Lab - Assignment - 04 - Spring 2020

Signals & Systems

Anirudh Jakhotia || UG-1 || S20190010007

Indian Institute of Information Technology,Sri City.

Q1) To write a matlab code for linear convolution of two time-limited continuous signals and demonstrate its application on the following pair of signals along with a theoretical validation.

Function x(t) is given as follows:

$x(t) = \exp(-2t)(u(t) - u(t-4))$

Function h(t) is given as follows:

$$h(t) = \begin{cases} 1 - t & 0 \le t < 1 \\ t - 1 & 1 \le t < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Now, we have to do the linear convolution of these signals :

Here's the code :

Function a4(t) which resembles the h(t)

```
function h=a4(t)
h=zeros(size(t));
h(t>=0 & t<1)=1-t(t>=0 & t<1);
h(t>=1 & t<=2)=t(t>=1 & t<=2)-1;
end
```

Function rect(t) which generates a rectangle within the specified range and amplitude

```
function r=rect(t,a,b,A)
t1=t-a;
x1=unit(t1);
t2=t-b;
x2=unit(t2);
r=A*(x1-x2);
end
```

Here’s the code for question 1 :

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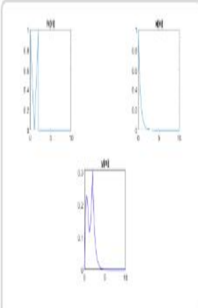
rect.mlx | assign04.mlx * | assign04_2_1.mlx | a5.mlx | tutorial1.mlx | unit.mlx | h5.mlx | a4.mlx | y4.mlx | +

```
close all

t=0:0.01:10;
h=h4(t);
subplot(2,3,1);plot(t,h);title('h[n]');
x=exp(-2*t).*(rect(t,0,4,1));
subplot(2,3,3);plot(t,x);title('x[n]');
tau=-6:0.01:20;
h1=h4(tau);
x1=exp(-2*tau).*(rect(tau,0,4,1));
for i=1:length(t)
    tt=t(i);
    x1_tt=exp(-2*(tt-tau)).*(rect((tt-tau),0,4,1));
    y_tt(i)=trapz(tau,x1_tt.*h1);
end
subplot(2,3,5);
plot(t,y_tt,"Color",'r');title('y[n]');
hold on;
r=a5(t);plot(t,r,"Color",'b');
```



Workspace

Name	Value
h	1x1001 double
h1	1x2601 double
i	1001
r	1x1001 double
t	1x1001 double
tau	1x2601 double
tt	10
x	1x1001 double
x1	1x2601 double
x1_tt	1x2601 double
y_tt	1x1001 double

Here's also the theoretical code as function a5(t)

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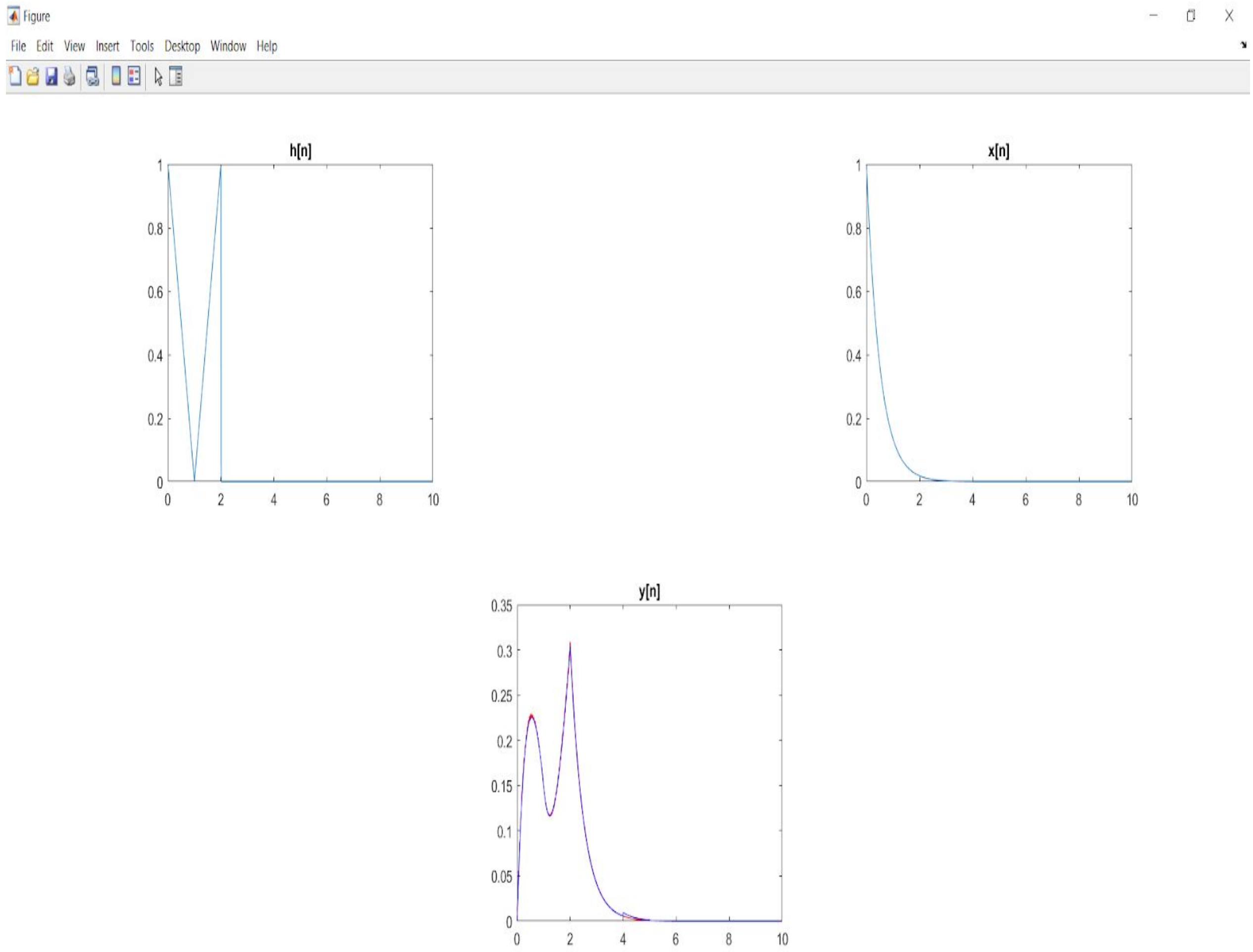
rect.mlx | assign04.mlx | assign04_2_1.mlx | a5.mlx | tutorial1.mlx | unit.mlx | h5.mlx | a4.mlx | y4.mlx | +

```
function  r=a5(t)
r=zeros(size(t));
r(t>=0 & t<1) = (3-2*t(t>=0 & t<1)-3*exp(-2*t(t>=0 & t<1)))/4;
r(t>=1 & t<2) = (2*t(t>=1 & t<2)-3+(2*exp(2)-3)*exp(-2*t(t>=1 & t<2)))/4;
r(t>=2 & t<4) = exp(-2*t(t>=2 & t<4))*(exp(4)+2*exp(2)-3)/4;
r(t>=4 & t<5) = (5*exp(-8)+exp(4-2*t(t>=4 & t<5))-5*exp(2-2*t(t>=4 & t<5)))/4;
r(t>=5 & t<6) = (exp(4-2*t(t>=5 & t<6))+exp(-8))/4;
r(t>=6)=0;
end
```

Workspace

Name	Value
h	1x1001 double
h1	1x2601 double
i	1001
r	1x1001 double
t	1x1001 double
tau	1x2601 double
tt	10
x	1x1001 double
x1	1x2601 double
x1_tt	1x2601 double
y_tt	1x1001 double

Here's the output :



Theoretical verification of Q1 is given at the last :
As,we can see that the theoretical and matlab code matches .

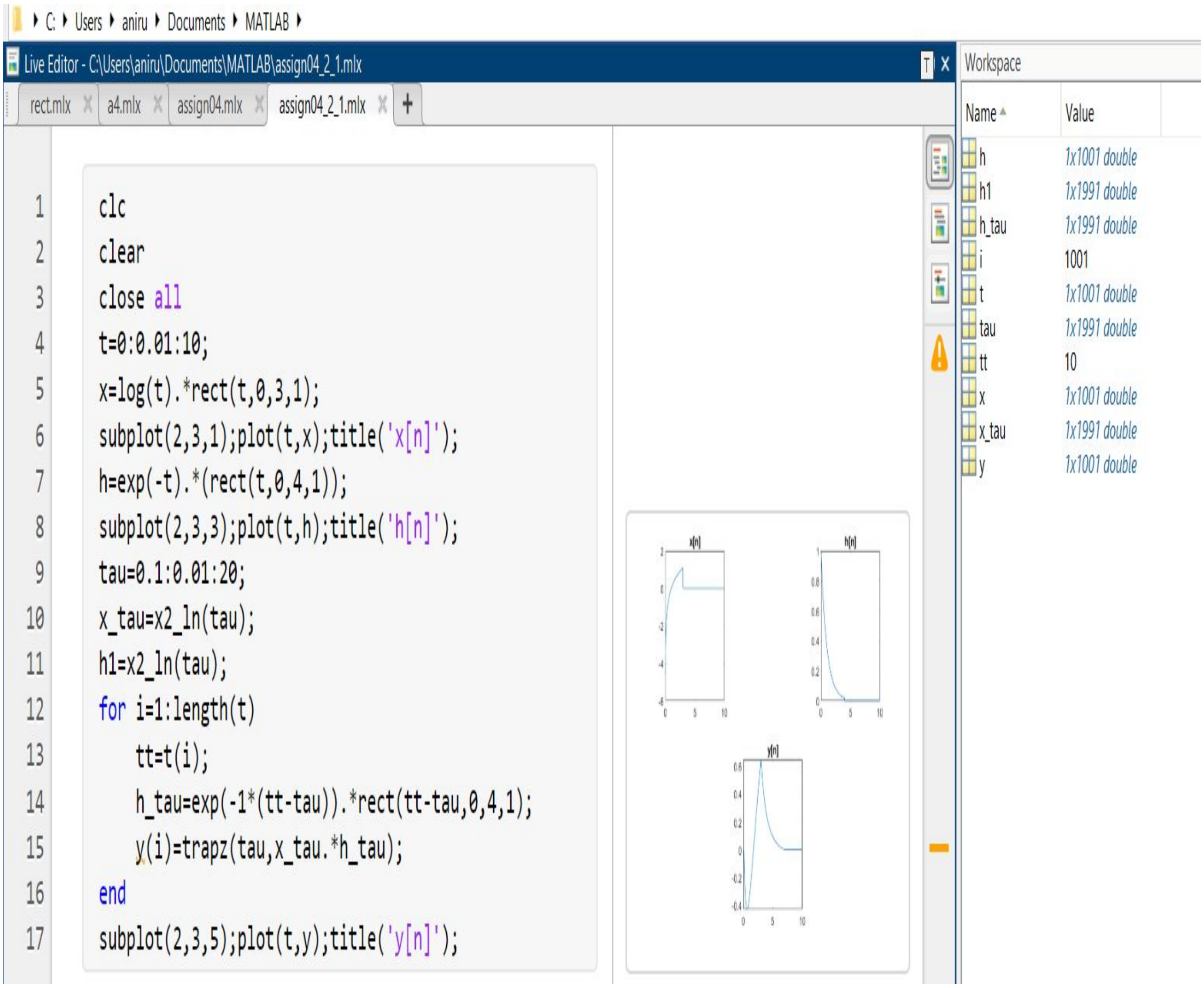
Q2) Use the same convolution function to convolve the following signals. (Theoretical verification is not necessary)

$x(t) = \ln(t)(u(t) - u(t - 3))$

and

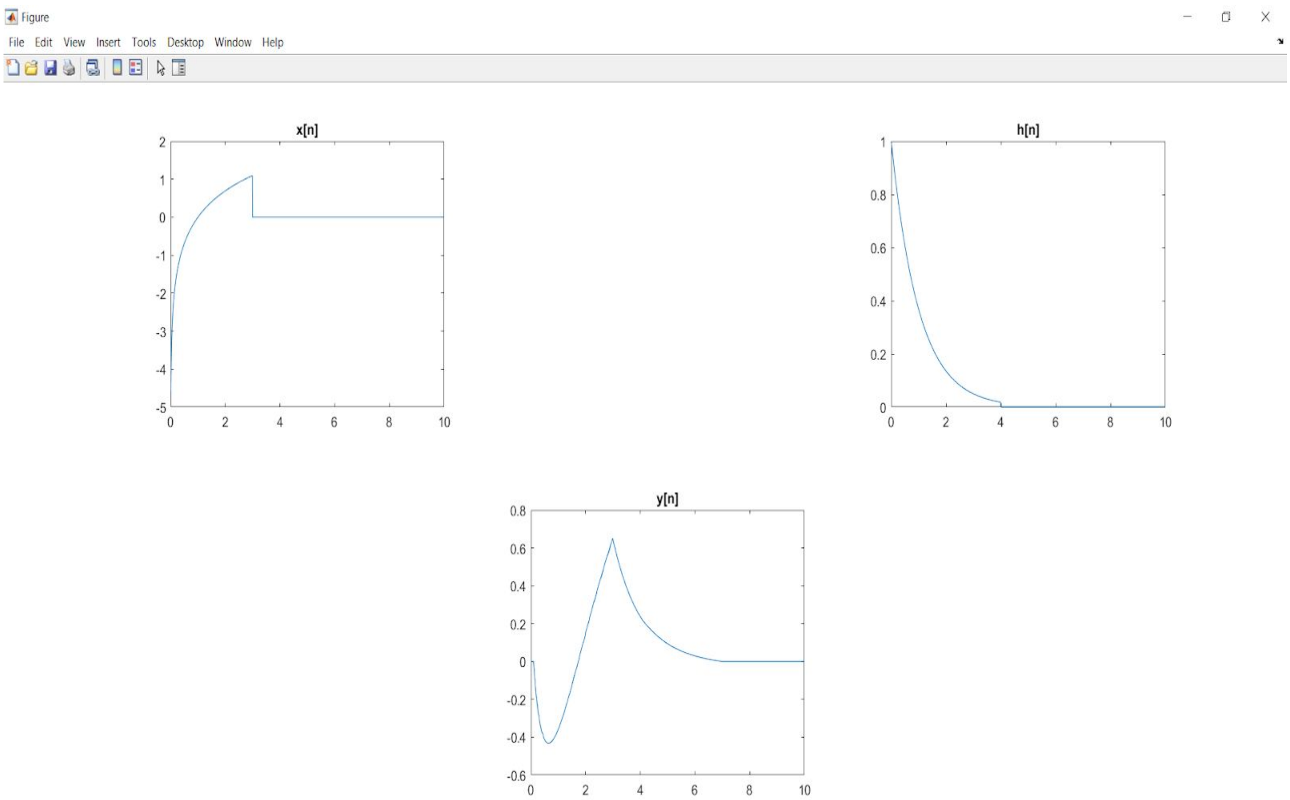
$h(t) = \exp(-t)(u(t) - u(t - 4))$

Here’s the code :




```
clc
clear
close all
t=0:0.01:10;
x=log(t).*rect(t,0,3,1);
subplot(2,3,1);plot(t,x);title('x[n]');
h=exp(-t).*(rect(t,0,4,1));
subplot(2,3,3);plot(t,h);title('h[n]');
tau=0.1:0.01:20;
x_tau=x2_ln(tau);
h1=x2_ln(tau);
for i=1:length(t)
    tt=t(i);
    h_tau=exp(-1*(tt-tau)).*rect(tt-tau,0,4,1);
    y(i)=trapz(tau,x_tau.*h_tau);
end
subplot(2,3,5);plot(t,y);title('y[n]');
```

Here’s the output :

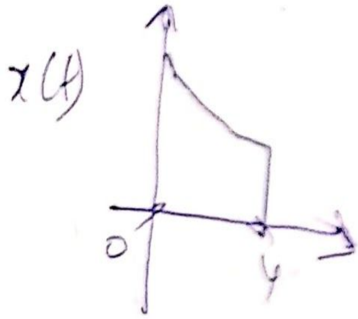


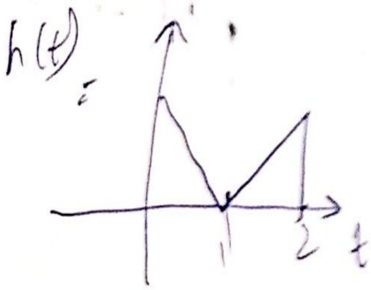
Theoretical explanation of Q1) :

(a)(b)

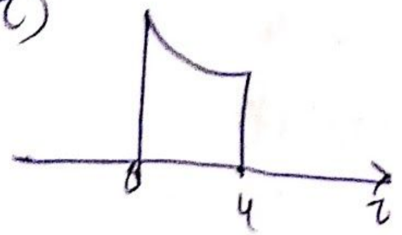
$$x(t) = e^{-2t}\left(u(t) - u(t-4)\right)$$

$$h(t) = \begin{cases} 1-t & 0 \le t < 1 \\ t-1 & 1 \le t < 2 \\ 0 & \text{else} \end{cases}$$



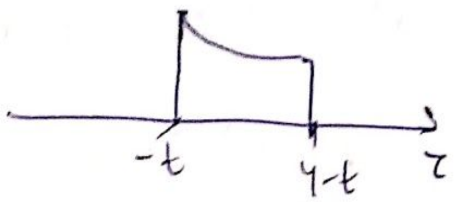


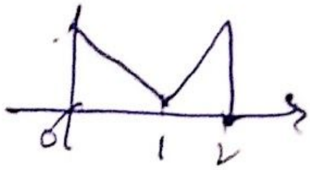
$x(\tau)$



$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

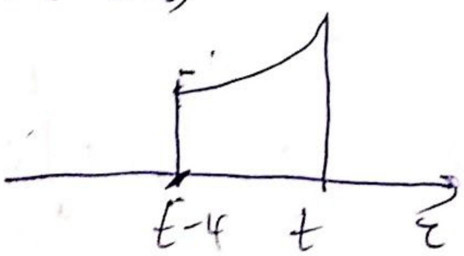
$x(\tau+t)$



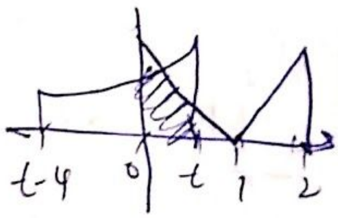


So, $t<0$, $y(t)=0$

$x(-\tau+t)$

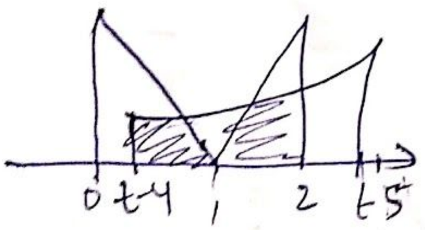


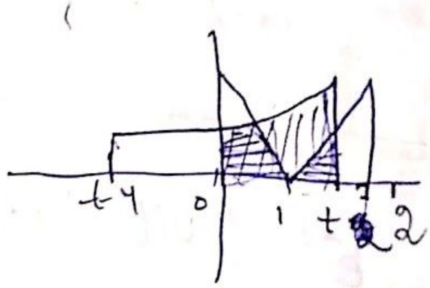
for $t \geq 0$, $t<1$, $0 \leq t < 1$



$4 \leq t < 5$

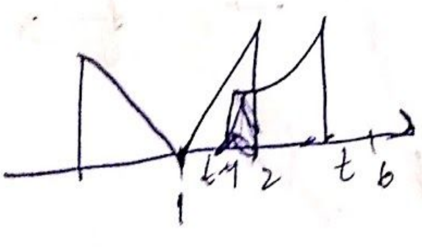
$1 \leq t < 2$

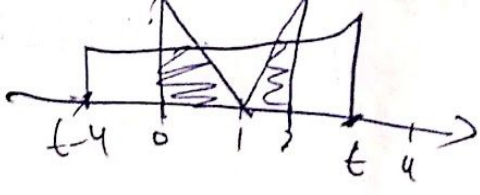




$5 \leq t < 6$

$2 \leq t < 4$





$5 \leq t < 6$

$$y(t) = \int_{t-4}^{2} e^{-2(t-\tau)} (\tau-1)\, d\tau + \ldots$$

① $y(t) = \int_0^t x(t-\tau)h(\tau)d\tau$

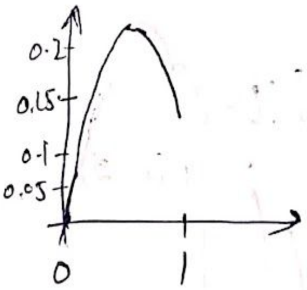
$$= \int_0^t e^{-2(t-\tau)}(1-\tau)d\tau = \int_0^t e^{-2t}e^{2\tau}(1-\tau)d\tau$$

$$= e^{-2t}\int_0^t e^{2\tau}d\tau - \int_0^t \tau e^{2\tau}d\tau.$$

$$= e^{-2t}\left(\left[\frac{e^{2\tau}}{2}\right]_0^t - \left(\left[\frac{\tau e^{2\tau}}{2}\right]_0^t - \left[\frac{e^{2\tau}}{4}\right]_0^t\right)\right)$$

$$= e^{-2t}\left[\frac{e^{2t}-1}{2}\right] - \left(\frac{te^{2t}-\cancel{1}}{\cancel{2}} + \frac{e^{2t}}{4} + \cancel{\frac{1}{2}}\right)$$

$$= \frac{1-e^{-2t}}{2} - \frac{t}{2} + \frac{1-e^{-2t}}{4}$$



$$y(t) = \frac{3}{4}(1-e^{-2t}) - \frac{t}{2}, \quad 0 < t < 1$$

$1 \le t < 2$

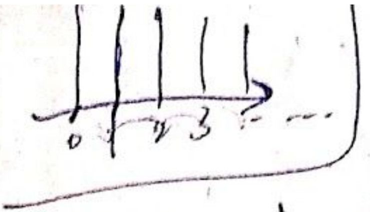
$$y(t) = \int_0^1 e^{-2(t-\tau)}(1-\tau)d\tau + \int_1^t e^{-2(t-\tau)}(\tau-1)d\tau.$$

$2 \le t < 4$

$$y(t) = \int_{t-4}^0 (0) + \int_0^1 e^{-2(t-\tau)}(1-\tau)d\tau + \int_1^2 e^{-2(t-\tau)}(\tau-1)d\tau + \int_2^t (0)$$

$4 \le t < 5$

$$y(t) = \int_{t-4}^1 e^{-2(t-\tau)}(1-\tau)d\tau + \int_1^2 e^{-2(t-\tau)}(\tau-1)d\tau + \int_2^t (0)\,dt$$

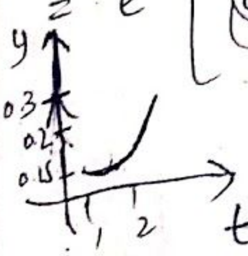


$1 \le t \le 2$

$$e^{-2t}\int_0^1 e^{2\tau}(1-\tau)\,d\tau + e^{-2t}\int_1^t e^{2\tau}(\tau-1)\,d\tau$$

$$= e^{-2t}\int_0^1 e^{2\tau}d\tau - \int_0^1 \tau e^{2\tau}d\tau + e^{-2t}\int_1^t \tau e^{2\tau}d\tau - \int_1^t e^{2\tau}d\tau$$

$$= e^{-2t}\left[\left(\frac{e^{2\tau}}{2}\right)_0^1 - \left[\tau\frac{e^{2\tau}}{2}\right]_0^1 - \left[\frac{e^{2\tau}}{4}\right]_0^1\right]$$



$$+ e^{-2t}\left[\left[\frac{\tau e^{2\tau}}{2}\right]_1^t - \left[\frac{e^{2\tau}}{4}\right]_1^t - \left[\frac{e^{2\tau}}{2}\right]_1^t\right]$$

$$= e^{-2t}\left[\frac{e^2-1}{2} - \left[\frac{e^2-0}{2} - \left(\frac{e^2-1}{4}\right)\right]\right]$$

$\frac{1}{2}+\frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$

$$+ e^{-2t}\left[\frac{te^{2t}-e^2}{2} - \left(\frac{e^{2t}-e^2}{4}\right) - \left(\frac{e^{2t}-e^2}{2}\right)\right]$$

$$= e^{-2t}\left(\frac{\cancel{e^2}-1}{2} - \frac{\cancel{e^2}}{2} + \frac{e^2-1}{4}\right) + e^{-2t}\left(\frac{te^{2t}-\cancel{e^2}-e^{2t}+\cancel{e^2}}{2} - \frac{e^{2t}}{4} + \frac{e^2}{4}\right)$$

$$= e^{-2t}\left(-\frac{1}{2} + \frac{e^2}{4} - \frac{1}{4}\right) + e^{-2t}\left(\frac{te^{2t}}{2} - \frac{e^{2t}}{2} - \frac{e^{2t}}{4} + \frac{e^2}{4}\right)$$

$$= e^{-2t}\left(\frac{-3+e^2}{4}\right) + e^{-2t}\left(\frac{te^{2t}}{2} - \frac{3e^{2t}}{4} + \frac{e^2}{4}\right) - \frac{1}{2}e^2$$

$$= e^{-2t}\left(\frac{e^2-3}{4}\right) + e^{-2t}\left(\frac{2te^{2t}-2e^2}{4}\right)$$

$$= e^{-2t}\left(\frac{e^2}{4}\right) - \frac{3}{4}e^{-2t} + \frac{t}{2} - \frac{3}{4} + e^{-2t}\left(\frac{e^2}{4}\right)$$

$$= e^{-2t}\cdot\frac{e^2}{2} - \frac{3}{4}e^{-2t} \Rightarrow \boxed{e^{-2t}\left(\frac{2e^2-3}{4}\right) + \frac{t}{2} - \frac{3}{4}}$$

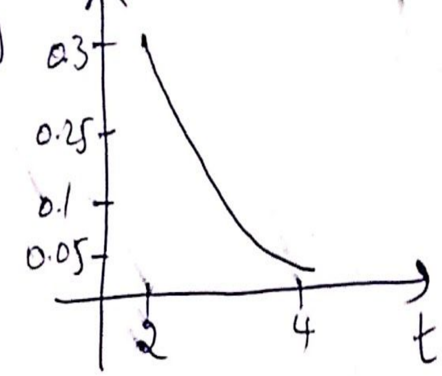
iii) for $2 \le t < 4$

$$y(t) = \int_0^1 e^{-2(t-\tau)}(\tau-1)d\tau + \int_1^2 e^{-2(t-\tau)}(\tau-1)d\tau$$

$$= e^{-2t}\left\{(\tau-1)\frac{e^{2\tau}}{2} - \frac{e^{2\tau}}{4}\right\}\Big|_0^1 + (\tau-1)\frac{e^{2\tau}}{2} - \frac{e^{2\tau}}{4}\Big|_1^2$$

$$= e^{-2t}\left(-\frac{e^2}{4} + \frac{3}{4}\right) + e^{-2t}\left(\frac{e^4}{2} - \frac{e^4}{4} + \frac{e^2}{4}\right)$$

$$= e^{-2t}\left(\frac{e^4}{4} + \frac{2e^2}{4} - \frac{3}{4}\right) = \frac{e^{-2t}}{4}\left\{e^4 + 2e^2 - 3\right\}$$



for t $4 \le t < 5$

$$= \int_{t-4}^{1} e^{-2(t-\tau)} (1-\tau)\, d\tau + \int_{1}^{2} e^{-2(t-\tau)} (\tau-1)\, d\tau$$

$$= -e^{-2t} \left\{ (\tau-1)\frac{e^{2\tau}}{2} - \frac{e^{2\tau}}{4} \right\} \Bigg|_{t-4}^{1} + e^{-2t} \left\{ (\tau-1)\frac{e^{2\tau}}{2} - \frac{e^{2\tau}}{4} \right\} \Bigg|_{1}^{2}$$

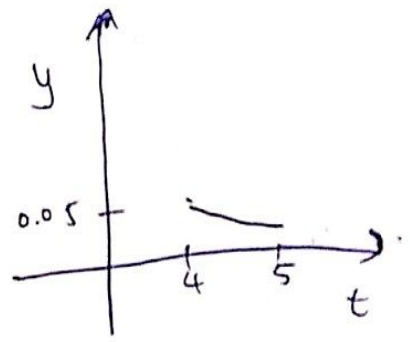
$$+ e^{-2t} \left\{ \frac{e^4}{4} + \frac{e^2}{4} \right\}$$

$$= e^{-2t} \left\{ -\frac{e^2}{2} - \left\{ (1+t+5) \frac{e^{2(t-4)}}{2} - \frac{e^{2(t-4)}}{4} \right\} \right\}$$

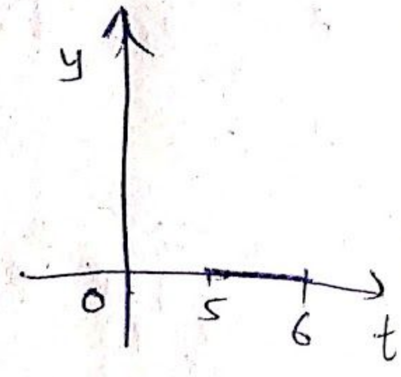
$$+ e^{-2t} \left\{ \frac{e^4}{4} + \frac{e^2}{4} \right\}.$$

$$= e^{-2t} \left\{ -\frac{e^2}{4} + \frac{(2t-11)}{4} e^{2t-8} + \frac{e^4}{2} \right\}.$$

$$= \frac{e^{4-2t}}{2} + \frac{e^{2-2t}}{4} - \frac{11}{4} e^{-8} + \frac{t}{2} e^{-8}.$$



$5 \le t < 6$



$$y(t) = \int_{t-4}^{2} e^{-2(t-\tau)} (\tau - 1)\, d\tau$$

$$= e^{-2t} \int_{t-4}^{2} e^{2\tau} (\tau - 1)\, d\tau$$

$$= e^{-2t} \int_{t-4}^{2} (\tau e^{2\tau} - e^{2\tau})\, d\tau$$

$$= e^{-2t} \left[\left[\frac{\tau e^{2\tau}}{2} \right]_{t-4}^{2} - \left[\frac{e^{2\tau}}{4} \right]_{t-4}^{2} - \left[\frac{e^{2\tau}}{2} \right]_{t-4}^{2} \right]$$

$$= e^{-2t} \left[\frac{2e^4 - (t-4)e^{2(t-4)}}{2} - \frac{e^4 + e^{2(t-4)}}{4} - \frac{e^4}{2} + \frac{e^{2(t-4)}}{2} \right.$$

$$= e^{-2t} \left[e^4 - \frac{t}{2} e^{2(t-4)} + 2e^{2(t-4)} - \frac{e^4}{4} + \frac{e^{2(t-4)}}{4} \right.$$

$$\left. - \frac{e^4}{2} + \frac{e^{2(t-4)}}{2} \right]$$

$$= e^{-2t} \left(\frac{e^4}{4} - \frac{t}{2} e^{2(t-4)} + \frac{11}{4} e^{2(t-4)} \right)$$

$-\frac{3}{4}e^4 + e^4$ $\quad$ $\frac{1}{4} + \frac{1}{2}$

$\frac{1+2}{4}$

$e^4(1 - \frac{3}{4})$ $= e^{-2t} \left(\frac{e^4}{4} - \frac{t e^{2t} \cdot e^{-8}}{2} + \frac{11}{4} e^{2t} \cdot e^{-8} \right)$ $\quad$ $\frac{3}{4}$ $+2$

$\frac{3+8}{4}$

$$= e^{-2t} \frac{e^4}{4} - 2t \frac{\cdot e^{-8}}{\cancel{2}4} + \frac{11}{4} e^{-8}$$

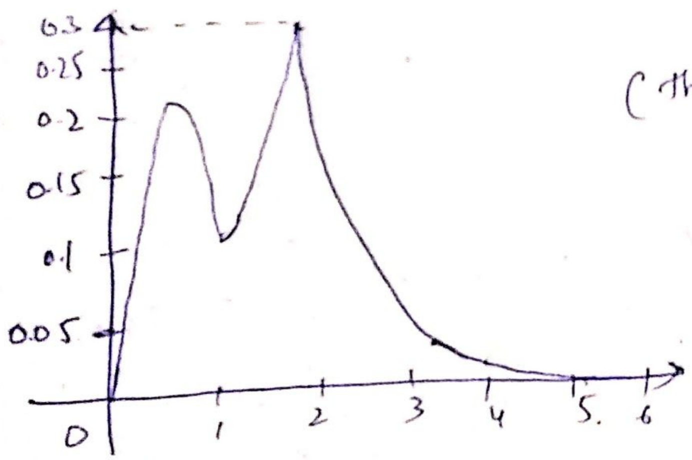
$$\boxed{y(t) = \frac{1}{4} \left(11 e^{-8} - 2t e^{-8} + e^{-2t} e^{4} \right)}$$

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$$

$$= \underbrace{y(t)}_{0 \le t < 1} + \underbrace{y(t)}_{1 \le t < 2} + \underbrace{y(t)}_{2 \le t < 4} + \underbrace{y(t)}_{4 \le t < 5} + \underbrace{y(t)}_{5 \le t < 6}$$

So, Graph of y(t) vs t (Summation of all such intervals).



(thearetically)

Hence, from the above observations it can be verified that both theoretically and practically(using code) we obtained the same results .

---------------------------THE END-----------------------------------