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Comparison of linear, nonlinear, and network flow programming techniques in fuel scheduling☆

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Abstract

The objective of fuel scheduling is to minimize the operating cost over a given time period while satisfying several constraints including fuel supply, fuel inventory, total power requirement, and generating unit power limits. This paper presents a comparison of network flow programming (NFP) the out-of-kilter method, the linear programming (LP) revised simplex method, and the nonlinear programming (NLP) generalized reduced gradient method in order to solve the fuel scheduling problem. For application in NFP and LP, a separable programming technique is used to linearize the objective function and the constraints. Results based on sample data obtained from Omaha Public Power District are presented to demonstrate the differences between NFP, LP, and NLP application to this problem.

Keywords: Fuel scheduling; Programming techniques

1. Introduction

Fuel scheduling is a vital part of electric utility operation and planning. The fuel used by a generating unit may be obtained from various contacts at various prices. Moreover, the price of fuel is dependent upon availability and demand. Fuel contracts are generally under a take-or-pay agreement. These contracts include both a minimum and a maximum limit on delivery of fuels over the life of the contract. The fuel inventories are usually within a specified range in order to allow for inaccurate load forecasts or the inability of suppliers to deliver on time. Fuel scheduling is composed of a hierarchy of long-term, mid-term, and short-term scheduling, that yields a satisfactory optimal strategy. In long-term fuel scheduling which is the subject of this paper, the scheduling period is divided into smaller subperiods (months or weeks) in order to obtain the 'optimal' fuel strategy.

The objective of fuel scheduling is to minimize the operating cost over a given time period while satisfying several constraints including fuel supply, fuel inventory, total power requirement, and generating unit power limits [1, 2].

The problem of fuel scheduling can be treated as a long-term (one year) [3], short-term (one week), daily [4, 5], or real-time dispatching [6]. In long-term fuel scheduling the study period is divided into subperiods and the optimal fuel use strategy is obtained for the subperiods while satisfying the constraints. Seymore et al. [3] used long-term fuel scheduling in an example problem when considering prespecified plant fuel requirements. This was done for each interval of the study period, and the resulting network flow was then solved. Lamont et al. [4] presented a daily fuel scheduling problem that considers the network flow algorithm. In order to satisfy the daily fuel supply limits of the contract and other constraints, this algorithm uses hourly scheduling of mixed fuels to units.

2. Methodology

To solve the long-term fuel scheduling problem, the period under study is divided into several time periods that can vary from a few minutes to as long as a year. The objective function of the fuel scheduling problem is to minimize the total system fuel cost over the study period by optimally allocating fuels from various contracts on fuel supply, fuel consumption, and net system load.

The linear programming solution will then consist of an objective function that is made up of the sum of the linear cost functions, each of which is a function of one or more variables from only one time interval. The fuel

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scheduling constraints are designed to help the utility to plan for transportation and storage of the fuel. The purpose of these constraints is to allocate fuel from suppliers to the plants and to satisfy plant energy requirements and inventory levels. These constraints will be linear functions of variables such as the power output of the units and fuel delivered to units.

2.1. Problem formulation

Assuming that all generating units are available and remain on-line for T periods, the fuel scheduling problem can be expressed as minimizing the system operating cost over the T periods such that the constraints for generation–load, fuel supply–delivery, and fuel consumption–inventory for any given period as well as maximum–minimum power output limit constraints for each generating unit are satisfied. The fuel scheduling problem can then be written mathematically as

$$\text{minimize} \quad F_T = \sum_{i=1}^{N} \sum_{j=1}^{T} F_i[P_i(j)] \tag{1}$$

subject to

$$\sum_{i=1}^{N} P_i(j) = L(j) \qquad j = 1, 2, \ldots, T \tag{2}$$

$$\sum_{i=1}^{K} D_i(j) = S(j) \qquad j = 1, 2, \ldots, T \tag{3}$$

$$\sum_{m=1}^{K} \left[V_m(j) + D_m(j) - \sum_{i=1}^{N} q_i(j) \right] = V_m(j+1) \tag{4}$$

$$P_i^{\mathrm{MIN}} \leqslant P_i \leqslant P_i^{\mathrm{MAX}} \tag{5}$$

The heat rate curves of the generating units, $H_i[P_i(j)]$, are normally nonlinear or piecewise linear functions. As a result, the fuel consumptions $q_i(j)$ are not linear functions. The cost curve of each generating unit, $F_i[P_i(j)]$, is the product of the heat rate curve and the fuel cost (\$/MBtu[1]) of each generating unit. In order to solve the fuel scheduling problem by linear programming, the objective function and the constraints must all be linear functions. In order to linearize these functions, a separable programming technique is utilized.

2.2. Nonlinear programming

A nonlinear programming problem is to find an extremum of an objective function $f(\boldsymbol{x})$ subject to equality and/or inequality constraints in which either $f(\boldsymbol{x})$, or at least one of the functions appearing in the constraint set, or both, is a nonlinear function. In this paper the generalized reduced gradient (GRG) algorithm is utilized to find the optimum solution. The GRG algorithm is an extension of the Wolfe algorithm [7] to accommodate both a nonlinear objective function and nonlinear constraints. The concept of generalized reduced gradient (GRG) was first introduced in the 1960s by Abadie [7, 8]. Since then, other versions of GRG have been developed and implemented [9]. In general, the optimization problem to be solved can be written mathematically as

$$\text{minimize} \quad f(\boldsymbol{x}) \tag{6}$$

subject to

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{b} \tag{7}$$

$$\boldsymbol{c} \leqslant \boldsymbol{h}(\boldsymbol{x}) \leqslant \boldsymbol{d} \tag{8}$$

$$\boldsymbol{l} \leqslant \boldsymbol{x} \leqslant \boldsymbol{u} \tag{9}$$

By introducing the appropriate slack variables, the inequality constraints can be converted to equalities. Therefore, the problem at hand includes only equality constraints. In the fuel scheduling problem the inequality constraint might be the amount of fuel supplied or stored.

The GRG algorithm is initiated at a feasible point and the gradient of f at the current point is computed. If the current point is close enough to being optimal, then stop. Otherwise, a search direction using the gradient is computed. Then it is determined how far to move along this direction starting from the feasible point. A new point is established and the above procedure is repeated.

2.3. Linear programming

A linear programming problem is one in which a linear objective function is to be minimized or maximized subject to a set of linear constraints. There are several algorithms available to solve the linear programming problem [10]. In this paper, the revised simplex method is used to solve this minimization problem. The method uses the same basic principles as the standard simplex method, but at each iteration it uses a reduced tableau which results in considerable saving in computer storage and time. If the linear programming problem has a feasible solution, then there must be a feasible region in which the objective function can move. The goal is to move the objective function line in the feasible region in such a way as to continually decrease the value of the objective function until a minimum is reached. In the fuel scheduling problem the objective function as well as the fuel consumption inventory constraints are nonlinear. Therefore, a separable programming technique is utilized to convert the nonlinear function to a piecewise linear function to be used by linear programming.

[1] 1 MBtu = 1055 MJ.

2.4. Separable programming

Using regression analysis, the incremental heat rate curve break points of each generating unit are fitted into a least-square parabola of the following form:

$$H_i = \alpha_0 + \alpha_1 P_i + \alpha_2 P_i^2 \tag{10}$$

The input–output cost of each generating unit is, therefore, a third-order polynomial obtained by integrating the corresponding incremental heat rate curve, and multiplying by its cost C_i and the number of hours per period.

Separable programming is used to linearize this third-order polynomial for the cost curve of each generating unit in the objective function and the fuel consumption for each generating unit in the constraint set of the problem [11]. Generally, in separable programming, the nonlinear function is approximated by a piecewise linear function. We assume that the following continuous nonlinear separable function of nth order is to be approximated by a linear function:

$$f(P) = aP^n + bP^{n-1} + \cdots + uP + v \tag{11}$$

The range of values for P can be divided into M segments of equal length. A linear equation for each of the segments connecting point k to $k+1$ can be determined by the following relationships:

$$\hat{f}(P) = f_k + \frac{f_{k+1} - f_k}{P_{k+1} - P_k}(P - P_k) \tag{12}$$

Any P between P_k and P_{k+1} can be expressed as a linear combination of P_k and P_{k+1}, that is,

$$P = \lambda_k P_k + \lambda_{k+1} P_{k+1} \tag{13}$$

and

$$\hat{f}(P) = \lambda_k f_k + \lambda_{k+1} f_{k+1} \tag{14}$$

such that

$$\lambda_k + \lambda_{k+1} = 1 \tag{15}$$

and

$$\lambda_k, \lambda_{k+1} \geqslant 0 \tag{16}$$

Similarly, for M line segments, any nonlinear separable function $f(P)$ can be approximated by

$$f(P) = \hat{f}(P) = \sum_{k=0}^{M} \lambda_k f_k \tag{17}$$

such that

$$\sum_{k=0}^{M} \lambda_k = 1 \tag{18}$$

and

$$\lambda_k \geqslant 0 \tag{19}$$

2.5. Network flow programming

Network flow programming is an application of the simplex method to network structured linear programming problems. This method performs the simplex operations directly on the network. Therefore, the method is a lot more efficient than the regular simplex approach.

The network consists of a finite set of nodes and a set of arcs joining pairs of nodes. With each node i, a number b_i is associated which represents the available supply of fuel (if $b_i > 0$) or the required demand (if $b_i < 0$). Associated with each arc (i, j) is the amount of flow on the arc, x_{ij}, and the unit shipping cost along the arc, c_{ij}. The minimum cost flow problem is to find a set of arc flows that minimizes a linear cost function subject to the demand constraint. Mathematically this problem can be written as

$$\text{minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij} \tag{20}$$

subject to

$$\sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = b_i \qquad i = 1, \ldots, m \tag{21}$$

$$x_{ij} \geqslant l_{ij} \qquad i, j = 1, \ldots, m \tag{22}$$

$$x_{ij} \leqslant u_{ij} \qquad i, j = 1, \ldots, m \tag{23}$$

In this paper, the model is solved by the 'out-of-kilter' method. This method is more efficient than the simplex algorithm since the solution procedure exploits the network structure of the model and its relationship to the complementary slackness condition of linear programs [12, 13]. This algorithm starts with a solution in which the conservation of flow is satisfied. It then identifies the out-of-kilter states and determines whether to increase or decrease the flows to bring these arcs into kilter such that the conservation of flow is maintained. This procedure is repeated until an optimal solution is obtained, if a feasible solution exists.

The piecewise linear cost curve of each generating unit is represented in the network by a set of parallel arcs, one for each segment of the cost function. The cost function of a generating unit is, therefore, given as

$$a_{ij} x_{ij}$$

if $\quad l_{ij} \leqslant x_{ij} \leqslant r_{ij}$

$$a_{ij} r_{ij} + a_{ij}^2 (x_{ij} - r_{ij})$$

if $\quad r_{ij} \leqslant x_{ij} \leqslant s_{ij}$

$$a_{ij} r_{ij} + a_{ij}^2 s_{ij} + a_{ij}^3 (x_{ij} - s_{ij})$$

if $\quad s_{ij} \leqslant x_{ij} \leqslant u_{ij}$

such that the cost coefficient $a_{ij} \leqslant a_{ij}^2 \leqslant a_{ij}^3$ and the scalar $l_{ij} \leqslant r_{ij} \leqslant s_{ij} \leqslant u_{ij}$.

Table 1
Description of nodes

Node	Description
S	source
FS	fuel suppliers
DP	delivery points
SF	storage facilities
G	generating units
P	plants
T	terminal

Table 2
Description of arcs

From node	To node	Description
S	FS	available supply
FS	DP	fuel provided to each delivery point
DP	SF	fuel stored
SF	G	fuel consumption by unit G
G	P	output power of each generating unit
P	T	required demand

The bounds on each parallel arc are

(r_{ij}, l_{ij}, a_{ij})

$(s_{ij} - r_{ij}, 0, a_{ij}^2)$

$(u_{ij} - s_{ij}, 0, a_{ij}^3)$

The fuel scheduling problem can be represented by the network shown in Fig. 1. The role of each node and arc in the network is described in Tables 1 and 2, respectively.

3. System study

Sample data were obtained from Omaha Public Power District (OPPD). The system comprises two coal fired plants. The first plant includes one dispatchable generating unit and the second plant includes five dispatchable generating units, resulting in a total of six dispatchable generating units in the system. The maximum and minimum power output of each generating unit and the coal cost ($/MBtu) are given in Table 3. The input–output characteristic of each generating unit is given in Table 4. The period under study was assumed to be three weeks and the load for each week was 800, 1000, and 1050 MW, respectively. It was assumed that all six units would be on-line for the three-week period. The coal supplier was to deliver a total of 50 000 tonnes of coal at the beginning of each week to the two power plants, and the initial inventories were assumed to be 40 000 and 90 000 tonnes at the beginning of the first week.

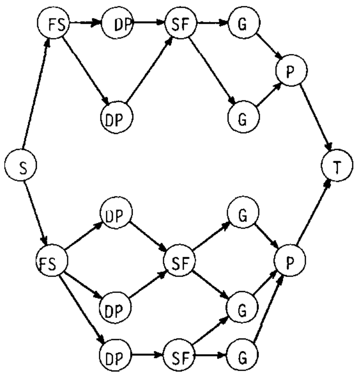

Fig. 1. Network representation of the fuel scheduling problem.

Table 3
Generating unit data

Unit	Plant	Min. (MW)	Max. (MW)	Cost ($/MBtu)
1	1	120	600	0.73
2	2	15	76	0.74
3	2	20	102	0.74
4	2	20	102	0.74
5	2	30	131	0.74
6	2	50	218	0.74

Table 4
Generalizing unit characteristics

Input–output cost curve ($/week)
$F(P_1) = 215 + 1020P_1 - 0.1943P_1^2 + 0.00152P_1^3$
$F(P_2) = 29.96 + 1133P_2 - 1.54P_2^2 + 0.1082P_2^3$
$F(P_3) = 39.16 + 1096P_3 - 1.195P_3^2 + 0.056P_3^3$
$F(P_4) = 39.03 + 1092P_4 + 1.223P_4^2 + 0.055P_4^3$
$F(P_5) = 52.8 + 1042P_5 - 0.8389P_5^2 + 0.0309P_5^3$
$F(P_6) = 81.5 + 1051P_6 - 0.5836P_6^2 + 0.0139P_6^3$

Table 5
Output results for week 1

	Number of linear segments			
	3	4	5	9
P_1	440.0	418.0	408.0	415.9
P_2	35.33	45.5	39.4	42.1
P_3	47.33	61.0	69.2	65.6
P_4	74.0	61.0	69.4	65.6
P_5	97.33	80.5	90.6	86.1
P_6	106.0	134.0	123.6	124.7
D_1	42550	11640	50000	38889
D_2	7445	38360	0	11111
V_1	40000	40000	40000	40000
V_2	90000	90000	90000	90000

Table 6
Output results for week 2

	Number of linear segments			
	3	4	5	9
P_1	535.0	494.5	504.0	506.1
P_2	55.67	60.75	51.6	55.7
P_3	74.67	81.5	85.6	83.8
P_4	74.67	81.5	85.6	83.8
P_5	97.33	105.75	110.8	108.6
P_6	162.0	176.0	162.4	162.0
D_1	0	12500	10000	27778
D_2	50000	37500	40000	22222
V_1	70176	40000	78870	67431
V_2	87568	117844	79091	90533

Table 7
Output results for week 3

	Number of linear segments			
	3	4	5	9
P_1	585.67	544.5	504.0	546.7
P_2	55.67	60.75	63.8	55.7
P_3	74.67	81.5	85.6	83.8
P_4	74.67	81.5	85.6	83.8
P_5	97.33	105.75	126.6	118.0
P_6	162.0	176.0	184.4	162.0
D_1	50000	50000	50000	0
D_2	0	0	0	50000
V_1	53333	42584	73792	80000
V_2	123848	140408	103942	97756

Table 8
Output results for week 1

	NLP	NFP	LP
P_1	408.0	408.0	408.0
P_2	43.7	39.0	39.4
P_3	63.7	69.0	69.2
P_4	64.4	69.0	69.4
P_5	88.0	91.0	90.6
P_6	132.2	124.0	123.6

Table 9
Output results for week 2

	NLP	NFP	LP
P_1	506.7	504.0	504.0
P_2	55.7	52.0	51.6
P_3	80.7	86.0	85.6
P_4	81.4	86.0	85.6
P_5	110.4	111.0	110.8
P_6	165.0	162.0	162.4

Table 10
Output results for week 3

	NLP	NFP	LP
P_1	530.6	520.0	504.0
P_2	59.7	64.0	63.8
P_3	85.2	86.0	85.6
P_4	85.9	86.0	85.6
P_5	116.6	111.0	126.6
P_6	172.0	184.0	184.4

Table 11
Comparison of total cost and CPU time

Method	No. of segments	Total cost ($)	CPU time (s)
LP	3	3779001	14
LP	4	3754123	17
LP	5	3740975	18
LP	9	3736273	35
NFP	5	3738266	NA[a]
NLP	NA[a]	3729585	111

[a]NA = not applicable.

The linear programming method was tested with a varying number of piecewise linear segments representing the nonlinear cost curves. In this study, three, four, five and nine line segments were used. The generating unit base points using different linear segments for different periods are given in Tables 5–7.

Results using NLP and NFP are given in Tables 8–10. The results of LP using five linear segments are also shown for comparison. All of the calculations were obtained on an IBM compatible PC-386, 20 MHz. The comparison of total system fuel costs and CPU time for different line segments is given in Table 11.

4. Conclusions

A comparison of linear, nonlinear, and network flow programming methods for solving the fuel scheduling problem has been presented. The LP and NFP methods require a linear objective function and constraints, therefore a separable programming technique was used to formulate the objective function and constraints as linear functions. The methods were tested by sample data obtained from OPPD. For LP and NFP, better results are achieved with more linear segments as an approximation of the cost curve, while the CPU time is moderately increased. The LP and NFP methods appear to be more suitable than the NLP with regard to the CPU time and convergence. The results of the LP and NLP methods were obtained by a production grade software, whereas the results of the NFP method were obtained by a research grade software, therefore a meaningful comparison of the CPU times cannot be made among the three methods.

5. Nomenclature

a_{ij}^k	cost coefficient of arc (i, j) on kth segment
b_i	available supply at node i
c_{ij}	generating unit cost along arc (i, j)
C_i	fuel cost for generating unit i (\$/MBtu)
$D_m(j)$	fuel delivered to plant m during jth period (tonnes)
$f(\mathbf{x})$	objective function
$\hat{f}(P)$	piecewise linear approximation of $f(P)$
f_k	cost function corresponding to point k
F_{T}	total system fuel cost (\$)
$F_i[P_i(j)]$	cost curve of generating unit i during period j (\$/h)
$\mathbf{g}(\mathbf{x})$	vector of equality constraints
$\mathbf{h}(\mathbf{x})$	vector of inequality constraints
$H_i[P_i(j)]$	heat rate curve of generating unit i during period j (MBtu/h)
K	number of plants
l_{ij}, u_{ij}	lower and upper bound on flow
$L(j)$	load at period j (MW)
M	number of segments
N	total number of generating units
$P_i(j)$	power output of ith unit during jth period (MW)
P_i^{MAX}	maximum power output for unit i (MW)
P_i^{MIN}	minimum power output for unit i (MW)
$q_i(j)$	fuel consumption for unit i during jth period (tonnes/h)
$S(j)$	fuel delivered at period j (tonnes)
T	number of periods
$V_i(j)$	initial inventory of mth fuel supply at start of jth period (tonnes)
$\mathbf{x}$	vector of variables
x_{ij}	amount of flow on arc (i, j)

Greek letters

$\alpha_0, \alpha_1, \alpha_2$	cost curve coefficients
λ_k	fraction of total power output in domain of interest

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