

CON101 : Match Making

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1 Protocol for magic trick

- To communicate suit: As there are five cards and 4 suits, by pigeonhole principle there exists 2 cards with same suite. So select those 2 cards. Assistant can show any of them to the magician and hence he can communicate the suit of the card.
- To communicate number: again by pigeonhole principle there are 13 cards and hence there exists at 6 cards gap between a and b or b and a when arranged by wrapping up after Q card. Assistant shows the card which is behind by at most 6 cards. To communicate the exact number by which the hidden card from shown card we can assign a number to each ordering based on rank and suit. So magician needs to memorize 6 combinations.
- Hence final approach: Choose 2 cards with same suit. Display card which lags at most by 6 cards from other. Display remaining 3 cards in a order corresponding to gap between ranks. Magician simply adds corresponding number to displayed card.

2 Why same trick with 4 cards will not work

- The previous protocol will not work. As for determining suit we need at least 5 cards to ensure there is repetition of suits.
- Brute force approach: Consider maximum possible card choosing with 4 cards. Its $52C4$. On the other hand Maximum possible sequence generation with 3 cards is $52P3=52C3*3!$. Even if we assume that magician can remember all these sequences and assign a card corresponding to each we will have $52C4/52P3=270725/132600=3$ cards per sequence. Hence there will be $1/3$ uncertainty to predict the correct card.
- Hence not possible with just displaying 3 cards with some order. We also need some other information to be communicated.