass of coal required to get an output electrical energy of 1 kWh from the station, if its overall efficiency is 18%.

Solution : Given : m = 1 kg, Heat energy = 16000 kJ, Output = 1 kWh, efficiency = η = 18%.

Calorific value of coal = Heat energy/kg = 16000 kJ/kg.

$$\text{Total input energy} = m \times 16000 \times 10^3 \text{ J} \qquad \ldots\text{(i)}$$

$$m = \text{mass of coal burned}$$

$$\text{Output required} = 1 \text{ kWh} = 1 \times 10^3 \text{ Wh}$$

$$= 1 \times 10^3 \times 3600 \text{ W sec. i.e. J}$$

$$\text{Input required} = \frac{\text{Output energy}}{\eta} \text{ in J.}$$

$$= \frac{1 \times 10^3 \times 3600}{0.18}$$

$$= 20 \times 10^6 \text{ J} \qquad \ldots\text{(ii)}$$

Equating (i) and (ii),

$$\therefore \quad m \times 16000 \times 10^3 = 20 \times 10^6$$

$$m = 1.25 \text{ kg}$$

Example 1

An electric pump lifts 12 m^3 of water per minute to a height of 15 m. If its overall efficiency is 60%, find the input power. If the pump is used for 4 hours a day, find the daily cost of energy at Rs. 2.25 per unit.

Solution : Given : m = 12 m^3 = 12 × 1000 kg (1 m^3 = 1000 kg), h = 15 m, η = 60%, time = 1 minute = 60 sec.

$$\begin{aligned}
\text{Energy output} &= m.g.h \\
&= 12 \times 1000 \times 9.81 \times 15 \\
&= 1.7658 \times 10^6 \text{ J} \\
\text{Energy input} &= \frac{\text{Output}}{\eta} \\
&= \frac{1.7658 \times 10^6}{0.6} \\
&= 2.943 \times 10^6 \text{ J} \\
\text{Time for lifting} &= 1 \text{ min.} = 60 \text{ sec.} \\
P_{in} &= \frac{\text{Input}}{\text{Time}} \\
&= \frac{2.943 \times 10^6}{60} = 49.05 \text{ kW} \\
\text{For 4 hours, pump consumes } P_{in} \times 4 &= 196.2 \text{ kWh} \\
\text{Thus, total units per day} &= 196.2 \\
\text{Daily cost} &= 196.2 \times 2.25 \\
\therefore \quad &= 441.45 \text{ Rs.}
\end{aligned}$$

Example 2

The effective head of 100 MW power station is 220 m. Station supplies full load for 12 hours a day. The overall efficiency of power station is 86.4%. Find the volume of water used.

Solution : Given : $h = 220$ m, $P_{out} = 100$ MW $= 100 \times 10^6$ W, $t = 12$ hours, $\eta = 86.4\%$

$$\therefore \quad \text{Energy output for full load} = P_{out} \times \text{Hours per day}$$
$$= 100 \times 10^6 \times 12$$
$$= 1200 \times 10^6 \text{ Wh}$$
$$= 1200 \times 10^6 \times 3600 \text{ J}$$
$$= 4.32 \times 10^{12} \text{ J}$$

$$\therefore \quad \text{Input energy} = \frac{\text{Energy output}}{\eta}$$
$$= \frac{4.32 \times 10^{12}}{0.864} = 5 \times 10^{12} \text{ J}$$

This is the potential energy of water (mgh).

$$\therefore \quad \text{P.E. of water} = \text{Input energy supplied}$$
$$\therefore \quad m \times g \times h = 5 \times 10^{12}$$
$$\therefore \quad m = \frac{5 \times 10^{12}}{9.81 \times 220} = 2.3167 \times 10^9 \text{ kg}$$

But, 1m^3 of water $= 1000$ kg

$$\therefore \quad \text{Volume of water used} = \frac{2.3167 \times 10^9}{1000} \text{ m}^3 = 2.3167 \times 10^6 \text{ m}^3$$

Batteries

Cell: A cell is a basic element of a battery. It consists of two suitable dissimilar metallic plates, called 'electrodes' immersed into a liquid or paste of a certain chemical material, called as 'electrolyte'. The chemical reactions between the electrodes and the electrolyte create a potential difference across the two electrodes. Thus, a cell is an electrochemical device that transforms chemical energy into electrical energy.

Types of Cells:

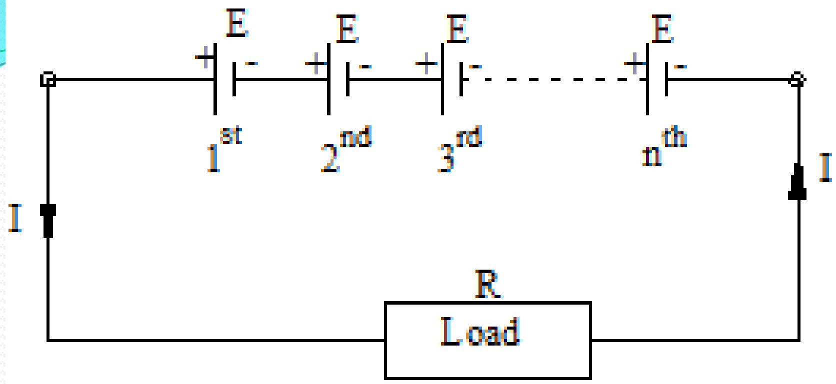
A primary cell: The cell, which supplies the energy only once until it is fully discharged is called as the primary cell. It is not rechargeable and hence can be used only once. A Leclanche cell, Voltaic cell, Daniel cell, carbon-zinc cell, zinc chloride cell, silver oxide cell etc. are the examples of primary cells.

A secondary cell: The cell, which can be recharged and used several times is called as a secondary cell. Secondary cells can be acid cells or alkaline cells. A lead-acid cell is a secondary acid cell whereas, a nickel-iron cell and a nickel-cadmium cell are the secondary alkaline cells.

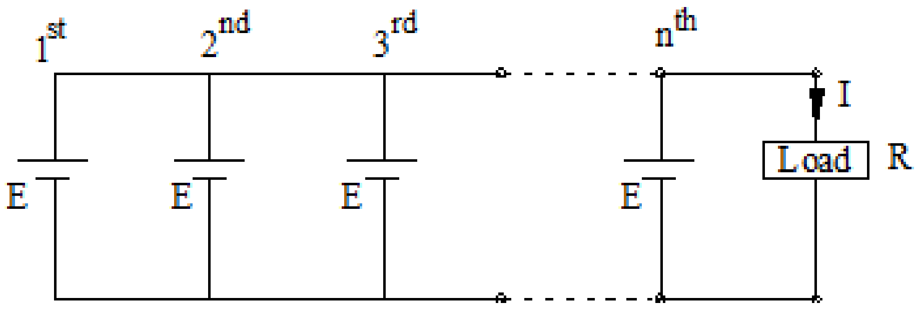
Battery:

A battery consists of a number of identical cells connected together inside a container. As a single cell can develop very small potential difference, the appropriate interconnection of many cells can give a sufficient value of potential difference, which can be useful for practical applications. The cells in a battery may be connected in series, parallel or series-parallel combinations depending on what voltage and current output is required from a battery. The series combination of cells increases the voltage whereas; the parallel combination increases the current delivering capacity.

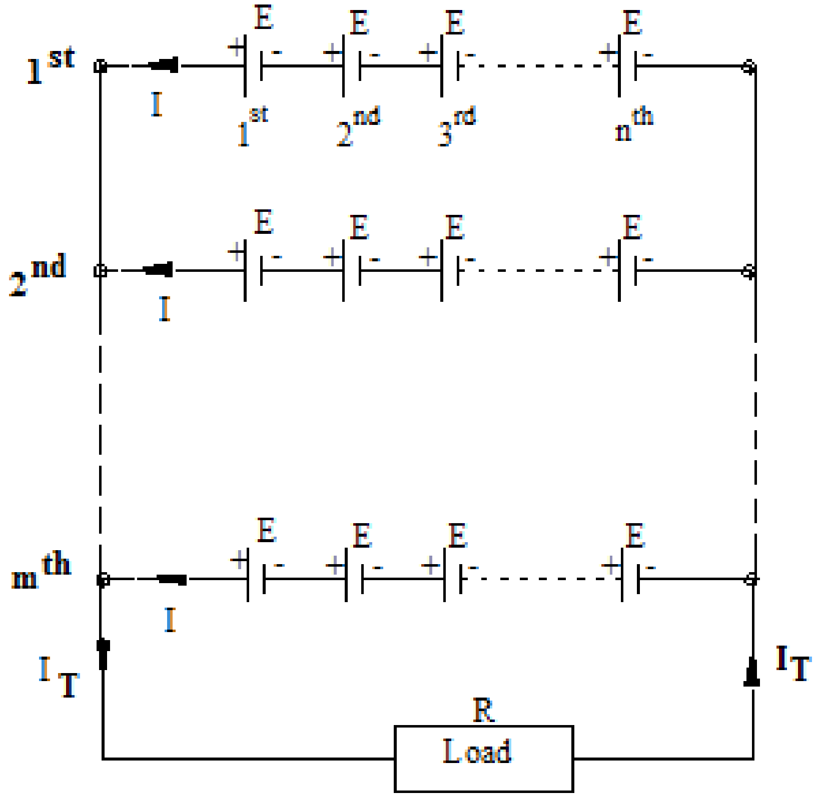
Series Grouping



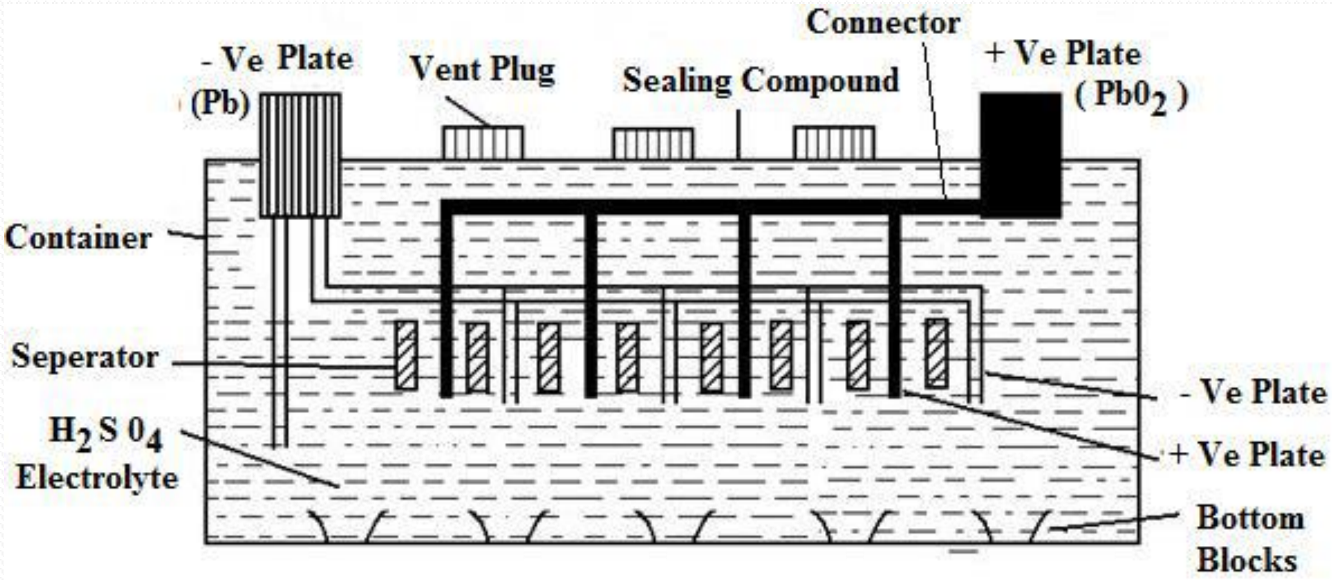
Parallel Grouping



Series-Parallel Grouping



Lead Acid battery



1. Plates

The plates, which are actually electrodes are
a) Anode: Lead Peroxide (PbO_2) of Chocolate brown colour
b) Cathode: Spongy Lead (Pb) with slate grey colour

2. Electrolyte: Aqueous solution of Sulphuric acid (H_2SO_4).

3. Separators: The separators are used to separate positive and negative plates, preventing them to come in contact with each other. These are made of either specially treated wood or perforated rubber. The common separator is wood, since it is the cheapest of all separators.

4. Container : The container is made of vulcanized rubber, or glass. Glass containers are normally used for light duty work and rubber container for portable work. Entire assembly of plates along with the solution is placed in the container.

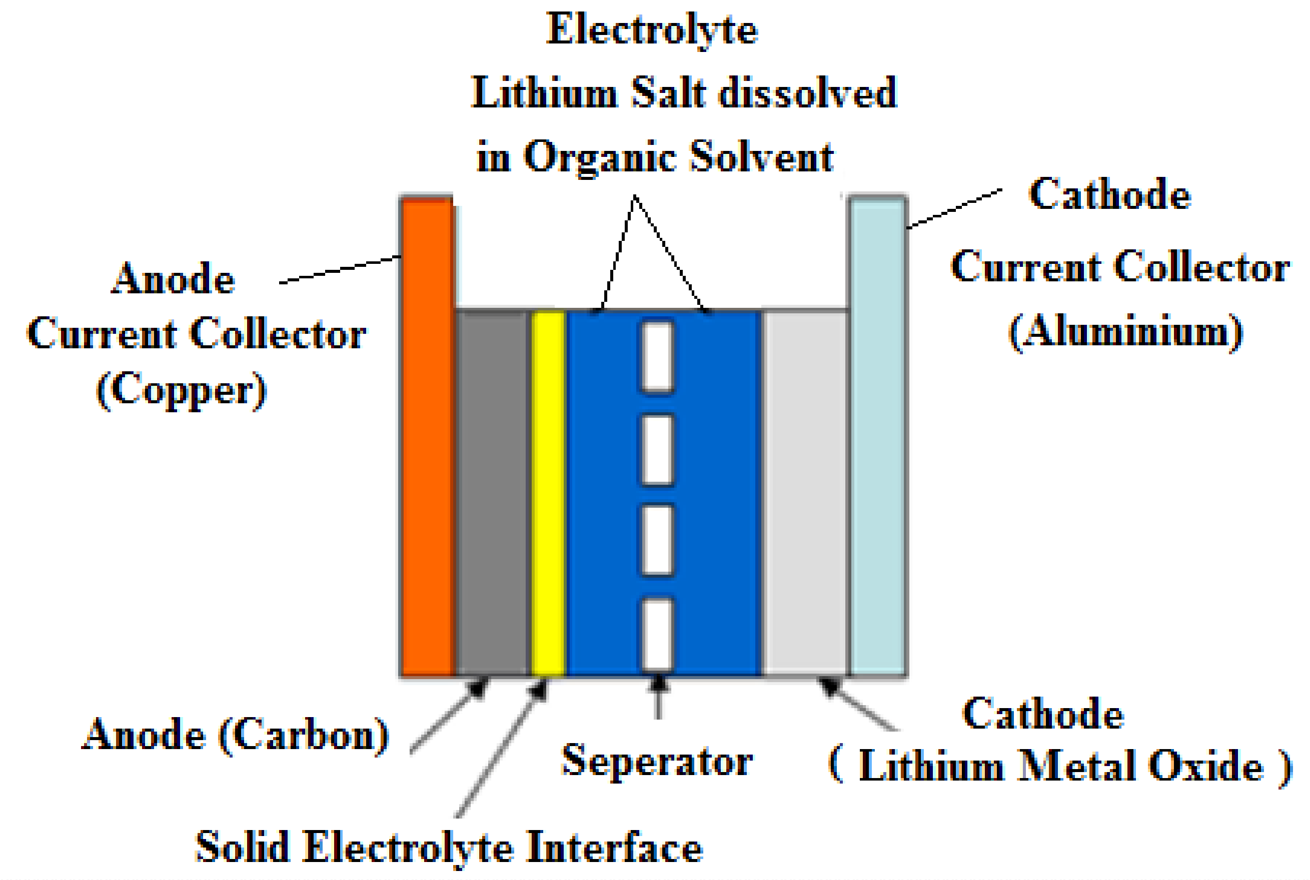
5. Bottom Blocks: These are fitted at the bottom of the container to prevent short circuiting of cell due to the active material fallen from the plates.

6. Plate connector : Separate connectors are used to connect all positive plates to one bar and negative plates to another common bar

7. Cover with vent Plug :The covers are generally of vulcanized rubber. A hole is provided for pouring the electrolyte and this can be closed by a screwed cap. The cap is provided with minute holes for gases to escape. This is also known as 'Vent Cap'. The function of the vent cap is to allow the escape of the gases.

Lithium Ion battery

The lithium ion battery was introduced in the early 1990s. This batteries consist of largely four main components: cathode, anode, electrolyte, and separator.



a) Cathode: A Positive electrode is made with Lithium Cobalt Oxide ($LiCoO_2$ has a current collector made of thin aluminum foil.

b) Anode: A negative electrode made with specialty carbon has current collector of thin copper foil.

c) Separator: It is a fine porous polymer film.

d) Electrolyte: Lithium salt in an organic solvent. Electrolyte is selected in such a way that there should be an effective transport of Li-ion to the cathod during discharge. Type of conductivity is ionic in nature rather than electronic.