wishes How many possible orders can Saleswomen use when visiting these cities?

Sol) $1 \times 7! = \boxed{7!}$

15-11-19

1) Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is tautology using truth table.

2) Show " $(p \wedge q) \rightarrow r$ & $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically Equivalent Using truth table.

Sol)

| p | q | r | $p \vee q$ | $\neg p \vee r$ | $q \vee r$ | R |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

2 Sol)

| p | q | r | $p \wedge q$ | R | $p \rightarrow r$ | $q \rightarrow r$ | R |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

3.) $(p \to q) \to (r \to s)$ & $(p \to r) \to (q \to s)$ are not logically Equivalent.

Sol.) 01 10 $p = q = s = 0;\ r = 1$.

00 10

4.) Determine whether given compound proposition is satisfiable using truth table.

$(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$.

Sol.) 0.

5.) Suppose that the domain of proportional function $P(x)$ consists of 1, 2, 3, 4, 5. Express these statements without using quantifiers, instead using only negations, disjunctions and functions

(a) $\exists x P(x)$

(b) $\forall x P(x)$

(c) $\neg \exists x P(x)$

(d) $\neg \forall x P(x)$

(e) $\forall x (x \neq 3 \to P(x)) \vee \exists x \neg P(x)$

Sol.) $P(1) \vee P(2) \vee P(3) \vee (P(4) \vee P(5))$.

$P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$.

$(\neg P(1)) \wedge (\neg P(2)) \wedge (\neg P(3)) \wedge (\neg P(4)) \wedge (\neg P(5))$.

$(\neg P(1)) \vee (\neg P(2)) \vee (\neg P(3)) \vee (\neg P(4)) \vee (\neg P(5))$.

$(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee ((\neg P(1)) \vee \neg P(2) \vee (\neg P(3)) \vee (\neg P(4)) \vee (\neg P(5)))$

6.) Translate the statement

$\exists x \forall y \forall z (F(x,y) \wedge F(x,z) \wedge (y \neq z)) \rightarrow \neg F(x,y,z)$

into English where $F(a,b)$ means 'a' & 'b' are friends & domain for $x, y$ & $z$ consists of all students in your school.

Sol:) There is student $x$ such that for all students $y$ & for all students $z$ if $x$ & $z$ are friends & $x, y$ are friends & if $y, z$ is not the same person then $y$ & $z$ are not friends.

7.) $P(x)$: $x + 2 = 2x$, $x \in \{1, 2, 3\}$

(1) $\forall x P(x)$ (2) $\exists x P(x)$

translate & state T (or) F.

(1) For Every $x$ in $\{1, 2, 3\}$ $x + 2 = 2x$

False

(2) There Exists $x$ in $\{1, 2, 3\}$

$F(x)$: $x$ is a fox.

$S(x)$: $x$ is sly

$T(x)$: $x$ is trustworthy.

~~convert~~ into

Everything is a fox

All foxes are sly

If any fox is sly, then if ~~it~~ is not trustworth convert into equiv.

(Q1) Translate every real number except zero has multiplicative inverse $x$, if $xy = 1$.

Sol:) $\forall x((x \neq 0) \Rightarrow \exists y(xy = 1))$.

(Q2) Translate statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$ where $C(x)$ is $x$ has a computer $F(x,y)$ is $x$ & $y$ are friends.

Sol:) For every student in a class every $x$ ~~student~~ $x$ have computer (or) he has a friend who have computer

Sol:) For every student $x$ in school, $x$ has computer (or) there is a student $y$ such that $y$ has a computer and $x$ and $y$ are friends.

(Q3) Express as logical statement "If a person is female & is a parent, then this person is someone's mother".

~~Sol) domain all people,~~

~~$F(x)$ – person is female~~

~~$P(x)$ – person is parent.~~

~~$m(x)$ – person is someone's mother.~~

~~$\forall_x [F(x) \wedge P(x)] \Rightarrow \exists_x m(x)$.~~

Sol:) $F(x)$ $x$ is female $M(x,y) \rightarrow x$ is mother of $y$).

$P(x)$ $x$ is parent.

$\forall x((F(x) \wedge P(x)) \Rightarrow \exists y\, M(x,y))$

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4.) Everyone has Exactly one best friend
domain → all people.

Sol). $p(x, y)$ – $x$ & $y$ are best friend.

$\cancel{\forall_x (\exists_y P(x,y) \wedge \neg \forall_z P(x,z))}$.

$\cancel{\forall_x (\exists_y P(x,y) \wedge \exists_y P(x, \neg y))}$.

$\cancel{\forall x (\exists_y P(x,y) \wedge \neg \forall_{\neg y} P(x, y))}$.

$\forall_x \exists y ( B(x,y) \wedge \forall z ((z \neq y) \Rightarrow \neg B(x,z)))$.

5.) Express as logical statement "there is a women who has taken a flight on Every airline in the world"

Sol) ~~$W(x, y)$ – women $x$ has taken $y$ flight~~

~~$x(y, z)$ – $y$ flight on $z$ airline.~~

~~$\exists_x W(x,y)$~~

Sol.) w – women in world.
f – all flights.
a – all airlines.
$p(w,f)$ → w has taken f.
$Q(f,a)$ → f is flight on a.

$\exists w \forall a \exists f (P(w, f) \wedge Q(f,a))$.

6.) use quantifier to Express definition of limit of a real valued function $f(x)$ & of real variable $x$ at point a in its domain.

Sol) def – for Every real number $\epsilon > 0$ there Exists a real number $\delta > 0$ Such that $|f(x) - L| < \epsilon$, whenever $0 < |x-a| < \delta$

$\exists w \forall a \exists f (P(w, f) \wedge Q(f,a))$.

① Prove that there are infinitely many prime numbers by contradiction.

Sol) let us suppose that there are finite no. of prime numbers.

Say $a = (P_1 \cdot P_2 \cdot P_3 \ldots P_{k-1} P_k P_{k+1} \ldots, P_n) + 1$

let $P_k$ be prime divisor of $a$.

$a = C \cdot P_k, \quad C \in N$.

$C \cdot P_k = (P_1 P_2 \ldots P_{k-1} \cdot P_k \cdot P_{k+1} \ldots P_n) + 1$

divide by $P_k$.

$C = (P_1 \cdot P_2 \ldots P_{k-1} \cdot P_{k+1} \ldots P_n) + \frac{1}{P_k}$

$\frac{1}{P_k} = C - (P_1 \cdot P_2 \ldots P_{k+1} \ldots P_n)$ which is contradicition.

$\because$ L.H.S is a non-integer & R.H.S is an integer.

~~③ Prove by~~ Contradici

② Prove by contrapositive for $a, b, n \in \mathbb{Z}$ if $n \nmid ab$ then $n \nmid a$ & $n \nmid b$

Sol) $\neg(n \nmid a \text{ and } n \nmid b) \Rightarrow \neg(n \nmid ab)$.

$n/a$ or $n/b \Rightarrow n/ab$.

$n/a$ meansd $n$ is divisible by $a$.

~~$n = ak$~~

$a = nk$. $\quad b = nl$.

$ab = n^2 k l$.

$= n l'$.

3) Prove (or) disprove the validity of following. Every living thing is a plant (or) animal. David's dog is alive and its not a plant. All animals have hearts. Therefore, David's dog has a heart.

Sol)

$L(x)$: $x$ is a living thing.

$P(x)$: $x$ is a plant.

$A(x)$: $x$ is an animal.

$H(x)$: $x$ has heart.

$d$: David's dog.

1. $\forall(x)(L(x) \to P(x) \vee A(x))$ premise.
2. $L(d) \wedge \neg P(d)$ premise.
3. $\forall x (A(x) \to H(x))$ premise.
4. $L(d) \to (P(d) \vee A(d))$ Universal instantiation from (1).
5. $L(d)$ } Simplification from (2).
6. $\neg P(d)$ }
7. $P(d) \vee A(d)$ Modus Ponens (4) & (5).
8. $A(d)$ disjunctive syllogism (6) & (7)
9. $A(d) \to H(d)$ Universal Instantation (3).
10. $H(d)$ modus Ponens 8 & 9.

4) Show that "it is not sunny this afternoon and it is colder than yesterday" we will go swimming only if it is sunny. if we do not go swimming, then we will take a canoe trip. we take a