SEAT NO. : S190248566

Name, Signature of Staff & Date

Name : Aniruddha Shivaji Repale.

Roll No.: SIT-55

Class : S.E. Subject : Discrete Mathematics

Day & Date : Monday, 18/12/23 Examination : Prelims

Section : — Course / Paper No.: —

Medium of Answer : English

Main Ans. Book + No. of Supplements = Total

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	Total	Signature of Examiner
Marks														

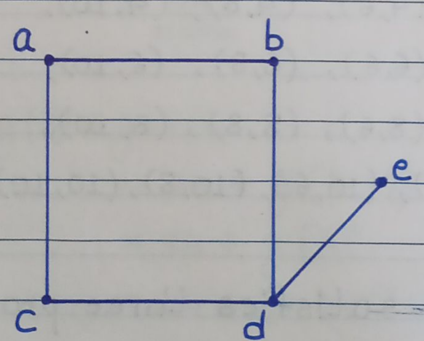
Use of Coloured pencil or ink is Strictly prohibited except in case of Diagrams and Sketches
(Write on both sides and start writing on this page)

Que.1] a] Find self-complementary simple graph with five vertices.

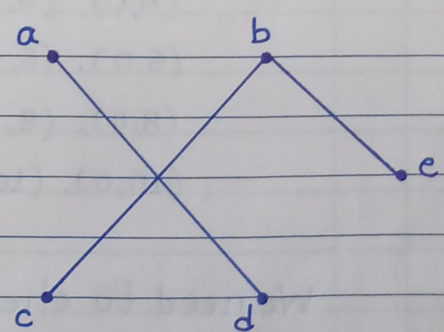
⇒ Ans :

Let G is a simple graph. Then the complement of G denoted by $\bar{G}$ is the graph whose vertex set is same as the vertex set in G and in which, two vertices are adjacent iff they are not adjacent in graph G.

e.g.

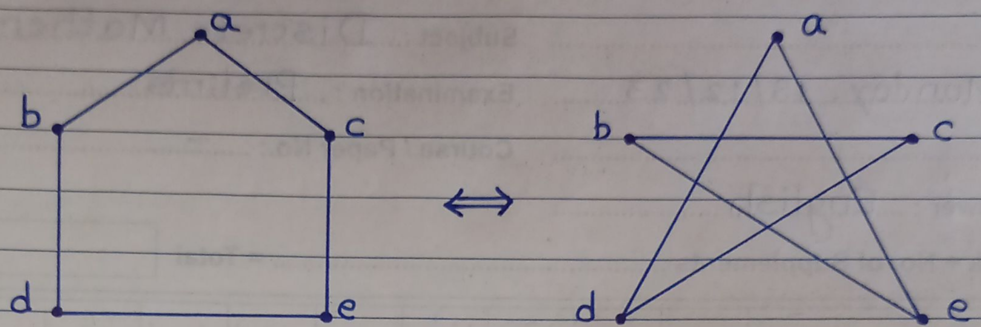


G



$\bar{G}$

A graph is said to be self complementary if it is isomorphic to its complement.
A self complementary simple graph with 5 vertices is as follows :-



G $\qquad\qquad$ $\overline{G}$

---

b] Let $A = \{0, 2, 4, 6, 8, 10\}$ and Relation aRb defined on set A as $aRb = \{(a,b) \mid (a-b) \% 2 == 0 \; ; \forall a, b \in A\}$. Find aRb is equivalence relation or not.

$\Rightarrow$ Ans :

$A = \{0, 2, 4, 6, 8, 10\}$

$R = \{(a,b) \mid (a-b) \% 2 == 0\}$

$\therefore R = \{(0,0), (0,2), (0,4), (0,6), (0,8), (0,10),$
$(2,0), (2,2), (2,4), (2,6), (2,8), (2,10),$
$(4,0), (4,2), (4,4), (4,6), (4,8), (4,10),$
$(6,0), (6,2), (6,4), (6,6), (6,8), (6,10),$
$(8,0), (8,2), (8,4), (8,6), (8,8), (8,10),$
$(10,0), (10,2), (10,4), (10,6), (10,8), (10,10)\}$

We need to check if aRb satisfies three properties of equivalence.

i) Reflexivity :-

In this case, aRa , $\forall a \in A$

Thus, aRb is Reflexive.

ii) Symmetry :-

aRb , bRa i.e. (a-b) % 2 == 0 Then (b-a) % 2 == 0.

Thus, aRb is symmetric.

iii) Transitivity :-

If aRb, bRc then aRc , $\forall a, b, c \in A$

Thus, aRb is Transitive.

Hence, given Relation 'R' satisfied all conditions for equivalen

$\therefore$ R is an Equivalence Relation.

c] If $f(x) = 2x$ and $g(x) = x^2 + 2$, Find (i) fog(5), (ii) fog(5)+gof(2).

$\Rightarrow$ Ans:

i) $fog(5) = f[g(5)]$

$= f[5^2 + 2]$

$= f(27)$

$= 2(27)$

$= \underline{\underline{54}}$

ii) $fog(5) + gof(2)$

$= 54 + g[f(2)]$

$= 54 + g[4]$

$= 54 + [4^2 + 2]$

$= 54 + 18$

$= \underline{\underline{72}}$

Que.2] a] $X=\{2,3,6,12,24,36\}$ and $x<=y$ iff x divides y.
Find i) Maximal element, ii) Minimal element,
iii) Draw Graph and its equivalent hasse diagram for divisibility on set X.

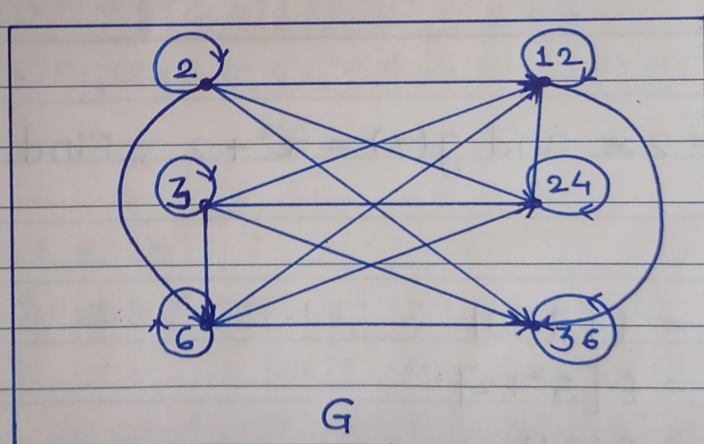
$\Rightarrow$ Ans:

$X=\{2,3,6,12,24,36\}$

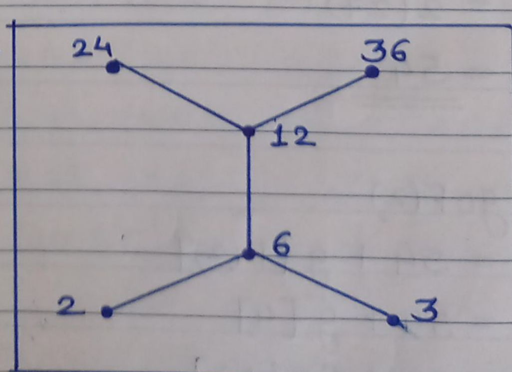
$x<=y$ iff x divides y

Let, $R=\{(2,2),(2,6),(2,12),(2,24),(2,36),$
$(3,3),(3,6),(3,12),(3,24),(3,36),$
$(6,6),(6,12),(6,24),(6,36),$
$(12,12),(12,24),(12,36),$
$(24,24),(36,36)\}$

Graph :-



Hasse Diagram :-



$\therefore$ Maximal elements = 24, 36
$\therefore$ Minimal elements = 2, 3

b] If $X = \{10, 20, 30, 40, 50\}$ & Relation on set X is represented as $aRb = \{(a,b) \mid a \text{ divide } b \; ; \forall a,b \in X\}$. Find a relation aRb is Partial Order Relation or not.

$\Rightarrow$ Ans :

$aRb = \{(10,10), (10,20), (10,30), (10,40), (10,50), (20,20), (20,40), (30,30), (40,40), (50,50)\}$

We need to check if aRb satisfies three properties of Partial Order Relation.

i) Reflexive :-

$aRa, \forall a \in X$

Here, $aRa = \{(10,10), (20,20), (30,30), (40,40), (50,50)\}$

Thus, R is Reflexive.

ii) Antisymmetric :-

If aRb and bRa then $a = b$

Thus, R is Antisymmetric.

iii) Transitivity :-

If aRb, bRc then aRc, $\forall a,b,c \in X$

Thus, R is Transitive.

Hence, aRb is a Partial Order Relation.

---

c] If $f(x) = 16x^2 + 12$, Find Inverse of $f(x)$ a function? Justify.

$\Rightarrow$ Ans:

$f(x) = 16x^2 + 12$

Let, $y = f(x) = 16x^2 + 12$

To find x in terms of y,

$$y = 16x^2 + 12$$

$$\therefore y - 12 = 16x^2$$

$$\therefore \frac{y-12}{16} = x^2$$

$$\therefore x = \pm\sqrt{\frac{y-12}{16}}$$

$x \leftrightarrow y$, we get inverse of $f(x)$.

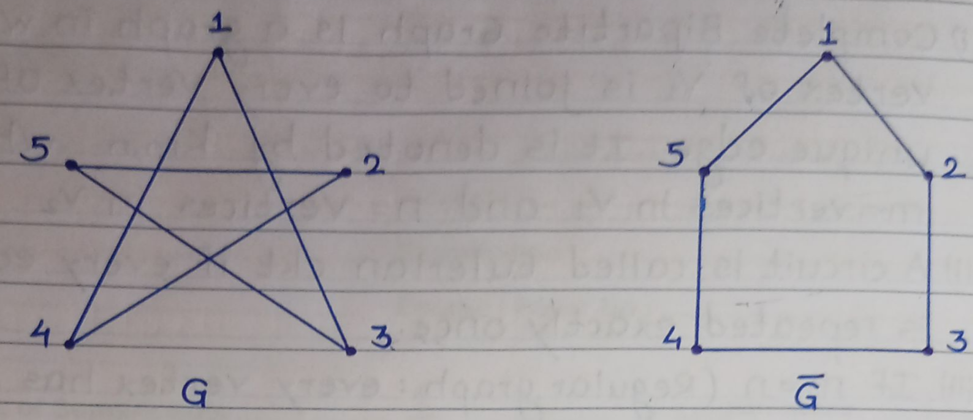
$$\therefore y = f^{-1}(x) = \pm\sqrt{\frac{x-12}{16}}$$

A function is defined as a relation in which each element of domain is mapped to exactly one element in co-domain.

Here, Inverse function has two values i.e. $\pm$ sign indicating two possible values for y, for given x.

$\therefore f^{-1}(x)$ is not a function in given example.

Que.3] a] Show that following graphs are Isomorphic.



⇒ Ans:

i) Two Graphs are said to be Isomorphic if there is one-one correspondence between their vertices and their edges such that incidences & adjacency are preserved.

ii) Conditions for Isomorphic Graphs :-

a] same no. of vertices.

b] Same no. of edges.

c] same degree of vertices.

iii) In given graphs,

Graph G and $\bar{G}$ has same number of vertices and same number of edges as well. i.e. 5 vertices & 5 edges in both.

Degree of vertices is also same in correspondence.

$(1 \rightarrow 1,\ 2 \rightarrow 2,\ 3 \rightarrow 3,\ 4 \rightarrow 4,\ 5 \rightarrow 5)$: Degree = 2 of each vertex.

Hence, it is proved that Graphs G & $\bar{G}$ are isomorphic i.e. $G \cong \bar{G}$.

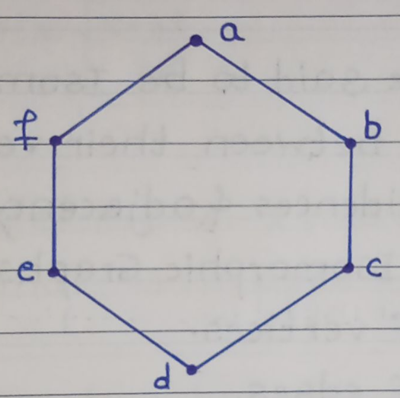
b] Under what condition $K_{m,n}$ will have Eulerian circu[it]

⇒ Ans :

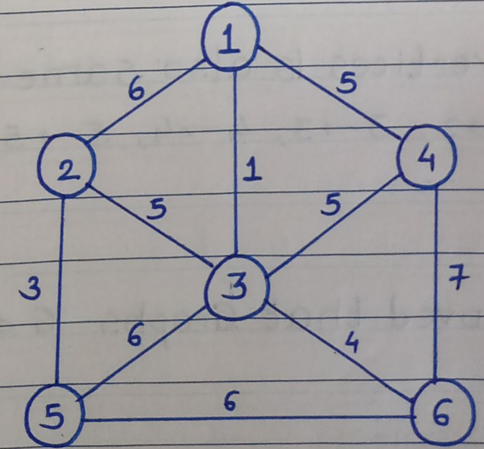
i) Complete Bipartite Graph is a graph in which each vertex of V_1 is joined to every vertex of V_2 by an unique edge. It is denoted by $K_{m,n}$ where, m = vertices in V_1 and n = vertices in V_2.

ii) A circuit is called Eulerian ckt if every edge of graph is repeated exactly once.

iii) If $m = n$ (Regular graph : every vertex has same degree) then $K_{m,n}$ will have Eulerian circuit.



c] Construct Minimal spanning Tree for following graph using kruskals Algorithm.



Name, Signature of Staff & Date

Name : Aniruddha

Roll No.:

Class :

Subject :

Day & Date :

Examination :

Section :

Course / Paper No.:

Medium of Answer :

Main Ans. Book + No. of Supplements 1 = Total

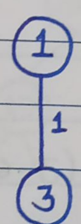
Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	Total	Signature of Examiner
Marks														

Use of Coloured pencil or ink is Strictly prohibited except in case of Diagrams and Sketches
(Write on both sides and start writing on this page)

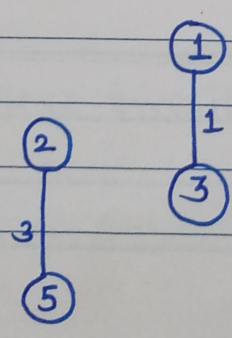
⇒ Ans :

In kruskal's Algorithm, we have to select min. weighted edge first & Arrange all edges in increasing order & select one by one & avoid those edges which will make circuit in tree.

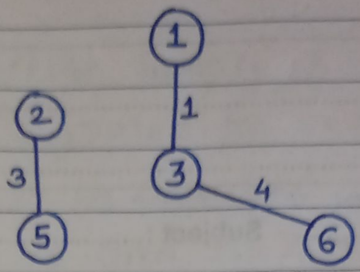
Step-1] Select edge (1-3) with min. weight '1'.



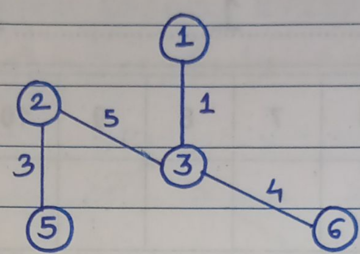
Step-2] Select edge (2,5) with min. weight '3'.



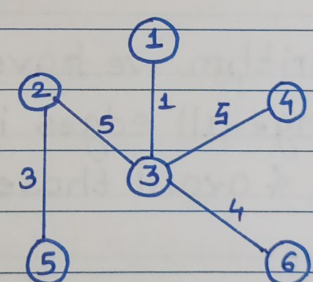
Step-3] Select edge (3,6) with weight '4'.



Step-4] Select edge (2,3) with weight '5'.



Step-5] Select edge (3,4) with weight '5'.



This is required minimum spanning tree.

Cost of above tree is –

$= w(1-3) + w(2-5) + w(3-6) + w(2-3) + w(3-4)$

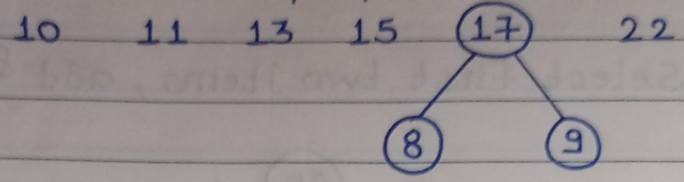
$= 1 + 3 + 4 + 5 + 5$

$= 18$

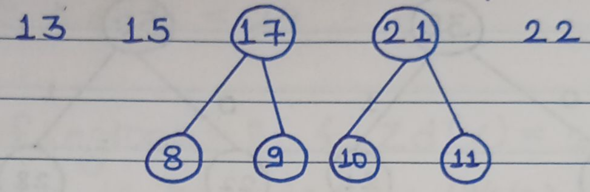
Que.4] a] Construct an optimal tree for 8, 9, 10, 11, 13, 15, 22 using Huffman coding.

⇒ Ans :

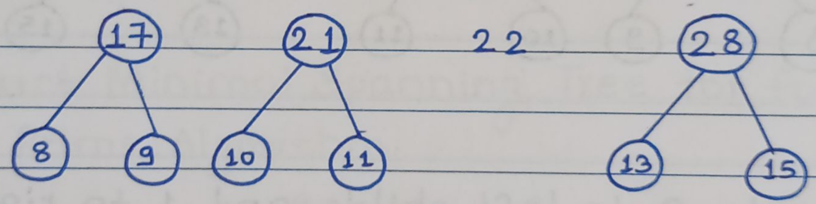
Step-1] All the given weights are already in ascending sequence, so select first two weights, add them & store result in proper sequence.



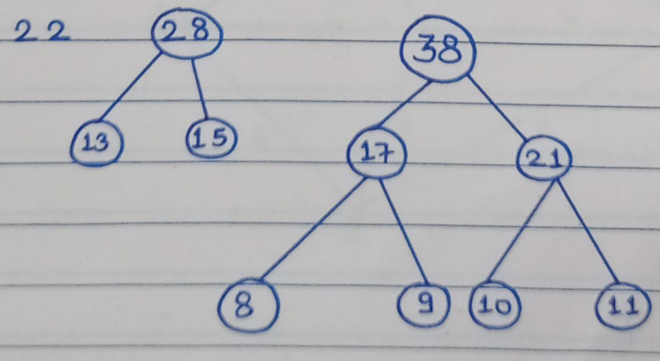
Step-2] Select first two items ; perform same procedure



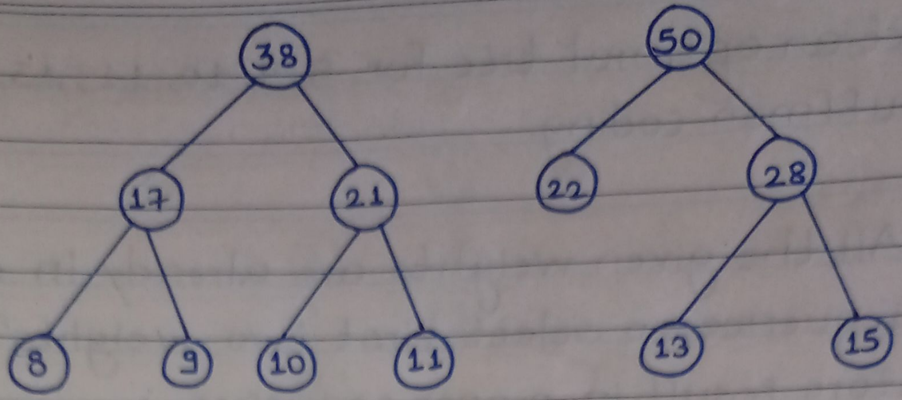
Step-3] Select first two items.



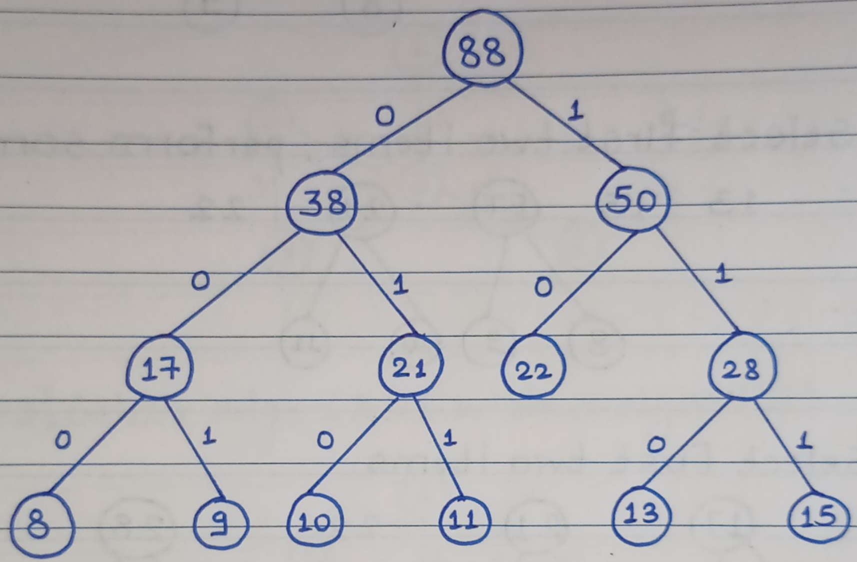
Step-4] Select first two items.



Step-5] Select first two items (22 & 28).



Step-6] Select first two items, add & arrange sequen



We apply 0 to left childs and 1 to right childs.

∴ Binary Prefix codes are,

17 – 00

21 – 01

22 – 10

28 – 11

8 – 000

9 – 001

10 – 010

11 – 011

13 – 110

15 – 111

b] Show that in a connected planar graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges.

⇒ Ans :

Here, $v = 6$ and $e = 12$.

According to Euler's Theorem,

$v - e + r = 2$

$\therefore 6 - 12 + r = 2$

$\therefore r = 8$

Now,

According to Handshaking Lemma,

$\sum d(v_i) = 2e$

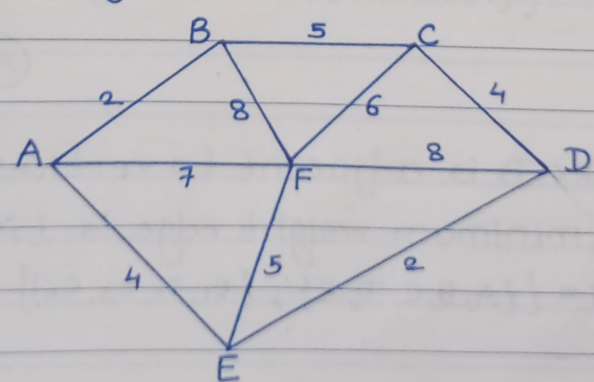
$\therefore \sum d(v_i) = 2 \times 12 = 24$

Since no. of regions $= 8$ & $\sum d(v) = 24$

$\therefore$ Each region is bounded by $(24/8) = 3$ edges.

---

c] Construct Minimal Spanning Tree for following graph using Prims Algorithm.



⇒ Ans :

Using Prim's Algorithm.

Initially minimum spanning tree is empty.

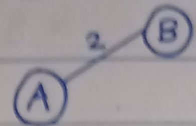
MST = $\{\phi\}$

Step-1] Select any arbitrary vertex of graph G as a starting of MST.

∴ MST = { {A}, {ϕ} }

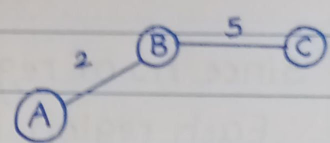
Step-2] Vertex A is adjacent to vertices B, E, F.
But the minimum weight is (A to B) = 2.
So add it into MST.

∴ MST = { {A, B}, {e_1} }



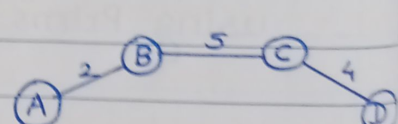
Step-3] Vertex B is adjacent to A, F, C vertices.
But, minimum weight edge is (B-C) = 5

∴ MST = { {A, B, C}, {e_1, e_2} }



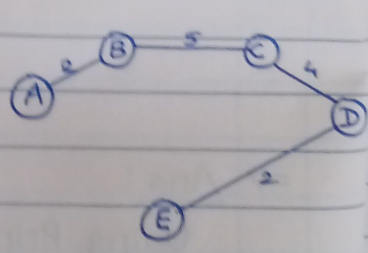
Step-4] Vertex C is adjacent to vertices B, F, D.
But, minimum weight edge is (C-D) = 4

∴ MST = { {A, B, C, D}, {e_1, e_2, e_3} }



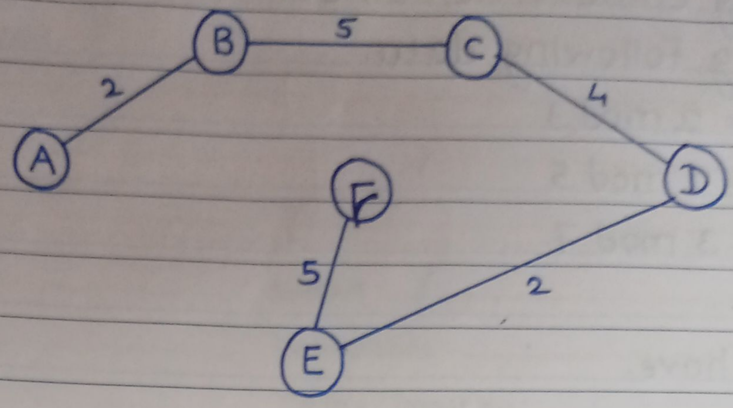
Step-5] Vertex D is adjacent to vertices C, F, E.
But, minimum weight edge is (D-E) = 2

∴ MST = { {A, B, C, D, E}, {e_1, e_2, e_3, e_4} }



Step-6] Vertex E is adjacent to the vertices A, F, D.
But, min. weighted edge ~~is~~ except D is (E-A) = 4. But, we can't add it into MST as it will a circuit. So, we add edge (E-F) = 5

∴ MST = { {A, B, C, D, E, F}, {e_1, e_2, e_3, e_4, e_5} }



This is the required Minimum spanning Tree.

$$W(MST) = W(A\text{-}B) + W(B\text{-}C) + W(C\text{-}D) + W(D\text{-}E) + W(E\text{-}F)$$
$$= 2 + 5 + 4 + 2 + 5$$
$$= 18$$

Que.5] a] Using chinese Remainder Theorem, find the value of using following data.

$p = 2 \bmod 3$

$p = 2 \bmod 5$

$p = 3 \bmod 7$

$\Rightarrow$ Ans :

We have,

$a_1 = 2, \ a_2 = 2, \ a_3 = 3$

$m_1 = 3, \ m_2 = 5, \ m_3 = 7$

$gcd(3,5) = gcd(3,7) = gcd(5,7) = 1$

$\therefore m_1, m_2, m_3$ are Relatively Prime.

i) $M = m_1 \times m_2 \times m_3 = 3 \times 5 \times 7 = 105$

ii) $M_1 = \frac{M}{m_1} = \frac{105}{3} = 35$

$M_2 = \frac{M}{m_2} = \frac{105}{5} = 21$

$M_3 = \frac{M}{m_3} = \frac{105}{7} = 15$

iii) $M_1 \cdot M_1^{-1} = 1 \bmod (m_1)$

$\therefore 35 \cdot M_1^{-1} = 1(\bmod (3))$

$\therefore (35 \cdot M_1^{-1} - 1) = 3k$

$\therefore M_1^{-1} = 2$

$M_2 \cdot M_2^{-1} = 1 \bmod (m_2)$

$\therefore M_2 \cdot M_2^{-1} = 1 \bmod (5)$

$\therefore 21 \cdot M_2^{-1} = 1 (\bmod 5)$

$\therefore (21 \cdot M_2^{-1} - 1) = 5k$

$\therefore M_2^{-1} = 1$

Name, Signature of Staff & Date

Name : Aniruddha

Roll No.:

Class :

Subject :

Day & Date :

Examination :

Section :

Course / Paper No.:

Medium of Answer :

Main Ans. Book + No. of Supplements 2 = Total

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	Total	Signature of Examiner
Marks														

Use of Coloured pencil or ink is Strictly prohibited except in case of Diagrams and Sketches
(Write on both sides and start writing on this page)

$M_3 . M_3^{-1} = 1 \bmod (m_3)$

$\therefore 15 \cdot M_3^{-1} = 1 \pmod 7$

$\therefore (15 \cdot M_3^{-1} - 1) = 7k$

$\therefore M_3^{-1} = 1$

<u>Now,</u>

$P = [a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + a_3 M_3 M_3^{-1}] \bmod M$

$\therefore P = [(2 \times 35 \times 2) + (2 \times 21 \times 1) + (3 \times 15 \times 1)] \bmod 105$

$\therefore P = [140 + 42 + 45] \bmod 105$

$\therefore p = 227 \pmod{105}$

$\therefore p = 17$

b] Compute GCD of following using Euclid Algorithm.

i) GCD (2071, 206)

⇒ Ans :

$2071 = 206(10) + 11$

$206 = 11(18) + 8$

$11 = 8(1) + 3$

$8 = 3(2) + 2$

$3 = 2(1) + 1$

$2 = 1(2) + 0$

$\boxed{\therefore GCD(2071, 206) = 1}$

ii) GCD (1276, 244)

⇒ Ans :

$1276 = 244(5) + 56$

$244 = 56(4) + 20$

$56 = 20(2) + 16$

$20 = 16(1) + 4$

$16 = 4(4) + 0$

$\boxed{\therefore GCD(1276, 244) = 4}$

c] Solve the congruence $8x \equiv 13 \pmod{29}$.

⇒ Ans :

i) gcd (8, 29)

$29 = 8(3) + 5$

$8 = 5(1) + 3$

$5 = 3(1) + 2$

$3 = 2(1) + 1$

$2 = 1(2) + 0$

$\therefore gcd(8, 29) = 1$ ..... (co prime) ..... ($\because$ 1 solution exists)

ii) $1 | 13 \Rightarrow 13 = 1k$, $\exists k \in I$

iii) $\frac{8}{1}x = \frac{13}{1} \left(\text{mod } \frac{29}{1}\right)$

$\therefore 8x = 13 \pmod{29}$

Now, to find inverse of 8,

$8a \equiv 1 \pmod{29}$

$\therefore (8a - 1) \equiv 29k$ OR $29 | (8a - 1)$

$\therefore a = 11$

Now,

$11 \times 8x \equiv 11 \times 13 \pmod{29}$

$\therefore 88x \equiv 143 \pmod{29}$

Reducing to smallest residue,

$\therefore x \equiv 143 \pmod{29}$

This means that x & 143 have same remainder when divided by 29. We can express 143 in terms of 29 as,

$143 = 29 \times 4 + 27$ .... (Quotient = 4 & Remainder = 27, on dividing 143 by 29)

$\therefore x \equiv 27 \pmod{29}$

Now, 27 is smallest non-negative residue because its betn 0 & 28 (since we're working on 29).

$\boxed{\therefore x \equiv 27 \pmod{29}}$

Que.6] a] Find the Totient function of following numbers:

i) 75

⇒ Ans :

The totient function / Euler's phi function, is a function that counts the positive integers up to given Integer 'n' that are relatively prime to 'n'. (i.e. Numbers that share no common factor with 'n' except 1).

Here, to find totient function (ϕ) of 75, we need to calculate the count of positive integers less than or equal to 75 that are relatively prime to 75.

Prime factorization of 75 = $3 \times 5 \times 5$

The totient function for product of distinct primes is given by,

$$\phi(n) = n \times \left(1 - \frac{1}{P_1}\right) \times \left(1 - \frac{1}{P_2}\right) \times \cdots\cdots\cdots \times \left(1 - \frac{1}{P_n}\right)$$

where, $P_1, P_2, \ldots\ldots P_n$ are prime factors of n.

∴ For 75,

$$\phi(75) = 75 \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right)$$

$$\therefore \phi(75) = 75 \times \left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right)$$

$$\therefore \phi(75) = \overset{5}{\cancel{75}} \times \left(\frac{8}{\cancel{15}}\right)$$

$$\boxed{\therefore \phi(75) = 40}$$

ii) 143

$\Rightarrow$ Ans :

Prime factorization of 143 = 11×13

We know,

$$\phi(n) = n \times \left(1-\frac{1}{P_1}\right) \times \left(1-\frac{1}{P_2}\right) \times \cdots\cdots \times \left(1-\frac{1}{P_n}\right)$$

$$\therefore \phi(143) = 143 \times \left(1-\frac{1}{11}\right) \times \left(1-\frac{1}{13}\right)$$

$$\therefore \phi(143) = 143 \times \left(\frac{10}{11}\right) \times \left(\frac{12}{13}\right)$$

$$\therefore \phi(143) = \frac{\cancel{143} \times 120}{\cancel{143}}$$

$$\boxed{\therefore \phi(143) = 120}$$

iii) 108

$\Rightarrow$ Ans :

Prime factorization of 108 = $2 \times 2 \times 3 \times 3 \times 3$

We know,

$$\phi(n) = n \times \left(1-\frac{1}{P_1}\right) \times \left(1-\frac{1}{P_2}\right) \times \cdots\cdots \times \left(1-\frac{1}{P_n}\right)$$

$$\therefore \phi(n) = 108 \times \left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{3}\right)$$

$$\therefore \phi(108) = 108 \times \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}$$

$$\therefore \phi(108) = \frac{108}{3}$$

$$\boxed{\therefore \phi(108) = 36}$$

b] Find the multiplicative inverse of 37 (mod 26) using Extended Eucliedian Algorithm.

⇒ Ans :

i) Find GCD (37, 26)

$37 = 26(1) + 11$ —— (1)

$26 = 11(2) + 4$ —— (2)

$11 = 4(2) + 3$ —— (3)

$4 = 3(1) + 1$ —— (4)

$3 = 1(3) + 0$

$\therefore$ GCD = 1

ii) Substituting backwards from (4) to (1),

$1 = 4 - 3\times1$ ..... ($\because 4 = 3(1) + 1$)

$\therefore 1 = 4 - (11 - 4\times2)1$

$= 4 - 1\cdot11 + 2\cdot4$

$= 4(1+2) - 1\cdot11$

$= 4(3) - 1\cdot11$

$= 3(26 - 11\times2) - 1\cdot11$ ....($\because 4 = 26 - 11\times2$)

$= 3\cdot26 - 6\cdot11 - 1\cdot11$

$= 3\cdot26 - 7\cdot11$

$= 3\cdot26 - 7(37 - 26\times1)$

$= 3\cdot26 - 7\cdot37 + 7\cdot26$

$= 26(10) - 37(7)$

$= 26(10) + 37(-7)$

~~Additive~~ Multiplicative Inverse of 37 (mod 26) is coefficient of 37. i.e. -7

We can also write -7 as 19 (mod 26) i.e. $-7 = 19 \pmod{26}$

$\therefore$ Multiplicative inverse of 37 (mod 26) is 19 also!

c] Using Euclidean Algorithm find GCD of 268 & 884.

⇒ Ans:

According to Euclidean Algorithm.

$a = bq + r \; ; \; 0 \leq r < b$

Now, GCD(268, 884) is:-

$884 = 268(3) + 80$

$268 = 80(3) + 28$

$80 = 28(2) + 24$

$28 = 24(1) + 4$

$24 = 4(6) + 0$

$\boxed{\therefore GCD(884, 268) = 4}$

Que.7] a] Define and give example of following:

i) Semigroup.

⇒ Ans:

An Algebraic system $(A, *)$ with a binary operation $*$ is said to be a semigroup if it satisfies –

1] closure Property ⇒ $a, b \in A$, $a * b \in A$

2] Associative Property ⇒ $a*(b*c) = (a*b)*c$, $\forall a, b, c \in$

e.g.

$(R,+)$, $(R,\times)$, $(Z,+)$ .... (R = Set of Real no., Z = Set of

ii) Field.

⇒ Ans:

A commutative ring with unity in which every non-z element possesses their multiplicative inverse is cal as field.

e.g.

$(R,+,\cdot)$, $(Q,+,\cdot)$ .... (Q = Set of Rational Numbers)

iii) Zero divisors.

⇒ Ans:

Let $(R,+,\cdot)$ be a commutative ring. An element $a \neq 0$ in R is said to be a zero divisor if $\exists\, b \neq 0$ in R such that $a \cdot b = 0$

e.g.

2 is a zero divisor in $(Z_4, +, \cdot)$ as $2 \cdot 2 = 4 = 0$

e.g. $((M_{2\times 2} R), +, \cdot)$ is a ring with zero divisors,

as $A \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$, but $A \neq 0$ & $B \neq$

...udha

Roll No.: ...

Class : ...

Day & Date : ... Subject : ...

Section : ... Examination : ...

Medium of Answer : ... Course / Paper No.: ...

Main Ans. Book + No. of Supplements ... 3 ... = Total

No.	1	2	3	4	5	6	7	8	9	10	11	12	Total	Signature of Examiner
arks														

Use of Coloured pencil or ink is Strictly prohibited except in case of Diagrams and Sketches
(Write on both sides and start writing on this page)

b] Find the hamming distance between code words of :
$C = \{(0000), (0101), (1011), (0111), (1111)\}$

⇒ Ans :

Hamming. dist. betn two code words is the no. of positions at which corresponding symbols differ.

Here,

$d[(0000),(0101)] = 2$

$d[(0000),(1011)] = 3$

$d[(0000),(0111)] = 3$

$d[(0000),(1111)] = 4$

$d[(0101),(1011)] = 3$

$d[(0101),(0111)] = 1$

$d[(0101),(1111)] = 2$

$d[(1011),(0111)] = 2$

$d[(1011),(1111)] = 1$

$d[(0111),(1111)] = 1$

c] Let I be the set of all integers. For each of following, determine whether * is commutative operation or not.

i) $a*b = \max(a,b)$

⇒ This operation is commutative because,

$\max(a,b) = \max(b,a)$

$\therefore a*b = b*a \;, \; \forall a,b \in I$

ii) $a*b = \min(a+2, b)$

⇒ This operation is not commutative because,

$a*b = \min(a+2, b)$

$b*a = \min(b+2, a)$

$\therefore a*b \neq b*a$

iii) $a*b = 2a-2b$

⇒ This operation is not commutative because,

$a*b = 2a-2b$

$b*a = 2b-2a$

$\therefore a*b \neq b*a$

iv) $a*b = \min(2a-b, 2b-a)$

⇒ This operation is commutative because,

$a*b = \min(2a-b, 2b-a)$

$b*a = \min(2b-a, 2a-b)$

$\therefore a*b = b*a, \; \forall a,b \in I$

v) $a*b = LCM(a,b)$

⇒ This operation is commutative because,

$LCM(a,b) = LCM(b,a)$

$\therefore a*b = b*a, \; \forall a,b \in I$

vi) $a * b = a/b$

$\Rightarrow$ This operation is <u>not commutative</u> because,

$a * b = a/b$

$b * a = b/a$

$\therefore a * b \neq b * a$

vii) $a * b = $ power (a, b)

$\Rightarrow$ This operation is <u>not commutative</u> because,

power $(a, b) \neq$ power (b, a) i.e. $a^b \neq b^a$

$\therefore a * b \neq b * a$

viii) $a * b = a^2 + 2b + ab$

$\Rightarrow$ This operation is <u>not commutative</u> because,

$a * b = a^2 + 2b + ab$

$b * a = b^2 + 2a + ab$

$\therefore a * b \neq b * a$

Que.8] a] The set Z of all integers with binary operation $*$ defined by $a*b = a+b+1$ such that for all a,b belonging to Z, Is $(Z,*)$ an Abelian Group?

⇒ Ans :

To determine if the set Z of all integers with binary operation $*$ defined by,

$$a*b = a+b+1$$

forms an Abelian group, we need to check group properties.

i) Closure :-

For all $a,b \in Z$, $a*b = a+b+1 \in Z$

$\therefore$ $*$ holds closure property.

ii) Associativity :-

$$a*(b*c) = a*(b+c+1)$$
$$= a+(b+c+1)+1$$
$$= a+b+c+2 \quad \text{——} (1)$$

$$(a*b)*c = (a+b+1)*c$$
$$= a+b+1+c+1$$
$$= a+b+c+2 \quad \text{——} (2)$$

From (1) & (2), we get

$a*(b*c) = (a*b)*c$

$\therefore$ $*$ holds Associativity.

iii) Identity Element :-

$a*e = e*a = a$, $\forall a \in Z$

$\therefore a+e+1 = e+a+1 = a$

$\therefore e = -1$

$\therefore -1$ is an identity element.

iv) Inverse Element :-

$$a*a^{-1} = a^{-1}*a = e$$

$$\therefore a+a^{-1}+1 = e$$
$$\therefore a+a^{-1}+1 = -1$$
$$\therefore a+a^{-1} = -2$$
$$\therefore a^{-1} = -2-a$$
$$\therefore a^{-1} = -a-2$$

$\therefore$ Inverse exists for $a \in Z$.

v) Commutativity :-

$$a*b = a+b+1$$
$$b*a = b+a+1$$

$\therefore a*b = b*a \;,\; \forall a,b \in Z$

$\therefore$ $*$ is commutative.

Hence, from all above properties, we conclude that $(Z,*)$ is an Abelian group.

---

b] Let $Z_n = \{0,1,2,\dots\dots,n-1\}$. Construct multiplication table for $n=6$. Is $(Z_n,+,*)$ an integral domain? where $*$ is a binary operation of Z_n such that $a*b$ = remainder of $a*b$ divided by n & $a+b = a+b \pmod n$.

⇒ Ans :

Here,

$$Z_n = \{0,1,2,\dots\dots,n-1\}$$
$$n = 6$$

∴ To construct multiplication table for $Z_6 = \{0,1,2,3,4,5\}$ with the operation $a * b = (a \cdot b) \bmod 6$, we can create table where each entry (a,b) contains result of $a*b$.

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Now Let's analyze whether $(Z_6, +, *)$ is an integral domain.
An Integral Domain is commutative ring with unity (a ring where mult. is commutative & has mult. identity) and no zero divisors.

Commutative Ring with Unity :-

i) Addition operation $(+) \Rightarrow Z_6$ is closed under addition, is associative & has identity element (0). It is commutative.

ii) Multiplication operation $(*) \Rightarrow Z_6$ is closed under multiplication, is associative & has identity element (1). It is commutative.

Zero Divisors :-

Zero Divisors are elements a & b such that $a \neq 0$, $b \neq 0$ & $a \cdot b = 0$. In this case, $2*3=0$, $3*2=0$; $2 \neq 0, 3 \neq 0$

Therefore, $(Z_6, +, *)$ is not an Integral Domain because it contains zero divisors.

c] Define and give examples.

i) Ring.

⇒ Ans:

Let R be a non-empty set equipped with two binary operations called addition and multiplication & denoted by '+' and '·' respectively. Then, An Algebraic structure $(R, +, \cdot)$ is called as Ring if it satisfies following properties-

1] $(R, +)$ is an Abelian group.

2] $(R, \cdot)$ is a semigroup.

3] Distributive Property.

$a \cdot (b+c) = a \cdot b + a \cdot c$ ----(Right distribution law)

$(a+b) \cdot c = a \cdot c + b \cdot c$ -----(Left distribution law)

e.g. $(Z, +, \cdot)$, $(2Z, +, \cdot)$, $(R, +, \cdot)$

ii) Ring Homomorphism.

⇒ Ans:

Let $(R, +, *)$ and $(S, +', *')$ be two rings.

A function $f: R \to S$ is called as Ring Homomorphism if for any $a, b \in R$,

1] $f(a+b) = f(a) +' f(b)$

2] $f(a * b) = f(a) *' f(b)$

iii) Integral Domain.

⇒ Ans:

A Commutative Ring without zero divisors is called as Integral Domain.

e.g. $(Z, +, \cdot)$, $(R, +, \cdot)$