BITS Pilani
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CS/IS F214
Logic in Computer Science

MODULE: PROPOSITIONAL LOGIC

Syntax – Order of Evaluation

RECALL: Syntax and Order of Evaluation

- One way to eliminate ambiguity in the grammar of a language is
  - to insist that the user (i.e. one who writes sentences) *must eliminate all ambiguity by parenthesizing everything*

- Alternatively, one can capture precedence and associativity rules in the grammar!

Propositional Logic: Typical Order of Evaluation

- The following *precedence* and *associativity* are conventional in **propositional logic**:
  - --> has the lowest precedence
  - $\vee$ has the next higher precedence
  - $\wedge$ has the next higher precedence
  - $\neg$ has the highest precedence
  - --> is right-associative
    - What about $\vee$ and $\wedge$?

Grammar – Approach 2

- Capturing precedence and associativity rules in the grammar:
  - --> has the lowest precedence
  - $\vee$ has the next higher precedence
  - $\wedge$ has the next higher precedence
  - $\neg$ has the highest precedence
  - all operators are right-associative

Rules of the grammar

Grammar – Approach 2

(**Gr-PropL-OE-2**): *incomplete*

1. **Form ---> DisForm '-->' Form**
2. **Form ---> DisForm**

We want to capture precedence and associativity rules:

--> has the lowest precedence and is right-associative

These two rules state that

any formula (Form) is in one of two forms

1. a *disjunctive formula* (**DisForm**), followed by the symbol '-->', and by another formula (**Form**)
2. only a *disjunctive formula* (**DisForm**)

where a disjunctive formula *does not include an implication*.

(**Gr-PropL-OE-2**): *incomplete*

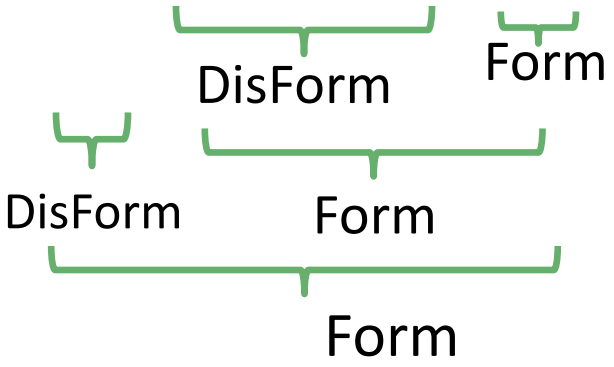
1. **Form ---> DisForm '-->' Form**
2. **Form ---> DisForm**

i.e. if *a formula includes* '-->' then *we separate the formula into*

(i) a left sub-formula that does not contain an '-->' **and**

(ii) a right sub-formula

e.g. **p --> q ∨ r ∧ s --> t**



Recursive Rule – Sentences Generated

- Give a rule of the form:
  - X --> a X

  what are the sentences that can be generated?
- Given a pair of rules of the form:
  - X --> a X
  - X --> a

  what are the sentences that can be generated?

Rules of the grammar

Precedence rules:
--> has the lowest precedence
$\vee$ has the next higher precedence
both are right-associative

(Gr-PropL-OE-2): *incomplete*

1. **Form ---> DisForm '-->' Form**
2. **Form ---> DisForm**
3. **DisForm ---> ConForm '$\vee$' DisForm**
4. **DisForm ---> ConForm**

The last two rules state that

any disjunctive formula (DisForm) is in one of two forms

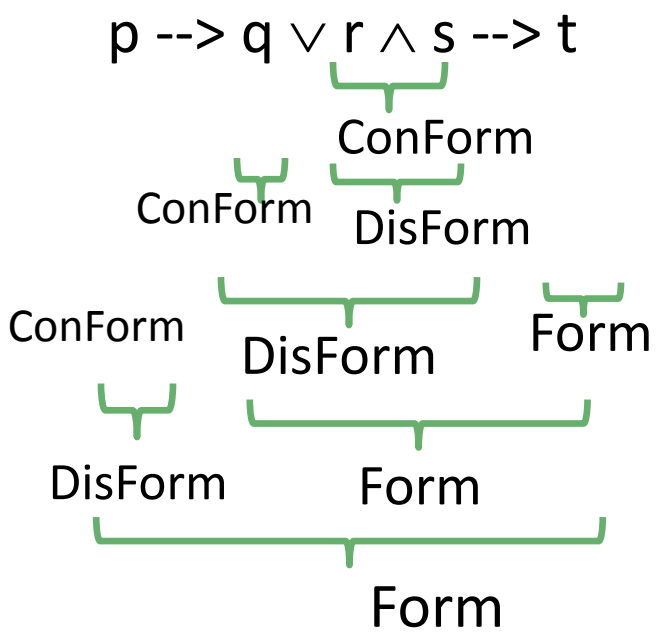
3. *a conjunctive formula* (**ConForm**), followed by disjunction symbol ('$\vee$'), and then by *a disjunctive formula* (**DisForm**)
4. *only a conjunctive formula* (**ConForm**)

where a conjunctive formula does not include a disjunction (i.e. '$\vee$')

Rules of the grammar

(Gr-PropL-OE-2): *incomplete*

1. Form ---> DisForm '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '∨' DisForm
4. DisForm ---> ConForm



Rules of the grammar

(**Gr-PropL-OE-2**): *incomplete*

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm

Grammar – Approach 2

We capture precedence and associativity rules:

--> has the lowest precedence

$\vee$ has the next higher precedence

$\wedge$ has the next higher precedence

All three operators are right-associative

Rules of the grammar

(**Gr-PropL-OE-2**):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm --> '$\neg$' NegForm
8. NegForm --> p

where p *is any propositional atom*

Grammar – Approach 2

We capture precedence and associativity rules:

--> has the lowest precedence

$\vee$ has the next higher precedence

$\wedge$ has the next higher precedence

$\neg$ has the highest precedence

All the binary operators are right-associative

Rules of the grammar

(**Gr-PropL-OE-2**):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm ---> '$\neg$' NegForm
8. NegForm ---> p

where p *is any propositional atom*

Grammar – Approach 2

We capture precedence and associativity rules:

--> has the lowest precedence

$\vee$ has the next higher precedence

$\wedge$ has the next higher precedence

$\neg$ has the highest precedence

All the binary operators are right-associative

Question: What is the drawback of the approach/grammar?

Question : Are there formulas that cannot be generated using this grammar? Is this question relevant?

Approach 2 - Limitations

While this grammar captures a set of precedences ($\neg$ over $\wedge$, $\wedge$ over $\vee$, $\vee$ over -->), there are limitations:

1. Precedence cannot be overruled

   i.e. $\neg p \wedge q$ is interpreted as

   (NOT p) AND q

   but there is *no way to generate the form* NOT (p AND q)

Grammar – Approach 2

(**Gr-PropL-OE-2**):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm ---> '$\neg$' NegForm
8. NegForm ---> p

   where p *is any propositional atom*

Approach 2 - Limitations

Issue: *Precedence cannot be overruled*

i.e. **$\neg p \wedge q$** is interpreted as

(NOT p) AND q

Solution: NOT (p AND q)

can be generated from this grammar using rule 8

Grammar – Approach 2A

(**Gr-PropL-OE-3**):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm ---> '$\neg$' NegForm
8. NegForm ---> '(' Form ')
9. NegForm ---> p

where p *is any propositional atom*

Exercise

- Argue that *rule* 8 allows: *precedence to be over-ruled by parenthesizing sub-expressions*.
- For instance, generate / parse formulas
  - $\neg(p \wedge q)$ and
  - $((p \dashrightarrow q) \vee r) \wedge s)$

  using this grammar.

Grammar – Approach 2A

(Gr-PropL-OE-3):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm ---> '$\neg$' NegForm
8. NegForm ---> '(' Form ')
9. NegForm ---> p

where p *is any propositional atom*

Propositional Logic – Grammar – Approach 2A.

(Gr-PropL-OE-3):

1. Form ---> Disform '-->' Form
2. Form ---> DisForm
3. DisForm ---> ConForm '$\vee$' DisForm
4. DisForm ---> ConForm
5. ConForm ---> NegForm '$\wedge$' ConForm
6. ConForm ---> NegForm
7. NegForm ---> '$\neg$' NegForm
8. NegForm ---> '(' Form ')
9. NegForm ---> p

where p *is any propositional atom*

Exercise: *Write formulas that cannot be generated from this grammar .* OR
Prove that this grammar can generate all well-formed-formulas.
[Hints:
•Use **Gr-PropL-AMB** as the "correct" definition of *well-formed formulas*.
•Use induction to prove:
• *all formulas generated by* ***Gr-PropL-AMB*** *can be generated by* ***Gr-PropL-OE-3*** *with appropriate parentheses.* **End of Hints.]**