M3 ODE Assignment :-

Ordinary Differential Equations

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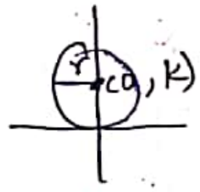
(1) Find the differential eq for the family of circles having their centre on the y-axis. Find the order and the degree of the eq formed. Is it linear and homogeneous differential equation ?



Sol:- Given eq is

$$(x-a)^2+(y-b)^2=r^2$$

put $a=0$, $b=k$ in eq [as centre on y axis]

Diff on both sides.

$$x^2+(y-k)^2=r^2$$

$$\cancel{2}x+\cancel{2}(y-k)y'=0$$

$$k=\frac{x}{y'}+y$$

Again diff,

$$0=\frac{y'(1)-xy''}{(y')^2}+y'$$

Order = 2

Degree = 1

$$y'-xy''+(y')^3=0$$

$$\boxed{xy''-(y')^3-y'=0}$$

So, clearly it is homogeneous eq. Eq is not linear as it involves $(y')^3$

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2) Reduce the following eq to the lebinitz form and solve.

Sol:- Given that

$$\tan y \frac{dy}{dx} = \cos^2 x \cos y - \tan x.$$

Devide by $\cos y$ on both sides.

$$\frac{\tan y}{\cos y} \frac{dy}{dx} = \frac{\cos^2 \cos y}{\cos y} - \frac{\tan x}{\cos y}$$

$$\sec y \tan y \frac{dy}{dx} + \sec y \tan x = \cos^2 x$$

Let $\sec y = t$

$$\sec y \tan y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + \tan x (t) = \cos^2 x$$

It is of form $\frac{dy}{dx} + Py = Q$.

$$IF = e^{\int P dx}.$$

So, here $IF = e^{\int \tan x dx} = e^{\log \sec x} = \sec x.$

General Sol is $y(IF) = \int Q \, IF dx.$

Here, $t(\sec x) = \int \cos^2 x (\sec x) dx$

$$t \sec x = \int \cos x \, dx + c$$

$$\downarrow$$

$$\sec y \sec x = \sin x + c$$

$\therefore$ General sol is $\boxed{\sec y \sec x = \sin x + c}$

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3) Solve the following equation by regrouping.

Sol: Given that

$$y(2xy + e^x)\,dx = e^x dy$$

Let us write it in $Mdx + Ndy = 0$ form

So, $(2xy^2 + ye^x)\,dx - e^x dy = 0$.

$M = 2xy^2 + ye^x \qquad N = -e^x$

$\frac{\delta M}{\delta y} = 2x(2y) + e^x(1) \qquad \frac{\delta N}{\delta x} = -e^x$

$\frac{\delta M}{\delta y} = 4xy + e^x \qquad \frac{\delta N}{\delta x} = -e^x$

As, $\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$, it is an inexact eq

Solving the inexact eq by Regrouping,

$$2xy^2 + ye^x dx = e^x dy$$

$$2xy^2 = e^x dy - ye^x dx$$

$$-2x = \frac{ye^x dx - e^x dy}{y^2}$$

$$-2x = D\left(\frac{e^x}{y}\right)$$

Integrating, $-2\int x\,dx = \int D\left(\frac{e^x}{y}\right)$

$$-\frac{\cancel{2}x^2}{\cancel{2}} = \frac{e^x}{y} + c$$

$$\therefore \boxed{-x^2 = \frac{e^x}{y} + c}$$

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4) Find the solution of following equation

$$(xy^3+y)\,dx + 2(x^2y^2+x+y^4)\,dy = 0$$

Sol:- Comparing with $M\,dx + N\,dy = 0$.

$M = xy^3+y \qquad N = 2(x^2y^2+x+y^4)$

$\frac{\delta M}{\delta y} = x(3y^2)+1 \qquad \frac{\delta N}{\delta x} = 2(2xy^2+1)$

$\frac{\delta M}{\delta y} = 3xy^2+1 \qquad \frac{\delta N}{\delta x} = 4xy^2+2$

$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x} \rightarrow$ Not an exact DE

Case (iv)

If $\dfrac{\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}}{M}$ is a f^n of y alone or const

then $IF = e^{\int f(y)\,dy}$

So, $\dfrac{\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}}{M} = \dfrac{4xy^2+2-(3xy^2+1)}{xy^3+y}$

$= \dfrac{xy^2+1}{y(xy^2+1)} = \frac{1}{y} = f(y)$

$IF = e^{\int f(y)\,dy} = e^{\int \frac{1}{y}\,dy} = e^{\ln y} = y$

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Multiplying IF with DE.

$y(xy^3+y)\,dx + 2y(x^2y^2+x+y^4)\,dy = 0$

$(xy^4+y^2)\,dx + (2x^2y^3+2xy+2y^5)\,dy = 0$

Now, $\frac{\partial M}{\partial y} = x(4y^3)+2y$

$\frac{\partial N}{\partial x} = 2y^3(2x)+2y$

$\frac{\partial M}{\partial y} = 4xy^3+2y$ $\frac{\partial N}{\partial x} = 4xy^3+2y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ exact equation.

So, $\underset{y\ const}{\int M\,dx} + \underset{no\ x\ term}{\int N\,dy} + C$

$\int (xy^4+y^2)\,dx + \int 2y^5\,dy = C$

$\frac{x^2y^4}{2} + xy^2 + \frac{2y^6}{6} = C$

$\therefore$ Eq is $\boxed{\frac{x^2y^4}{2} + xy^2 + \frac{y^6}{3} = C}$

5) Solve the following equation

$(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$

Sol: Comparing with $M\,dx + N\,dy = 0$

$M = xy^2\sin xy + y\cos xy$

$N = x^2y\sin xy - x\cos xy$

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$$\frac{\delta M}{\delta y} = x(y^2 \cos xy (x) + \sin xy (2y)) + y(-\sin xy)(x) + \cos xy (1)$$

$$\frac{\delta M}{\delta y} = x^2y^2\cos xy + 2xy\sin xy - xy\sin xy + \cos xy$$

$$\frac{\delta N}{\delta x} = y(x^2\cos xy(y) + \sin xy(2x)) - [x(-\sin xy)y + \cos xy(1)]$$

$$= x^2y^2\cos xy + 2xy\sin xy + xy\sin xy + \cos xy$$

$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$, it is an inexact eq.

Case (iii): Eq of type $f(xy)\,y\,dx + g(xy)\,x\,dy = 0$.

If, then $IF = \frac{1}{Mx - Ny}$

$$= \frac{1}{(xy^2\sin xy + y\cos xy)x - (x^2y\sin xy - x\cos xy)y}$$

$$= \frac{1}{x^2y^2\sin xy + xy\cos xy - x^2y^2\sin xy + xy\cos xy}$$

$$= \frac{1}{2xy\cos xy}$$

Multiply IF with DE.

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$$\frac{1}{2xy\cos xy}\left[(xy\sin xy + \cos xy)y\,dx + (xy\sin xy - \cos xy)x\,dy\right] = 0$$

$$\frac{(xy\sin xy + \cos xy)}{2x\cos xy} + \frac{(xy\sin xy - \cos xy)}{2y\cos xy}$$

$$= \left(\frac{y}{2}\tan xy + \frac{1}{2x}\right]dx + \left(\frac{x}{2}\tan xy - \frac{1}{2y}\right)dy = 0$$

$$M = \frac{y}{2}\tan xy + \frac{1}{2x} \qquad N = \frac{x}{2}\tan xy - \frac{1}{2y}$$

$$\frac{\delta M}{\delta y} = \frac{1}{2}\left(y\sec^2(xy)\,x + \tan xy\right)$$

$$\frac{\delta N}{\delta x} = \frac{1}{2}\left(x\sec^2(xy)\,y + \tan xy\right)$$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$, it is an exact eq.

$$\Rightarrow \underset{y\text{ const}}{\int M\,dx} + \underset{\text{free of } x}{\int N\,dy} = c$$

$$\Rightarrow \int\left(\frac{y}{2}\tan xy + \frac{1}{2x}\right)dx + \int\frac{-1}{2y}\,dy = c$$

$$\boxed{\because \int \tan ax\,dx = \frac{-1}{a}\ln|\cos(ax)| + c}$$

$$\Rightarrow (-1)\frac{y}{2}\cdot\left(\frac{1}{y}\right)\ln|\cos(xy)| + \frac{1}{2}\ln x - \frac{1}{2}\ln y = \ln c$$

$$= -\frac{1}{2}\ln|\cos xy| + \frac{1}{2}\ln x - \frac{1}{2}\ln y = 2\ln c$$

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$\ln|\cos xy|^{-1} + \ln\left(\frac{x}{y}\right) = \ln c_1^2$

$\ln\left|\sec xy \cdot \frac{x}{y}\right| = \ln c_1^2$

$x \sec xy = c_1^2 y$

$x \sec xy = cy.$

$\therefore$ Sol of DE = $\boxed{x \sec xy = cy}$.

6) Find the general solution of the following equation

Sol :- Given that.

$$y^2 \log y = xyp + p^2 \quad \text{where } p = \frac{dy}{dx}.$$

It is a first order, higher degree eq.

$$x = \frac{y^2 \log y - p^2}{py}$$

$$x = \frac{y \log y}{p} - \frac{p}{y}$$

It cannot be factorised.

So, the above eq is solvable for x

$$\frac{1}{p} \Leftarrow \frac{dx}{dy} = \frac{1}{p}\left(y \times \frac{1}{y} + \log y\right) + y \log y\left(-\frac{1}{p^2}\right)\frac{dp}{dy} - \left[p\left(-\frac{1}{y^2}\right) + \frac{1}{y}\frac{dp}{dy}\right]$$

$$\frac{1}{p} = \frac{1}{p}(1+\log y) - \frac{y\log y}{p^2}\frac{dp}{dy} + \frac{p}{y^2} - \frac{1}{y}\frac{dp}{dy}$$

$$\frac{1}{p} - \frac{1}{p} = \frac{\log y}{p} - \frac{y\log y}{p^2}\frac{dp}{dy} + \frac{p}{y^2} - \frac{1}{y}\frac{dp}{dy}$$

$$0 = \frac{\log y}{p} + \frac{p}{y^2} - \left(\frac{y\log y}{p^2} + \frac{1}{y}\right)\frac{dp}{dy}$$

$$\left(\frac{\log y}{p} + \frac{p}{y^2}\right) = \frac{y}{p}\left(\frac{\log y}{p} + \frac{p}{y^2}\right)\frac{dp}{dy}$$

$$\frac{dp}{dy} = \frac{p}{y}$$

$$\int \frac{dp}{p} = \int \frac{dy}{y}$$

$$\ln p = \ln y + \ln c_1$$

$$\ln\left(\frac{p}{y}\right) = c_1$$

$$p = ye^{c_1}$$

$$\boxed{p = cy}$$

Substituting in original eq

$y = p^2 x$ $\quad y^2\log y = xyp + p^2$

$$y^2\log y = xy(cy) + c^2y^2$$

$$y^2\log y = xcy^2 + c^2y^2$$

$$\log y = xc + c^2$$

$\therefore$ General solution of the differential eq

is $\boxed{\log y = xc + c^2}$

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7) Solve the following first order higher degree equation

Sol: Given that

$$4p^3 + 3xp = y \quad \text{where } p = \frac{dy}{dx}$$

It is a first order higher degree eq.

It cannot be factorized

$$y = 4p^3 + 3xp$$

So, eq is solvable for y.

$$\frac{dy}{dx} = 4(3p^2)\frac{dp}{dx} + 3\left(x\frac{dp}{dx} + p\right)$$

$$p = 12p^2\frac{dp}{dx} + 3x\frac{dp}{dx} + 3p$$

$$(12p^2 + 3x)\frac{dp}{dx} = -2p$$

$$\frac{\overset{6}{\cancel{12}}p^2}{-\cancel{2}p} + \frac{3x}{-2p} = \frac{dx}{dp}$$

$$-6p - \frac{3}{2}\frac{x}{p} = \frac{dx}{dp}$$

$$\frac{dx}{dp} + 6p = -\frac{3}{2}\frac{x}{p}$$

$$\frac{dx}{dp} + \frac{3}{2}\frac{x}{p} = -6p$$

It is a lebinitz equation.

$$IF = e^{\int \frac{3}{2}\frac{dp}{p}} = e^{\frac{3}{2}\ln p} = p^{3/2}.$$

$$x\,p^{3/2} = -\int 6p \cdot p^{3/2}\,dp + c$$

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$$x p^{3/2} = -6\int p^{5/2}\, dp + c$$

$$x p^{3/2} = (2) -6\frac{p^{7/2}}{7} + c$$

$$x p^{3/2} = \frac{-12}{7} p^{7/2} + c$$

$$x = \frac{-12}{7}\frac{p^{7/2}}{p^{3/2}} + \frac{c}{p^{3/2}}$$

$$\boxed{x = \frac{-12}{7} p^2 + \frac{c}{p^{3/2}}}$$

$$y = 4p^3 + 3xp$$

$$y = 4p^3 + 3p\left(\frac{-12}{7}p^2 + \frac{c}{p^{3/2}}\right).$$

$$\boxed{y = -\frac{8p^3}{7} + \frac{3c}{p^{1/2}}}$$

Sol x and y constitutes towards general sol^n.

8). Solve the given eq.

Sol:

$$xy\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}\left(3x^2 - 2y^2\right) - 6xy = 0$$

Devide with xy on both sides

$$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}\left(\frac{3x}{y} - \frac{2y}{x}\right) - 6 = 0$$

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$\Rightarrow (c\cdot p^2) + p\left(\frac{3x}{y}\right) - p\left(\frac{2y}{x}\right) - 6 = 0.$

$\left[\because \frac{dy}{dx} = p\right]$

$\Rightarrow p^2 + p\left(\frac{3x}{y}\right) - p\left(\frac{2y}{x}\right) - 6 = 0$

$p\left(p + \frac{3x}{y}\right) - 2\left(\frac{py}{x} + 3\right) = 0$

$p\left(p + \frac{3x}{y}\right) - \frac{2y}{x}\left(p + \frac{3x}{y}\right) = 0$

$p = \frac{2y}{x}, \quad p = -\frac{3x}{y}$

$\frac{dy}{dx} = \frac{2y}{x}, \quad \frac{dy}{dx} = -\frac{3x}{y}$

Integrating,

$\int \frac{dy}{2y} = \int \frac{dx}{x}, \quad \int y\,dy = -\int 3x\,dx$

$\frac{1}{2}\ln y = 2\ln x + \ln c \qquad \frac{y^2}{2} = -\frac{3x^2}{2} + c_1$

$\ln y - \ln x^2 = \ln c \qquad y^2 = -3x^2 + 2c_1$

$\ln\left(\frac{y}{x^2}\right) = \ln c \qquad \boxed{y^2 = -3x^2 + c}$

$\boxed{y = cx^2}$

$\therefore$ Solutions of diff eq are $y = cx^2$,

$y^2 = -3x^2 + c$

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q) Find the singular solution and the general solution of following eq reducing to clairauts form using the substitution

$x^2 = u$ and $y^2 = v$

Soln Given

$$xyp^2 - (x^2+y^2+1)p + xy = 0 \qquad p = \frac{dy}{dx}$$

Let $x^2 = u$, $y^2 = v$

$\therefore$ $2x\,dx = du$ $\qquad$ $2y\,dy = dv$

$dx = \frac{du}{2x}$ $\qquad$ $dy = \frac{dv}{2y}$

$$\frac{dy}{dx} = \frac{dv}{du}\frac{x}{y}$$

Let $q = \frac{dv}{du}$

$$p = \frac{x}{y}q$$

Substituting $p = \frac{x}{y}q$ in given eq.

$$xyp^2 - (x^2+y^2+1)p + xy = 0$$

$$xy\frac{x^2}{y^2}q^2 - (x^2+y^2+1)q\frac{x}{y} + xy = 0.$$

Multiply by y/x on both sides

$$x^2q^2 - (x^2+y^2+1)q + y^2 = 0$$

$$uq^2 - (u+v+1)q + v = 0.$$

$$-q + uq(q-1) = vq - v$$

$$-q + uq(q-1) = v(q-1)$$

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$$-q + uq(q-1) = v(q-1)$$

$$-q = v(q-1) - uq(q-1)$$

$$-q = (v-uq)(q-1)$$

$$\frac{-q}{q-1} + uq = v$$

$$v = uq - \frac{q}{q-1}$$

this eq is in the form of $y = p(x) + f(p)$

which is Clairauts eq.

So, general solution is $q = c$

$$\boxed{v = uc - \frac{c}{c-1}}$$

Singular sol^n

$$v = uc - \frac{c}{c-1}$$

Diff w.r.to c

$$0 = u - \left(-\frac{1}{(c-1)^2}\right)$$

$$u + \left(\frac{1}{(c-1)^2}\right) = 0$$

$$u = -\frac{1}{(c-1)^2} \quad [\because u = x^2]$$

$$x = \frac{\pm i}{c-1}$$

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$$xc - x = \pm i$$

$$xc = \pm i + x$$

$$c = \frac{\pm i + x}{x}$$

$$\boxed{c = \frac{\pm i}{x} + 1}$$

Substitute c. in given eq., $v = y^2$, $u = x^2$

$$v = uc - \frac{c}{(c-1)}$$

$$y^2 = \left(1 \pm \frac{i}{x}\right) x^2 - \frac{1 \pm i/x}{1 \pm \frac{i}{x} - 1}$$

$\therefore$ General sol $\Rightarrow$ $y^2 = cx^2 - \frac{c}{c-1}$

Singular sol $\Rightarrow$ $y^2 = \left(1 \pm \frac{i}{x}\right) x^2 - \frac{1 \pm i/x}{1 \pm i/x - 1}$

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10) Find the solution of.

$$\left(\frac{d^2y}{dx^2} + \frac{dy}{dx} + y\right)^2 \left(\frac{dy}{dx} - 2y\right)^3 = 0$$

Sol:- $(D^2y + Dy + y)^2 (Dy - 2y)^3 = 0$

$$((D^2 + D + 1)y)^2 ((D-2)y)^3 = 0$$

$D = 2, 2, 2.$

$D^2 + D + 1 = 0$

$a = 1,\ b = 1,\ c = 1$

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$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

So, roots are $2, 2, 2, \frac{-1 \pm \sqrt{3}i}{2}$

y_{cf_1} = For Roots are 2,2,2

Roots are repeating

So, $y_{cf_1} = (c_1 + c_2x + c_3x^2)e^{mx}$

$y_{cf_1} = (c_1 + c_2x + c_3x^2)e^{2x}$

y_{cf_2} = For Roots $\frac{-1 + \sqrt{3}i}{2}, \frac{-1 - \sqrt{3}i}{2}$

repeating 2 times.

So, $y_{cf_2} = e^{\alpha x}[(c_4 + c_5x)\cos\beta x + (c_6 + c_7x)\sin\beta x]$

$y_{cf_2} = e^{-\frac{1}{2}x}[(c_4 + c_5x)\cos\frac{\sqrt{3}}{2}x + (c_6 + c_7x)\sin\frac{\sqrt{3}}{2}x]$

$\therefore$ $y_{cf} = y_{cf_1} + y_{cf_2}$

$\therefore$ $y_{cf} = (c_1 + c_2x + c_3x^2)e^{2x}$

$+ e^{-\frac{x}{2}}[(c_4 + c_5x)\cos\frac{\sqrt{3}}{2}x + (c_6 + c_7x)\sin\frac{\sqrt{3}}{2}x]$

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11) Solve

$$(D^2-1)y = \sin^2 x + (1+x^2)e^x$$

Sol:

$y_{complete} = y_{complementary} + y_{particular}$

For, $y_{complementary}$ $(D^2-1)y = 0$

Roots are $1, -1$

$$\boxed{y_{cf} = c_1 e^x + c_2 e^{-x}}$$

For y_{P1}:

$$y_{P1} = \frac{1}{(D^2-1)}\left[\sin^2 x + (1+x^2)e^x\right]$$

$$y_{P1} = \frac{\sin^2 x}{(D^2-1)} + \frac{(1+x^2)e^x}{(D^2-1)}$$

$$y_{P1} = \frac{1+\cos 2x}{2(D^2-1)} + \frac{e^x}{D^2-1} + \frac{x^2 e^x}{D^2-1}$$

$$y_{P1} = \underset{(1)}{\frac{1}{2(D^2-1)}} + \underset{(2)}{\frac{\cos 2x}{2(D^2-1)}} + \underset{(3)}{\frac{e^x}{D^2-1}} + \underset{(4)}{\frac{x^2 e^x}{D^2-1}}$$

(1) $y_{P1} = \frac{1}{2}\left[e^x \int \frac{1}{2} e^{-x}\,dx - e^{-x}\int \frac{1}{2} e^x\,dx\right]$

$= \frac{1}{2}\left[\frac{-1}{2} - \frac{1}{2}\right] = \frac{1}{2}[-1] = \frac{-1}{2}.$

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(2) $y_{P1} = \frac{1}{2(D^2-1)}[\cos 2x]$

$= \frac{\cos 2x}{2(-4-1)}$ $a=2$, $D^2=-a^2=-4$

$= \frac{\cos 2x}{2(-5)} = \frac{\cos 2x}{-10}$ //

(3) $y_{P1} = \frac{e^x}{D^2-1}$

$y_{P1} = \frac{1}{(D-1)}\left[\frac{e^x}{(D+1)}\right]$ $a=1$

$= \frac{1}{(D-1)}\left[\frac{e^x}{(1+1)}\right]$

$= \frac{1}{2(D-1)}[e^x]$

$a=1$

$P(D) = D-1$

$P(a) = 1-1 = 0.$

$P'(a) = 1$

$P'(a) = 1.$

So, $P(a)=0$ and $P'(a) \neq 0$

then $\frac{x e^{ax}}{P'(a)}$

$y_{P1} = \frac{xe^x}{2(P'(1))} = \frac{xe^x}{2(1)} = \frac{xe^x}{2}.$

(4) $\frac{x^2e^x}{(D^2-1)}$

As it is form $e^{ax}V(x)$ $\boxed{y_{P1} = e^{ax}\frac{1}{P(D+a)} \cdot V(x)}$

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$$So,\ y_{P1} = \frac{e^x\, x^2}{((D+1)^2-1)} \ [a=1] = e^x \frac{1(x^2)}{D(D+2)}$$

$$= \frac{e^x}{2}\left[\frac{1}{D}\left(1+\frac{D}{2}\right)^{-1}(x^2)\right]$$

$$= \frac{e^x}{2}\left[\frac{1}{D}\left[1-\frac{D}{2}+\frac{D^2}{4}-\frac{D^3}{8}\right]x^2\right]$$

$$= \frac{e^x}{2}\left[\left(\frac{1}{D}-\frac{1}{2}+\frac{D}{4}-\frac{D^2}{8}\right)x^2\right]$$

$$\therefore 4^{th}\ term = \frac{e^x}{2}\left[\frac{x^3}{3}-\frac{x^2}{2}+\frac{x}{2}-\frac{1}{4}\right]$$

$$y_{comp} = y_{cf} + y_{P1}$$

$$= c_1 e^x + c_2 e^{-x} + \frac{-1}{2} + \frac{1}{10}\cos 2x + \frac{xe^x}{2}$$

$$+ \frac{e^x}{2}\left[\frac{x^3}{3}-\frac{x^2}{2}+\frac{x}{2}-\frac{1}{4}\right]$$

12) Find the solution of the following

$$(D^2-2D+1)y = xe^x \sin x.$$

Sol: Given

$$(D^2-2D+1)y = xe^x \sin x$$

For y_{cf}, we take $(D^2-2D+1)y = 0$

Roots are $1, 1$.

$$\boxed{y_{cf} = (c_1+c_2x)e^x}$$

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$$y_{P_1} = \frac{1}{(D-1)^2} \cdot x e^x \sin x$$

$$= e^x \cdot \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x$$

$$\left[\because \text{ If } Q(x) = e^{ax} V(x) \quad y_{P_1} = e^{ax} \frac{1}{P(D+a)} V(x) \right]$$

$$= e^x \frac{1}{D^2 + \cancel{1} + \cancel{2D} - \cancel{2D} - \cancel{2} + \cancel{1}} x \sin x$$

$$= e^x \frac{1}{D^2} x \sin x$$

$$\left[\text{Now,} \quad Q(x) = x V(x) \right.$$

$$\left. y_{P_I} = \left[x - \frac{P'(D)}{P(D)} \right] \cdot \frac{V(x)}{P(D)} \right].$$

$$\text{So,} \quad = e^x \left[x \frac{1}{D^2} \sin x - \cancel{e^x} \frac{2D \sin x}{(D^2)^2} \right]$$

$$= e^x \left[x \iint \sin x - \frac{2 \cancel{e^x} \cos x}{(D^2)^2} \right]$$

$a = 1$

$D^2 = -a^2$

$D^2 = -1$

$$= e^x \left[x \int \cos x - \frac{2 \cancel{e^x} \cos x}{(-1)^2} \right]$$

$$= e^x \left[-x \sin x - 2 \cancel{e^x} \cos x \right]$$

$$= -e^x \left[x \sin x + 2 \cos x \right]$$

$$\boxed{y_{compl} = (c_1 + c_2 x) e^x - e^x (x \sin x + 2 \cos x)}$$

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13) Solve the following

$$(D^2+6D+9)y = \frac{e^{3x}}{x^2}$$

Sol :- Given eq is

$$(D^2+6D+9)y = \frac{e^{3x}}{x^2}$$

$$(D+3)^2 y = \frac{e^{3x}}{x^2}.$$

For complementary eq, $(D+3)^2 y = 0$.

$-3, -3$

$$y_{cf} = (c_1+c_2x)e^{mx}$$

$$\therefore \boxed{y_{cf} = (c_1+c_2x)e^{-3x}}$$

For particular integral.

$$y_{p1} = \frac{1}{(D+3)^2}\frac{e^{3x}}{x^2}$$

$$\left[y_{p1} = \frac{1}{P(D)} e^{ax} V(x) = e^{ax}\frac{1}{P(D+a)} V(x) \right]$$

$a = 3$

$$y_{p1} = e^{3x}\frac{1}{(D+3+3)^2} x^{-2}$$

$$y_1 = e^{-3x} \qquad y_2 = xe^{-3x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x}+xe^{-3x} \end{vmatrix} = e^{-6x}$$

$$y_{p1} = -y_1\int\frac{y_2 Q(x)}{W}dx + y_2\int\frac{y_1 Q(x)}{W}dx$$

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$$y_{P1} = xe^{-3x}\int \frac{e^{-3x}}{e^{-6x}}\frac{e^{3x}}{x^2}dx - e^{-3x}\int \frac{xe^{-3x}e^{3x}}{e^{-6x}x^2}dx$$

$$y_{P1} = xe^{-3x}\int \frac{e^{6x}}{x^2}dx - e^{-3x}\int \frac{xe^{6x}}{x^2}dx.$$

$$y_{P1} = e^{-3x}\left[x\int \frac{e^{6x}}{x^2} - \int \frac{xe^{6x}}{x^2}dx\right]$$

$$-\int \frac{e^{6x}}{6x}dx - \frac{1}{6}\left[\ln|6x| + \sum_{n=1}^{\infty}\frac{(6x)^n}{n.n!}\right].$$

$$y_{P1} = -e^{-3x}\left[\ln|6x| + \sum_{n=1}^{\infty}\frac{(6x)^n}{nn!}\right] + \frac{e^{3x}}{6x} + \frac{xe^{-3x}}{6}\left[\ln|6x| + \sum_{n=1}^{\infty}\frac{(6x)^n}{nn!}\right].$$

$$\therefore y_{cs} = (c_1 + c_2x)e^{-3x} + \frac{e^{3x}}{6x} + \left(\frac{xe^{-3x}}{6} - e^{-3x}\right)\left[\ln|6x| + \sum_{n=1}^{\infty}\frac{(6x)^n}{n.n!}\right]$$

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14) Solve the following

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y = \log x$$

Sol)

Given that

$$(x^2D^2+7xD+13)y = \log x, \quad D=\frac{d}{dx}$$

this is cauchy's Euler diff eq.

So, $x^2D^2 = D(D-1)y$

$xD = Dy$, $e^t = x$, $t=\log x$

Substituting,

$$D(D-1)y + 7Dy + 13y = t$$

$$(D^2-D+7D+13)y = t$$

$$(D^2+6D+13)y = t \Rightarrow \text{Reduced to const co-eff}$$

$$y_{cf} = (D^2+6D+13)y = 0$$

$a=1, b=6, c=0$

$$= \frac{-6 \pm \sqrt{36-4(13)}}{2(1)} = \frac{-6 \pm \sqrt{16}}{2} = \frac{-6 \pm 4i}{2}$$

$$= -3 \pm 2i$$

Let $D_1 = \frac{d}{dt}$.

$$y_{cf} = e^{-3\log x}\left[c_1\cos(2\log x) + c_2\sin(2\log x)\right]$$

$$= x^{-3}\left[c_1\cos(2\log x) + c_2\sin(2\log x)\right]$$

$$y_{PI} = \frac{1}{(D_1^2+6D+13)}t = \frac{1}{13\left(\frac{D_1^2}{13}+\frac{6D_1}{13}+1\right)}t$$

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$$= \frac{1}{13}\left[\frac{D_1^2}{13} + \frac{6D_1}{13} + 1\right]^{-1} t$$

$$= \frac{1}{13}\left[1 + \left(\frac{D_1^2}{13} + \frac{6D_1}{13}\right)\right]^{-1} t$$

$$= \frac{1}{13}\left[1 - \left(\frac{D_1^2}{13} + \frac{6D_1}{13}\right)\right] t$$

$$= \frac{1}{13}\left[1 - \frac{6D_1}{13}\right] t$$

$$= \frac{1}{13}\left[t - \frac{6}{13} D_1 t\right]$$

$$= \frac{1}{13}\left[\log x - \frac{6}{13}\right]$$

$$\boxed{y_{PI} = \frac{1}{169}(13\log x - 6)}$$

$$y_{comp} = y_{cf} + y_{PI}$$

$$\therefore y_{comp} = x^{-3}\left[c_1 \cos(2\log x) + c_2 \sin(2\log x)\right] + \frac{1}{169}(13\log x - 6).$$

15) Slope of any point (x,y) of a curve is given by $1 + \frac{y}{x}$. If curve passes through $(1,2)$. Find particular sol^n.

Sol^t Given

$$\text{slope} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{y}{x}.$$

To solve this,

This is a homogeneous eq.

So, $y = vx$.

As $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x}$$

$$v + x\frac{dv}{dx} = 1 + v$$

$$x\frac{dv}{dx} = 1$$

$$x\,dv = dx$$

$$\int dv = \int \frac{dx}{x}$$

$$v = \ln x + c$$

General sol $\boxed{\frac{y}{x} = \ln x + c}$

as the curve passes through $(1,2)$,

$$\frac{2}{1} = \ln(1) + c$$

$$\boxed{c = 2}$$

$\therefore$ Particular sol $= \boxed{\frac{y}{x} = \ln x + 2}$