



BITS Pilani
Pilani Campus



CS/IS F214 Logic in Computer Science

MODULE: PROPOSITIONAL LOGIC

Natural Deduction: Derived Rules, Proofs - Examples

Modus Tollens

$$\phi \rightarrow \psi$$

$$\neg \psi$$

$$\neg \phi$$

MT (modus tollens)

Questions: What is the relation between this and *modus ponens*?

How do you derive this rule (from the basic rules)?



Proof by Contradiction

 $\neg \phi$
 \vdots
 \vdots
 \vdots
 \perp
 ϕ
PbC

- One can infer anything from a contradiction.
- But in this case the contradiction resulted from an assumption i.e. $\neg \phi$
- Therefore it is meaningful to infer ϕ that the assumption led to the contradiction
- In fact one must infer ϕ to eliminate the assumption $\neg \phi$.

• Why?

Exercise: Derive PbC from the $\neg i$ rule.

Exercises

- Prove the following:

$$1. \neg p \vee q \mid\!\!\vdash p \rightarrow q \quad \square$$

$$2. q \rightarrow (p \rightarrow r), \neg r, q \mid\!\!\vdash \neg p$$

$$3. p \rightarrow q \mid\!\!\vdash \neg p \vee q$$

$$4. \text{ Use 3. to prove LEM}$$

[These examples were worked out in class – on the board]



Take-home Exercises

Prove the following sequents

1. $\neg(p \wedge q) \quad | \! - \quad \neg p \vee \neg q$
2. $q \rightarrow (p \rightarrow r), \neg r, q \quad | \! - \quad \neg p$
3. $p \rightarrow q, r \rightarrow s \quad | \! - \quad p \vee r \rightarrow q \vee s$
4. $(p \rightarrow r) \wedge (q \rightarrow r) \quad | \! - \quad p \wedge q \rightarrow r$

