Lab Assignment – 05 – Spring 2020

Signal & Systems

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Summary:

- This assignment mainly dealt with finding the Fourier coefficients and reconstructing the original signal using the obtained Fourier coefficients.
- The main thing we learnt from this exercise was that we can create a signal from a given signal, which has discontinuities at certain points, by eliminating them.

1. In the fundamental interval the signal x1(t) is defined as

$x1(t) = (1 - |t/2|)\ (u\ (t + 1) - u\ (t - 1))$

2. In the fundamental interval the signal x2(t) is defined as

$x2(t) = t^2(u\ (t + 1) - u\ (t - 1))$

-> First, we'll write the code for finding the Fourier coefficients for both x1 & x2.

->Then we'll verify the real and imaginary components of coefficients of both x1 & x2 with that of exponential (Euler's form) one and the theoretical one.

->Finally, we'll reconstruct x1 & x2 using their coefficients.

Here's the code:

Editor - C:\Users\atharva deshpande\Documents\MATLAB\lab5.m

lab4one.m | lab4two.m | lab5.m | quiz1.m | even.m | odd.m

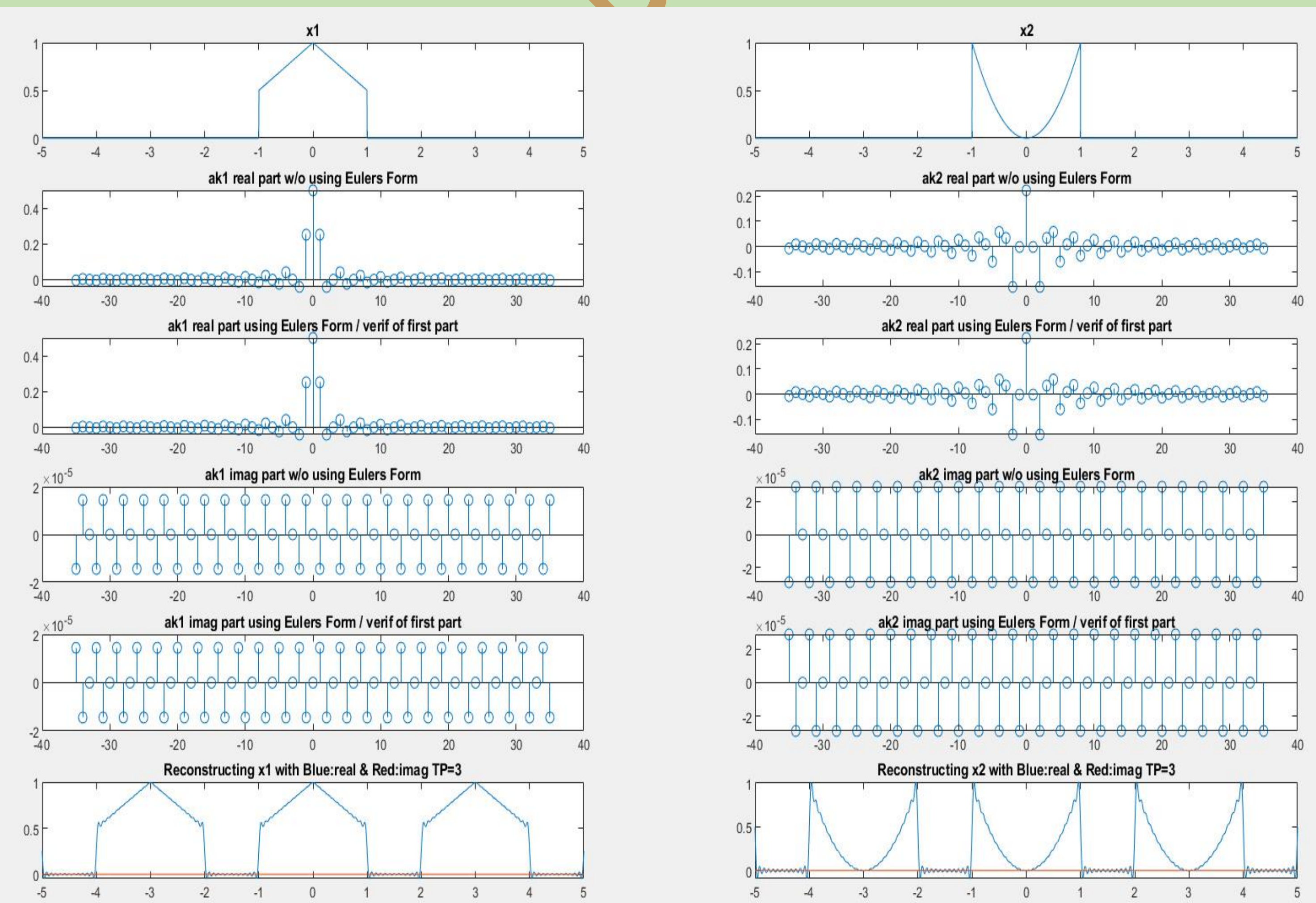
```
del=0.0001;m=5;t=-m:del:m;
x1=zeros(size(t));x1(t<-1 & t>=1)=0;x1(t>=-1 & t<1)=1-abs(t(t>=-1 & t<1)/2);
x2=zeros(size(t));x2(t<-1 & t>=1)=0;x2(t>=-1 & t<1)=t(t>=-1 & t<1).*t(t>=-1 & t<1);
subplot(6,2,1),plot(t,x1),title('x1');subplot(6,2,2),plot(t,x2),title('x2');
tp=3;w=2*pi/tp;%Given time period=3 %Here tp>2
new_t=-tp/2:del:tp/2;
x1_new=x1(((m-(tp/2))/del)+1:((m-(tp/2))/del)+1+(tp/del));
x2_new=x2(((m-(tp/2))/del)+1:((m-(tp/2))/del)+1+(tp/del));
av1=trapz(new_t,x1_new)/tp; %Calculating a0 for x1
av2=trapz(new_t,x2_new)/tp; %Calculating a0 for x2
numcoef=35;n=-numcoef:1:numcoef;ak1=zeros(size(n));ak1and2_verif=zeros(size(n));
for i=-numcoef:1:numcoef
    ak1(i+numcoef+1)=trapz(new_t,x1_new.*cos(w*i*new_t))/tp;
    ak1and2_verif(i+numcoef+1)=trapz(new_t,x1_new.*exp(-1i*i*w*new_t))/tp;
end
ak1(numcoef+1)=av1;%Accomodating a0 constructively in ak1 i.e real part of ak for x1
subplot(6,2,3);stem(n,ak1),ylim([-0.6,0.6]);title('ak1 real part w/o using Eulers Form');
subplot(6,2,5);stem(n,real(ak1and2_verif)),ylim([-0.6,0.6]);title('ak1 real part using Eulers Form / verif of first part');
ak2=zeros(size(n));
for i=-numcoef:1:numcoef
    ak2(i+numcoef+1)=trapz(new_t,x1_new.*sin(w*i*new_t))/tp;
end
ak2(numcoef+1)=0;%As sin(0)=0
subplot(6,2,7);stem(n,ak2),ylim([-0.6,0.6]);title('ak1 imag part w/o using Eulers Form');
subplot(6,2,9);stem(n,imag(ak1and2_verif)),ylim([-0.6,0.6]);title('ak1 imag part using Eulers Form / verif of first part');
ak3=zeros(size(n));ak3and4_verif=zeros(size(n));
```

Editor - C:\Users\atharva deshpande\Documents\MATLAB\lab5.m

lab4one.m | lab4two.m | lab5.m | quiz1.m | even.m | odd.m

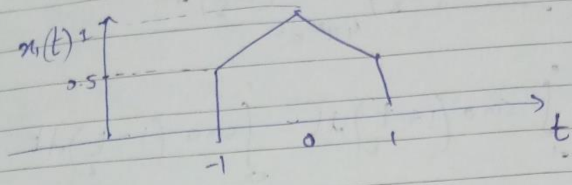
```matlab
for i=-numcoef:1:numcoef
    ak3(i+numcoef+1)=trapz(new_t,x2_new.*cos(w*i*new_t))/tp;
    ak3and4_verif(i+numcoef+1)=trapz(new_t,x2_new.*exp(-1i*i*w*new_t))/tp;
end
ak3(numcoef+1)=av2;%Accomodating a0 constructively in ak2 i.e real part of ak for x2
subplot(6,2,4);stem(n,ak3),ylim([-0.6,0.6]);title('ak2 real part w/o using Eulers Form');
subplot(6,2,6);stem(n,real(ak3and4_verif)),ylim([-0.6,0.6]);title('ak2 real part using Eulers Form / verif of first part');
ak4=zeros(size(n));
for i=-numcoef:1:numcoef
    ak4(i+numcoef+1)=trapz(new_t,x2_new.*sin(w*i*new_t))/tp;
end
ak4(numcoef+1)=0;%As sin(0)=0

subplot(6,2,8);stem(n,ak4),ylim([-0.6,0.6]);title('ak2 imag part w/o using Eulers Form');
subplot(6,2,10);stem(n,real(ak3and4_verif)),ylim([-0.6,0.6]);title('ak2 imag part using Eulers Form / verif of first part');
%Reconstructing the original signal
x_1=zeros((size(t)));
for i=-numcoef:1:numcoef
    x_1=x_1+ak1(i+numcoef+1).*exp(1i*i*w*t);
end
subplot(6,2,11),plot(t,real(x_1));hold('on');plot(t,imag(x_1));hold('off');title('Reconstructing x1 with Blue:real & Red:imag
x_2=zeros((size(t)));
for i=-numcoef:1:numcoef
    x_2=x_2+ak3(i+numcoef+1).*exp(1i*i*w*t);
end
subplot(6,2,12),plot(t,real(x_2));hold('on');plot(t,imag(x_2));hold('off');title('Reconstructing x2 with Blue:real & Red:imag
```

Here’s the output:



Theoretical part:

$\underline{x1}$



$$x1(t) = \begin{cases} 1 - \frac{t}{2} & 0 < t < 1 \\ 0 & \text{otherwise} \\ 1 + \frac{t}{2} & -1 < t < 0 \end{cases}$$

$$a_k = \int_{T_0} x(t)\, e^{-jk\omega t}\, dt \quad \ldots [T_0 = 3]$$

$$= \int_{-1.5}^{1.5} x(t)\, e^{-jk\omega t}\, dt$$

$$= \cancel{\int_{-1.5}^{-1}} \to 0 + \int_{-1}^{0} + \int_{0}^{1} + \cancel{\int_{1}^{1.5}} \to 0$$

$$= \int_{-1}^{0} \left(1+\frac{t}{2}\right)(\cos\theta - i\sin\theta)\, dt + \int_{0}^{1} \left(1-\frac{t}{2}\right)(\cos\theta - i\sin\theta)\, dt$$

$$\ldots \left[\text{Where } \theta = \frac{2\pi k t}{3}\right]$$

$$= \int_{-1}^{0} \cos\theta \left(1+\frac{t}{2}\right) dt - i\int_{-1}^{0} \sin\theta \left(1+\frac{t}{2}\right) dt + \int_{0}^{1}\left(1-\frac{t}{2}\right)\cos\theta\, dt - i\int_{0}^{1} \sin\theta \left(1-\frac{t}{2}\right) dt$$

$$Re = \int_{-1}^{0} \cos\theta \left(1+\frac{t}{2}\right) dt + \int_{0}^{1}\left(1-\frac{t}{2}\right)\cos\theta\, dt$$

$$= \int_{-1}^{0} \cos\left(\frac{2\pi k t}{3}\right)\left(1+\frac{t}{2}\right) dt + \int_{0}^{1} \cos\left(\frac{2\pi k t}{3}\right)\left(1-\frac{t}{2}\right) dt$$

$$= \frac{6\pi k \sin\left(\frac{2\pi k}{3}\right) + 9 - 9\cos\left(\frac{2\pi k}{3}\right) + 6\pi k \sin\left(\frac{2\pi k}{3}\right) + 9 - 9\cos\left(\frac{2\pi k}{3}\right)}{8\pi^2 k^2}$$

$$\therefore \boxed{Re(a_k) = \frac{6\pi k \sin\left(\frac{2\pi k}{3}\right) - 9\cos\left(\frac{2\pi k}{3}\right) + 9}{4\pi^2 k^2}}$$

$$Img = \int_{-1}^{0} \sin\theta\left(1 + \frac{t}{2}\right) dt + \int_{-1}^{0} (\sin\theta)\left(1 - \frac{t}{2}\right) dt$$

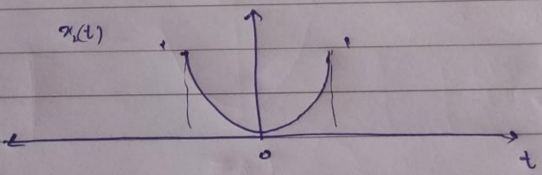
$$= 2\left[\frac{9\sin\left(\frac{2\pi k}{3}\right) + 6\pi k \cos\left(\frac{2\pi k}{3}\right) - 12\pi k}{8\pi^2 k^2}\right]$$

$$\boxed{Img = \frac{9\sin\left(\frac{2\pi k}{3}\right) + 6\pi k \cos\left(\frac{2\pi k}{3}\right) - 12\pi k}{4\pi^2 k^2}}$$

$\forall\ k \neq 0$

for $k = 0$, $Img = 0$

x_2



$$x_2(t) = \begin{cases} t^2 & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$a_k = \int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt \qquad \because [T_0 = 3]$$

$$= \int_{-1.5}^{1.5} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \cancel{\int_{-1.5}^{-1}}^{\,0} + \int_{-1}^{0} + \int_{0}^{1} + \cancel{\int_{1}^{1.5}}^{\,0}$$

$$= \int_{-1}^{1} t^2(\cos\theta - i\sin\theta)\, dt$$

$$\left[\theta = \frac{2\pi k t}{3}\right]$$

$$Re = \int_{-1}^{1} t^2 \cos\theta \, dt$$

$$= \int_{-1}^{1} t^2 \cos\left(\frac{2\pi k t}{3}\right) dt$$

$$= \frac{(6\pi^2 k^2 - 27)\sin\left(\frac{2\pi k}{3}\right) + 18\pi k \cos\left(\frac{2\pi k}{3}\right)}{2\pi^3 k^3}$$

$$\therefore \boxed{Re = \frac{(6\pi^2 k^2 - 27)\sin\left(\frac{2\pi k}{3}\right) + 18\pi k \cos\left(\frac{2\pi k}{3}\right)}{2\pi^3 k^3}}$$

$$Img = \int_{-1}^{1} t^2 \sin\left(\frac{2\pi k t}{3}\right) dt$$

$$= 2\left[\frac{18\pi k \sin\left(\frac{2\pi k}{3}\right) + (27 - 6\pi^2 k^2)\cos\left(\frac{2\pi k}{3}\right) - 27}{4\pi^3 k^3}\right]$$

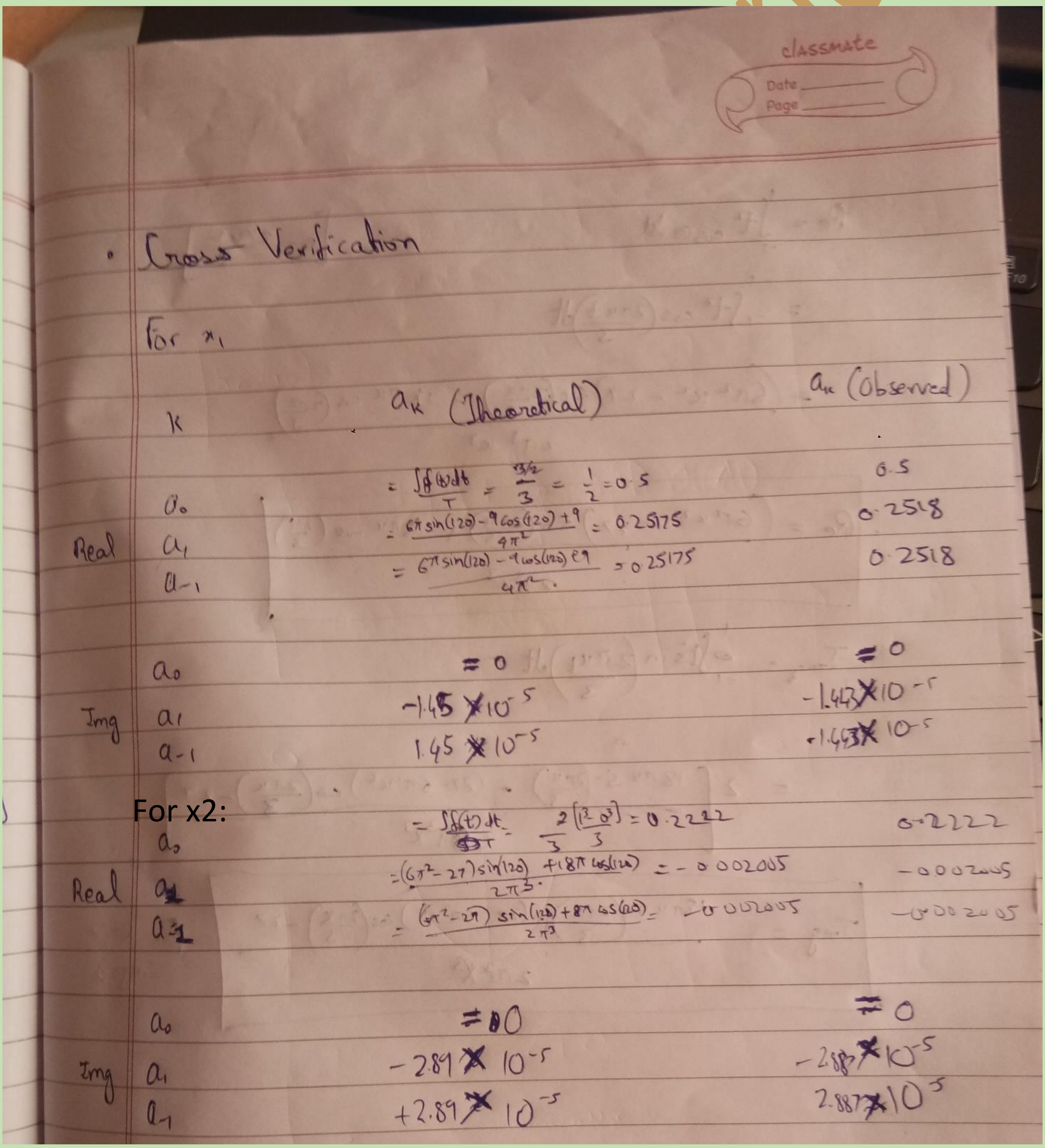
$$\boxed{Img = \frac{18\pi k \sin\left(\frac{2\pi k}{3}\right) + (27 - 6\pi^2 k^2)\cos\left(\frac{2\pi k}{3}\right) - 27}{2\pi^3 k^3}}$$

For $K=0$

$Img = 0$

Observation :

- We derived the expression to find the real and imaginary components of the Fourier coefficients for both x1 & x2.
- We found out that the values obtained on paper matched with those we got from our output. Here's the proof:



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• Cross Verification

For x_1

	k	a_k (Theoretical)	a_k (Observed)
Real	a_0	$= \frac{\int f(t)\,dt}{T} = \frac{3/2}{3} = \frac{1}{2} = 0.5$	0.5
	a_1	$= \frac{6\pi\sin(12\theta) - 9\cos(12\theta) + 9}{4\pi^2} = 0.25175$	0.2518
	a_{-1}	$= \frac{6\pi\sin(12\theta) - 9\cos(12\theta) + 9}{4\pi^2} = 0.25175$	0.2518
Img	a_0	$= 0$	$= 0$
	a_1	-1.45×10^{-5}	-1.443×10^{-5}
	a_{-1}	1.45×10^{-5}	$+1.443 \times 10^{-5}$

For x2:

	k	a_k (Theoretical)	a_k (Observed)
Real	a_0	$= \frac{\int f(t)\,dt}{T} = \frac{2}{3}\frac{[1^3 - 0^3]}{3} = 0.2222$	0.2222
	a_1	$= \frac{(6\pi^2 - 27)\sin(12\theta) + 18\pi\cos(12\theta)}{2\pi^3} = -0.002005$	-0.002005
	a_{-1}	$= \frac{(6\pi^2 - 27)\sin(12\theta) + 8\pi\cos(12\theta)}{2\pi^3} = -0.002005$	-0.002005
Img	a_0	$= 0$	$= 0$
	a_1	-2.89×10^{-5}	-2.88×10^{-5}
	a_{-1}	$+2.89 \times 10^{-5}$	2.887×10^{-5}

Convergence:

- To analyse the convergence of the reconstructed signal with that of the original signal we subtracted the values of original signal from the real values of the reconstructed signal i.e. we ignored the imaginary components from the reconstructed signal. Let's name this observation as 'subconvg'.
  Here's the appended code snippet:

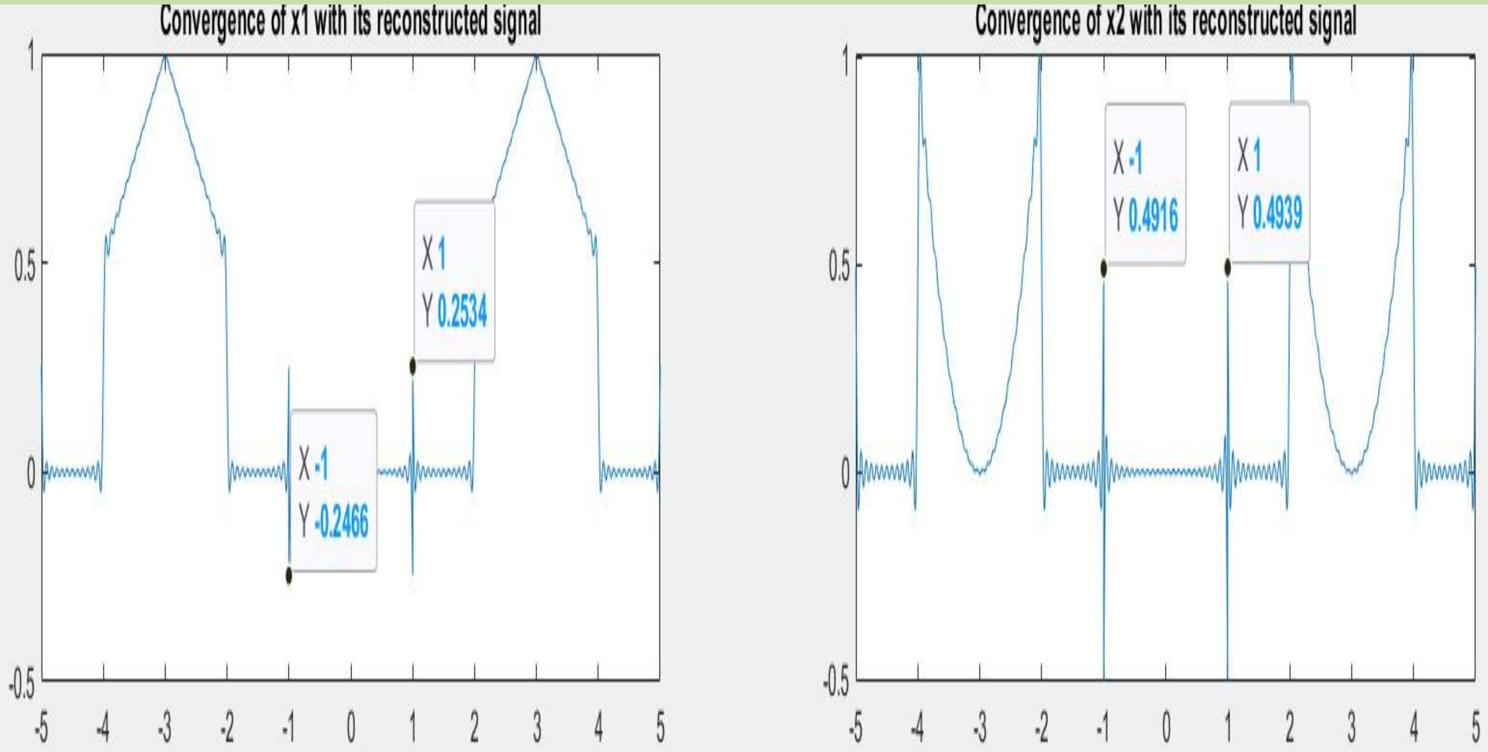
```
subplot(6,2,[7,9]);plot(t,(x_1)-x1);title('Convergence of x1 with its reconstructed signal');
subplot(6,2,[8,10]);plot(t,(x_2)-x2);title('Convergence of x2 with its reconstructed signal');
```

Command Window

- Basically, we have to look out for the critical points where we have discontinuities in the original signal.
- One of our main mottos to reconstruct the signal was to eliminate the discontinuous points. So that's the major thing we need to be concerned about the reconstructed signal.
- In short, if the reconstructed signal comes out to be same as that of the original signal the critical point should show 'zero' at its respective position. If not then we have

achieved our task provided the value is much closer to zero.

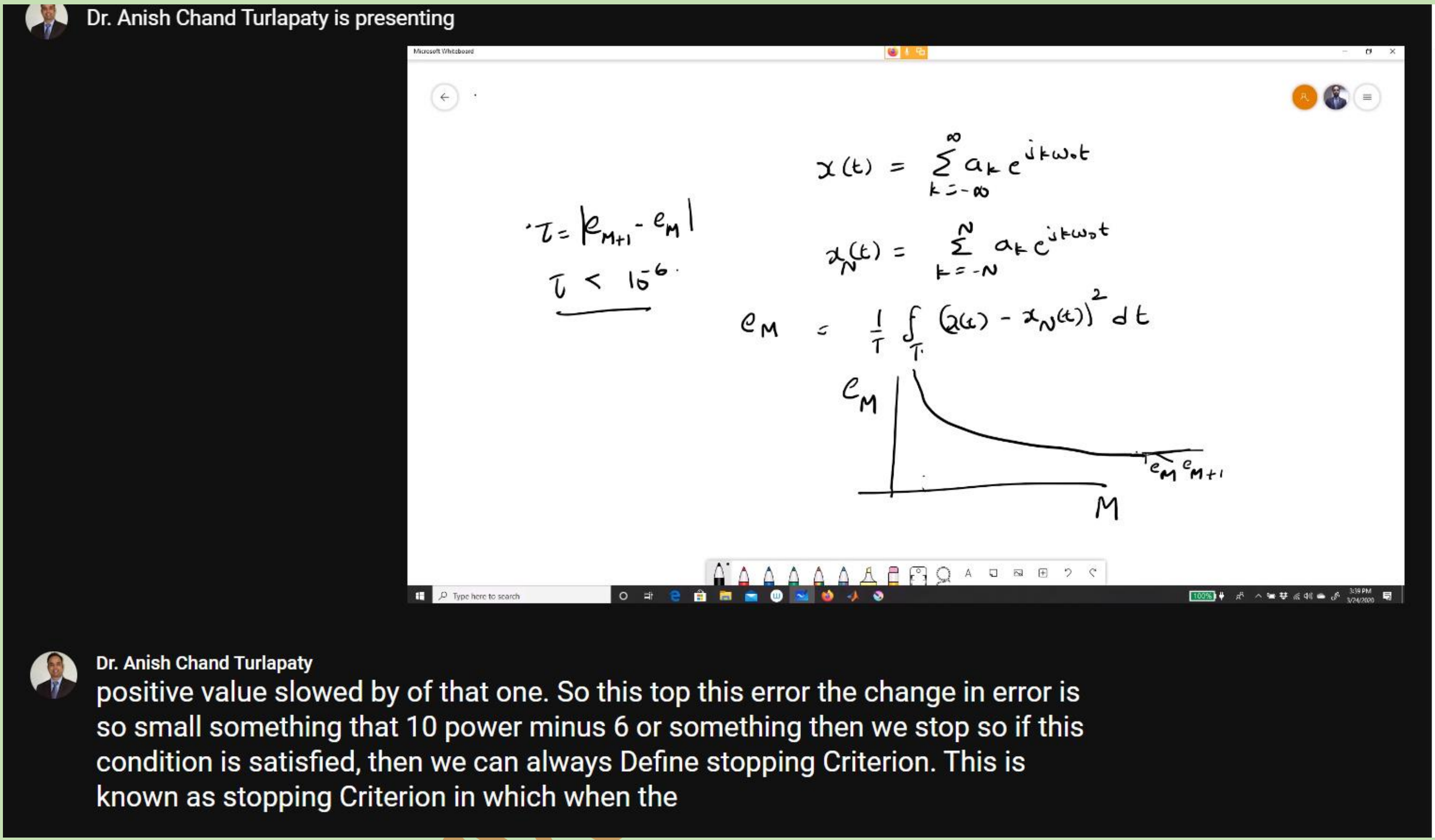
- After running the code here's what we have got:



- Now the precision of these critical values can be increased by using subsequently large number of coefficients but that would kill much run time of the code. Still, given a powerful processor we can achieve our task meticulously.
- Given these values we can conclude that since we didn't come across any discontinuity especially at the critical points, all we can say is that both x1 and x2 are convergent.

-> Then the best method to find the convergence of the original and reconstructed signal is given here

(Look for tau<10^-6):



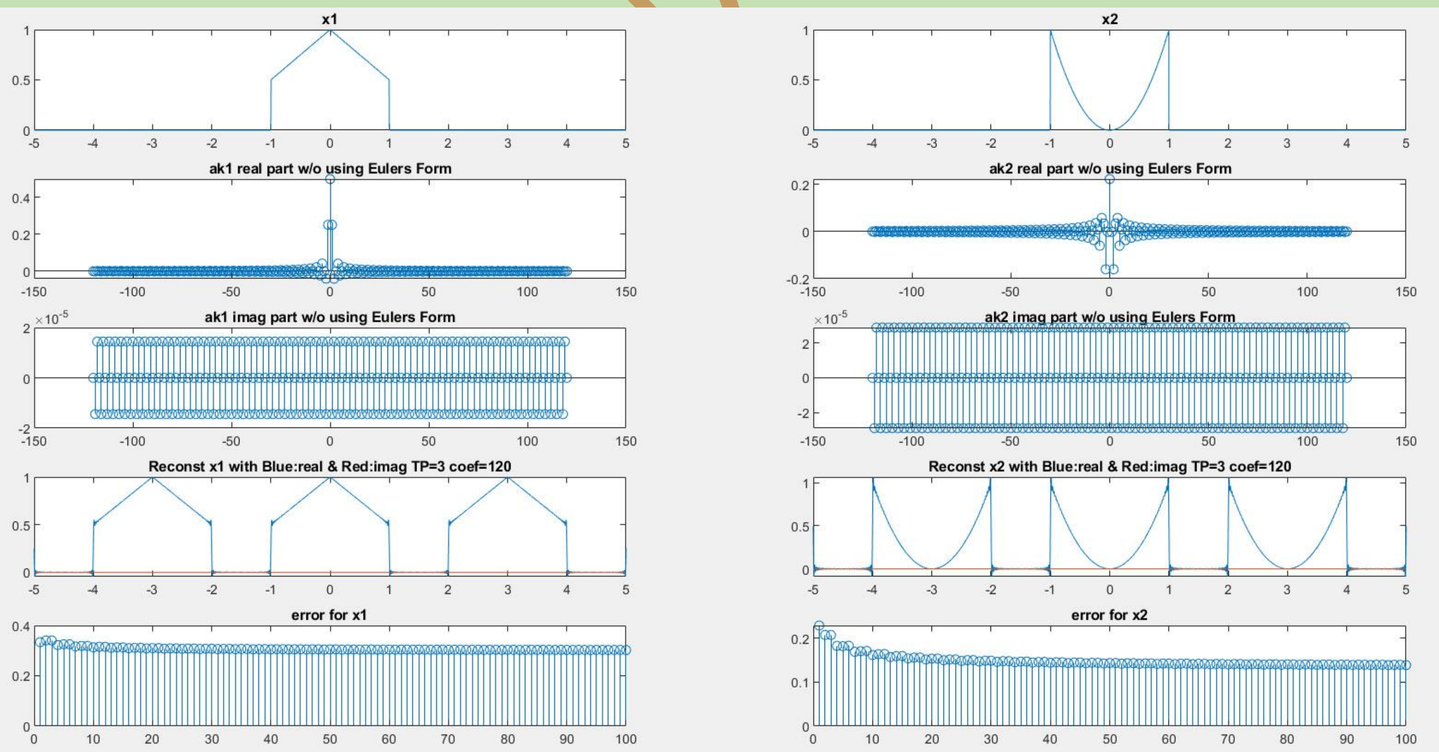
So basically, we find the error for each value of number of coefficients till a particularly large value like 100. Here's the code appended to the previous one to demonstrate the convergence:

Editor - C:\Users\atharva deshpande\Documents\MATLAB\lab5.m

lab4one.m | lab4two.m | lab5.m | quiz1.m | even.m | odd.m

```
%Demonstrating Convergence for x1 & x2
M=100;
mx=1:M;err_1=zeros(size(mx));err_2=zeros(size(mx));
for k=1:M
numcoef=k;n=-numcoef:1:numcoef;akrc1=zeros(size(n));akrc2=zeros(size(n));
for i=-numcoef:1:numcoef
    akrc1(i+numcoef+1)=trapz(new_t,x1_new.*exp(-1i*i*w*new_t))/tp;
end
akrc1(numcoef+1)=av1;
xrc_1=zeros(size(t));
for i=-numcoef:1:numcoef
    xrc_1=xrc_1+akrc1(i+numcoef+1).*exp(1i*i*w*t);
end
err_1(k)=mean((abs(x1-xrc_1)).^2);
for i=-numcoef:1:numcoef
    akrc2(i+numcoef+1)=trapz(new_t,x2_new.*exp(-1i*i*w*new_t))/tp;
end
akrc2(numcoef+1)=av2;
xrc_2=zeros(size(t));
for i=-numcoef:1:numcoef
    xrc_2=xrc_2+akrc2(i+numcoef+1).*exp(1i*i*w*t);
end
err_2(k)=mean((abs(x2-xrc_2)).^2);
end
subplot(5,2,9);stem(mx,err_1);title('error for x1');
subplot(5,2,10);stem(mx,err_2);title('error for x2');
%Let's take two random errors for x1 & x2
reconerr_1=abs(err_1(M)-err_1(M-1));disp('reconerr_1=');disp(reconerr_1);
reconerr_2=abs(err_2(M)-err_2(M-1));disp('reconerr_2=');disp(reconerr_2);
```

Here's the output:



Command Window

New to MATLAB? See resources for Getting Started.

```
reconerr_1=
   1.4906e-06

reconerr_2=
   5.9849e-06
```

Three points to be considered:

1. We want to know how accurate our reconstructed signal is with respect to the original signal. As we want to observe the phenomenon for different number of coefficients, all we do is apply a for loop on the variable numcoef which decides the number of coefficients.
2. As we can see in the first picture, ideally the error must fall significantly as we increase the number of coefficients. Accordingly, our output is showing the same graph for both x1 & x2.
3. Next, we choose to find the difference between two consecutive errors for large number of coefficients (e(M+1)-e(M), M=99). As clarified in the notes (first picture), tau<10^-6 and we have achieved that as our variables reconerr_1 and reconerr_2 depict tau1 & tau2 respectively. Hence, it is proved that the reconstructed signals for both x1 & x2 are convergent.