Lab Assignment - 05 - Spring 2020

Signals and systems

Anirudh Jakhotia || UG-1 || S20190010007
Indian Institute of Information Technology, Sri city.

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1) In the fundamental interval the signal x1(t) is defined as :

$x1(t) = (1 - |t/2|)(u(t + 1) - u(t - 1))$

2) In the fundamental interval the signal x2(t) is defined as :

$x2(t) = t^2(u(t + 1) - u(t - 1))$

- First, we will write the code to plot x1(t) and then compute its fourier coefficients .
- Then, we will verify the real and imaginary components of coefficients of x1 with that of exponential (Euler's form) one and the theoretical one.
- Finally, we will reconstruct x1 using its coefficients.

Here's the code :

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assign05.mlx | assign05_2.mlx | assign05_3.mlx | assign05_4.mlx | unit.mlx | +

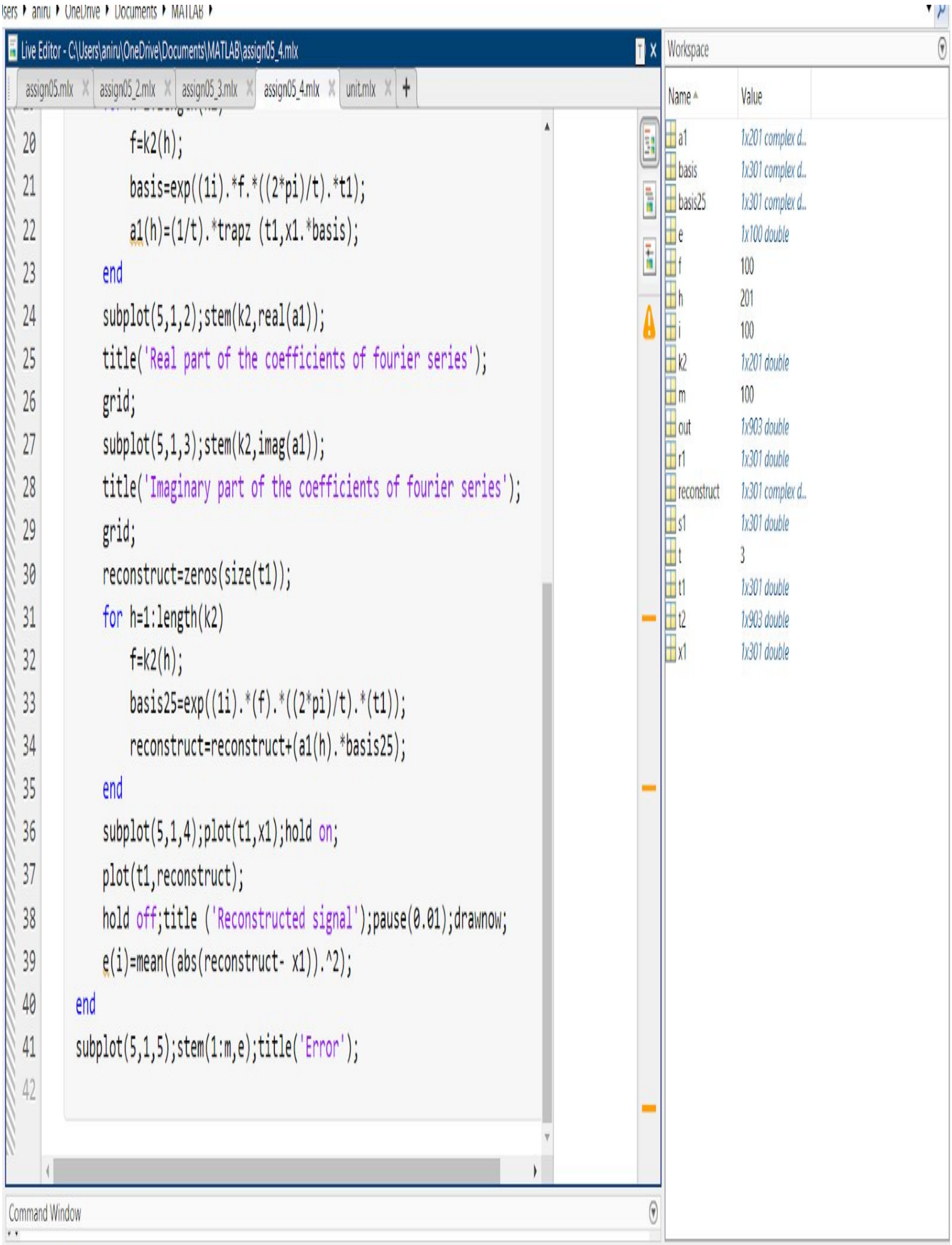
```
clc
close all
clear

t=3;
t1=-t/2:0.01:t/2;
t2=-3*t/2-0.01:0.01:3*t/2+0.01;
s1=unit(t1+1)-unit(t1-1);
r1=1-abs(t1/2);
x1=s1.*r1;
out=[];
for i=1:3
    out=[out x1];
end
subplot(5,1,1);plot(t2,out);title('original signal');
grid;m=100;
for i=1:m
    k2=-i:i;
    for h=1:length(k2)
        f=k2(h);
        basis=exp((1i).*f.*((2*pi)/t).*t1);
        a1(h)=(1/t).*trapz (t1,x1.*basis);
    end
```

Workspace

Name ▲	Value
a1	1x201 complex d...
basis	1x301 complex d...
basis25	1x301 complex d...
e	1x100 double
f	100
h	201
i	100
k2	1x201 double
m	100
out	1x903 double
r1	1x301 double
reconstruct	1x301 complex d...
s1	1x301 double
t	3
t1	1x301 double
t2	1x903 double
x1	1x301 double

Command Window



- Now, we again do the same thing for x2. We will write the code to plot x2(t) and then compute its fourier coefficients .

- Then, we will verify the real and imaginary components of coefficients of x2 with that of exponential (Euler's form) one and the theoretical one.

- Finally, we will reconstruct x2 using its coefficients.

Here's the code :

```
clc
clear
close all

t=3;t1=-t/2:0.01:t/2;
t2=-3*t/2-0.01:0.01:3*t/2+0.01;
s1=unit(t1+1)-unit(t1-1);
r1=t1.*t1;
x1=s1.*r1;
out=[];
for i=1:3
out=[out x1];
end
subplot(5,1,1);plot(t2,out);title('Original Signal');
grid;
m=100;
for i=1:m
k2=-i:i;
for h=1:length(k2)
f=k2(h);
basis=exp((1i).*f.*((2*pi)/t).*t1);
a1(h)=(1/t).*trapz(t1,x1.*basis);
end
subplot(5,1,2);
```

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assign05.mlx | assign05_2.mlx | assign05_3.mlx | assign05_4.mlx | unit.mlx

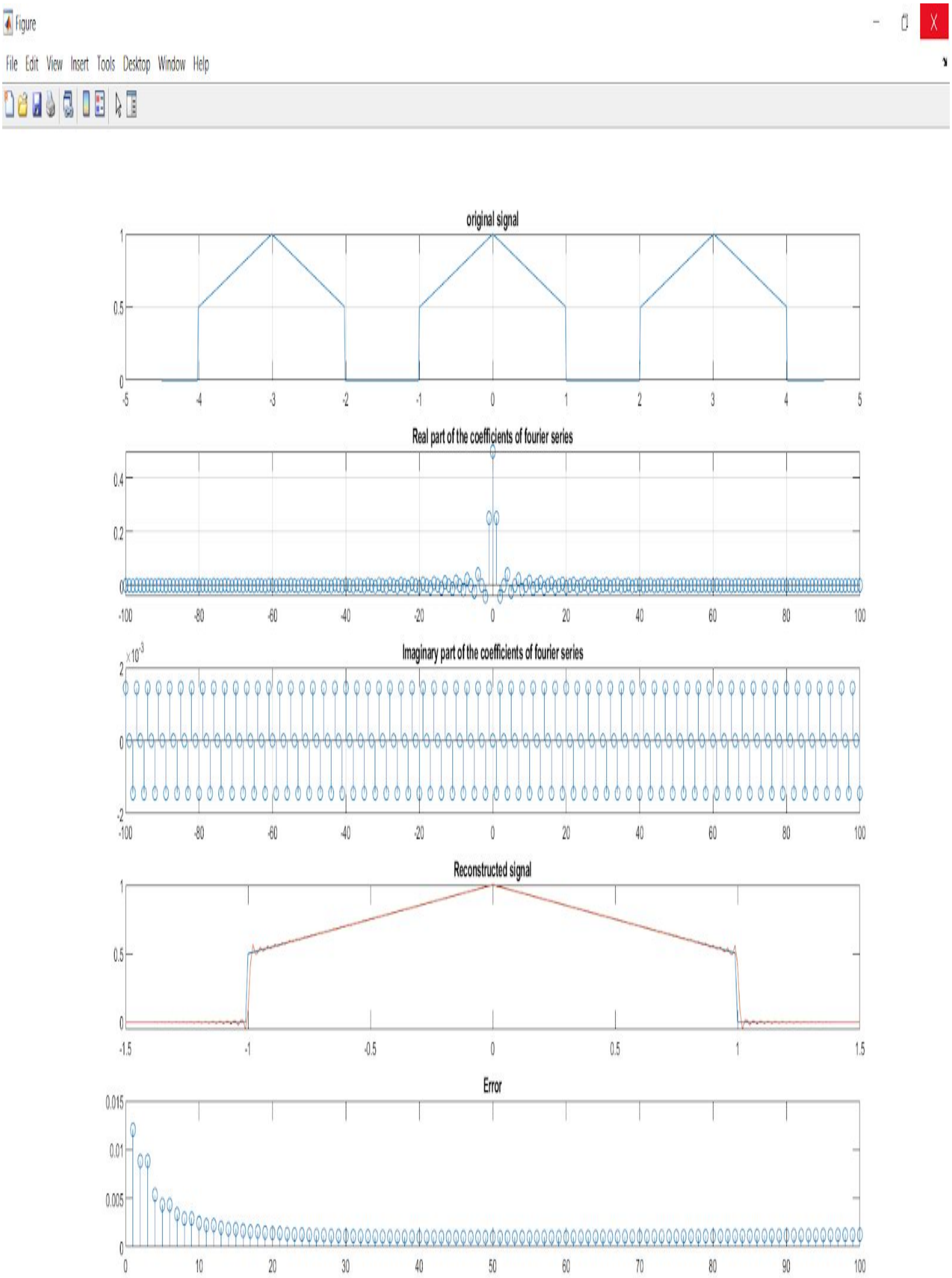
```
f=k2(h);
basis=exp((1i).*f.*((2*pi)/t).*t1);
a1(h)=(1/t).*trapz(t1,x1.*basis);
end
subplot(5,1,2);
stem(k2,real(a1));
title('Real part of the coefficients of fourier series');
grid;subplot(5,1,3);
stem(k2,imag(a1));
title('Imaginary part of the coefficients of Fourier series');
grid;
reconstruct=zeros(size(t1));
for h=1:length(k2)
f=k2(h);
basis25=exp((1i).*(f).*((2*pi)/t).*(t1));
reconstruct=reconstruct+(a1(h).*basis25);
end
subplot(5,1,4);
plot(t1,x1);hold on;
plot(t1,reconstruct);
hold off;title('Reconstructed signal');pause(0.01);drawnow;
e(i)=mean((abs(reconstruct- x1)).^2);
end
subplot(5,1,5);stem(1:m,e);title('Error');
```

Workspace

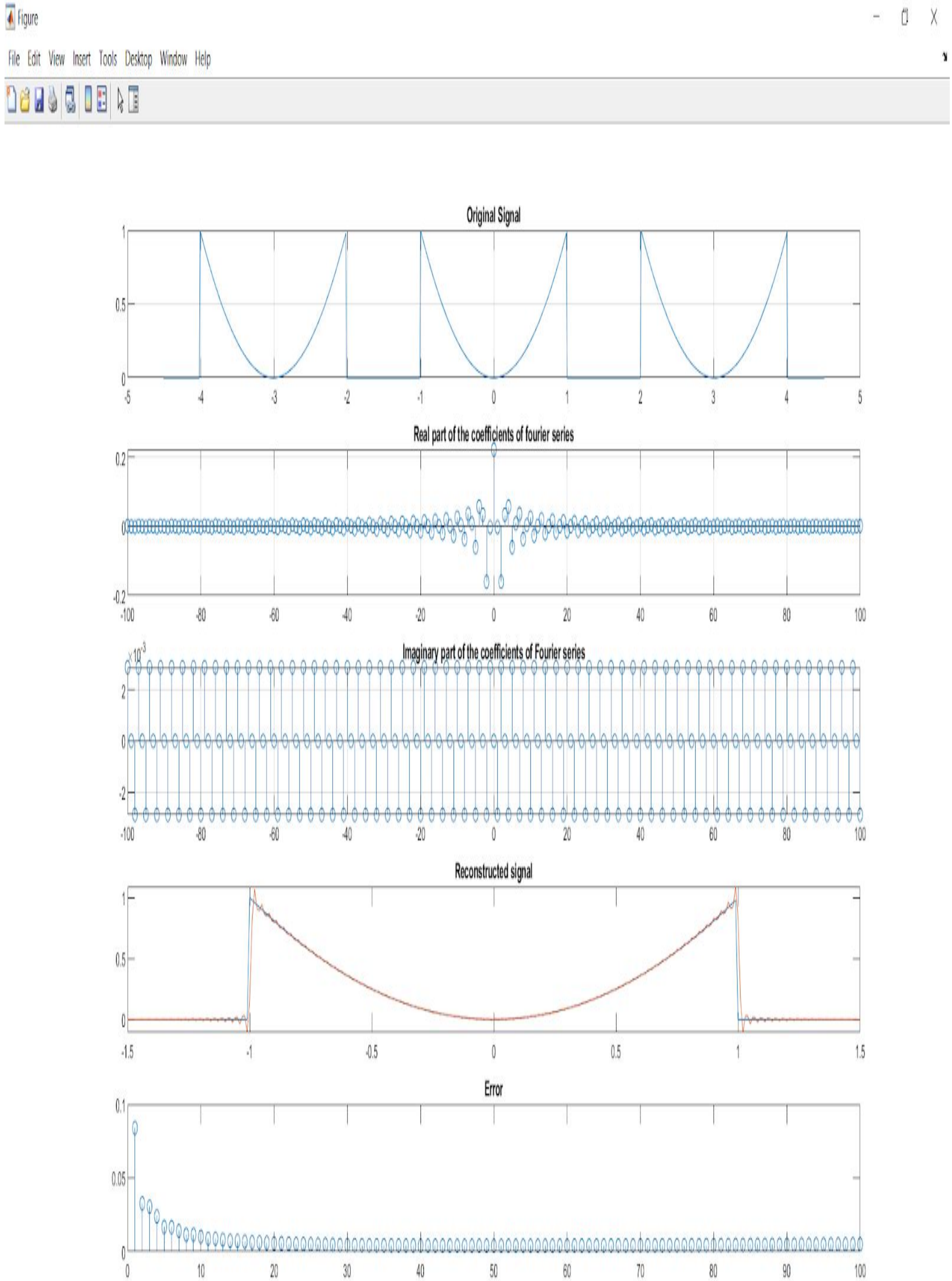
Name	Value
a1	1x201 complex d...
basis	1x301 complex d...
basis25	1x301 complex d...
e	1x100 double
f	100
h	201
i	100
k2	1x201 double
m	100
out	1x903 double
r1	1x301 double
reconstruct	1x301 complex d...
s1	1x301 double
t	3
t1	1x301 double
t2	1x903 double
x1	1x301 double

Command Window

Here's the output for 1)

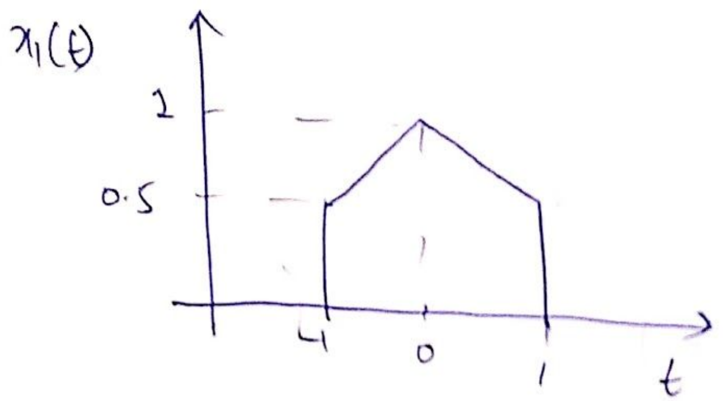


Here's the output for 2)



Theoretical part :

$\underline{x_1}$



$$x_1(t) = \begin{cases} 1 - \frac{t}{2} & 0 < t < 1 \\ 0 & \text{otherwise} \\ 1 + \frac{t}{2} & -1 < t < 0 \end{cases}$$

$$a_k = \frac{1}{T}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt \qquad [T_0 = 3]$$

$$= \int_{-1.5}^{1.5} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$\boxed{\theta = k\omega_0 t} = \cancel{\int_{-1.5}^{-1}}^{\,0} + \int_{-1}^{0} + \int_{0}^{1} + \cancel{\int_{1}^{1.5}}^{\,0}$$

$$= \int_{-1}^{0}\left(1+\frac{t}{2}\right)(\cos\theta - i\sin\theta)\,dt + \int_{0}^{1}\left(1-\frac{t}{2}\right)(\cos\theta - i\sin\theta)\,dt$$

$$\left[\text{where } \theta = \frac{2\pi k t}{3}\right]$$

$$= \int_{-1}^{0}\cos\theta\left(1+\frac{t}{2}\right)dt - i\int_{-1}^{0}\sin\theta\left(1+\frac{t}{2}\right)dt + \int_{0}^{1}\left(1-\frac{t}{2}\right)\cos\theta\, dt$$

$$- i\int_{0}^{1}\sin\theta\left(1-\frac{t}{2}\right)dt$$

$$Re = \int_{-1}^{0} \cos\theta \left(1 + \frac{t}{2}\right) dt + \int_{0}^{1} \left(1 - \frac{t}{2}\right) \cos\theta \, dt$$

$$= \int_{-1}^{0} \cos\left(\frac{2\pi k t}{3}\right)\left(1 + \frac{t}{2}\right) dt + \int_{0}^{1} \cos\left(\frac{2\pi k t}{3}\right)\left(1 - \frac{t}{2}\right) dt$$

$$= \frac{6\pi k \sin\left(\frac{2\pi k}{3}\right) + 9 - 9\cos\left(\frac{2\pi k}{3}\right) + 6\pi k \sin\left(\frac{2\pi k}{3}\right) + 9 - 9\cos\left(\frac{2\pi k}{3}\right)}{8\pi^2 k^2}$$

$$\boxed{Re\,(a_k) = \frac{6\pi k \sin\left(\frac{2\pi k}{3}\right) - 9\cos\left(\frac{2\pi k}{3}\right) + 9}{4\pi^2 k^2}}$$

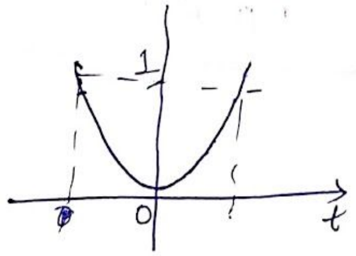
$$\underline{\underline{Img}} = \int_{-1}^{0} \sin\theta \left(1 + \frac{t}{2}\right) dt + \int_{0}^{1} \sin\theta \left(1 - \frac{t}{2}\right) dt$$

$$= 2\left[\frac{9\sin\left(\frac{2\pi k}{3}\right) + 6\pi k \cos\left(\frac{2\pi k}{3}\right) \cancel{+} 12\pi k}{8\pi^2 k^2}\right]$$

$$\boxed{Img = \frac{9\sin\left(\frac{2\pi k}{3}\right) + 6\pi k \cos\left(\frac{2\pi k}{3}\right) - 12\pi k}{4\pi^2 k^2}} \quad \forall k \neq 0$$

for $k = 0$
$Img = 0$

$\underline{\underline{x_2}}$: $x_2(t)$



$$x_2(t) = \begin{cases} t^2 & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$a_k = \frac{1}{T}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt \qquad [T_0 = 3]$$

$$= \int_{-1.5}^{1.5} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \int_{-1.5}^{0} + \int_{-1}^{0} + \int_{0}^{1} + \int_{1}^{1.5}$$

$$= \int_{-1}^{x} t^2(\cos\theta - i\sin\theta)\, dt$$

$$Re = \int_{-1}^{1} t^2 \cos\theta\, dt \qquad \theta = \frac{2\pi k t}{3}$$

$$= \int_{-1}^{1} t^2 \cos\left(\frac{2\pi k t}{3}\right) dt$$

$$= \frac{(6\pi^2 k^2 - 27)\sin\left(\frac{2\pi k}{3}\right) + 18\pi k\cos\left(\frac{2\pi k}{3}\right)}{2\pi^3 k^3}$$

$$\therefore \boxed{Re = \frac{(6\pi^2 k^2 - 27)\sin\left(\frac{2\pi k}{3}\right) + 18\pi k\cos\left(\frac{2\pi k}{3}\right)}{2\pi^3 k^3}}$$

$$Img = \int_{-1}^{1} t^2 \sin\left(\frac{2\pi k t}{3}\right) dt$$

$$= 2\left[\frac{18\pi k \sin\left(\frac{2\pi k}{3}\right) + (27 - 6\pi^2 k^2)\cos\left(\frac{2\pi k}{3}\right) - 27}{4\pi^3 k^3}\right]$$

$$\boxed{Img = \frac{18\pi k \sin\left(\frac{2\pi k}{3}\right) + (27 - 6\pi^2 k^2)\cdot\cos\left(\frac{2\pi k}{3}\right) - 27}{2\pi^3 k^3}}$$

Observation :

- We derived the expression to find the real and imaginary components of the Fourier coefficients for both x1 & x2.
- We found out that the values obtained on paper matched with those we got from our output.

Here’s the proof :

Cross verification :

For x_1 ;

	K	a_k (Theoretical)	a_k (observed)
Real	a_0	$= \frac{1}{T}\int x(t) e^{-0} dt = \frac{3/2}{\frac{1}{3}} = \frac{1}{2} = 0.5$	0.5
	a_1	$\frac{6\pi \sin(120) - 9\cos(120) + 9}{4\pi^2}$ $= 0.25175$	0.2518
	a_{-1}	$\frac{6\pi \sin(120) - 9\cos(120) + 9}{4\pi^2}$ $= 0.25175$	0.2518
Img	a_0	$= 0$	$= 0$
	a_1	-1.45×10^{-5}	-1.443×10^{-5}
	a_{-1}	$+1.45 \times 10^{-5}$	-1.443×10^{-5}

For x_2 :-

	k	a_k (Theoretical)	a_k (observed)
Real	a_0	$\int \frac{x(t)dt}{T} = \frac{2}{3}\left(\frac{1^3-0^3}{3}\right)$ $= 0.222$	0.222
	a_1	$\frac{(6\pi^2-27)\sin(120)+18\pi\cos(120)}{2\pi^3}$ $= -0.002005$	-0.002005
	a_{-1}	$\frac{(6\pi^2-27)\sin(120)+18\cos(120)}{2\pi^3}$ $= -0.002005$	-0.002005
Img	a_0	$= 0$	$= 0$
	a_1	-2.89×10^{-5}	-2.88×10^{-5}
	a_{-1}	$+2.89\times10^{-5}$	2.887×10^{-5}

Convergence:

- To analyse the convergence of the reconstructed signal with that of the original signal we subtracted the values of original signal from the real values of the reconstructed signal i.e. we ignored the imaginary components from the reconstructed signal.
- Basically, we have to look out for the critical points where we have discontinuities in the original signal.
- One of our main mottos to reconstruct the signal was to eliminate the discontinuous points. So that’s the major thing we need to be concerned about the reconstructed signal.
- In short, if the reconstructed signal comes out to be the same as that of the original signal the critical point should show ‘zero’ at its respective position. If not then we have achieved our task provided the value is much closer to zero.
- In the above output figure 1 and 2, we can see that in subplot 4 of each figure we have plotted the reconstructed signal against the original signal.
- We got the values of reconerr_1=1.4906e-06 and reconerr_2=5.9849e-06 i.e., they are the values of difference between two errors at two points for x1 and x2.
- Given these values we can conclude that since we didn’t come across any discontinuity especially at the critical points, all we can say is that both x1 and x2 are convergent.

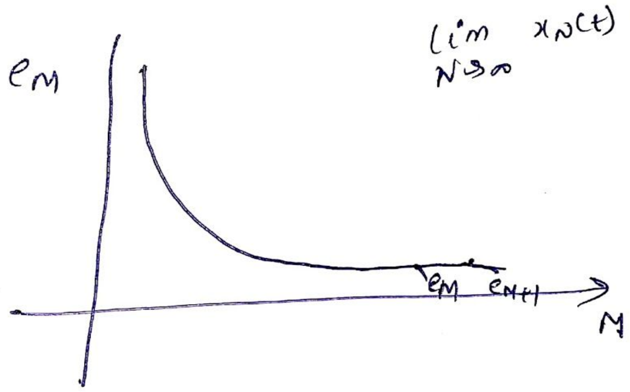
    - Then, the best method to find the convergence of the original and reconstructed signal is given here (Look for tau<10^-6):

Given signal $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

Reconstructed signal $x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$

$$e_M = \frac{1}{T}\int_T |x(t) - x_N(t)|^2 dt$$

$\lim_{N\to\infty} x_N(t) \to x(t)$ convergence



$$\tau = |e_{M+1} - e_M|$$

$$\tau < 10^{-6}.$$

So basically, we find the error for each value of the number of coefficients till a particularly large value like 100.

---------------THE END--------------