BEE, Unit No: 05
DC Circuits
Solved Networks (Numericals)

*Contents:

1. Series-Parallel Combination of resistances & star delta Conversion
2. Kirchhoff's Laws
3. Superposition Theorem
4. Thevenins Theorem

1. Series-Parallel combination of resistances and star Delta Conversion Technique

While dealing with various complex electric networks, it is often required to reduce the network during solution procedure. This task becomes easy, if one knows how to convert given star arrangement of resistances into equivalent delta arrangement and vice-versa.

[Note: Remember that by using star-delta conversion technique, the complex network has to be reduced to a single equivalent resistance between any two asked terminals]

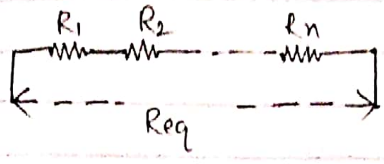
Steps to be followed while solving complex networks using star-delta conversion technique

1. The two terminals between which we have to find equivalent resistance should remain as it, till we get single equivalent resistance between them
2. Observe the network carefully. If any series/parallel arrangement of resistances is observed, find out total series or parallel resistance
3. Again redraw the network and observe it carefully
4. If star arrangement of resistances is observed, convert into delta or vice-versa.
5. Repeat step no. 2 [Step no. 4 may need to be repealed in case of more complex network]
6. Finally reduce the network to get single equivalent resistance between the desired two terminals.

PTO...

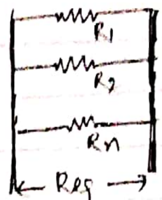
Series- Parallel Combination of resistances

Series Combination



$R_{eq} = R_1 + R_2 + \cdots + R_n$

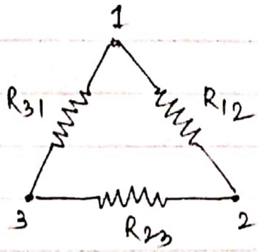
Parallel combination



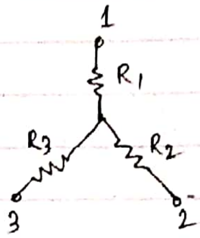
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Star - Delta convesion Technique

1. Delta to star



$\Rightarrow$

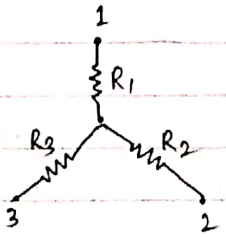


$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

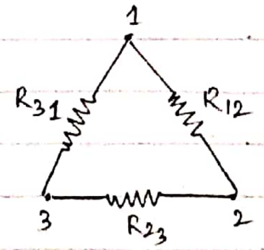
$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

2. Star to Delta



$\Rightarrow$

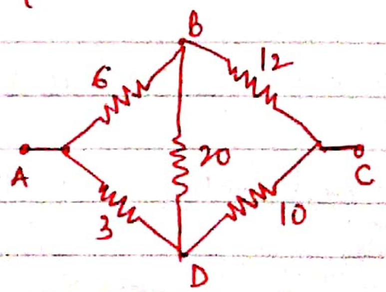


$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

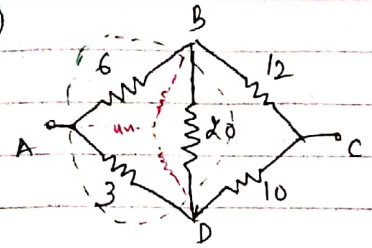
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

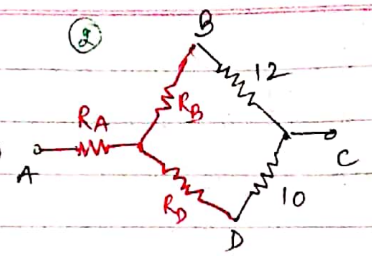
Q1. Find the equivalent resistance between terminals A and C



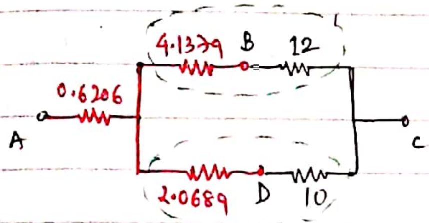
Solution.

Converting delta ABD into star

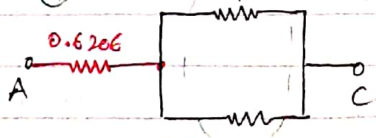
① 

② 

$$R_A = \frac{6\times 3}{6+3+20} = 0.6206\,\Omega \quad R_B = \frac{6\times 20}{6+3+20} = 4.1379\,\Omega, \quad R_D = \frac{3\times 20}{6+3+20} = 2.0689\,\Omega$$

③ 

④ $4.1379 + 12 = 16.1379\,\Omega$



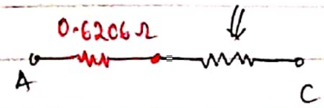
$2.0689 + 10 = 12.0689$

$12\,\Omega$ and $4.1379\,\Omega$ are in series

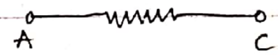
$2.0689\,\Omega$ & $10\,\Omega$ are in series

$16.1379\,\Omega$ and $12.0689\,\Omega$ are connected in parallel

⑤ $$\frac{16.1379\times 12.0689}{16.1379+12.0689} = 6.905\,\Omega$$



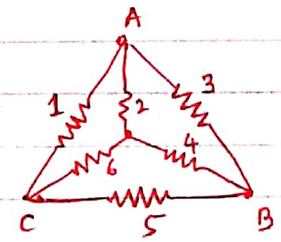
⑥ $0.6206 + 6.905 = 7.525\,\Omega$



0.6206 and 6.905 are in series

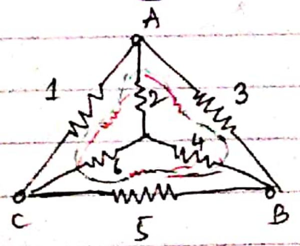
$$\boxed{R_{AC} = 7.525\,\Omega}$$

Q 2 Find the resistance between terminals B and C



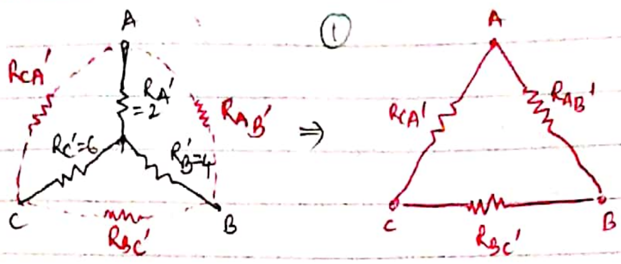
Solution:

Converting inner star into delta



Consider only inner star

①



$$R_{AB}' = R_A' + R_B' + \frac{R_A' R_B'}{R_C'}$$

$$= 2 + 4 + \frac{2\times4}{6}$$

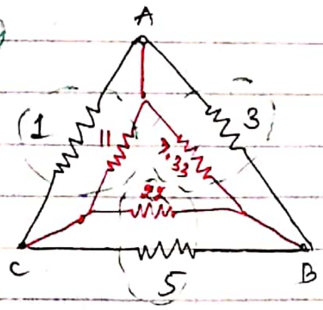
$$= 7.33\,\Omega$$

$$R_{BC}' = R_B' + R_C' + \frac{R_B' R_C'}{R_A} = 4 + 6 + \frac{4\times6}{2} = 22\,\Omega$$

$$R_{CA}' = R_C' + R_A' + \frac{R_C' R_A'}{R_B'} = 6 + 2 + \frac{6\times2}{4} = 11\,\Omega$$

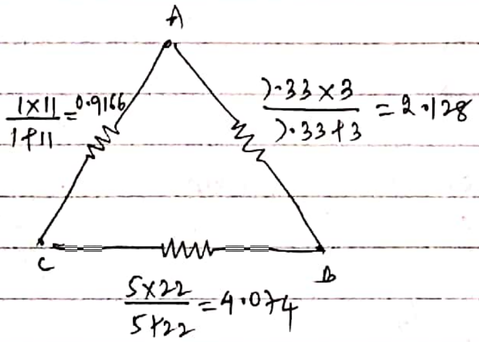
Consider whole network

②

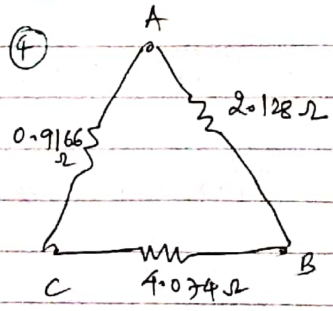


$7.33\,\Omega$ & $3\,\Omega$, $5\,\Omega$ and $22\,\Omega$, $11\,\Omega$ & $1\,\Omega$ are connected in parallel. seperately

③

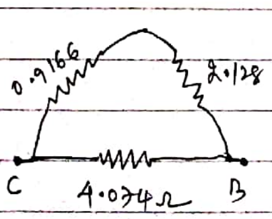


④

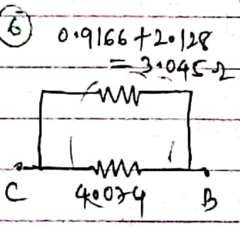


As we have to find equivalent resistance between B & C, resistances 0.9166 and $2.128\,\Omega$ ~~are~~ will be in series and this series combination will be in parallel with $4.074\,\Omega$

⑤

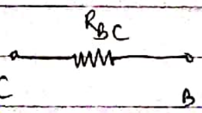


⑥



3.045 and 4.074 are connected in parallel

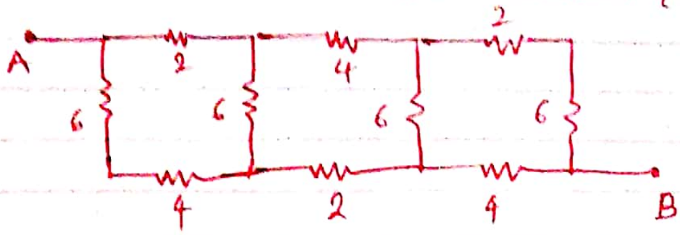
⑦



$$R_{BC} = \frac{3.045 \times 4.074}{3.045 + 4.074} = \underline{\underline{1.742\,\Omega}}$$

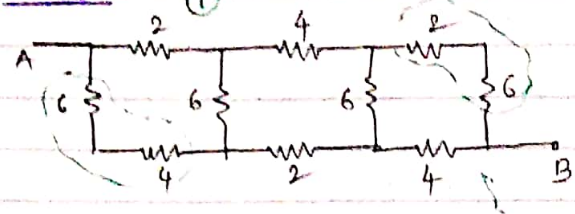
(9)

Q3 Find the resistance between terminals A & B



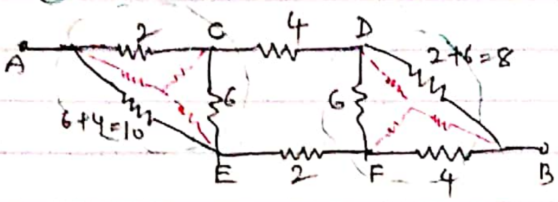
Solution:

①



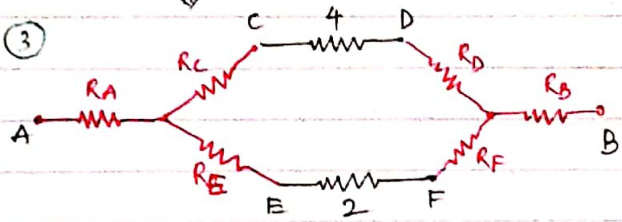
6Ω and 4Ω are in series
2Ω and 6Ω are in series

②



Converting delta ACE and delta DFB into star

③

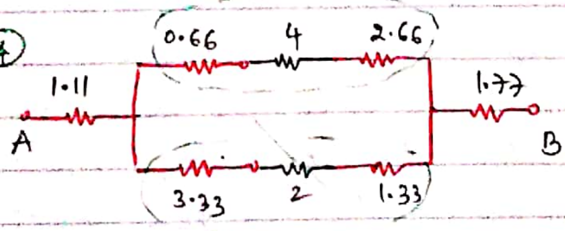


$$R_A = \frac{2\times10}{2+6+10} = 1.11\Omega \qquad R_D = \frac{6\times8}{6+8+4} = 2.66\Omega$$

$$R_C = \frac{2\times6}{2+6+10} = 0.66\Omega \qquad R_F = \frac{4\times6}{6+8+4} = 1.33\Omega$$

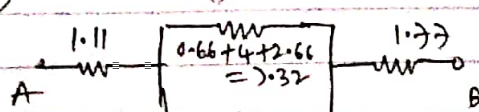
$$R_E = \frac{6\times10}{2+6+10} = 3.33\Omega \qquad R_B = \frac{4\times8}{6+8+4} = 1.77\Omega$$

④



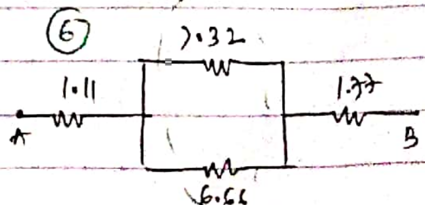
0.66, 4 and 2.66 are in series
3.33, 2 & 1.33 are in series

⑤



3.33+2+1.33 = 6.66

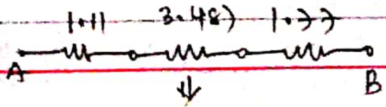
⑥



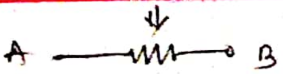
7.32 & 6.66 are in parallel

$$\frac{7.32\times6.66}{7.32+6.66} = 3.487\Omega$$

⑦



⑧



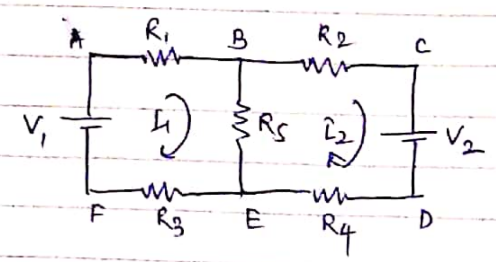
$$\underline{R_{AB} = 6.367\Omega}$$

(8)

2. Kirchhoff's Laws

Solution of the network using Kirchhoffs laws.

Only loop current method has to be used to find out current flowing through a particular resistance.



Consider the network as shown above.

Steps to be followed while getting solution using Kirchoffs ~~laws~~ laws.

1. Observe the network carefully.
2. Consider no of loop currents. In above network two individual loops are observed. Hence two loop currents are considered.
3. As generalized method is to be used consider clockwise direction for considered two loop currents. (no. of loop currents = no. of Individual loops)
4. Write voltage equations
5. Solve the equations by using calculator.

Generalized method

The voltage equations can be written as

$$R_{11} I_1 + R_{12} I_2 = V_1 \quad -(I)$$

$$R_{21} I_1 + R_{22} I_2 = V_2 \quad -(II)$$

where (I) R_{11} & R_{22} are the values obtained by adding all resistances present in loop 1 and loop 2 respectively.

(II) R_{12} and R_{21} are the values of the resistances common to first and second loop, second and first loop respectively.

(III) V_1 and V_2 are the source values. It will be considered as positive ~~Hence~~ when the current approaches negative terminal of battery first. It will be considered negative when the loop current approaches positive terminal first.

Hence the voltage equations will be

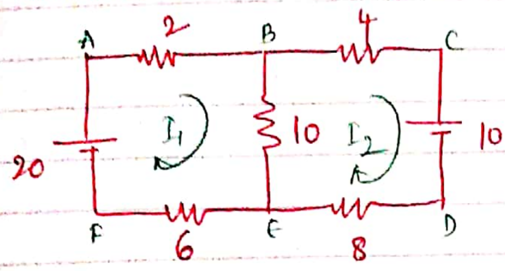
$$(R_1 + R_3 + R_5) I_1 - R_5 I_2 = V_1 \quad -(I)$$

$$-(R_5) I_1 + (R_2 + R_4 + R_5) I_2 = -V_2 \quad -(II)$$

R_{12} and R_{21} will be negative as the currents flowing through them will be in opposite directions.

[Note: Always consider $R_{11}, R_{22}, \ldots R_{nn}$ as positive and $R_{12}, R_{21}, R_{31}, R_{13}, R_{23}, R_{32}$ as negative]

Q1 find the current flowing through 10Ω resistance.



$I_{10\Omega} = (I_1 - I_2) = ?$

Solution:

Voltage equation for the loop with loop current as I_1

$R_{11} I_1 + R_{12} I_2 = V_1$

$(2+6+10) I_1 - 10 I_2 = 20$

$18 I_1 - 10 I_2 = 20 \quad — (1)$

Voltage equation for the loop with loop current as I_2

$R_{21} I_1 + R_{22} I_2 = V_2$

$-10 I_1 + (4+8+10) I_2 = -10$

$-10 I_1 + 22 I_2 = -10 \quad — (11)$

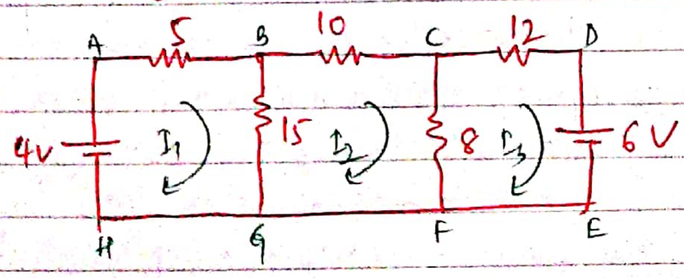
Solve equation (1) and (11) using calculator

$x = I_1 = 1.1486 \text{ Amp} \quad y = I_2 = 0.0675 \text{ Amp}$

Current through 10Ω

$I_{10\Omega} = (I_1 - I_2) = 1.1486 - 0.0675 = \underline{1.0811 \text{ Amp}}$

Q2 Find Current flowing through 10Ω resistance using loop current method



$I_{10} = I_2 = ?$

Solution

Voltage equation for the loop with loop current as I_1

$R_{11} I_1 + R_{12} I_2 + R_{13} I_3 = V_1$

$(5+15) I_1 - 15 I_2 - 0 = 4$

$$20I_1 - 15I_2 - 0 = 4 \quad -(\mathrm{i})$$

Voltage equation for the loop with loop current as I_2

$$R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = V_2$$

$$-15I_1 + (15+10+8)I_2 - 8I_3 = 0$$

$$-15I_1 + 33I_2 - 8I_3 = 0 \quad -(\mathrm{ii})$$

Voltage equation for the loop with loop current as I_3

$$R_{31}I_1 + R_{32}I_2 + R_{33}I_3 = V_3$$

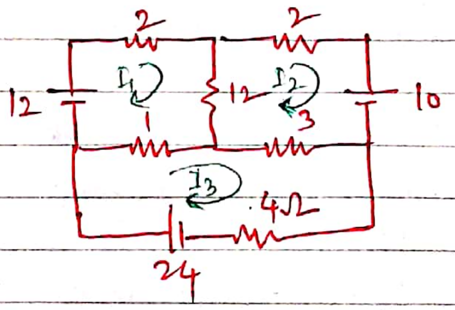
$$-0I_1 - 8I_2 + (8+12)I_3 = -6$$

$$0 - 8I_2 + 20I_3 = -6 \quad -(\mathrm{iii})$$

Solve the equation (i), (ii) & (iii) and find value of I_2 using calculator directly

$$y = I_2 = 0.0323 \text{ Amp}$$

Q3 find current through 4Ω resistance using loop current method



$$I_{4\Omega} = I_3 = ?$$

Solution:

Voltage equation for the loop with loop current I_1

$$(2+12+1)I_1 - 12I_2 - 1I_3 = 12$$

$$15I_1 - 12I_2 - I_3 = 12 \quad -(\mathrm{i})$$

Voltage equation for the loop with loop current I_2

$$-12I_1 + (2+12+3)I_2 - 3I_3 = -10$$

$$-12I_1 + 17I_2 - 3I_3 = -10 \quad -(\mathrm{ii})$$

Voltage equation for the loop with loop current I_3

$$-1I_1 - 3I_2 + (1+3+4)I_3 = 24$$

$$-I_1 - 3I_2 + 8I_3 = 24 \quad -(\mathrm{iii})$$

Solve equations (i) (ii) & (iii) and find value of I_3 using calculator

$$z = I_3 = 4.11 \text{ Amp}$$

3. Superposition Theorem

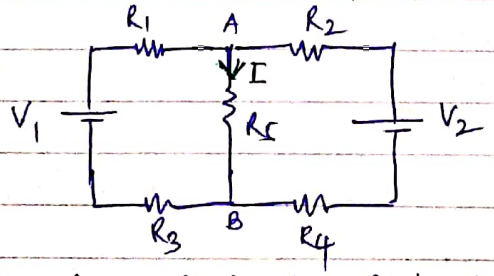
This theorem is used to find current through particular branch in multisource network.

Steps to be followed while solving examples using superposition theorem

1. Assume current flowing through the particular branch (branch through which current has to be calculated) as I Amp.
2. Consider only one source at one time. Replace the remaining sources by their respective internal resistances (Voltage source has to be replaced by zero resistance i.e. short circuit and current source by open circuit (infinite resistance))
3. When first source is considered, assume current through the particular branch as I'
4. When second source is considered, assume current through that particular branch as I''
5. If more than two sources are present in the network, assume current as $I', I'', I''', ...$
6. Current I will be algebraic sum of the current $I', I''...$

(Note: As per our syllabus, numericals will be asked on the networks with Voltage source only)

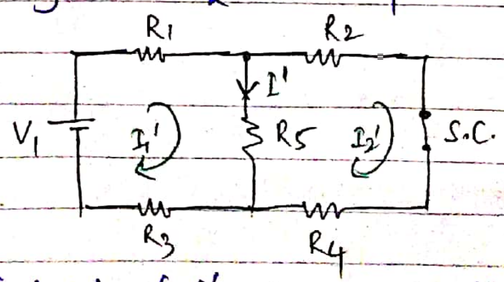
Consider the network as shown below. [Same network which was considered for kirchhoff's law]



Say the particular branch is, branch AB carrying R_5. Assume current through it as I Amp

Step 1. Consider voltage source V_1 and current through AB is I'.

Voltage source V_2 has to replaced by short circuit.



To find value of I', we can use kirchoff's loop current method.

Current $I' = I_1' - I_2'$

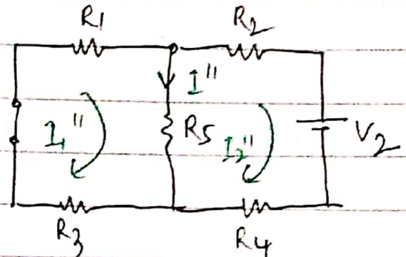
$$(R_1+R_3+R_5)\,I_1' - R_5 I_2' = V_1 \quad \text{--- (i)}$$

$$-R_5 I_1' + (R_2+R_4+R_5)\,I_2' = 0 \quad \text{--- (ii)}$$

By solving equations (i) & (ii) find I_1' & I_2'

$$I' = I_1' - I_2' \quad \text{--- (iii)}$$

Step 2. Consider second source V_2, replace V_1 by short circuit. Current through AB is I''



Using kirchoffs loop current method, $I'' = I_1'' - I_2''$

$$(R_1+R_3+R_5)\,I_1'' - R_5 I_2'' = 0 \quad \text{--- (iv)}$$

$$-R_5 I_1'' + (R_2+R_4+R_5)\,I_2'' = -V_2 \quad \text{(v)}$$

By solving equation (iv) & (v), find I_1'', I_2''

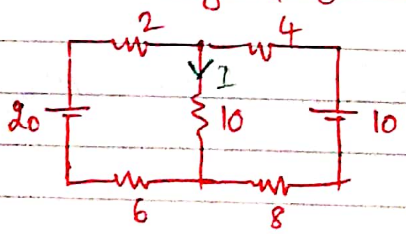
$$I'' = I_1'' - I_2'' \quad \text{--- (vi)}$$

Step 3.

Current through branch AB is

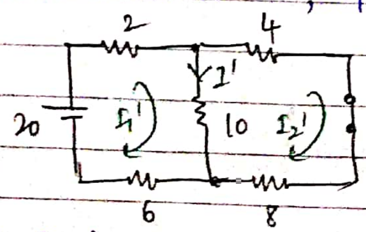
$$I = I' + I''$$

Q.1 Find current flowing through 10Ω using superposition theorem



Solution

Consider 20 Volt source, replace 10volt source by short circuit



$$I' = I_1' - I_2'$$

Voltage equations:

1st loop $\quad (2+10+6)\,I_1' - 10\,I_2' = 20$

$$18\,I_1' - 10\,I_2' = 20 \quad \text{--- (i)}$$

2nd loop $-10I_1' + (4+10+8) I_2' = 0$

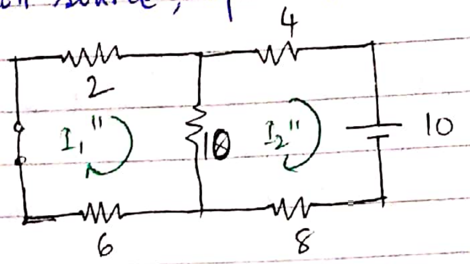
$-10 I_1' + 22 I_2' = 0$ — (II)

By solving equation (I) and (II)

$I_1' = 1.486$ Amp $\quad I_2' = 0.675$ Amp

$I' = 1.486 - 0.675 = 0.811$ Amp

Consider 10 Volt source, replace 20 Volt Source by short circuit



Voltage equations

1st loop $\quad (2+6+10) I_1'' - 10 I_2'' = 0$

$18 I_1'' - 10 I_2'' = 0$ — (III)

2nd loop $\quad -10 I_1'' + (4+10+8) I_2'' = -10$

$-10 I_1'' + 22 I_2'' = -10$ — (IV)

By solving equations (III) and (IV)

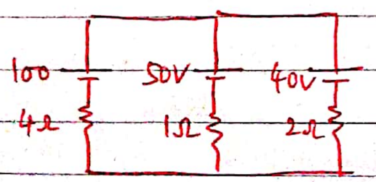
$I_1'' = -0.3378$ Amp $\quad I_2'' = 0.6081$ Amp

$I'' = I_1'' - I_2'' = -0.3378 + 0.6081 = 0.2703$ Amp

Hence the current flowing through 10Ω resistance when both the sources are acting simultaneously

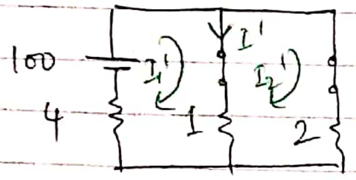
$I = I' + I'' = 0.811 + 0.2703 = \underline{\underline{1.0813 \text{ Amp}}}$

Q 2. Find current flowing through 1Ω resistance using superposition theorem.



$I_{1\Omega} = ?$

1. Consider a source of 100 volt is acting alone



$I' = I_1' - I_2'$

Voltage equations

$(4+1) I_1' - 1 I_2' = 100 \Rightarrow 5 I_1' - I_2' = 100$ — (i)

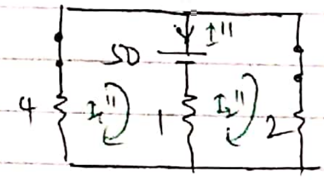
$-1 I_1' + (1+2) I_2' = 0 \Rightarrow -I_1' + 3 I_2' = 0$ — (ii)

Solving equations (i) and (ii)

$I_1' = 21.428$ A $\quad I_2' = 7.142$ Amp

$I' = I_1' - I_2' = 14.286$ Amp

2. Consider a source of 50 volt acting alone



$I'' = I_1'' - I_2''$

Voltage equations

$(4+1) I_1'' - 1 I_2'' = -50 \Rightarrow 5 I_1'' - I_2'' = -50$ — (iii)

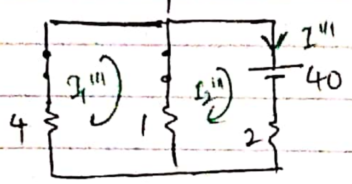
$-1 I_1'' + (1+2) I_2'' = 50 \Rightarrow -I_1'' + 3 I_2'' = 50$ — (iv)

Solving equations (iii) and (iv)

$I_1'' = -7.1428$ A $\quad I_2'' = 14.285$ A

$I'' = I_1'' - I_2'' = -21.427$ Amp

3. Consider a source of 40 volt acting alone



$I''' = I_1''' - I_2'''$

Voltage equations

$5 I_1''' - I_2''' = 0$ — (v)

$-I_1''' + 3 I_2''' = -40$ — (vi)

Solving equation (v) & (vi)

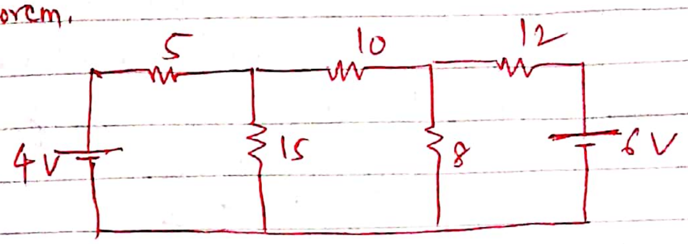
$I_1''' = -2.857$ A $\quad I_2''' = -14.285$ Amp

$I''' = I_1''' - I_2''' = 11.428$ Amp

(4) when all the sources are acting simultaneously

$$I = I' + I'' + I''' = \underline{4.287 \text{ Amp}}$$

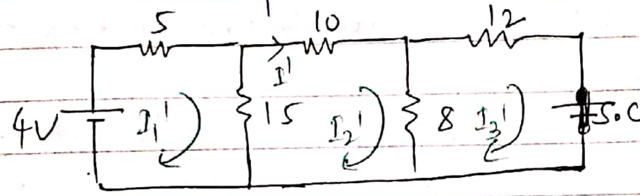
Q.3 Find the current flowing through 10Ω resistance using superposition theorem.



$I_{10\Omega} =$

Solution:

(1) Consider a source of 4 Volt is acting alone)



$I' = I_2'$

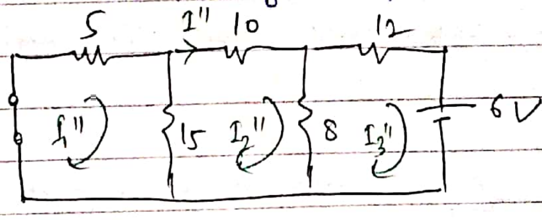
Voltage equations

$$20 I_1' - 15 I_2' - 0 = 4 \quad -(\text{i})$$
$$-15 I_1' + 33 I_2' - 8 I_3' = 0 \quad -(\text{ii})$$
$$-0 - 8 I_2' + 20 I_3' = 0 \quad -(\text{iii})$$

By solving equation (i), (ii) & (iii)

$I_1' = 0.3213$ Amp, $\underline{I_2' = 0.1617}$ Amp, $I_3' = 0.0646$ Amp

(ii) Consider 6 Volt source (acting alone)



$I'' = I_2''$

Voltage equations

$$20 I_1'' - 15 I_2'' - 0 = 0 \quad -(\text{iv})$$
$$-15 I_1'' + 33 I_2'' - 8 I_3'' = 0 \quad -(\text{v})$$
$$-0 - 8 I_2'' + 20 I_3'' = -6 \quad (\text{vi})$$

By solving equations (iv), (v) & (vi)

$I_1'' = -0.0970$ A, $I_2'' = -0.1293$ A, $I_3'' = -0.3517$ Amp

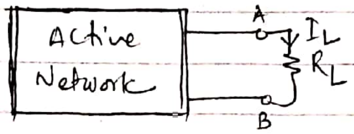
(3) When both the sources are acting simultaneously

$$I = I' + I'' = 0.1617 - 0.1293 = \underline{0.0324 \text{ Amp}}$$

3. Thevenin's Theorem:

This theorem is again used to find the current flowing through a particular resistance.

Consider the network as shown below. The resistance R_L is connected between terminals A & B.



Using thevenin's theorem the current through R_L is given by

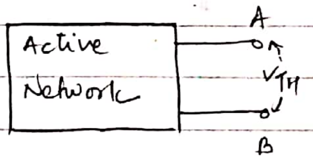
$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

where V_{TH} = Thevenin's equivalent voltage, R_{TH} = Thevenins equivalent resistance

* Steps to be followed while solving numericals using thevenin's theorem *

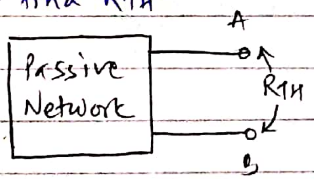
To find current 'I_L', we need to find value of 'V_{TH}' and 'R_{TH}'

1. To find V_{TH}



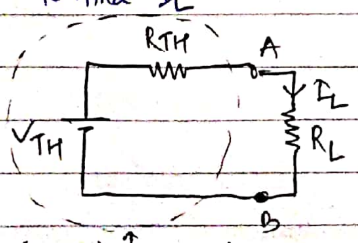
Remove resistance 'R_L' and calculate the voltage between the open circuited terminals A and B. This voltage is called as 'V_{TH}'

2. To find R_{TH}



Remove resistance R_L. Make the network as passive Network. It will be made as passive network by replacing all sources present in the network by their respective internal resistances.
(Simply replace voltage source by short circuit and current source by open circuit)

3. To find I_L



Thevenin's equivalent ckt

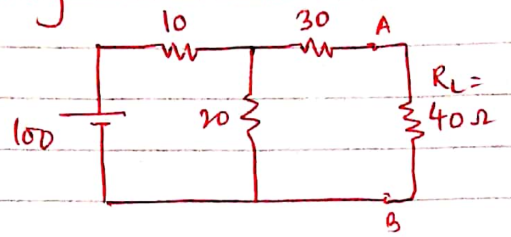
Once we get value of V_{TH} and R_{TH}. Draw thevenin's equivalent circuit and again connect resistance 'R_L' between A and B.

Hence the current through R_L is

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

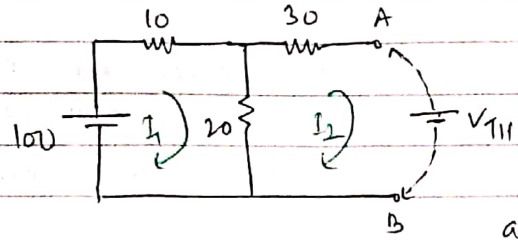
[when V_{TH} will be as per diagram shown above, current flows from A to B But when B is +ve with respect to A, current flows from B to A.]

Q.1 find the current flowing through the resistance of 40Ω using Thevenin's Theorem.



Solution

1. To find V_{TH}



Remove R_L of 40Ω

Assume A is +ve w.r.to B

(If value of V_{TH} obtained after calculation is +ve, then the polarities assumed are correct otherwise vice-versa means B is +ve w.r.to A

Using KVL we can find find value of V_{TH}. Here $I_2 = 0$

Voltage equations

$$30 I_1 - 20 I_2 = 100 \quad \& \quad -20 I_1 + 50 I_2 = -V_{TH}$$

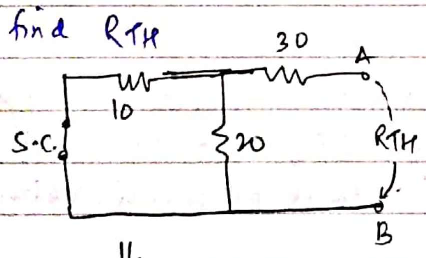
as $I_2 = 0$ $\quad 30 I_1 = 100 \quad$ and $\quad -20 I_1 = -V_{TH}$

OR $V_{TH} = 20 I_1$

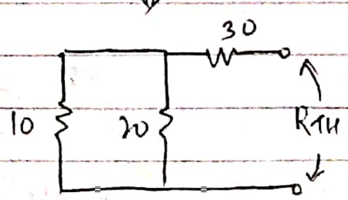
$$I_1 = \frac{100}{30} = 3.33 \text{ Amp}$$

$$V_{TH} = 20 I_1 = 20 \times 3.33 = 66.66 \text{ Volt} \quad -(1)$$

2. To find R_{TH}



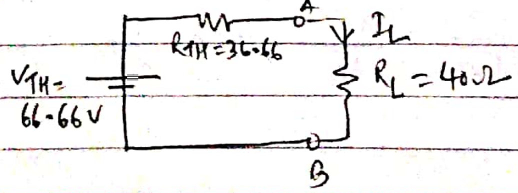
voltage source of 100 Volt, need to be replaced by Short circuit.



10Ω & 20Ω are in parallel. And this parallel combination will be in series with 30Ω

$$R_{TH} = \frac{10 \times 20}{10+20} + 30 = 36.66\,\Omega$$

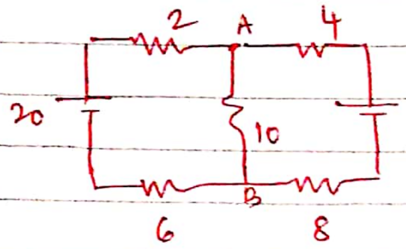
3. To find I_L



$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{66.66}{36.66 + 40}$$

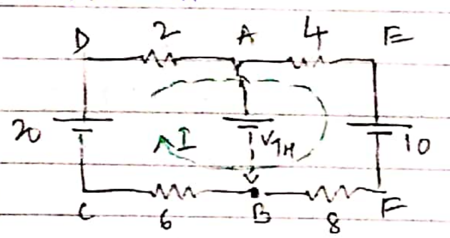
$$I_L = 0.869 \text{ Amp}$$

Q2 find the current flowing through 10Ω using thevenins theorem.



Solution.

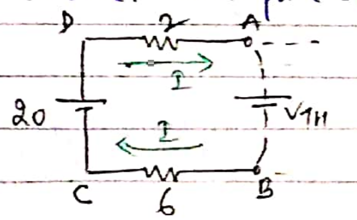
1. To find V_{TH}



As 10Ω resistance between A and B is removed. Only one loop will be their. Hence current I will flow as shown in figure.

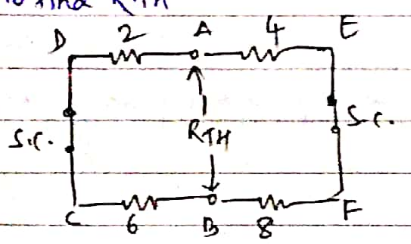
$$(2+4+8+6)I = -10+20 = 10$$
$$20I = 10$$
$$I = 10/20 = 0.5 \text{ Amp}$$

Consider left half part of the circuit. [Right half can be considered]

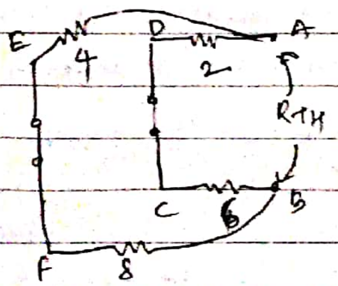


$$(2+6)I = -V_{TH} + 20$$
$$V_{TH} = 20 + 8I = 20 \cancel{+} 8(0.5)$$
$$= 20 \cancel{+} 4 = 16 \text{ volt}$$

2. To find R_{TH}



20V and 10V sources are replaced by short circuit as well have make the network as passive network



4 and 8 are in series $4+8=12$

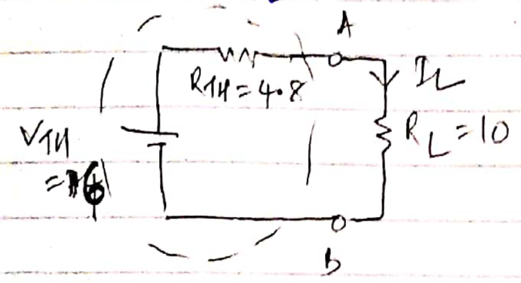
2 and 6 are in series $2+6=8$

These series combinations will be parallel to eachother

12Ω is in parallel with 8Ω

$$R_{TH} = \frac{12\times 8}{12+8} = \frac{96}{20} = 4.8\ \Omega$$

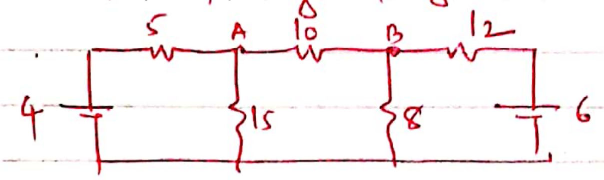
3. To find I_L



Thevenin's eq. ckt

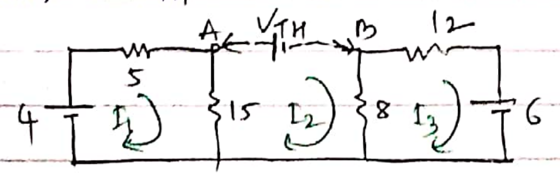
$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{16}{4.8+10} = \underline{1.081 \text{ Amp}}$$

Q3 find the current flowing through 10Ω using Thevenins Theorem.



Solution:

1. To find V_{TH}



As 10Ω resistance is removed
$I_2 = 0$ Amp

Voltage equations.

$$20 I_1 - 15 I_2 - 0 I_3 = 4 \quad \text{—(I)}$$
$$-15 I_1 + 23 I_2 - 8 I_3 = -V_{TH} \quad \text{—(II)}$$
$$-0 I_1 - 8 I_2 + 20 I_3 = -6 \quad \text{—(III)}$$

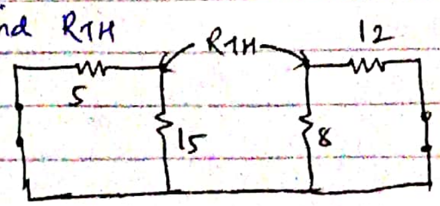
from equation no. (I)
$20 I_1 = 4$
$I_1 = 4/20 = 0.2$ Amp

from equation no. (III)
$20 I_3 = -6$
$I_3 = -6/20 = -0.3$ Amp

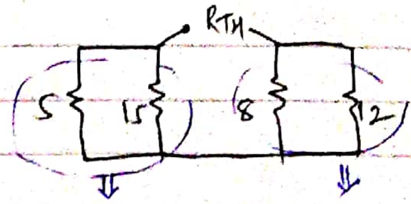
Putting value of I_1 and I_3 in equation no. (II)

$$-15 I_1 + 23 I_2 - 8 I_3 = -V_{TH}$$
$$-15(0.2) + 0 - 8(-0.3) = -V_{TH}$$
$$-3 + 2.4 = -V_{TH}$$
$$\underline{V_{TH}} = 3 - 2.4 = \underline{0.6} \text{ Volt}$$

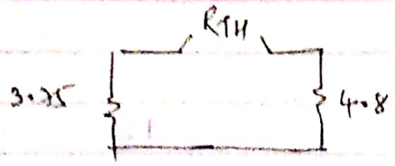
2. To find R_{TH}



⇒

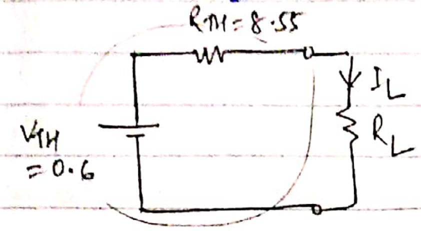


$\frac{5 \times 15}{5+15} = 3.75$ $\qquad$ $\frac{8 \times 12}{8+12} = \frac{96}{20} = 4.8$



$R_{TH} = 3.35 + 4.08 = 8.55\,\Omega$

3. To find I_L



Thevenins eq. ckt

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{0.6}{8.55+10}$$

$= 0.0323$ Amp