INTRODUCTION TO DATA ANALYTICS

Class # 17

Classification: Naïve Bayes' Classifier

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AN INTERESTING FACT..

No number before 1,000 contains the letter A.
But there are plenty of E's, I's, O's, U's, and Y's.

TODAY'S DISCUSSION…

- Introduction to Classification
- Classification Techniques
  - Supervised and unsupervised classification
- Formal statement of supervised classification technique
- Bayesian Classifier
  - Principle of Bayesian classifier
  - Bayes' theorem of probability
- Naïve Bayesian Classifier

Identify the objects

INTRODUCTION TO CLASSIFICATION

Example 17.1

- Teacher classify students as A, B, C, D and F based on their marks. The following is one simple classification rule:

Mark $\geq$ 90	**:**	**A**
90 > Mark $\geq$ 80	**:**	**B**
80 > Mark $\geq$ 70	**:**	**C**
70 > Mark $\geq$ 60	**:**	**D**
60 > Mark	**:**	**F**

Note:

Here, we apply the above rule to a specific data

(in this case a table of marks).

EXAMPLES OF CLASSIFICATION IN DATA ANALYTICS

- Life Science: Predicting tumor cells as benign or malignant
- Security: Classifying credit card transactions as legitimate or fraudulent
- Prediction: Weather, voting, political dynamics, etc.
- Entertainment: Categorizing news stories as finance, weather, entertainment, sports, etc.
- Social media: Identifying the current trend and future growth

CLASSIFICATION : DEFINITION

- Classification is a form of data analysis to extract models describing important data classes.

- Essentially, it involves dividing up objects so that each is assigned to one of a number of mutually exhaustive and exclusive categories known as classes.

  - The term "mutually exhaustive and exclusive" simply means that each object must be assigned to precisely one class

    - That is, never to more than one and never to no class at all.

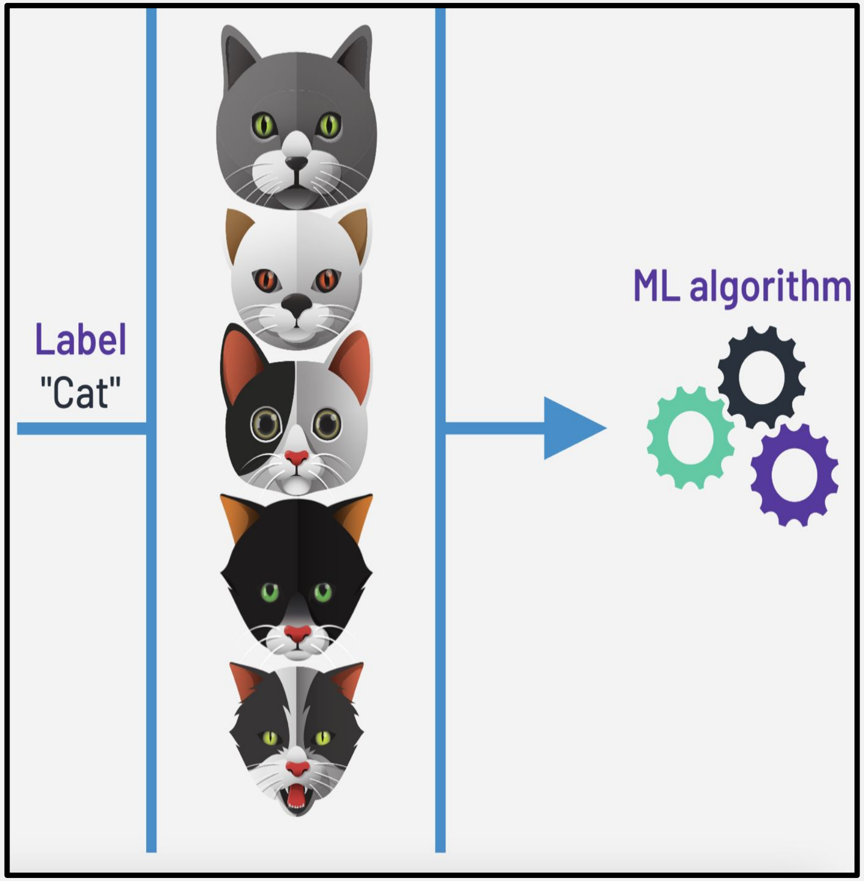
CLASSIFICATION TECHNIQUES

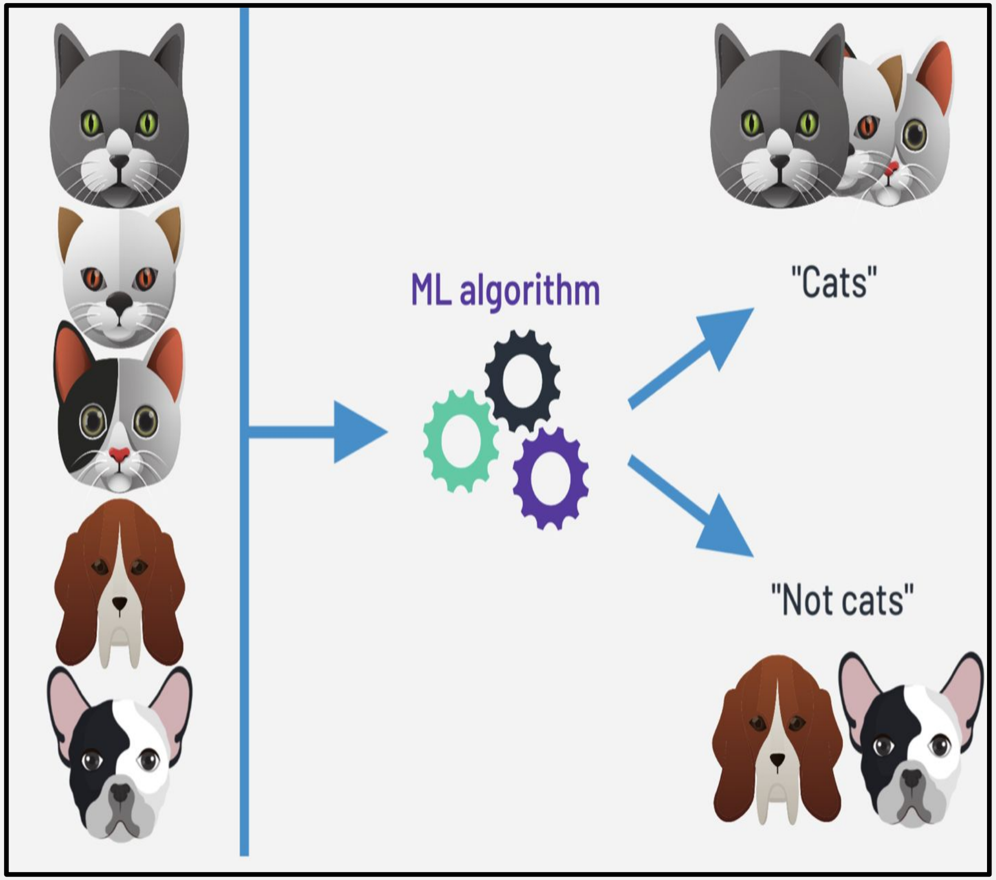
- Classification consists of assigning a class label to a set of unclassified cases.

- **Supervised Classification**
  - The set of possible classes is known in advance.

- **Unsupervised Classification**
  - Set of possible classes is not known. After classification we can try to assign a name to that class.
    - Unsupervised classification is called **clustering**.

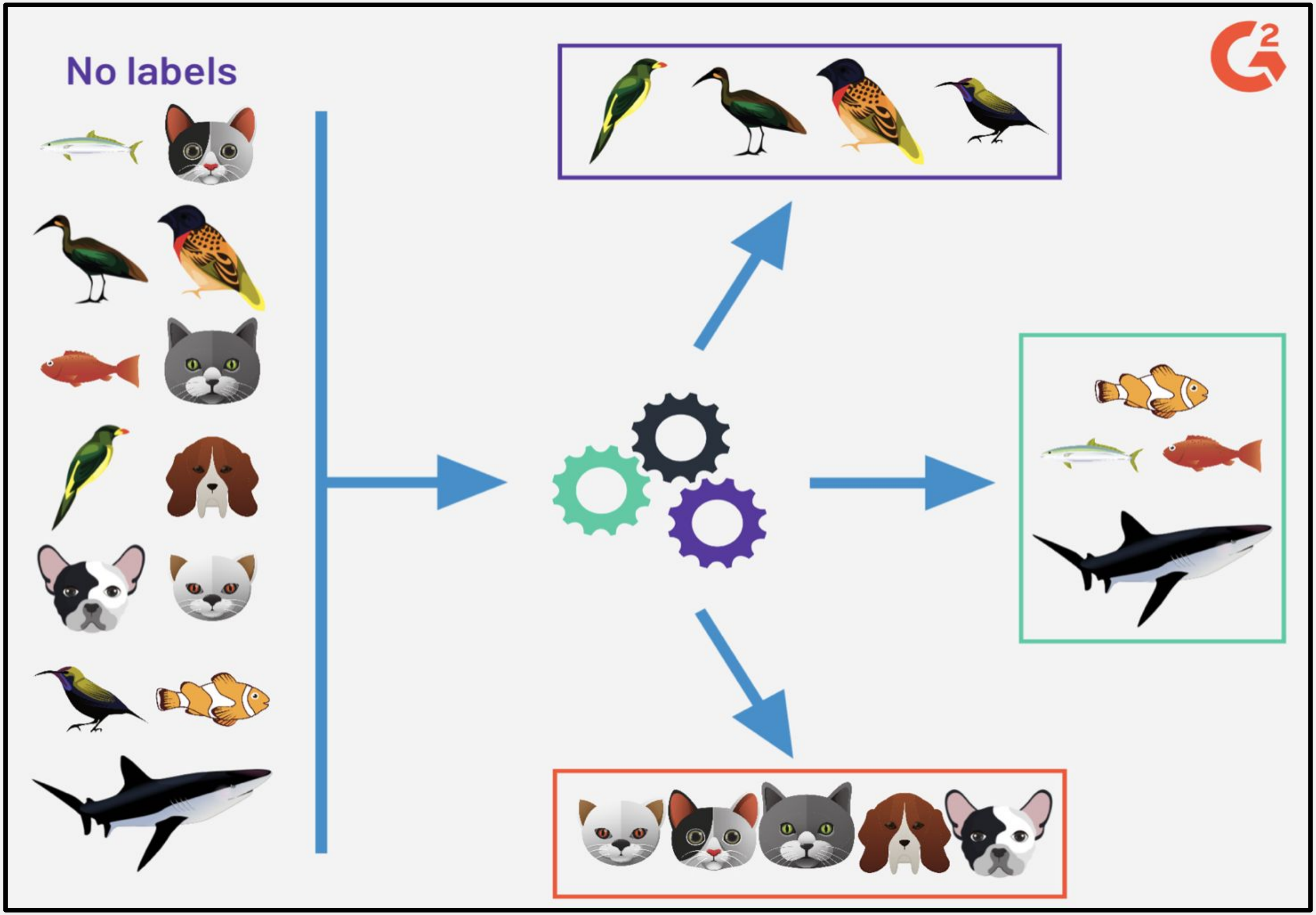
SUPERVISED CLASSIFICATION





Source: https://www.g2.com/articles/supervised-vs-unsupervised-learning

UNSUPERVISED CLASSIFICATION



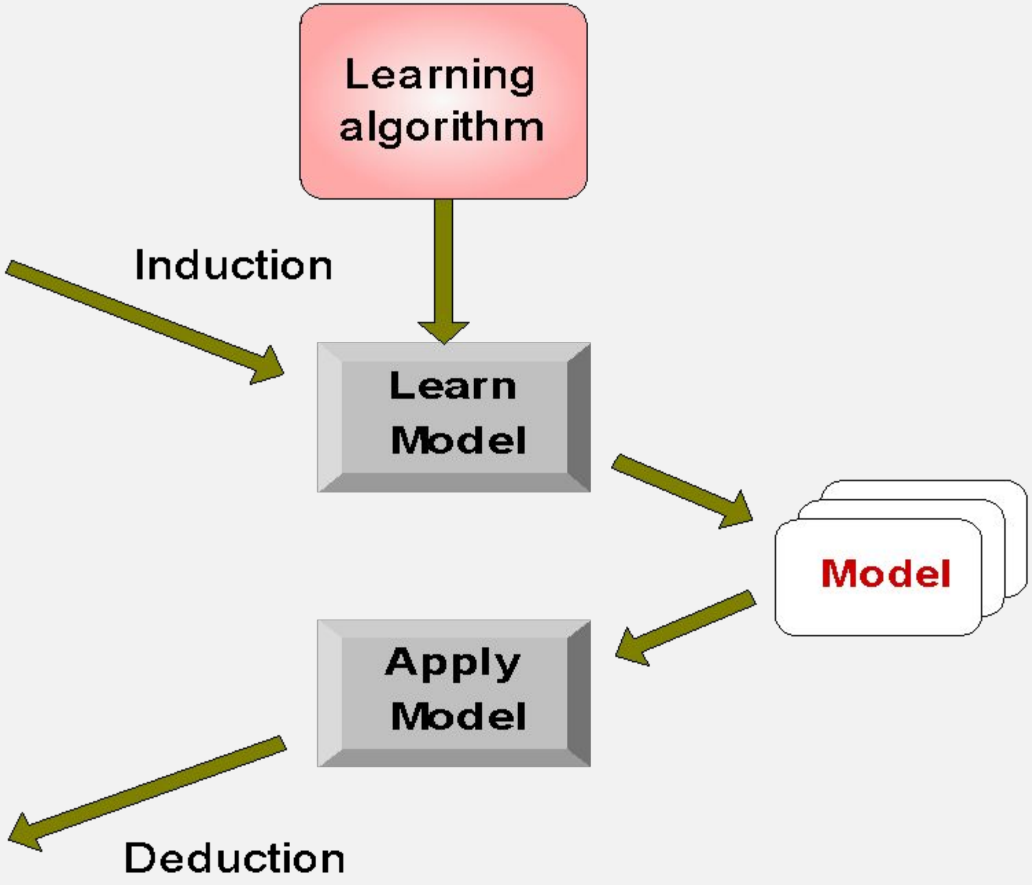
SUPERVISED CLASSIFICATION TECHNIQUE

- Given a collection of records (*training set*)
  - Each record contains a set of *attributes*, one of the attributes is the *class*.
- Find a *model* for class attribute as a function of the values of other attributes.
- Goal: Previously unseen records should be assigned a class as accurately as possible.
  - Satisfy the property of "mutually exclusive and exhaustive"

ILLUSTRATING CLASSIFICATION TASKS

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

CLASSIFICATION PROBLEM

- More precisely, a classification problem can be stated as below:

Definition: **Classification Problem**

Given a database D = $\{t_1, t_2, \ldots\ldots, t_m\}$ of tuples and a set of classes C = $\{c_1, c_2, \ldots\ldots, c_k\}$, the classification problem is to define a mapping f : D $\rightarrow C$,

Where each t_i is assigned to one class.

Note that tuple $t_i \in D$ is defined by a set of attributes $A = \{A_1, A_2, \ldots\ldots, A_n\}$.

CLASSIFICATION TECHNIQUES

- A number of classification techniques are known, which can be broadly classified into the following categories:

1. Statistical-Based Methods
    - Bayesian Classifier

2. Distance-Based Classification
    - K-Nearest Neighbours

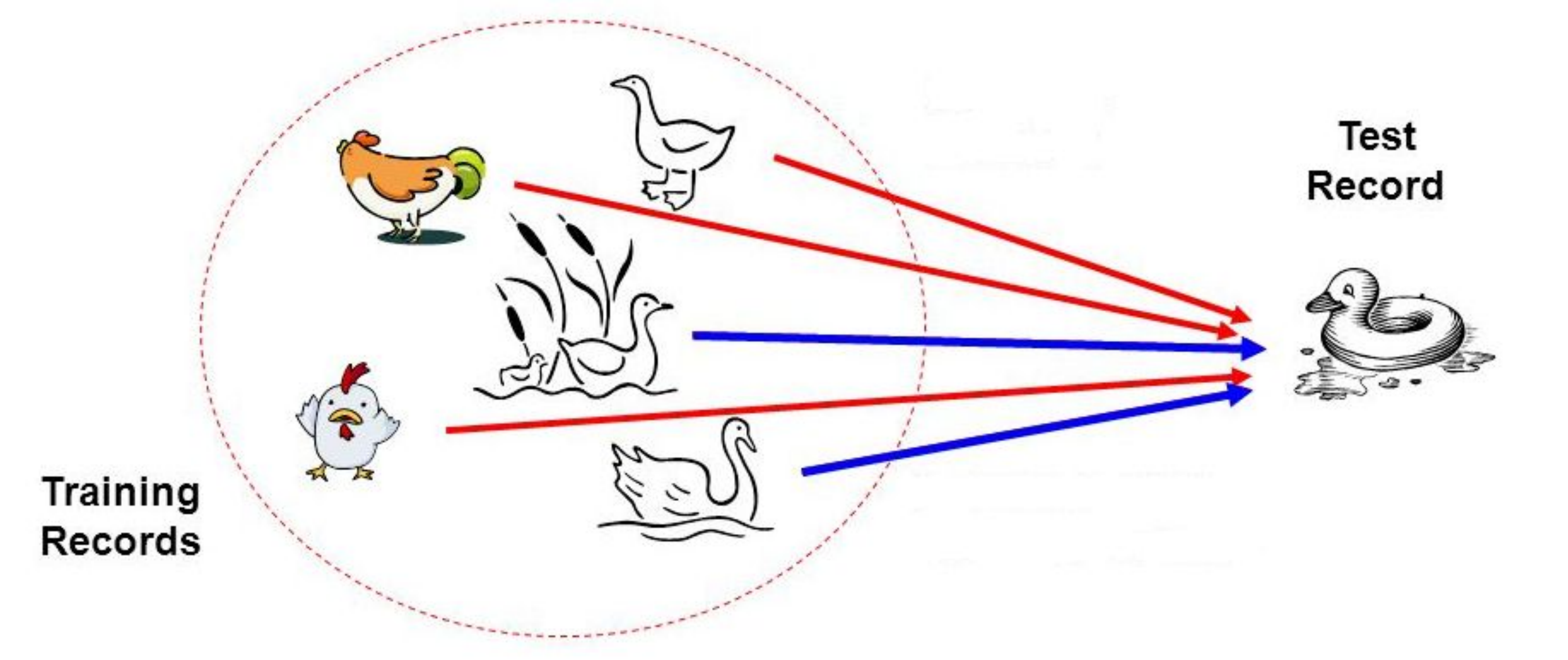
3. Decision Tree-Based Classification
    - ID3, C 4.5, CART

4. Support vector machine
5. Classification using Neural Network (ANN)

Bayesian Classifier

BAYESIAN CLASSIFIER

- Principle
  - If it walks like a duck, quacks like a duck, then it is probably a duck

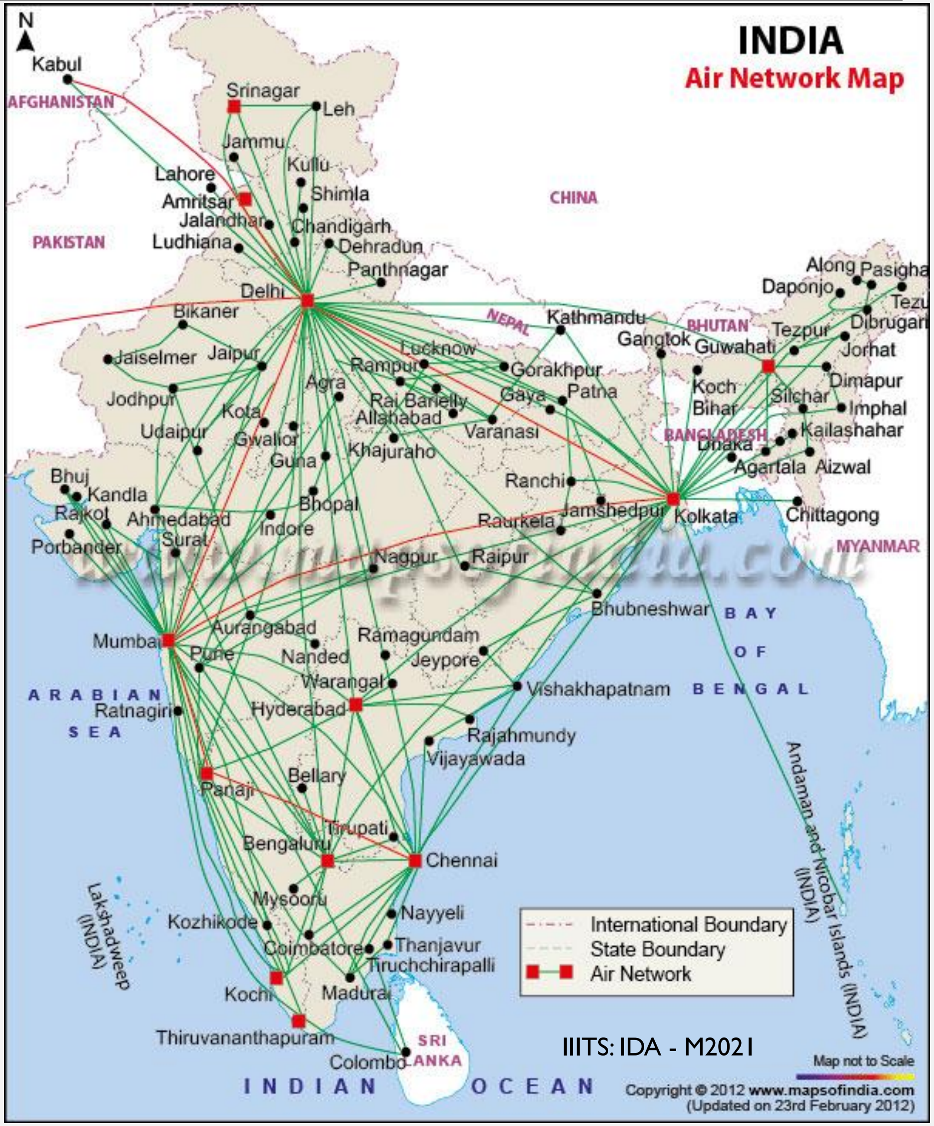


BAYESIAN CLASSIFIER

- A statistical classifier
  - Performs *probabilistic prediction*, *i.e.*, predicts class membership probabilities
- Foundation
  - Based on Bayes' Theorem.
- Assumptions
  1. The classes are mutually exclusive and exhaustive.
  2. The attributes are independent given the class.
- Called "Naïve" classifier because of these assumptions.
  - Empirically proven to be useful.
  - Scales very well.

EXAMPLE: BAYESIAN CLASSIFICATION

- **Example 17.2:** Air Traffic Data
  - Let us consider a set of observation recorded in a database
    - Regarding the arrival of airplanes in the routes from any airport to New Delhi under certain conditions.



AIR-TRAFFIC DATA

Days	Season	Fog	Rain	Class
Weekday	Spring	None	None	On Time
Weekday	Winter	None	Slight	On Time
Weekday	Winter	None	None	On Time
Holiday	Winter	High	Slight	Late
Saturday	Summer	Normal	None	On Time
Weekday	Autumn	Normal	None	Very Late
Holiday	Summer	High	Slight	On Time
Sunday	Summer	Normal	None	On Time
Weekday	Winter	High	Heavy	Very Late
Weekday	Summer	None	Slight	On Time

Cond. to next slide…

AIR-TRAFFIC DATA

Cond. from previous slide…

Days	Season	Fog	Rain	Class
Saturday	Spring	High	Heavy	Cancelled
Weekday	Summer	High	Slight	On Time
Weekday	Winter	Normal	None	Late
Weekday	Summer	High	None	On Time
Weekday	Winter	Normal	Heavy	Very Late
Saturday	Autumn	High	Slight	On Time
Weekday	Autumn	None	Heavy	On Time
Holiday	Spring	Normal	Slight	On Time
Weekday	Spring	Normal	None	On Time
Weekday	Spring	Normal	Heavy	On Time

AIR-TRAFFIC DATA

- In this database, there are four attributes

  A = [Day, Season, Fog, Rain]

  with 20 tuples.
- The categories of classes are:

  C= [On Time, Late, Very Late, Cancelled]

- Given this is the knowledge of data and classes, we are to find most likely classification for any other unseen instance, for example:

Week Day	**Winter**	**High**	**None**	**???**

- Classification technique eventually to map this tuple into an accurate class.

BAYESIAN CLASSIFIER

- In many applications, the relationship between the attributes set and the class variable is non-deterministic.
  - In other words, a test cannot be classified to a class label with certainty.
  - In such a situation, the classification can be achieved probabilistically.
- The Bayesian classifier is an approach for modelling probabilistic relationships between the attribute set and the class variable.
- More precisely, Bayesian classifier use Bayes' Theorem of Probability for classification.
- Before going to discuss the Bayesian classifier, we should have a quick look at the Theory of Probability and then Bayes' Theorem.

Bayes' Theorem of Probability

SIMPLE PROBABILITY

Definition: **Simple Probability**

If there are n elementary events associated with a random experiment and m of n of them are favorable to an event A, then the probability of happening or occurrence of A is

$$P(A) = \frac{m}{n}$$

MUTUALLY EXCLUSIVE EVENTS

- Suppose, A and B are any two events and *P(A)*, *P(B)* denote the probabilities that the events *A* and *B* will occur, respectively.

- **Mutually Exclusive Events:**
  - Two events are mutually exclusive, if the occurrence of one precludes the occurrence of the other.

Example: Tossing a coin (two events)

Tossing a ludo cube (Six events)

Kings and Aces are Mutually Exclusive

- Can you give an example, so that two events are not mutually exclusive?

Hint: Hearts and Kings, Weather (sunny, foggy, warm)

INDEPENDENT EVENTS

- **Independent events:** Two events are independent if occurrences of one does not alter the occurrence of other.

Example: Tossing both coin and ludo cube together.

(How many events are here?)

- Can you give an example, where an event is dependent on one or more other events(s)?

Hint: Receiving a message (A) through a communication channel (B) over a computer (C), rain and train.

JOINT PROBABILITY

Definition: **Joint Probability**

If $P(A)$ and $P(B)$ are the probability of two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive, then $P(A \cap B) = 0$
If A and B are independent events, then $P(A \cap B) = P(A).P(B)$

Thus, for mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

CONDITIONAL PROBABILITY

Definition: **Conditional Probability**

If events are dependent, then their probability is expressed by conditional probability. The probability that A occurs given that B is denoted by $P(A|B)$.

Suppose, A and B are two events associated with a random experiment. The probability of A under the condition that B has already occurred and $P(B) \neq 0$ is given by

$$P(A|B) = \frac{\text{Number of events in } B \text{ which are favourable to } A}{\text{Number of events in } B}$$

$$= \frac{\text{Number of events favourable to } A \cap B}{\text{Number of events favourable to } B}$$

$$= \frac{P(A \cap B)}{P(B)}$$

CONDITIONAL PROBABILITY

Corollary: **Conditional Probability**

$$P(A \cap B) = P(A).P(B|A), \quad if\ P(A) \neq 0$$

or $$P(A \cap B) = P(B).P(A|B), \quad if\ P(B) \neq 0$$

For three events A, B and C

$$P(A \cap B \cap C) = P(A).P(B|A).P(C|A \cap B)$$

For n events $A_1, A_2, \ldots, A_n$ and if all events are mutually independent to each other

$$P(A_1 \cap A_2 \cap \ldots \ldots \ldots \ldots \cap A_n) = P(A_1).P(A_2) \ldots \ldots \ldots \ldots P(A_n)$$

Note:

$P(A|B) = 0$ if events are **mutually exclusive**

$P(A|B) = P(A)$ if A and B are **independent**

$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$ otherwise,

$\mathrm{P(A \cap B) = P(B \cap A)}$

CONDITIONAL PROBABILITY

- **Generalization of Conditional Probability:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(B)}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)} \qquad \because P(A \cap B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

By the law of total probability : P(B) = $P[(B \cap A) \cup (B \cap \bar{A})]$, where $\bar{A}$ denotes the compliment of event A. Thus,

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P[(B \cap A) \cup (B \cap \bar{A})]}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

CONDITIONAL PROBABILITY

In general,

$$P(A|D) = \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D\,|\,C)}$$

Total Probability

Definition: Total Probability

Let $E_1, E_2, \ldots \ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 *or* E_2 *or* $\ldots \ldots E_n$, then

$$P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + \cdots \ldots \ldots \ldots . + P(E_n).P(A|E_n)$$

Total Probability: An Example

Example 17.2

A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. What is the probability that the ball drawn is red?

This problem can be answered using the concept of Total Probability

E_1 =Selecting bag I

E_2 =Selecting bag II

A = Drawing the red ball

Thus, $P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2)$

where, $P(A|E_1)$ = Probability of drawing red ball when first bag has been chosen

and $P(A|E_2)$ = Probability of drawing red ball when second bag has been chosen

Reverse Probability

Example 17.3:

A bag (Bag I) contains 4 red and 3 black balls. A second bag (Bag II) contains 2 red and 4 black balls. You have chosen one ball at random. It is found as red ball. What is the probability that the ball is chosen from Bag I?

Here,

E_1 = Selecting bag I

E_2 = Selecting bag II

A = Drawing the red ball

We are to determine P(E_1|A). Such a problem can be solved using Bayes' theorem of probability.

BAYES' THEOREM

Theorem: **Bayes' Theorem**

Let $E_1, E_2, \ldots \ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or $\ldots \ldots E_n$, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{i=1}^{n} P(E_i).P(A|E_i)}$$

PRIOR AND POSTERIOR PROBABILITIES

- P(A) and P(B) are called prior probabilities
- P(A|B), P(B|A) are called posterior probabilities

Example: Prior versus Posterior Probabilities

- This table shows that the event Y has two outcomes namely A and B, which is dependent on another event X with various outcomes like x_1, x_2 and x_3.
- **Case1:** Suppose, we don't have any information of the event A. Then, from the given sample space, we can calculate $P(Y = A) = \frac{5}{10} = 0.5$
- **Case2:** Now, suppose, we want to calculate $P(X = x_2 | Y = A) = \frac{2}{5} = 0.4$

The later is the conditional or posterior probability, where as the former is the prior probability.

X	Y
	A
	A
	B
	A
	B
	A
	B
	B
	B
	A

NAÏVE BAYESIAN CLASSIFIER

- Suppose, Y is a class variable and $X = \{X_1, X_2, \ldots\ldots, X_n\}$ is a set of attributes, with instance of Y.

INPUT (X)	CLASS(Y)
… … …	
… … …	…
… … …	…

- The classification problem, then can be expressed as the class-conditional probability

$$P\big(Y = y_i | (X_1 = x_1) \text{ AND } (X_2 = x_2) \text{ AND } \ldots\ldots (X_n = x_n)\big)$$

NAÏVE BAYESIAN CLASSIFIER

- Naïve Bayesian classifier calculate this posterior probability using Bayes' theorem, which is as follows.

- From Bayes' theorem on conditional probability, we have

$$P(Y|X) = \frac{P(X|Y) \cdot P(Y)}{P(X)}$$

$$= \frac{P(X|Y) \cdot P(Y)}{P(X|Y = y_1) \cdot P(Y = y_1) + \cdots + P(X|Y = y_k) \cdot P(Y = y_k)}$$

where,

$$P(X) = \sum_{i=1}^{k} P(X|Y = y_i) \cdot P(Y = y_i)$$

Note:

- $P(X)$ is called the evidence (also the total probability) and it is a constant.

- The probability $P(Y|X)$ (also called class conditional probability) is therefore proportional to $P(X|Y) \cdot P(Y)$.

- Thus, $P(Y|X)$ can be taken as a measure of Y given that X.

$$P(Y|X) \approx P(X|Y) \cdot P(Y)$$

NAÏVE BAYESIAN CLASSIFIER

- Suppose, for a given instance of X (say $x = (X_1 = x_1)$ and ..... $(X_n = x_n)$).
- There are any two class conditional probabilities namely $P(Y= y_i|X=x)$ and $P(Y= y_j \mid X=x)$.
- If $P(Y= y_i \mid X=x) > P(Y= y_j \mid X=x)$, then we say that y_i is more stronger than y_j for the instance $X = x$.
- The strongest y_i is the classification for the instance $X = x$.

NAÏVE BAYESIAN CLASSIFIER

- **Example:** With reference to the Air Traffic Dataset mentioned earlier, let us tabulate all the posterior and prior probabilities as shown below.

Attribute		On Time	Late	Very Late	Cancelled
		Class			
Day	Weekday	9/14 = 0.64	½ = 0.5	3/3 = 1	0/1 = 0
	Saturday	2/14 = 0.14	½ = 0.5	0/3 = 0	1/1 = 1
	Sunday	1/14 = 0.07	0/2 = 0	0/3 = 0	0/1 = 0
	Holiday	2/14 = 0.14	0/2 = 0	0/3 = 0	0/1 = 0
Season	Spring	4/14 = 0.29	0/2 = 0	0/3 = 0	0/1 = 0
	Summer	6/14 = 0.43	0/2 = 0	0/3 = 0	0/1 = 0
	Autumn	2/14 = 0.14	0/2 = 0	1/3= 0.33	0/1 = 0
	Winter	2/14 = 0.14	2/2 = 1	2/3 = 0.67	0/1 = 0

NAÏVE BAYESIAN CLASSIFIER

Attribute		Class: On Time	Class: Late	Class: Very Late	Class: Cancelled
Fog	None	5/14 = 0.36	0/2 = 0	0/3 = 0	0/1 = 0
	High	4/14 = 0.29	1/2 = 0.5	1/3 = 0.33	1/1 = 1
	Normal	5/14 = 0.36	1/2 = 0.5	2/3 = 0.67	0/1 = 0
Rain	None	5/14 = 0.36	1/2 = 0.5	1/3 = 0.33	0/1 = 0
	Slight	8/14 = 0.57	0/2 = 0	0/3 = 0	0/1 = 0
	Heavy	1/14 = 0.07	1/2 = 0.5	2/3 = 0.67	1/1 = 1
Prior Probability		14/20 = 0.70	2/20 = 0.10	3/20 = 0.15	1/20 = 0.05

NAÏVE BAYESIAN CLASSIFIER

Instance:

Week Day	Winter	High	Heavy	???

Case1: Class = On Time : $0.70 \times 0.64 \times 0.14 \times 0.29 \times 0.07 = 0.0013$

Case2: Class = Late : $0.10 \times 0.50 \times 1.0 \times 0.50 \times 0.50 = 0.0125$

Case3: Class = Very Late : $0.15 \times 1.0 \times 0.67 \times 0.33 \times 0.67 = 0.0222$

Case4: Class = Cancelled : $0.05 \times 0.0 \times 0.0 \times 1.0 \times 1.0 = 0.0000$

Case3 is the strongest; Hence correct classification is **Very Late**

NAÏVE BAYESIAN CLASSIFIER

- **Algorithm: Naïve Bayesian Classification**

Input: Given a set of k mutually exclusive and exhaustive classes $C = \{c_1, c_2, \ldots\ldots, c_k\}$, which have prior probabilities $P(C_1), P(C_2), \ldots\ldots P(C_k)$.

There are n-attribute set $A = \{A_1, A_2, \ldots\ldots, A_n\}$, which for a given instance have values $A_1 = a_1$, $A_2 = a_2, \ldots\ldots, A_n = a_n$

Step: For each $c_i \in C$, calculate the class condition probabilities, $i = 1,2,\ldots\ldots,k$

$$p_i = P(C_i) \times \prod_{j=1}^{n} P(A_j = a_j | C_i)$$

$$p_x = \max\{p_1, p_2, \ldots\ldots, p_k\}$$

Output: C_x is the classification

Note: $\sum p_i \neq 1$, because they are not probabilities rather proportion values (to posterior probabilities)

NAÏVE BAYESIAN CLASSIFIER

Pros and Cons

- The Naïve Bayes' approach is a very popular one, which often works well.
- However, it has a number of potential problems
  - It relies on all attributes being categorical.
  - If the data is less, then it estimates poorly.

NAÏVE BAYESIAN CLASSIFIER

- **Approach to overcome the limitations in Naïve Bayesian Classification**

- Estimating the posterior probabilities for continuous attributes

  - In real life situation, all attributes are not necessarily be categorical, In fact, there is a mix of both categorical and continuous attributes.

  - In the following, we discuss the schemes to deal with continuous attributes in Bayesian classifier.

1. We can discretize each continuous attributes and then replace the continuous values with its corresponding discrete intervals.

2. We can assume a certain form of probability distribution for the continuous variable and estimate the parameters of the distribution using the training data. A Gaussian distribution is usually chosen to represent the posterior probabilities for continuous attributes. A general form of Gaussian distribution will look like

$$P(x: \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where, μ and σ^2 denote mean and variance, respectively.

NAÏVE BAYESIAN CLASSIFIER

- For each class C_i, the posterior probabilities for attribute A_j (it is the numeric attribute) can be calculated following Gaussian normal distribution as follows.

$$P(A_j = a_j | C_i) = \frac{1}{\sqrt{2\pi}\sigma_{ij}} e^{-\frac{(a_j - \mu_{ij})^2}{2\sigma_{ij}^2}}$$

Here, the parameter μ_{ij} can be calculated based on the sample mean of attribute value of A_j for the training records that belong to the class C_i.

Similarly, σ_{ij}^2 can be estimated from the calculation of variance of such training records.

NAÏVE BAYESIAN CLASSIFIER

M-estimate of Conditional Probability

- The M-estimation is to deal with the potential problem of Naïve Bayesian Classifier when training data size is too poor.
  - If the posterior probability for one of the attribute is zero, then the overall class-conditional probability for the class vanishes.
  - In other words, if training data do not cover many of the attribute values, then we may not be able to classify some of the test records.
- This problem can be addressed by using the M-estimate approach.

M-ESTIMATE APPROACH

- M-estimate approach can be stated as follows

$$P(A_j = a_j|C_i) = \frac{n_{c_i} + mp}{n + m}$$

where, n = total number of instances from **class** C_i

n_{c_i} = number of training examples from class C_i that take the value $A_j = a_j$

m = it is a parameter known as the equivalent sample size, and

p = is a user specified parameter.

Note:

If $n = 0$, that is, if there is no training set available, then $P(a_i|C_i) = p$,

so, this is a different value, in absence of sample value.

A PRACTICE EXAMPLE

Example 8.4

Class:
C1:buys_computer = 'yes'
C2:buys_computer = 'no'

Data instance
X = (age <=30,
Income = medium,
Student = yes
Credit_rating = fair)

age	income	student	credit rating	com
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

A PRACTICE EXAMPLE

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 × 0.444 × 0.667 × 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 × 0.4 × 0.2 × 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Therefore, X belongs to class ("buys_computer = yes")

REFERENCE

- The detail material related to this lecture can be found in

Data Mining: Concepts and Techniques, (3rd Edn.), Jiawei Han, Micheline Kamber, Morgan Kaufmann, 2015.

Introduction to Data Mining, Pang-Ning Tan, Michael Steinbach, and Vipin Kumar, Addison-Wesley, 2014

Any question?