

# CS/IS F214 Logic in Computer Science

#### MODULE: PREDICATE LOGIC

#### **Predicate Logic**

Proofs and Proof Rules: Introduction

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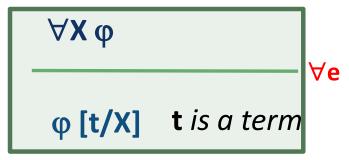
### **Universally Quantified Variables and Inference**

- Given a statement of the form:
  - ∀X (bird(X) --> fly(X))
  - one can infer the following:
    - bird(fancy\_piggy) --> fly(fancy\_piggy)
    - bird(seagull(jonathon)) --> fly(seagull(jonathon))
- This amounts to an <u>inference rule</u>:
  - If a <u>statement with a universally quantified variable</u>, say
    X, is true
    - then <u>any instance of that statement</u> i.e. the <u>statement with any term substituted for X</u> – should be true.



## **Inference Rule for Universally Quantified Formulas**

- The inference rule can be stated formally as follows:
  - Proof Rule (<u>Elimination of universal quantifier</u>):



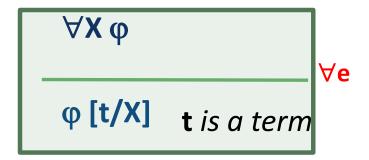
- where  $\varphi[t/X]$  denotes
  - the <u>formula</u>  $\varphi$  in which <u>each occurrence</u> of variable X has been <u>replaced</u> with term t.



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## Inference Rule for Universally Quantified Formulas

Proof Rule (<u>Elimination of universal quantifier</u>):



- φ[t/X] is usually read as:
  - φ with **t** (*substituted*) for **X**
- Note that φ[t/X] is not a formula (in our language):
  - it denotes the substitution operation that results in a formula



#### **Existentially Quantified Variables and Inference**

- Given a statement of the form:
  - mammal(platypus) ∧ lays\_eggs(platypus)
  - one can infer the following:
    - ∃X (mammal(X) ∧ lays\_eggs(X))
- Given a statement of the form:
  - ugly(duckling) \( \triangle \text{becomes(swan(duckling))} \)
  - one can infer the following:
    - ∃X (ugly(X) ∧ becomes(swan(X)))



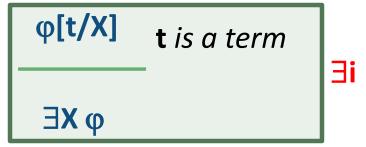
#### **Existentially Quantified Variables and Inference**

- From: mammal(platypus) ∧ lays\_eggs(platypus)
  - infer :  $\exists X \text{ (mammal(X)} \land lays\_eggs(X))$
- From: ugly(duckling) \( \triangle \text{becomes(swan(duckling))} \)
  - infer : ∃X (ugly(X) ∧ becomes(swan(X)))
- This kind of reasoning amounts to an <u>inference rule</u>:
  - If a statement is true for a concrete value
    - then the <u>same statement existentially quantified by a</u> <u>variable</u>, say X, <u>is true</u>
      - where X replaced by the concrete instance yields the original statement.



### **Inferring Existentially Quantified Formulas**

- The inference rule can be stated formally as follows:
  - Proof Rule (Introduction of existential quantifier):



- where denotes  $\phi[t/X]$  the formula  $\phi$  in which each occurrence of variable X has been replaced with term t.
- Note that the *substitution* operation referred here is the same one in the proof rule  $\forall e$ 
  - but this proof rule uses it in reverse

