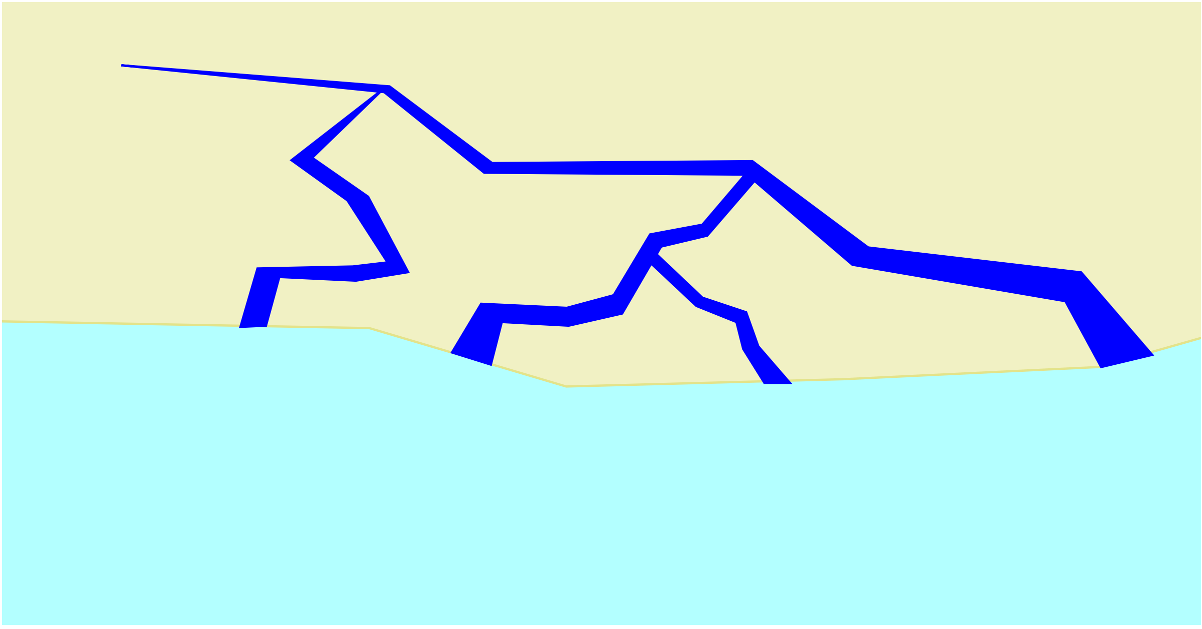
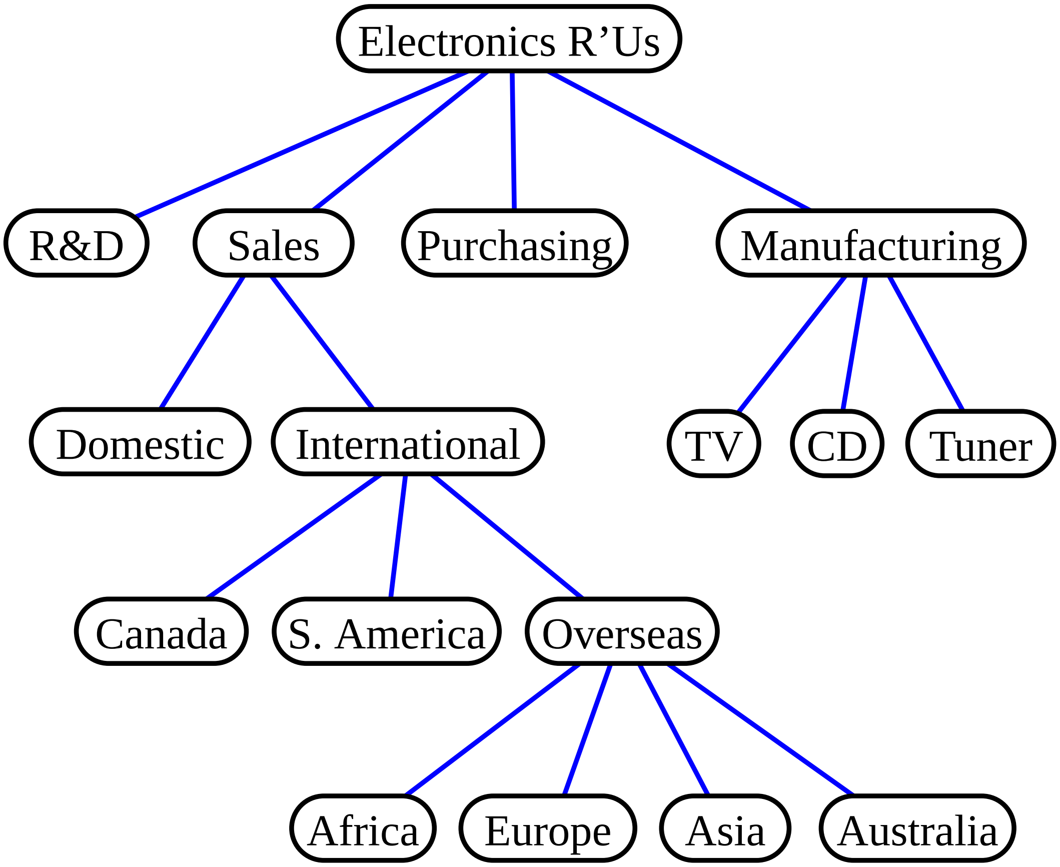
# TREES

- trees
- binary trees
- traversals of trees
- template method pattern
- data structures for trees

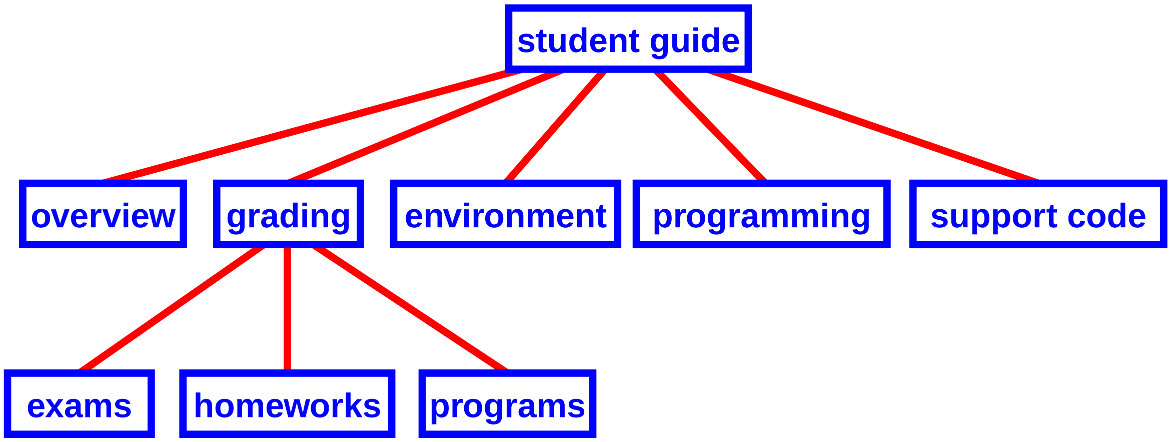


# Trees

- a tree represents a hierarchy
  - organization structure of a corporation

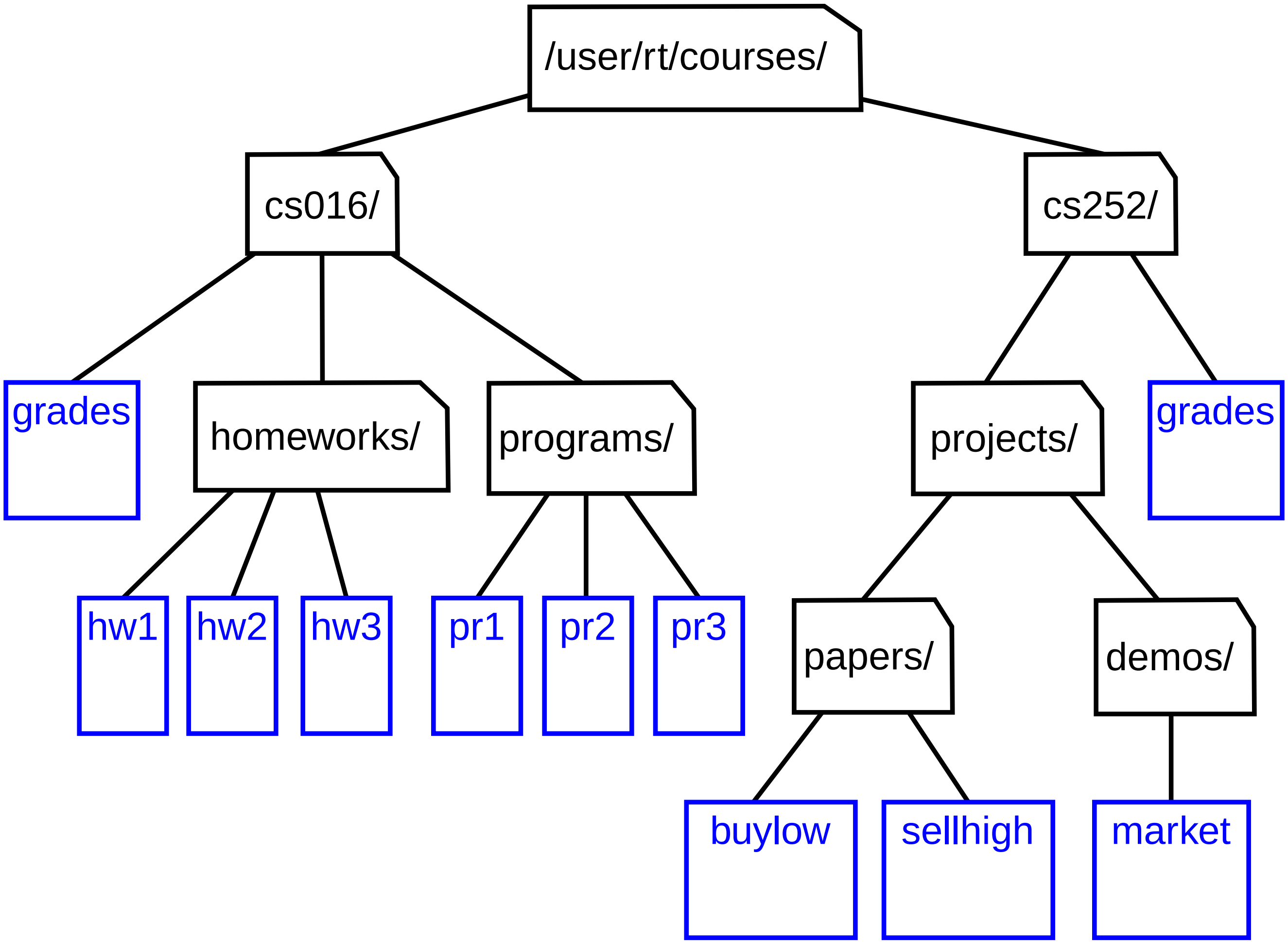


  - table of contents of a book



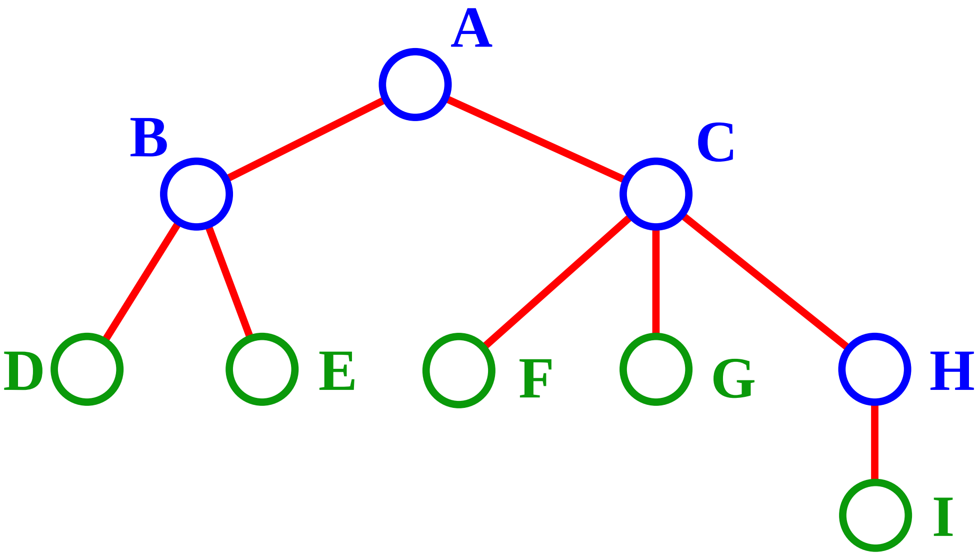
# Another Example

- Unix or DOS/Windows file system



# Terminology

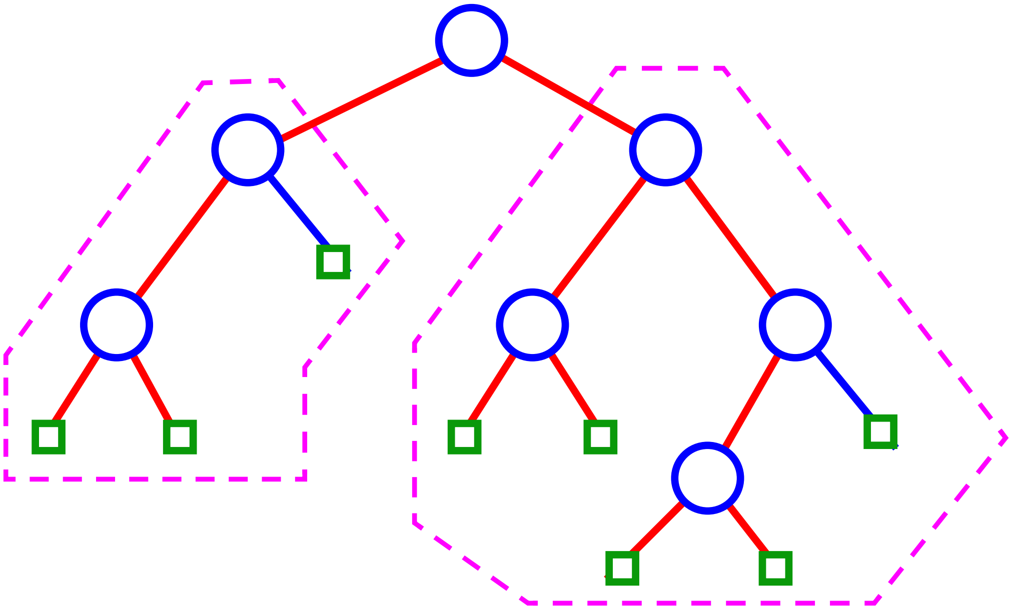
- ***A*** is the ***root*** node.
- ***B*** is the ***parent*** of D and E.
- ***C*** is the ***sibling*** of B
- ***D*** and ***E*** are the ***children*** of B.
- ***D, E, F, G***, I are ***external nodes***, **or** ***leaves***.
- ***A, B, C, H*** are ***internal nodes*****.**
- The ***depth*** (***level***) of ***E*** is ***2***
- The ***height*** of the tree is ***3***.
- The ***degree*** of node ***B*** is ***2***.



**Property:** (*# edges*) = (*#nodes*) – 1

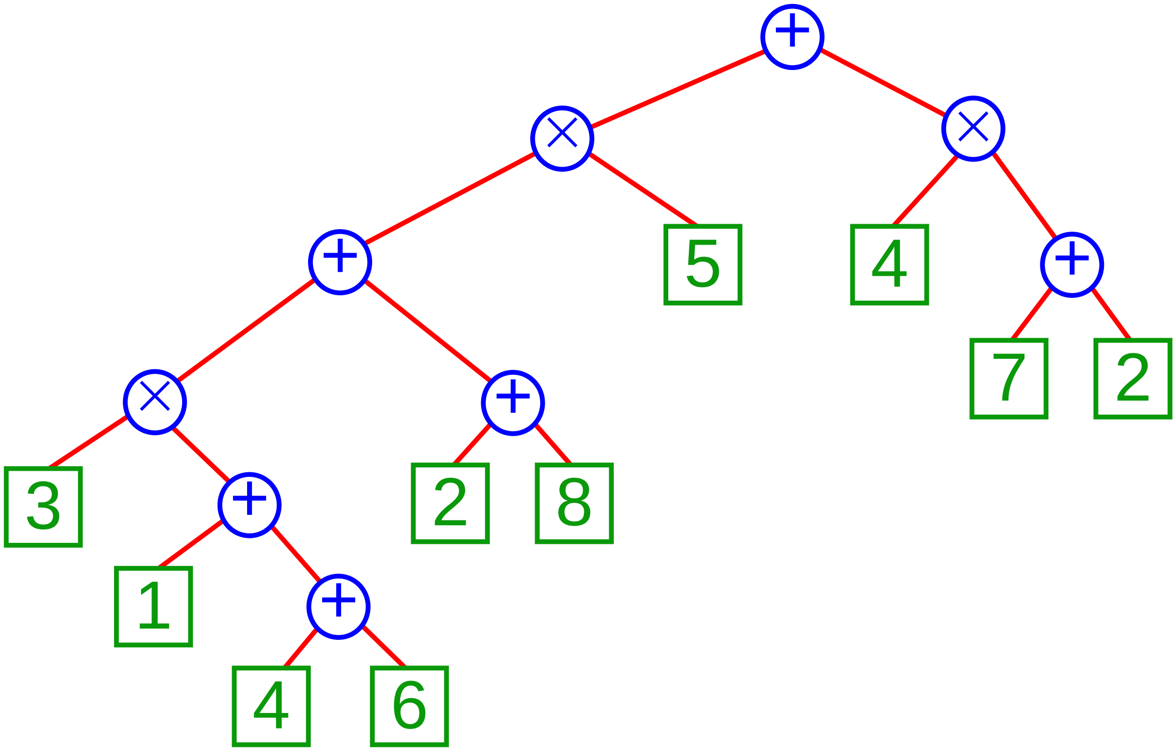
# Binary Trees

- ***Ordered tree:*** the children of each node are ordered.
- ***Binary tree:*** ordered tree with all internal nodes of ***degree 2***.
- Recursive definition of binary tree:
- A ***binary tree*** is either
  - an external node (leaf), **or**
  - an internal node (the ***root***) and two binary trees (***left subtree*** and ***right subtree***)



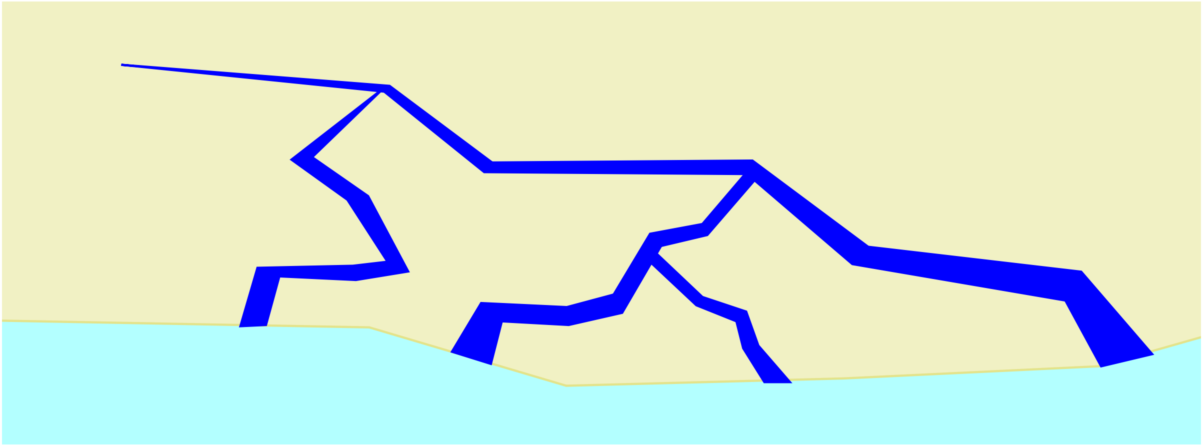
# Examples of Binary Trees

- arithmetic expression



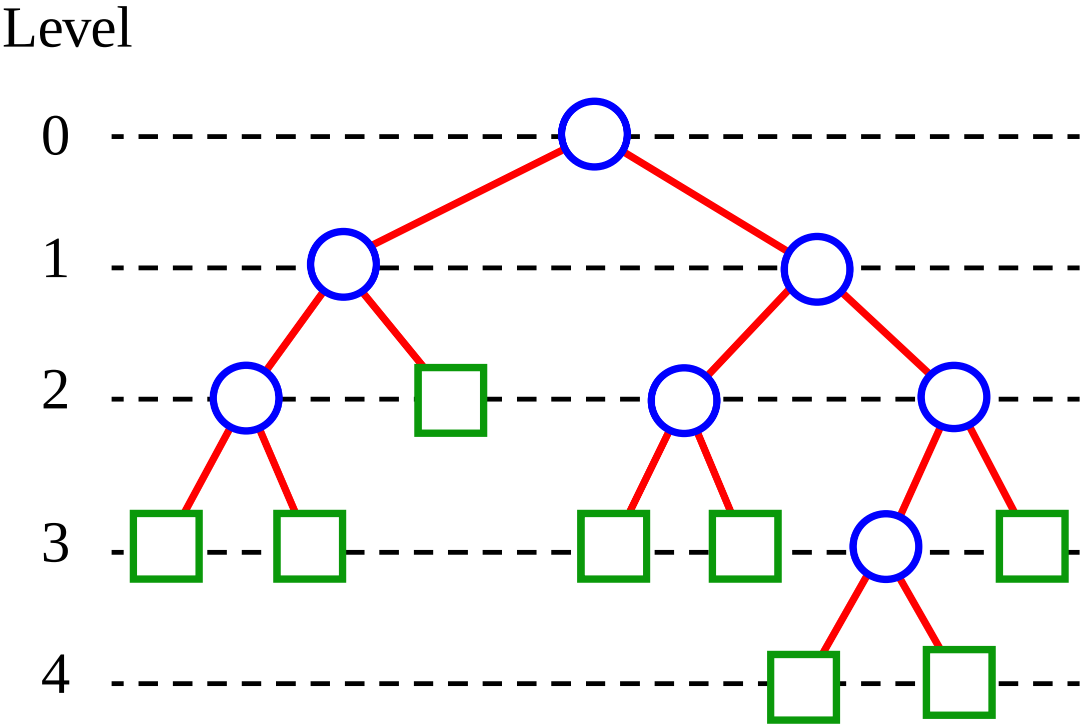
((((3 × (1 + (4 + 6))) + (2 + 8)) × 5) + ( 4 × (7 + 2)))

- river



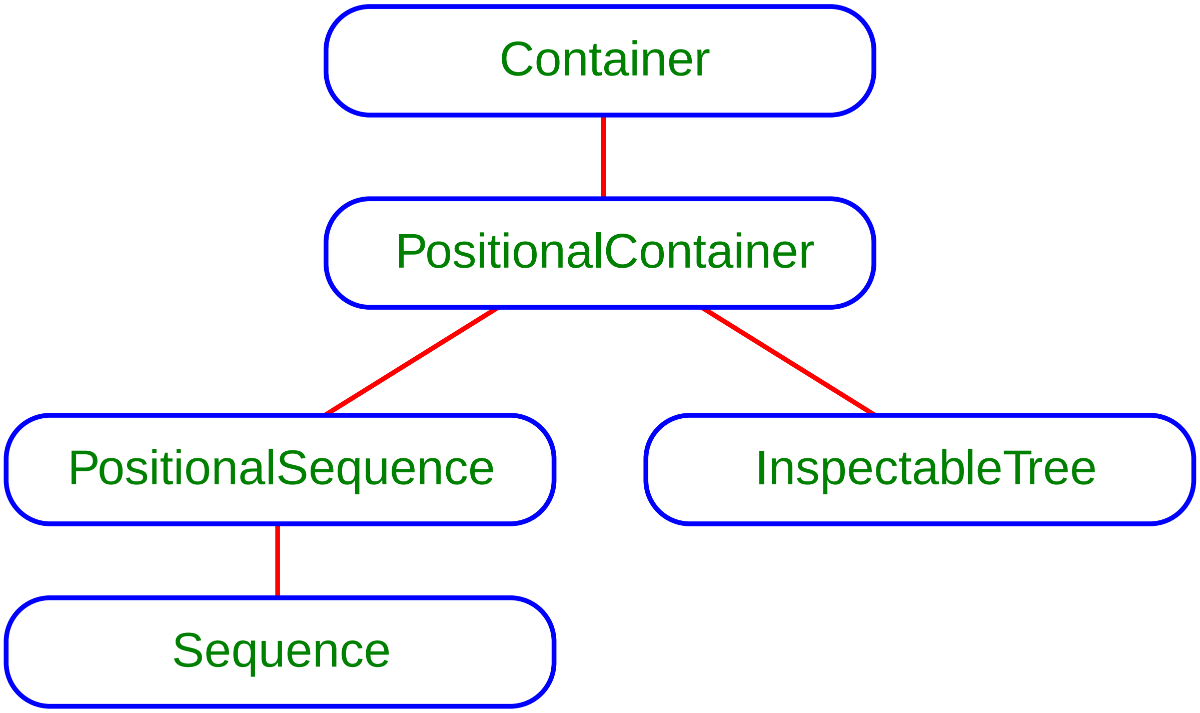
# Properties of Binary Trees

- (# external nodes ) = (# internal nodes) + 1
- (# nodes at level **i**) $\leq$ $2^{\mathbf{i}}$
- (# external nodes) $\leq$ $2^{(\text{height})}$
- (height) $\geq$ $\log_2$ (# external nodes)
- (height) $\geq$ $\log_2$ (# nodes) – 1
- (height) $\leq$ (# internal nodes) = ((# nodes) – 1)/2



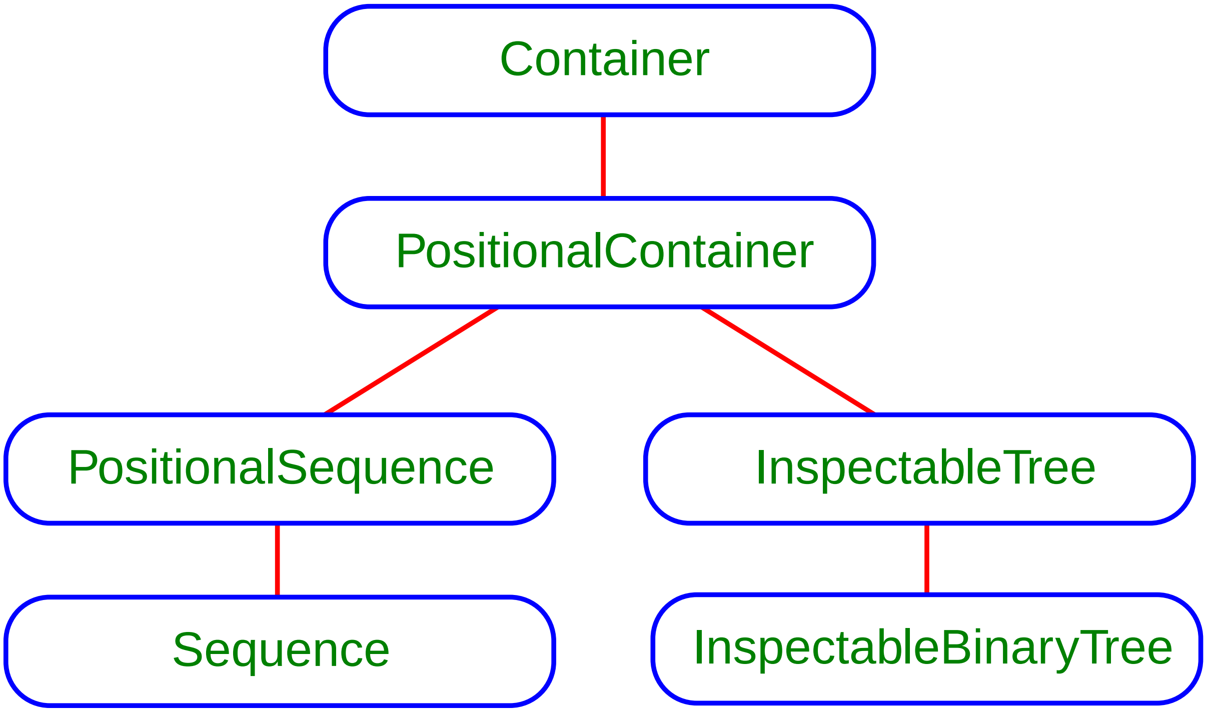
# The Tree ADT

- the nodes of a tree are viewed as positions
- generic container methods
  - size(), isEmpty(), elements(), newContainer()
- positional container methods
  - positions(), replace(p,e), swap(p,q)
- query methods
  - isRoot(p), isInternal(p), isExternal(p)
- accessor methods
  - root(), parent(p), children(p), siblings(p)
- update methods (application specific)



# The Binary Tree ADT

- extends the tree ADT
- accessor methods
  - leftChild(p), rightChild(p), sibling(p)
- update methods
  - expandExternal(p), removeAboveExternal(p)
  - other application specific methods
- interface hierarchy of positional containers

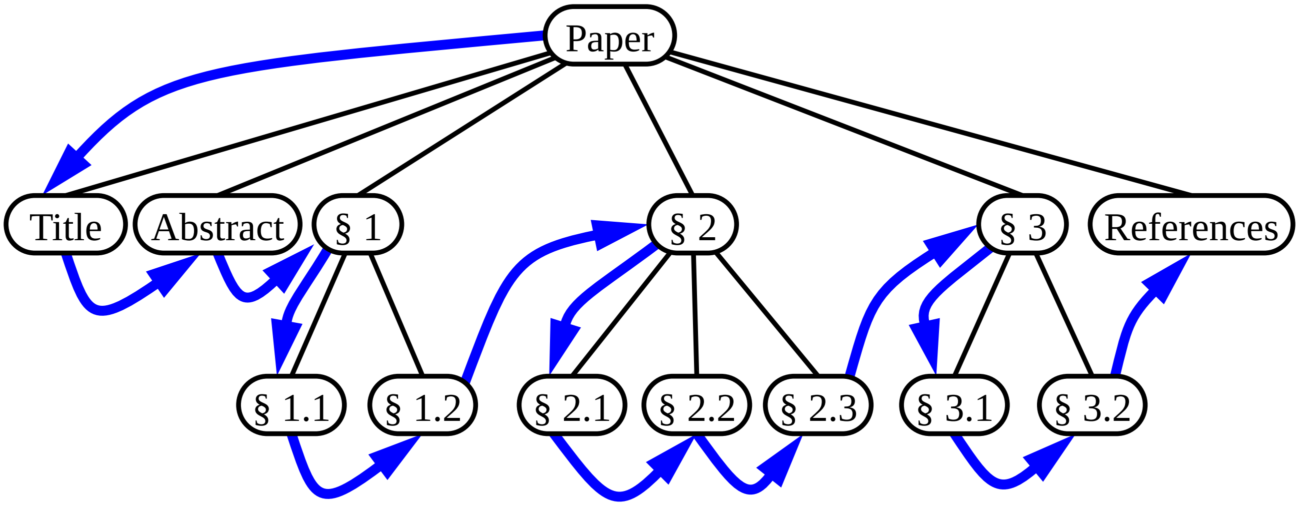


# Traversing Trees

- preorder traversal

```
Algorithm preOrder(v)
  “visit” node v
  for each child w of v do
    recursively perform preOrder(w)
```

- reading a document from beginning to end

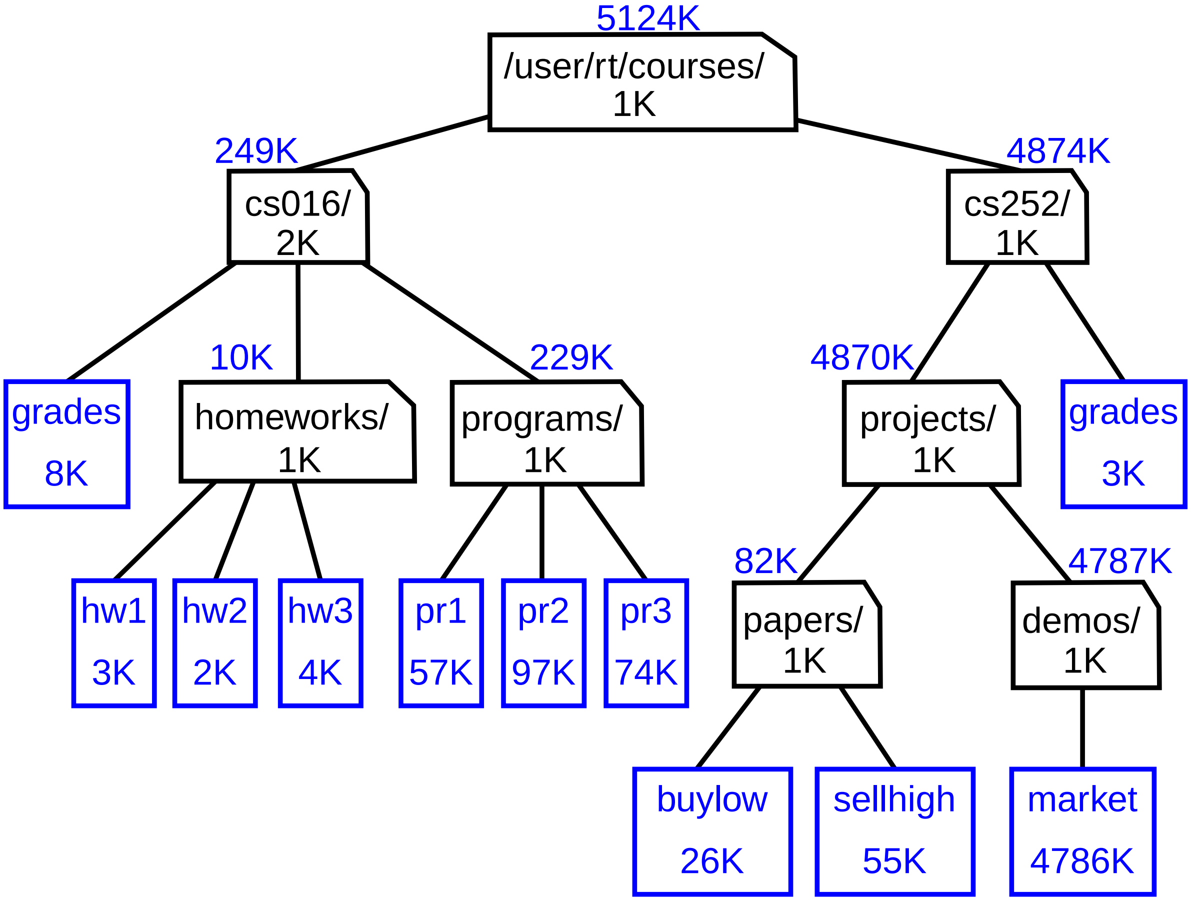


# Traversing Trees

- postorder traversal

```
Algorithm postOrder(v)
  for each child w of v do
    recursively perform postOrder(w)
  "visit" node v
```

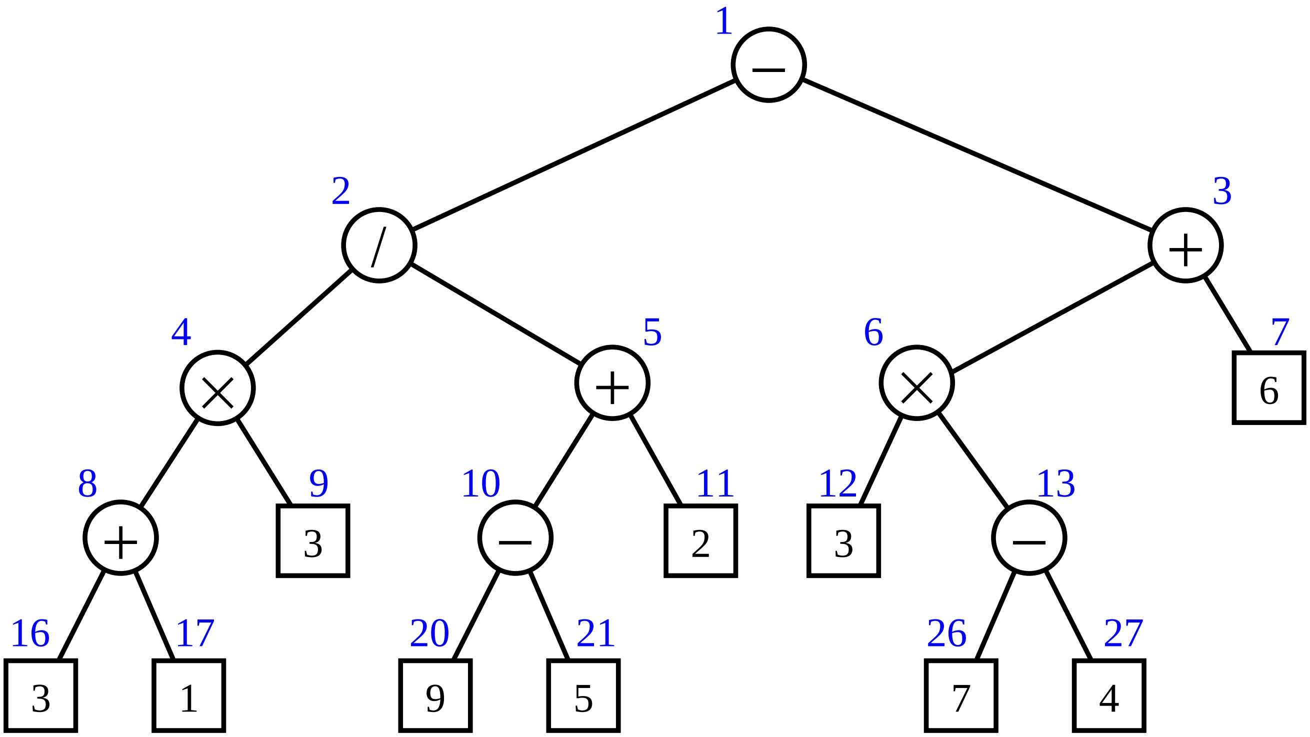
- du (disk usage) command in Unix



# Evaluating Arithmetic Expressions

- specialization of a postorder traversal

```
Algorithm evaluateExpression(v)
  if v is an external node
    return the variable stored at v
  else
    let o be the operator stored at v
    x ← evaluateExpression(leftChild(v))
    y ← evaluateExpression(rightChild(v))
    return x o y
```

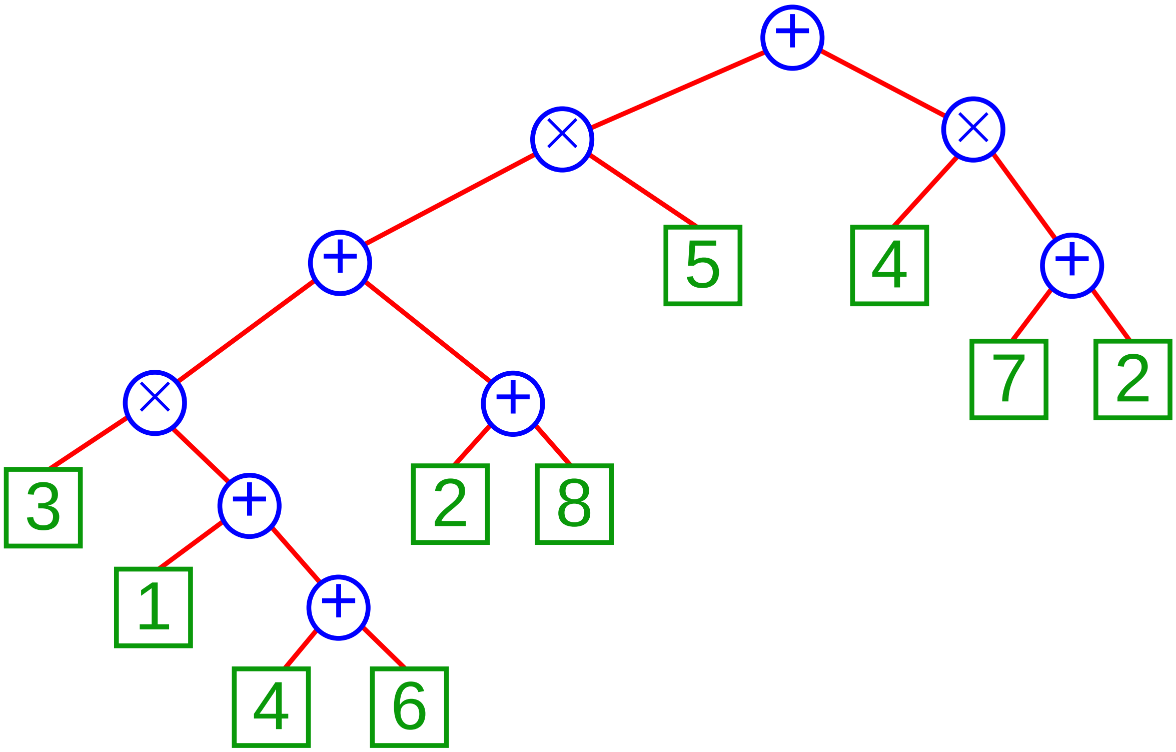


# Traversing Trees

- inorder traversal of a binary tree

```
Algorithm inOrder(v)
    recursively perform inOrder(leftChild(v))
    "visit" node v
    recursively perform inOrder(rightChild(v))
```

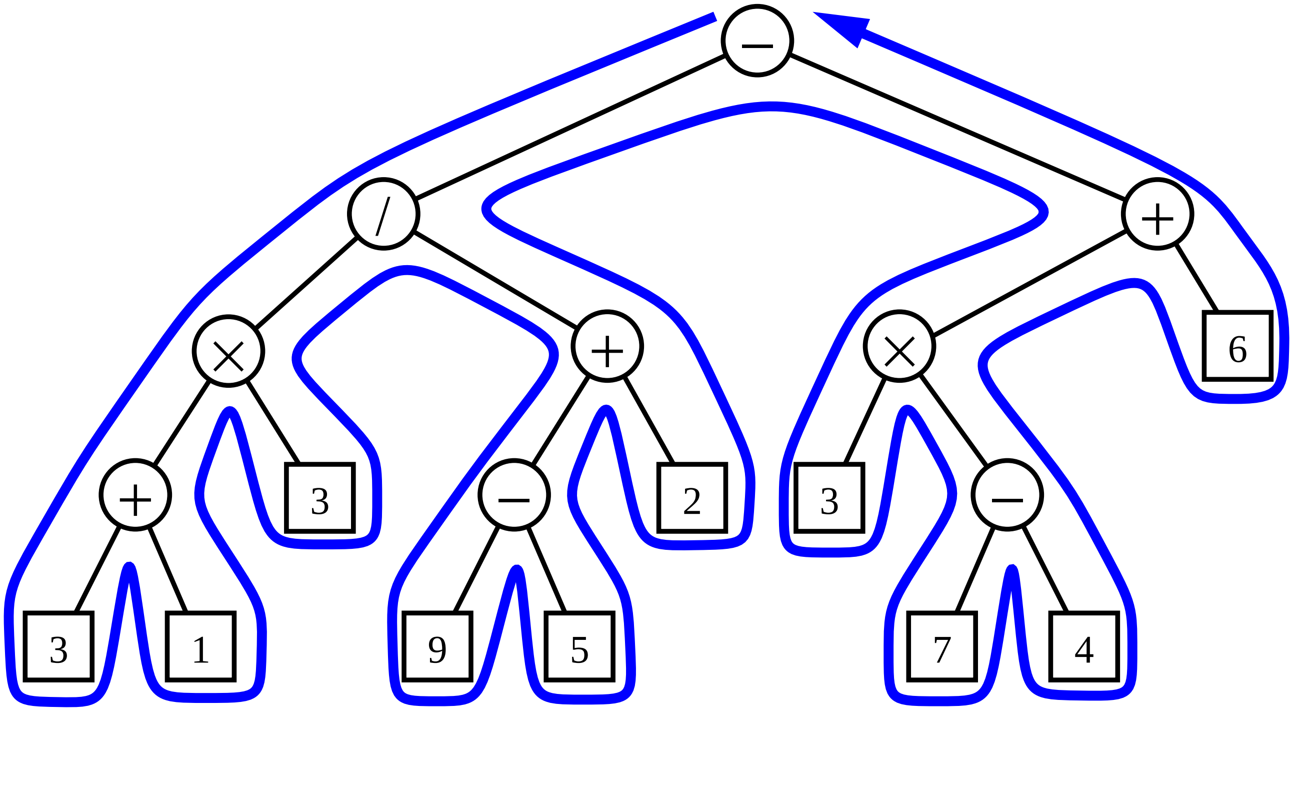
- printing an arithmetic expression
  - specialization of an inorder traversal
  - print “(“ before traversing the left subtree
  - print “)” after traversing the right subtree



((((3 × (1 + (4 + 6))) + (2 + 8)) × 5) + ( 4 × (7 + 2)))

# Euler Tour Traversal

- generic traversal of a binary tree
- the preorder, inorder, and postorder traversals are special cases of the Euler tour traversal
- “walk around” the tree and visit each node three times:
  - on the left
  - from below
  - on the right



# Template Method Pattern

- generic computation mechanism that can be specialized by redefining certain steps
- implemented by means of an abstract Java class with methods that can be redefined by it subclasses

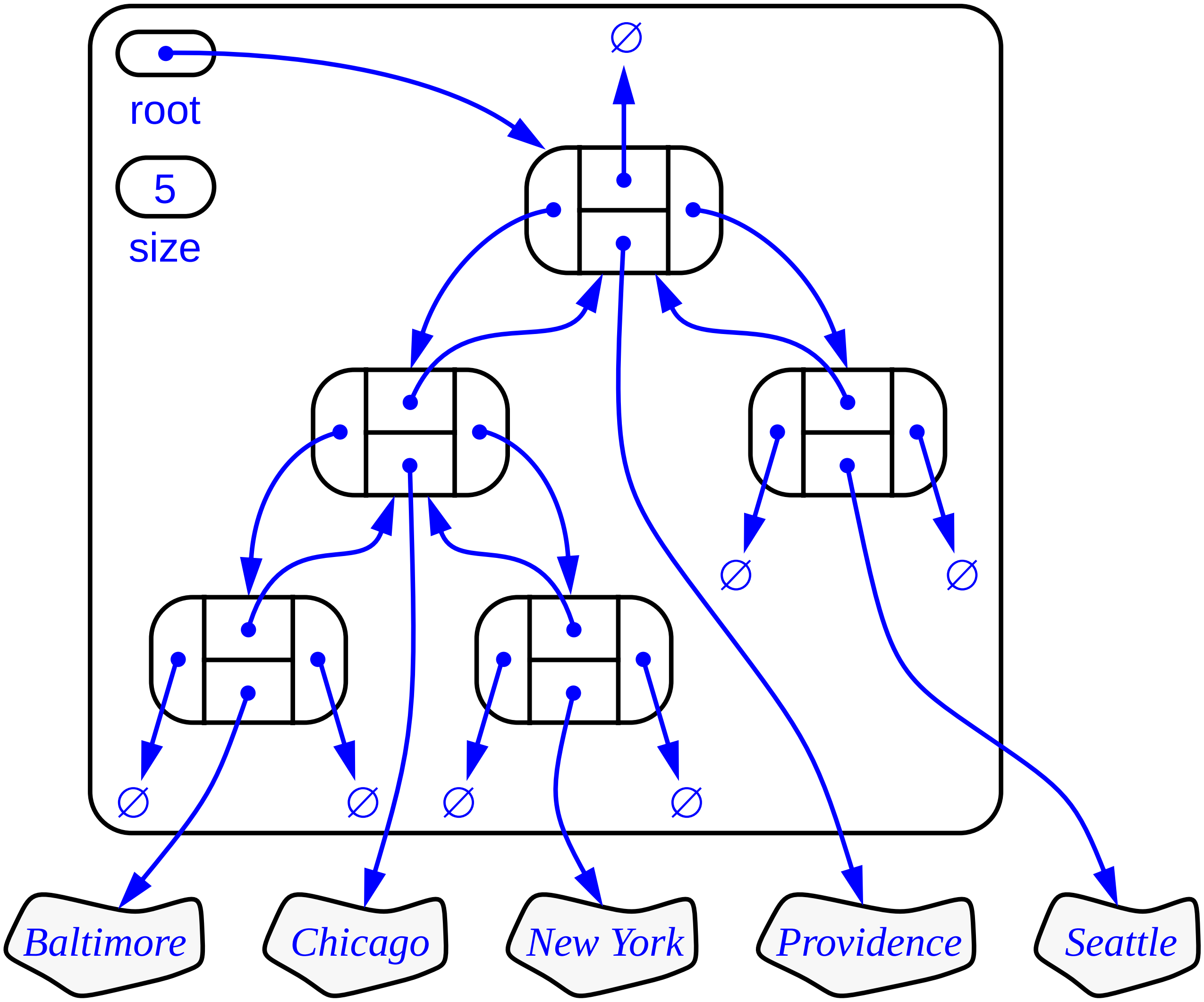
```
public abstract class BinaryTreeTraversal {

protected BinaryTree tree;

...

protected Object traverseNode(Position p) {
  TraversalResult r = initResult();
  if (tree.isExternal(p)) {
   external(p, r);
  } else {
   left(p, r);
   r.leftResult = traverseNode(tree.leftChild(p));
   below(p, r);
   r.rightResult = traverseNode(tree.rightChild(p));
   right(p, r);
  }
  return result(r);
 }
```

# Specializing the Generic Binary Tree Traversal

- printing an arithmetic expression

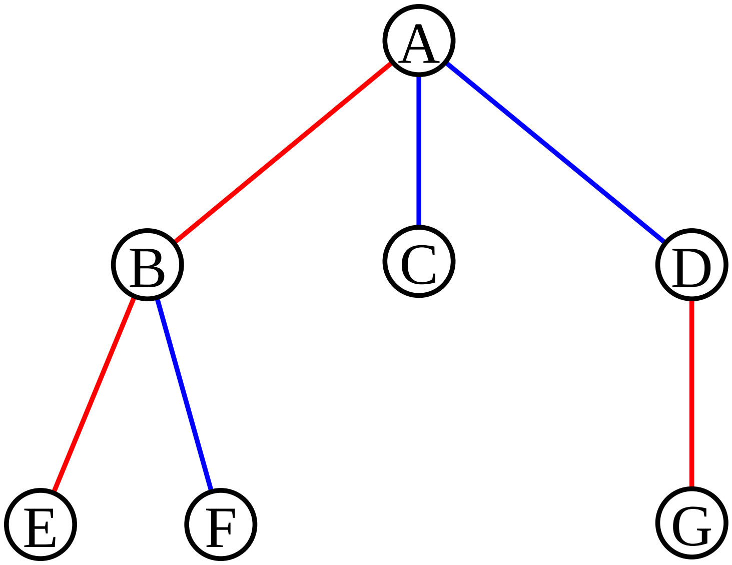
```
public class PrintExpressionTraversal
    extends BinaryTreeTraversal {

...

  protected void external(Position p, TraversalResult r) {
   System.out.print(p.element());
  }

  protected void left(Position p, TraversalResult r) {
   System.out.print("(");
  }

  protected void below(Position p, TraversalResult r) {
   System.out.print(p.element());
  }

  protected void right(Position p, TraversalResult r) {
   System.out.print(")");
  }

}
```

# Linked Data Structure for Binary Trees



# Representing General Trees

- tree T



- binary tree T' representing T