

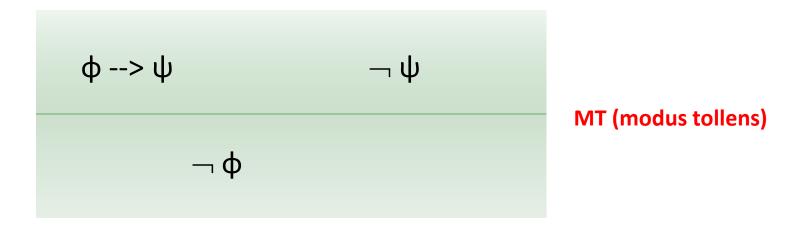
CS/IS F214 Logic in Computer Science

MODULE: PROPOSITIONAL LOGIC

Natural Deduction: Derived Rules, Proofs - Examples

21-08-2018 Sundar B. CS&IS, BITS Pilani 0

Modus Tollens

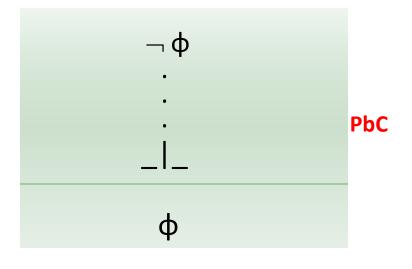


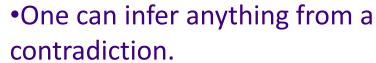
<u>Questions</u>: What is the relation between this and *modus ponens*?

How do you derive this rule (from the basic rules)?



Proof by Contradiction





- •But in this case the contradiction resulted from an assumption i.e. $\neg \phi$
- •Therefore it is meaningful to infer φ that the assumption led to the contradiction
- In fact one must infer φ to eliminate the assumption $\neg \varphi$.
 - Why?

Exercise: Derive PbC from the \neg i rule.



Exercises

• Prove the following:

1.
$$\neg p \lor q \mid -- p \rightarrow q$$

2.
$$q --> (p --> r), \neg r, q \mid -- \neg p$$

3.
$$p-->q \mid -- \neg p \lor q \mid$$

4. Use 3. to prove LEM

[These examples were worked out in class – on the board]



Take-home Exercises

Prove the following sequents

1.
$$\neg(p \land q) \mid -- \neg p \lor \neg q$$

2.
$$q \rightarrow (p \rightarrow r)$$
, $\neg r$, $q \mid --- \neg p \mid$

3.
$$p --> q, r --> s \mid -- p \lor r --> q \lor s$$

4.
$$(p --> r) \land (q --> r) \mid -- p \land q --> r$$

