Strings

- The set of all possible strings over Σ is denoted by Σ^*.
- We define $\Sigma^0 = \{\epsilon\}$ and $\Sigma^n = \Sigma^{n-1} \cdot \Sigma$
  - with some abuse of the concatenation notation applying to sets of strings now
- So $\Sigma^n = \{\omega | \omega = xy \text{ and } x \in \Sigma^{n-1} \text{ and } y \in \Sigma\}$
- $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \cdots \Sigma^n \cup \cdots = \bigcup_0^\infty \Sigma^i$
  - Alternatively, $\Sigma^* = \{x_1 x_2 \ldots x_n | n \geq 0 \text{ and } x_i \in \Sigma \text{ for all } i\}$
- Φ denotes the empty set of strings $\Phi = \{\}$,
  - but $\Phi^* = \{\epsilon\}$

- $\Sigma = \{ a, b, c\}$
- $\Sigma^1 = \{ a, b, c\}$
- $\Sigma^2 = \{ aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

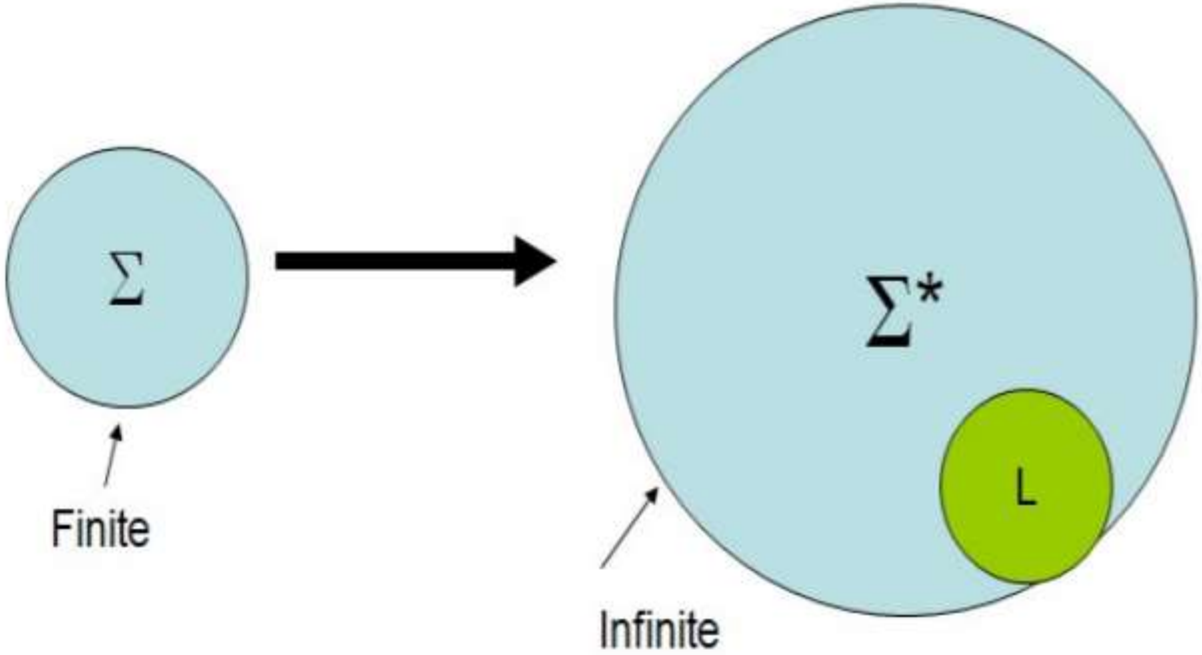
$\Sigma^0 = \varepsilon$ for any Σ

Strings

- Σ^* is a countably infinite set of finite length strings
- If x is a string, we write x^n for the string obtained by concatenating n copies of x.
  - $(aab)^3 = aabaabaab$
  - $(aab)^0 = \epsilon$

Languages

- A language L over Σ is any subset of Σ^*



- L can be finite or (countably) infinite

Some Languages

- $L = \Sigma^*$ – The mother of all languages!
- $L = \{a, ab, aab\}$ – A fine finite language.
  - Description by enumeration
- $L = \{a^n b^n : n \geq 0\} = \{\epsilon, \text{ ab}, \text{ aabb}, \text{ aaabbb}, \ldots\}$
- $L = \{\omega | n_a(\omega) \text{ is even}\}$
  - $n_x(\omega)$ denotes the number of occurrences of x in ω
  - all strings with even number of a's.
- $L = \{\omega | \omega = \omega^R\}$
  - All strings which are the same as their reverses – palindromes.
- $L = \{\omega | \omega = xx\}$
  - All strings formed by duplicating some string once.
- $L = \{\omega | \omega$ is a syntactically correct Java program $\}$

Languages

- Since languages are sets, all usual set operations such as intersection and union, etc. are defined.
- Complementation is defined with respect to the universe $\Sigma^* : \overline{L} = \Sigma^* - L$

Languages

- If L, L_1 and L_2 are languages:
  - $L_1 \cdot L_2 = \{xy | x \in L_1 \text{ and } y \in L_2\}$
  - $L^0 = \{\epsilon\}$ and $L^n = L^{n-1} \cdot L$
  - $L^* = \bigcup_0^\infty L^i$
  - $L^+ = \bigcup_1^\infty L^i$

Finite State Automata

- Finite State Automata (FSA) are the simplest automata.
- Only the finite memory in the control unit is available.
- The memory can be in one of finite states at a given time – hence the name.
  - One can remember only a (fixed) finite number of properties of the past input.
  - Since input strings can be of arbitrary length, it is not possible to remember unbounded portions of the input string.
- It comes in Deterministic and Nondeterministic flavors.

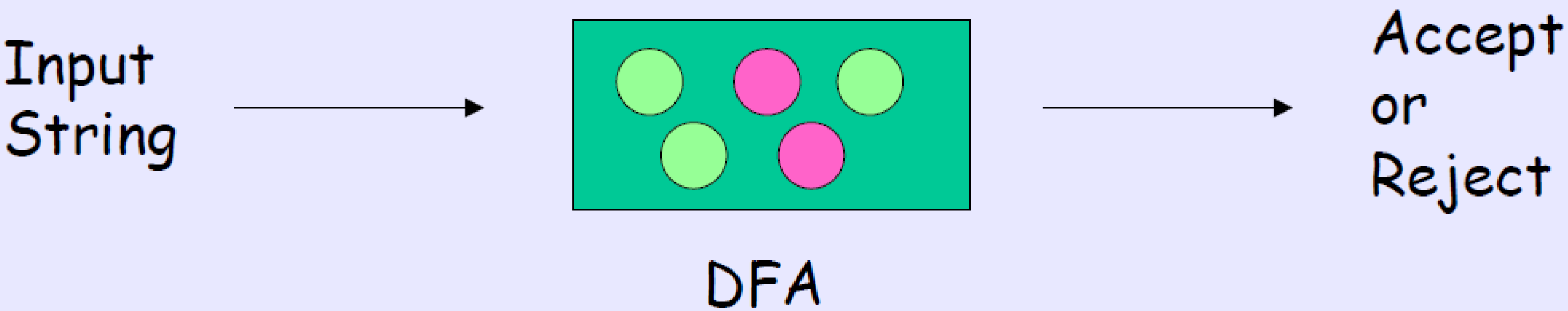
Example: Switch

DETERMINISTIC FINITE STATE AUTOMATA (DFA)

- A DFA starts in a start state and is presented with an input string.
- It moves from state to state, reading the input string one symbol at a time.
- What state the DFA moves next depends on
  - the current state,
  - current input symbol
- When the last input symbol is read, the DFA decides whether it should accept the input string

Finite State Machines

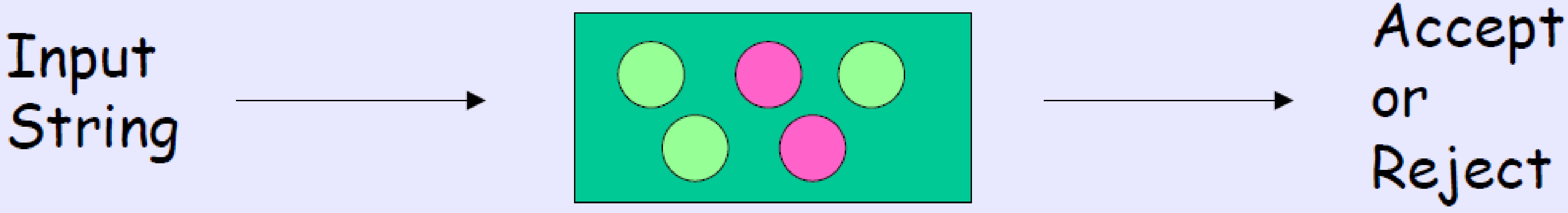
DFA



- A machine with finite number of states, some states are accepting states, others are rejecting states
- At any time, it is in one of the states
- It reads an input string, one character at a time

DFA

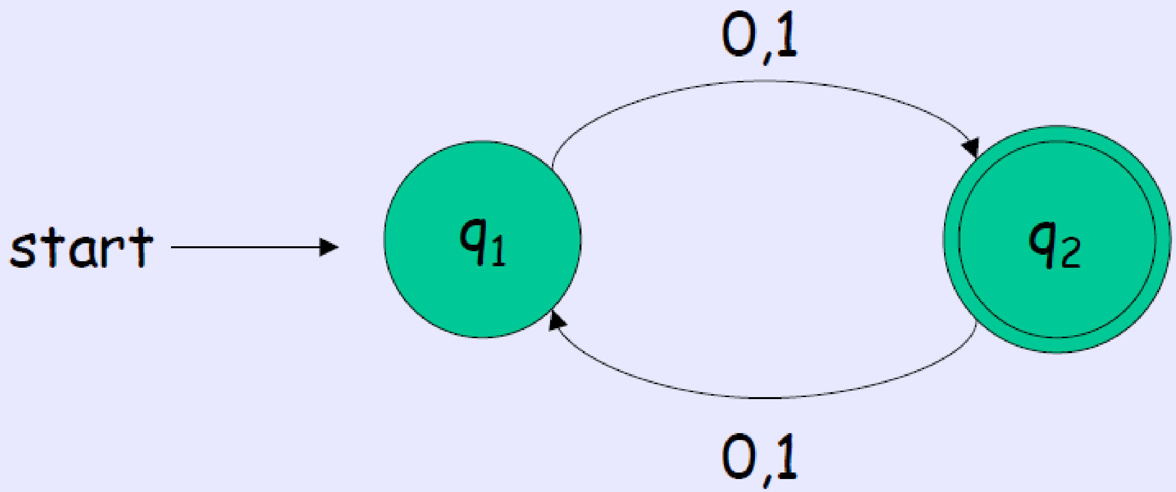
Example: Pattern recognition



DFA

- After reading each character, it moves to another state depending on what is read and what is the current state
- If reading all characters, the DFA is in an accepting state, the input string is accepted.
- Otherwise, the input string is rejected.

Example of DFA



- The circles indicates the states
- If accepting state is marked with double circle
- The arrows pointing from a state q indicates how to move on reading a character when current state is q

DFA – Formal Definition

- A Deterministic Finite State Acceptor (DFA) is defined as the 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where
  - Q is a finite set of states
  - Σ is a finite set of symbols – the alphabet
  - $\delta : Q \times \Sigma \rightarrow Q$ is the next-state function
  - $q_0 \in Q$ is the (label of the) start state
  - $F \subseteq Q$ is the set of final (accepting) states

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Note, there must be exactly one start state.
Final states can be many or even empty !

Some Terminology

Let M be a DFA

- Among all possible strings, M will accept some of them, and M will reject the remaining
- The set of strings which M accepts is called the language recognized by M
- That is, M recognizes A if

$$A = \{ w \mid M \text{ accepts } w \}$$