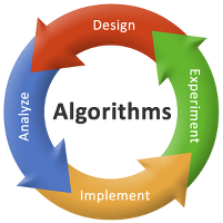
Assignment – Transportation Algorithms

Course: Algorithms

Faculty: Dr. Rajendra Prasath



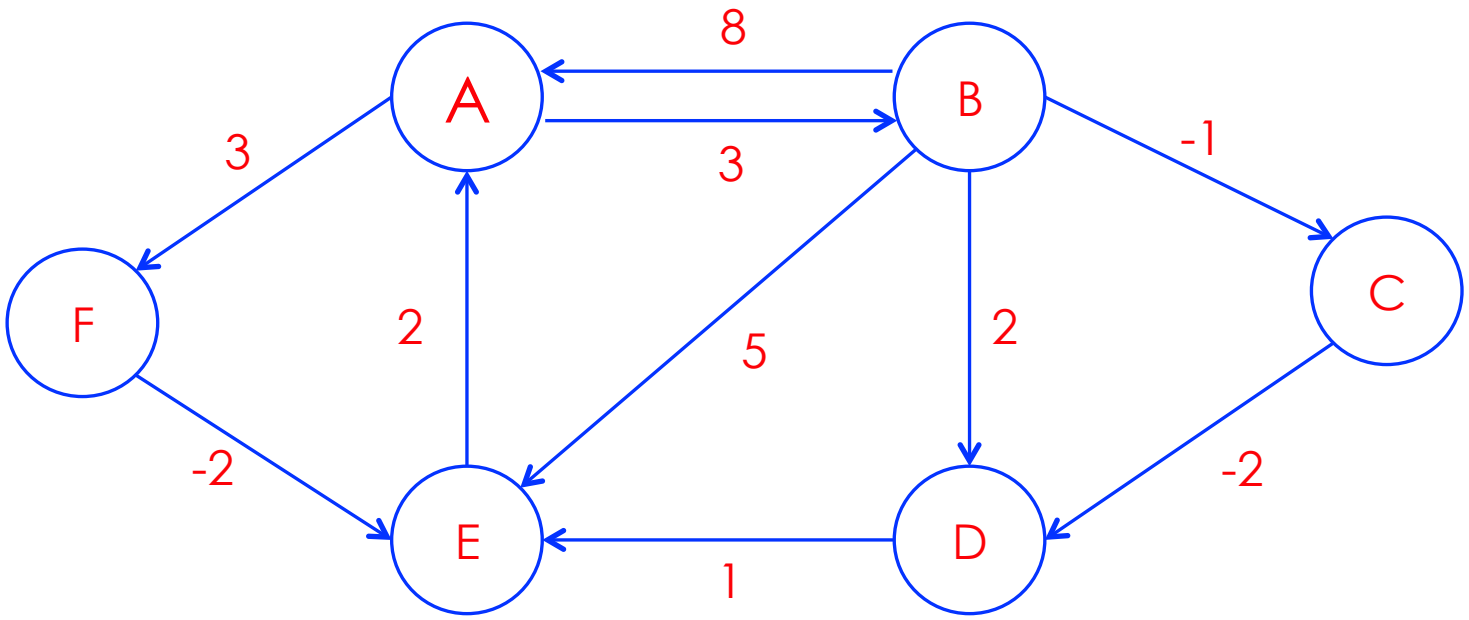
Autumn 2018

An Overview of Assignment and Transportation Algorithms

This lecture covers an interesting overview of assignment and transportation algorithms. In this class of algorithms, we will explore Assignment problems and Transportation problems. We will look at one algorithm each and their illustrations with many examples.

Recap: Floyd-Warshall Algo

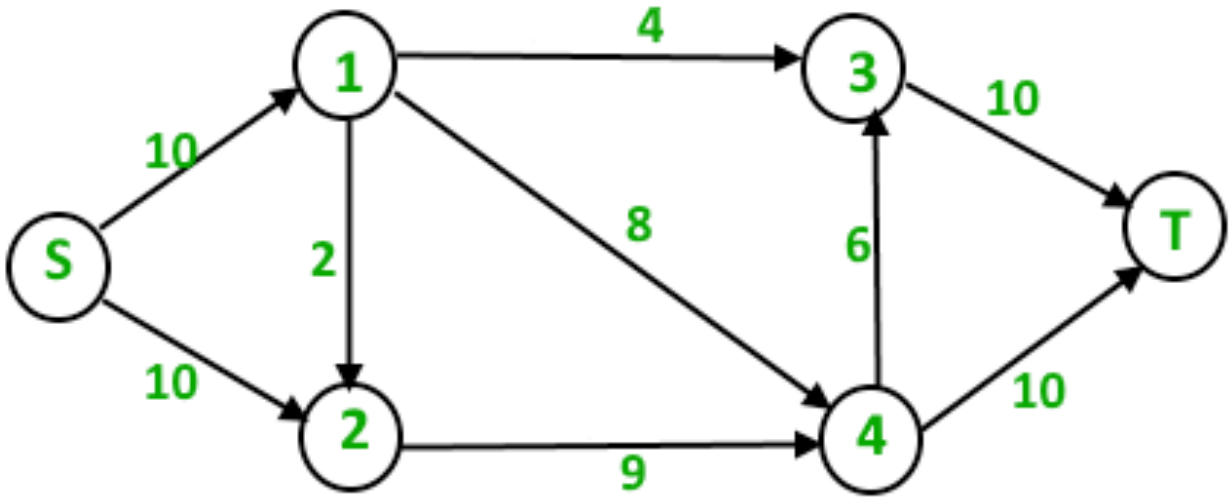
- **Compute All Pairs Shortest Paths**
  - Did you work our this for different ordering?
    - **For Example, Consider the ordering: B, D, E, F, C, A**



Network Flow Algorithms

An Example

- **S = Source** Node
- **T = Sink** Node
- Several other nodes connected with edges



Recap:Optimization Problems

Optimization is the main concern in this class of problems:

- Given the data, how do we manage the tasks in an optimized manner?
- How do we maximize the outcome with the given resources and with the imposed constraints

Popular Algorithms:

- Ford – Fulkerson's Algorithm (Max Flow Algorithm)
- Hungarian Method (Assignment Problem)
- North West Cornet Rule (Transportation Problems)
- Simplex Method (for LPP)

Assignment Problems

Problem Definition

- Given a set of tasks to be carried out by a set of employees
- Find an assignment that minimizes the overall expense when different employees may cost different amounts based upon the task to which they are assigned

- Popular Approach:
  - Hungarian Method

Assignment Problem

- Assign n persons to n jobs
  - Obtain a Square matrix
- **Hungarian Method: Algorithm**

1. Subtract row minima – subtract the smallest entry in each row
2. Subtract column minima - Subtract the smallest entry in each column
3. Cover all zeros with the minimum number of lines
4. If the number of lines is equal to the number of rows in your square matrix, stop here, otherwise goto next step
5. Create additional zeros - Find the smallest element, call it c, that is not covered by a line. Subtract c from all uncovered elements in the matrix and add it to any element that is covered twice. Go back to 3rd step

Hungarian Method: Exercise 1

- How to solve this assignment problem?

Jobs →

Persons →

	1	2	3	4
A	90	75	75	80
B	35	85	55	65
C	125	95	90	105
D	45	110	95	115

Hungarian Method: Exercise 2

- How to solve this assignment problem?

Jobs →

Persons →

	1	2	3	4
A	20	35	47	102
B	15	28	52	95
C	78	21	13	112
D	113	84	112	309

Exercise - 3

- How to solve this unbalanced assignment problem?

Jobs →

	1	2	3
A	482	437	512
B	421	399	432
C	502	407	518
D	414	402	411

Persons →

Exercise - 4

- How to solve this unbalanced maximization assignment problem?

Machines →

Job ↓	A	B	C	D	E
1	5	7	11	6	7
2	8	5	5	6	5
3	6	7	10	7	3
4	10	4	8	2	4

Transportation Algorithms

Problem Definition

- Determine the most cost-effective way to ship goods from a set of supplying factories to a set of retail stores selling these goods.
- Units flow from Supply nodes (factories) to Demand nodes (retail shops)
- Popular Approach:
  - North West Corner Rule

NorthWest Corner Rule

Transportation Problem

- Applied to a special type of Linear Programming Problem
- Used to compute a basic feasible solution of a transportation problem
- Balanced Demand – Supply Problem

North West Corner Rule (contd.)

Steps in the Algorithm:

- Select the upper left-hand corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand
- Adjust the supply and demand numbers in the respective rows and columns
- If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column
- If the supply for the first row is exhausted, then move down to the first cell in the second row
- If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column
- Continue the process until all supply and demand values are exhausted

NWCR - Example

- How to solve this Transportation problem?

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1	2	1	4	30
F2	3	3	2	1	50
F3	4	2	5	9	20
Demand	20	40	30	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2	1	4	~~30~~ 10
F2	3	3	2	1	50
F3	4	2	5	9	20
Demand	~~20~~	40	30	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~
F2	3	3	2	1	50
F3	4	2	5	9	20
Demand	~~20~~	~~40~~ 30	30	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~
F2	3	3 (30)	2	1	~~50~~ 20
F3	4	2	5	9	20
Demand	~~20~~	~~40~~ ~~30~~	30	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~
F2	3	3 (30)	2 (20)	1	~~50~~ ~~20~~
F3	4	2	5	9	20
Demand	~~20~~	~~40~~ ~~30~~	~~30~~ 10	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~
F2	3	3 (30)	2 (20)	1	~~50~~ ~~20~~
F3	4	2	5 (10)	9	~~20~~ 10
Demand	~~20~~	~~40~~ ~~30~~	~~30~~ ~~10~~	10	100

NWCR - Illustration

Warehouses →

Factories →

	W1	W2	W3	W4	Supply
F1	1 (20)	2 (10)	1	4	~~30~~ ~~10~~
F2	3	3 (30)	2 (20)	1	~~50~~ ~~20~~
F3	4	2	5 (10)	9 (10)	~~20~~ ~~10~~
Demand	~~20~~	~~40~~ ~~30~~	~~30~~ ~~10~~	~~10~~	100

Northwest Corner Rule (contd.)

Initial Feasible Solution:

- Number of basic variables

  $= m + n - 1 = 3 + 4 - 1 = 6$

- Initial basic feasible solution

  $20 \times 1 + 10 \times 2 + 30 \times 3$

  $+ 20 \times 2 + 10 \times 5 + 10 \times 9 = 310$

NWCR – Exercise 1

- How to solve this Transportation problem?

Shops →

Plants →

	S1	S2	S3	S4	Supply
P1	3	5	7	6	50
P2	2	5	8	2	75
P3	3	6	9	2	25
Demand	20	20	50	60	150

NWCR – Exercise 2

- Solve this problem with NWCR

Sites →

Cranes →

	S1	S2	S3	S4	S5	Supply
C1	3	2	4	7	2	80
C2	4	2	1	5	6	120
C3	7	2	3	1	5	140
C4	3	6	3	2	1	70
Demand	80	70	90	60	110	410

Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)

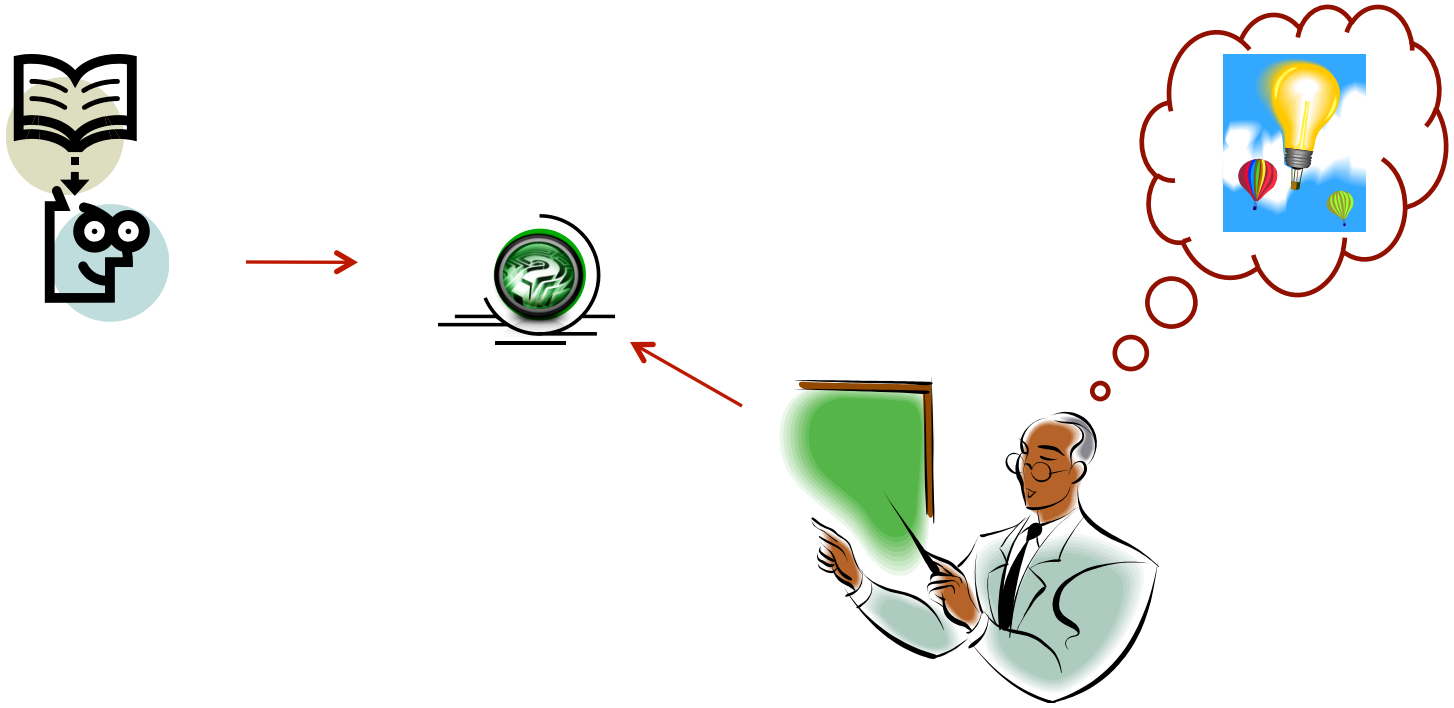
- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)

- You may grow a culture of **collaborative learning** by helping the needy students

Assistance

- You may post your questions to me at any time
- You may meet me in person on available time or with an appointment
- TA s would assist you to clear your doubts.
- You may leave me an email any time (email is the best way to reach me faster)

Thanks …



… Questions ???