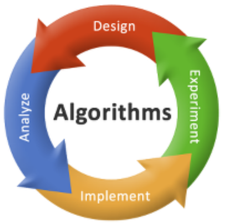
Small World Networks – A Simple Overview

Course: Algorithms



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Small World Network – A Simple Overview

This lecture covers the interesting aspects of small world and scale free networks. We will also look at the practical applications with their limitations.

Dijkstra’s Algorithm

- Single Source Shortest Path Algorithm proposed by Dijkstra in 1956 (Originally conceived)

- Basic Idea:
  - Two sets are maintained:
  - one set contains vertices included in shortest path tree and the other set includes vertices not yet included in shortest path tree
  - At every step of the algorithm, we find a vertex which is in the other set (set of not yet included) and has a minimum distance from the source

- Similar to Prim’s algorithm for MST

Key Aspect in Bellman-Ford

- Dynamic programming
  - Explore All possible solutions and find the best solution to the given problem
  - Relaxation criteria

  Consider an edge connecting a pair of vertices: (u, v)

  If (d[u] + c[u,v] < d[v]) then
  d[v] = d[u] + c[u,v]

  How many times do we relax all edges?
  - (n-1) times (Why?)
  - the longest possible path connecting n edges

Floyd – Warshall Algorithm

- For every pair (i, j) of vertices, there are two cases:
  - k is not an intermediate vertex in shortest path: i → j
    We keep the value of dist[i][j] unchanged

  - k is an intermediate vertex in shortest path: i → j
    Update the value of dist[i][j] as follows:

```
if dist[i][j] > dist[i][k] + dist[k][j] then
        dist[i][j]  = dist[i][k] + dist[k][j]
```

- Choose the **minimum** and store it in dist[i][j]

- Explore optimal substructure property in the all-pairs shortest path problem

Floyd–Warshall: Fact

- We follow the simple relaxation formula:

$$M^k[i, j] = \min\{M^{k-1}[i, k], M^{k-1}[i, k] + M^{k-1}[k, j]\}$$

- Code:

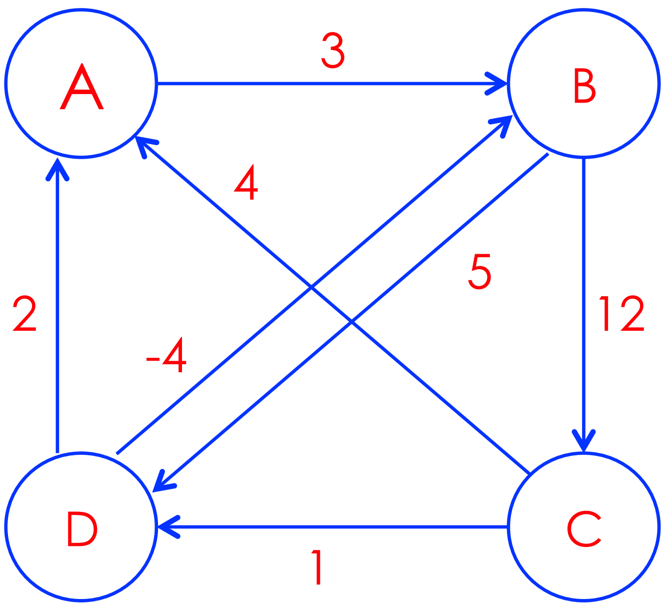
```
for (k =1; k <=n; k++) {
      for (i =1; i <=n; i++) {
            for (j =1; j <=n; j++) {
                  M[i,j] = min{ M[i,j], M[i,k] + M[k,j]}
            }
      }
}
```

Exercise 1

- Compute All Pairs Shortest Paths

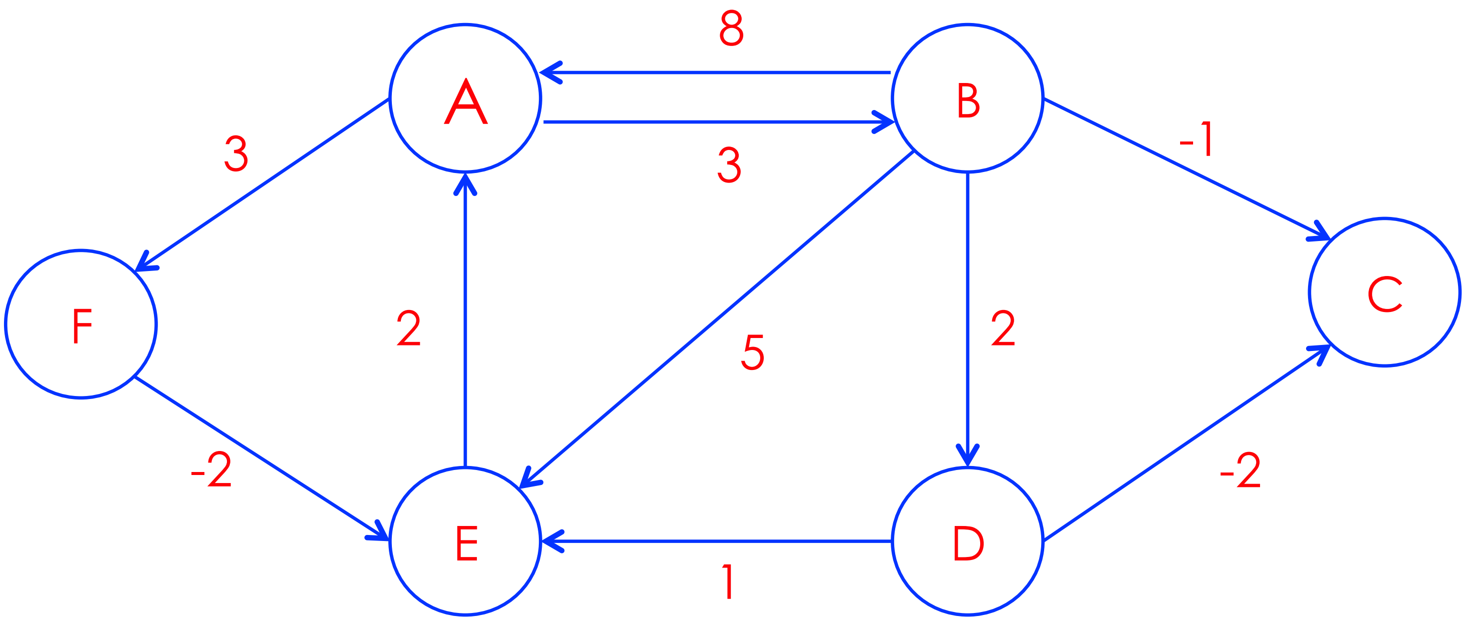
- The Final Solution

	A	B	C	D
A	0	3	∞	7
B	8	0	2	∞
C	5	∞	0	1
D	2	∞	∞	0



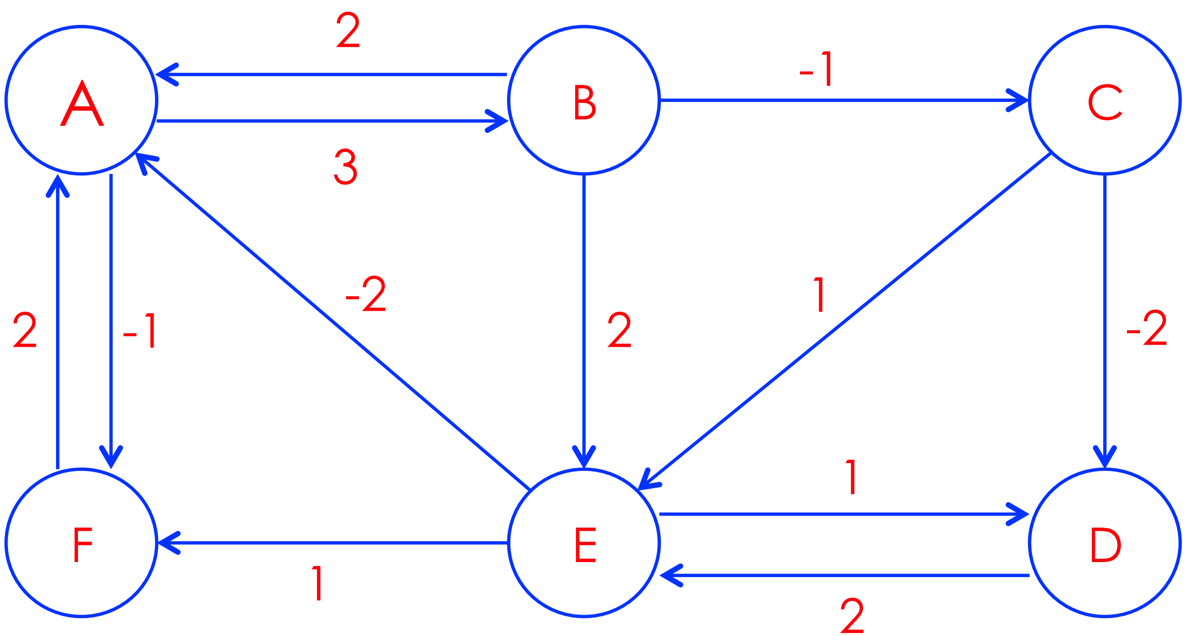
Exercise 1

- Compute All Pairs Shortest Paths



Exercise 3

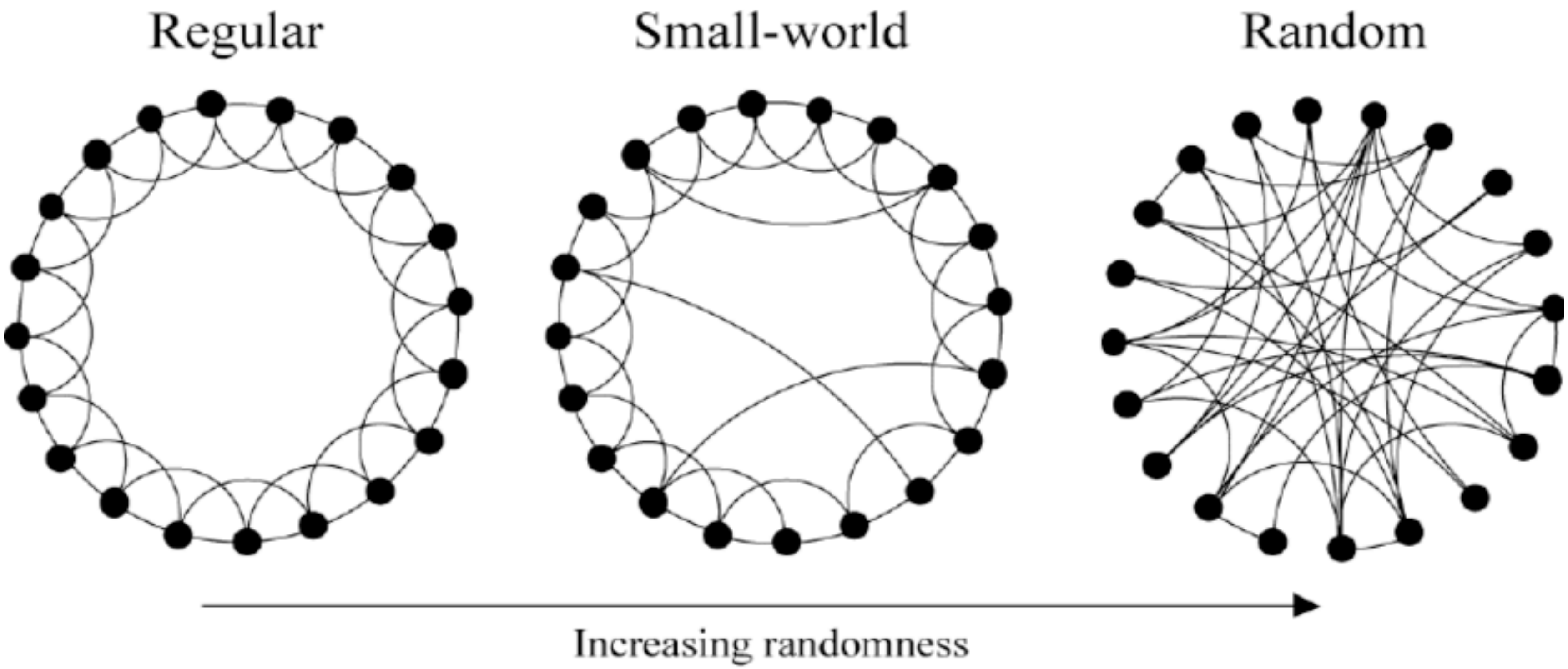
- Compute All Pairs Shortest Paths



Small World Networks

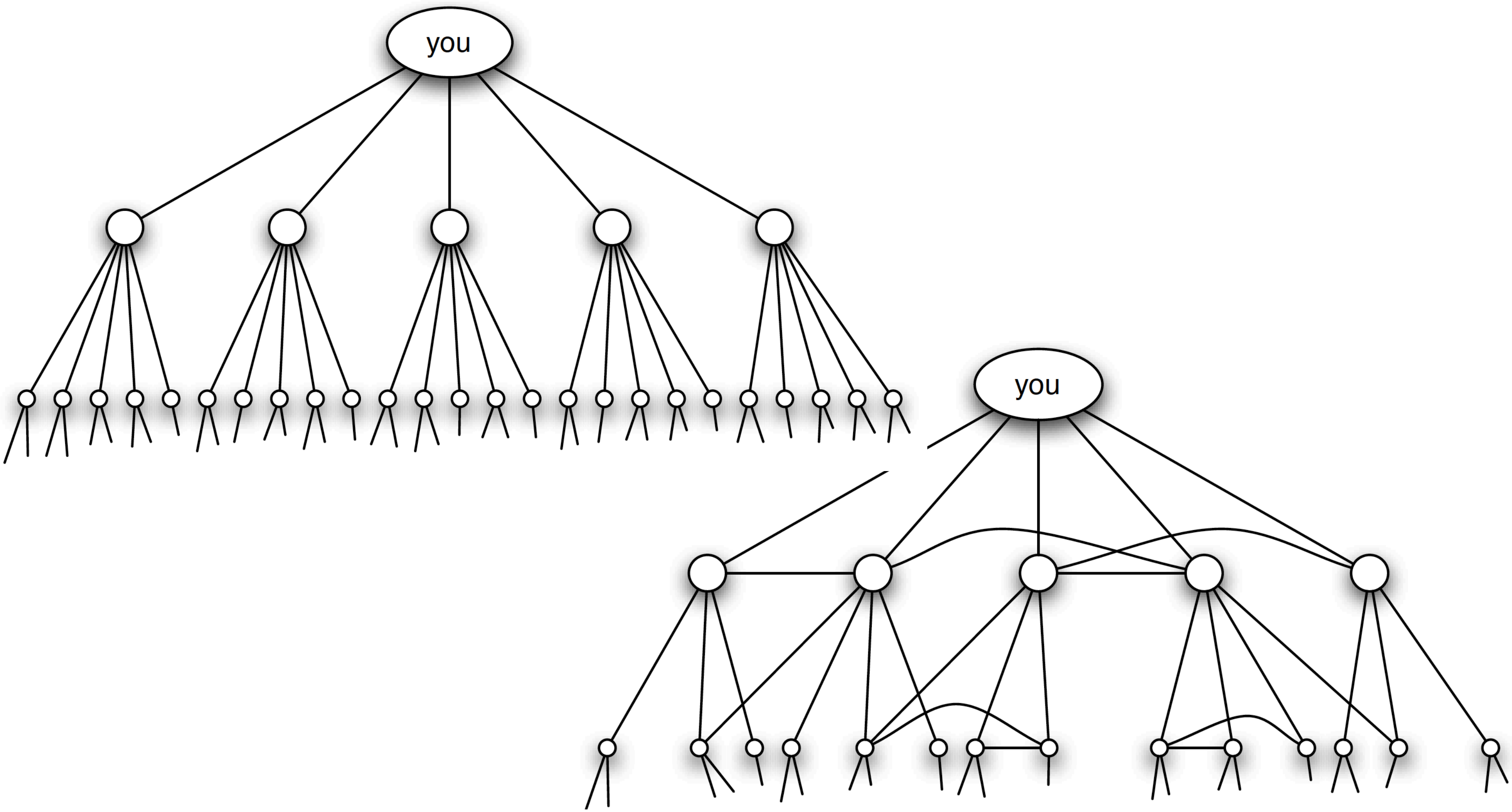
- Type of Mathematical Graph
  - Most nodes are not neighbors of one another
  - but the neighbors of any given node are likely to be neighbors of each other
  - Most nodes can be reached from every other node by a small number of hops or steps

Small World Networks



- Six Degrees of Separation

Small World Networks



Small World Net – Examples

Spread of Infectious Disease

- Type of Distributed Dynamic System
- Disease spreads from a small set of initiators to a much larger population

- At time (t = 0), single infective introduced into a healthy population

- After 1 unit of time, infective is “removed
  - (dies or becomes immune), but in that interval can infect (with some probability) each of its neighbors

Small World Net – Examples

Spread of Infectious Disease (contd)

- Three distinct regimes of behavior :
  - Diseases with Low infectious ness (Infects Little population, then dies)
  - Diseases with High infectious ness (Infects Entire population, function of 'L' !!)
  - Diseases with Medium infectious ness (Complicated relationship between Structure and Dynamics, not completely characterized

Small World Networks

Properties

- High clustering coefficient
- Most pairs of nodes will be connected by at least one short path

- Airline Flight Networks:
  - A small mean-path length
  - Between any two cities, one can likely to take three or fewer flights
  - Why? many flights are routed through hub cities

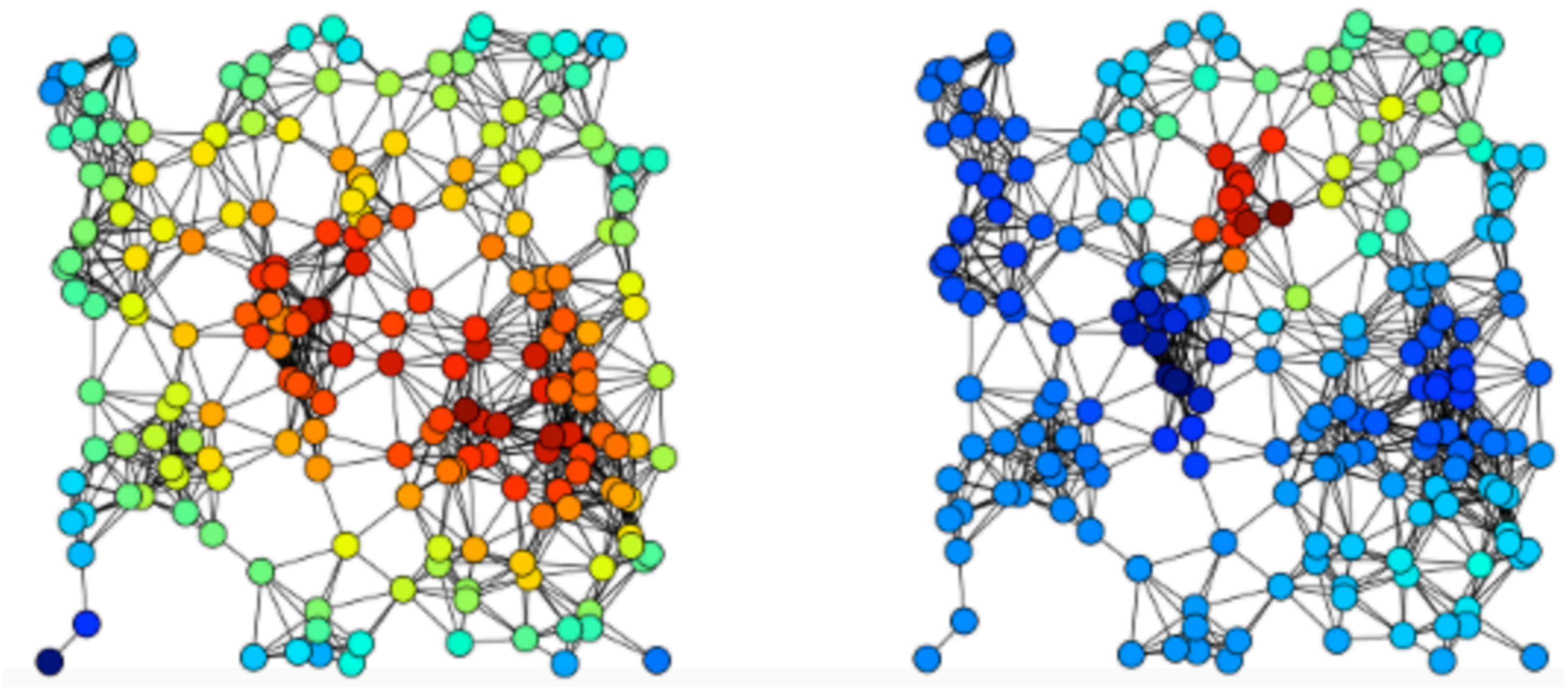
Metrics

Small-world properties are found in many real-world phenomena:

- Degree Centrality
- Degree Distribution
- Betweenness Centrality
- Closeness
- Motif
- Clustering Coefficient
- Degree distribution
- Assortativity
- Distance Modularity
- Efficiency

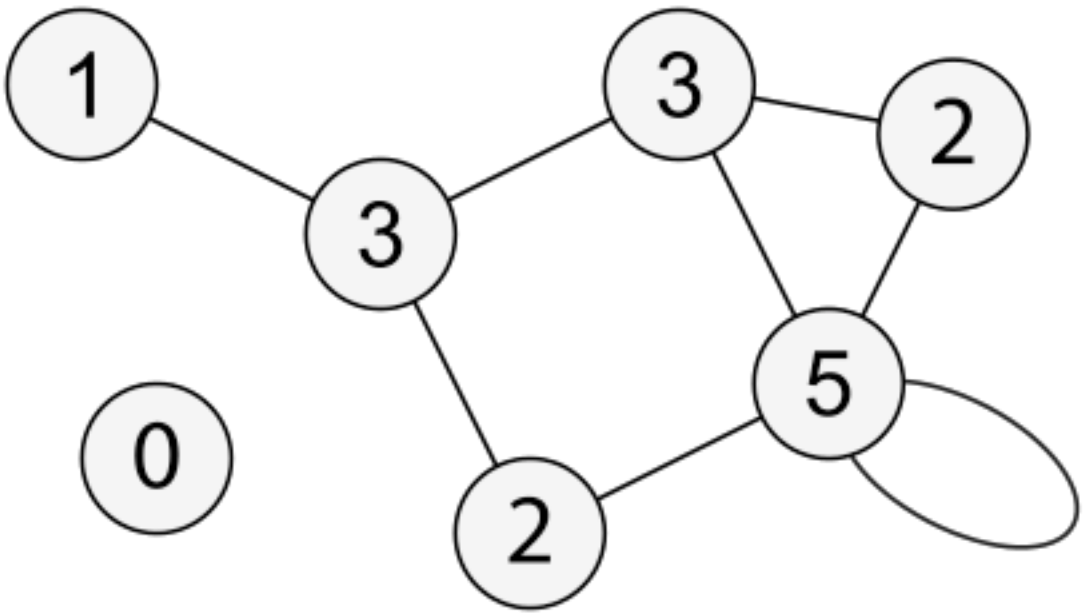
Centrality

Finding the most important vertices:



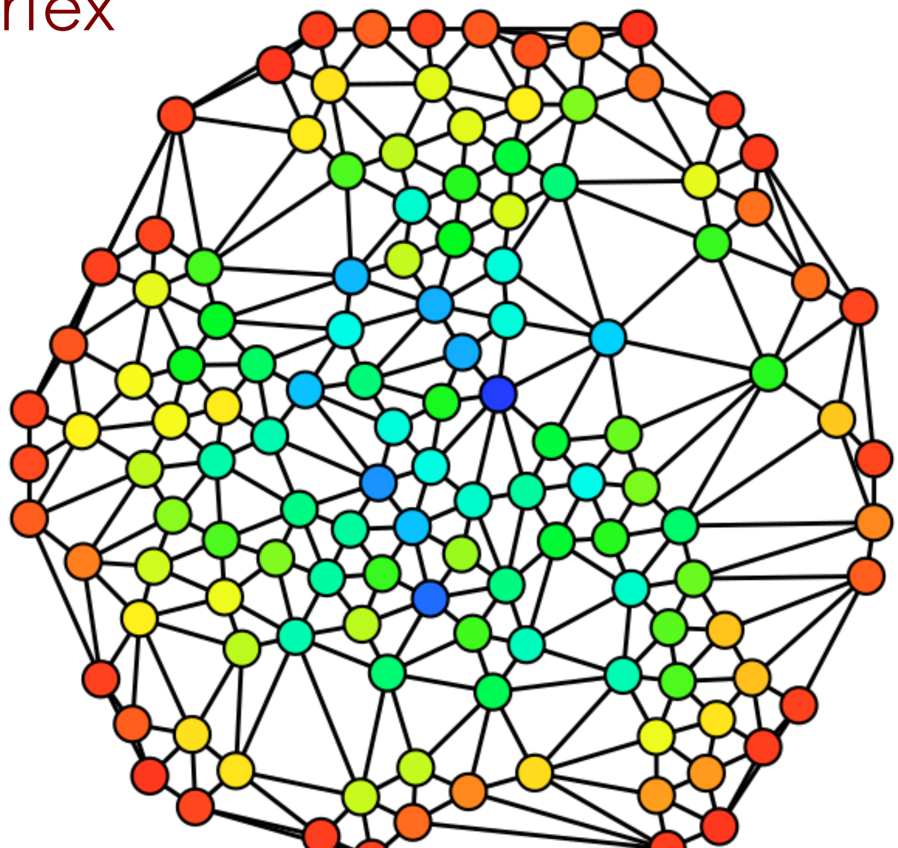
Degree

Finding the number of edges incident on a vertex



Betweenness

The number of the shortest paths that pass through each vertex



Degree Distribution

The probability distribution of these degrees over the whole network

- Two different degrees (Directed Graphs):
  - in-degree - the number of incoming edges, and
  - out-degree - the number of outgoing edges.

- The degree distribution P(k) of a network is defined to be the fraction of nodes in the network with degree k
- If there are n nodes in total in a network and

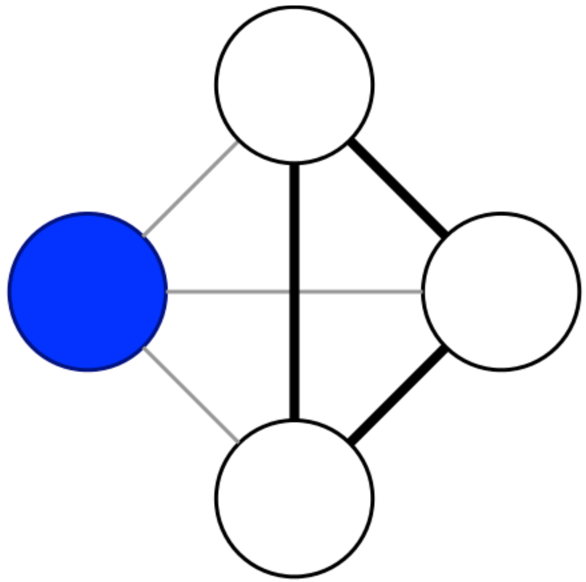
  n_k of them have degree k,

  Then $P(k) = n_k/n$

Clustering Coefficient

A measure of the degree to which nodes in a graph tend to cluster together

- C = 1



A Few Examples

Small-world properties are found in many real-world phenomena:

- Websites with navigation menus
- Food webs
- Electric power grids
- Metabolite processing networks
- Networks of brain neurons
- Voter networks
- Telephone call graphs and
- Social influence networks

- Cultural networks
- Word co-occurrence networks and so on

Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)

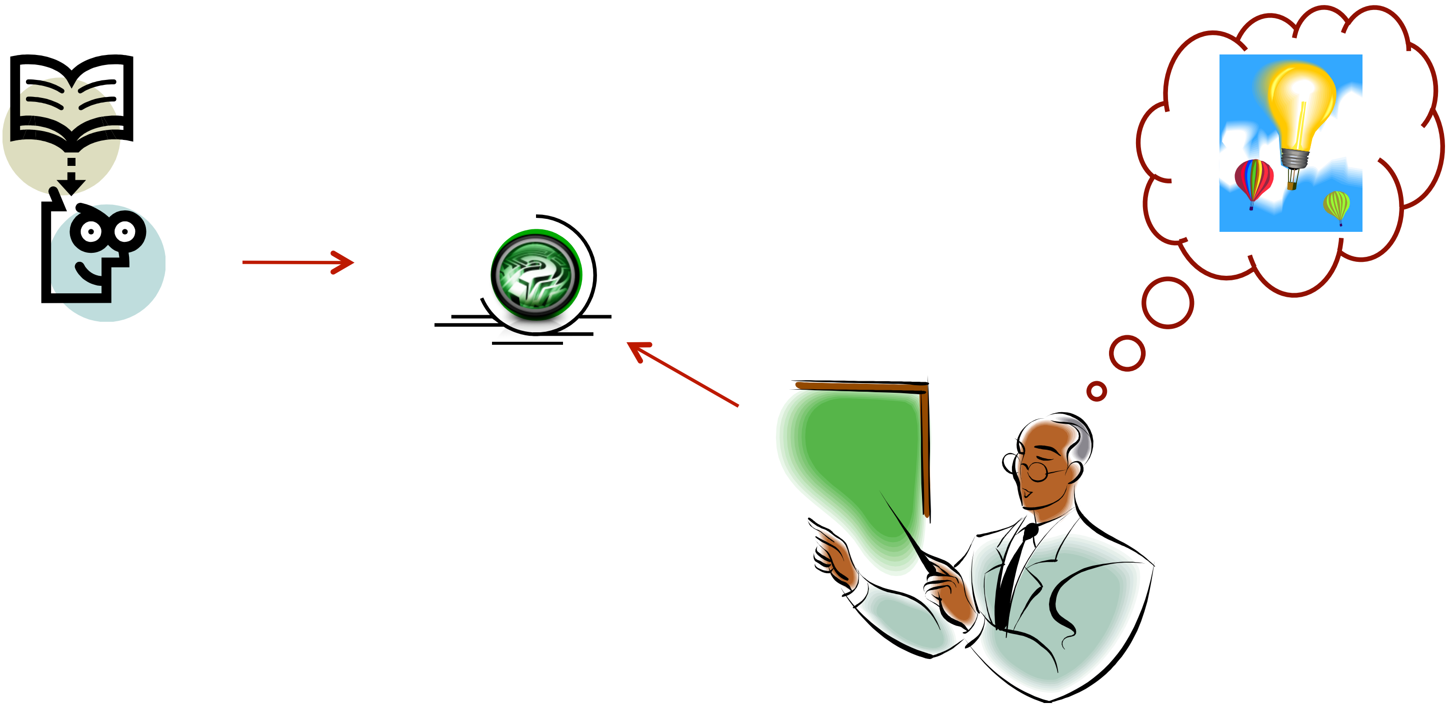
- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)

- You may grow a culture of **collaborative learning** by helping the needy students

Assistance

- You may post your questions to me at any time
- You may meet me in person on available time or with an appointment
- TA s would assist you to clear your doubts.
- You may leave me an email any time (email is the best way to reach me faster)

Thanks ...



... Questions ???