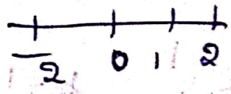
Q) $P(X = -2) = P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{1}{3}$, $P(X=2) = \frac{1}{6}$. Find Median.

Sol) $E(X) = -\frac{1}{2} + \frac{1}{3} + \frac{1}{3} = \frac{2}{3} - \frac{1}{2} = \frac{+1}{6}$.



$P(X \le M) \ge \frac{1}{2}$.

$P(X \ge M) \ge \frac{1}{2}$.

(Q) $f_X(x) = 1, \quad 0 \le x \le 1$.

$f_X(x) = 0$, otherwise.

$F_X(x) = \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{x} f_X(x)\,dx = \frac{1}{2}$ $\quad x = \frac{1}{2}$.

Sol) $E(X) = \frac{1}{2}$. $\quad \cancel{F_X(x) = \int_{-\infty} dx}$

$= \int_0 x^2 \cdot dx$.

$var(X) = ?$

$var(X) = E(X^2) - [E(X)]^2$

$= E(X^2) - \frac{1}{4}$

$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$.

$E(M) = \frac{1}{2}$

M.G.F. (Moment Generating $f^n$)

$M_X(t) = E(e^{tX})$.

Let X be random variable, the $M_X(t)$ is called MGF provided the function exists in the neighbourhood of t.

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\,dx.$$

→ M.G.F uniquely determines $-x$ a) c.d.f and Conversely if M.G.F Exists its Unique.
If the M.G.F $M_x(t)$ Exists for $|t|<t_0$ the derivatives of all orders Exists at $t=0$ and can be Evaluated under integral sign (or) Summation sign

$$\left.\frac{d^k M_x(t)}{dt^k}\right|_{t=0} = \mu_k' , \quad k=1,2,3\ldots$$

$$\frac{d}{dt}(M_x(t)) = \frac{d}{dt}\left[E(e^{tx})\right] = E\left(\frac{d}{dt}e^{tx}\right)$$

$$= E(xe^{tx})\Big|_{t=0} = E($$

$$E(e^{tx}) = E\left(1+\frac{tx}{1!}+\frac{(tx)^2}{2!}+\ldots\right)$$

$$= 1+\frac{t}{1!}\cdot\mu_1' + \frac{t^2}{2!}\mu_2' + \ldots$$

(Q) $f_x(x) = \lambda e^{-\lambda x}$, $x>0$, $\lambda>0$
find M.G.F

Sol) $M_x(t) = E(e^{tx}) = \int e^{tx} f_x(x)\,dx$

$$= \int e^{tx}\cdot\lambda\cdot e^{-\lambda x}\,dx$$

$$\frac{d}{dt}(M_x(t)) = E(x)$$

$$= \lambda\int_0^\infty e^{(t-\lambda)x}\,dx$$

(If $t<\lambda$ it Exists).

$$= \lambda\cdot\left[\frac{e^{(t-\lambda)x}}{t-\lambda}\right]_0^\infty$$

$$= \lambda\cdot\left[0-\frac{1}{t-\lambda}\right] = \boxed{\frac{\lambda}{\lambda-t}}$$

$$\left.\frac{d}{dt}(M_x(t))\right|_{t=0} = \left.\frac{\lambda}{(\lambda-t)^2}\right|_{t=0} = \boxed{\frac{1}{\lambda} = E(X)}$$

$$\left.M_x''(t)\right|_{t=0} = \frac{2\cdot\lambda}{(\lambda-t)^3} = \frac{2}{\lambda^2} = E(X^2).$$

$$Var(X) = E(X^2) - [E(X)]^2$$
$$= \boxed{\frac{1}{\lambda^2}}.$$

## Discrete dist$^n$s –

## Discrete Uniform Dist. –

$$X \rightarrow 1, 2, \ldots, N$$

$$P(X = i) = \frac{1}{N} \quad (i = 1, 2, 3, 4 \ldots, N)$$

$$P(X \neq i) = 0$$

$$E(X) = 1 \cdot \frac{1}{N} + 2 \cdot \frac{1}{N} + \ldots$$
$$= \frac{N(N+1)}{2N} = \boxed{\frac{N+1}{2}}.$$

$$E(X^2) = 1 \times \frac{1}{N} + 4 \times \frac{1}{N} + \ldots$$
$$= \boxed{\frac{1}{N} \times \frac{n(n+1)(2n+1)}{6}}.$$

$$Var(X) = \frac{\cancel{N}(N+1)(2N+1)}{6\cancel{N}} - \frac{(N+1)^2 \cancel{N}}{4\cancel{N}}.$$
$$= \frac{2(2N^2+3N+1) - 3(N^2+1+2N)}{12} = \boxed{\frac{N^2-1}{12}}$$

$$M_X(t) = E(e^{tX}) = \int$$
$$= \sum_{x=0}^{\infty} e^{tx} \cdot E(x)$$
$$= e^t \cdot \frac{1}{N} + e^{2t} \cdot \frac{1}{N} + \ldots$$
$$= \boxed{\frac{e^t (e^{Nt} - 1)}{N(e^t - 1)}}. \quad ; \quad t \neq 0.$$

$t = 0$

$1 = 1$

Rough –

$$\lambda \int_0^{\infty} x^2 \cdot e^{-\lambda x} dx$$
$$\left[ \cancel{\lambda} \cdot x^2 \cdot \frac{e^{-\lambda x}}{\cancel{\lambda}} \right]_0^{\infty}$$
$$+ \frac{2}{\lambda} \int_0^{\infty} x \cdot e^{-\lambda x} dx$$

$\rightarrow P(X = c) = 1$
$\qquad = 0$, otherwise.

- $E(X) = c$ ; $\mu_k' = c^k$ , $k = 1, 2, \ldots$

## Bernoulli Distribution –

A Bernoulli trial is an experiment with two possible outcomes success (or) failure. Success happens with probability P and failure happen with probability 1−P

Success $\downarrow$ $P$ ; Failure $\downarrow$ $1-P$

$X \rightarrow 1$ ; $0$

$P_X(0) = 1-P$, $P_X(1) = P$, $0<p$
$P_X(x) = 0$, $\forall x \neq 0, 1$

$E(X) = 0(1-P) + 1(P) = P$

$\mu_k' = E(X^k) = \boxed{P}$

$Var(X) = p - p^2 = p(1-p) = \boxed{pq}$

$M_X(t) = \int e^{tx} E(x)\, dx = e^{t(x)}(P) + \ldots = E(e^{tX})$

$= e^t(P) + e^{t(0)}(1-P) = \boxed{pe^t + (1-P)}$

## DMPT-15 –

Binomial Dist – $X \sim Bin(n, p)$.

$\rightarrow$ consider n independent and identical bernoullion trials with prob of success in each trial as 'p'.

$X \rightarrow$ no of successes in n trials
$\rightarrow 0, 1, 2, \ldots, n$.

$P(X = j) = P_X(j) = {}^nC_j \cdot p^j (1-p)^{n-j}$ ; $j = 0, 1, 2 \ldots n$

$P(X = x) = P_X(x) = {}^nC_x \cdot p^x (1-p)^{n-x}$ ; $x = 0, 1, \ldots n$

$$\sum_{j=0}^{n} P_X(j) = \sum_{j=0}^{n} {}^nC_j\, p^j (1-p)^{n-j}$$

$$= {}^nC_1\, p(1-p) + {}^nC_0 (1-p)^n + {}^nC_2\, p^2 (1-p)^{n-2}$$

$$= (1-p+p)^n = \boxed{1}$$

$$E(X) = \sum_{j=0}^{n} j\, {}^nC_j\, p^j (1-p)^{n-j}$$

$$= 0 + {}^nC_1\, p(1-p)^{n-1} + 2\, {}^nC_2\, p^2 (1-p)^{n-2}$$

$$+ 3\, {}^nC_3\, p^3 (1-p)^{n-3}$$

$$+ \cdots + n\, {}^nC_n\, p^n (1-p)^0$$

$$= \sum_{j=1}^{n} \frac{n!}{(n-j)!\,(j-1)!} p^j (1-p)^{n-j}$$

$$= \sum_{i=0}^{n-1} \frac{n!}{(i)!\,(n-i-1)!} (p)^{i+1} (1-p)^{n-i-1} \quad i = (j-1)$$

$$= n \cdot p \sum_{i=0}^{n-1} \frac{(n-1)!}{i!\,(n-1-i)!} p^i (1-p)^{n-i-1}$$

$$= np\,(p+1-p)^{n-1} = \boxed{np}$$

$$E(X(X-1)) = n(n-1)p^2$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= \boxed{np(1-p)} = \boxed{npq} \quad (q = 1-p).$$

$$\beta_1 = \frac{\mu_3}{\sigma^3} = \frac{1-2p}{(npq)^{1/2}}$$

$$= 0 \quad p = \frac{1}{2}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{1-6pq}{npq}.$$

$M_x(t) = E(e^{tx}) = \sum_{j=0} e^{tj}\ {}^nC_j\ p^j (1-p)^{n-j}$

$= (1-p+pe^t)^n$

$= (q+pe^t)^n$

<u>Geometric</u> <u>Distribution</u> - "$X \sim Geo(p)$"

Suppose independent Bionomial trial is conducted till success is acheived.

$X \rightarrow$ no. of trials needed for first success.

$X \rightarrow 1, 2, 3 \ldots\ldots$

$$P(X=j) = (1-p)^{j-1} \cdot p \quad ; \quad j = 1, 2, \ldots\ldots$$

$$\sum_{x=1}^{\infty} P_X(x) = (1-p)^0 \cdot p + (1-p)^1 \cdot p + (1-p)^2 p + \ldots$$

$$= p\left(\frac{1}{1-1+p}\right) = p \times \frac{1}{p} = \frac{p}{p} = \boxed{1}$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \boxed{\frac{q}{p^2}}$$

$\left(E(x) = \frac{1}{p}\right)$

(Refer - Tut)

$$M_X(t) = E(e^{tx}) = \sum_{x=1}^{\infty} e^{tx} (1-p)^{x-1} p$$

$$= p \cdot \frac{e^t}{1-(1-p)e^t} = \boxed{\frac{pe^t}{1-qe^t}}$$

$0 < qe^t < 1$

$e^t < \frac{1}{q}$

$\boxed{t < -\log q}$

$$P(X > m) = (1-p)^m \cdot p + (1-p)^{m+1} \cdot p + (1-p)^{m+2} \cdot p$$

$$= p \cdot \frac{(1-p)^m}{p} = (1-p)^m = \boxed{q^m}$$

P( X>

"(ACB)"

(Inverse (-ve))

consider

identical

X →

P(X = k

E(X) =

<u>Markov's</u>

that

for

P{X

<u>proof</u>-

Valu

conside

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$$P(\underset{A}{X > m+n} / \underset{B}{X > n}) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{q^{m+n}}{q^n}$$

$$= \boxed{q^m}$$

"$(A \subset B)$" $\downarrow$ Memoryless prop.

(starting point doesn't matter).

(Inverse) – Binomial –
(–ve) – Binomial –

→ consider independent Bernoullis trials under identical conditions till $r^{th}$ success achieved

$$\underbrace{- - - - - - -}_{\substack{k-1 \\ r-1}} \text{\textcircled{}}\, p$$

$X \longrightarrow r, r+1, - - - - -$

$$P(X = k) = {}^{k-1}C_{r-1}\, q^{k-r} p^r \; ; \quad k = r, r+1 - - -$$

$$E(X) = \frac{r}{p} \; ; \quad V(X) = \frac{rq}{p^2}.$$

Markov's inequality – If $X$ is a random variable that takes only non-negative values, then for any value $a > 0$,

$$P\{X \geq a\} \leq \frac{E(X)}{a}.$$

proof – let us suppose that R.V $X$ takes values, $x_1 < x_2 \cdots x_{j-1} < x_j = a < x_{j+1} < x_n < \cdots$

consider $E(X) = \sum_{i=1}^{\infty} x_i P(x_i)$

$$= \sum_{i=1}^{j-1} x_i P(x_i) + \sum_{i=j}^{\infty} x_i P(x_i)$$

$$\geq \sum_{i=j}^{\infty} x_i P(x_i) \geq \sum_{i=j}^{\infty} a P(x_i)$$

$$\Rightarrow E(X) \geq a P(X \geq a)$$

$$\Rightarrow \boxed{P(X \geq a) \leq \frac{E(X)}{a}}.$$

For continuous

$$E(X) = \int_0^{\infty} x \cdot f(x) = \int_0^{a} x f(x)\, dx + \int_a^{\infty} x f(x)\, dx$$

$$\geq \int_a^{\infty} x f(x)\, dx \geq \int_a^{\infty} a f(x)\, d$$

$$\Rightarrow \boxed{\frac{E(x)}{a} \geq P\{x \geq a\}}.$$

Chebyshev's Inequality - If $X$ is a random variable with finite mean $\mu$ and vari $\sigma^2$ then for any value $k>0$;

$$P\{|X-\mu| \geq k\} \leq \frac{\sigma^2}{k^2}.$$

Proof - Since $(x-\mu)^2$ is non-negative and applying Markov's inequality by taking $a=$ we get $P\{(x-\mu)^2 \geq k^2\} \leq \frac{E[(X-\mu)^2]}{k^2} = \frac{\sigma^2}{k^2}$

Since $(x-\mu)^2 \geq k^2$ if and if $|X-\mu| \geq k$ we have

$$P\{|X-\mu| \geq k\} \leq \frac{\sigma^2}{k^2}.$$

① Suppose that if it is known that the n of items produced in a factory during a week is a R.V with mean 50.

ⓐ what can be said about prob. that we production will exceed 75?

Sol) $P\{X > 75\} \leq \frac{E(X)}{75} = \frac{50}{75} = \frac{2}{3}$.

ⓑ If variance is 25, what can be said abo prob. this week's production will b b/w 40 & 60?

Sol) $P\{|x-50| \geq 10\} \leq \frac{25}{10^2} = \frac{1}{4}.$

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$1 - P\{|X-50| \geq 10\} \geq 1 - \frac{1}{4}$

$P\{|X-50| < 10\} \geq \frac{3}{4}$.

② If $X$ is uniformly distributed over interval $(0, 10)$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

$$E[X] = \int_0^{10} \frac{1}{10-0} x\,dx = \frac{1}{10}\left[\frac{x^2}{2}\right]_0^{10} = 5.$$

$$var[X] = E(X^2) - (E(x))^2 = \frac{1}{10}\int_0^{10} x^2 dx - (5)^2 = \frac{25}{3}.$$

ⓐ $P\{|X-5| \geq 4\} = ?$

$$P\{|X-5| \geq 4\} \leq \frac{25}{3(4^2)} = 0.52.$$

$$1 - P\{|X-5| < 4\} = 1 - P\{-4 < x-5 < 4\}$$

$$= 1 - P\{1 < x < 9\}$$

$$= 1 - \int_1^9 \frac{1}{10-0} dx = 1 - \frac{1}{10}[9-1]$$

$$= 1 - \frac{8}{10} = 0.2.$$

∴ we can't expect the actual value and bound to be equal (or) closer.

**Theorem -**

If $var(X) = 0$, then $P[x = E[x]] = 1$.

for any value $n \geq 1$, by chebeshev's inequality

$$P\{|X-\mu| \geq \frac{1}{n}\} \leq \frac{\sigma^2}{\frac{1}{n^2}} = \sigma^2(n^2)$$

$$P\{|X-\mu| \geq \frac{1}{n}\} = 0 \quad (\because var(X) = 0).$$

let $n \to \infty$ and apply the continuity prop. we have

$$\lim_{n\to\infty} P\{|X-\mu| > \frac{1}{n}\} = 0 \quad P\{\lim_{n\to\infty} |X-\mu| > \frac{1}{n}\} = 0$$

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$P\{|x-M|>0\}=0$

$P\{x \neq M\}=0$

$P\{x=M\}=1 \qquad P\{x=E(x)\}=1.$

→ **weak law of large numbers**: Let $x_1, x_2, \ldots$ be sequence of independent and identically dist. R.V each having finite mean $E(x_i)$, then, for any $E>0$,

$$P\left\{\left|\frac{x_1+x_2 \ldots x_n}{n}-M\right| \geq E\right\} \to 0 \text{ as } n\to\infty$$

let $X = \frac{x_1+x_2+\ldots x_n}{n}$

$$= \frac{1}{n} E(x_1+x_2+\ldots x_n)$$

$$= \frac{1}{n}\left(E(x_1)+E(x_2)+\ldots E(x_n)\right)$$

$$= \frac{1}{n}[Mn] = \boxed{M}$$

$$var\left(\frac{x_1+x_2+\ldots x_n}{n}\right) = \frac{1}{n^2}\left(var(x_1)+var(x_2)+\ldots var(x_n)\right)$$

$$= \frac{1}{n^2} \times n \times \sigma^2 = \frac{\sigma^2}{n}$$

by applying chebeshey's inequality

$$P\left\{\left|\frac{x_1+x_2+\ldots+x_n}{n}-M\right| \geq E\right\} \leq \frac{\sigma^2}{n(E^2)}$$

letting $n\to\infty$

$$P\left\{\left|\frac{x_1+x_2+\ldots x_n}{n}\right|-M\right\} \to 0 \text{ as } n\to\infty$$

DMPT - 17 -

observations/occurences/happening over a time/space.
poisson process - provided they satisify below

① No. of outcomes (or) occurences during disjoint time intervals are intependent.

② prob. of single occurence during a small time int. is proportional to leng. of int.

③ The prob. of more than one occurences during a small time int. is negligible.

$X \to$ no. of occurences in an interval of length $t$.

$X(t) \to (0, t]$ $P_n(t) = P(X(t) = n) = \dfrac{e^{-\lambda t} \cdot (\lambda t)^n}{n!}, n = 0, 1, 2$

② $P_1(h) = P(X(h) = 1) = \lambda h$.

③ $P(X(h) \geq 2) = o(h)$.

The dist. of occurences in a fixed time interval during a poisson process is called poisson's distribution.

$$p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2 \ldots \quad \lambda > 0$$

$= 0$, elsewhere.

$$\sum_{x=-\infty}^{\infty} P_X(x) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} \cdot e^{\lambda} = \boxed{1}.$$

$$\mu_1' = E(X) = e^{-\lambda} \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{x!} \cdot x$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \cdot (\lambda)$$

$$= e^{-\lambda} \cdot e^{\lambda} \cdot \lambda = \boxed{\lambda}.$$

$$E(X^2) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \cdot x^2 = e^{-\lambda} \sum_{x=0}^{\infty} \frac{x \cdot \lambda^x}{(x-1)!}$$

$Var(X) = E(X^2)$

~~Var(x)~~ $E(X(X-1)) = \alpha_2 = \boxed{\lambda^2}$.

$E(X^2) = E(X(X-1)) + E(X) = \boxed{\lambda^2 + \lambda}$

$= \lambda^2 + \lambda - [E(X)]^2$

$= \boxed{\lambda}$. $- \lambda^2$

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$\alpha_k = E(x(x-1)\ldots(x-k+1)) = \lambda^k$

$\beta_1 = \dfrac{\mu_3}{\sigma^3} = \dfrac{\lambda}{\lambda^{3/2}} = \dfrac{1}{\sqrt{\lambda}} > 0$

$\beta_2 = \dfrac{\mu_4}{\mu_2^2} - 3 = \dfrac{\lambda + 3\lambda^2}{\lambda^2} - 3 = \dfrac{1}{\lambda} > 0$

$M_x(t) = E(e^{tx}) = e^{-\lambda} \sum_{x=1}^{\infty} \dfrac{e^{tx} \cdot \lambda^x}{x!}$

$= e^{-\lambda} \sum_{x=1}^{\infty} \dfrac{(\lambda e^t)^x}{x!} = e^{-\lambda} \cdot e^{\lambda e^t}$

$= \boxed{e^{\lambda(e^t - 1)}}$

<u>Theorem</u> – Let $X \sim Bin(n, p)$.

Let $n \to \infty$, $p \to 0$ $\ni np \to \lambda$ then

$P_x(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$

$M_x(t) = (q + pe^t)^n$

$= (1 - p + pe^t)^n$

$= [1 + p(e^t - 1)]^n$

$= [1 + \frac{\lambda}{n}(e^t - 1)]^n$

$= Lt_{n \to \infty} \, n \cdot [1 + \frac{\lambda}{n}(e^t - 1)]^{-1}$

$= \boxed{\lambda(e^t - 1)}$

<u>Continuous Distribution</u> –

<u>cont.</u> <u>Uniform Dist</u>$^n$ –

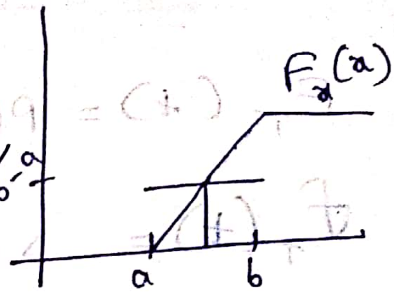
$f_x(x) = k \quad , \quad a \leq x \leq b$

$\quad = 0 \quad , \quad$ elsewhere.

$$\int_a^b f_X(x) + \int_{-\infty}^{0} f_x(x) + \int_b^{\infty} f_x(x) = 1$$

$$\Rightarrow \quad k(b-a) = 1$$

$$\boxed{K = \frac{1}{b-a}}$$




$$F_X(x) = 0, \quad \text{if} \quad x \leq a.$$
$$= \frac{x-a}{b-a}, \quad \text{if} \quad a \leq x \leq b$$
$$= 1, \quad x > b.$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_x(x)\, dx$$
$$= \int_a^b x \cdot k\, dx = \frac{k \cdot (b^2 - a^2)}{2} = \frac{1}{\cancel{(b-a)}} \cdot \frac{(b+a)\cancel{(b-a)}}{2}$$
$$= \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

$$E(X^k) = \frac{b^{k+1} - a^{k+1}}{(k+1)(b-a)}, \quad E(X^2) = \frac{b^3 - a^3}{3(b-a)}$$

$$[E(X)]^2 = \left(\frac{a+b}{2}\right)^2$$

$$M_X(t) = E(e^{tx})$$
$$= \int_{x=-\infty}^{x=\infty} e^{tx} \cdot f_x(X=x)\, dx$$
$$= \int_a^b e^{tx} \cdot k\, dx$$
$$= \boxed{\frac{1}{b-a} \cdot \frac{e^{t(b)} - e^{t(a)}}{t}}, \quad t \neq 0$$
$$= 1 \quad \text{if} \quad t = 0.$$

DMPT - 18

Consider a Poisson's process with rate $(\lambda) > 0$ let $t$ be time of $1^{st}$ occurence

$P(T > t) = P(X(t) = 0) = e^{-\lambda t}, \quad t > 0$

$= 1 \quad ; \quad t \leq 0$

$F_T(t) = P(T \leq t) = 1 - P(T > t) = 0, \quad t \leq 0$

$= 1 - e^{-\lambda t}, \quad t > 0.$

$f_T(t) = \lambda e^{-\lambda t}, \quad t > 0.$

$= 0, \quad t \leq 0.$

$\mu_k' = E(T^k)$

$= \int_0^\infty t^k \lambda e^{-\lambda t} dt$

$= \dfrac{\lambda \Gamma(k+1)}{\lambda^{k+1}} = \dfrac{k!}{\lambda^k}$

$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}$

$\Gamma(\alpha+1) = \alpha \Gamma(\alpha)$

$\Gamma(n+1) = n!$

$\Gamma 1 = 1$

$\Gamma \frac{1}{2} = \sqrt{\pi}.$

$\mu_1' = E(T) = \dfrac{1}{\lambda}.$

$Var(T) = E(T^2) - [E(T)]^2 = \dfrac{1}{\lambda^2}$

$\mu_3' = \dfrac{6}{\lambda^3}, \quad \mu_4' = \dfrac{24}{\lambda^4}.$

$\beta_1 = 2 > 0$

$\beta_2 = 6 > 0.$

$P(T > a+b / T > b) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{e^{-\lambda(a+b)}}{e^{-\lambda(b)}} = \boxed{e^{-\lambda a}}$

memoryless property.

Me

**Normal** Distribution — $X \sim N(\mu, \sigma^2)$

$f_X(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma > 0$

$$\int_{-\infty}^{\infty} f_X(x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx, \qquad z = \frac{x-\mu}{\sigma}, \quad dz = \frac{dx}{\sigma}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz.$$

$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz, \qquad t = \frac{z^2}{2}.$$

$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t} \cdot \frac{1}{\sqrt{2t}} dt = \frac{1}{\sqrt{\pi}} \int_0^{\infty} t^{\frac{1}{2}-1} e^{-t} dt = 1$$

$$E\left(\frac{x-\mu}{\sigma}\right)^k = \int_{-\infty}^{\infty} \left(\frac{x-\mu}{\sigma}\right)^k \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_{-\infty}^{\infty} z^k \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \qquad \boxed{z = \frac{x-\mu}{\sigma}}$$

$k = 2m+1$, $E\left(\frac{x-\mu}{\sigma}\right)^k = 0$

$k = 2m$, $E\left(\frac{x-\mu}{\sigma}\right)^k = (2m-1)(2m-3)\ldots 5.3.1.$

$k = 1$, $E\left(\frac{x-\mu}{\sigma}\right) = 0$

$E(X) = \mu$.

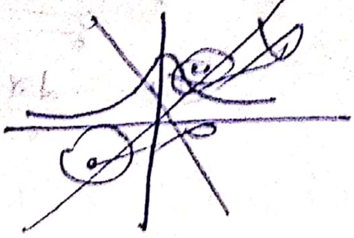
$E\left(\frac{X-\mu}{\sigma}\right)^2 = 1. \quad k = 2.$

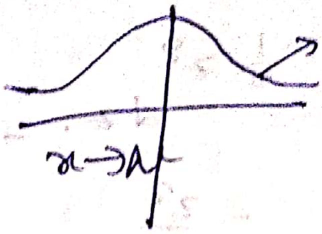
~~$E(X^2)$~~

$E(X-\mu)^2 = \sigma^2 = \mu_2.$

All odd-ordered central Moments for normal dis. are 0.

$\beta_1 = 0, \quad \beta_2 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{3\sigma^4}{\sigma^4} - 3 = \boxed{0}$






$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= e^{\mu t + \frac{1}{2}\sigma^2 z^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2 - 2t\sigma z + \sigma^2 z^2)} dz.$$

$$= e^{\mu t + \frac{1}{2}\sigma^2 z^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-\sigma t)^2} dz$$

$$= \boxed{e^{\mu t + \frac{1}{2}\sigma^2 t^2}}$$

$X \sim N(\mu, \sigma^2)$

$Y = aX + b, \quad a \neq 0$

$b \in R.$

$$M_Y(t) = E(e^{tY}) = E(e^{t(aX+b)})$$

$$= e^{bt} E(e^{at(X)}).$$

$$= e^{bt} M_X(at).$$

$$= e^{bt} e^{\mu at + \frac{1}{2}\sigma^2 t^2 a^2}$$

$$= e^{t(a\mu + b + \cancel{\frac{1}{2}\sigma^2 a^2 t^2}) + \frac{1}{2}\sigma^2 a^2}$$

$Y \sim N(a\mu + b, a^2\sigma^2)$.

$Y = ax + b$

$E(Y) = aE(x) + b$

$= a\mu + b$.

$Var(Y) = var(ax+b) = a^2 Var(x) = a^2\sigma^2$.

DMPT-19-

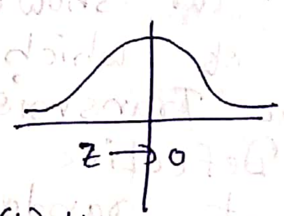
$X \sim N(\mu, \sigma^2)$ and $Y = ax + b, a \neq 0, b \in R$ then

$Y \sim N(a\mu + b, a^2\sigma^2) \rightarrow$ Linearity prop. of Normal Dist$^n$.

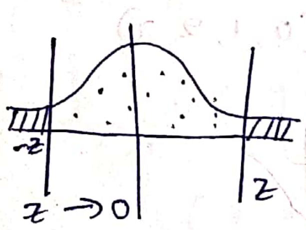
$z = \frac{x - \mu}{\sigma}$. $a = \frac{1}{\sigma}$ ; $b = -\frac{\mu}{\sigma}$.

$Z \sim N(0,1) \rightarrow$ standard Normal variable.

$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$.




cdf of $z$ $\Phi(z) = \int_{-\infty}^{z} \phi(t)\,dt$.




$1 - \Phi(z) = \Phi(-z)$.

$\Rightarrow \Phi(z) + \Phi(-z) = 1$.

of $z = 0$ ; $\Phi(0) = \frac{1}{2}$.

$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$

$= P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$

$\Phi\left(\frac{b-\mu}{\sigma}\right) = \int_{-\infty}^{\frac{b-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$.

$\Phi(-0.5) = 1 - \Phi(0.5)$

Joint Distribution -

Bivariate Distribution -

$X, Y \quad \Omega \to R^2 \; (\Omega \to R^p)$

$X_1$ - weight, $X_2 \to$ pulse rate

$X_3$ - height, $X_4 \to$ B.P

$F_{XY}(x,y) = P(X \le x, Y \le y)$,

$((x,y) \in R^2)$.

$X \to (X_1, X_2, X_3, X_4) \in R^4$

Random vector

Both X and Y are descrete

$(X, Y) \in \mathcal{X} \times \mathcal{Y}$. $\to$ finite (or) Countable.

p.d.f of x, y (X, Y) satisfies,

① $P(X = x, Y = y) = P_{X,Y}(x_i, y_i)$.

② $P_{XY}(x_i, y_i) \ge 0 \quad \forall (x_i, y_i) \in \mathcal{X} \times \mathcal{Y}$.

③ $\sum_x \sum_y P_{X,Y}(x_i, y_i) = 1$.

(Q) Suppose in a car showroom has 10 cars of a brand out of which 5 are good 2 are having Defective Transmission problem and 3 ared having Defective steering. If 2 cars are selected at random

$X \to$ No. of ~~w~~cards with DT (0, 1, 2)

$Y \to$ No. of cars with D.S (0, 1, 2, 3)

Sol) $P_{X,Y}(0,0) = \frac{^5C_2}{^{10}C_2} = \frac{\frac{\cancel{5} \times 4^2}{\cancel{2}}}{\frac{\cancel{10}_2 \times 9}{\cancel{2}}} = \boxed{\frac{2}{9}}$

$P_{X,Y}(0,1) = \frac{^5C_1 \times {^3C_1}}{^{10}C_2} = \frac{\cancel{5} \times \cancel{3}}{\frac{\cancel{10}^2 \times \cancel{9}3}{\cancel{2}}} = \boxed{\frac{1}{3}}$

| X \ Y | 0 | 1 | 2 | $P_X(x)$ |
|---|---|---|---|---|
| 0 | $\frac{10}{45}$ | $\frac{1}{3}$ | $\frac{3}{45}$ | $\frac{28}{45}$ |
| 1 | $\frac{10}{45}$ | $\frac{6}{45}$ | 0 | $\frac{16}{45}$ |
| 2 | $\frac{1}{45}$ | 0 | 0 | $\frac{1}{45}$ |
| $P_Y(y)$ | $\frac{21}{45}$ | $\frac{21}{45}$ | $\frac{3}{45}$ | |

$P(X \le 1, Y \le 1)$

$= P(X \le 0, Y \le 1) +$

$P(X \le 1, Y \le 0) +$

$P(X \le 0, Y \le 0)$

$= \boxed{\frac{41}{45}}$

$P_X(x) =$

$P_Y(y) =$

The condi

$P_{X/Y=y_j}(x_i)$

$P_{Y/X=x}$

$P_{Y/X=}$ (o

X, Y —

joint p.d.f $f$

(i) $f$

(ii) $\int_{-\infty}^{\infty}$

(iii) $= \int_a^b$

Margi

$f_X$

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$$P_X(x) = \sum_{y_j \in \mathcal{Y}} P_{X,Y}(x_i, y_j)$$ — Marginal Distributions.

$$P_Y(y) = \sum_{x_i \in \mathcal{X}} P_{X,Y}(x_i, y_j)$$

The conditional pdf of X given $Y = y_j$ :

$$P_{X/Y=y_j}(x_i) = \frac{P_{X,Y}(x_i, y_j)}{P_Y(y_j)} ; \quad x_i \in \mathcal{X}$$

$$P_{Y/X=x_j}(y_i) = \frac{P_{X,Y}(x_i, y_j)}{P_X(x_i)} ; \quad y_j \in \mathcal{Y}.$$

$$P_{Y/X=0}(0) = \frac{P_{X,Y}(0,0)}{P_X(0)} \quad \cancel{; \; y.}$$

$$= \boxed{\frac{10}{28}}.$$

$$\cancel{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$ joint

Marginal

$X, Y \rightarrow$ cont.

joint p.d.f $f_{X,Y}(x, y)$

(i) $f_{X,Y}(x,y) \geq 0$

(ii) $\int_{-\infty}^{\infty} \int f_{X,Y}(x,y)\, dx\, dy = 1.$

(iii) $\int_a^b \int_c^d f_{X,Y}(x, y) = P\begin{pmatrix} a < X < b \\ c < Y < d \end{pmatrix}$

Marginal distn

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy. \qquad f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx.$$

We say that random variables $X$ & $Y$ are independently distributed.

$F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y)$, $\forall (x,y) \in \mathbb{R}^2$.

If $X, Y$ both are discrete. $X$ & $Y$ are independent.

$p_{X,Y}(x_i, y_j) = p_X(x_i) \cdot p_Y(y_j)$, $\forall (x_i, y_j) \in \mathcal{X}_{XY}$.

$X$ & $Y$ are independent if

1. $X$ & $Y$ are continuous.

$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$, $\forall (x,y) \in \mathbb{R}^2$

$$f_{X,Y}(x,y) = 1, \quad 0<x<1, \quad 0<y<1.$$
$$= 0, \quad \text{elsewhere}.$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_0^1 1\,dy = 1.$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_0^1 1\,dx = 1.$$

$X, Y \to$ descrete $0, 1$

$P(X=1, Y=1) = \frac{1}{4}$, $P(X=1, Y=0) = \frac{1}{4}$.

$P(X=0, Y=1) = \frac{1}{4}$, $P(X=0, Y=0) = \frac{1}{4}$.

$P(X=1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\qquad P(Y=1) = P(Y=0) = \frac{1}{2}$.

$P(X=0) = \frac{1}{2}$.

Moments -

$$E(g(X,Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) \cdot f_{XY}(x,y)\,dx\,dy.$$

product Moments -

$\mu'_{rs} = E(X^r Y^s)$ $\quad (r,s)^{th}$ non-central prod moment

$\mu'_{1,1} = E(XY)$ $\quad \mu'_{1,0} = E(X)$

$\mu'_{0,1} = E(Y)$.

$\mu_{r,s} = E\left[(X-\mu_X)^r (Y-\mu_Y)^s\right]$ $(r,s)^{th}$ central prod. moment

$r=1,\ s=1$

$\mu_{1,1} = E\left[(X-\mu_X)(Y-\mu_Y)\right] =$ Co-variance of $X, Y$ $Cov(X,Y)$.

$= E(XY - \mu_X Y + \mu_X \mu_Y - \mu_Y X)$.

$E(XY) - \mu_X\mu_Y + \mu_X\mu_Y - \mu_X \mu_Y$

$= \boxed{E(XY) - E(X)E(Y)}$ = covariance of $X, Y$

If X & Y are independent,

$E(X^r Y^s) = E(X^r) \cdot E(Y^s)$.

$r=1, s=1$; $E(XY) = E(X)\cdot E(Y) \Rightarrow cov(X,Y) = 0$.

let X & Y be independent R.V then

$E(g(X) \cdot h(Y)) = E(g(X)) \cdot E(h(Y))$.

The coef. of correlation b/w X and Y

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$-1 \le \rho_{X,Y} \le 1$. $\quad -1 \le \frac{cov(X,Y)}{\sigma_X \sigma_Y} \le 1$.

If X, Y are independent

$\rho_{X,Y} = 0$.

If X and Y are independent then $\rho_{X,Y} = 0$ but converse is not true.

X and Y have joint pmf,

| X \ Y | -1 | 0 | 1 | $P_X(x)$ |
|---|---|---|---|---|
| 0 | 0 | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ |
| 1 | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $2/3$ |
| $P_Y$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | |

$E(X) = 1 \times \frac{2}{3} = \boxed{\frac{2}{3}}$

$E(Y) = \boxed{0}$

$E(x,y) = 0$.
$X, Y$

Discrete Maths - (DM) R. B - Kenelh Rosen

PHP - PGN Hole principle

If $k$ is a +ve integer and $k+1$ (or) more objects are placed in $k$ boxes then there is atleast one box containing two (or) more objects.

- Generalised

If $n$ objects are placed in $k$ boxes then there is atleast one box containing

$\lceil N/k \rceil$

Q) If there is sequence of 1, 11, 111, 1111, ... prove that there is one num. which is divisible by 7777.

Sol:)

1, 11, 111, 1111, ----, 1 ... 1 (7778)

There are at least two no's having same remainder by PHP.

$111\ldots1 = m_1 \times 7777 + r$

$11\ldots1 = m_2 \times 7777 + r$

$11111\ldots10000 = (m_1 - m_2)\,7777$

clearly there is one num. which is divisible by 7777.

(Q) Given m integers $a_1, a_2, \ldots a_m$ there exists $k, l$ with $1 \le k \le l \le m$ ~~Sum~~ Such that $a_{k+1} + a_{k+2} + \ldots + a_l$ is divisible by m. Use PHP

Sol)

1, 2, ----, m-1

$a_1,\ a_1+a_2,\ a_1+a_2+a_3,\ \ldots,\ a_1+a_2+\ldots+a_m$

$a_1 + a_2 + \ldots + a_k + a_{k+1} + \ldots + a_l = C_1 m + r$

$a_1 + a_2 + \ldots + a_k = C_2 m + r$

$a_{k+1} + \ldots + a_l = (C_1 - C_2)\,m$

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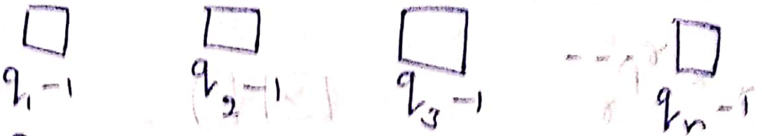
**Strong PHP** -

let $q_1, q_2, \ldots, q_n$ be +ve integers. If $q_1+q_2+\ldots+q_n - n+1$ no. of objects are put into $n$ boxes then either $1^{st}$ box contains atleast $q_1$ objects (or) second box contain atleast $q_2$ objects (or) $n^{th}$ box contains atleast $q_n$ objects.

proof -




$q_1 + q_2 + \ldots + q_n - n$

**Counting** -

**product Rule** -

Suppose that a procedure can be broken down into a sequence of two tasks if there are $n_1$ ways to do $1^{st}$ task and for Each of the ways doing $1^{st}$ tasks, there are $n_2$ ways for doing $2^{nd}$ task, there are $n_1 n_2$ ways to do procedure.

(Q) The chairs of an Auditorium is labels with upper case Alphabet followed by +ve integer not Exceeding 100. what is ~~lar~~ highest no. of chairs in Auditorium.

sol) 2600.

**Sum rule** - If a task can be done either in one of $n_1$ ways (or) in one of $n_2$ ways where none of set of $n_1$ ways is same as any of set of $n_2$ ways then there are $n_1 + n_2$ ways to do task.

(Q) In how many ways can you choose two books of different languages among 5 books in ~~lating~~ latin, 10 books in French, 7 books in greek.

Sol) $50+70+35 = \boxed{155}$

$^5C_1 \cdot {}^{10}C_1 + {}^{10}C_1 \cdot {}^7C_1 + {}^5C_1 \cdot {}^7C_1$

$$^nP_r = {}^nC_r \times {}^rP_r$$

$$^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

$$^nP_r = \frac{n!}{(n-r)!} = n(n-1)\ldots(n-r+1)$$

$n$

$(n)\ (n-1)(n-2)\ (n-r+1)$

(Q) $(k!)!$ is divisible by $(k!)^{(k-1)!}$

Sol) $((k)(k-1)(k-2)\ldots \cancel{(k} \cdot 3 \cdot 2 \cdot 1))!$

$$\frac{(k!)(k!-1)(k!-2)\ldots\ldots 3\cdot 2\cdot 1}{(k!)^{(k-1)!}}$$

1 2 ... $k!$

k k ... k

$(k-1)!$

$$\frac{[k!]!}{k!k!\ldots k!} = \frac{(k!)!}{(k!)^{(k-1)!}}$$

let there be $n$ objects that are not are distinct. Specifically there are $q_1$ objects of 1st kind, $q_2$ objects of 2nd kind, then no. of permutations of $n$ objects

$$= \frac{n!}{q_1! q_2! \cdots q_n!}$$

when repetitions in selection of objects are allowed the no. of ways of selecting $r$ objects from $n$ distinct objects is

$^{n+r-1}C_r$. *

proof -

$1, 2, \ldots n$

$\{i_1, i_2, \ldots i_r\}$ — Not distinct

$1, 2, \ldots n+r-1$ Distinct

$\{i_1+0, i_2+1, \ldots i_r+r-1\}$

- when 3 distinct dies are rolled the total no. of outcomes are $6^3 = 216$. If 3 dies are indistinguishable the no. of outcomes are $^{6+3-1}C_3$

when the objects are not all distinct the no. of ways to select one (or) more objects from them is equal to $(q_1+1)(q_2+1)\cdots(q_t+1) - 1$ where there are $q_1$ objects of 1st kind, ...

(Q) How many divisors of 1400 are there?

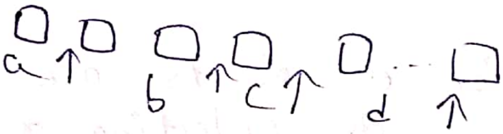
Sol) $1400 = 2^3 \times 5^2 \times 7$.

$4 \times 3 \times 2 = 24$

Urn and ball problems -

$a+b+c+d = r$

$z \geq 1$




${}^{n-1}C_{r-1}$

${}^{n-1}C_3$

$z \geq 0$


${}^{n+3}C_3$

${}^{n+r-1}C_{r-1}$

(Q) $x+y+z = 1000$

$x, y, z \geq 1$

Sol) ${}^{999}C_2$

**principle of Inclusion - Exclusion - / Subtraction**

If a task can be done in either $n_1$ way $n_2$ ways then the no. of ways to d that task is $n_1+n_2-$ (no. of ways to task that are common to diff. ways

(Q) A Computer company receives 350 a from industries. Suppose 220 of appli majored in CSE, 147 majored in business & 51 majored in both. How m of them majored neither C.S nor application.

Sol) $A_1 = 220$ $A_2 = 147$ $A_1 \cap A_2 = 51$.

$A_1 \cup A_2 = 316$. Req =1

DMPT -23-

## propositional logic-

**proposition**- An Assertion with some truth value is a proposition.

Assertion - It is a statement.

(or)

A proposition is a statement (i.e) Assertion which is either true (or) false but not both

Eg:

① 2 is prime number.

② $4+4=8$.

③ The moon is made of cheese.

Not a proposition

① $x+y=3$

② $x=5$.

③ Buy a book in amazon.

④ what time is it now.

⑤ This statement is false. (Paradox).

## Propositional Variable -

It denoteds an arbitary proposition with Unspecified truth values $p, q, r$

p : John is 6' tall

Q : There are 4 cows in field.

## Logical connectives

| | | |
|---|---|---|
| $\wedge$ | $P \wedge Q$ | $\wedge$ = and |
| $\vee$ | $P \vee Q$ | $\vee$ = or. |
| | $\neg P$ | $\neg$ = not. |

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preposition logic -

P : T or F

Q : T or F

| P | Q | $P \wedge Q$ | $P \vee Q$ | $P \oplus Q$ |
|---|---|---|---|---|
| F | F | F | F | F |
| F | T | F | T | T |
| T | F | F | T | T |
| T | T | T | T | F |

Disjunction  inclusive or

$\wedge$ - conjunction

$\vee$ - Disjunction.

~~Ex~~ propositional form -

An assertion which contains at least one propositional variable is called propositional form.

Any proposition form connecting variable is called well formed formula'

$P \Rightarrow Q$

| P | Q | $P \Rightarrow Q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | T | F |
| T | F | T |

In $P \Rightarrow Q$ P is called premise/Hypothesis/Ant-
Q is called Conclusion/consequence

If P, then Q  $P \Rightarrow Q$ (or) $P \rightarrow Q$

P only if Q  $P \Leftrightarrow Q$.  $P \rightarrow Q$

P is sufficient condition for Q.  $P \Rightarrow Q$

Q is necessary condition P  $P \Rightarrow Q$

Q if P

Q follows from P

Q provided P

Q is logical consequence of P.

Q whenever P ..

If $P \Rightarrow Q$.

then, (i) $Q \Rightarrow P$ converse.

(ii) $\neg Q \Rightarrow \neg P$ contrapositive

(iii) $\neg P \Rightarrow \neg Q$ inverse.

| $P$ | $Q$ | $P \Rightarrow Q$ | $\neg Q$ | $\neg P$ | $\neg Q \Rightarrow \neg P$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |

## Bi-Conditional

$P \Leftrightarrow Q$

(or)

$P \longleftrightarrow Q$

| $P$ | $Q$ | $P \Leftrightarrow Q$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

P if and only if Q

P is a necessary and sufficient condition for Q.

| $Q \wedge \neg P \Rightarrow P$ | $P$ | $Q$ | ~~R~~ | |
|---|---|---|---|---|
| 1 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

| $P$ | $Q$ | $R$ | $[(P\land Q)\lor \lnot R]\Leftrightarrow P$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Tautology - It is propositional form whose truth value is true for all possible values of its propositional variables.
$(P\lor\lnot P)$

Contradicition: $(P\land\lnot P)$
A contridiction (or) is propositional form which is always false

A propositional form which is neither Tautology nor contradicition is contingency.

precedence - $\lnot \;\land\; \lor \;\Rightarrow\; \Leftrightarrow$ $\longrightarrow$

If $P \Leftrightarrow Q$ is a tautology then $P \equiv Q$.

Logical identities -

$P \equiv P\lor P$ Idempotence of $\lor$

$P \equiv P\land P$ " of $\land$

$P\lor Q \equiv Q\lor P$ commutative of $\lor$

$P\land Q \equiv Q\land P$ " " $\land$.

$(P\lor Q)\lor R \equiv P\lor(Q\lor R)$
$(P\land Q)\land R \equiv (P\land Q)\land R$
} Associative law

$\lnot(P\lor Q) \equiv \lnot P\land\lnot Q$
$\lnot(P\land Q) \equiv (\lnot P)\lor(\lnot Q)$
} Demorgans law.

| P | Q | R | $[(P\land Q)\lor \neg R] \Leftrightarrow P$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Tautology - It is propositional form whose truth value is true for all possible values of its propositional variables.
$(P\lor\neg P)$

Contradicition - $(P\land\neg P)$
A contridiction (or) is propositional form which is always false

A propositional form which is neither Tautology nor Contradicition is contingency.

precedence - $\neg\ \land\ \lor\ \Rightarrow\ \Leftrightarrow$

If $P \Leftrightarrow Q$ is a tautology then $P\equiv Q$.

Logical identities -

$P\equiv P\lor P$ — Idempotence of $\lor$

$P\equiv P\land P$ — " of $\land$

$P\lor Q\equiv Q\lor P$ — commutative of $\lor$

$P\land Q\equiv Q\land P$ — " $\land$.

$$\left.\begin{array}{l}(P\lor Q)\lor R\equiv P\lor(Q\lor R)\\ (P\land Q)\land R\equiv(P\land Q)\land R\end{array}\right\}\text{Associative law}$$

$$\left.\begin{array}{l}\neg(P\lor Q)\equiv\neg P\land\neg Q\\ \neg(P\land Q)\equiv(\neg P)\lor(\neg Q).\end{array}\right\}\text{Demorgans law}$$

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ } Distributive

$P \vee 1 \equiv 1$

$P \wedge 1 \equiv P$.

$P \wedge \neg P \equiv 0$

$P \vee \neg P \equiv 1$

$P \equiv \neg(\neg P)$ $\neg$ Double negation

## Identities Using Conditional statement -

$(P \Rightarrow Q) \equiv (\neg P \vee Q) \rightarrow$ Implication

$(P \Leftrightarrow Q) \equiv [(P \Rightarrow Q) \wedge (Q \Rightarrow P)] \rightarrow$ Equivalence

$[(P \wedge Q) \Rightarrow R] \equiv [P \Rightarrow (Q \Rightarrow R)] \rightarrow$ Exportation

$[(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)] \equiv \neg P$ Absordity

$(P \Rightarrow Q) \equiv (\neg Q \Rightarrow \neg P)$ Contraposition.

1.)

| P | Q | $P \Rightarrow Q$ | $\neg P$ | $\neg P \vee Q$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

2.)

| P | Q | $P \Leftrightarrow Q$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

DMPT - 25 -

prove. $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$ Using Identities

$\underbrace{\neg(\neg P \vee Q)}_{\text{Implication}} \underbrace{= P \wedge \neg Q}_{\text{Demorgans law}}$.

prove $\neg(P \vee (\neg P \wedge Q)) = \neg P \wedge \neg Q$ Using Identities

$\left.\begin{array}{l} \neg P \wedge \neg(\neg P \wedge Q) \\ \neg P \wedge (P \vee \neg Q) \end{array}\right\}$ Demorgans law.

$(\neg P \wedge P) \vee (\neg P \wedge \neg Q)$.

$= F \vee (\neg P \wedge \neg Q)$ $(\neg P \wedge P \equiv F)$

$= \boxed{\neg P \wedge \neg Q}$

Ex -

preposition P is saying 'it is snowing'.
preposition Q is saying 'I will go to town'.
preposition R is saying 'I have time'.

(a) If it is not snowing & I have time then I will go to town.

Sol.) $(\neg P \wedge R) \Rightarrow Q$.

(a) I will go to town only if I have time.

Sol.) $Q \Rightarrow R$.

(a) It is not snowing.

Sol.) $\sim P$

(Q) It is snowing & I will not go to town.

Sol) $[P \cap (\sim Q)]$.

(Q) $Q \Leftrightarrow (R \cap \neg P)$.

Sol) I will go to town if & only if I have time and it is not snowing.

(Q) $R \cap Q$.

Sol) I have time & I will go to town.

(Q) $(Q \Rightarrow R) \cap (R \Rightarrow Q)$

$Q \Leftrightarrow R \;/\; R \Leftrightarrow Q$

Sol) I will go to town if and only if I have time.

(Q) $\neg(R \vee Q) \equiv \neg R \cap \neg Q$.

Sol) I don't have time & I will not go to town.

<u>other Logical Implication</u> -

$P \Rightarrow (P \vee Q)$ — Addition — Tautology

$(P \cap Q) \Rightarrow P$. — Simplification — Tautology

$(P \wedge [P \Rightarrow Q]) \Rightarrow Q$ — Modusponens — Tautology.

$[(P \Rightarrow Q) \cap \sim Q] \Rightarrow \neg P$ — Modus Tollens.

$[\neg P \wedge (P \vee Q)] \Rightarrow Q$ — Disjunctive syllogism.

$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$ — Hypothetical syllogism.

$[(P \Rightarrow Q) \cap (R \Rightarrow S)] \Rightarrow [(P \cap R) \Rightarrow (Q \cap S)]$ — Tautology.

$[(P \Leftrightarrow Q) \cap (Q \Leftrightarrow R)] \Rightarrow (P \Leftrightarrow R)$.

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DMPT-26 -

Brown, joines and Smith are suspected off income tax Evation. They testify under oath as follows,

→ Brown says jones is guilty & S is inno.

→ Jone says if Bro is guil. then so is Smith

→ Smith says I am inno. but atleast one of the others is guilty.

Assuming all told truth.

Sol)

B : Brown is innocent.

$\neg B$ : " is guilty.

J : Jones is innocent.

$\neg J$ : " is guilty.

S : Smith is innocent.

$\neg S$ : Smith is guilty.

$B : \neg J \wedge S$ T

$J : \neg B \Rightarrow \neg S \quad S \Rightarrow B$ T

$S : S \wedge (\neg B \vee \neg J)$ T

J - guilty

S - innocent.

B - innocent.

Predicate & Quantifier -

unary predicate $x > 3$

$p(x) : x > 3$

$p(2)$ F

$P(C) \rightarrow$ preposition

$P(x)$ : Computer $x$ is under attack

CS 1, CS 2, MATH 1

CS 2, MATH 2

$P(CS1)$ : F

$P(CS1), P(MATH1)$ T

$P(x, y): x+y>0 \rightarrow$ Binary predicate.

$P(x_1, x_2, \ldots, x_n) \rightarrow$ n-ary predicate.
n-place predicate.

$Sum(x, y, z): x+y=z$ predicate constant.

→ If $P(x_1, x_2 \ldots, x_n)$ is true for all values $c_1, c_2, \ldots c_n$ from universe $U$ then you say $P(x_1, x_2 \ldots, x_n)$ is valid in $U$.

$$\left[\begin{array}{l} Q(x, y): x+y>0 \\ u: x>0 \\ \quad y>0 \\ x \in R^+ \\ y \in R^+ \end{array}\right]$$

→ If $P(x_1, x_2, \ldots, x_n)$ is true for som $c_1, c_2, \ldots c_n$ in $U$ then it is said that $P$ is satisfiable in $U$. → If $P$ is not true for any values $c_1, c_2 \ldots c_n$ in $U$ then $P$ is said to be unsatisfiable in $U$

$$\left[\begin{array}{l} Q(x, y): x+y>0 \\ u: x \in R \\ \quad y \in R \downarrow \end{array}\right] \quad \left[\begin{array}{l} Q(x, y): x+y>0 \\ u: x<0 \\ \quad y<0 \downarrow \end{array}\right]$$

satisfiable — Unsatisfiable

→ Giving value to variable is known as Binding the variable.

Quantifer -

Universal Quantifier ($\forall$) - For any $x$, for all $x$, for each $x$, for aribitary $x$ $P(x)$.

$U$: Set of integers $I$

$\forall x\, [x < x+1]$ — T

$\forall x\, [x = 3]$ — F

$\forall x\, [x = x+1]$ — F

predicate

Existential Quantifier - for some $x$ $P(x)$

$\exists x \; P(x)$

$U$ : set of integers $I$.

$\exists x \, [x < x+1]$ T

$\exists x \, [x = 3]$ T.

$\exists x \, [x = x+1]$ F

$U: \{1, 2, 3\}$

$\forall x \; P(x) : P(1) \land P(2) \land P(3)$

$\exists x \; P(x) : P(1) \lor P(2) \lor P(3)$

Uniqueness Quantifier -

$\exists ! \, x \; P(x)$

$U$ : set of integers $I$.

$\exists ! x \, [x < x+1]$ F

$\exists ! x \, [x = 3]$ T

$\exists ! x \, [x = x+1]$ F.

$$\exists ! x \; P(x) : [P(1) \land \neg P(2) \land \neg P(3)]$$
$$\lor [\neg P(1) \land \neg P(2) \land P(3)]$$
$$\lor [\neg P(1) \land P(2) \land \neg P(3)].$$

$\forall$ and $\exists$ have higher precidence while compared to $\land \lor \Rightarrow$

$\forall x \; P(x, y) \longrightarrow$ Unary predicate.

$\forall x \forall y \; P(x, y) \longrightarrow$ preposition.

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u: set of married person

$\forall x \exists y$ [x is married to y] T

$\exists y \forall x$ [x is married to y] f.

u: the integer.

$\forall x \exists y \; [x + y = 0]$ True

$\exists y \forall x \; [x + y = 0]$ False.

$\left.\begin{array}{l}\forall x \forall y \\ \exists x \exists y\end{array}\right\}$ Interchangable.

$\forall x \exists y \not\equiv \forall y \exists x$

$\forall y \; p(x_1, x_2, x_3 \ldots x_n) \equiv p(x_1, x_2, \ldots x_n)$

DMPT-27-

$\forall x \forall y \forall z \; P(x, y, z)$

$\forall y \, Sum(x, y, z)$

$R(x, z)$

$x \leq z$.

If Domain is Empty $\forall x P(x)$ is considered to be true.

$u: \phi$

$\forall x P(x)$ T

$\exists x P(x)$ F

$u: R$

$\forall x \; x^2 \geq x$ F

$u: I$

$\forall x \; x^2 \geq x$ T

$\underline{u: R}$

$\forall x < 0 \; (x^2 > 0)$ $\quad \forall y \neq 0 \; (y^3 \neq 0)$ $\quad \exists z > 0 \; (z^2 = 2$

$\equiv \forall x (x < 0 \Rightarrow x^2 > 0)$ $\quad \equiv \forall y (\neg y = 0 \Rightarrow \neg y^3 = 0)$ $\quad \equiv \exists z (z > 0 \wedge z^2$

$\equiv \forall y (y \neq 0 \Rightarrow y^3 \neq 0)$ $\quad \equiv \exists z (z > 0 \wedge z^2 = 2)$

## Logical Equivalences involving Quantifier -

Statements involving predicates and Quantif-iers are logically Equivalent if they have same truth value no matter which predi-cates are substituted into statement and which domains of this course is used for variable in these preposition functions.

$\forall x\,(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x\, Q(x).$

$\forall x\,[P(x) \vee Q(x)) \equiv \forall x\ P(x) \vee \forall x\ Q(x).$

$\exists x\,[P(x) \vee Q(x)) = \exists x\ P(x) \vee \exists x\ Q(x).$

## Negating Quantifiar Expression -

Every student in class has taken a course in calculus then take $P(x)$ $x$ as a course in caluculus. Universe is all Student in class $\forall x P(x)$.

The Negating Quantifiar is

$\neg \forall x P(x) \equiv \exists x\ \neg P(x).$

$\neg \exists x\, P(x) \equiv \forall x\ \neg P(x).$

(Q) Expression the statement Every student in this class has studied calculus into logical notation

Sol) $C(x)$: $x$ has studied calculus.

U: all students in class.

$\forall x\ C(x)$.

$S(x)$: student $x$ in class

$C(x)$: $x$ has studied calculus.

$\forall x\,(S(x) \Rightarrow C(x)).$

$Q(x, y)$: student $x$ has studied the subject $y$

$\forall x\ Q(x, \text{calculus})$. u: all students in class.

$u$ : all students

$\forall x\,(S(x) \Rightarrow a(x, \text{calculus}))$.

(a) Some students in this ~~t~~ class has visited Mexico.

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