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BITS Pilani
Pilani Campus

CS/IS F214
Logic in Computer Science

MODULE: INTRODUCTION

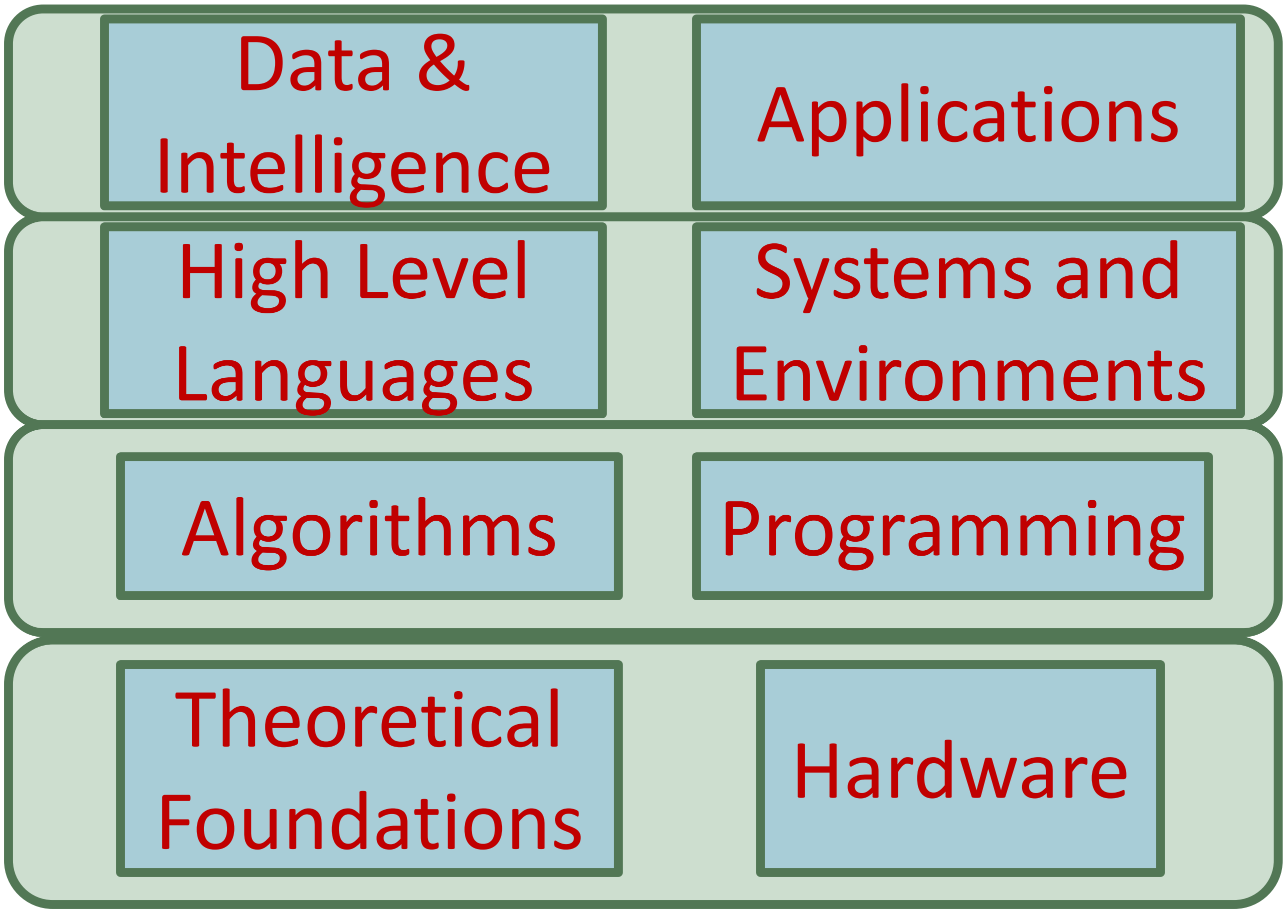
Course Context & Administrivia

Why learn logic?

COURSE CONTEXT & ADMINISTRIVIA

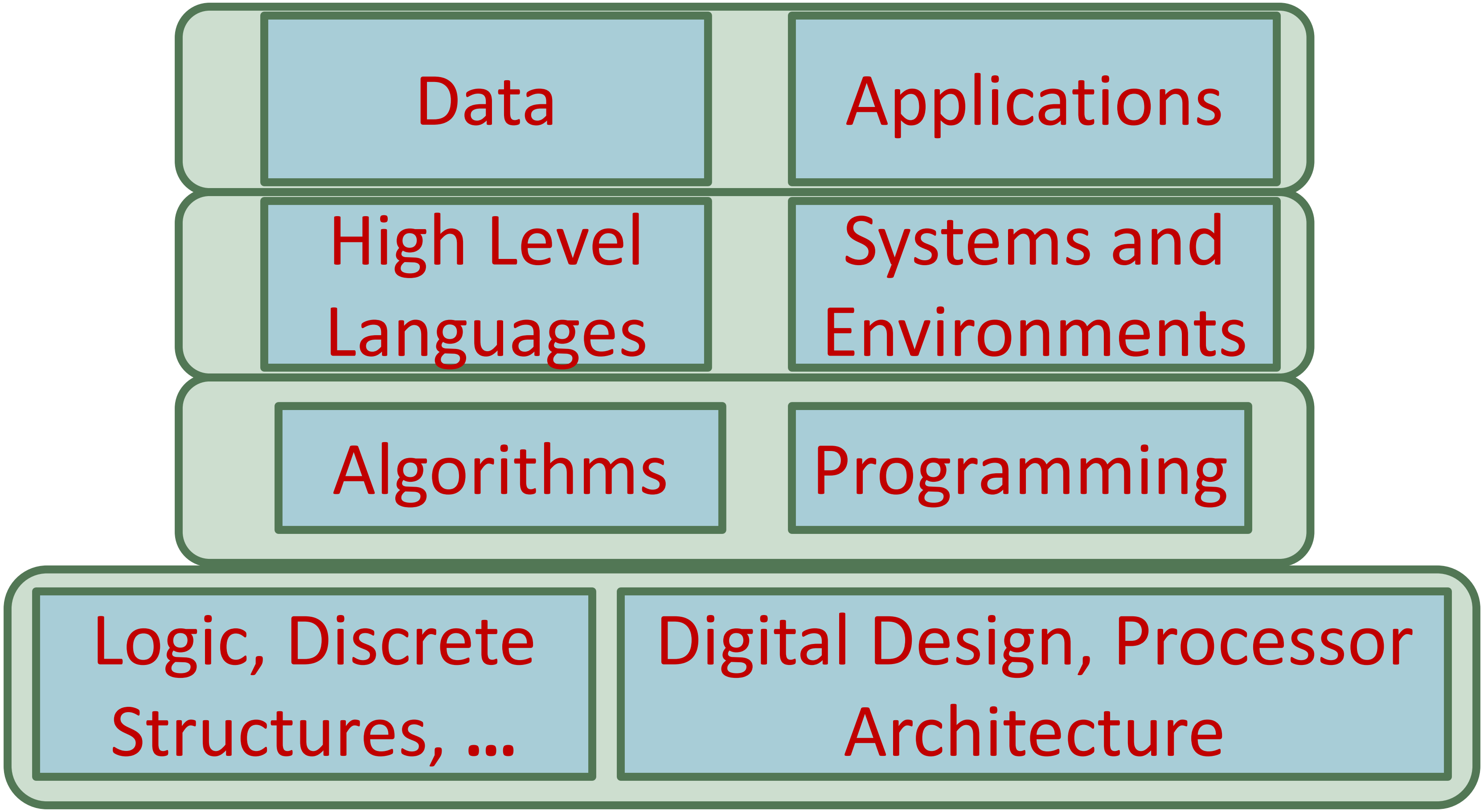
Computing

- Computing as a subject of study can be broadly portrayed as:



Computing: Context for studying Logic

- Computing as a subject of study can be broadly portrayed as:



Course – Nature and Style

- Content:
  - theoretical (i.e. mathematical) and foundational

- Theme:
  - Primary:
    - **Proofs and Proof Systems**
  - Secondary:
    - Formal Languages,
    - Models of Computation and Programming,
    - Applications of Logic

- Style:
  - elementary but rigorous

Course – Operations

- Lectures and Tutorials are important
  - Course Material is simple enough
    - to learn on one’s own and get a decent grade
  - but
    - you will learn more (and better) in class than from the text book (or from the Web)
    - tutorials are meant for your practice with our help (i.e. not intended to be packaged)

Course Operations - Evaluation

- 3 quizzes (all Open Book)
  - Conducted during tutorial sections (see Handout for schedule)
  - Each of 10% weight
  - Only one make-up!!
- 1 Assignment (20% weight)
  - Will be scheduled post mid-term test
  - Will require programming in Prolog
- Mid-Term Test and Comprehensive Exam (both Open Book)
  - see Handout for dates
  - scheduled centrally by Instruction Division

WHY LEARN LOGIC?

Why learn Logic? – A Motivating Example

- Consider the following question:
  - Is 0.999999… (ad infinitum) < 1.0 ?

- How do you resolve this question?

A typical solution:

Consider 0.9999….
as an infinite geometric progression:
9/10, 9/100, 9/1000, ….
and compute the sum (of the progression).

Why learn Logic? – A motivating example

- Alternative approach to resolve the question:
  - Is 0.999999… (ad infinitum) < 1.0 ?

- What we know (about ***real numbers***):

1. The relation < is defined to be **total** on real values i.e. for real numbers x and y, **x < y** or **y < x** or **x = y**.
2. For real numbers x and y, if **x < y** then there is a real number z such that **x + z = y**

Why learn Logic? – A motivating example

- Properties of Real Numbers:
  1. The relation < is defined to be total on real values i.e. for real numbers x and y, **x < y** or **y < x** or **x=y**
  2. For real numbers x and y, if x < y then there is a real number z such that x + z = y
- We can infer:
  - **1 < 0.999… is not true** (Why?)
  - But **1 > 0.999.. is not true** either:
    - Consider any z (say 0.00000..0001):

    adding it to 0.999… will make it

    larger than 1 (say 1.0000..000999…)
  - What (choice) is left?

Logic and Proofs

- What is it we have done?
- Alternative approach:

Axioms

1. The relation < is defined to be total on real numbers i.e. for real numbers x and y, either x < y or y < x unless x equals y.
2. For real numbers x and y, if x < y then there is a real number z such that x + z = y

Proof

- We observe: 1 < 0.999… is not true
- Consider any z (say 0.00000..0001): adding it to 0.999… will make it larger than 1 (say 1.0000..000999…)
  - And so 1 > 0.999… is not true

What is Logic?

- Logic is useful to do "formal" reasoning:
  - i.e. structured, un-ambiguous reasoning
- What is the outcome of such reasoning?
  - Typically, a proof.

What is Logic?

- Logic is useful to do “formal” reasoning:
  - i.e. structured, un-ambiguous reasoning
- What is the outcome of such reasoning?
  - Typically, a proof.
- What is the object of such reasoning?

  i.e. what do we reason about?
  - Any subject – real or imagined, that can be captured using the same logic
    - This is referred to as “modeling”
- Typically,
  - **axioms model “what we observe” *as-is***
  - **proofs provide reasoning / justification to conclusions.**

A BRIEF – AND SELECTIVE – HISTORY OF LOGIC – PART I

Euclid – the first logician!

- Euclid wrote "Elements" - a 10-volume book on Geometry (circa 300 B.C)
  - Several of the "theorems" in his book were discovered by his predecessors (e.g. Pythagoras, Hippocrates, and others).
  - But Euclid's contribution (through the book) was in *structuring the proofs*.

Euclid's axioms

- Euclid defined five ***axioms*** - he called them ***postulates*** - about
  - points and a straight line (segment),
  - infinite line,
  - circle,
  - right angles, and
  - parallel lines

modeling "real world" plane geometry.

- Everything else we know about plane geometry was "provable" from these axioms:
  - e.g. sum of angles in a triangle is 180 degrees

Euclid – the first logician!

- Later, (in the 18th century) – Gauss, Bolyai, and Lobachevsky (independently) redefined one of the axioms:
  - introducing ***non-Euclidean geometry***
    - e.g. ***geometry of a spherical surface***:
      - Q: What is the sum of angles in a triangle on a spherical surface? Is there an easy proof?

Hilbert's Program

- David Hilbert, a well know mathematician around the turn of the last century, i.e. circa 1900

  identified a set of 23 problems to be "grand challenge" problems of math.
- One of Hilbert's problems was to <u>"formalize" all of math</u>
  - i.e. *put mathematics on a solid footing of well-defined (systems of) axioms and unambiguous proofs*.

Russell's Paradox

- Bertrand Russell - a philosopher - took Hilbert's formalization challenge seriously:
  - He was inspired by Euclid's Elements
  - He started by defining "Sets" among other things
- Russell's paradox:
  - Consider
    - a *catalog C of catalogs that lists all those catalogs that do not list themselves*.
  - Does C list itself?
- Where lies the contradiction?
  - This led to axiomatic set theory and a specific restriction on "recursive" or "self-referential" definitions

END

Additional – Euclid's Axioms

- Euclid's Postulates (Source: **Wolfram MathWorld**)

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate