DMPT -23-

propositional logic -

proposition - An Assertion with some truth value is a proposition.

Assertion - It is a statement.

or)

A proposition is a statement (i.e) Assertion which is either true (or) false but not both.

e.g.

1) 2 is prime number.
2) $4+4=8$.
3) The moon is made of cheese.

Not a proposition

1) $x+y=3$
2) $x=5$.
3) Buy a book in amazon.
4) what time is it now.
5) This statement is false. (Paradox).

propositional Variable -

It denoteds an arbitary proposition with unspecified truth values p, q, r

p : John is 6' tall

q : There are 4 cows in field.

Logical Connectives

$\wedge$	$P \wedge Q$	$\wedge$ = and
$\vee$	$P \vee Q$	$\vee$ = or.
$\neg$	$\neg P$	$\neg$ = not.

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preposition logic -

Disjunction inclusive or

P	Q	$P \wedge Q$	$P \vee Q$	$P \oplus Q$
F	F	F	F	F
F	T	F	T	T
T	F	F	T	T
T	T	T	T	F

P : T or F

Q : T or F

$\wedge$ - conjunction

$\vee$ - Disjunction.

~~Ex~~ propositional form -

An assertion which contains atleast one propositional variable is called propositional Form.

Any proposition form connecting variables is called well formed formula'

$P \Rightarrow Q$

P	Q	$P \Rightarrow Q$
F	F	T
F	T	T
T	T	F
T	F	T

In $P \Rightarrow Q$ P is called premise/Hypothesis/Anteccident

Q is called Conclusion/consequence.

If P, then Q $P \Rightarrow Q$ (or) $P \rightarrow Q$

P only if Q $P \Leftrightarrow Q$. $P \rightarrow Q$

P is sufficient condition for Q. $P \Rightarrow Q$

Q is necessary condition P $P \Rightarrow Q$

Q if P

Q follows from P

Q provided P

Q is logical consequence of P.

Q whenever P ..

If $P \Rightarrow Q$.

then, (i) $Q \Rightarrow P$ converse.

(ii) $\neg Q \Rightarrow \neg P$ contrapositive

(iii) $\neg P \Rightarrow \neg Q$ inverse.

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

Bi-Conditional

$P \Leftrightarrow Q$

(or)

$P \longleftrightarrow Q$

P if and only if Q

P is a necessary and sufficient condition for Q

P	Q	$P \Leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

$Q \wedge \neg P \Rightarrow P$	P	Q	~~R~~	
1	0	0		
0	0	1		
1	1	0		
1	1	1		

P	Q	R	$[(P \wedge Q) \vee \neg R] \Leftrightarrow P$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Tautology - It is propositional form whose truth value is true for all possible values of its propositional variables.
$(P \vee \neg P)$

Contradicition - $(P \wedge \neg P)$
A contridiction (or) is propositional form which is always false

A propositional form which is neither Tautology nor contradicition is contingency.

precedence - $\neg \;\; \wedge \;\; \vee \;\; \Rightarrow \;\; \Leftrightarrow$

If $P \Leftrightarrow Q$ is a tautology then $P \equiv Q$.

Logical identities -

$P \equiv P \vee P$ — Idempotence of $\vee$

$P \equiv P \wedge P$ — " of $\wedge$

$P \vee Q \equiv Q \vee P$ — commutative of $\vee$

$P \wedge Q \equiv Q \wedge P$ — " " $\wedge$.

$\left.\begin{array}{l}(P \vee Q) \vee R \equiv P \vee (Q \vee R) \\ (P \wedge Q) \wedge R \equiv (P \wedge Q) \wedge R\end{array}\right\}$ Associative law

$\left.\begin{array}{l}\neg (P \vee Q) \equiv \neg P \wedge \neg Q \\ \neg (P \wedge Q) \equiv (\neg P) \vee (\neg Q).\end{array}\right\}$ Demorgans law.

$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ } Distributive

$P \vee 1 \equiv 1$

$P \wedge 1 \equiv P$.

$P \wedge \neg P \equiv 0$

$P \vee \neg P \equiv 1$

$P \equiv \neg(\neg P)$ Double negation

Identities Using Conditional statement -

$(P \Rightarrow Q) \equiv (\neg P \vee Q)$ → Implication

$(P \Leftrightarrow Q) \equiv [(P \Rightarrow Q) \wedge (Q \Rightarrow P)]$ → Equivalence

$[(P \wedge Q) \Rightarrow R] \equiv [P \Rightarrow (Q \Rightarrow R)]$ → Exportation

$[(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)] \equiv \neg P$ Absordity

$(P \Rightarrow Q) \equiv (\neg Q \Rightarrow \neg P)$ Contraposition.

1.)

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

2.)

P	Q	$P \Leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

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DMPT - 25 -

prove.

$\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$ using Identities

$\underbrace{\neg(\neg P \vee Q)}_{\text{Implication}} = \underbrace{P \wedge \neg Q}_{\text{Demorgans law}}$.

prove

$\neg(P \vee (\neg P \wedge Q)) = \neg P \wedge \neg Q$ using Identiti

$\left.\begin{array}{l}\neg P \wedge \neg(\neg P \wedge Q) \\ \neg P \wedge (P \vee \neg Q)\end{array}\right\}$ Demorgans law.

$(\neg P \wedge P) \vee (\neg P \wedge \neg Q)$.

$= F \vee (\neg P \wedge \neg Q)$ $(\neg P \wedge P \equiv F)$

$= \boxed{\neg P \wedge \neg Q}$

Ex -

preposition P is saying 'it is snowing'.

preposition Q is saying 'I will go to town'.

preposition R is saying 'I have time'.

(Q) If it is not snowing & I have time then I will go to town.

Sol) $(\neg P \wedge R) \Rightarrow Q$.

(a) I will go to town only if I have time.

Sol) $Q \Rightarrow R$.

(a) It is not snowing.

Sol) $\sim P$.

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(Q) It is snowing & I will not go to town.

Sol:) $[P \cap (\sim Q)]$.

(Q) $Q \Leftrightarrow (R \cap \neg P)$.

Sol:) I will go to town if & only if I have time and it is not snowing

(Q) $R \cap Q$.

Sol:) I have time & I will go to town.

(Q) $(Q \Rightarrow R) \cap (R \Rightarrow Q)$

$Q \Leftrightarrow R$ / $R \Leftrightarrow Q$

Sol:) I will go to town if and only if I have time.

(Q) $\neg(R \vee Q) \equiv \neg R \cap \neg Q$.

Sol:) I don't have time & I will not go to town.

<u>other Logical Implication</u> -

$P \Rightarrow (P \vee Q)$ Addition Tautology.

$(P \cap Q) \Rightarrow P$. Simplification Tautology.

$(P \wedge [P \Rightarrow Q]) \Rightarrow Q$ Modusponens Tautology.

$[(P \Rightarrow Q) \cap \sim Q] \Rightarrow \neg P$ Modus Tollens

$[\neg P \wedge (P \vee Q)] \Rightarrow Q$ Disjunctive syllogism.

$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$ Hypothetical syllogism.

$[(P \Rightarrow Q) \cap (R \Rightarrow S)] \Rightarrow [(P \cap R) \Rightarrow (Q \cap S)]$ Tautology.

$[(P \Leftrightarrow Q) \cap (Q \Leftrightarrow R)] \Rightarrow (P \Leftrightarrow R)$.

DMPT-26 -

Brown, joones and Smith are suspected off inco-me tax Evation. They testify under oath as follows,

→Brown says jones is guilty & S is inno.

→Jone says if Bro is guil. then So is Smit

→Smith says I am inno but atleast one of the others is guilty.

Assuming all told truth.

Sol)

B : Brown is innocent.

$\neg B$: " is guilty.

J : Jones is innocent.

$\neg J$: " is guilty.

S : Smith is innocent.

$\neg S$: Smith is guilty.

$B : \neg J \wedge S$ T

$J : \neg B \Rightarrow \neg S$ $\quad S \Rightarrow B$ T

$S : S \wedge (\neg B \vee \neg J)$ T

J - guilty

S - innocent.

B - innocent.

Predicate & Quantifier -

unary predicate $x > 3$

$P(x) : x > 3$

$P(2)$ F

$P(C) \rightarrow$ preposition

$P(x)$: ~~Computer~~ x is under attack.

CS 1, CS 2, MATH 1

CS 2, MATH 2

$P(CS 1)$: F

$P(CS 1), P(MATH 1)$ T

$P(x, y) : x + y > 0 \rightarrow$ Binary predicate.

$p(x_1, x_2, \dots, x_n) \rightarrow$ n-ary predicate.

$sum(x, y, z) : x + y = z$ n-place predicate. predicate constant.

→ If $P(x_1, x_2 \dots, x_n)$ is true for all values $c_1, c_2, \dots c_n$ from Universe U then you say $P(x_1, x_2 \dots, x_n)$ is valid in U.

$$\left[\begin{array}{l} Q(x, y) : x + y > 0 \\ u : x > 0 \\ \quad y > 0 \\ x \in R^+ \\ y \in R^+ \end{array}\right]$$

→ If $P(x_1, x_2, \dots, x_n)$ is true for som $c_1, c_2, \dots c_n$ in U then it is said that P is satisfiable in U. → If P is not true for any values $c_1, c_2 \dots c_n$ in U then P is said to be Unsatisfiable in U

$$\left[\begin{array}{l} Q(x, y) : x + y > 0 \\ u : x \in R \\ \quad y \in R \end{array}\right] \downarrow$$

Satisfiable

$$\left[\begin{array}{l} Q(x, y) : x + y > 0 \\ u : x < 0 \\ \quad y < 0 \end{array}\right] \downarrow$$

Unsatisfiable

$\forall$

→ Giving value to variable is known as Binding the variable.

Quantifier -

Universal Quantifier - for any x, for all x, for each x, for arbitrary x $P(x)$.

Quantifier

U : Set of Integers I

$\forall x\ [x < x + 1]$ T

$\forall x\ [x = 3]$ F

$x\ [x = x + 1]$ P — predicate.

Existential Quantifier - for some x $P(x)$

$\exists x \; P(x)$

U : set of integers I.

$\exists x \, [x < x+1]$ T

$\exists x \, [x = 3]$ $\cancel{F}$T.

$\exists x \, [x = x+1]$ F

U : $\{1, 2, 3\}$

$\forall x \; P(x) : P(1) \wedge P(2) \wedge P(3)$

$\exists x \; P(x) : P(1) \vee P(2) \vee P(3)$

Uniqueness Quantifier -

$\exists ! \, x \; P(x)$

U : set of integers I.

$\exists ! x \, [x < x+1]$ F

$\exists ! x \, [x = 3]$ T

$\exists ! x \, [x = x+1]$ F.

$$\exists ! x \; P(x) : [P(1) \wedge \neg P(2) \wedge \neg P(3)] \vee [\neg P(1) \wedge \neg P(2) \wedge P(3)] \vee [\neg P(1) \wedge P(2) \wedge \neg P(3)].$$

$\boxed{\forall \text{ and } \exists \text{ have higher precidence while compared to } \wedge \vee \Rightarrow}$

$\forall x \; p(x, y) \rightarrow$ Unary predicate

$\forall x \forall y \; p(x, y) \rightarrow$ preposition.

u: Set of married person

$\forall x \exists y$ [x is married to y] T

$\exists y \forall x$ [x is married to y] F

u: the integer

$\forall x \exists y \; [x+y=0]$ True

$\exists y \forall x \; [x+y=0]$ False

$\left.\begin{array}{l}\forall x \forall y \\ \exists x \exists y\end{array}\right\}$ Interchangable

$\forall x \exists y \not\equiv \forall y \exists x$

$\forall y \; P(x_1, x_2, x_3 \ldots x_n) \equiv P(x_1, x_2, \ldots x_n)$

DMPT-27-

$\forall x \forall y \forall z \; P(x, y, z)$

$\forall y \, Sum(x, y, z)$

$R(x, z)$

$x \leq z$

$\forall x P(x)$ is considered to be true.

is Empty

of Domain

u: ϕ
$\forall x P(x)$ T
$\exists x P(x)$ F

u: R	
$\forall x \; x^2 \geq x$	F

u: I	
$\forall x \; x^2 \geq x$	T

u: R

$\forall x < 0 \; (x^2 > 0)$

$\equiv \forall x (x < 0 \Rightarrow x^2 > 0)$

$\forall y \neq 0 \; (y^3 \neq 0)$

$\equiv \forall y (\neg y = 0 \Rightarrow \neg y^3 = 0)$

$\equiv \forall y (y \neq 0 \Rightarrow y^3 \neq 0)$

$\exists z > 0 \; (z^2 = 2)$

$\equiv \exists z (z > 0 \wedge z^2 = 2)$

$\equiv \exists z (z > 0 \wedge z^2 = 2)$

0

1

Logical Equivalences involving Quantifier -

Statements involving predicates and Quantifiers are logically Equivalent if they have same truth value no matter which predicates are substituted into statement and which domains of this discourse is used for variable in these preposition functions.

$$\forall x\,(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x).$$

$$\forall x\,(P(x) \vee Q(x)) \equiv \forall x\, P(x) \vee \forall x\, Q(x).$$

$$\exists x\,(P(x) \vee Q(x)) \equiv \exists x\, P(x) \vee \exists x\, Q(x).$$

Negating Quantifiar Expression -

1. Every student in class has taken a course in calculus then take $P(x)$ x as a course in caluculus. Universe is all Students in class $\forall x P(x)$.

The Negating Quantifiar is

$$\neg \forall x P(x) \equiv \exists x \neg P(x).$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x).$$

(Q) Expression the statement Every student in this class has studied calculus into logical notation

Sol) $C(x)$: x has studied calculus.

U : all students in class.

$\forall x\, C(x)$.

$S(x)$: student x in class

$C(x)$: x has studied calculus.

$\forall x\,(S(x) \Rightarrow C(x))$.

$Q(x, y)$: student x has studied the subject y

$\forall x\, Q(x, \text{calculus})$. u: all students in class.

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ier -
and Quantif.
they have
ich predi-
which
for variable.

$Q(x)$. T

$\forall x\, Q(x)$. F

$\exists x\, Q(x)$. T.

ken a course
x as a
is all Student

student in
lus into
culus.
in class.

culus.

died the subject
ll students in
class.

U : all students

$\forall x\, (S(x) \Rightarrow Q(x, \text{calculus}))$.

(2) Some students in this class has visited Mexico.

$(P(1) \wedge Q(1)) \wedge (P(2) \wedge Q(2)) \wedge \ldots$

$(P(1) \wedge P(2) \wedge P(3) \ldots) \wedge (Q(1) \wedge Q(2) \ldots)$

$\neg (P(1) \wedge P(2) \wedge \ldots)$

$= (\neg P(1)) \vee (\neg P(2)) \vee \ldots$

DMPT-29-

Translate

(1) Not all cars have carbonators

car(x) : x is a car.

carburator(x) : x has carburator.

Sol) $\neg \forall x (car(x) \Rightarrow Carburator(x))$

(or)

$\exists x \neg (car(x) \Rightarrow carburator(x))$.

$\exists x \neg (\neg car(x) \vee carburator(x))$.

$\exists x (car(x) \wedge \neg carburator(x))$.

(2) No dogs are intelligent.

Sol) dog(x) : x is a dog.

intelligent(x) : x is intelligent.

$\cancel{\forall} \forall x (dog(x) \Rightarrow \neg intelligent(x))$.

$\forall x \neg\neg (dog(x) \Rightarrow \neg intelligent(x))$

$\neg \exists x \neg (\neg dog(x) \vee \neg int(x))$

$\neg \exists x (dog(x) \wedge int(x))$

(3) Some numbers are not real.

Sol) num(x) : x is number

real(x) : x is real.

$\exists x (num(x) \wedge \neg real(x))$.

$\neg \forall x (num(x) \Rightarrow real(x))$.

~~An~~

An argument in propositinal logic is sequence of proposition, all but final proposition of arguments are called promises & final proposition is called conclusion.

An arg is "valid" if truth values of all its premises implies that conclusion is true.

$(P_1 \wedge P_2 \wedge P_3 \cdots P_m) \Rightarrow q$ — conclusion

$\downarrow$ T $\qquad$ T

Rules of inference related to language of proposition—

(i) Addition Rule — If P is true then PVQ is also true.

$P \Rightarrow (P \vee Q)$

(ii) Simplification Rule: $(P \wedge Q) \Rightarrow P$

(iii) Modus Ponens: $P \wedge (P \Rightarrow Q) \Rightarrow Q$.

(iv) Modus Tollens: $\neg Q \wedge (P \Rightarrow Q) \Rightarrow \neg P$.

(v) Disjunctive Syllogism:

$(P \vee Q) \wedge \neg P \Rightarrow Q$.

(vi) Hypothetical syllogism:

$$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow [P \Rightarrow R].$$

(vii) $\begin{array}{l} P \\ Q \\ \hline \therefore P \wedge Q \end{array}$ Conjunction.

(viii) $(P \Rightarrow Q) \wedge (R \Rightarrow S)$

$\neg Q \vee \neg S$

$\overline{\therefore \neg P \vee \neg R}$

$[(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge (\neg Q \vee \neg S)] \Rightarrow [\neg P \vee \neg R]$

Disjunctive Dilemma.

(ix) $(P \Rightarrow Q) \wedge (R \Rightarrow S)$

$P \vee R$

$\therefore Q \vee S$

$(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge (P \vee R) \Rightarrow (Q \vee S)$

Conjunctive Dilemma

(Q) If horses fly (or) cows eat grass, then the Mosquito is national bird. If Mosquito ~~for~~ is national bird then peanut butter tastes good on hot dogs. But peanut butter tastes terrible on hot dogs. Therefore, cows don't eat grass.

Sol) Horses fly — H

Cows eat grass — G

Mosquito is N.B — M

Peanut Butter tastes good on hot doges — P

① $(H \vee G) \Rightarrow M$

② $M \Rightarrow P$

③ $\neg P$

$\therefore \neg G$

Hypothetical Syllogism.

1 & 2 — 4 $(H \vee G) \Rightarrow P$

4 & 3 — 5 Modus Tollens

$\neg (H \vee G)$.

Demorgan's law — 6. $\neg H \wedge \neg G$.

Simplification — 7. $\neg G$

(Q) If ~~tw~~ today is tuesday, then I have a test in C.S.E (or) test in Economics. If my economics professor is sick then ~~H~~ I will not have Exam in Economics. Today is tuesday & my Economics professor is sick. $\therefore$ I have Exam on C.S.E.

Sol) Today is tuesday — T

Test in C.S.E — C

Test in Economics — E.

Economics Sick — S

$T \Rightarrow C$

$S \Rightarrow \neg E$

$T \wedge S$

$\therefore C$

from 3 Simplification 4. $\neg T$

1 & 4 Modus Ponens 5. S.

2 & 5 Modus Ponens 6. $CSVE$.

from 6 & 7 Disjunctive syllogism 7. $\neg E$.

8. CS.

Recurrence Relation,

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$

If $c_1, c_2, c_3, \ldots, c_k \in R$ such that $r^k - c_1 r^{k-1} - c_2 r^{k-2} \cdots c_k = 0$ has k distinct roots. Then, $\{a_n\}_{n \in N}$ is sol^n of relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots$, if

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \cdots + \alpha_k r^k$

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ where $a_0 = 2$, $a_1 = 5$, $a_2 = 15$.

$r^3 - 6r^2 + 11r - 6 = 0$.

$a_n = \alpha_1 1^n + \alpha_2 2^n + \alpha_3 3^n$.

$a_0 = \alpha_1 + \alpha_2 + \alpha_3$

$2 = \alpha_1 + \alpha_2 + \alpha_3$

$5 = \alpha_1 + 2\alpha_2 + 3\alpha_3$

$15 = \alpha_1 + 4\alpha_2 + 9\alpha_3$

$\alpha_1 = 1$; $\alpha_2 = -1$; $\alpha_3 = 2$

$$\boxed{a_n = 1^n - 2^n + 2 \cdot 3^n}$$

If roots aren't distinct then;

$a_n = (\alpha_{1,0} + \alpha_{1,1} n + \alpha_{1,2} n^2 + \cdots \alpha_{1,m_1-1} n^{m_1-1}) r_1^n$

$a_0 = 1$ $\cancel{\alpha_1 + 2}$

$a_1 = 2$

$a_2 = -1$ $+ (\alpha_{2,0} + \alpha_{2,1} n + \cdots \alpha_{2,m_2-1} n^{m_2-1}) r_2^n +$

$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$

$r^3 + 3r^2 + 3r + 1 = 0$

$r = (-1)$

$a_n = (\alpha_{1,0} + \alpha_{1,1} n + \alpha_{1,2} n^2)(-1)^n$

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DMPT-31 -

The fallacy of affirming the consequence.

1. $P \Rightarrow Q$

2. Q

$\therefore$ X

1. If it rains I am not going to class

2. I am not going to class

The fallacy of denying the anticident.

1. $P \Rightarrow Q$

2. $\neg P$

$\therefore$ X

1. If it rains I am not going to class.

2. It's not raining.

Rules of inference for quantifier -

1. $P(c)$ is true for an arbitrary element of u

$\therefore \forall P(x)$ Universal generalization.

2. $\forall x\, P(x)$

$\therefore P(c)$ c is arbibrary element of u

Universal instantiation.

3. $P(c)$ for some $c \in u$

$\therefore \exists x\, P(x)$ Existential generalization.

4. $\exists x\, P(x)$

$\therefore P(c)$ c is some element in u existential instan.

→ All men are mortal
Socrates is a man
∴ Socrates is mortal

man(x) : x is a man

mortal(x): x is mortal

1. $\forall x\,(man(x) \Rightarrow mortal(x))$

2. man(Socrates)

∴ mortal(socrate).

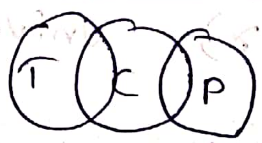
3. Universal instantat
man (socrates) ⇒
mortal(Socra

4. 2 & 3 Modus Pone.
mortal(Socrates).

→ Some Trig. fn^n are continuous
some continous functions are periodic
∴ Some Trig. functions are periodic.

T(x) $\quad \exists x\,(T(x) \wedge C(x))$
C(x) $\quad \exists x\,(C(x) \wedge P(x))$
P(x)

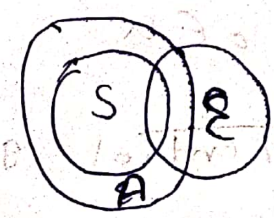
$\exists x\,(T(x) \wedge P(x))$



So It's false

→ Some Scientists are not Engineers.
Some astronauts are not Engineers

∴ Some scientists are not astronauts.



A Theorem is a statement that can be shown to be true less important theorems sometimes are called propositions. we demon-strate that a theorem is true with a proof. A proof is a valid Argument that establishes the truth of theorem. The statements used in a proof can include axioms (or) postulation which are state-ments we assumed to be true, the premises if any of theorem (or) pervi-ously proved theorems.

Less imp theorem that is helpful in proof of other results is called a lemma.

A corollary is a theorem that can be established directly from theorem that had been proved.

A conjucture is a statement that is being proposed to be a true statement, usually on basis of parial evidence (or) intution of expecti.

when proof of conjucture is found, the conjucture becomes theorem. Many times conjucture are shown to be false, so they are not theorem.

DMPT-32 -

$\forall x\,(P(x) \Rightarrow Q(x))$

n is an arbitrary Element in Universe

$P(n) \Rightarrow Q(n)$ (Universal generalization).

$$\frac{P(c)}{\therefore \forall x\, P(x)}$$

Direct proof –

$p \Rightarrow q$

Assume premise is true using axioms, previously proved theories & rules of inference show conclusion is true.

If n is an odd integer then n^2 is odd.

Sol) Direct proof –
premise: n is odd ↑
conclusion: n^2 is odd (Have to prove)

$n = 2k+1$ $\quad n^2 = (2k+1)^2 = 4k^2+4k+1$
$= 2k'+1$.

(ii) Indirect proof –! (contrapositive Method)

$p \Rightarrow q$

$\neg q \Rightarrow \neg p$

Eg- If n is an integer and $3n+2$ is odd then n is odd.

Sol) premise: $3n+2$ is odd (we have to prove F)
conclusion: n is odd. F

In this problem we can't use direct proof.

$n = 2k$

$3n+2 = 3(2k)+2 = 2(3k+1) = 2k'$

Its Even. So premise is false.

$\neg q \Rightarrow \neg p \equiv p \Rightarrow q$

proof by contradiction -

p: $\sqrt{2}$ is Irrational

$\neg p$: $\sqrt{2}$ is rational

$\sqrt{2} = \frac{a}{b}$, $b \neq 0$ & no common divisor b/w a & b.

$a^2 = 2b^2 \rightarrow a = 2k$, Even

$4k^2 = 2b^2$

$b^2 = 2k^2 \rightarrow$ Even.

$\therefore$ There is common divisor 2 b/w a & b

$\neg p \Rightarrow (r \wedge \neg r)$

$\neg p$ is false.

$\therefore$ p is true.

Equivalence method: $\neg p \Rightarrow (r \wedge \neg r)$.

$$(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p).$$

p if and only if q.

Counter Example -

Every +ve integer is sum of squares of two integers.

Sol) False.

By its form -

x is Even (or) odd.

$E(8) \vee O(8)$

$\forall x (E(x) \vee O(x))$

$\forall x (E(x) \vee \neg E(x))$.

} Valid statements.

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(Q) give a direct proof that if m and n a
both perfect squares then m, n is als
a perfect square. (integer a is perfect sq
if $\forall x$ $a = x^2$).

Sol.) premise: m is perfect square
n is perfect square T.

Conclusion: mn is perfect square

$m = k^2$ $mn = (kl)^2 = k'^2$
$n = l^2$
It is true.

(Q) give a direct proof that if m and n are both perfect squares then m, n is also a perfect square. (integer a is perfect square if $\forall x$ $a = x^2$).

Sol.) premise: m is perfect square
n is perfect square

conclusion: mn is perfect square

$m = k^2$
$n = l^2$

$mn = (kl)^2 = k'^2$

It is true.

DMPT-33 -

(Q) Show that the preposition P(0) is true, where P(n) is if n > 1 then $n^2 > n$

Sol.) By vacious proof its true

proofs

1. Direct proof.
2. Indirect proof.
3. Vacious proof.
4. Trivial proof — Refer in T.B.
5. Proof by Cases

$(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \Rightarrow Q \equiv (P_1 \Rightarrow Q) \vee (P_2 \Rightarrow Q) \ldots \vee (P_n \Rightarrow Q)$.

$\neg Q \Rightarrow \neg(P_1 \wedge P_2 \wedge \ldots \wedge P_n)$.

$\neg(\neg Q) \vee \neg(P_1 \wedge P_2 \wedge \ldots \wedge P_n)$.

$Q \vee \neg(P_1 \wedge P_2 \wedge \ldots \wedge P_n)$.

$(Q \vee Q \vee Q \vee Q \ldots) \vee (\neg P_1 \vee \neg P_2 \vee \neg P_2 \ldots \vee \neg P_n)$.

$(Q \vee \neg P_1) \vee (Q \vee \neg P_2) \vee \ldots \vee (Q \vee \neg P_n)$.

$(\neg Q \Rightarrow \neg P_1) \vee (\neg Q \vee \neg P_2) \vee \ldots \vee (\neg Q \vee \neg P_n)$

$\equiv (P_1 \Rightarrow Q) \vee (P_2 \Rightarrow Q) \vee (P_3 \Rightarrow Q) \ldots \vee (P_n \Rightarrow Q)$"

$(P_1 \vee P_2 \vee \dots P_n) \Rightarrow Q.$

$\neg(P_1 \vee P_2 \vee \dots P_n) \vee Q.$

$(\neg P_1 \wedge \neg P_2 \wedge \neg P_3 \dots \neg P_n) \vee Q.$

$(\neg P_1 \vee Q) \wedge (\neg P_2 \vee Q) \wedge \dots \wedge (\neg P_n \vee Q).$

$(P_1 \Rightarrow Q) \wedge (P_2 \Rightarrow Q) \dots \wedge (P_n \Rightarrow Q).$

Q) let $\sqcup$ denotes ~~mean~~ (min) operates, $a \sqcup b$ gives minimum of a & b.

$a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c \quad \forall a, b, c \in I.$

Sol:) ① $a \le b \le c$
② $b \le c \le a$
③ $c \le a \le b$
④ $c \le b \le a$
? ⑤ $b \le a \le c$
⑥ $a \le c \le b$

(1.) $a \sqcup (b \sqcup c) = a \sqcup b = a$
$(a \sqcup b) \sqcup c = a \sqcup c = a.$

⑥ $a \sqcup (b \sqcup c) = a \sqcup c = c.$
$(a \sqcup b) \sqcup c = b \sqcup c = c.$

$\neg\exists x P(x)$ $\quad \exists x P(x)$ — constructive. $c \in u$ $P(c)$ / non-constructive.

$\forall x P(x)$ (Universal generalisation) take arbitrary $c \in u$ $P(c)$ is true

$\neg\forall x P(x)$

Q) Show that $\exists$ irrational numbers x & y such that x^y is irrational

Sol:)

Normal Forms -

Literal - A variable (or) negation of a variable, is called a literal.

product of variables and their negations in a formula is called an elementary product.

Eg- $P \wedge Q \wedge \neg R$.

similarly sum of variables and their negation is called elementary sum.

Eg- $P \vee Q \vee \neg R$.

A necessary and sufficient condition for an elementary product to be identically false is that it contains atleast one pair of factors in which one is negation of other

A necessary and sufficient condition for an Elementary Sum ---

Disjunctive Normal Form (D.N.F) - (S.O.P).

A formula ~~is~~ which is equivalent to give formula and which consists of sum of a elementary product is called D.N.F.

Any wff can be converted to D.N.F.

(wff - well formed formula).

(Q) Convert $(P \Rightarrow Q) \wedge \neg Q$ into D.N.F.

Sol) $(\neg P \vee Q) \wedge \neg Q$.

$(\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)$.

Conjuctive Normal form (C.N.F) - (P.O.S)

~~(Q) write~~ A formula which is Equivalent to given formula and which constists of product of Elementary Sum is called C.N.F.

(Q) Convert $(P \Rightarrow Q) \wedge \neg Q$ into C.N.F

Sol) ~~$(\neg P \vee Q) \wedge \neg Q$.~~

~~$(\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)$~~

~~$(P \wedge \neg Q) \vee (Q \wedge \neg$~~

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(Q) Convert $(P \Rightarrow Q) \wedge (\neg Q)$ into C.N.F.

Sol) $(\neg P \vee Q) \wedge (\neg Q) \equiv (\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)$.

→ Let P & Q be two prepositional variables.

$(P \wedge Q$ $\quad P \wedge \neg Q$ $\quad \neg P \wedge Q$ $\quad \neg P \wedge \neg Q)$ Min terms.

$(P \vee Q$ $\quad P \vee \neg Q$ $\quad \neg P \vee Q$ $\quad \neg P \vee \neg Q)$ Max. terms.

→ for a given formula an equivalent formula consists of disjunction of min terms only is known principle disjunctive ~~formu~~ normal form (P.D.N.F). Such a normal form is also called the sum of products canonical form.

→ (principle conjuctive normal form)
PCNF
(product of sum canonical form)

$\cancel{(Q)\ (P \Rightarrow Q) \wedge \neg q}$

$\cancel{\text{Sol)} \equiv (\neg P \vee q) \wedge \neg q}$

$\cancel{(\neg P \wedge \neg q) \wedge (q}$ (Page 820).

$\cancel{(Q)\ (P \Rightarrow Q) \wedge \neg q}$.

(Q) $(P \Rightarrow q) \wedge \neg q$

Sol) $(\neg P \vee q) \wedge \neg q$.

$\cancel{(\neg P \wedge \neg q) \vee (q \wedge \neg q)}$

$(\neg P \vee q) \wedge ((P \wedge \neg P) \vee \neg q)$. (PCNF).

$\equiv (\neg P \vee q) \wedge (P \vee \neg q) \wedge (\neg P \vee \neg q)$.

DMPT-34

ϕ: null set { }

$R = \{x / x$ is Real numb

set builder form

$S = \{\phi\}$

Two sets are Equal if and only if they have the same Elements. If a & b are sets then a & b are Equal $\Leftrightarrow \forall x \{x \in A \Leftrightarrow x \in B\}$.

The set A is subset of B if and only if Every Element in A is also Element in B.

$A \subseteq B$

$\forall x (x \in A \Rightarrow x \in B)$.

S

$\{\phi, S\}$

$\phi \subseteq A$

$\forall x (x \in \phi \Rightarrow x \in A) \rightarrow$ vacious proof premise is false

$A = B$

of $(A \subseteq B) \wedge (B \subseteq A)$

power set of S is set of all subsets of S

$S = \{1, 2\}$.

$P(S) = \{\{1\}, \{2\}, \{1, 2\}, \phi\}$. 2^n Elements

n-tuple - $\{a_1, a_2, ---- a_n\}$ we can't change order.

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let A & B two sets the cartision of A&B denoted by $A\times B$ is denoted by set of all

$$A\times B = \{(a,b) / a\in A \wedge b\in B\}$$

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) / a_i \in A_i \text{ forall } i=1,2 \ldots n\}$$

$$A^n = \{(a_1, a_2 \ldots a_n) / a_i \in A \ \forall\ i=1,2 \ldots n\}.$$

$$A\cup B = \{x / x\in A \vee x\in B\}$$

$$A\cap B = \{x / x\in A \wedge x\in B\}$$

$$A-B = \{x / x\in A \wedge x\notin B).$$

$$\bar{A} = \{x\in U / x\notin A\}.$$

U is universal set

$A\cup U = U$

$A\cap U = A$.

$$\overline{A\cap B} = \bar{A}\cup\bar{B}.$$

proof:-

$$\overline{A\cap B} = \{x | x\notin A\cap B\}$$

$$= \{x | \neg(x\in A\cap B)\}$$

$$= \{x / \neg(x\in A) \cup \neg(x\in B))$$

$$= \{x / x\notin A \cup x\notin B).$$

$$= \{x / x\in \bar{A}\cup\bar{B}\} = \bar{A}\cup\bar{B}.$$

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$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

$x \in A \cap (B \cup C)$.

$(x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$x \in (A \cap B) \cup x \in (A \cap C)$

$x \in (A \cap B) \cup (A \cap C)$.

let A & B be non-empty sets. A function $f: A \to B$ is an assignment of Exactly one Element of B to Each Element of A. whe write $f(a) = b$ If b is Unique Element of B assigned by ~~Eleme~~ function f to Element a of A.

$f : A \to B \longrightarrow$ Codomain.

Domain Image of a is b.

$f(a) = b$, ~~$b \in f$~~

There may be $x \in A$, $y \in B$.

ff Such that there is no pre image of y in Set A.

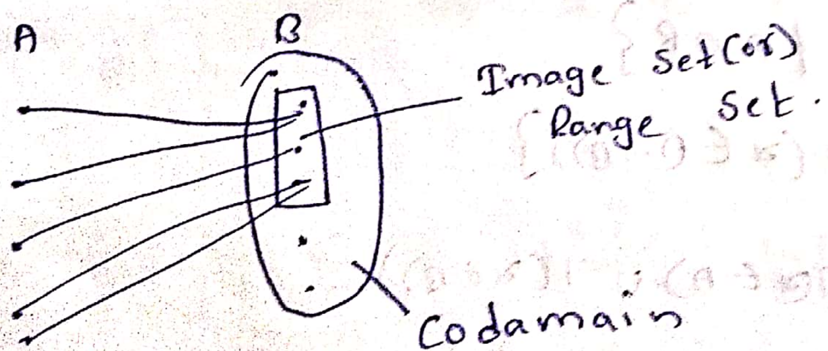


Image set $\subseteq$ Codamain

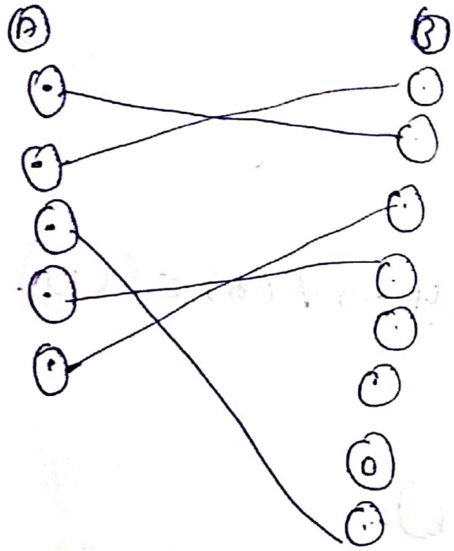
$f : Z^+ \to Z^+$

$f(x) = x^2$

$Z^+ \to$ codomain

range (or) Image $\to \{1, 4, 9, 16, \ldots\}$

<u>one to one function</u> -



$f(x_1) = y_1$

$f(x_2) = y_2$

If $y_1 = y_2$ then $x_1 = x_2$.

If $y_1 \neq y_2$ then $f(x_1) \neq f(x_2)$.

one to one & onto is known as one to one correspondence.

<u>D.M.P.T - 35</u> -

$A \times B \neq B \times A$.

$A = \{a, b, c\}$ $B = \{1, 2\}$

$\{(a,1), (a,2), \ldots, (c,1), (c,2)\}$

$\{(1,a), (1,b), (1,c), (2,a) \ldots\}$.

<u>proper subset</u> -

$A \subset B$ if $A \subseteq B$ & $A \neq B$

$A_i = \{i, i+1, i+2, \ldots\}$ $\bigcup_{i=1}^{n} A_i = A_1$

$A_1 = \{1, 2, 3, \ldots\}$ $\bigcap_{i=1}^{n} A_i = A_n$.

$A_2 = \{2, 3, \ldots\}$

$B_i = \{1, 2, 3, \ldots, i\}$.

$\bigcup_{i=1}^{\infty} B_i = N$.

$\bigcap_{i=1}^{\infty} B_i = \{1\}$.

Real valued function -

codomain (Real).

Strictly increasing function

$f : S_R \to S_R$ $\quad \forall x \forall y (x < y \Rightarrow f(x) < f(y))$

one to one $\to$ injection.

$\forall a \forall b (f(a) = f(b) \Rightarrow a = b)$.

$\forall a \forall b ((a \neq b) \Rightarrow f(a) \neq f(b))$.

onto - surjuction.

$\forall y \exists x (f(x) = y)$.

one to one correspondence (bijective)

Both one to one and Onto

$f : R \to R$

$f(x) = x + 1$ $\quad y = f(x) = x + 1$

$f(a) = f(b)$ $\quad x = y - 1$.

$a + 1 = b + 1$ $\quad$ Onto.

$a = b$

injuctive

Show function is not one to one

find $a_1 \neq a_2$

$f(a_1) = f(a_2)$

show function is not onto.

find $y \in$ codomain

s.t $f(x) \neq y$ for all x domain.

$f: A \to A$ finite

f is one to one $\Leftrightarrow$ f is onto.

one to one correspondence

$I_A : A \to A$. (Bijective).

$I_A(x) = x$. $\forall x \in A$

bijective.

Inverse: $f: A \to B$ one to one correspondence.

$f(a) = b$. $f^{-1} : B \to A$.

$f^{-1}(b) = a$.

$f: \mathbb{R} \to \mathbb{R}$ is invertable?

is $f(x) = x^2$ is

$f(-1) = f(1) = 1$

Sol) No as not one to one correspondence.

$f: \mathbb{R}^+ \to \mathbb{R}^+$ is invertable.

$f(x) = x^2$

$f^{-1} \neq \frac{1}{f}$.

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Sequence

$f : S_Z \to S$

$\{a_n\}$ $a_0, a_1, a_2, \ldots, a_n, \ldots$

$a_n = \frac{1}{n}$ $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$

$\sum_{i=1}^{4} \sum_{j=1}^{3} ij = \cancel{1+2+3+2+4+6+6+12+18+4+3+6+9}$

$= \cancel{6+12+36+18 = 60}$

$= 10 + 2\times 10 + 3 \times 10 = \boxed{60}$

Size of set -

S is finite with n elements

$|S| = N$

↑

cardinality of S

$|A| = |B|$ if there is one to one correspondence from A to B. (bijection)

if $f : A \to B$ is one to one

$|A| \leq |B|$

if $f : B \to A$ is one to one

$|B| \leq |A|$.

→ If a set is finite it is countable.

→ If a set is infinite it can be countable (or) non-countable.

→ A set finite (or) having same cardinality (set of all +ve Z) is called countable.

Set

finite — infinite

finite: countable

infinite: countable, uncountable.

Eg - All integers are countable.

$f : Z^+ \to Z^*$ bijection.

$\{1, 2, \ldots \infty\}$

$\{-\infty, \ldots, -2, -1, 0, 1, 2, \ldots\}$

$Z = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$

$$\left.\begin{aligned} f(x) &= \frac{n}{2}, \quad n \text{ is even.} \\ &= -\frac{n-1}{2}, \quad n \text{ is odd.} \end{aligned}\right\} \text{bijective.}$$

DMPT-36 -

Mathematical induction -

$P(n)$, n is positive integer.

$\forall n\, P(n)$ $n \in N$

$n! < n^n$, $\forall n \in N$

$n^3 - n$ is divisible by 3, $n \geq 2$. A set with n elements have 2^n subsets.

$n=1$, $P(1)$ is true.

Take arbitrary $= k$, $P(k) \Rightarrow P(k+1)$

$$\boxed{[P(1) \wedge \forall k(P(k) \Rightarrow P(k+1))] \Rightarrow \forall n\, P(n)}$$

domain is all +ve integers.

Basis step : $P(1)$ is true. (true)

Inductive step : from $P(k)$ is step and conclude $P(k+1)$ is true.

(Q) why Mathematical Induction works?

Sol) well ordering property of positive integer.

$P(1)$ is true $P(k) \Rightarrow P(k+1)$.

$S = \{$ all +ve integers for which $P(n)$ is false $\}$

m is least Element

$m \neq 1$

$P(m-1)$ is true $\Rightarrow P(m)$ is true

So it's contradiction.

$P(n) =$ $n = b, b+1, b+2, \ldots$

$b \neq 1$

Basis step: $P(b)$ is true.

Inductive Step: $P(k)$ is $P(k+1)$ is true for all $k = b, b+1, \ldots$

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$1+2+3 \ldots + n = \frac{n(n+1)}{2}$, for all +ve integers.

$p(n)$: $1+2+3+ \ldots + n = \frac{n(n+1)}{2}$.

domain is +ve integers.

Basis step: $P(1)$ is true

L.H.S = 1

R.H.S = 1

Inductive step: If $P(K)$ is true

$1+2+ \ldots K = \frac{K(K+1)}{2}$

$$P(K+1) = \frac{K(K+1)}{2} + K+1 = \frac{K^2+K+2K+2}{2}$$

$$= \frac{K(K+2)+(K+2)}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$

$\therefore$ L.H.S = R.H.S.

1	n	$n+1$
2	$n-1$	$n+1$
3	$n-2$	$n+1$
⋮	⋮	⋮
		$n(n+1)$

$R + R$

$$\boxed{R = \frac{n(n+1)}{2}}$$

Strong Induction:-

$p(n)$ is true $\forall n \in Z^+$.

Basis step : $P(1)$ is true.

Induction step: If $P(J)$ is true $\forall$ +ve integers not exceeding k then $P(K+1)$ is true.

$$[P(1) \wedge P(2) \wedge P(3) \ldots \wedge P(K)] \Rightarrow P(K+1)$$

$\forall k$ +ve integers.

This is also called as Second prinple of Mathematical Induction (or) Complete Induction.

(Q) Show that if n is integer greater than 1 then n can be written as product of primes.

P(n) is product of primes (n>1)

Sol.) Basis – P(2)

$2 = 2 \times 1$ is true

Induction step – If P(2), P(3), ... P(k) is true

Assume: P(j) is true ∀ +ve integers upto k

P(k+1) k+1 =

prime → Its proved

composite = a × b.

$2 \leq a \leq b < k+1$.

Recursive Def^n:

$f: Z^+ \to S$

Basic: $f(0) = 0$, $f(1) = 1$.

Recursive: $f(n) = f(n-1) + f(n-2)$.

Basis: $f(0) = 3$.

Recursive: $f(n+1) = 2f(n) + 3$.

$a_n = n!$

Basic: $a_0 = 1$

Recursive: $a_n = n a_{n-1}$

$S_n = a^n$

Basis: $S_0 = 1$.

Recursive: $S_n = a \cdot S_{n-1}$

Re

Bc

Rec

→ fu

Su

on

→ Rel

to

A

R =

Recursive Defn of set

Basis step: $3 \in S$.

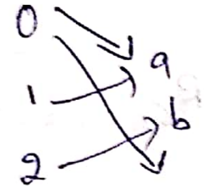
Recursive step: If $x \in S$, $y \in S$
then $x+y \in S$.

DMPT-37-

$A = \{0, 1, 2\}$; $B = \{a, b\}$

$R = \{(0,a), (0,b), (1,a), (2,b)\}$

$0 R a$. $1 \not R b$.



$f : A \to B$ $f(a) = b$.

$(a, f(a))$.

→function is a Relation R from a to b such that every element in a is the 1st element of exactly one ordered pair of R

→Relation of set A is a relation from A to A

$R \subseteq A \times A$.

$A = \{1, 2, 3, 4\}$ $R = \{(a,b) \mid a \text{ divides } b\}$

$R = \{(1,2), (1,3), (1,4), (1,1), (2,2), (2,4), (3,3), (4,4)\}$.

Relations on Set of integer, Z

$R_1 = \{(a,b) / a \leq b\}$

$R_2 = \{(a,b) / a > b\}$

$R_3 = \{(a,b) / a = b \text{ (or) } a = -b\}$.

$R_4 = \{(a,b) / a = b\}$

$R_5 = \{(a,b) / a = b+1\}$

$R_6 = \{(a,b) / a+b \leq 3\}$.

$(1,1) \rightarrow R_1, R_3, R_4, R_6$.

$(1,2) \rightarrow R_1, R_6$

$(2,1) \rightarrow R_2, R_5, R_6$

$(1,-1) \rightarrow R_2, R_3, R_6$

$(2,2) \rightarrow R_1, R_3, R_4$

R_1, R_2, R_4, R_5
$\downarrow$
anti-symmetric

R_1, R_3, R_4
$\downarrow$
Reflexive.

R_3, R_4, R_6
$\downarrow$
Symmetric.

→ A Relation R on a set A is called Reflexive for $(a,a) \in R$ $\forall a \in A$.

R is Reflexive if $\forall a \, ((a,a) \in R)$.

→ R is symmetric if $\forall a, \forall b \in A$ $(b,a) \in R$ whenever $(a,b) \in R$

→ A Relation R on Set A is called Symmetric if $(b,a) \in R$ when Ever $(a,b) \in R$.

$\forall a \forall b \, ((a,b) \in R \Rightarrow (b,a) \in R)$

→ A Relation R on Set A is Anti-Symmetric if $\forall a, \forall b \in A$ if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$

$\forall a \forall b \, (((a,b) \in R \wedge (b,a) \in R) \Rightarrow (a = b))$.

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Transitive Relation -

A Relation R on set A is transitive when

$$\forall a \forall b \forall c (((a,b) \in R \land (b,c) \in R) \Rightarrow (a,c) \in R)$$

$R = \{(a,b) / a \text{ divides } b\}$

$a R b \Rightarrow a = bk = cmk$

$b R c \Rightarrow b = cm$

Equivalence Relation - A Relation on set A ~~It is~~ is Equivalence Relation if it is Reflexive, Symmetric, Transitive.

$a \equiv b \pmod 4$

St $a-b$ is divisible by 4.

Equivalence Relation.

$a \sim b =$ Equivalence Relation

$R_1 = \{(x,y) / x < y\}$, $R_2 = \{(x,y) / x > y\}$.

$R_1 \cup R_2$ $\quad$ $R_1 - R_2$

$R_1 \cap R_2$ $\quad$ $R_2 - R_1$