Machine Learning

Neural Network

Indian Institute of Information Technology
Sri City, Chittoor

Previous Class

A linear function: $f(x_i, W) = Wx_i$

Loss: $L = \underbrace{\frac{1}{N}\sum_i L_i}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$ $\qquad R(W) = \sum_k \sum_l W_{k,l}^2$

SVM Loss:
Hinge Loss
Max-margin loss

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Softmax Loss:
Cross-entropy loss

$$L_i = -log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Today’s class

Optimization

Gradient Descent & Back propagation

Perceptron

Update rule

Neural networks

Optimization

Optimization is the process of finding the set of parameters ***W*** that minimize the loss function.

Source: http://cs231n.github.io

Optimization

Optimization is the process of finding the set of parameters ***W*** that minimize the loss function.

Strategy #1:First very bad idea solution: Random search:

Simply try out many different random weights and keep track of what works best.

Source: http://cs231n.github.io

Optimization

Optimization is the process of finding the set of parameters ***W*** that minimize the loss function.

Strategy #1:First very bad idea solution: Random search:

Simply try out many different random weights and keep track of what works best.

Strategy #2: Random local search:

Start out with a random ***W***, generate random changes ***δW*** to it and if the loss at the changed ***W+δW*** is lower, we will perform an update.

Source: http://cs231n.github.io

Optimization

Optimization is the process of finding the set of parameters ***W*** that minimize the loss function.

Strategy #1:First very bad idea solution: Random search:

Simply try out many different random weights and keep track of what works best.

Strategy #2: Random local search:

Start out with a random ***W***, generate random changes ***δW*** to it and if the loss at the changed ***W+δW*** is lower, we will perform an update.

Strategy #3: Following the gradients:

There is no need to randomly search for a good direction: this direction is related to the **gradient** of the loss function.

Source: http://cs231n.github.io

Gradient Descent

The procedure of repeatedly evaluating the **gradient of loss function** and then performing a **parameter update**.

Source: http://cs231n.github.io

Gradient Descent

The procedure of repeatedly evaluating the **gradient of loss function** and then performing a **parameter update**.

Vanilla (Original) Gradient Descent:

```
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Source: http://cs231n.github.io

Gradient Descent

The procedure of repeatedly evaluating the **gradient of loss function** and then performing a **parameter update**.

Vanilla (Original) Gradient Descent:

```
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

***Mini-batch Gradient Descent* (MGD):**

```
while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Source: http://cs231n.github.io

Gradient Descent

The procedure of repeatedly evaluating the **gradient of loss function** and then performing a **parameter update**.

Vanilla (Original) Gradient Descent:

```
while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Mini-batch Gradient Descent (MGD):

```
while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Stochastic Gradient Descent (SGD):

Special case of MGD when mini-batch contains only a single example

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = \qquad \frac{\partial f}{\partial y} =$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \to \qquad \frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = \qquad \frac{\partial f}{\partial y} =$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \to \qquad \frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

$$f(x,y) = xy \qquad \to \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$f(x,y) = \max(x,y) \qquad \to \qquad \frac{\partial f}{\partial x} = \qquad \frac{\partial f}{\partial y} =$$

Source: http://cs231n.github.io

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x,y) = x + y \qquad \to \qquad \frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

$$f(x,y) = xy \qquad \to \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$f(x,y) = \max(x,y) \qquad \to \qquad \frac{\partial f}{\partial x} = 1(x >= y) \qquad \frac{\partial f}{\partial y} = 1(y >= x)$$

Source: http://cs231n.github.io

Compound expressions with chain rule

$$f(x,y,z) = (x+y)z$$

Source: http://cs231n.github.io

Compound expressions with chain rule

$$f(x, y, z) = (x + y)z$$

$$\Downarrow$$

$q = x + y$ and $f = qz$

Source: http://cs231n.github.io

Compound expressions with chain rule

$$f(x, y, z) = (x + y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Source: http://cs231n.github.io

Compound expressions with chain rule

$$f(x, y, z) = (x + y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Chain rule: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$

Source: http://cs231n.github.io

Compound expressions with chain rule

$$f(x, y, z) = (x + y)z$$

$$q = x + y \text{ and } f = qz$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$

Source: http://cs231n.github.io

Compound expressions with chain rule

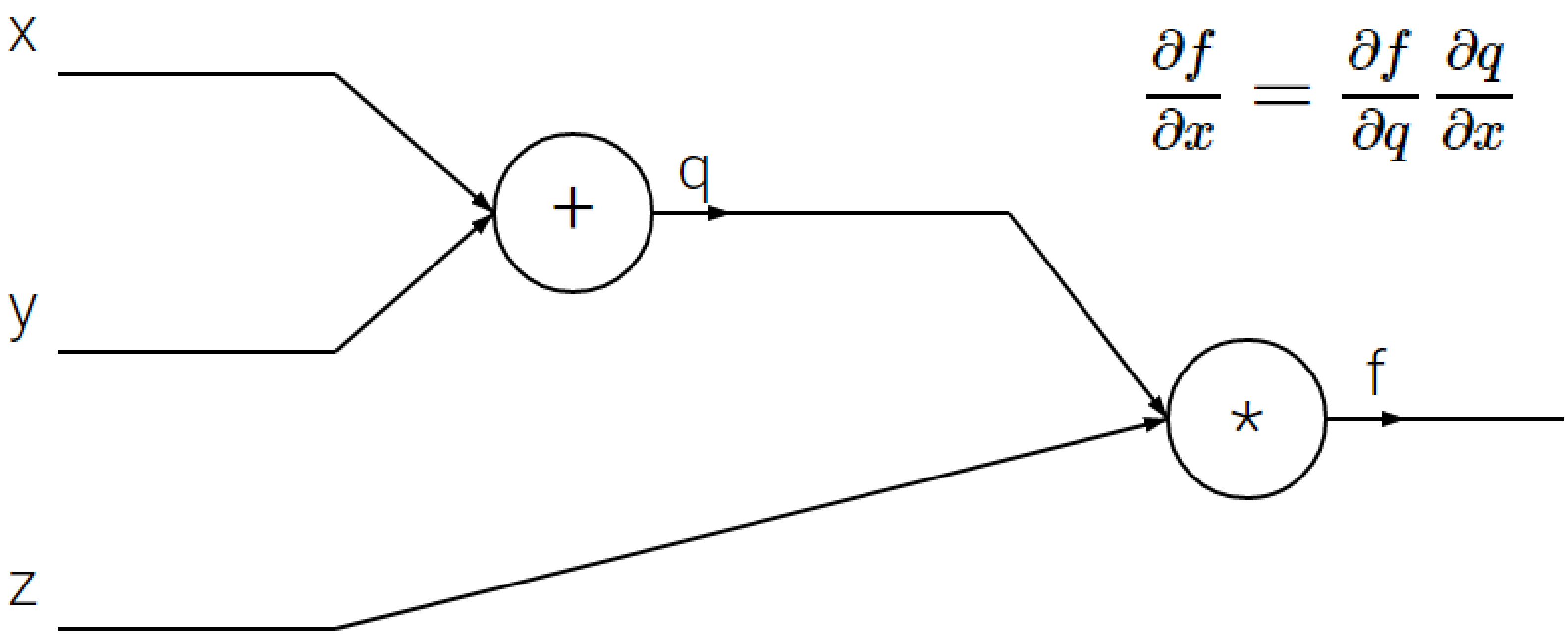
$$f(x, y, z) = (x + y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

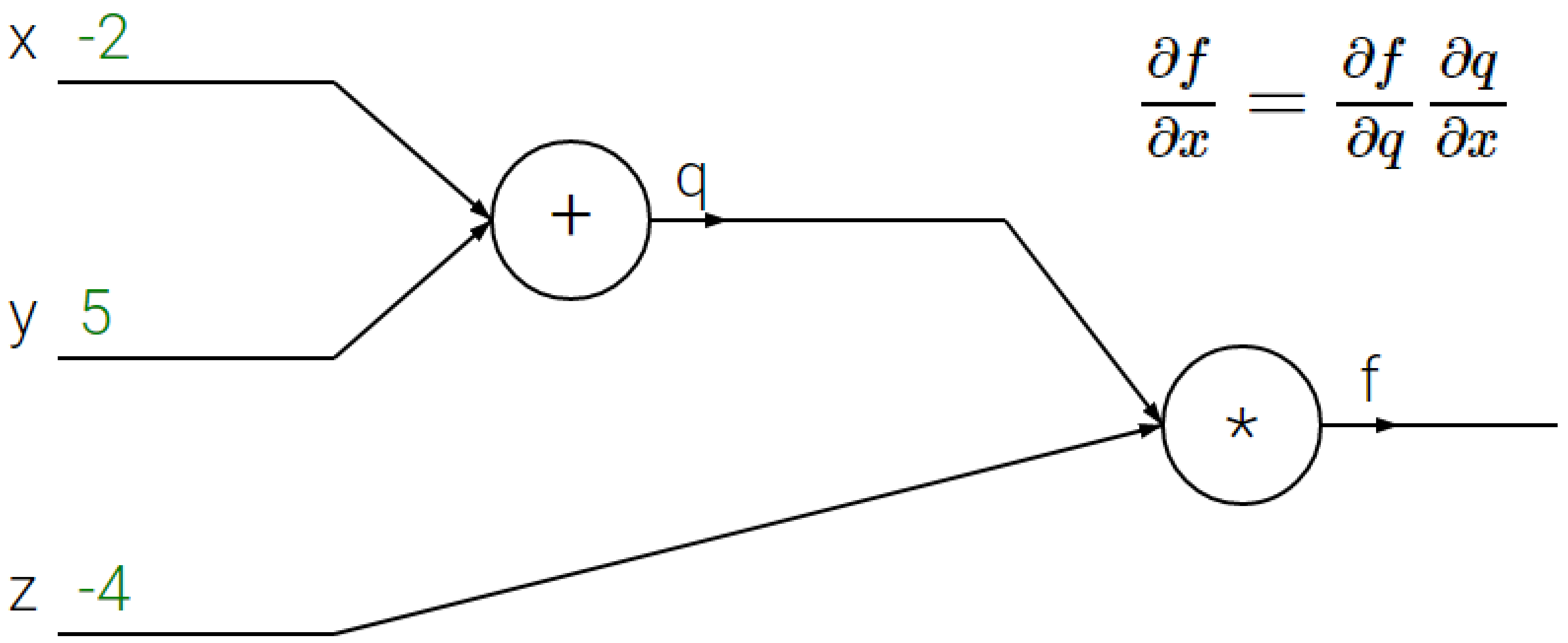
$$f(x, y, z) = (x + y)z$$

$$q = x + y \text{ and } f = qz$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

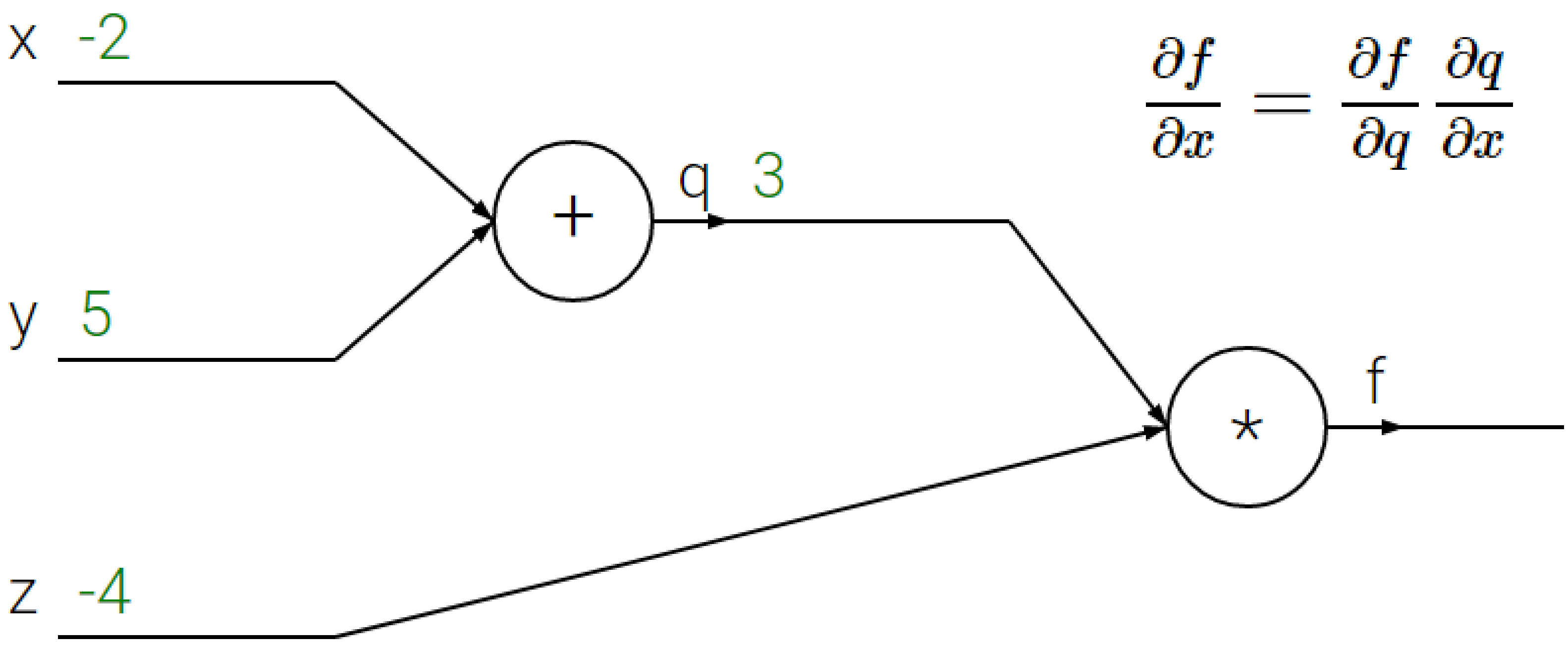
$$f(x, y, z) = (x + y)z$$

$$q = x + y \text{ and } f = qz$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

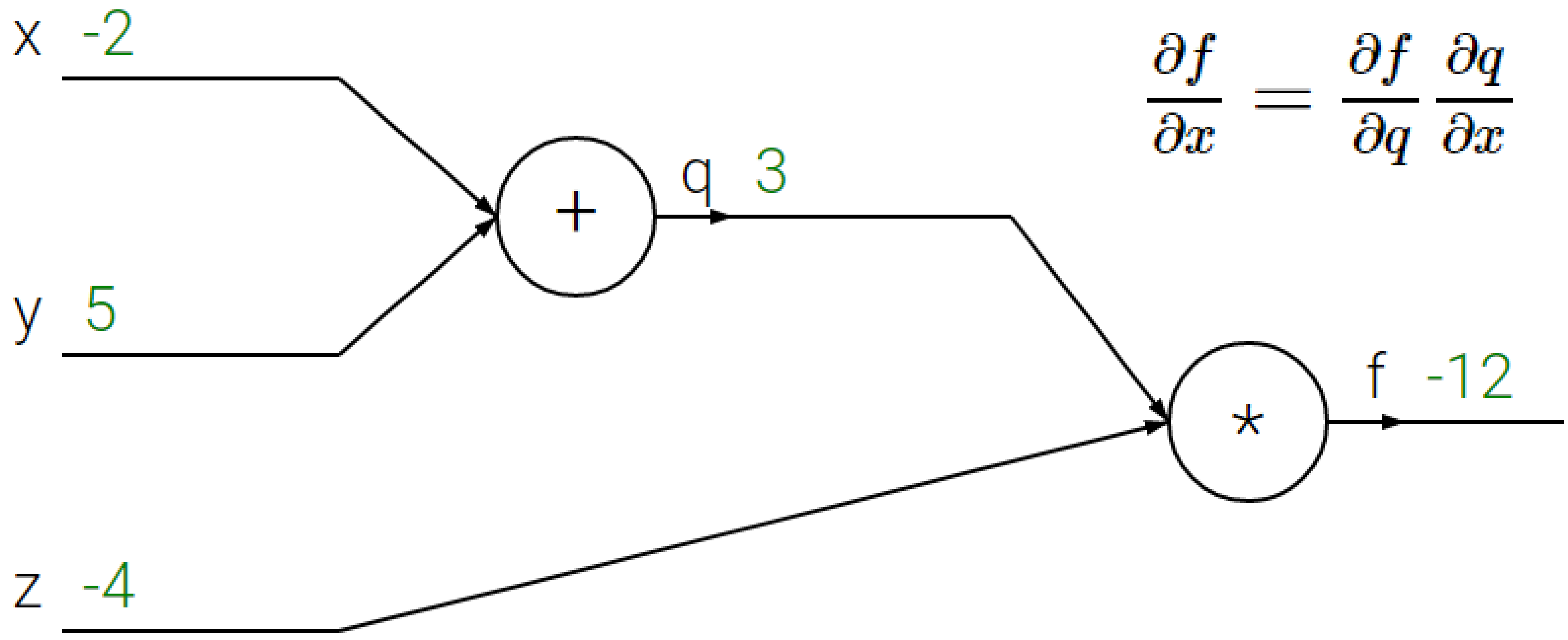
$$f(x,y,z) = (x+y)z$$

$$q = x + y \text{ and } f = qz$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

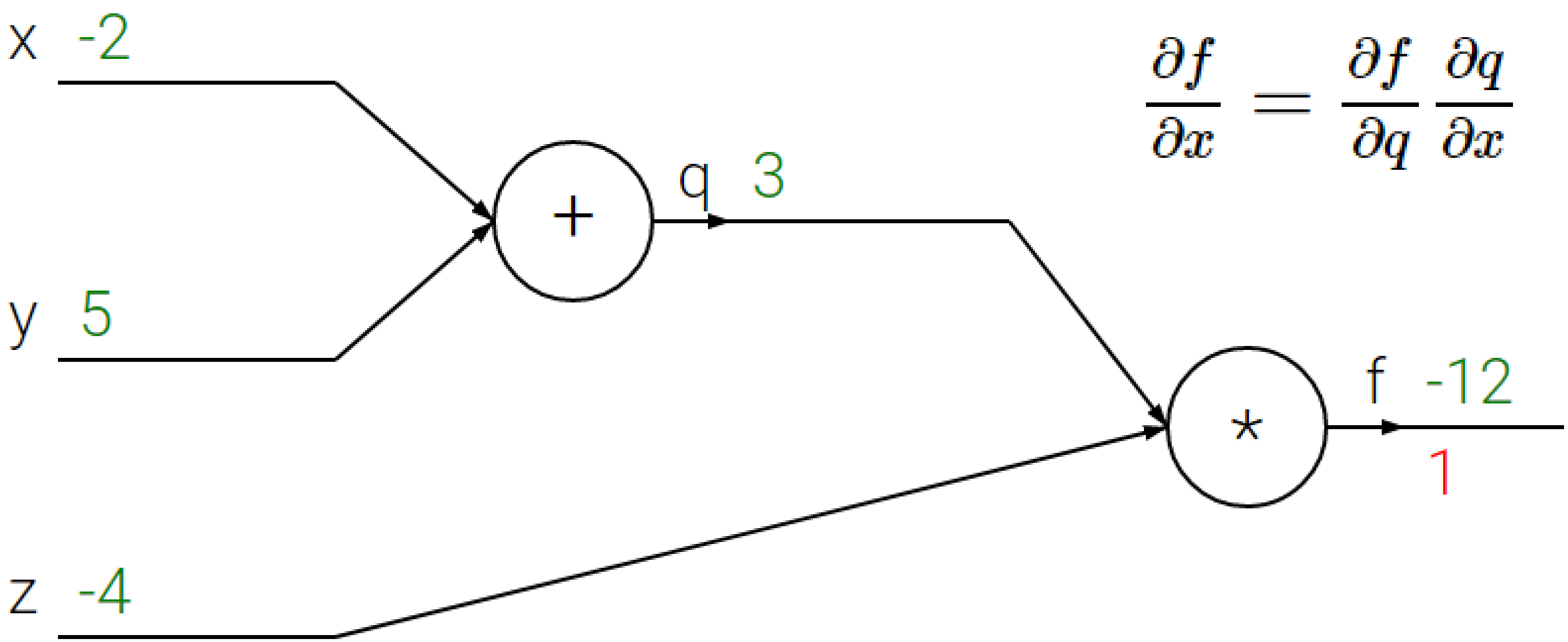
$$f(x, y, z) = (x + y)z$$

$$q = x + y \text{ and } f = qz$$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

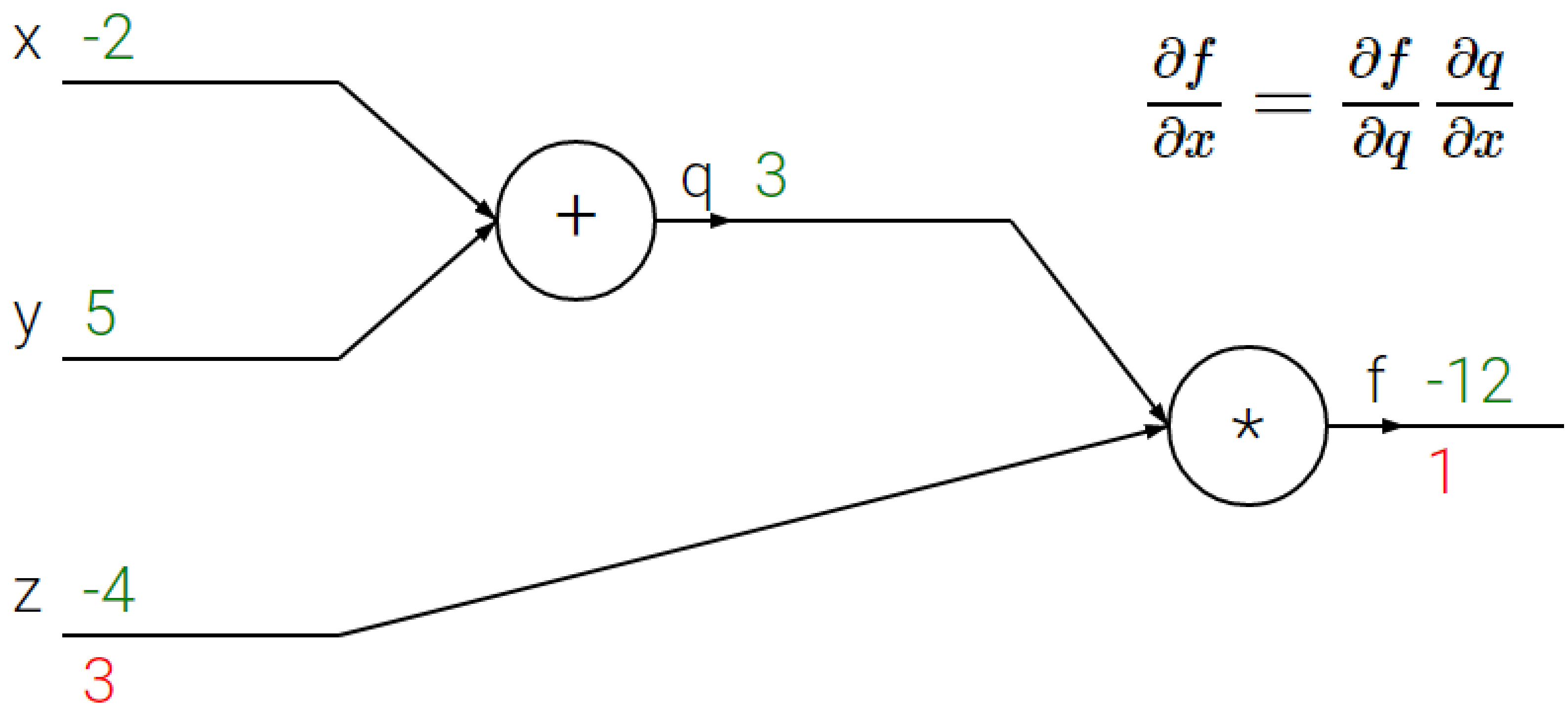
$$f(x,y,z) = (x+y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

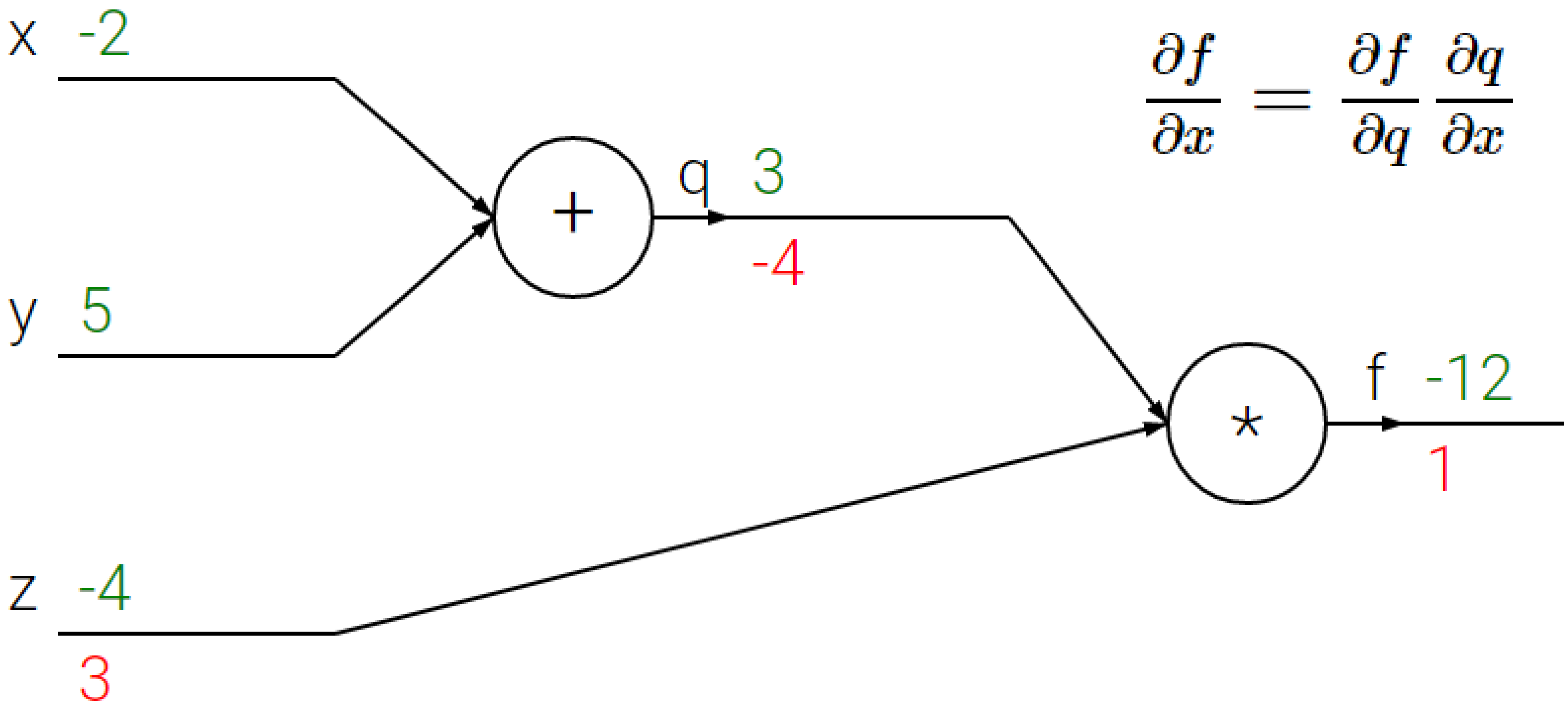
$$f(x,y,z) = (x+y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

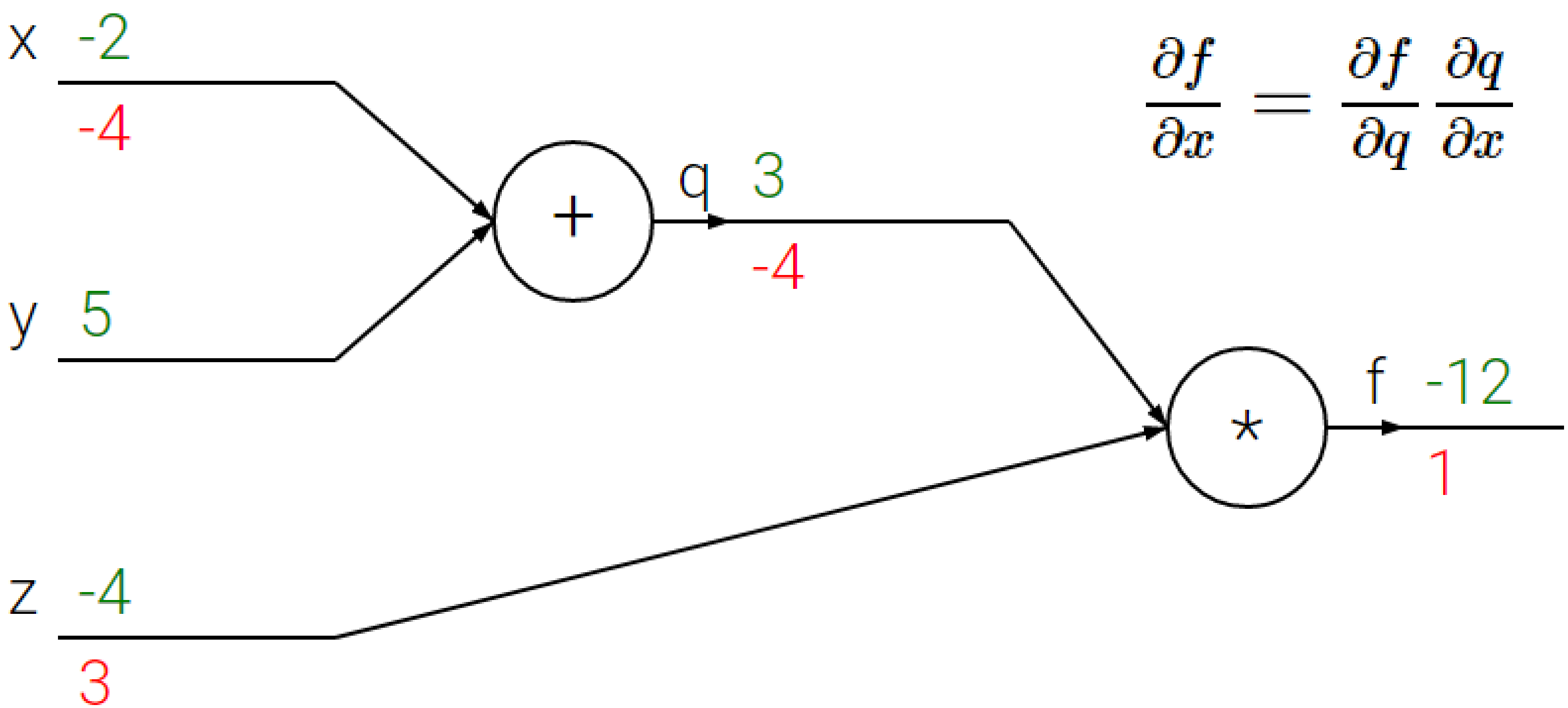
$$f(x, y, z) = (x + y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



Source: http://cs231n.github.io

Compound expressions with chain rule

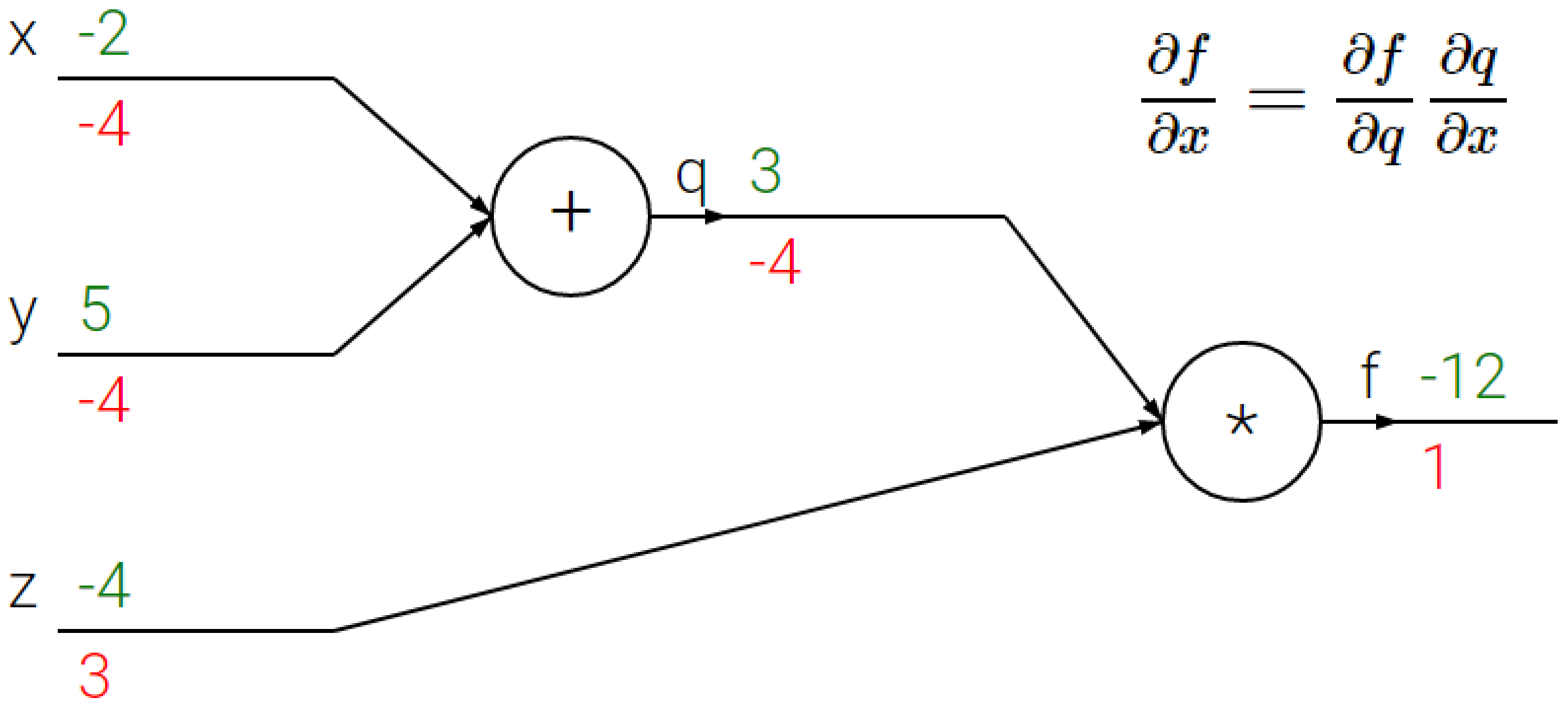
$$f(x, y, z) = (x + y)z$$

$q = x + y$ and $f = qz$

$$\frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

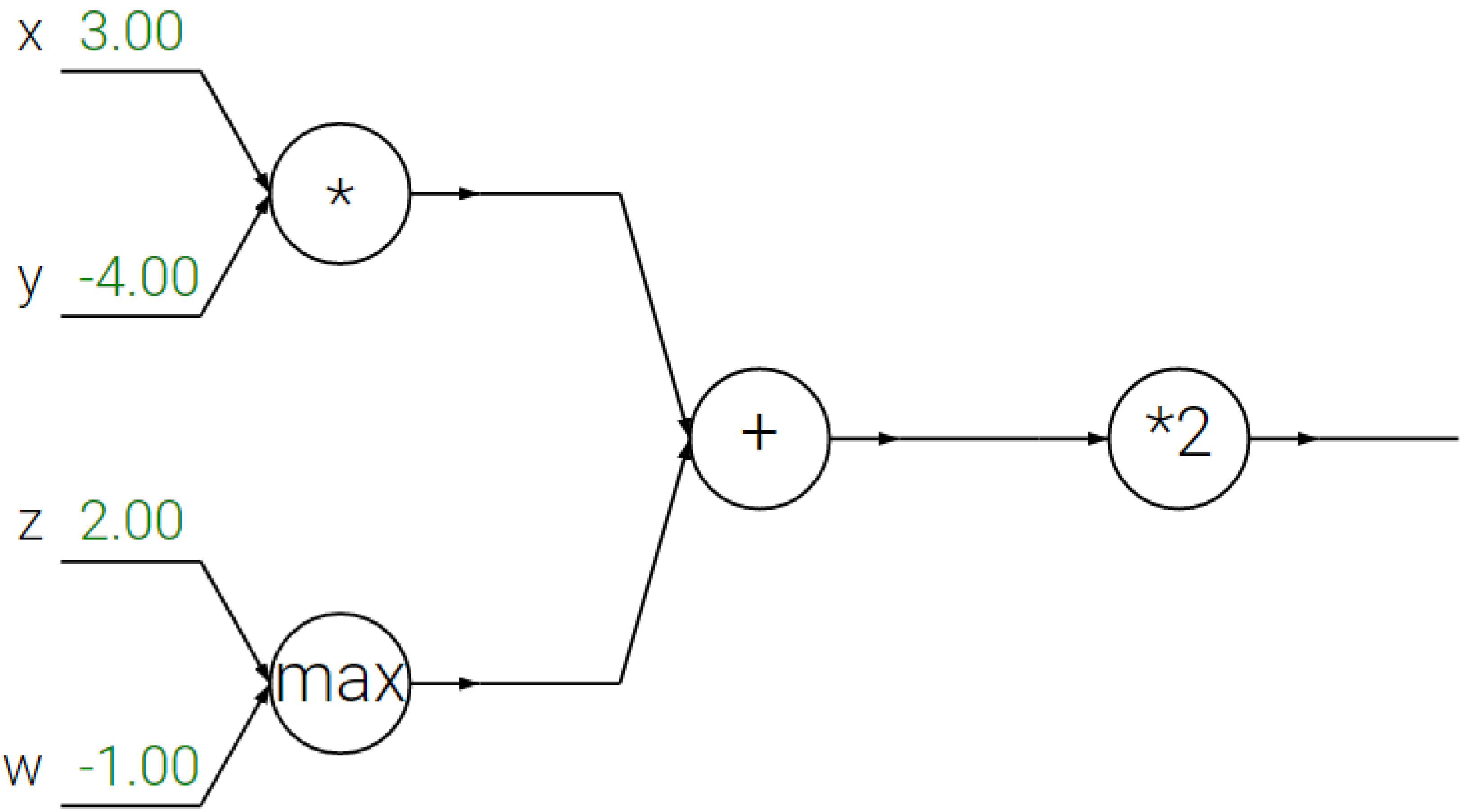
$$\frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$



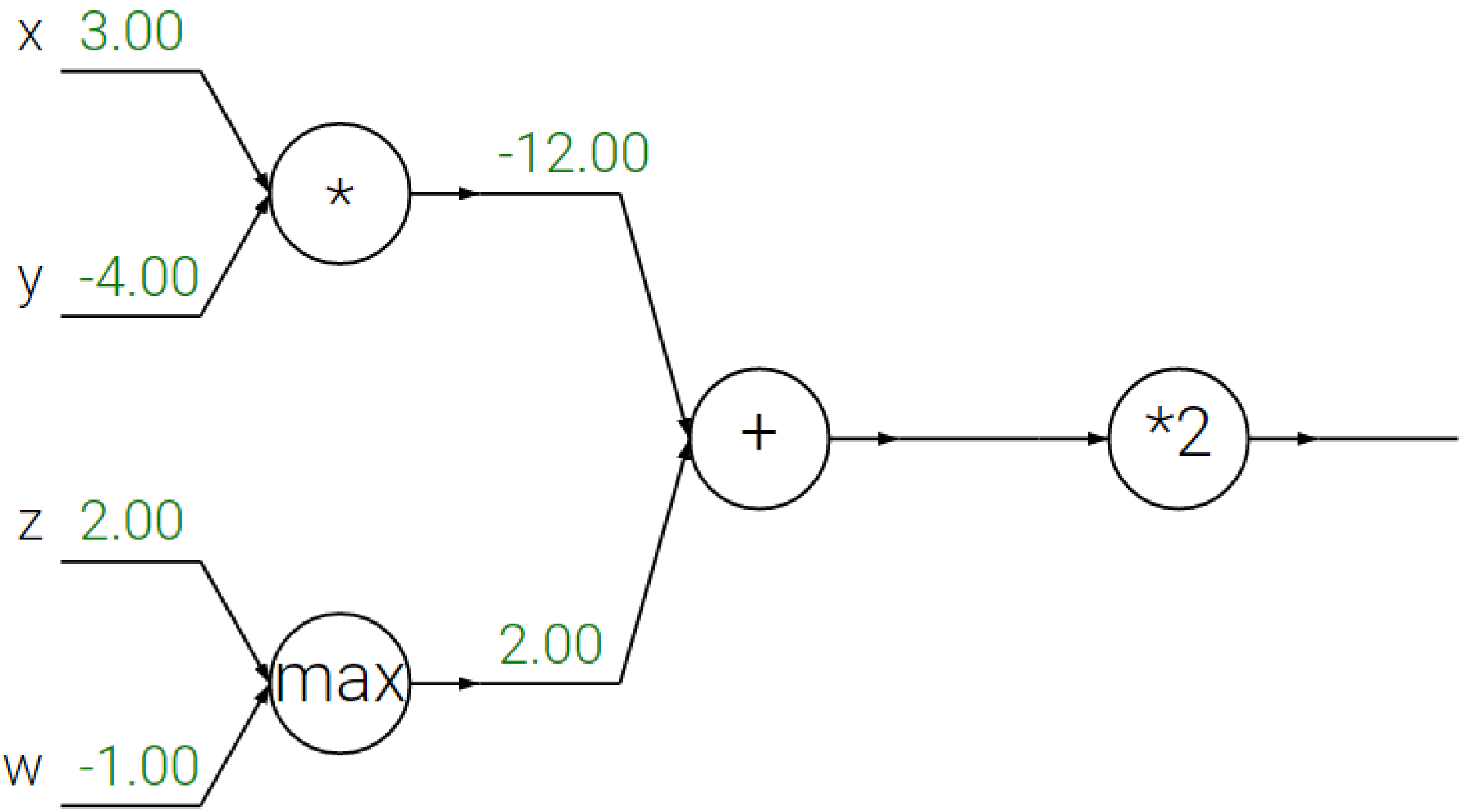
Source: http://cs231n.github.io

Forward and Backward Pass



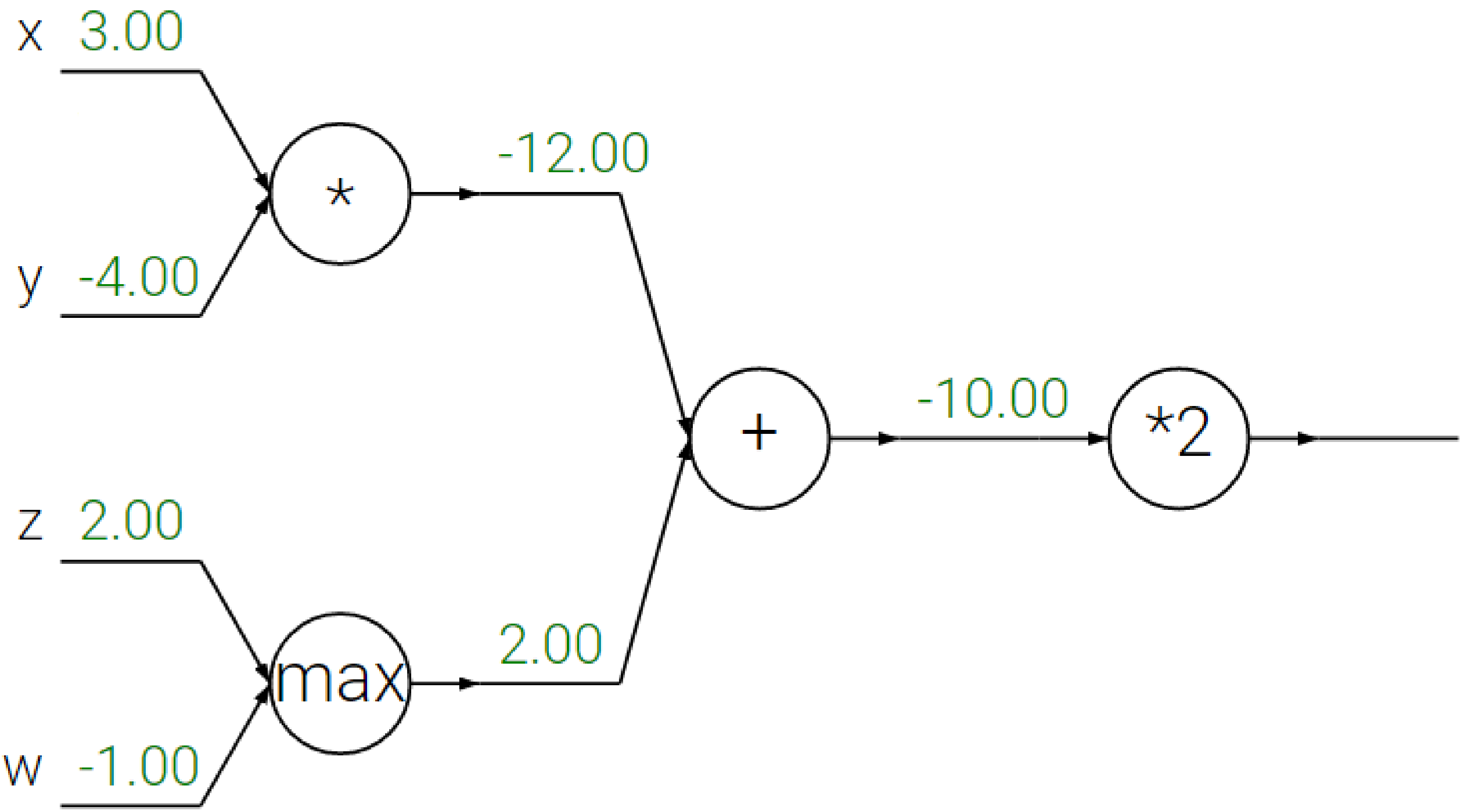
Source: http://cs231n.github.io

Forward and Backward Pass



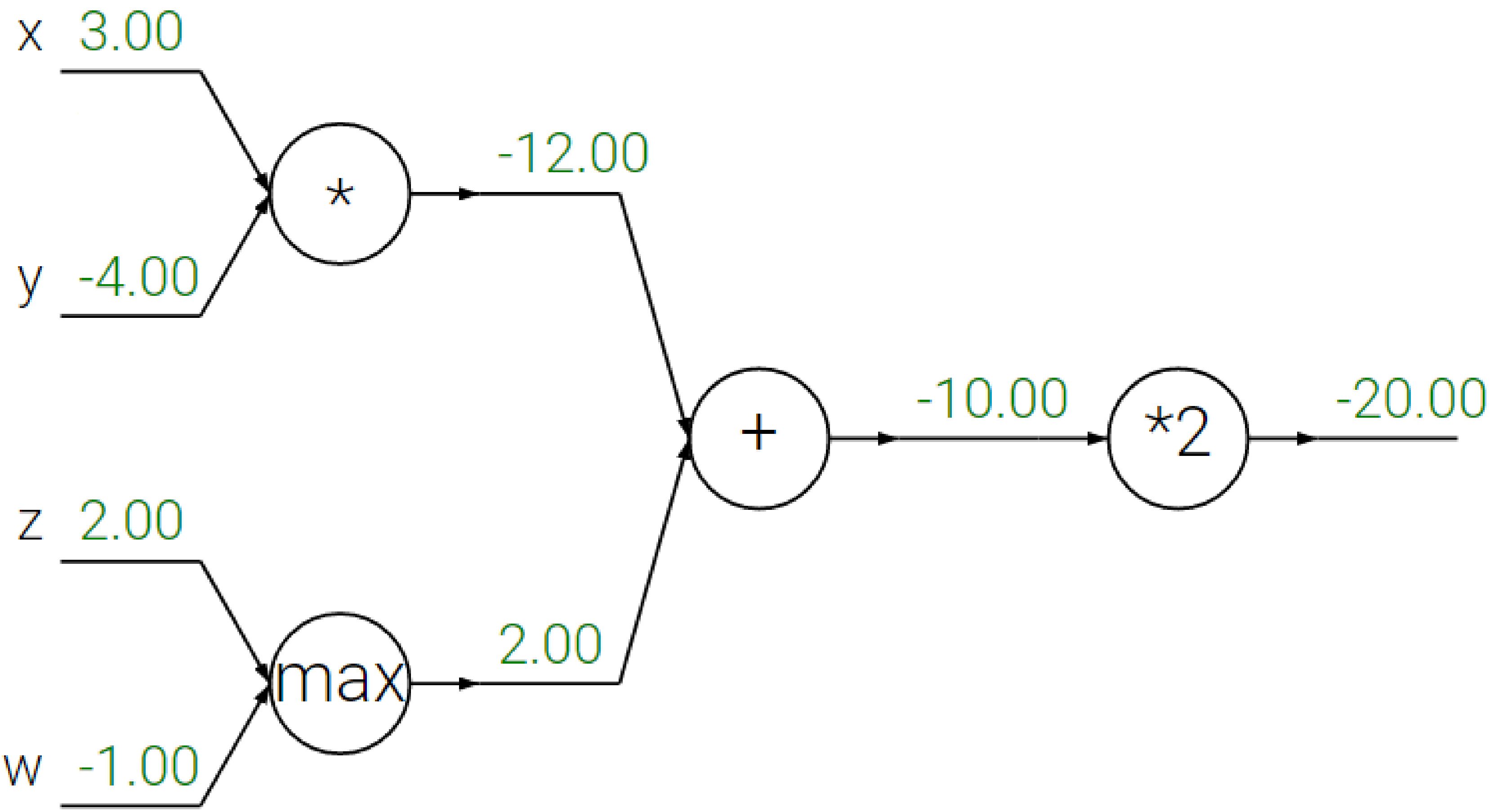
Source: http://cs231n.github.io

Forward and Backward Pass



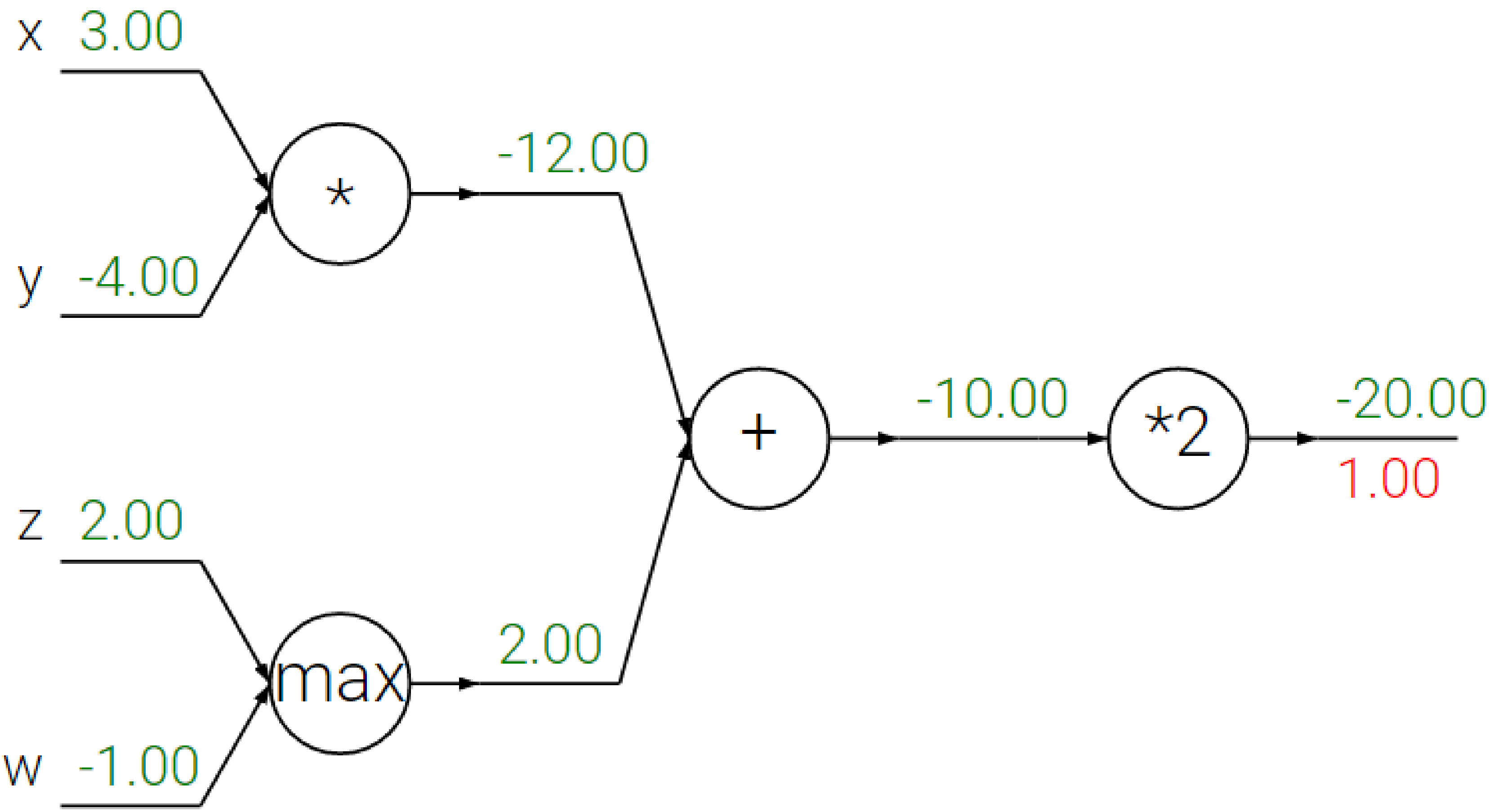
Source: http://cs231n.github.io

Forward and Backward Pass



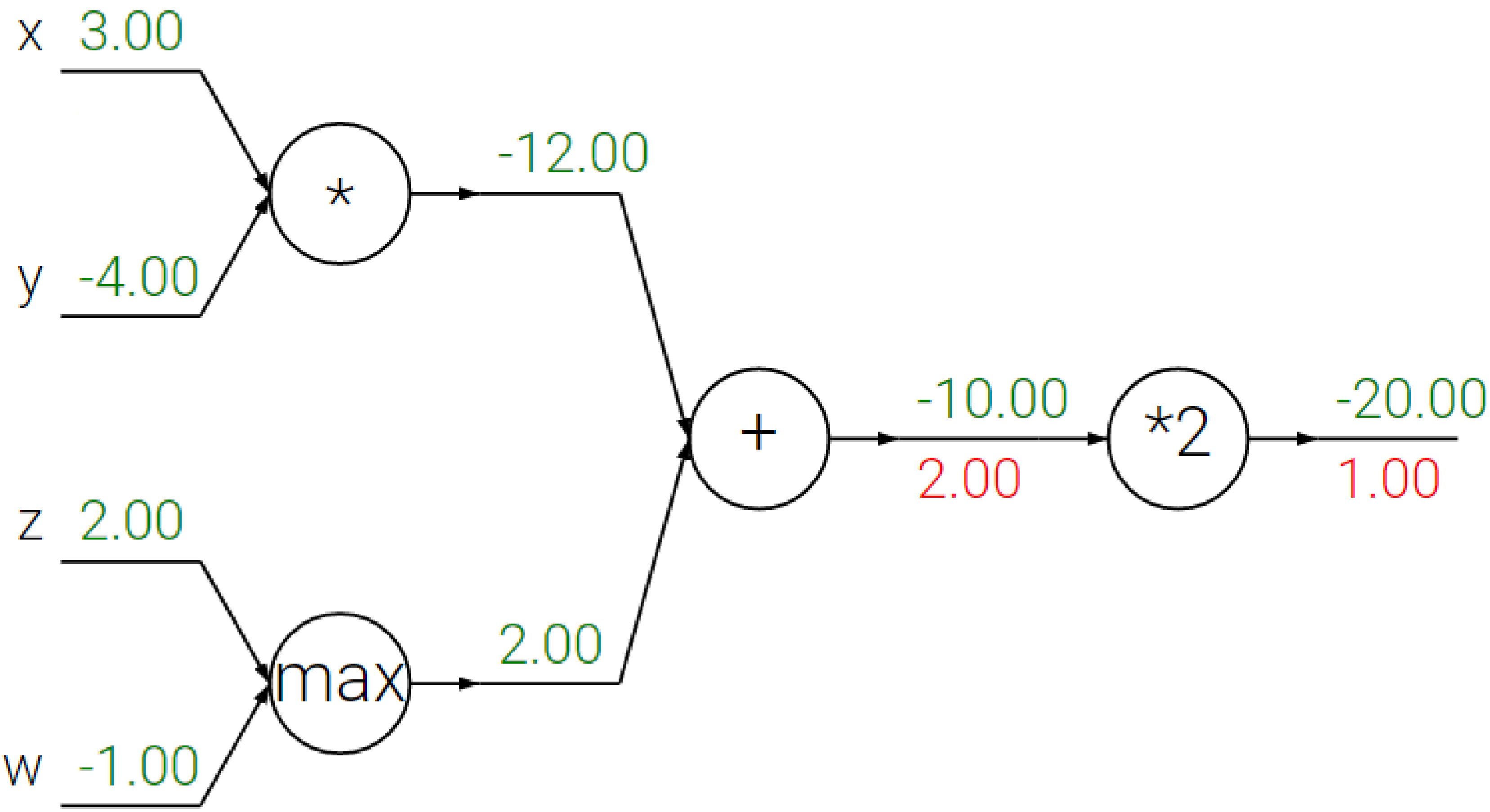
Source: http://cs231n.github.io

Forward and Backward Pass



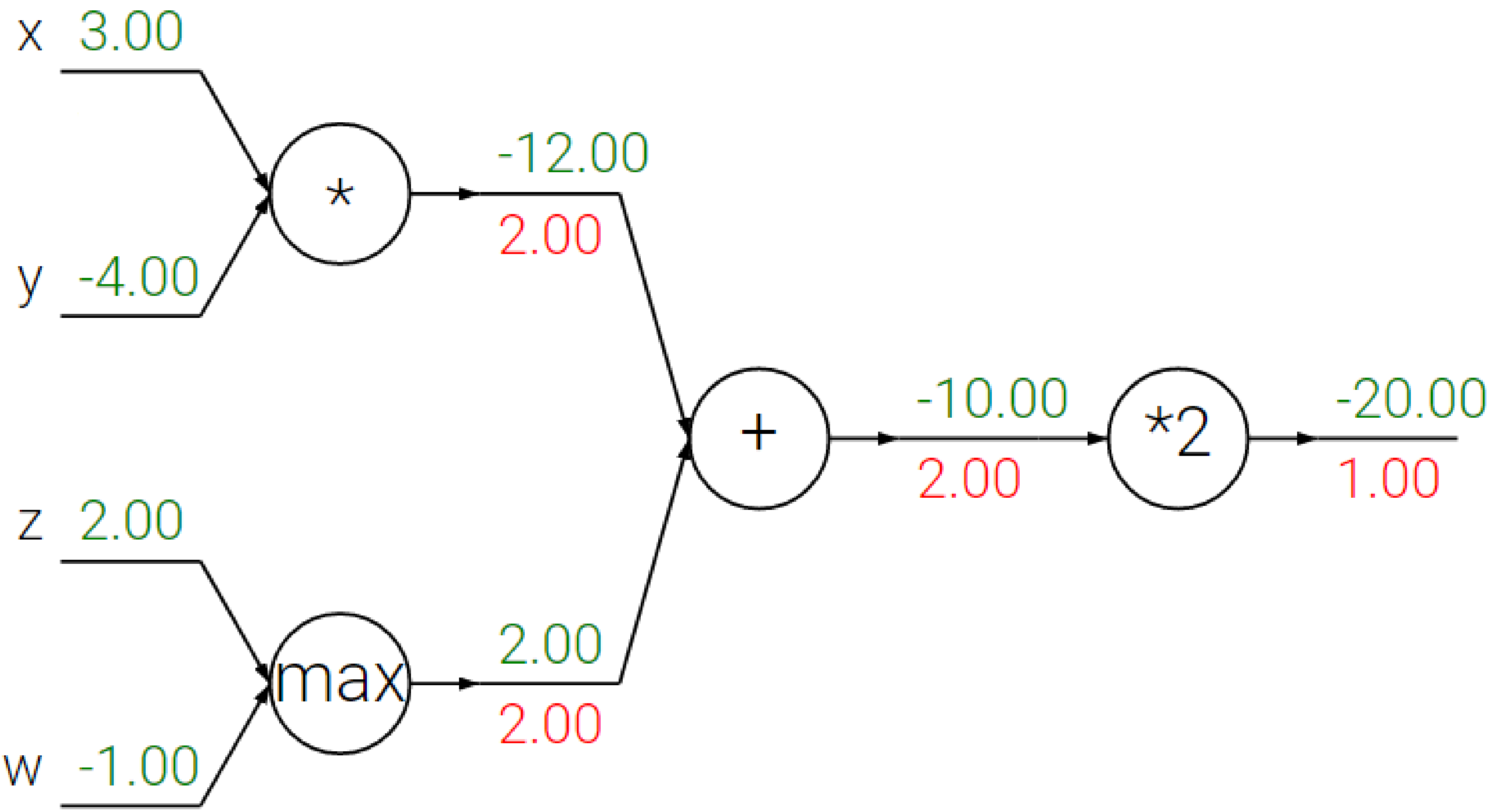
Source: http://cs231n.github.io

Forward and Backward Pass



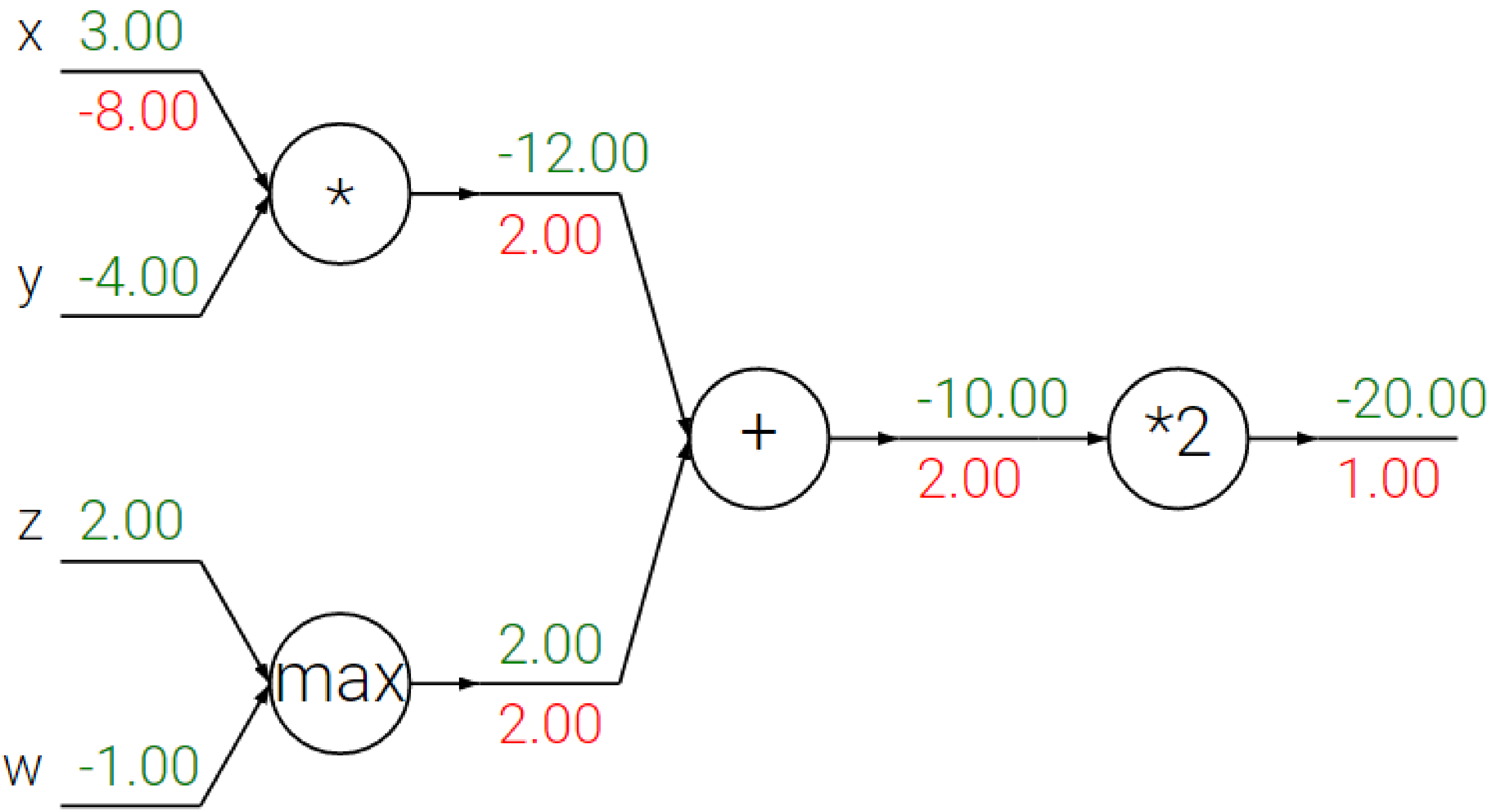
Source: http://cs231n.github.io

Forward and Backward Pass



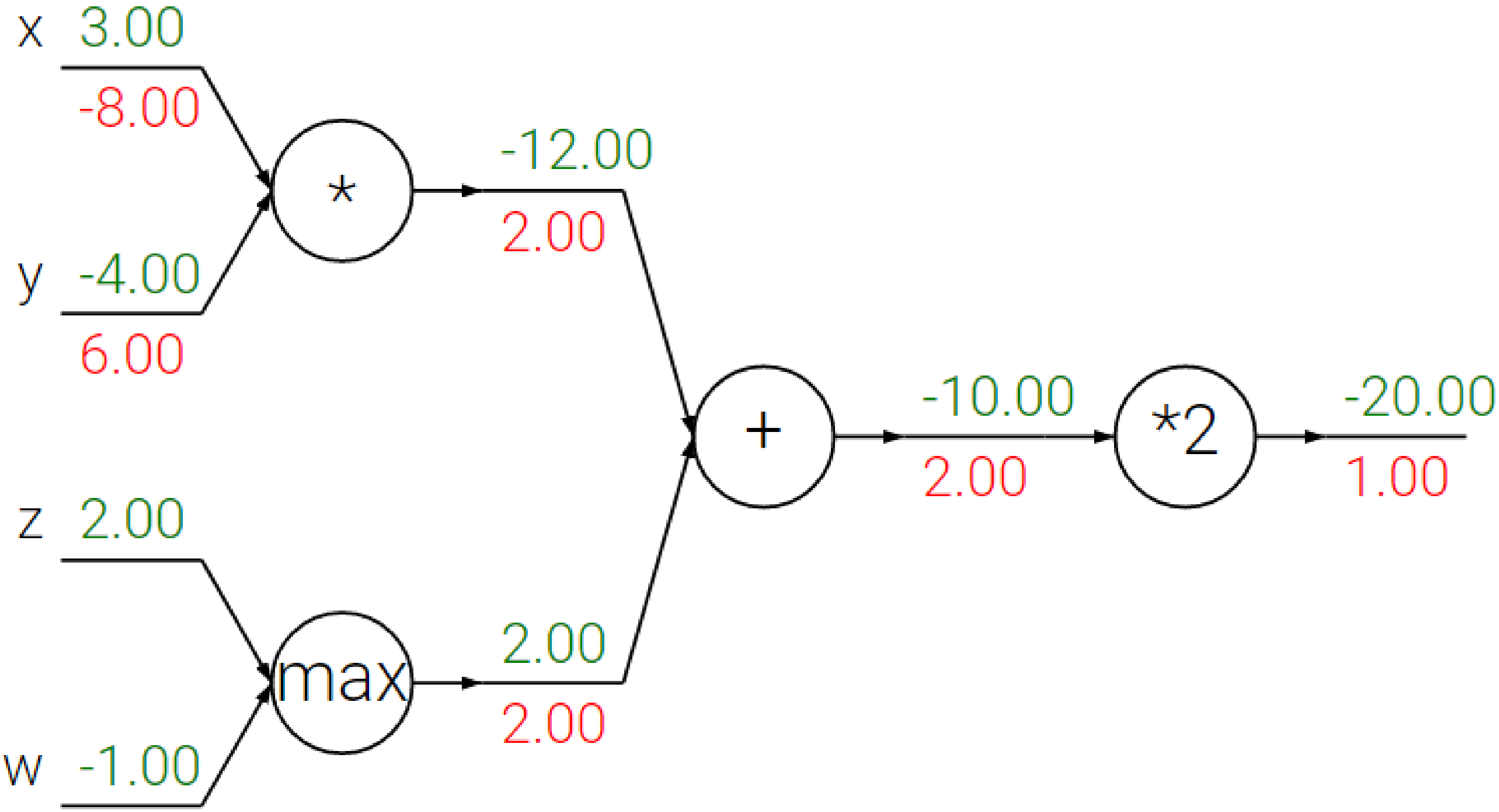
Source: http://cs231n.github.io

Forward and Backward Pass



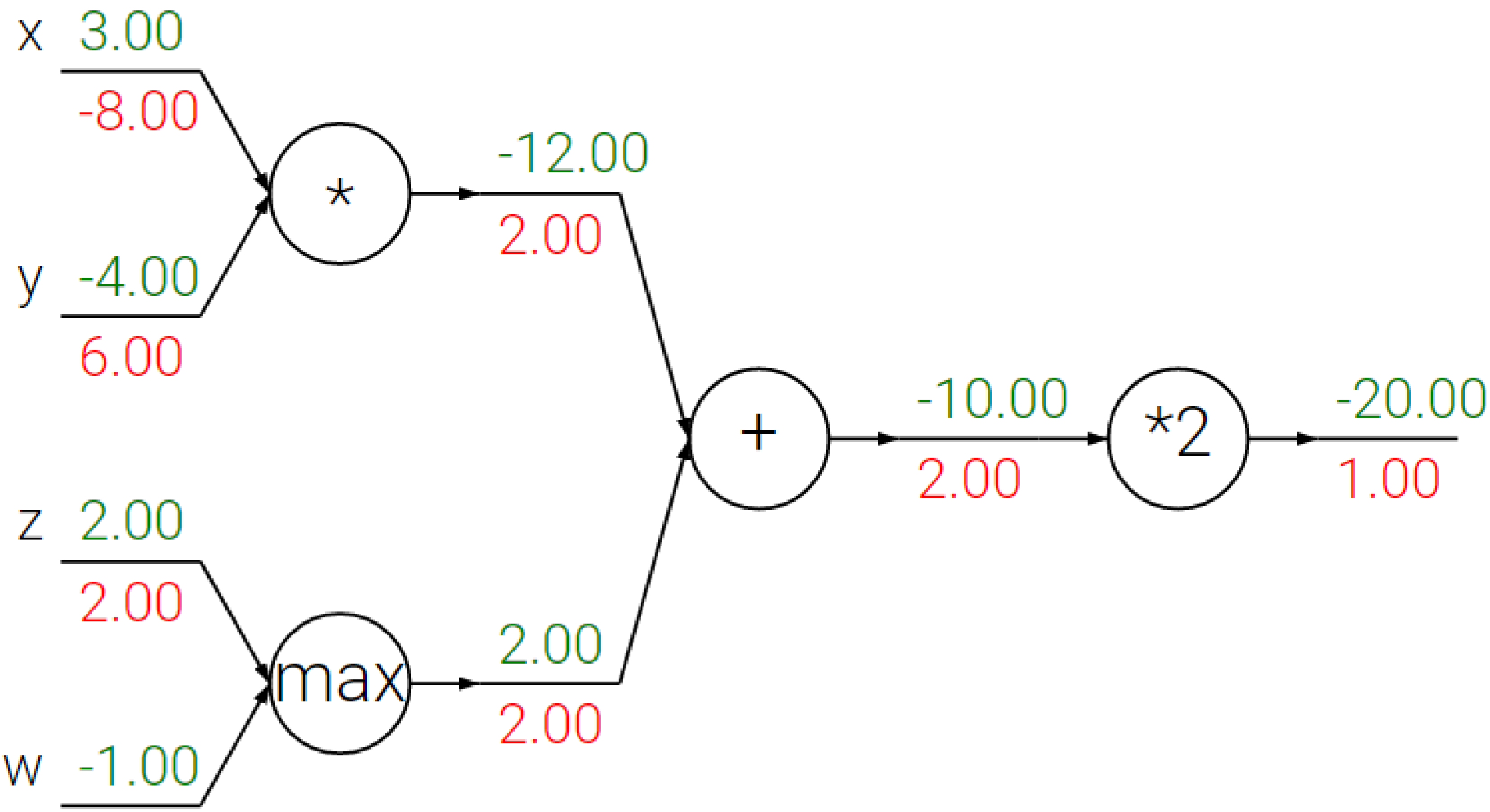
Source: http://cs231n.github.io

Forward and Backward Pass



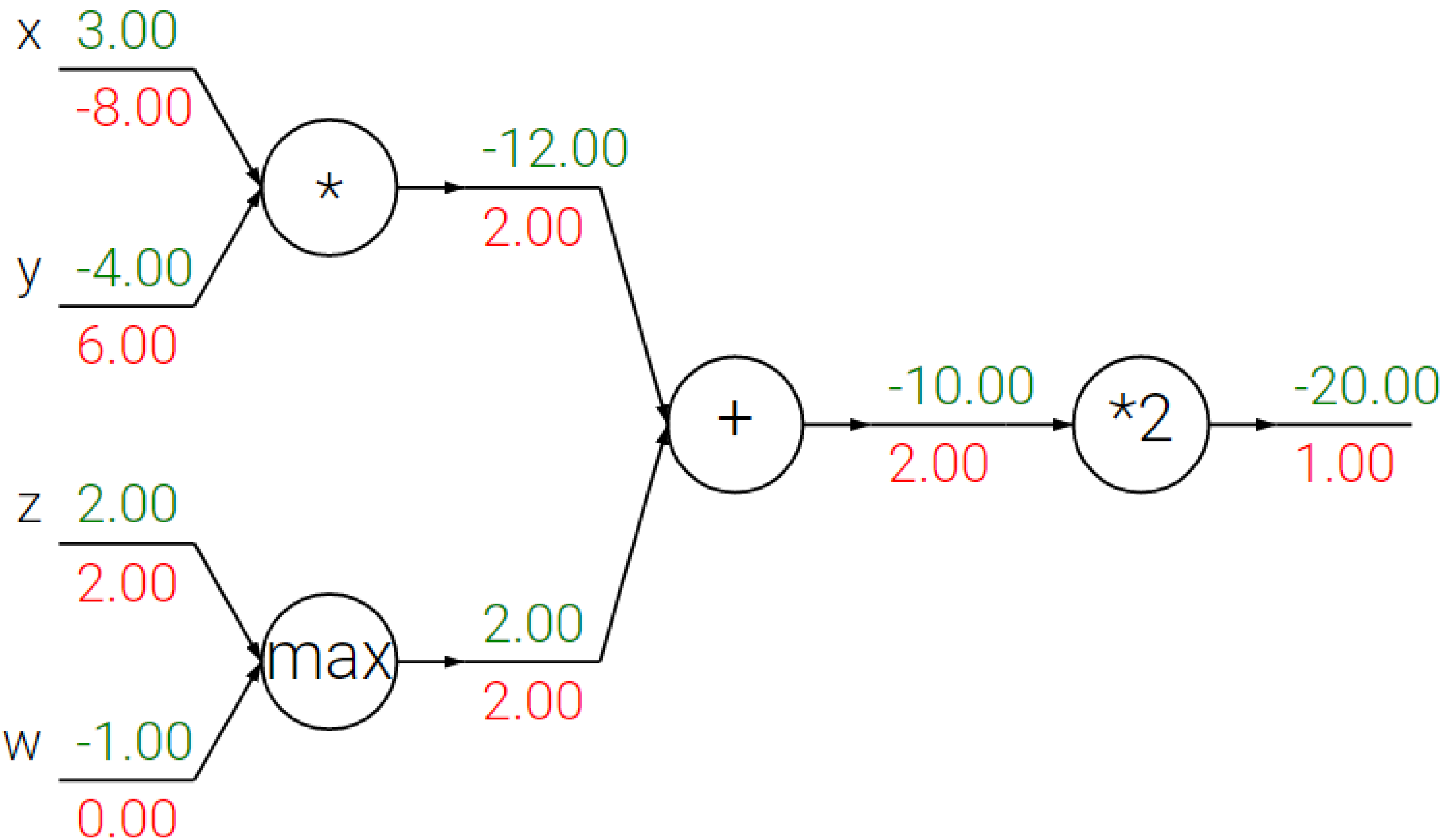
Source: http://cs231n.github.io

Forward and Backward Pass



Source: http://cs231n.github.io

Forward and Backward Pass



Source: http://cs231n.github.io

Sigmoid example

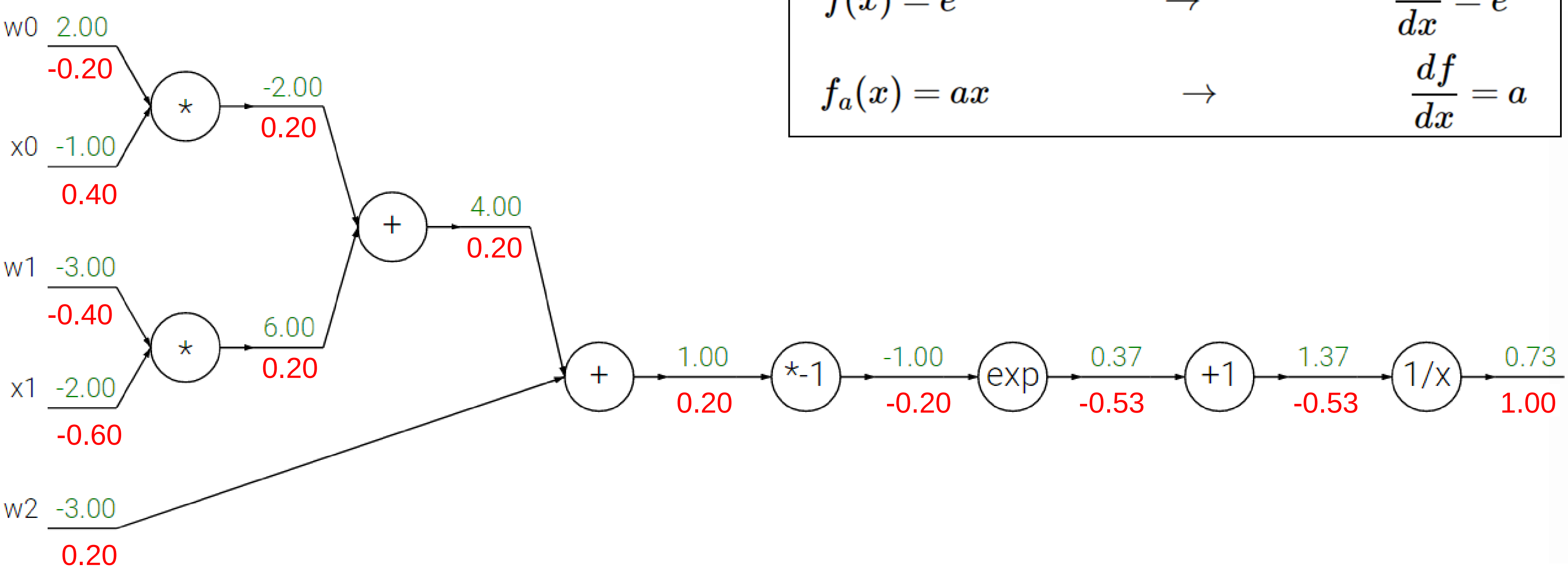
$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$



Modified from: http://cs231n.github.io

SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

Source: http://cs231n.github.io

SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Source: http://cs231n.github.io

SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Gradient w.r.t. w_{y_i}:

$$\nabla_{w_{y_i}} L_i = - \left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Gradient w.r.t. w_{y_i}:

$$\nabla_{w_{y_i}} L_i = - \underbrace{\left(\sum_{j \neq y_i} \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right)}_{\text{Count of the number of classes that didn't meet the desired margin}} x_i$$

Source: http://cs231n.github.io

SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Gradient w.r.t. w_{y_i} :

$$\nabla_{w_{y_i}} L_i = - \underbrace{\left(\sum_{j \neq y_i} \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right)}_{\text{Count of the number of classes that didn't meet the desired margin}} x_i$$

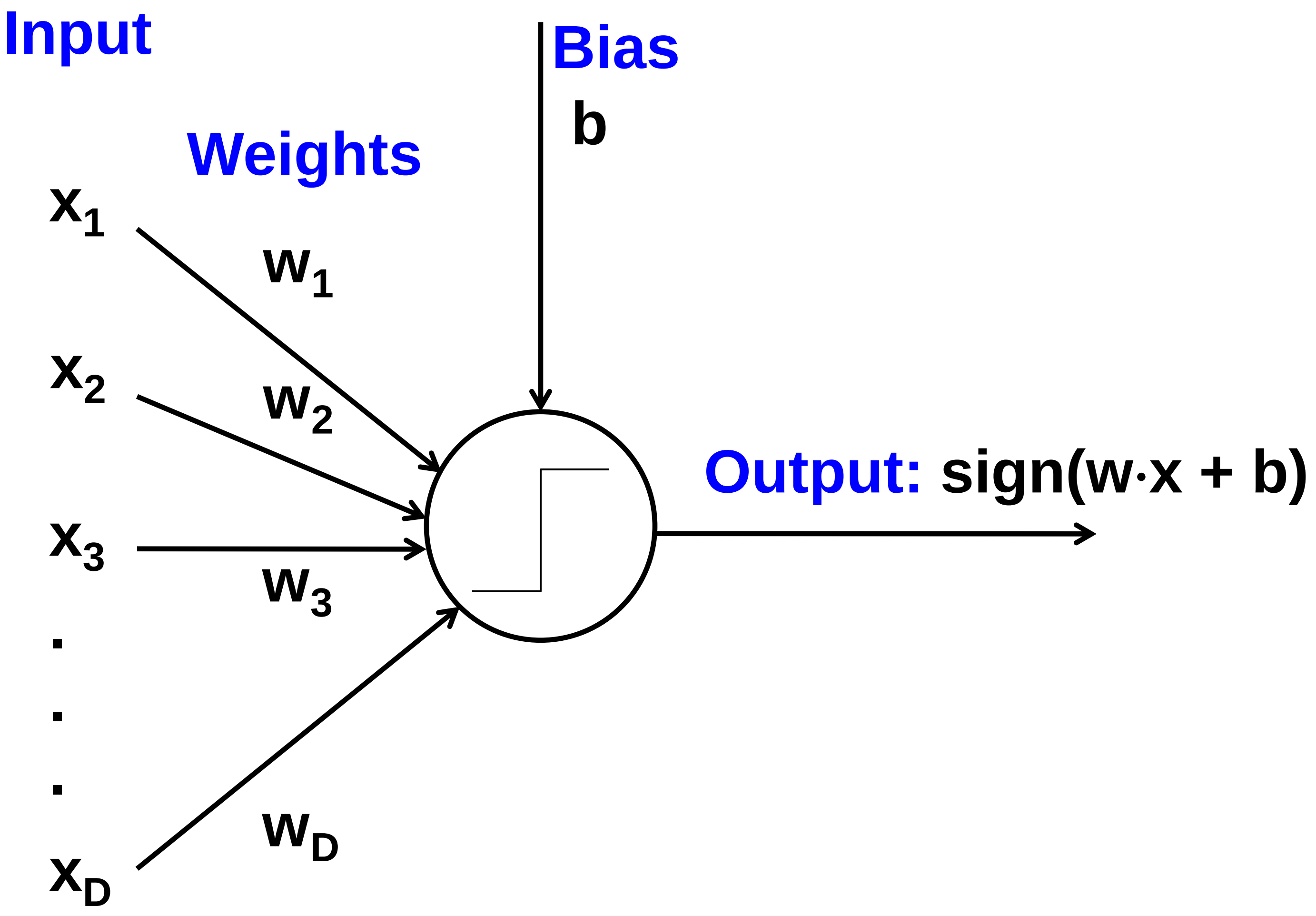
Gradient for the other rows where $j \neq y_i$:

$$\nabla_{w_j} L_i = \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) x_i$$

Source: http://cs231n.github.io

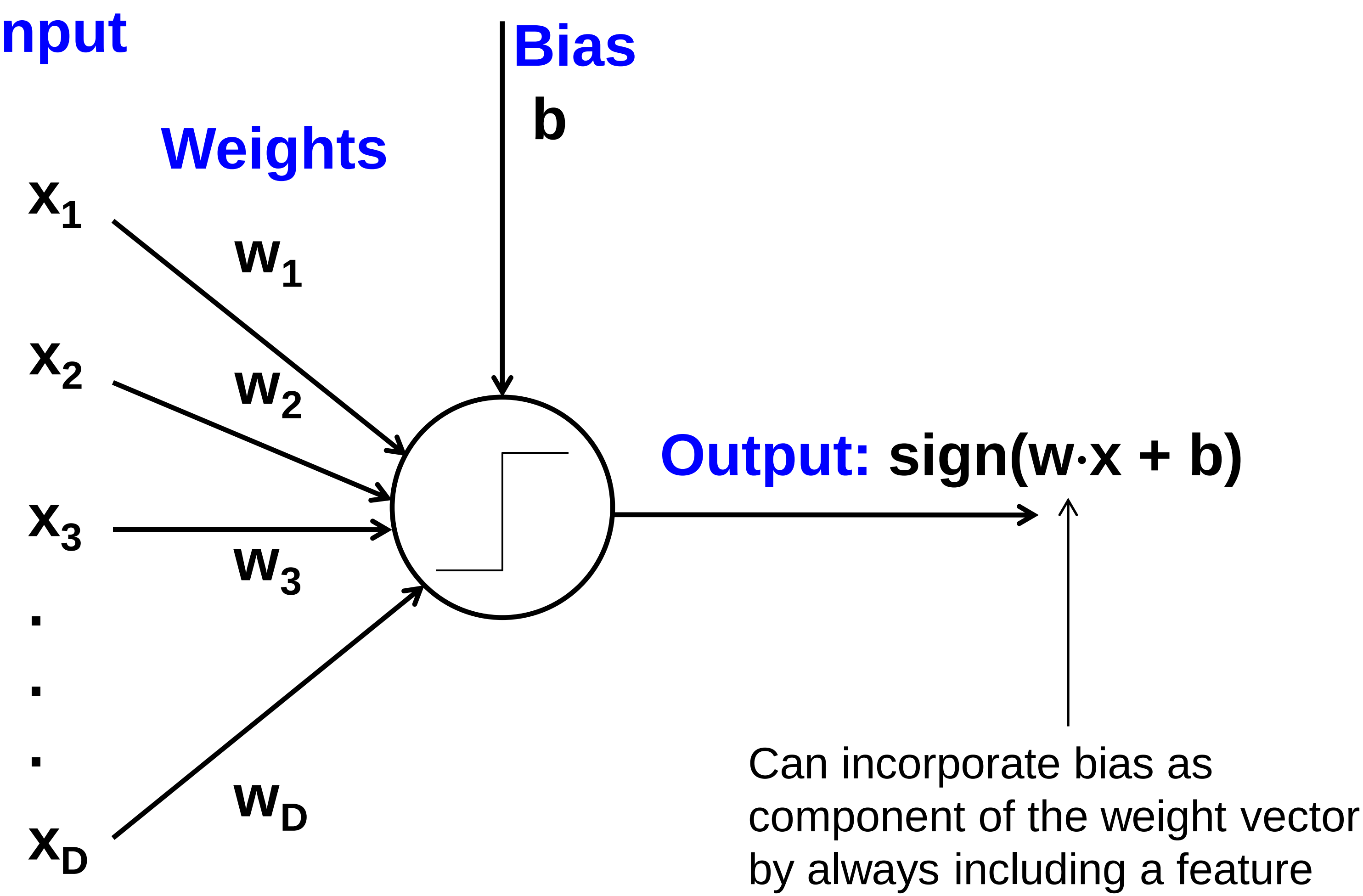
Perceptron

- Supervised learning of binary classifier



Perceptron

- Supervised learning of binary classifier



Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w}.\mathbf{x})$

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w}.\mathbf{x})$
  - Update weights: $\mathbf{w} \leftarrow \mathbf{w} + \alpha(y-y')\mathbf{x}$

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w} . \mathbf{x})$
  - Update weights: $\mathbf{w} \leftarrow \mathbf{w} + \alpha(y-y')\mathbf{x}$
  - **What happens if y' is correct?**

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w} . \mathbf{x})$
  - Update weights: $\mathbf{w} \leftarrow \mathbf{w} + \alpha(y-y')\mathbf{x}$
  - **What happens if y’ is correct?**
  - **Otherwise, if y’ is wrong -**

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w.x})$
  - Update weights: $\mathbf{w} \leftarrow \mathbf{w} + \alpha(y-y')\mathbf{x}$
  - **What happens if y' is correct?**
  - **Otherwise, if y' is wrong -**

  $w_i \leftarrow w_i + \alpha(y-y')x_i$

    – If $y = 1$ and $y' = -1$, w_i will be increased if x_i is positive or decreased if x_i is negative → $\mathbf{w \cdot x}$ will get bigger

Perceptron update rule

- Initialize weights randomly
- Cycle through training examples in multiple passes (*epochs*)
- For each training instance **x** with label y:
  - Classify with current weights: $y' = \text{sign}(\mathbf{w.x})$
  - Update weights: $\mathbf{w} \leftarrow \mathbf{w} + \alpha(y-y')\mathbf{x}$
  - **What happens if y' is correct?**
  - **Otherwise, if y' is wrong -**

  $w_i \leftarrow w_i + \alpha(y-y')x_i$

    - If y = 1 and y' = -1, w_i will be increased if x_i is positive or decreased if x_i is negative → **w·x** will get bigger
    - If y = -1 and y' = 1, w_i will be decreased if x_i is positive or increased if x_i is negative → **w·x** will get smaller

Single neuron as a linear classifier

Binary Softmax classifier (*Logistic Regression*)

$$\sigma(\sum_i w_i x_i + b)$$

Source: http://cs231n.github.io

Single neuron as a linear classifier

Binary Softmax classifier (*Logistic Regression*)

$$\sigma(\sum_i w_i x_i + b)$$

Probability of one of the classes: $P(y_i = 1 \mid x_i; w)$

Source: http://cs231n.github.io

Single neuron as a linear classifier

Binary Softmax classifier (*Logistic Regression*)

$$\sigma(\sum_i w_i x_i + b)$$

$$\downarrow$$

Probability of one of the classes: $P(y_i = 1 \mid x_i; w)$

Probability of the other class would be:

$$P(y_i = 0 \mid x_i; w) = 1 - P(y_i = 1 \mid x_i; w)$$

Source: http://cs231n.github.io

Single neuron as a linear classifier

Binary Softmax classifier (*Logistic Regression*)

$$\sigma(\sum_i w_i x_i + b)$$

$$\downarrow$$

Probability of one of the classes: $P(y_i = 1 \mid x_i; w)$

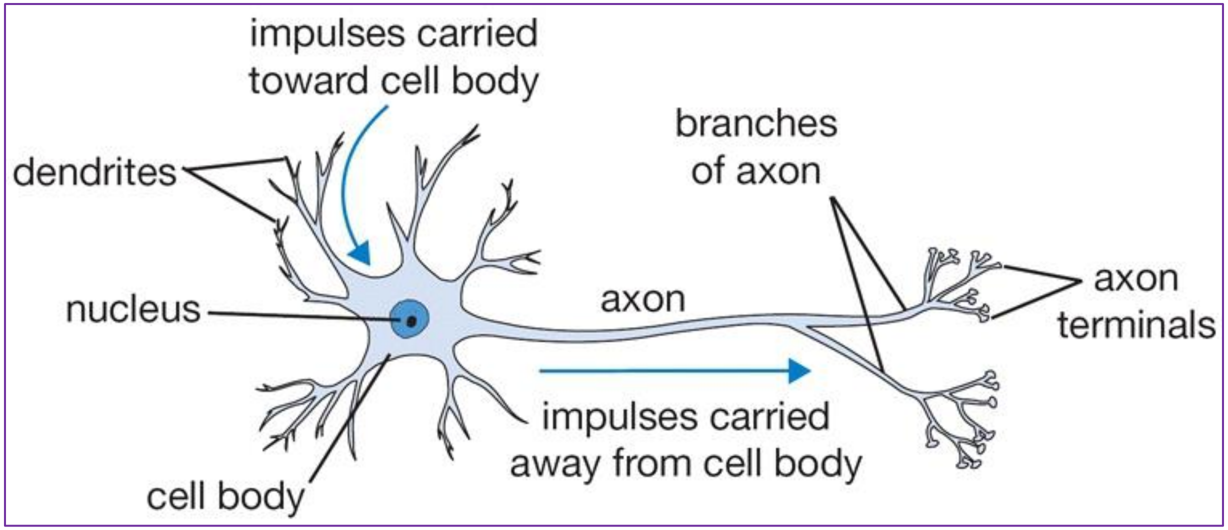
Probability of the other class would be:

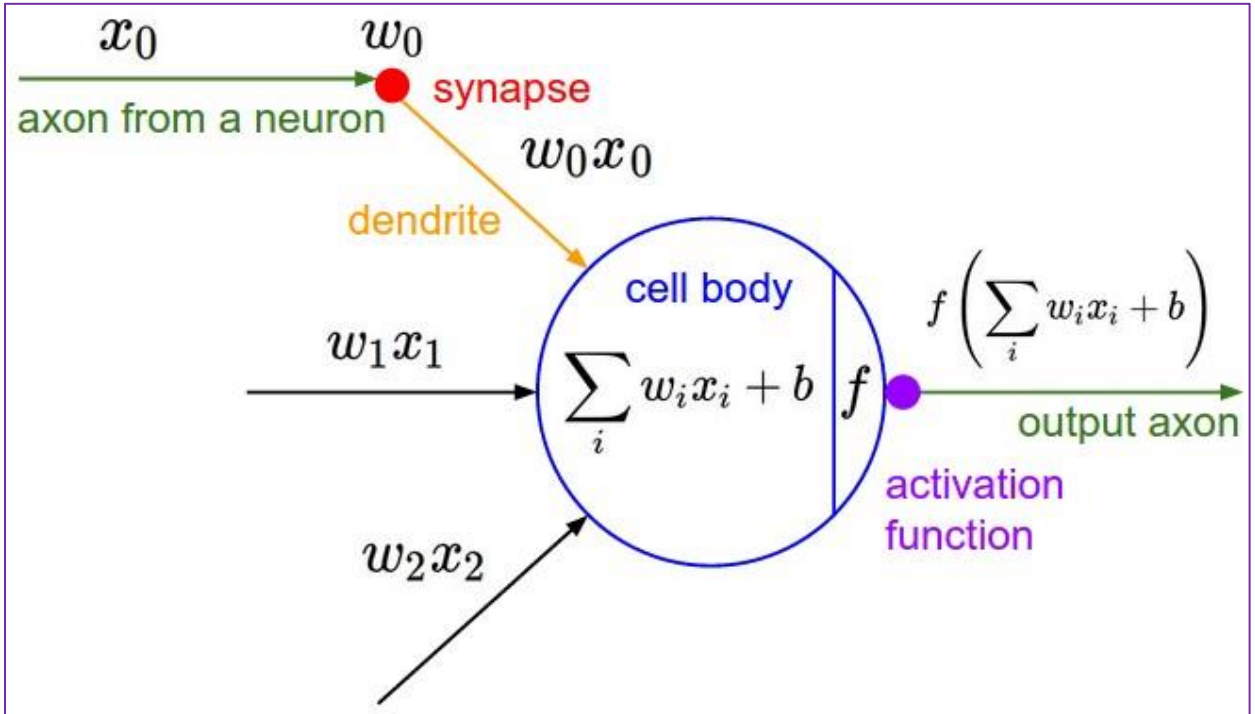
$$P(y_i = 0 \mid x_i; w) = 1 - P(y_i = 1 \mid x_i; w)$$

Binary SVM classifier.
Alternatively, we could attach a max-margin hinge loss to the output of the neuron and train it to become a binary Support Vector Machine.

Source: http://cs231n.github.io

Loose inspiration: Human neurons

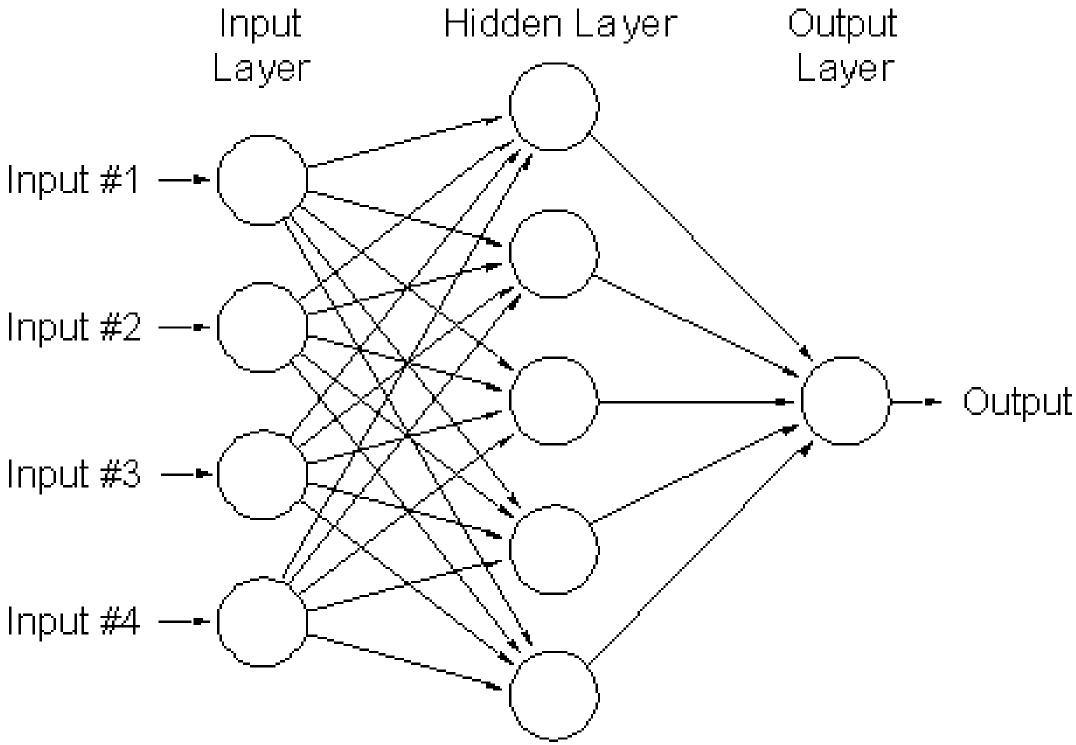




Source: http://cs231n.github.io

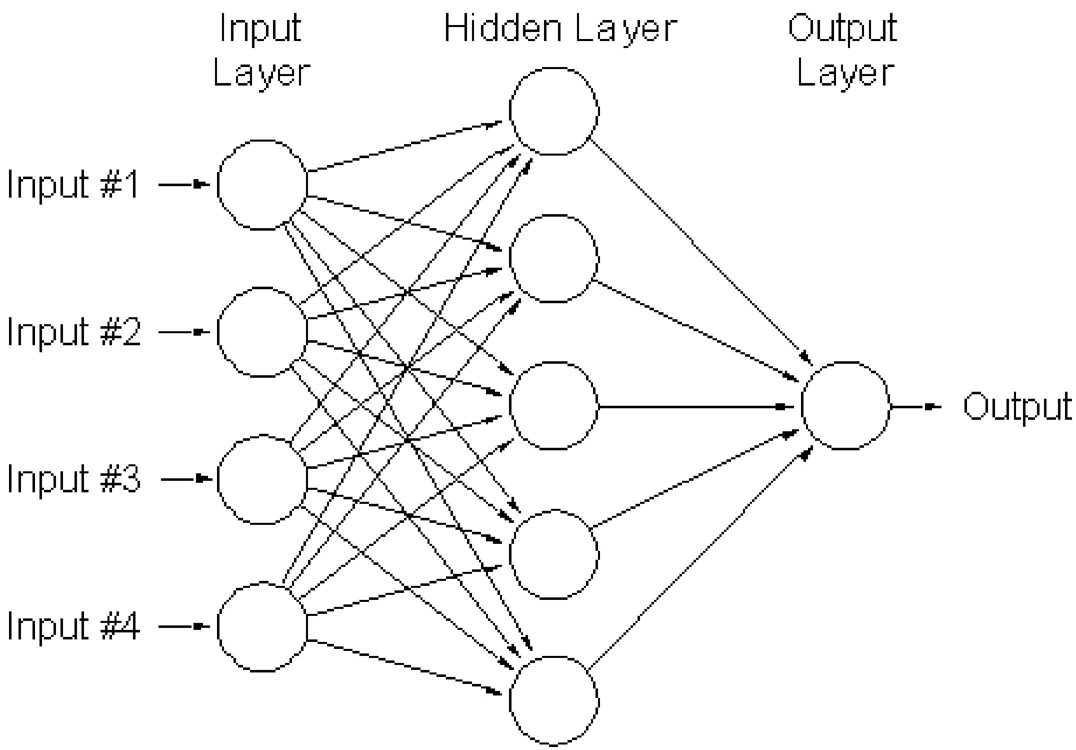
Multi-Layer Neural Networks

- Network with a hidden layer:



Multi-Layer Neural Networks

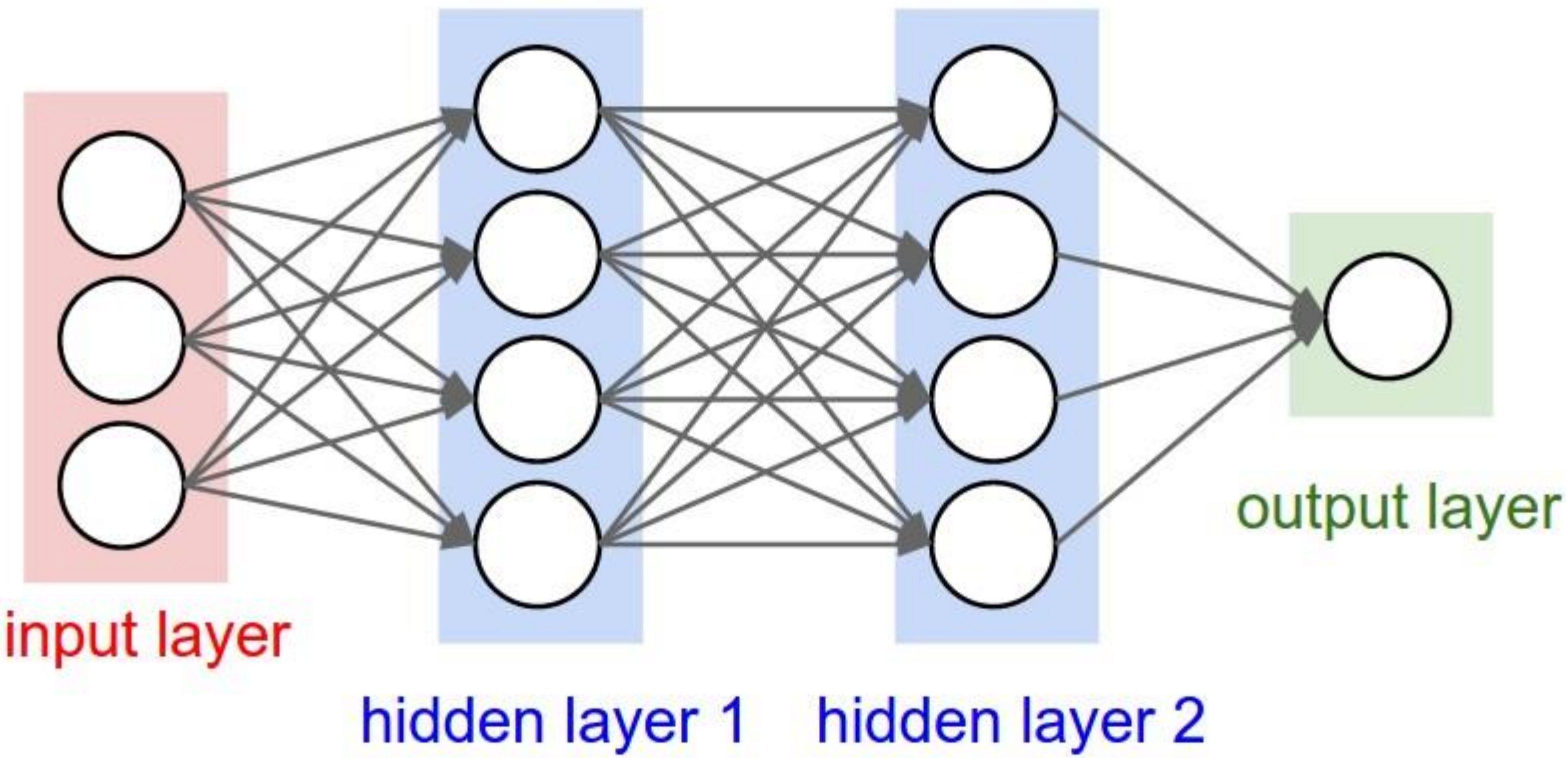
- Network with a hidden layer:



- Can represent nonlinear functions (provided each perceptron has a nonlinearity)

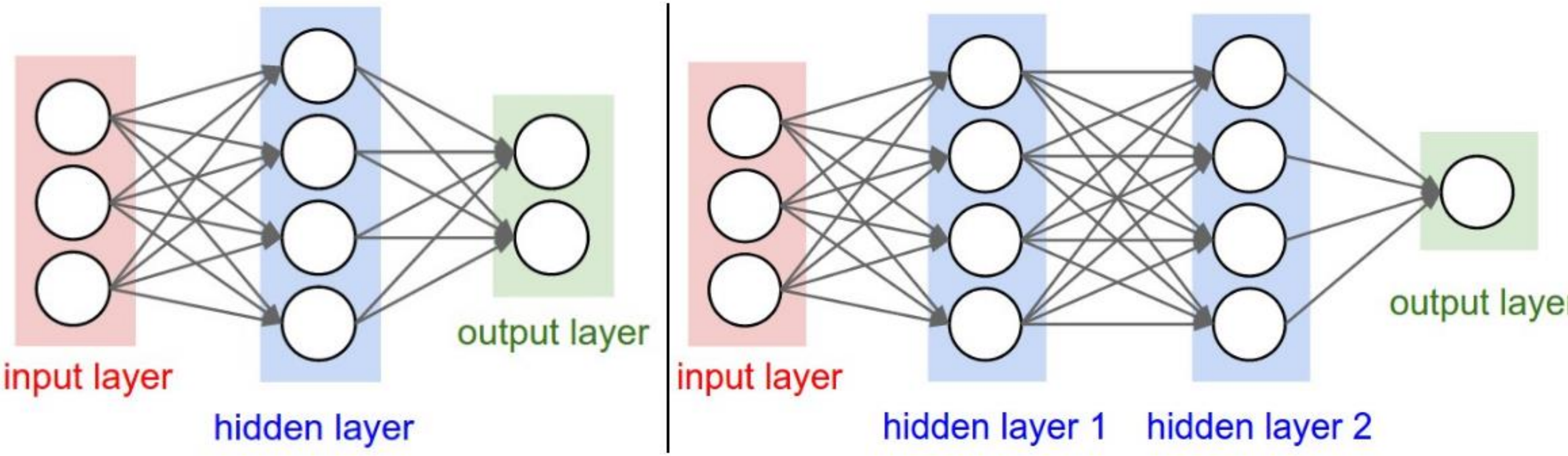
Multi-Layer Neural Networks

- Beyond a single hidden layer:



Source: http://cs231n.github.io

Sizing neural networks



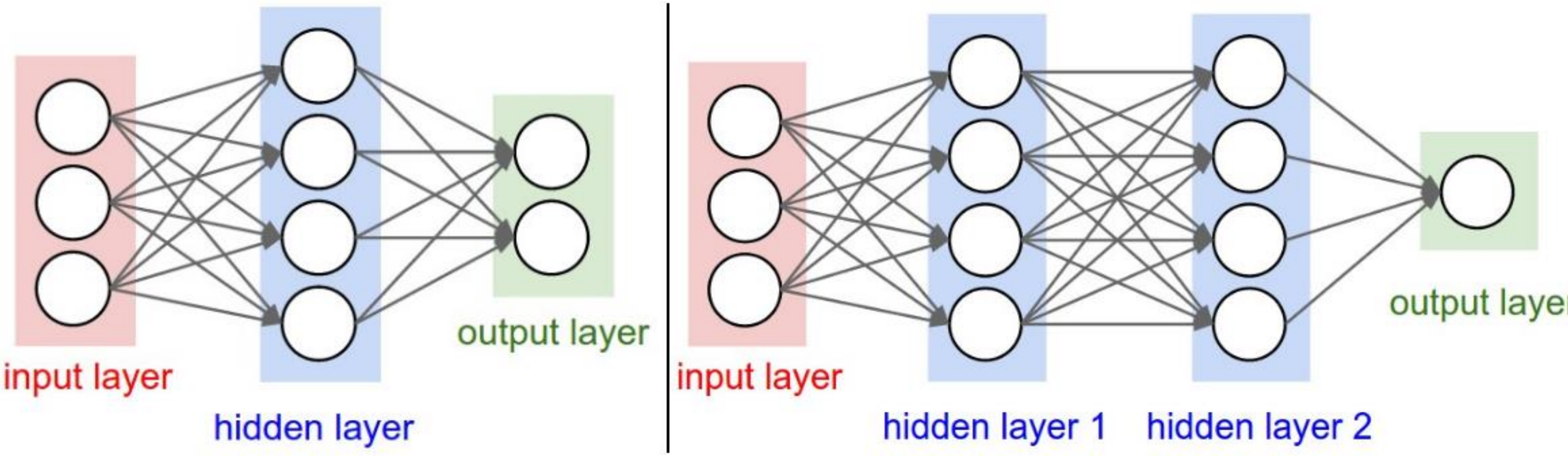
First network (left):

No. of neurons (not counting the inputs):

No. of learnable parameters:

Source: http://cs231n.github.io

Sizing neural networks



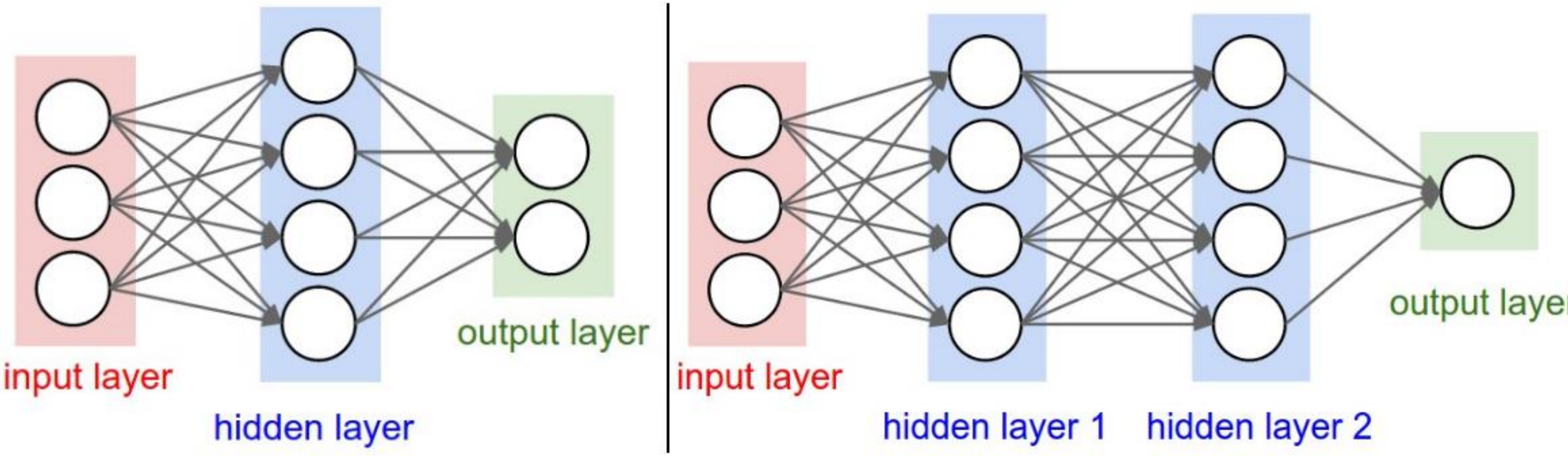
First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters:

Source: http://cs231n.github.io

Sizing neural networks



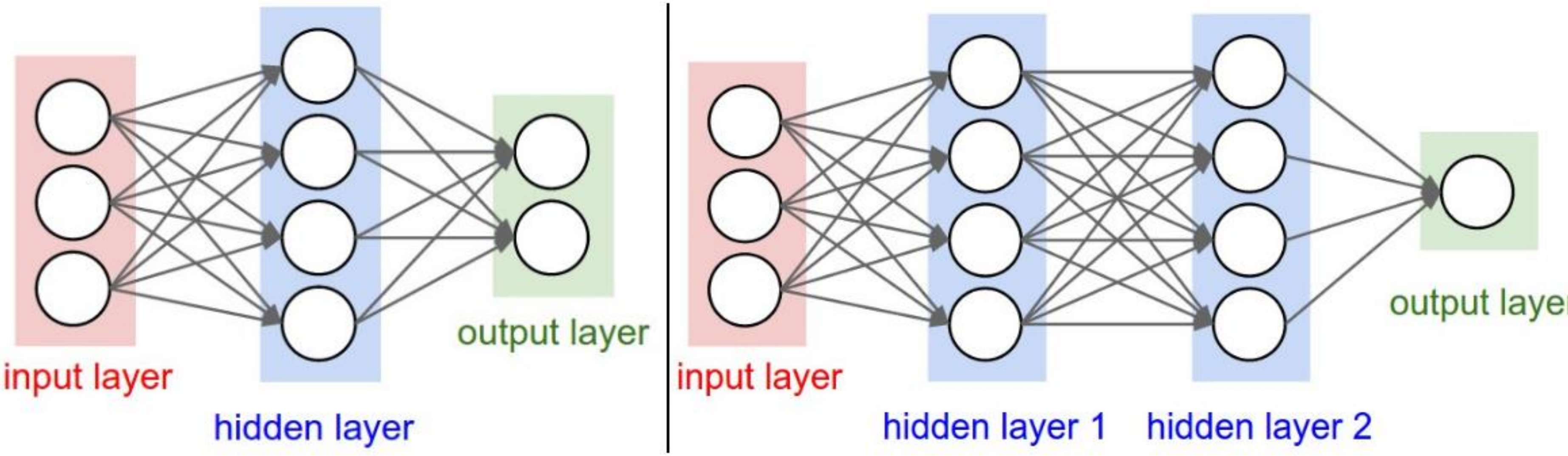
First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights +
4 + 2 = 6 biases = 26.

Source: http://cs231n.github.io

Sizing neural networks



First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights +
4 + 2 = 6 biases = 26.

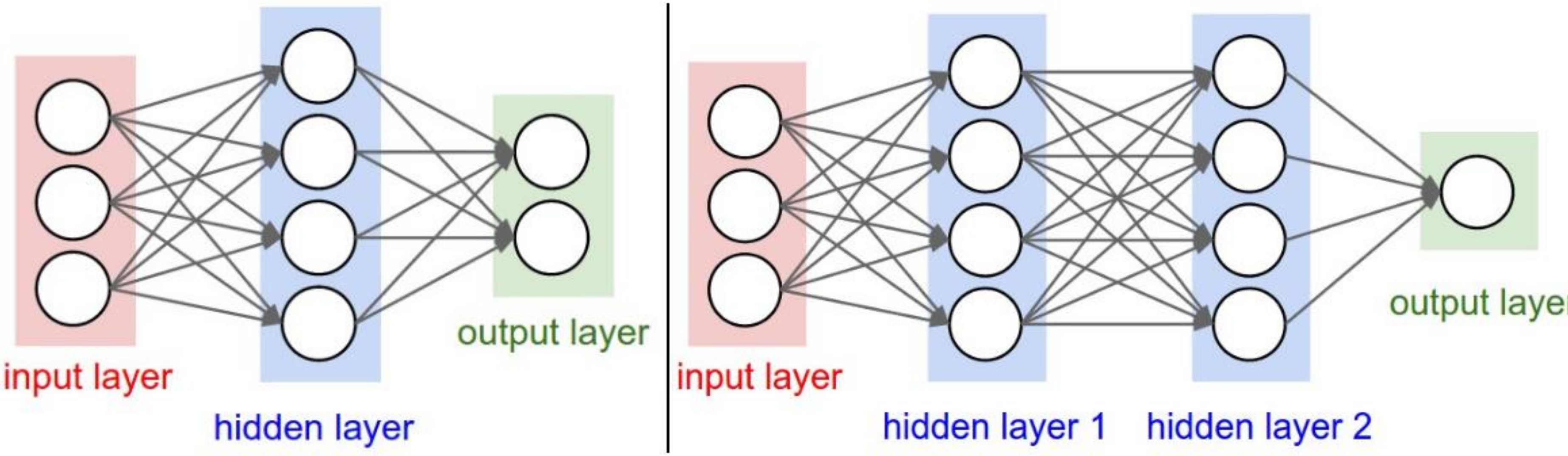
Second network (right):

No. of neurons (not counting the inputs):

No. of learnable parameters:

Source: http://cs231n.github.io

Sizing neural networks



First network (left):

No. of neurons (not counting the inputs): 4 + 2 = 6

No. of learnable parameters: [3 x 4] + [4 x 2] = 20 weights + 4 + 2 = 6 biases = 26.

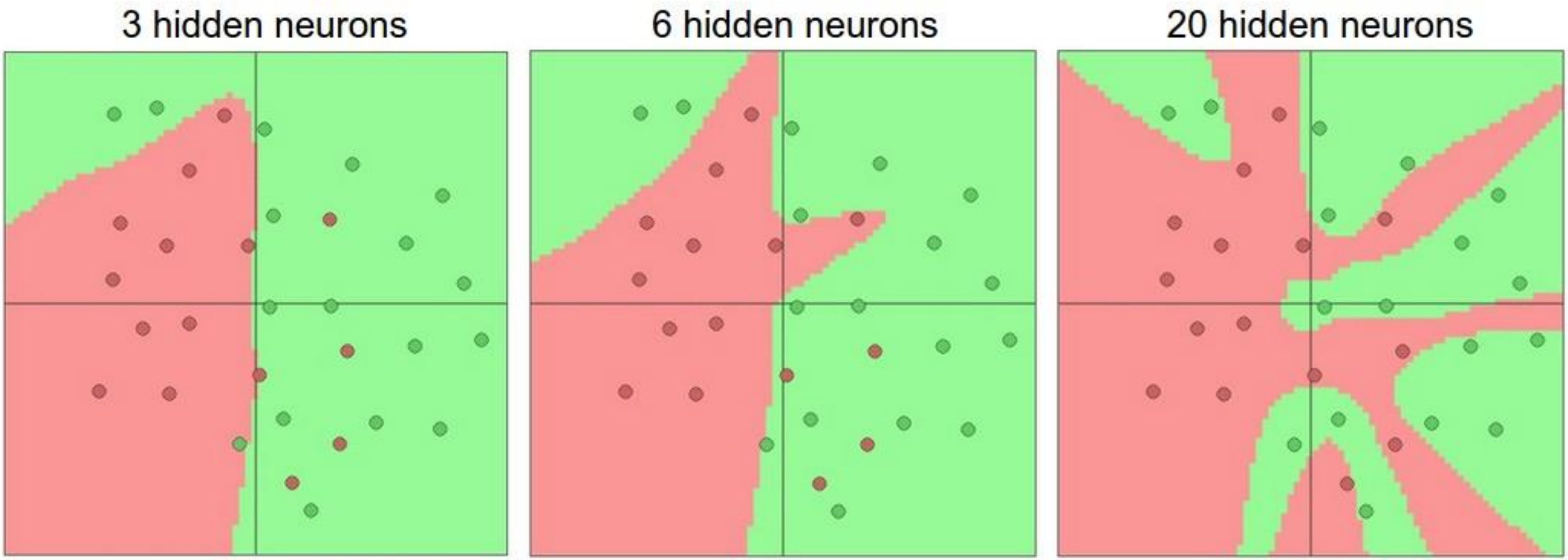
Second network (right):

No. of neurons (not counting the inputs): 4 + 4 + 1 = 9

No. of learnable parameters: [3x4]+[4x4]+[4x1] = 32 weights + 4 + 4 + 1 = 9 biases = 41.

Source: http://cs231n.github.io

Multi-Layer Neural Networks



Source: http://cs231n.github.io

Training of multi-layer networks

- Find network weights to minimize the error between true and estimated outputs of training examples:

$$E(\mathbf{w}) = \sum_{j=1}^{N} \left(y_j - f_{\mathbf{w}}(\mathbf{x}_j) \right)^2$$

Training of multi-layer networks

- Find network weights to minimize the error between true and estimated outputs of training examples:

$$E(\mathbf{w}) = \sum_{j=1}^{N} \left(y_j - f_{\mathbf{w}}(\mathbf{x}_j) \right)^2$$

- Update weights by **gradient descent:** $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial E}{\partial \mathbf{w}}$

Training of multi-layer networks

- Find network weights to minimize the error between true and estimated outputs of training examples:

$$E(\mathbf{w}) = \sum_{j=1}^{N} \left(y_j - f_{\mathbf{w}}(\mathbf{x}_j) \right)^2$$

- Update weights by **gradient descent:** $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial E}{\partial \mathbf{w}}$

- **Back-propagation:** gradients are computed in the direction from output to input layers and combined using chain rule

Neural networks: Pros and cons

- Pros
  - Flexible and general function approximation framework
  - Can build extremely powerful models by adding more layers

Neural networks: Pros and cons

- Pros
  - Flexible and general function approximation framework
  - Can build extremely powerful models by adding more layers

- Cons
  - Hard to analyze theoretically (e.g., training is prone to local optima)
  - Huge amount of training data, computing power may be required to get good performance
  - The space of implementation choices are huge (network architectures, parameters)

Acknowledgements

Thanks to the following researchers for making their teaching/research material online

- Forsyth
- Steve Seitz
- Noah Snavely
- J.B. Huang
- Derek Hoiem
- D. Lowe
- A. Bobick
- S. Lazebnik
- K. Grauman
- R. Zaleski
- Antonio Torralba
- Rob Fergus
- Leibe
- And many more ……….

Next Lecture

Convolutional Neural Networks