



BITS Pilani
Pilani Campus



CS/IS F214 Logic in Computer Science

MODULE: INTRODUCTION

A Brief – and Selective - History of Logic – Part II: Intuitionism

Intuitionism



2G Scam – Proof 1

Assume that *the Govt. acted honestly*:

- Then they should have *wanted to increase revenue for Govt.*
- Auctioning spectrum would have brought more revenue than awarding it on **First-Come-First-Serve** basis.
- Spectrum was awarded on a **FCFS** basis.
 - This is a contradiction.
- Therefore we infer *that the Govt. was dishonest*

2G Scam – Proof 2

- Company X hired lobbyist Y.
- Lobbyist Y paid Rs *rrrrrrrrrr* to Minister Z.
- Company X received preferential allocation of spectrum.
- Therefore we infer that *the Govt. was dishonest.*

Proof Techniques: *Proof by Contradiction*

- *Proof by Contradiction* [PbC] :
 - Assume that the *negation of a statement X* is true.
 - ...
 - Derive a contradiction
 - Conclude that X is true
- e.g. (see previous slide)
 - Consider the first “proof” of 2G scam



Brouwer's Intuitionism

- Intuitionism is a school of thought in logic and mathematics:
 - founded by Brouwer
 - in the early 20th century as a response to Hilbert's program
- Intuitionists rejected ***Proof by Contradiction [PbC]***
- Why did intuitionists reject **PbC**?
 - Consider the two “proofs” of 2G scam in the next two slides.
 - Intuition (no pun intended):
 - Negation of a negation does not prove existence!



Brouwer's Intuitionism – Another Illustration

- Consider the claim:
 - There exist irrational numbers a and b such that a^b is rational.
- Consider a proof of this claim:
 - Let $c = \sqrt{2}$
 - Consider c^c – it is either rational or irrational.
 - If it is: then let $c=a$ and $c=b$ and we are done.
 - If not: let $a=c^c$ and $b=c$ then a^b is rational.
- What is wrong with this proof?
 - How do we avoid such proofs?
 - Should we avoid such proofs?



Brouwer's Intuitionism

- The proof in the last slide is non-constructive:
 - i.e. it proves that *there exist irrational a and b such that a^b is rational but it does not produce (i.e. construct) such a and b .*
- Brouwer and intuitionists denounced such “***non-constructive***” ***proofs*** and argued that such proofs should be disallowed:
 - They identified the “**Law of Excluded Middle (LEM)**” as the root cause of such proofs:
 - **LEM:** *for any proposition A : A OR (NOT A) is true*
- Exercise:
 - Identify where LEM is used in the previous proof?



Brouwer's Intuitionism

- The philosophical underpinning of Brouwer's argument is this:
 - while $A \text{ OR } (\text{NOT } A)$ is always true it may not always be proven or provable for a specific A
 - and therefore it may not be used in a proof (by itself) unless one has proven A or one has proven $\text{NOT } A$.
- What is common in the critique of LEM and that of PbC by intuitionists?

