

INTRODUCTION TO DATA ANALYTICS

Class #7

Data Pre-processing – Data Transformation

Dr. Sreeja S R

Assistant Professor
Indian Institute of Information Technology
IIIT Sri City

QUOTE OF THE DAY..

Nothing great was ever achieved without enthusiasm.

• RALPH WALDO EMERSON, American philosopher

Data transformation are needed to bring uniformity to the data. In addition, it can be used to scale the data to a preferred range. Following are few methods as data transformation.

- Change of origin
- Change of scale
- Change of origin and scale
- Decimal scaling
- Min-Max normalization
- Standard normalization

1. Change of Origin:

- Arbitrarily choose a constant. If sample values are integers, an integer constant is preferred.
- Shift data point by subtracting (or adding) the chosen constant from each sample observation.
- This technique is useful when data values are large, and variability is not so large.

Lemma 6.1:

- a) The range of original data is preserved by a change of origin transformation.
- b) If \bar{x} is the old mean and c is the chosen constant, then the new mean of the transformed data is $\bar{x} = c + \bar{x}'$

Example:

Apply change of origin method to the following data and calculate the old and new means.

$$x = \{115, 128, 110, 104, 133\}$$

Let us arbitrarily chose the constant 120 and subtract this from each value in x to get the transformed data x' as

$$x' = \{-5, 8, -10, -16, 13\}$$

The new mean $\bar{x}' = \frac{\sum x_i'}{5} = -2$

The mean of the original data is

$$\bar{x} = c + \bar{x}' = 120 - 2 = 118$$

2. Change of scale:

- This method is used to shorten the range of large numbers or lengthen the range of very small numbers.
- Chose (arbitrarily) a non-zero constant c. If c is less then the minimum of the observation, each value will be transformed to a value greater than 1. On the other hand, if it is greater than the maximum of the observation, then each value will be transformed to a value less than 1.
- If the value is between min and max of the sample, then the transformed values lie on the real line (positive real line if all x_i 's are positive)
- If all values are small fractions, we may multiply a large constant to scale them up and vice-versa.

3. Change of origin and scale:

• This is the most frequently used method to standardize data values. Depending upon the constants used to change the origin and scale, a variety of transformation intervals can be obtained.

Example: A sample in the range (a, b) can be transformed to a new interval (c, d) by the following transformation.

Let x is the original and y is the transformed value. Then

$$y = c + \frac{(d-c)}{(b-a)} \times (x-a)$$

[Prove that all values in the range (a, b) are mapped onto the range (c, d)].

4. Min-Max Normalization:

- Min-Max normalization performs a linear transformation on the original data.
- Suppose, min_A and max_A are the minimum and maximum values of an attribute A. Min-Max normalization maps a value, v of A to v' in the range min_A' and max_A' using the following transformation:

$$v' = \frac{(v - min_A)}{(max_A - min_A)} \times (max_A' - min_A') + min_A'$$

If $[min_A', max_A'] = [0,1]$, then it is a special case of Min-max normalization.

5. Standard Normalization:

- This transformation is so called because it is extensively used in statistics in standardizing normal scores.
- Here, the origin is changed using the mean of the sample, and the scale is changed using the standard deviation of the sample.

$$v' = \frac{(v - \bar{A})}{\sigma_A}$$

where \overline{A} and σ_A are the mean and std for the attribute A.

• This method is also alternatively termed as z-score normalization (zero-mean normalization) and the transformed values are called z-scores.

5. Standard Normalization:

Example: Given $X = \{32, 80, 56, 75, 69, 26, 44, 50\}$. Apply the standard normalization.

Here, the mean $\bar{A} = 54$, $\sigma_A^2 = 390$. Thus the z-scores are $v' = \frac{(v - \bar{A})}{\sigma_A} = \{-1.1140133 \quad 1.3165612 \quad 0.1012739 \quad 1.0633763 \quad 0.7595545 \quad -1.4178351 \quad -0.5063697 \quad -0.2025479\}$

Note: It is very interesting to note that z-scores will always lie in the interval [-3, +3]

6. Decimal Scaling:

This data transformation method is same as the "change of scale" method by either scale up or scale down.

$$v' = \frac{v}{10^j}$$

Here j represents movement of decimal points. Decided based on the maximum value in the data.

Non-linear transformation

All the transformation methods discussed above are called linear transformation methods. Linear data transformation methods may be insufficient in some data missing applications. The popular non-linear transformations are:

- Square-root transformation
- Logarithmic and exponential transformation
- Hyperbolic transformation
- Trigonometric transformation
- Power transformation
- Polynomial transformation, etc.

These transformations are used either to stabilize the variance or bring the data into one of the well-known distributional form.