4-10-19

1) Calculate Var(X) if X represents the outcome when a fair die is rolled.

2) The following gambling game known as wheel of fortune is as follows: A player bets on one of no.s b/w 1 to 6. Three dices are rolled and if number bet by player appears i times, i = 1, 2, 3, then find the player wins

1.) There is a dinner party where n men check their hats. The hats are mixed up during dinner so that afterward each man receives a random hat. In particular each man gets his own hat with prob. $\frac{1}{n}$. What is Expected no. of men who get their own hat?

Sol) let G be total no. of men who get their own hat.

$$G_i = \begin{cases} 1 & \text{if person gets his own hat} \\ 0 & \text{otherwise.} \end{cases}$$

$$E(G) = E(G_1) + E(G_2) + \dots + E(G_n)$$

$$= \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} = n\left(\frac{1}{n}\right) = \boxed{1}.$$

$$E(G_i) = P(G_i).$$

2.) Game-A: we win \$2 with 2/3 and lose \$1 with prob. 1/3.

Game-B: we win \$1002 with prob. 2/3 and lose \$2001 prob. 1/3.

which game is better financially?

Sol) $E(x) = 2 \times \frac{2}{3} - 1 \times \frac{1}{3} = 1$

$E(Y) = 1002 \times \frac{2}{3} - 2001 \times \frac{1}{3}$

$= 1.$

$E(x^2) = 4 \times \frac{2}{3} + 1 \times \frac{1}{3} = \frac{\cancel{9}\,3}{\cancel{3}} = 3.$

$E(Y^2) = \frac{2}{3} \times (1002)^2 + \frac{1}{3} \times (2001)^2 =$

$Var(x) = 3 - 1 = \boxed{2}.$

$Var(Y) = \boxed{2.004002}.$

If var is High then it is more Risky.

3.) In a certain game, Each card has a point value
* 10 & (k q, j) worth 10 points each
* Numbered cards in the range 2-9 are worth 5 points each. * Aces are worth 15 points each
Suppose that you throughly shuffle a 52 cards deck. What is prob. Expected total value of three cards on top of deck after shuffle?

Sol) $E(x) = \left(5 \times \frac{8}{13}\right) + \left(10 \times \frac{4}{13}\right) + \left(15 \times \frac{1}{13}\right) = \boxed{\frac{95}{13}}$

let $x = x_1 + x_2 + x_3$

$E(x) = E(x_1) + E(x_2) + E(x_3)$

$= \frac{95}{13} + \frac{95}{13} + \frac{95}{13} = 3\left(\frac{95}{13}\right) = \boxed{\frac{285}{13}}$

4) let T be the time for first success in Bernoulli trials process. Then we take the sample space Ω the integers 1, 2, ... and assign the geometric dist.

$m(j) = P(T=j) = q^{j-1} \cdot p \quad E(T) = ?$

Sol) $E(x) = \sum_{j=1}^{\infty} j\, q^{j-1} \cdot p = \frac{p}{q} \sum_{j=1}^{\infty} j q^{j}$

$= p\left[\frac{1 \times \cancel{q}}{\cancel{q}} + 2q + 3q^2 + 4q^3 + 5q^4 + + \cdots\right]$

$A = q + 2q^2 + 3q^3 + 4q^4 + \cdots$

$Aq = \quad q^2 + 2q^3 + 3q^4 + \cdots$

$A(1-q) = \frac{q}{1-q}$

$\frac{a}{1-r} + \frac{rd}{(1-r)^2}$

$= \frac{q}{1-q} + \frac{q}{(1-q)^2}$

$\frac{q-q+q}{(1-q)^2} = \frac{q}{p^2} \times \frac{p}{q}$

$A = \frac{q}{(1-q)^2}$

$E(x) = \frac{p}{q} \times \frac{q}{(1-q)^2} = \boxed{\frac{p}{(1-q)^2}} = \boxed{\frac{1}{p}}$

5.) When looking at person's eyes colour (eye colour) it turns out that 1% of people in world has green eys. Consider a grp of 20 people.

(a) None have green Eyes – $^{20}C_0 \cdot \left(\frac{1}{100}\right)^0 \cdot \left(\frac{99}{100}\right)^{20}$

(b) Nine have green Eyes – $^{20}C_9 \left(\frac{1}{100}\right)^9 \left(\frac{99}{100}\right)^{11}$

(c) Atmost 3 have green Eyes – $^{20}C_0 \cdots + {}^{20}C_1 \cdots + {}^{20}C_2 \cdots + {}^{20}C_3$

(d) At least 4 have green Eyes – $1 - (\quad)$

6.) According to center for disease control (CDC) about 1 in 88 children in U.S have been diagnosed with autism. Consider a group of 10 children.

(a) None have autism.

(b) seven have autism.

(c) Atleast 5 have autism.

(d) At most 2 have autism.

7.) Two players A and B play a coin tossing game A gives B one doller if a head turns up, otherwise B pays A one doller. If prob. that coin shows a head is p, find Expected gain of A.

8.) A fair die is rolled n times. find the no. of trials needed for prob. of atleast one 6 to be $\geq \frac{1}{2}$ (n=?).

11-10-19

1.) A die is rolled unitil a 1 occurs. what is prob. that we have to roll the die (a) once (b) twice (c) thrice what is resulting dist.?

Sol) $p(X=1) = \left(\frac{5}{6}\right)^0 \left(\frac{1}{6}\right)^1$

$p(X=2) = \left(\frac{5}{6}\right)^1 \left(\frac{1}{6}\right)^1$

$p(X=3) = \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^1$

$p(X=k) = \left(\frac{5}{6}\right)^{k-1} \left(\frac{1}{6}\right)^1$

$= \boxed{q^{k-1} \cdot p}$

2.) 2 coins are tossed until HH occurs 3 times what is prob. that (a) Exactly 5 tosses of 2 coins are necessary (b) $p(X<6)$ (c) $E(X) = ?$, $var(x) = ?$

$p(X=x) = {}^{x-1}C_{r-1} \, p^r \cdot q^{x-r}$

Sol) $\cancel{P(X=5) = \left(\frac{3}{4}\right)^4}$

$\cancel{p(X=5) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2}$

(a) $p(X=5) = {}^4C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

(b) $p(X<6) = P(x=3) + p(x=4) + p(x=5)$

$= {}^2C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 + {}^3C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^1 + {}^4C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

(c) $E(x) = \frac{r}{p} = \frac{3}{\frac{1}{4}} = 12.$

$Var(x) = \frac{rq}{p^2} = \frac{(3)(3/4)}{(1/4)^2} = \frac{\frac{9}{4}}{\frac{1}{4 \times 4}} = \boxed{36}$

3) During a bad economy, a graduating ECE student goes to carrer fair booths in tech. Sector. His/her likelyhood of receiving an off-campus interview invitation after carreer fair booth visit depends on how well he/she did in ECE 313. An A in 313 results in prob. $P = 0.95$ of obtaining an invitation where as a C in 313 results in prob. $P = 0.15$

(a) Give the p.m.f in terms of

(b) $E(Y) = ?$

(c) i) Assume that each student visits 5 booths during a typical carrer fair find prob. that in A in 313 will not get an off-campus interview invitation.

ii) Similarly find prob that a C in 313 will get an invitation.

Sol:) (a) $P(Y=k) = (1-P)^{k-1} (P)^1$, for $k > 1$

(b) $E(Y) = \frac{1}{P} = \begin{cases} \frac{1}{0.95} & \text{for } A \\ \frac{1}{0.15} & \text{for } C. \end{cases}$

(c) (i) $(1 - 0.95)^5$

(ii) $1 - (1-0.95)^5$.

4.) Suppose that 105 passengers hold reservation for a 100 passenger-flight. The no. of passengers who show up at the gate can be modeled as binomial r.v with parameters (105, 0.9)

(a) on an average, how many passengers show up at gate.

Sol:) $E(x) = np = 105 \times 0.9 = \boxed{94.5}$.

(b) $P(X \leq 100)$?

Sol:) $1 - P(X > 100) = 1 - [P(X=101) + P(X=102) + P(X=103) + P(X=104) + P(X=105)]$

$= 1 - [{}^{105}C_{101} (0.9)^{101} (0.1)^4 + \dots]$.

(c) Explain why the no. of no-shows can be modeled as a binomial r.v with parameter (105, 0.1)

Sol) if x is a binomial variable with parameters (n, p), then $Y = n - x$ is b r.v with parameter $(n, 1-p)$ for this case no-show is a success.

5) An airline knows that 5% of people making reservations do not turn up for flight. So it sells 52 tickets for a 50 seat flight. what is prob. that every passenger who turns up will get a seat?

Sol) $X \sim Bin(52, 0.95)$. X – no. of passengers turning up.

$$P(X \leq 50) = 1 - [P(X=51) + P(X=52)]$$

$$= 1 - [{}^{52}C_{51} (0.95)^{51} (0.05) + {}^{52}C_{52} (0.95)^{52}]$$

$$\simeq \boxed{0.74}.$$

6) Suppose independent tests are conducted on monkey to develop a vaccine. If prob. of success is $\frac{1}{3}$ in each trial what is prob. that at least 5 trials are needed to get first success?

Sol) $X \to$ no. of tests.

$$P(X \geq 5) = 1 - [P(X=1) + P(X=2) + P(X=3) + P(X=4)]$$

$$= P(X=5) + P(X=6) + \cdots P(X=\infty)$$

$$= \sum_{n=5}^{\infty} \left(\frac{2}{3}\right)^{n-1} \left(\frac{1}{3}\right).$$

$$= \frac{1}{3} \cdot \left[\left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 + \cdots\right]$$

$$= \frac{1}{3} \cdot \frac{\left(\frac{2}{3}\right)^4}{1 - 2/3} = \boxed{(2/3)^4}.$$

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7.) An Urn contains N white and M black balls. Balls are randomly selected one at a time, until black one is obtained. If we assume that each ball is selected is replaced before next one is drawn. What is prob. that

(a) Exactly n draws are needed.

Sol.) $\left(\frac{N}{N+M}\right)^{n-1} \cdot \left(\frac{M}{N+M}\right)$

(b) atleast k draws are needed.

Sol.) $p(X \geq k) = \left(\frac{N}{N+M}\right)^{k-1} \left(\frac{M}{N+M}\right) + \left(\frac{N}{N+M}\right)^{k} \left(\frac{M}{N+M}\right) + \cdots$

$$= \left(\frac{M}{N+M}\right) \cdot \frac{\left(\frac{N}{N+M}\right)^{k-1}}{\left(\frac{M}{N+M}\right)} = \boxed{\left(\frac{N}{N+M}\right)^{k-1}}$$

8.) Find the expected value and variance of no. of times one must throw a die until outcome 1 has occured 4 times.

18-10-19

1) The lifetime T (years) of an electronic Component is a continuous R.V with prob. d.f given by $f(t) = e^{-t}$ $t \geq 0$

(a) Find lifetime 'L' which a typical component is 60% certain to Exceed

(b) If 5 components are sold to a manufacturer find prob. that atleast one of them will have a lifetime less than L years.

Sol.) (a) $P(T > L) = \int_L^\infty e^{-t}\,dt = \left[e^{-t}\right]_\infty^L = e^{-L} = 0.6.$

$\boxed{L = 0.51 \text{ years}}$

(b) $\cancel{P(T < L) = \int_0^L e^{-t}\,dt} = -e^{-L} + 1 = 1 - 0.6$

$\cancel{= 0.4}$

(b) $P(\ ) = 1 - P(\text{none have less than } L \text{ years})$

$= 1 - (0.6)^5 = \boxed{0.92}$

2.) Suppose that crowd size at home games for a particular football club follows a normal distribution with mean 26000 and standard deviation 5000. What percentage of crowds are b/w 31000 & 36000?

Sol.) $\mu = 26000$ $\sigma^2 = 5000.$

$P(31000 < X < 36000) = ?$ $\quad z_1 = \frac{x - \mu}{\sigma}$

$= \frac{1}{5000\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{31000 - 26000}{5000}\right]^2} + \frac{1}{5000\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{36000 - 26000}{5000}\right]^2}$

$= \frac{1}{5000\sqrt{2\pi}} \left[-e^{-\frac{1}{2}} + e^{-\frac{1}{2}\left(\frac{18}{25}\right)4}\right].$

$= \frac{1}{5000\sqrt{2\pi}} \left[e^{-2} - e^{-\frac{1}{2}}\right].$

$F_X(2) - F_X(1)$

$= 0.9772 - 0.8413$

$= \boxed{0.1359}$

3.) let $X \sim N(3,4)$ then find

$P(2 < X \leq 5) = ?$

Sol.) $\mu = 3$. $\sigma = 2$.

$\frac{5-3}{2} = 1$ $\cancel{2}$ $\frac{2-3}{2} = -0.5$

$F_X(1) - F_X(0.5) = 0.5328$

$= 0.8413 - 0.3085$

$\begin{array}{r} 5328 \\ 3085 \\ \hline 8413 \end{array}$

4.) Assume that the time required for dist. runner to run a mile is a normal random Variable with parameters $\mu = 4$ ~~miles~~ 1sec. and $\sigma = 2$ sec. what is prob. that this athelete will run a mile.

(i) in less than 4 min.?

(ii) in more 3 min, 55 sec.?

Sol.) (i) $\mu = 4$ $\sigma = 2$.

$P(0 < t < 240)$

$= \frac{-241 + 240}{2} = -\frac{1}{2}$

$F\left(-\frac{1}{2}\right) + \cancel{F\left(\frac{-241}{2}\right)} = \boxed{0.309}$

$\cancel{F\left(-\frac{1}{2}\right)}$

(ii) $1 - F\left(\frac{235-241}{2}\right)$

$1 - F(-3)$

$= 1 - 0.0013 = 0.9987$.

5) Assume that high price of commodity tommorrow is a normal variable with $\mu = 10$, $\sigma = \frac{1}{4}$. what is the shortest interval that has prob 0.95 of including tomorrow's high price for this commodity?

Sol) $\mu = 10$, $\sigma = \frac{1}{4}$.

$[10-a, 10+a]$.

$$F\left(\frac{10+a-10}{\frac{1}{4}}\right) - F\left(\frac{10-a-10}{\frac{1}{4}}\right) = 0.95.$$

$F(4a) - F(-4a) = 0.95$.

$F(4a) - (1 - F(4a)) =$

$F(4a) = 0.975$. $4a =$

25-10-19

1) A fair coin is tossed 3 times. Let x = no. of heads in 3 tossing & Y = Diff. in ab. value b/w no. of heads & no. of tails. Find joint PMF of (X, Y).

Sol)

	X	Y
HHH	3	3
HHT	2	1
HTH	2	1
THH	2	1
TTT	0	3
TTH	1	1
THT	1	1
HTT	1	1

Y\X	0	1	2	3	P(Y)
1	0	$\frac{3}{8}$	$\frac{3}{8}$	0	$\frac{6}{8}$
3	$\frac{1}{8}$	0	0	$\frac{1}{8}$	$\frac{2}{8}$
$P_X(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

$P_{X,Y}(3,3) = \frac{1}{8}$; $P_{X,Y}(0,3) = \frac{1}{8}$

$P_{X,Y}(2,1) = \frac{3}{8}$

$P_{X,Y}(1,1) = \frac{3}{8}$

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(a) $P\{X=i/Y=1\} = \dfrac{P\{X=i,\ Y=1\}}{P\{Y=1\}} = \dfrac{\frac{1}{6}}{\frac{6}{5}}$

(b) $P\{X=i/Y=3\} =$

(c) $P\{Y=j/X=0\} =$

(a) Sol.) $= \begin{cases} 0 & i=0,3 \\ \frac{1}{2} & i=1,2 \end{cases}$

(b) Sol.) $= \begin{cases} 0 & i=1,2 \\ \frac{1}{2} & i=0,3 \end{cases}$

(c) Sol.) $= \begin{cases} 0 & i=1 \\ 1 & j=3 \end{cases}$

2.) A Restaurant Serves 3 fixed price dinners costing \$12, \$15 & \$20. For a randomly selected couple dining at this Restaurant, let X = Cost of Man's dinner and Y = Cost of women's dinner. The joint PMF of X & Y is as follows. Find Co-variance and correlation coeff.

	$p(x,y)$	Y: 12	15	20	
	12	.05	.05	.10	.20
X	15	.05	.10	.35	.50
	20	0	.20	.10	.30
		.10	.35	.55	1

$\mathrm{cov}(X,Y) = E(XY) - E(X).E(Y)$

$E(X) = (12)(0.20) + (15)(0.50) + (20)(0.30)$
$= 2.4 + 7.5 + 6 = \boxed{15.9}$

$E(Y) = 12(0.10) + 15(0.35) + 20(0.55)$

$= \boxed{17.45}$

$E(XY) = 144(0.05) + (180)(0.05) + (240)(0.10)$
$+ 180(0.05) + 225(0.10) + 300(0.35)$
$+ 300(0.20) + 400(0.10) + 0.$

$= 7.2 + 9 + 24 + 9 + 22.5 + 40 + 60 + 105$

$= \boxed{276.7}$

$cov(x,y) = 276.7 - (17.45)(15.9)$

$= \boxed{0.755}$

$\sigma_x^2 = E(x^2) - [E(x)]^2 = 8.4 \Rightarrow \sigma_x = 2.91$

$\sigma_y^2 = E(Y^2) - [E(Y)]^2 = 8.64 \Rightarrow \sigma_Y = 2.94$

$\sigma_{xy} = \dfrac{-0.755}{2.91 \times 2.94} = -0.088 \simeq -0.09.$

b) change the scale from dollar to cent and find Co-rrelation coefficient.

Sol.) The values will be same.

3.) Suppose that $n+m$ independent trials having a common prob. of success p are performed. If x is no. of success in first n trials and Y is no. of successes in final m trials. find $P\{X = x, Y = y\}$?

Sol.)

$P\{x = x, Y = y\} = P\{x = x\} \cdot P\{Y = y\}$.

$= {}^nC_x \cdot p^x (1-p)^{n-x} \cdot {}^mC_y \, p^y (1-p)^{m-y}$

4.) Let X, Y be identically distributed with common PMF $P\{X=k\} = \frac{1}{N}$, $k = 1, 2, \ldots N$ $(N > 1)$

Find correlation co-efficient

Sol:)

$$E(X) = 1\cdot\frac{1}{N} + 2\cdot\frac{1}{N} + \cdots + N\cdot\frac{1}{N} = \frac{N+1}{2}.$$

$$E(X^2) = 1^2\cdot\frac{1}{N} + 2^2\cdot\frac{1}{N} + \cdots\ N^2\cdot\frac{1}{N} = \frac{(N+1)(2N+1)}{6}.$$

$$\text{var}(X) = E(X^2) - [E(X)]^2 = \frac{(N+1)(2N+1)}{6} - \left[\frac{N+1}{2}\right]^2$$

$$= \boxed{\frac{N^2-1}{12}}$$

$$E(XY) = \frac{1}{2}\left[E(X^2) + E(Y^2) - E(X-Y)^2\right],$$

$$= \frac{1}{2}\left[2\,\frac{(N+1)(2N+1)}{6} - E(X-Y)^2\right],$$

$$= \frac{(N+1)(2N+1)}{6} - \frac{E(X-Y)^2}{2}$$

$$\text{cov}(X,Y) = E(XY) - E(X)\cdot E(Y)$$

$$= \frac{(N+1)(2N+1)}{6} - \frac{E(X-Y)^2}{2} - \frac{(N+1)^2}{4}$$

$$\text{cov}(X,Y) = \frac{N^2-1}{12} - \frac{E(X-Y)^2}{2}$$

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X\,\sigma_Y} = \frac{\frac{N^2-1}{12} - \frac{E(X-Y)^2}{2}}{\frac{N^2-1}{12}}$$

$$= 1 - \frac{6\,E(X-Y)^2}{N^2-1}$$

$P\{X=Y\} = 1 \Leftrightarrow \rho_{XY} = 1.$

$P\{Y = N+1-X\} \Leftrightarrow \rho_{XY} = -1.$

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1/11/2019 -

(Q) In a chess competition including some men & some women, every player needs to play with exactly one game with every other player. It was found that in 45 games, both players were women & in 190 games both players were men. What is no. of games in which one person was men & other was women.

Sol) no. of women = w $\Rightarrow$ ${}^{w}C_2 = 45$

$$\Rightarrow \frac{w!}{(w-2)!\times 2} = 45 \qquad (w)(w-1) = 45$$

$$w^2 - w - 90 = 0$$

$$w^2 - 10w + 9w - 90 = 0$$

$$w(w-10) + 9(w-10) = 0$$

$$\boxed{w = 10}$$

45 = 3 × 3 × 5.

no. of men = m $\Rightarrow$ ${}^{m}C_2 = 190$

$$m(m-1) = 380.$$

$$m^2 - m - 380 = 0.$$

$$\boxed{m = 20}.$$

$${}^{20}C_1 \times {}^{10}C_1 = 20 \times 10 = \boxed{200}.$$

(Q) In how many diff. ways 3 persons A, B, C having 6, 7 & 8 one rupee coins respectively can donate ₹10 collectively?

Sol.) ~~x+y+z~~

$${}^{n+r-1}C_{r-1}$$

$$x_1 + x_2 + \dots + x_r = n \qquad {}^{n+r-1}C_{r-1}$$

$$x_1, x_2 \dots x_n \geq 0$$

$$x + y + z = 10.$$

$${}^{10+3-1}C_{3-1} - {}^{5}C_2 - {}^{4}C_2 - {}^{3}C_2.$$

$$\boxed{{}^{12}C_2 - {}^{5}C_2 - {}^{4}C_2 - {}^{3}C_2}$$

$x + y + z = 2$

$2 + 3 - 1$

$x + y + z = 10.$

7

$x + y + z = 3$

${}^{n+r-1}C_{r-1}$

0, 1, 2, 3

$x + y + z = 2$ $\quad {}^{3+3-1}C_2$

$${}^{2+3-1}C_{3-1} \quad {}^{5}C_2.$$

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(Q) How many students must be in a class to generate that at least two students receive the same score on final Exam, if Exam is graded on scale from 0 to 100 points?

Sol) n · 0, 1, 2, ..., 100 ⟶ $100+1+1 = \boxed{102}$

~~100~~ n

(Q) Among 100 people there at least how many of them have birthday in same month?

Sol) $\left\lceil \frac{100}{12} \right\rceil = 9$ ⟹ $12 \times 8 = 96$.

(Q) what is minimum no. of students required in a discrete mathematics class to be sure that atleast six will recieve the same grade, if there are five possible grades.

(sum written above "six")

Sol) $5 \times 5 + 1 = \boxed{26}$

(Q) (a) How many cards must be selected from a stadard deck of 52 cards to garentee that atleast 3 cards of same suit are choosen?

Sol) $4 \times 2 + 1 = \boxed{9}$.

(b) How many must be selected to garentee that at least 3 hearts are selected

Sol) $13 \times 3 + 3 = \boxed{42}$.

(Q) Suppose that a saleswomen has to visit eight diff. cities. She must begin her trip in specified city, but she can visit the other seven cities in any order she wishes How many possible orders can Saleswome use when visiting these cities?

Sol) $1 \times 7! = \boxed{7!}$