

Computer Graphics and Multimedia

Light: Radiometry and Reflectance

Dr. Mrinmoy Ghorai

**Indian Institute of Information Technology
Sri City, Chittoor**



Today's Agenda

- Light
 - Radiometry
 - Reflectance

Why should we care?

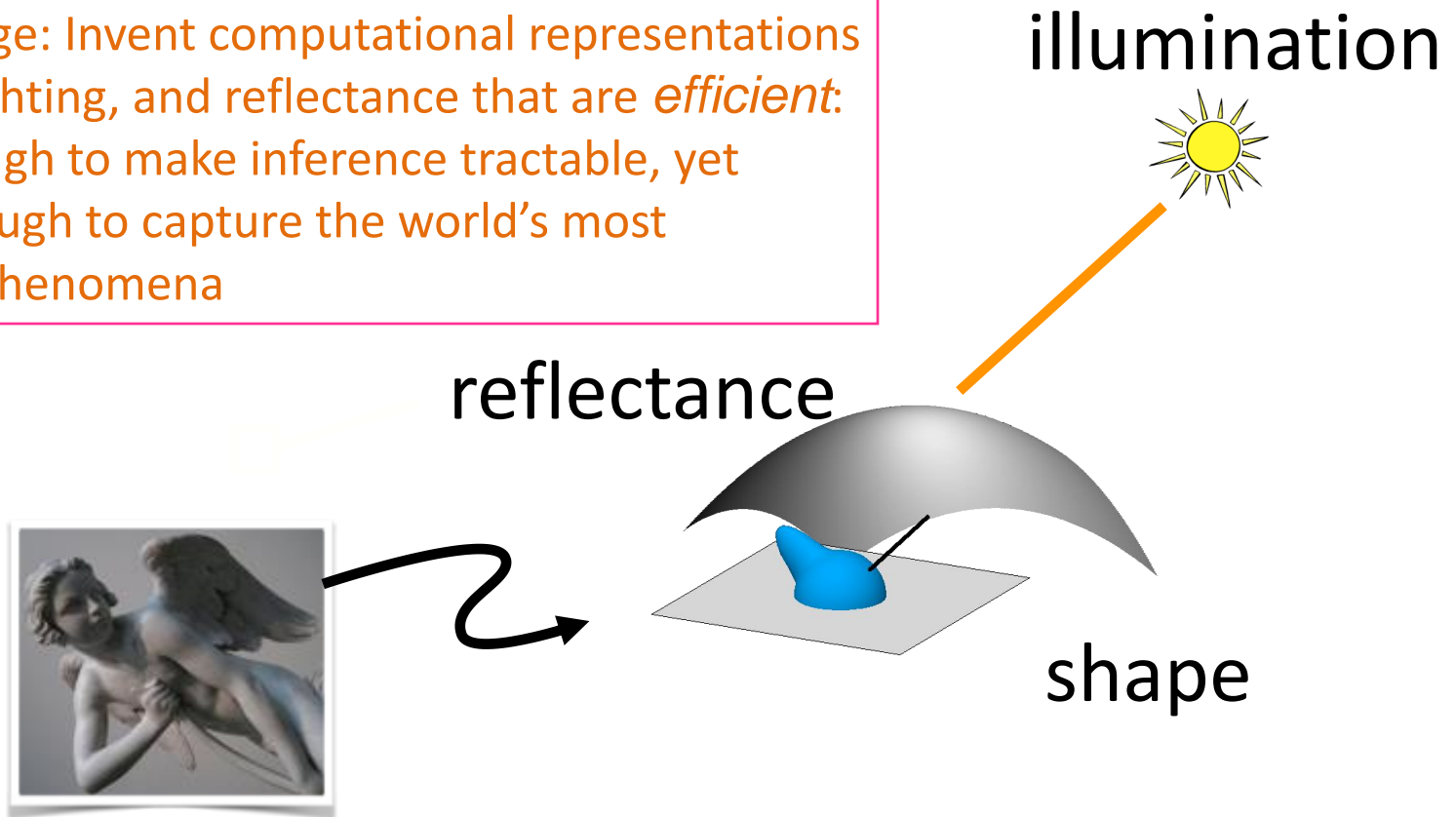
- The **appearance** of objects is given by the way in which they **reflect** and transmit **light**.
- The **color** of objects is determined by the parts of the **spectrum of** (incident white) **light** that are reflected or transmitted without being absorbed.

Appearance



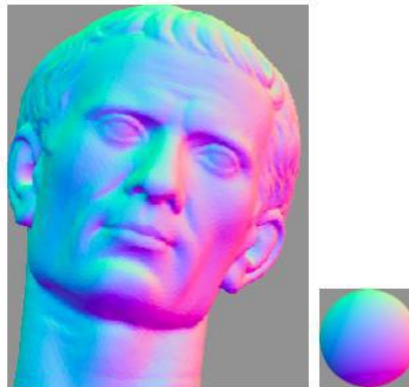
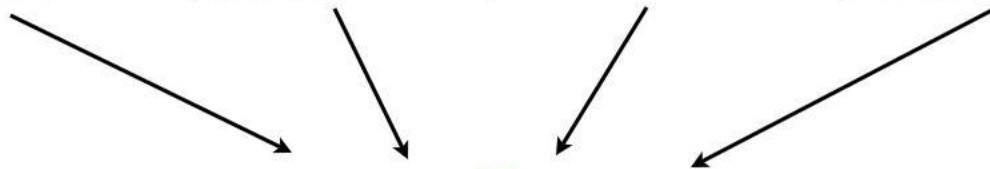
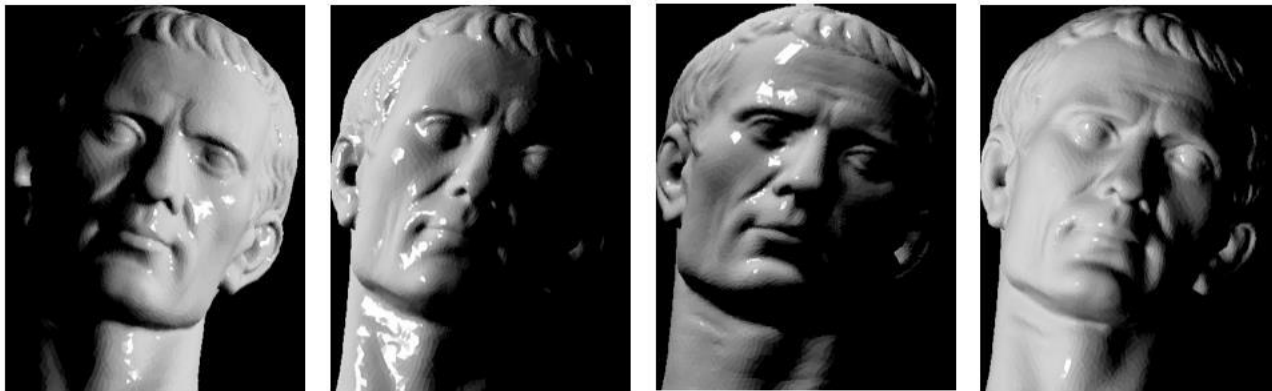
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are *efficient*: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



I \Rightarrow shape, illumination, reflectance

Application: Photometric Stereo



Analysis under different lighting conditions to estimate a normal direction at each pixel.

- Why study the physics (optics) of the world?
- Lets see some pictures!

Light and Shadows



Light and Shadows



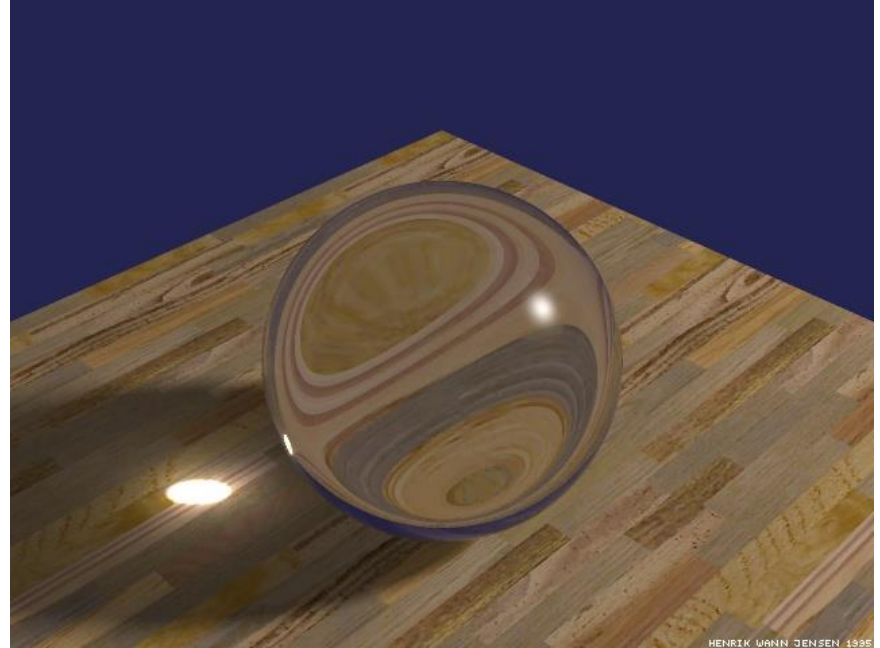
Reflections



Reflections



Refractions



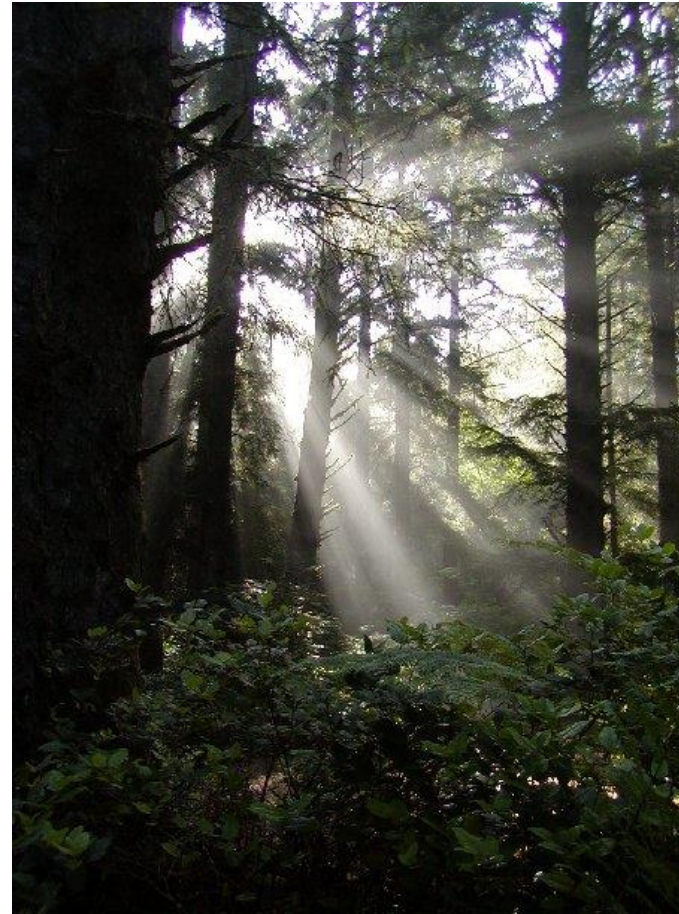
Refractions



Inter-reflections



Scattering



Scattering

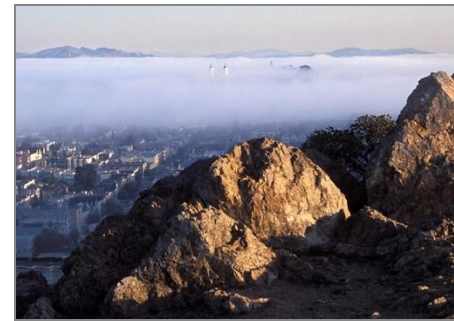


More Complex Appearances

More Complex Appearances

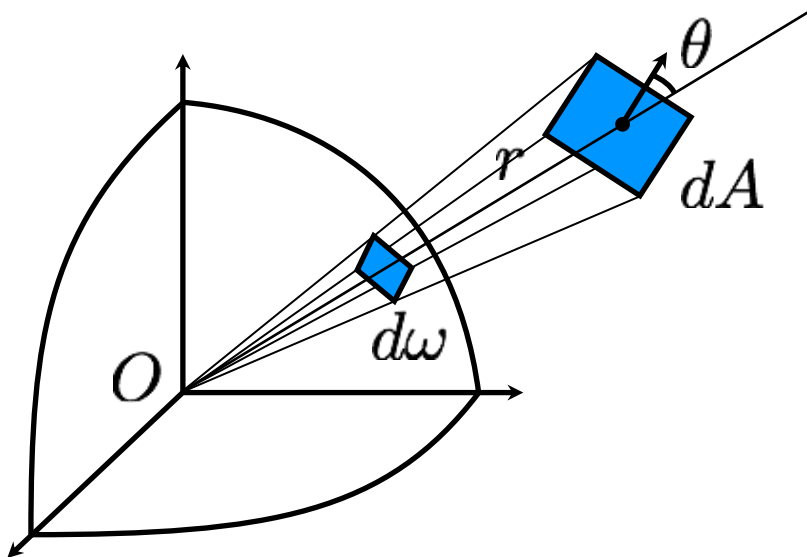


More Complex Appearances



Measuring Light and Radiometry

- **Solid angle:** The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

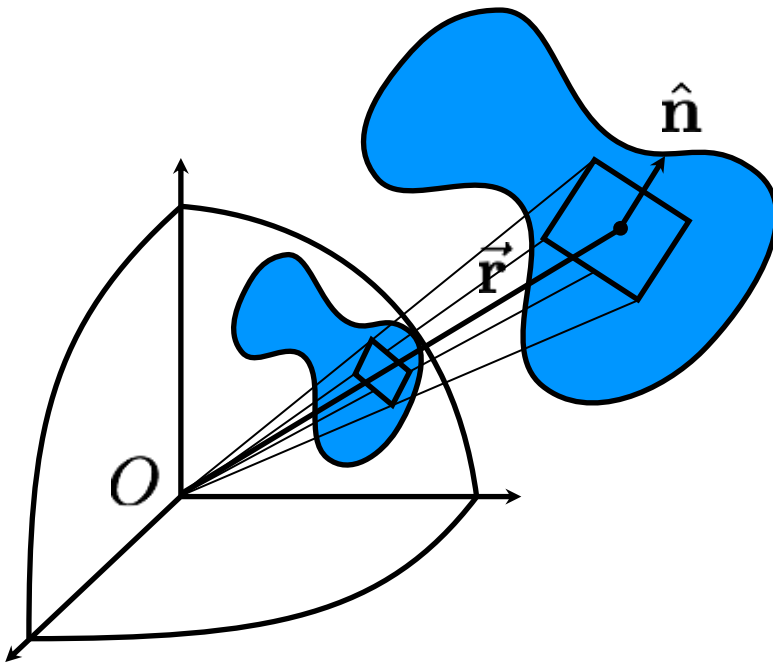
One can show:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Units: steradians [sr]

Measuring Light and Radiometry

- To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{r} \cdot \hat{n} dS}{|\vec{r}|^3}$$

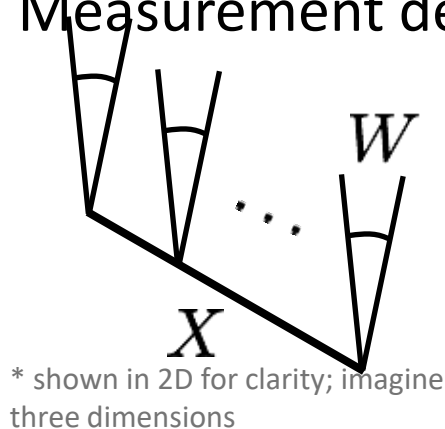
One can show:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Units: steradians [sr]

Quantifying light: flux, irradiance, and radiance

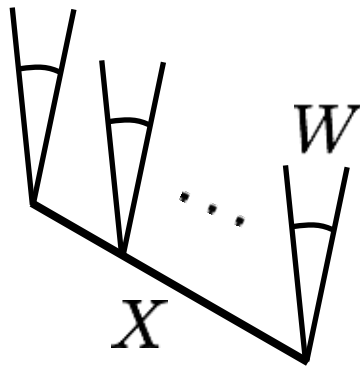
- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area $|X|$



radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

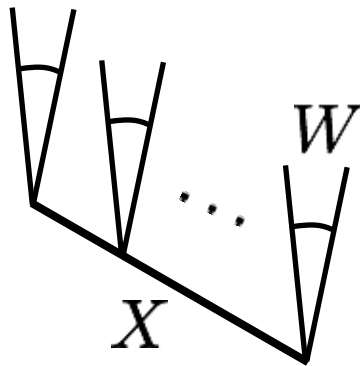
- *Irradiance*:
A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$



$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$

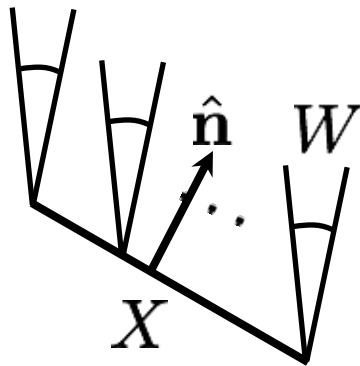


$$\lim_{X \rightarrow x}$$

$$\frac{\Phi(W, X)}{|X|}$$

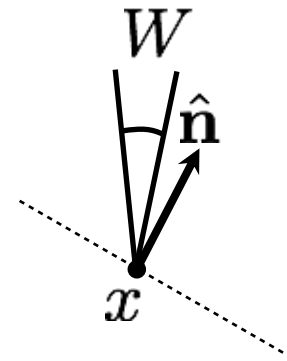
Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$
- Depends on sensor direction normal.



$$\frac{\Phi(W, X)}{|X|}$$

$$\lim_{X \rightarrow x}$$

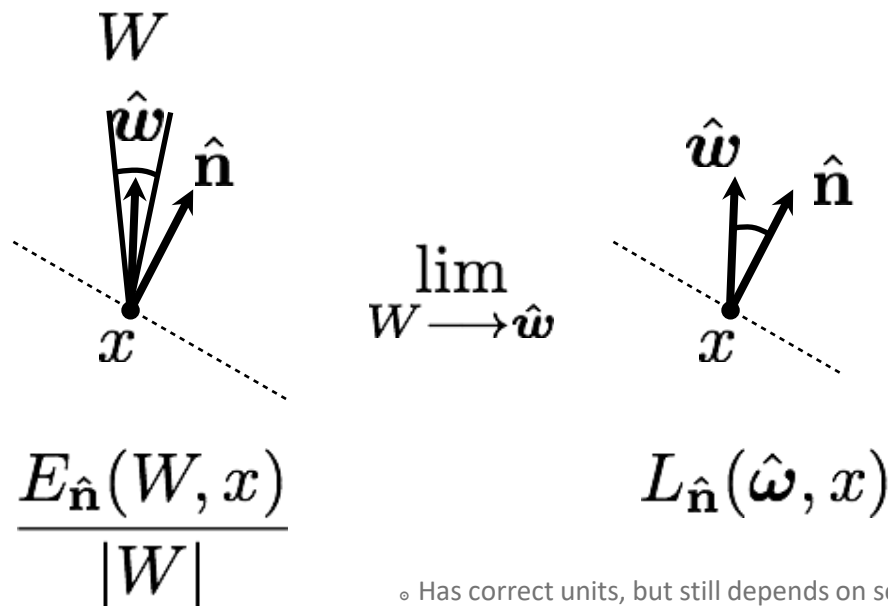


$$E_{\hat{n}}(W, x)$$

- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

Quantifying light: flux, irradiance, and radiance

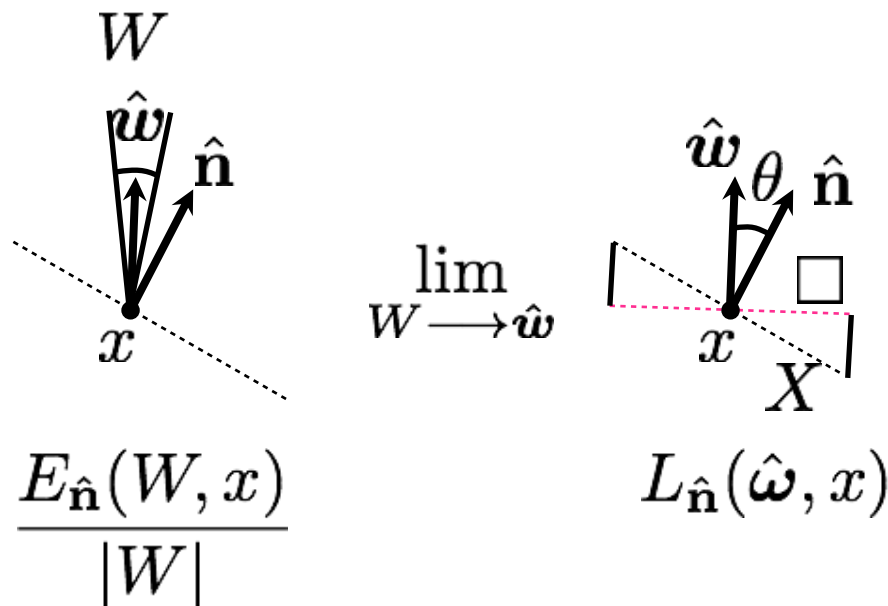
- *Radiance*:
A measure of incoming light that is independent of sensor area $|X|$, orientation \mathbf{n} , and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter $[W/(m^2 \cdot sr)]$



- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
A measure of incoming light that is independent of sensor area $|X|$, orientation \hat{n} , and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter $[W/(m^2 \cdot sr)]$



$$\cos \theta = \frac{\square/2}{|X|/2}$$

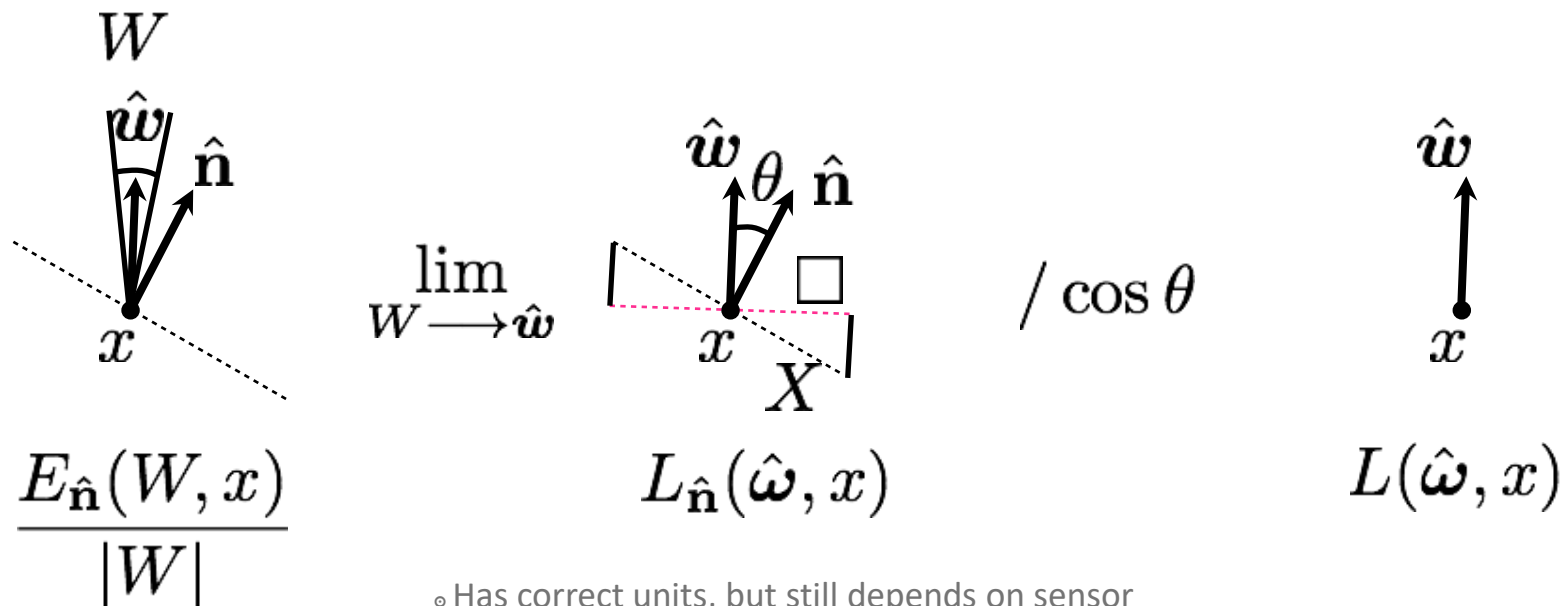
$$\rightarrow \square = |X| \cos \theta$$

“foreshortened area”

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

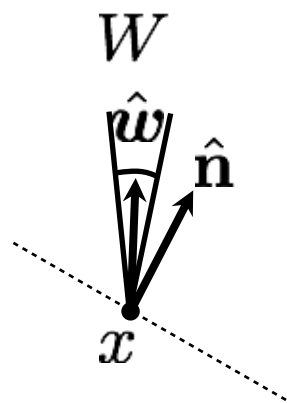
- *Radiance*:
A measure of incoming light that is independent of sensor area $|X|$, orientation \hat{n} , and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter $[W/(m^2 \cdot sr)]$



- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was

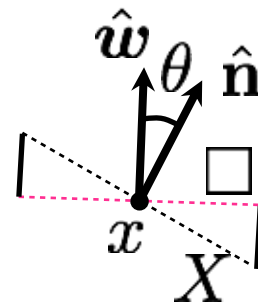
Quantifying light: flux, irradiance, and radiance

- *Radiance*:
A measure of incoming light that is independent of sensor area $|X|$, orientation \hat{n} , and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter $[W/(m^2 \cdot sr)]$



$$\frac{E_{\hat{n}}(W, x)}{|W|}$$

$\lim_{W \rightarrow \hat{\omega}}$



$$L_{\hat{n}}(\hat{\omega}, x)$$

$/ \cos \theta$



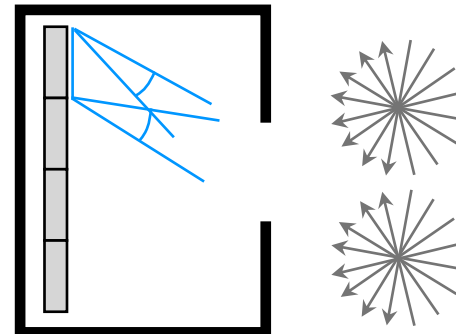
$$L(\hat{\omega}, x)$$

"foreshortened in the direction of travel"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

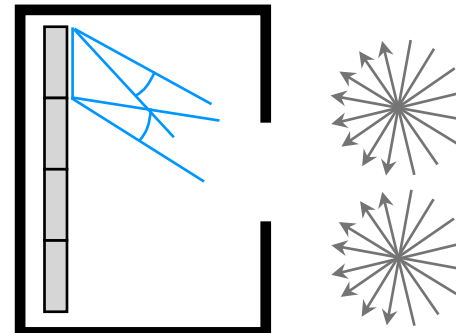
- Attractive properties of radiance:
 - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
 - Allows computing the radiant flux measured by *any* finite sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$



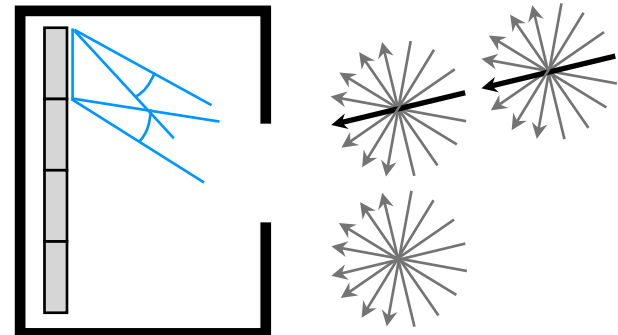
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
 - Allows computing the radiant flux measured by *any* finite sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$

- Constant along a ray in free space

$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$



Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
 - Allows computing the radiant flux measured by *any* finite sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$

- Constant along a ray in free space

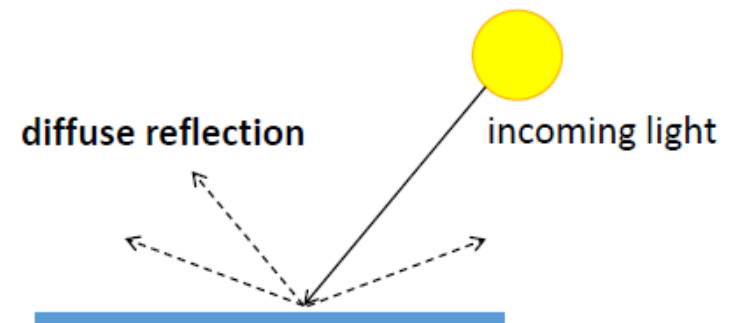
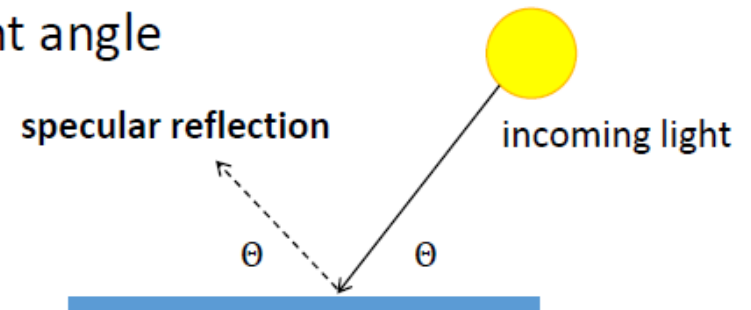
$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$

- A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
 - “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Reflectance and BRDF

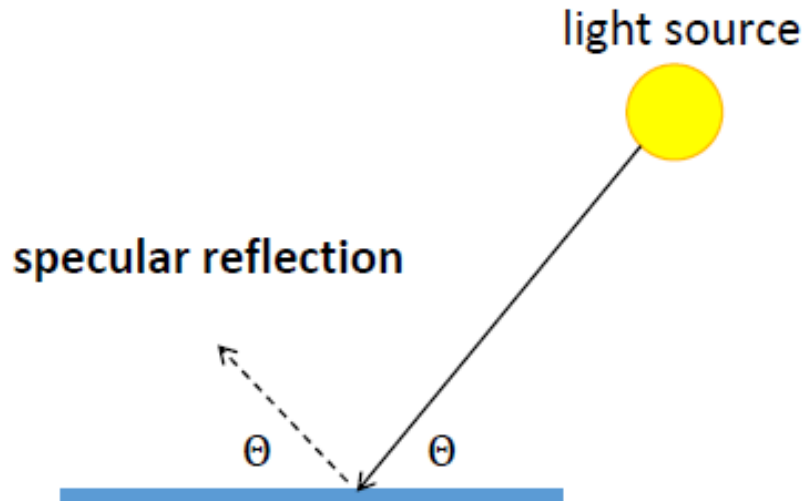
Basic models of reflection

- Specular: light bounces off at the incident angle
 - E.g., mirror
- Diffuse: light scatters in all directions
 - E.g., brick, cloth, rough wood



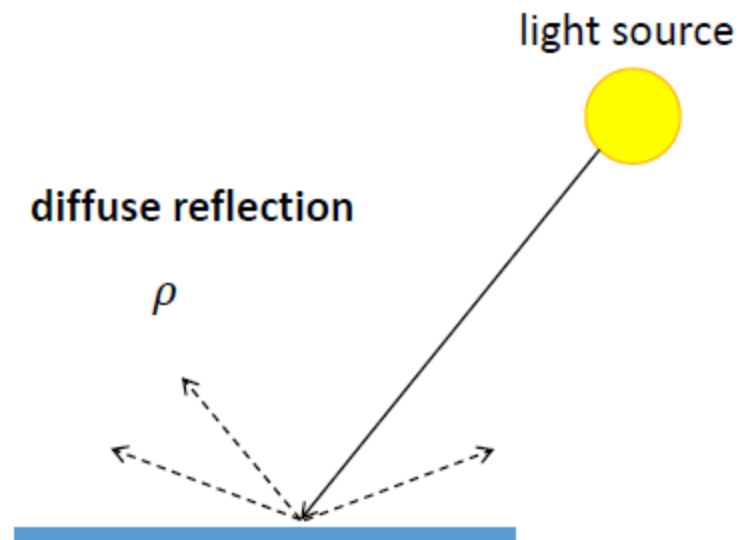
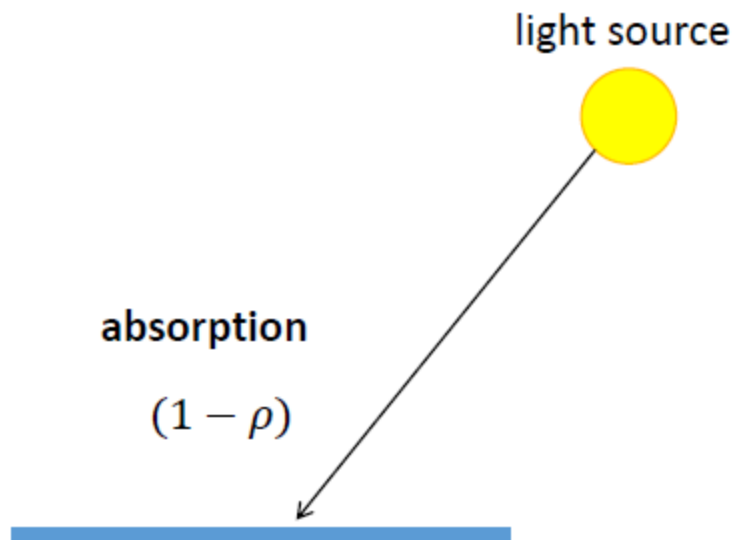
Specular Reflection

- Reflected direction depends on light orientation and surface normal
 - E.g., mirrors are fully specular



Lambertian reflectance model

- Some light is absorbed (function of albedo ρ)
- Remaining light is scattered (diffuse reflection)
- Examples: soft cloth, concrete, matte paints



Most surfaces have both specular and diffuse components

- Specularity = spot where specular reflection dominates (typically reflects light source)



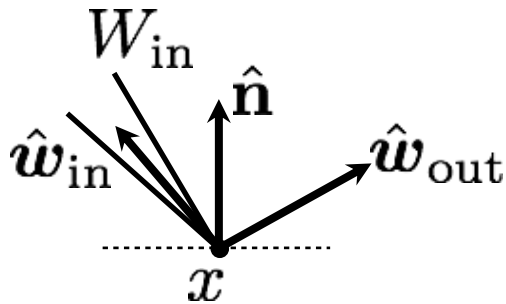
Photo: northcountryhardwoodfloors.com



Typically, specular component is small

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
 - converges as we use smaller and smaller incoming and outgoing wedges
 - does not depend on the size of the wedges (i.e. is intrinsic to the material)

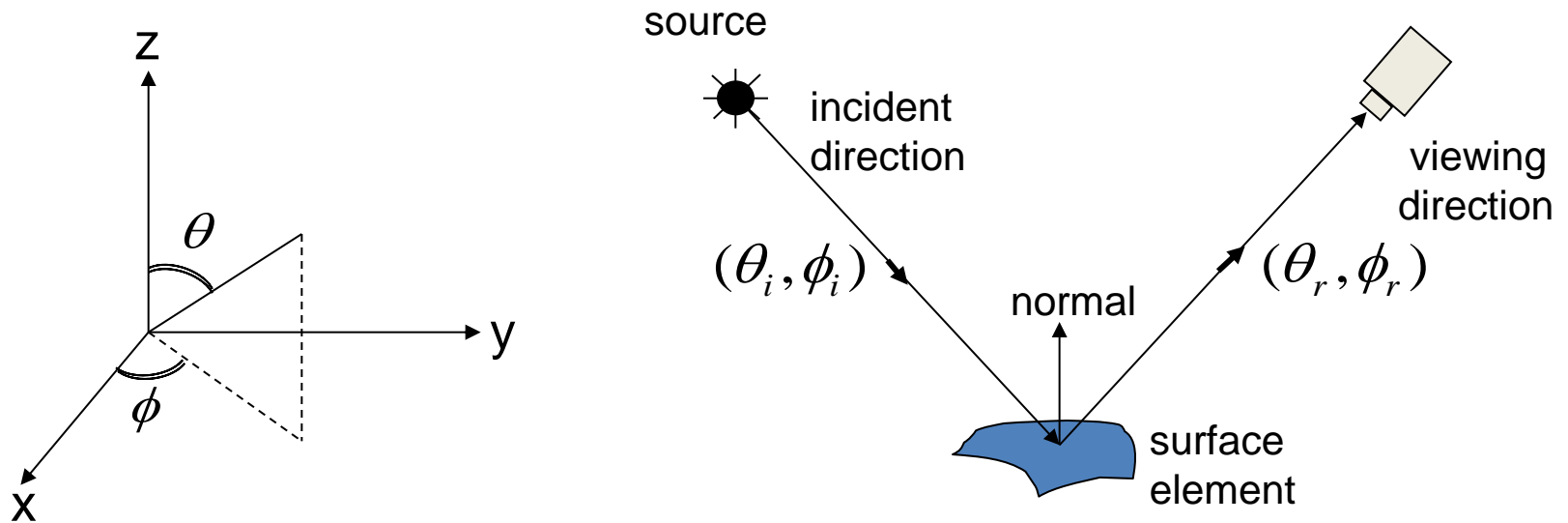


$$\lim_{W_{in} \rightarrow \hat{w}_{in}} f_{x, \hat{n}}(\hat{w}_{in}, \hat{w}_{out})$$

$$f_{x, \hat{n}}(W_{in}, \hat{w}_{out}) = \frac{L^{out}(x, \hat{w}_{out})}{E_{\hat{n}}^{in}(W_{in}, x)}$$

- Notations x and n often implied by context and omitted; directions ω are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function



$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

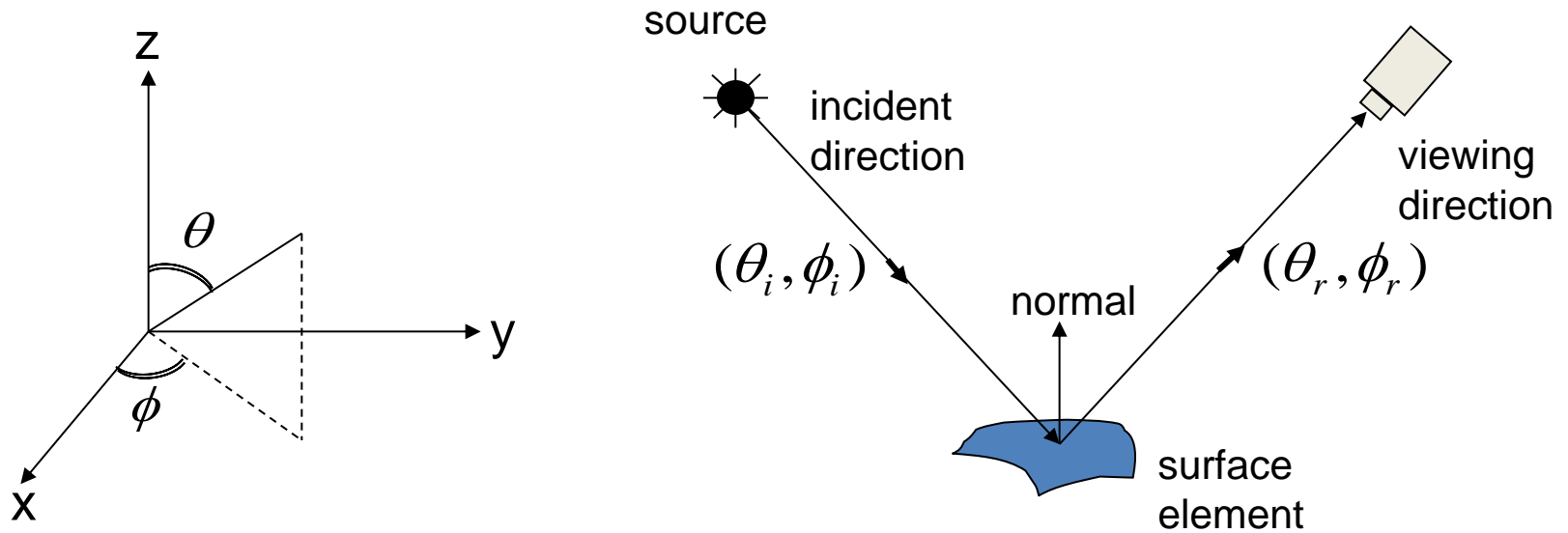
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF} : f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties

Important Properties of BRDFs

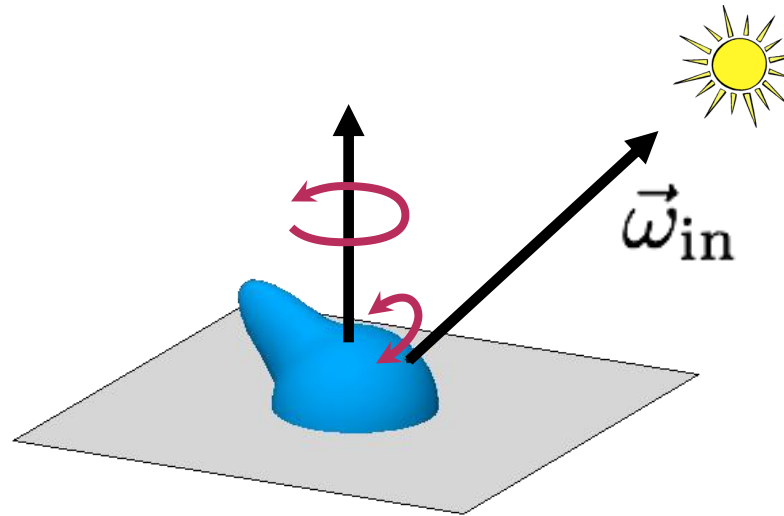
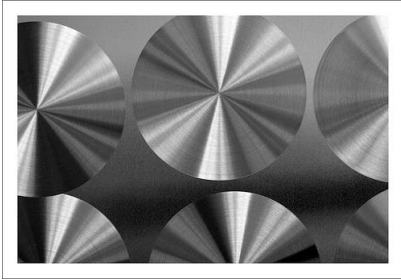


- Conservation of Energy:

$$\forall \hat{\omega}_{\text{in}}, \int_{\Omega_{\text{out}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) \cos \theta_{\text{out}} d\hat{\omega}_{\text{out}} \leq 1$$

Why smaller than or equal?

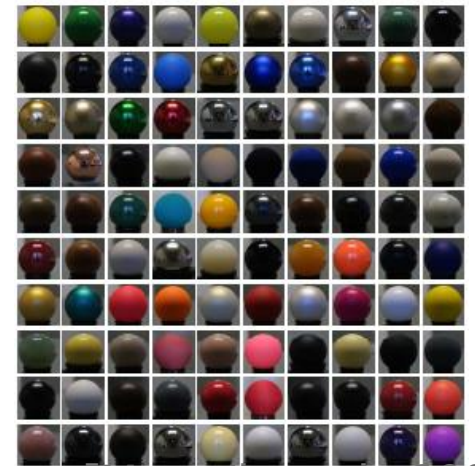
Common assumption: Isotropy



BRDF does not change
when surface is rotated
about the normal.

4D \rightarrow 3D

$$f_r(\vec{\omega}_{in}, \vec{\omega}_{out})$$



[Matusik et al., 2003]

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

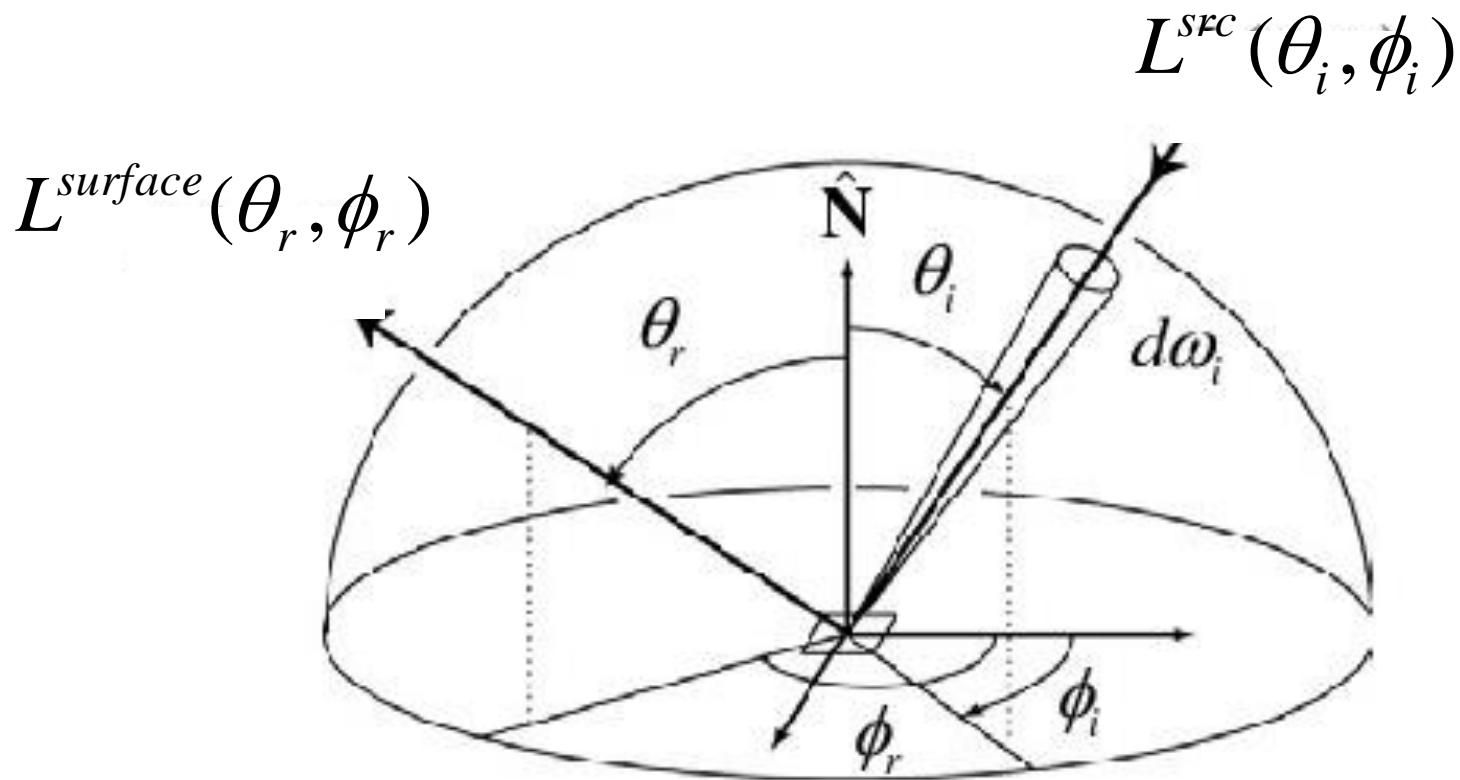
- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Reflectance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = E^{surface}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = \underline{E^{surface}(\theta_i, \phi_i)} f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r, \phi_r) = \underline{L^{src}(\theta_i, \phi_i)} f(\theta_i, \phi_i; \theta_r, \phi_r) \underline{\cos \theta_i d\omega_i}$$

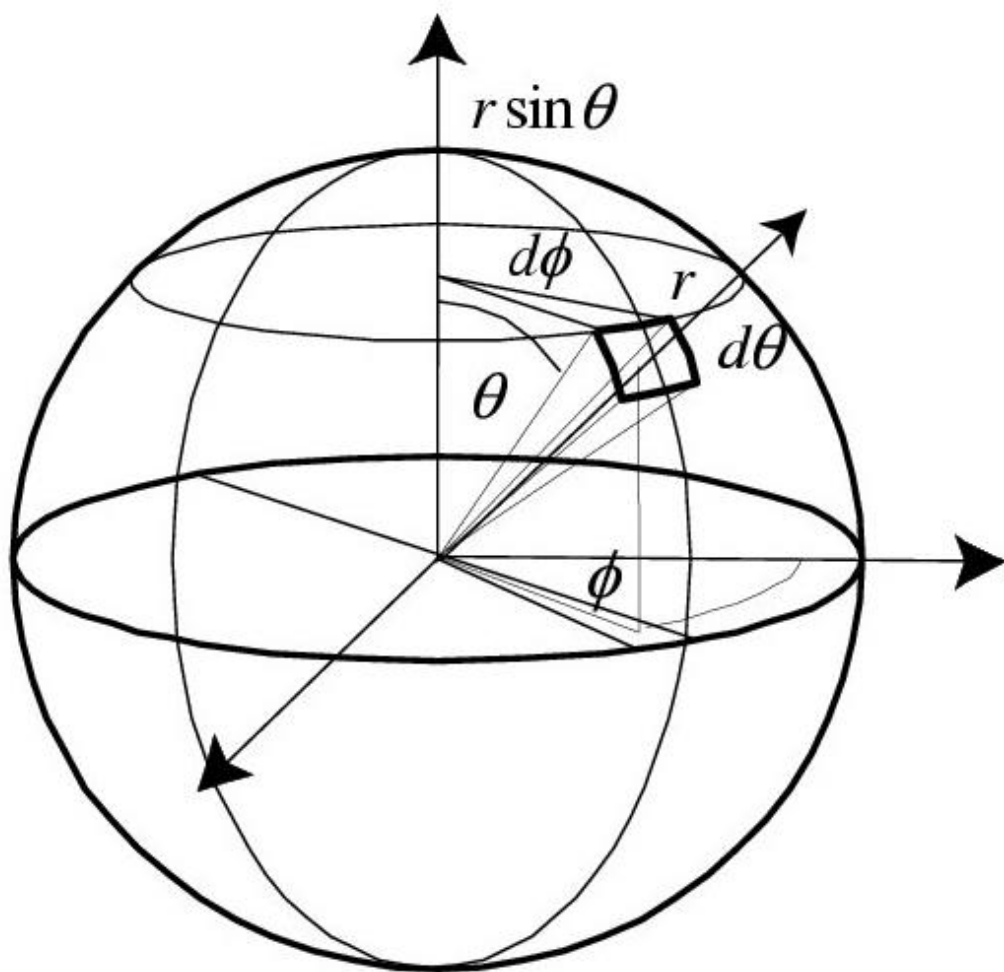
Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r, \phi_r) = \int_{2\pi} L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_0^{\pi/2} L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \underline{\sin \theta_i d\theta_i d\phi_i}$$

Differential Solid Angles

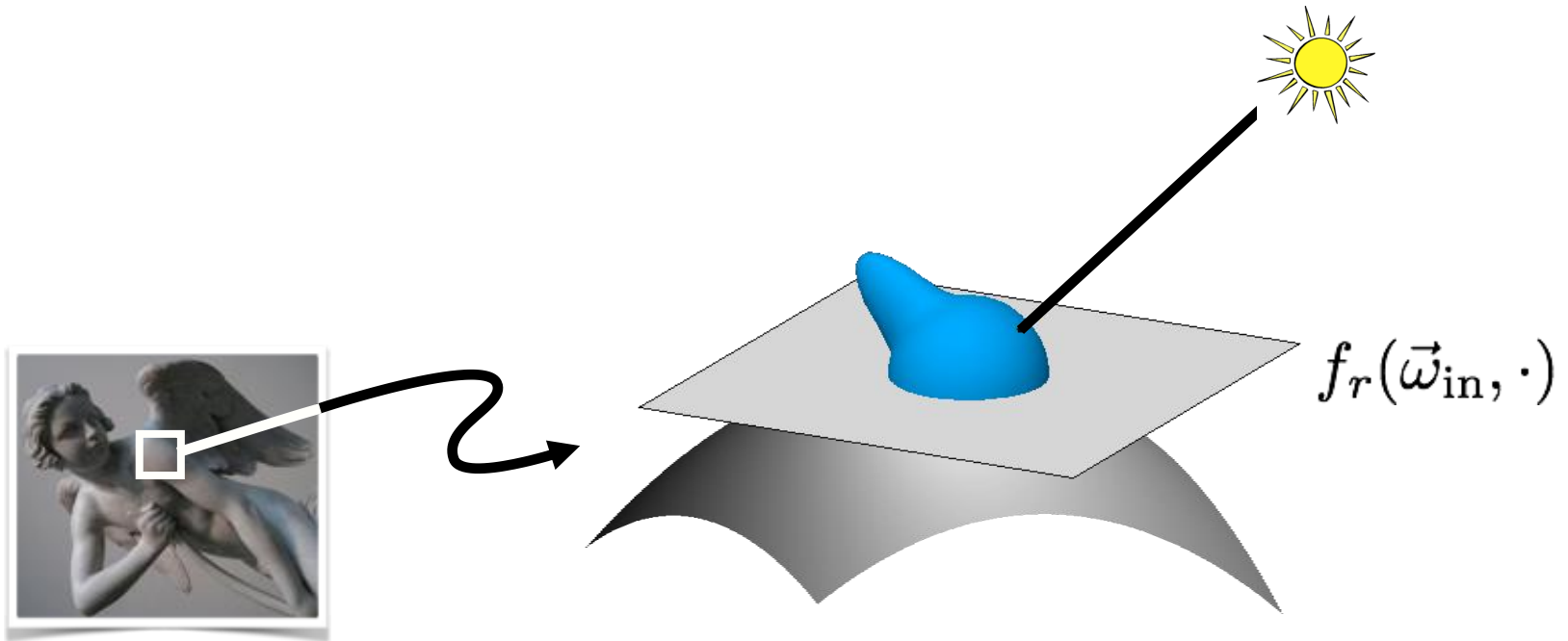


$$\begin{aligned} dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi \end{aligned}$$

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$S = \int_0^\pi \int_0^{2\pi} \sin \theta d\theta d\phi = 4\pi$$

BRDF



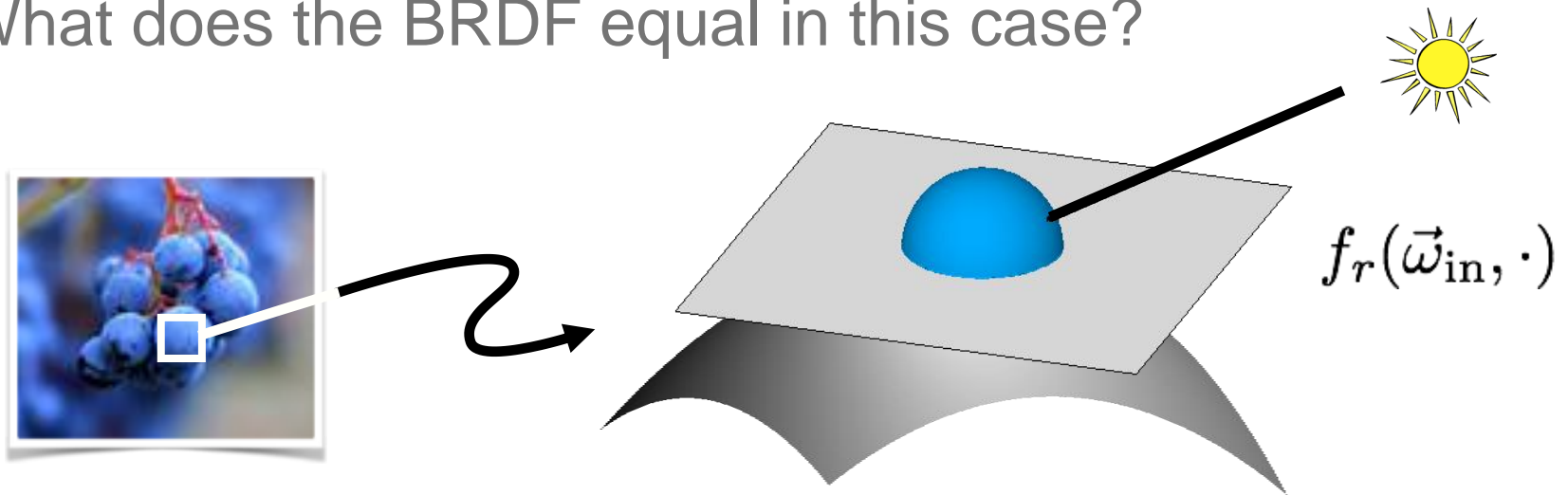
$$f_r(\vec{\omega}_{in}, \vec{\omega}_{out})$$

Bi-directional Reflectance Distribution Function (BRDF)

BRDF

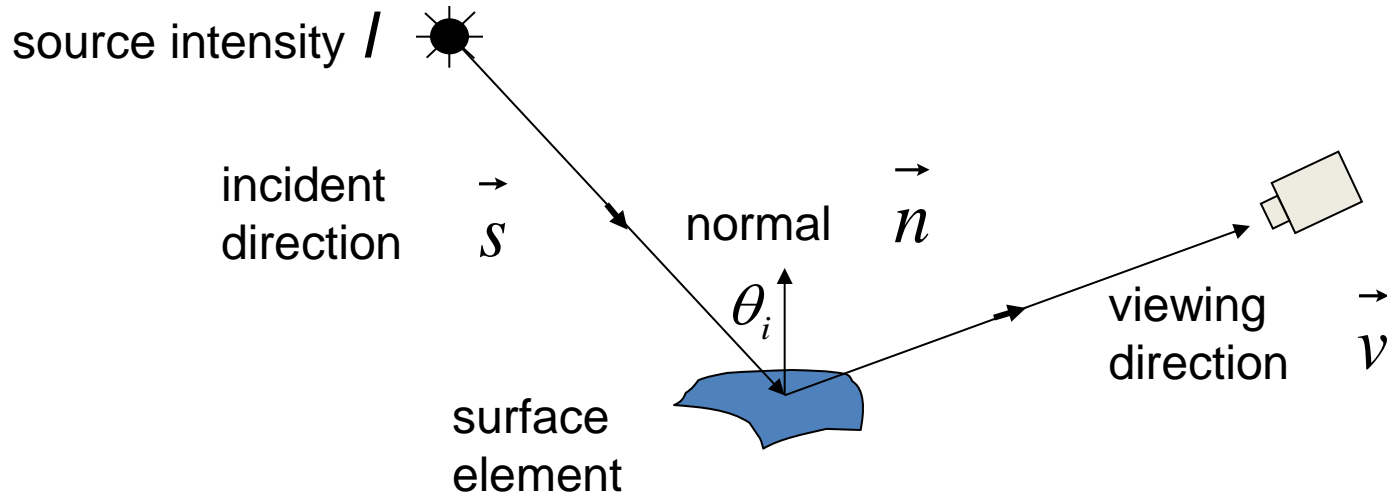
Lambertian (diffuse) BRDF: energy equally distributed in all directions

What does the BRDF equal in this case?



Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF

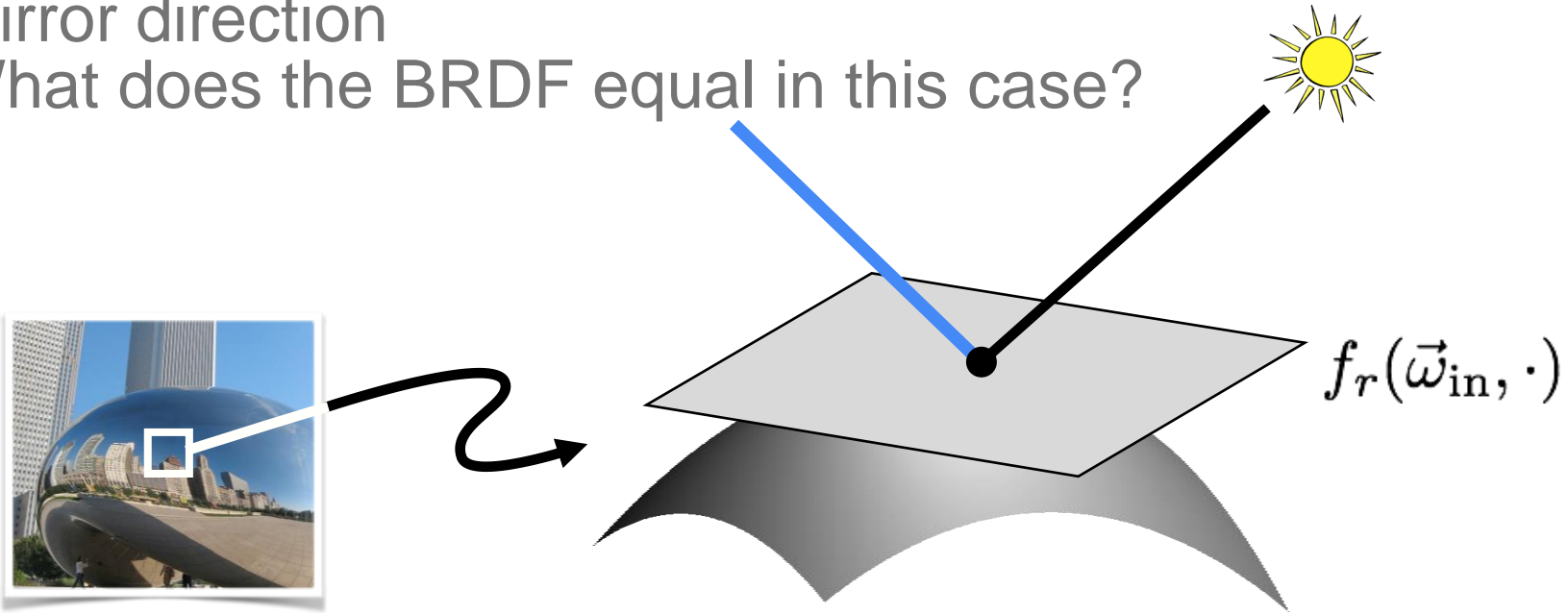


- Surface appears equally bright from ALL directions! (independent of \vec{v})
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ ↗ albedo
- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

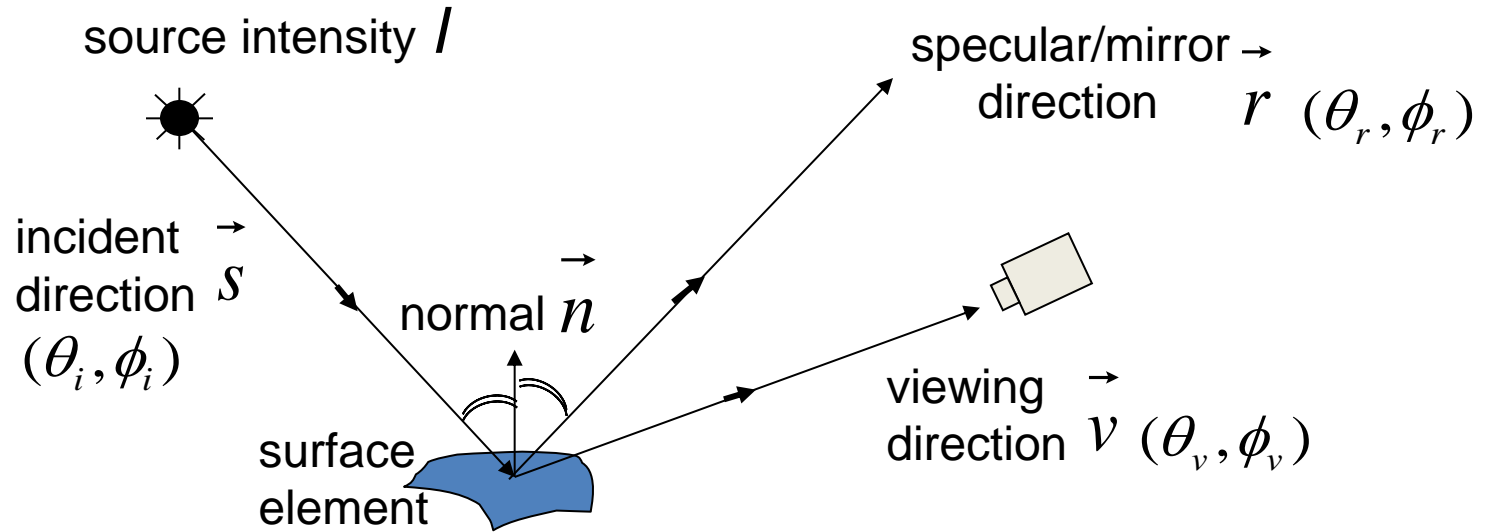
What does the BRDF equal in this case?



$$f_r(\vec{\omega}_{in}, \vec{\omega}_{out})$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function :

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$$

specular albedo

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

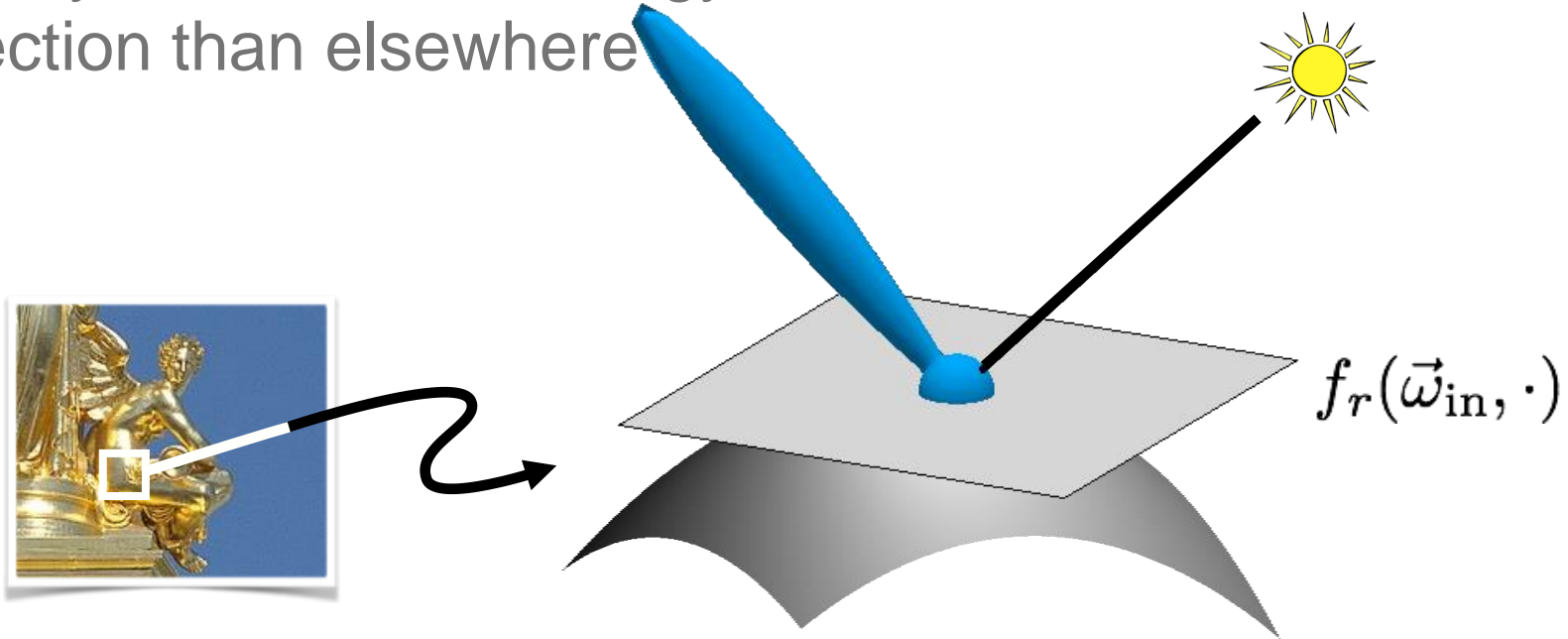


Many materials exhibit both Reflections:



BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Thank you: Question?