Relational Model

Dr. Odelu Vanga
Indian Institute of Information Technology Sri City
http://www.iiits.ac.in/people/regular-faculty/dr-odelu-vanga/

Additional Operations

- Set intersection
- Natural join
- Division

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \textbf{ and } t \in s \}$
- Assume:
  - r, s have the *same arity*
  - attributes of r and s are compatible
- Note: $r \cap s = r - (r - s)$

Set-Intersection Operation - Example

- Relation r, s:

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

r ∩ s

A	B
α	2

r

A	B
α	1
α	2
β	3

s

A	B
α	2
β	3

r ∩ s

A	B
α	2
β	3

Natural-Join Operation

- Notation: $r \bowtie s$
- Let r and s be relations on schemas R and S respectively.
- The result is a relation on schema $R \cup S$ which is obtained by considering each pair of tuples t_r from r and t_s from s.
- If t_r and t_s have the same value on each of the attributes in $R \cap S$, a tuple t is added to the result, where
  - t has the same value as t_r on r
  - t has the same value as t_s on s
- Example:

  $R = (A, B, C, D)$

  $S = (E, B, D)$

  Result schema $= (A, B, C, D, E)$

- $r \bowtie s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E} (\sigma_{r.B = s.B \ \ r.D = s.D} (r \ \text{x} \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B\ \ r.D\, =\, s.D}\,(r\ \ \mathrm{x}\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B\quad r.D\, =\, s.D}\,(r \text{ x } s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B \quad r.D\, =\, s.D}\,(r \text{ x } s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \ r.D\ =\ s.D}\ (r\ \ x\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B\quad r.D\, =\, s.D}\,(r \;\text{x}\; s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B \;\; r.D\, =\, s.D}\,(r \;\; \text{x} \;\; s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B \quad r.D\, =\, s.D}\,(r \ \mathrm{x} \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\left(\sigma_{r.B = s.B \quad r.D = s.D}\left(r \times s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B\,=\,s.B\ \ r.D\,=\,s.D}\left(r\ \mathrm{x}\ s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B = s.B \quad r.D = s.D}\,(r \times s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\left(\sigma_{r.B = s.B \quad r.D = s.D}\left(r \ \mathrm{x}\ s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B = s.B \;\; r.D = s.D}\,(r \text{ x } s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B\ =\ s.B\ \ r.D\ =\ s.D}\ (r \text{ x } s)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \ r.D\ =\ s.D}\ (r\ \text{x}\ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\quad r.D\ =\ s.D}\ (r\ \text{x}\ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B \quad r.D\, =\, s.D}\,(r \text{ x } s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B\,=\,s.B\quad r.D\,=\,s.D}\left(r\ \mathrm{x}\ s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B\,=\,s.B\quad r.D\,=\,s.D}\,(r\ \mathrm{x}\ s)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\left(\sigma_{r.B = s.B \quad r.D = s.D}\left(r \text{ x } s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\,=\,s.B\ \ r.D\,=\,s.D}\ (r\ \ \mathrm{x}\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \ r.D\ =\ s.D}\ (r\ \text{x}\ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B\,=\,s.B\quad r.D\,=\,s.D}\left(r \ \mathrm{x}\ s\right)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\left(\sigma_{r.B = s.B\ \ r.D = s.D}\,(r \text{ x } s)\right)$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r $\quad s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\,=\,s.B\quad r.D\,=\,s.D}\ (r\ \ \mathrm{x}\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	*1*	α	a
β	*2*	γ	a
γ	*4*	β	b
α	*1*	γ	a
δ	*2*	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	*1*	α	a	α
α	*1*	α	a	γ
α	*1*	γ	a	α
α	*1*	γ	a	γ
δ	*2*	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \ r.D\ =\ s.D}\ (r\ \ \text{x}\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- $r \quad s$ is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E} (\sigma_{r.B = s.B \quad r.D = s.D} (r \times s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\, =\, s.B \quad r.D\, =\, s.D}\,(r \;\text{x}\; s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r*   *s* is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\left(\sigma_{r.B\, =\, s.B \quad r.D\, =\, s.D}\,(r \;\text{x}\; s)\right)$$

Natural Join Operation – Example

- Relations r, s:

r

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

s

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\quad r.D\ =\ s.D}\ (r\ \text{x}\ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- r s is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\,(\sigma_{r.B\,=\,s.B\quad r.D\,=\,s.D}\,(r\ \mathrm{x}\ s))$$

Natural Join Operation – Example

- Relations r, s:

A	*B*	*C*	*D*
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	*D*	*E*
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	*B*	*C*	*D*	*E*
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

- *r* *s* is defined as:

$$\Pi_{r.A,\, r.B,\, r.C,\, r.D,\, s.E}\,(\sigma_{r.B\,=\,s.B \;\; r.D\,=\,s.D}\,(r \;\; \text{x} \;\; s))$$

Natural Join - Example Queries

Find all customers who have an account from at least the "Downtown" and the "Uptown" branches.

- Query :

$$\Pi_{\mathrm{CN}}(\sigma_{BN=\text{“Downtown”}}(depositor \bowtie account)) \cap$$

$$\Pi_{CN}(\sigma_{BN=\text{“Uptown”}}(depositor \bowtie account))$$

where *CN* denotes customer-name and *BN* denotes *branch-name*.

Division Operation

- Suited to queries that include the phrase "for all".
- Let *r* and *s* be relations on schemas R and S respectively where
  - $R = (A_1, \ldots, A_m, B_1, \ldots, B_n)$
  - $S = (B_1, \ldots, B_n)$

  The result of r ÷ s is a relation on schema

  $R - S = (A_1, \ldots, A_m)$

$$r \div s = \{\, t \mid t \in \Pi_{R\text{-}S}(r) \wedge \forall\, u \in s\, (\, tu \in r\,)\,\}$$

Division Operation – Example

Relations *r*, *s*:

r:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

s:

B
1
2

$$r \div s = \{ t \mid t \in \prod_{R-S}(r) \land \forall u \in s \, (tu \in r) \}$$

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α

(α) (α) (1) (2) $(\alpha, 1)$ $(\alpha, 2)$

$$r \div s = \{\, t \mid t \in \Pi_{R-S}(r) \wedge \forall\, u \in s\, (tu \in r)\,\}$$

Complete the process ?

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α

(β) (β) (1) (2) $(\beta, 1)$ $(\beta, 2)$

$$r \div s = \{\, t \mid t \in \Pi_{R\text{-}S}(r) \land \forall\, u \in s\ (tu \in r)\,\}$$

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α
β

(β) (β) (1) (2) $(\beta, 1)$ $(\beta, 2)$

$$r \div s = \{ t \mid t \in \Pi_{R\text{-}S}(r) \wedge \forall u \in s \, (tu \in r) \}$$

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α
β

(γ) (γ) (1) (2) $\cancel{(\gamma, 1)}$ $\cancel{(\gamma, 2)}$

$$r \div s = \{\, t \mid t \in \prod_{R\text{-}S}(r) \wedge \forall\, u \in s\ (\, tu \in r\,)\,\}$$

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α
β

(1) (2) $(\delta, 1)$ $(\delta, 2)$

(δ) (δ)

$$r \div s = \{\, t \mid t \in \Pi_{R\text{-}S}(r) \wedge \forall u \in s\ (tu \in r)\,\}$$

Division Operation – Example

Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α
β

$(\in)$ $(\in)$

(1)
(2)

$(\in, 1)$
$(\in, 2)$

$$r \div s = \{\, t \mid t \in \Pi_{R\text{-}S}(r) \wedge \forall\, u \in s\ (\, tu \in r\,)\,\}$$

Division Operation – Example

Relations *r, s*:

A	*B*
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$R = \{A, B\}$

$S = \{B\}$

$R\text{-}S = \{A\}$

$r \div s$:

A
α
β

Result

$$r \div s = \{ t \mid t \in \prod_{R\text{-}S}(r) \wedge \forall\, u \in s\ (tu \in r) \}$$

Another Division Example

Relations *r*, *s*:

A	*B*	*C*	*D*	*E*
α	a	α	a	*1*
α	a	γ	a	*1*
α	a	γ	b	*1*
β	a	γ	a	*1*
β	a	γ	b	*3*
γ	a	γ	a	*1*
γ	a	γ	b	*1*
γ	a	β	b	*1*

r

D	*E*
a	*1*
b	*1*

s

$r \div s$: **?**

$$r \div s = \{\, t \mid t \in \Pi_{R-S}(r) \wedge \forall\, u \in s\, (tu \in r)\, \}$$

Another Division Example

Relations *r, s*:

A	*B*	*C*	*D*	*E*
α	a	α	a	*1*
α	a	γ	a	*1*
α	a	γ	b	*1*
β	a	γ	a	*1*
β	a	γ	b	*3*
γ	a	γ	a	*1*
γ	a	γ	b	*1*
γ	a	β	b	*1*

r

D	*E*
a	*1*
b	*1*

s

$r \div s$:

A	*B*	*C*
α	a	γ
γ	a	γ

$$r \div s = \{\, t \mid t \in \Pi_{R-S}(r) \land \forall\, u \in s\ (tu \in r)\,\}$$

Example Queries

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.
  - Query 1

$$\Pi_{\mathrm{CN}}(\sigma_{BN=\text{“Downtown”}}(depositor \bowtie account)) \cap$$

$$\Pi_{CN}(\sigma_{BN=\text{“Uptown”}}(depositor \bowtie account))$$

where *CN* denotes customer-name and *BN* denotes *branch-name*.

  - Query 2

$$\Pi_{customer\text{-}name,\ branch\text{-}name}\ (depositor \bowtie account)$$

$$\div\ \rho_{temp(branch\text{-}name)}\ (\{(\text{“Downtown”}), (\text{“Uptown”})\})$$

Example Queries

Find all customers who have an account at all branches located in Brooklyn city.

$$\Pi_{customer\text{-}name,\, branch\text{-}name}(depositor \bowtie account)$$
$$\div \Pi_{branch\text{-}name}(\sigma_{branch\text{-}city = \text{"Brooklyn"}}(branch))$$

THANK YOU