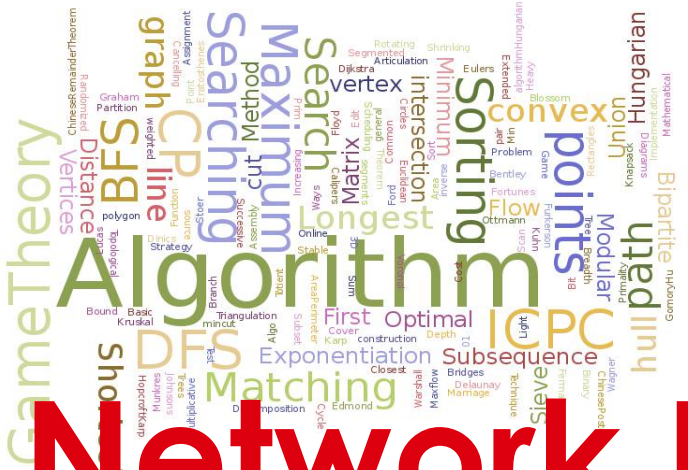
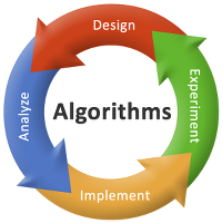


Network Flow Algorithms

Course: Algorithms

Faculty: Dr. Rajendra Prasath



Autumn 2018

An Overview of Network Flow Algorithms

This lecture covers an interesting overview of network flow algorithms. In this class of algorithms, we will explore problems like Maximum Flow, Bipartite Matching, Minimum Cost Flow, Transportation and Assignment Problems

Floyd – Warshall Algorithm

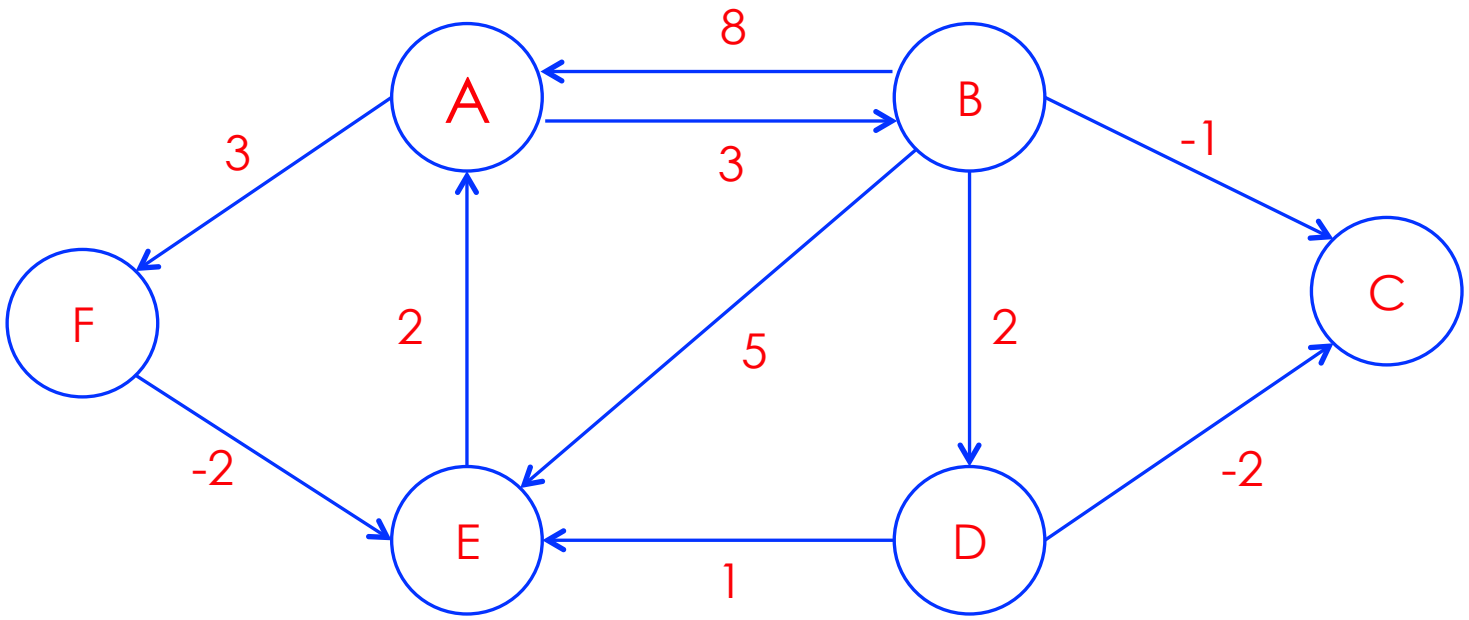
- For every pair (i, j) of vertices, there are two cases:
  - k is not an intermediate vertex in shortest path: i → j
    We keep the value of dist[i][j] unchanged
  - k is an intermediate vertex in shortest path: i → j
    Update the value of dist[i][j] as follows:

```
if dist[i][j] > dist[i][k] + dist[k][j] then
    dist[i][j]  = dist[i][k] + dist[k][j]
```

- Choose the **minimum** and store it in dist[i][j]
- Explore optimal substructure property in the all-pairs shortest path problem

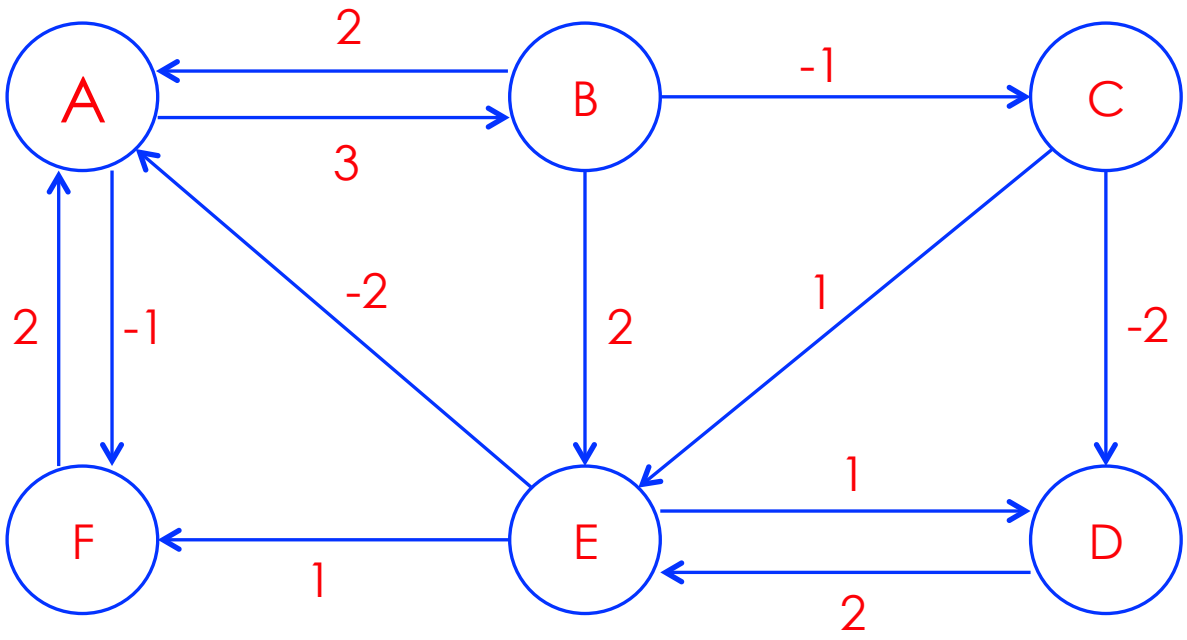
Exercise 1

- Compute All Pairs Shortest Paths



Exercise 3

- Compute All Pairs Shortest Paths



Small World Nets - Metrics

Properties are found in many Real-World Apps:

- Degree Centrality
- Degree Distribution
- Betweenness Centrality
- Closeness
- Motif
- Clustering Coefficient
- Degree distribution
- Assortativity
- Distance Modularity
- Efficiency

Small World Nets - Examples

Small-world properties are found in many real-world phenomena:

- Websites with navigation menus
- Food webs
- Electric power grids
- Metabolite processing networks
- Networks of brain neurons
- Voter networks
- Telephone call graphs and
- Social influence networks
- Cultural networks
- Word co-occurrence networks and so on

Network Flow Algorithms

An Overview

- Numerous problems can be modelled as graph problems
  - Nodes
  - Edges a capacity associated with each edge over which commodities flow

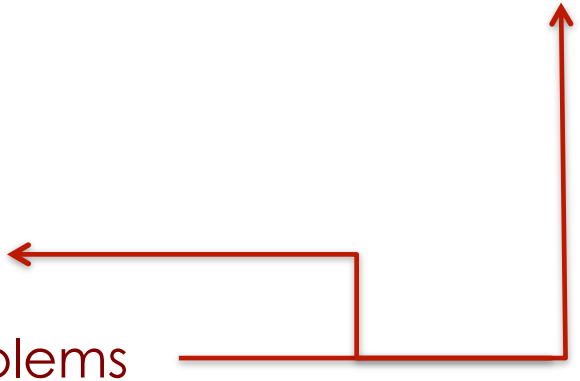
- Variations of Linear Programming Problems
  - Optimization Problems

- Real-World problems

Network Flow Algorithms

Problems in this Category:

- Assignment Problems
- Transportation Problems
- Bipartite Matching
- Maximum Flow
- Minimum Cost Flow
- Linear Programming Problems
  - Optimization subject to the constraints
  - Either Minimization of Maximization problem



Network Flow Algorithms

Linear Programming Problem

- General Purpose Solution using Simplex Method

Optimize the given objective function
z = Maximize 17.1667x + 25.8667y
subject to the constraints
13x + 19y <= 2400
20x + 29y <= 2100
x >= 10 and x, y >= 0

Find x and y that satisfies he above constraints:

Solution: x = 10 and y = 65.52 and z = 1866.45

Network Flow Algorithms

Define a Flow Network

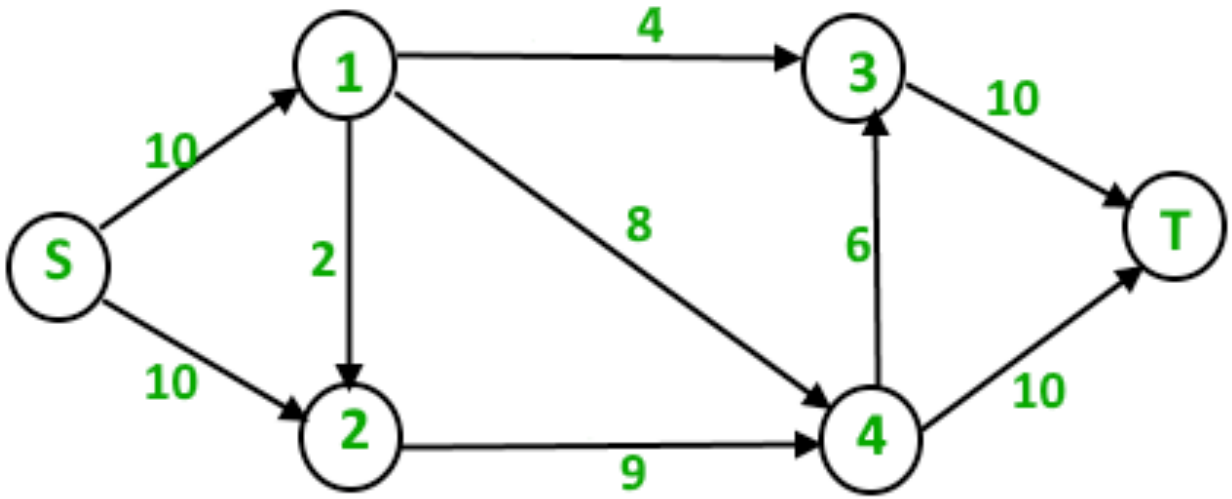
- A Flow Network is defined as a directed graph
  - **Source** Node
  - **Sink** Node and several other nodes connected with edges

- Each edge has an individual capacity
  - The maximum limit of flow that edge could allow

Network Flow Algorithms

An Example

- **S = Source** Node
- **T = Sink** Node
- Several other nodes connected with edges



Network Flow Algorithms

Maximum Flow Problem

- Given a network that shows the potential capacity over which goods can be shipped between two locations, compute the maximum flow supported by the network

Popular Algorithms:

- Ford – Fulkerson's Algorithm
- Dinic's algorithm

Network Flow Algorithms

Minimum Cost Flow

- Supply Nodes (S) produce units shipped over a network of distribution nodes to be consumed at demand nodes (T)
- Each edge has the following:
  - (low, high) capacity
  - An Actual Flow
  - Associated cost per unit flowing over the edge
- **Goal:**
  - Meet all demands and
  - minimize total cost of all edges

Network Flow Algorithms

Bipartite Matching Problem

- Source Nodes are to be matched with Sink Nodes
- Edge Capacity is uniform (weight = 1)

- **Goal:**
  - Maximize the number of pairs

Network Flow Algorithms

Transportation Problem

- Determine the most cost-effective way to ship goods from a set of supplying factories to a set of retail stores selling these goods.

- Units flow from Supply nodes (factories) to Demand nodes (retail shops)

- Popular Approach:
  - Northwest Corner Rule

Assignment Algorithms

Problem Definition:

- Given a set of tasks to be carried out by a set of employees
- Find an assignment that minimizes the overall expense when different employees may cost different amounts based upon the task to which they are assigned

- Popular Approach:
  - Hungarian Method

Assignment Problem

- Assign n persons to n jobs
  - Obtain a Square matrix

- **Hungarian Method: Algorithm**
  1. Subtract row minima – subtract the smallest entry in each row
  2. Subtract column minima - Subtract the smallest entry in each column
  3. Cover all zeros with the minimum number of lines
  4. If the number of lines is equal to the number of rows in your square matrix, stop here, otherwise goto next step
  5. Create additional zeros - Find the smallest element, call it c, that is not covered by a line. Subtract c from all uncovered elements in the matrix and add it to any element that is covered twice. Go back to 3rd step

Hungarian Method

- How to solve this assignment problem?

Jobs →

Persons →

	1	2	3	4
A	90	75	75	80
B	35	85	55	65
C	125	95	90	105
D	45	110	95	115

Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)

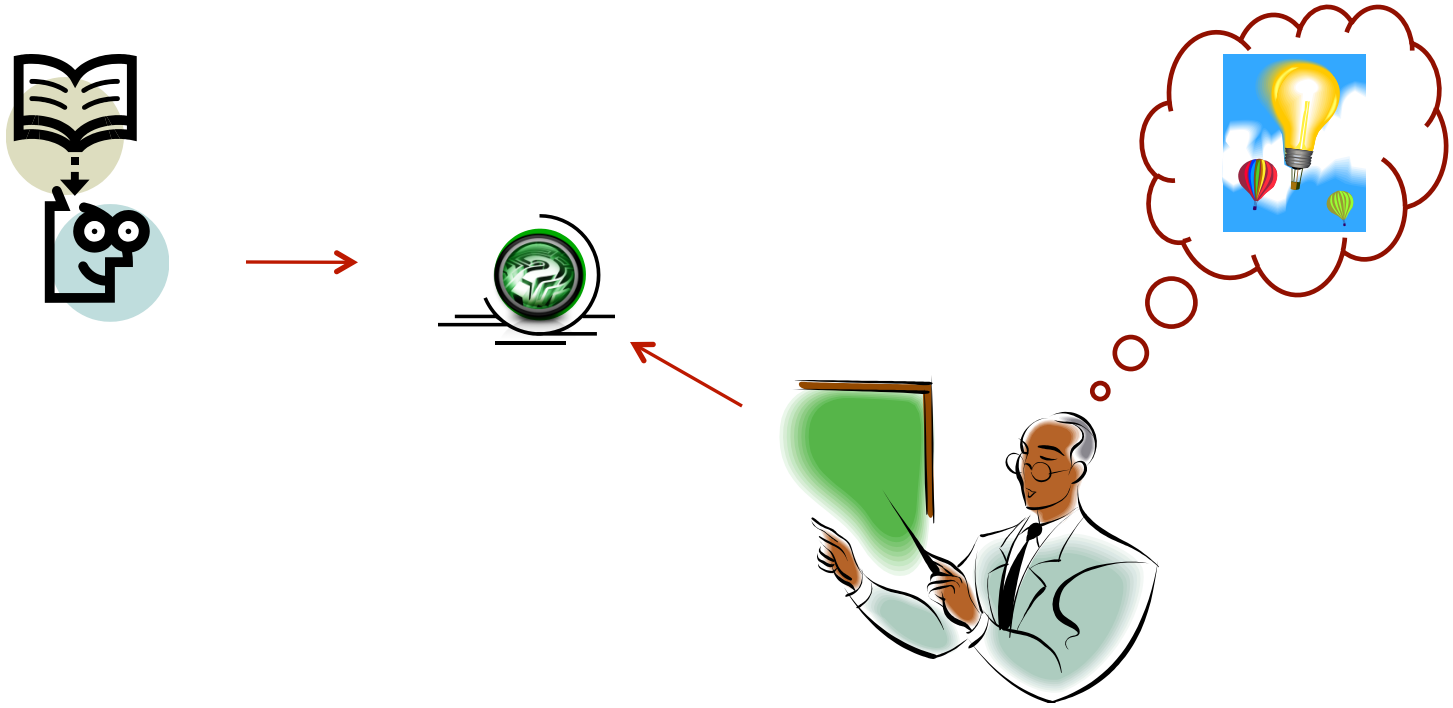
- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)

- You may grow a culture of **collaborative learning** by helping the needy students

Assistance

- You may post your questions to me at any time
- You may meet me in person on available time or with an appointment
- TA s would assist you to clear your doubts.
- You may leave me an email any time (email is the best way to reach me faster)

Thanks …



… Questions ???