INTRODUCTION TO DATA ANALYTICS

Class # 18

Naïve Bayes' Classifier: Example

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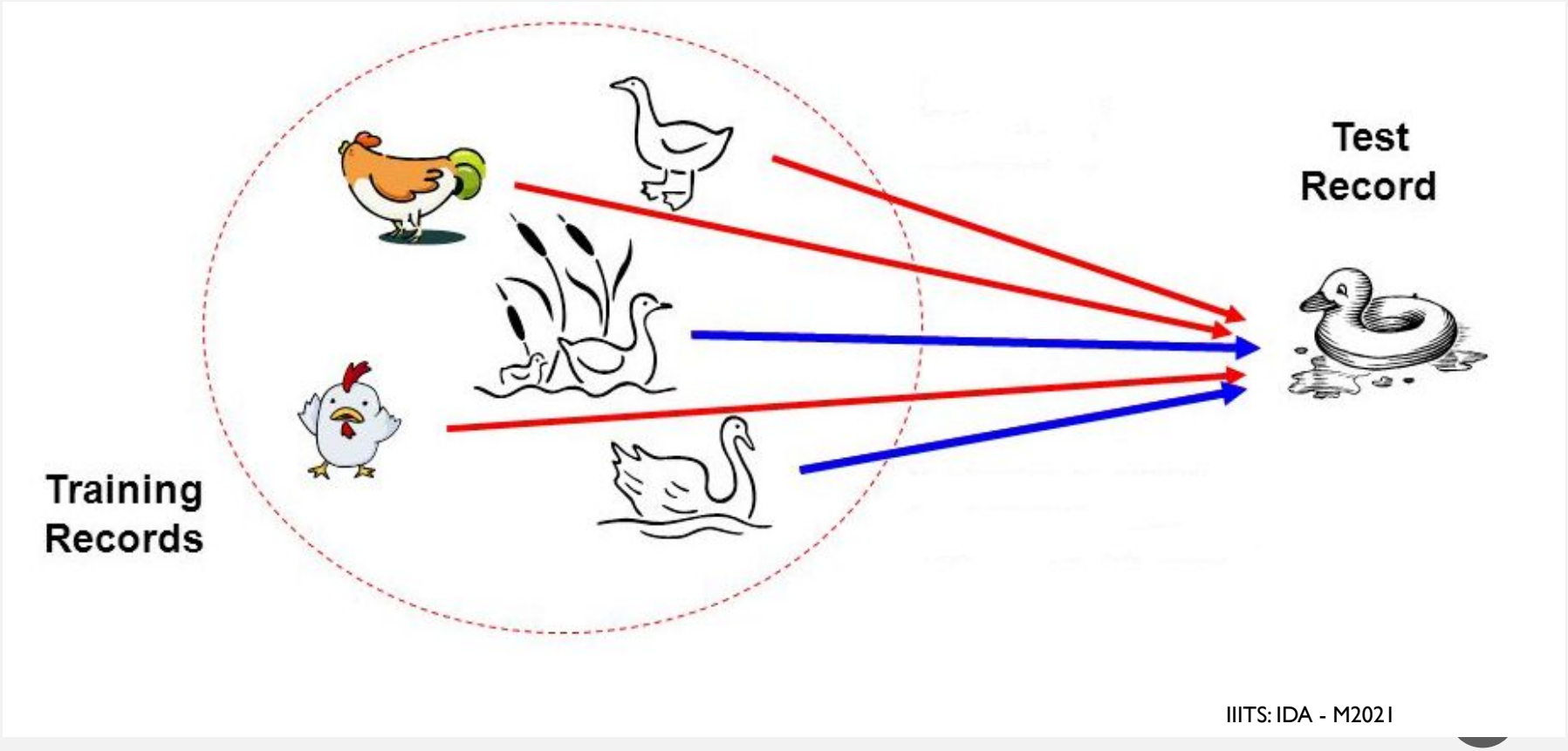
Bayesian Classifier

BAYESIAN CLASSIFIER

- Principle
  - If it walks like a duck, quacks like a duck, then it is probably a duck

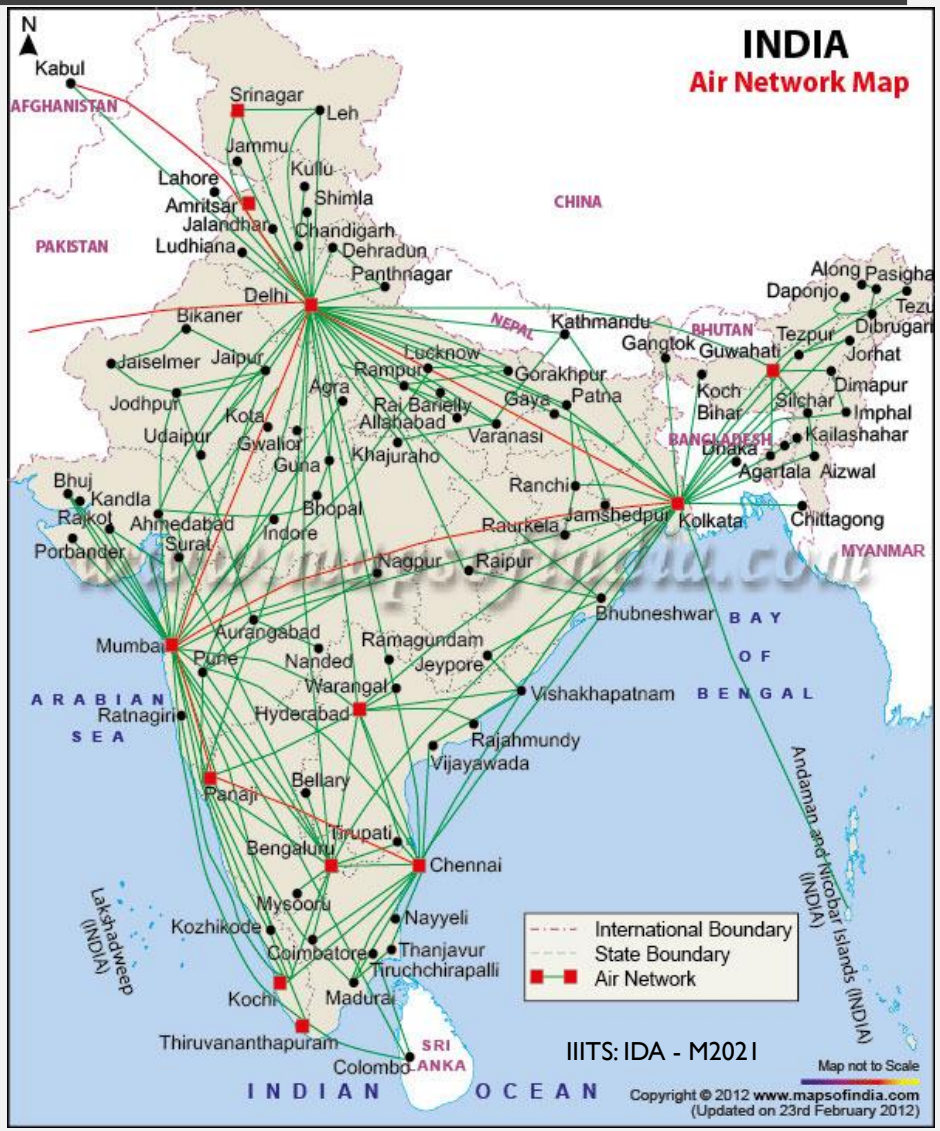


BAYESIAN CLASSIFIER

- A statistical classifier
  - Performs *probabilistic prediction*, *i.e.*, predicts class membership probabilities
- Foundation
  - Based on Bayes' Theorem.
- Assumptions
  1. The classes are mutually exclusive and exhaustive.
  2. The attributes are independent given the class.
- Called "Naïve" classifier because of these assumptions.
  - Empirically proven to be useful.
  - Scales very well.

EXAMPLE: BAYESIAN CLASSIFICATION

- **Example 8.2:** Air Traffic Data
  - Let us consider a set observation recorded in a database
    - Regarding the arrival of airplanes in the routes from any airport to New Delhi under certain conditions.



AIR-TRAFFIC DATA

Days	Season	Fog	Rain	Class
Weekday	Spring	None	None	On Time
Weekday	Winter	None	Slight	On Time
Weekday	Winter	None	None	On Time
Holiday	Winter	High	Slight	Late
Saturday	Summer	Normal	None	On Time
Weekday	Autumn	Normal	None	Very Late
Holiday	Summer	High	Slight	On Time
Sunday	Summer	Normal	None	On Time
Weekday	Winter	High	Heavy	Very Late
Weekday	Summer	None	Slight	On Time

Cond. to next slide…

AIR-TRAFFIC DATA

Cond. from previous slide…

Days	Season	Fog	Rain	Class
Saturday	Spring	High	Heavy	Cancelled
Weekday	Summer	High	Slight	On Time
Weekday	Winter	Normal	None	Late
Weekday	Summer	High	None	On Time
Weekday	Winter	Normal	Heavy	Very Late
Saturday	Autumn	High	Slight	On Time
Weekday	Autumn	None	Heavy	On Time
Holiday	Spring	Normal	Slight	On Time
Weekday	Spring	Normal	None	On Time
Weekday	Spring	Normal	Heavy	On Time

AIR-TRAFFIC DATA

- In this database, there are four attributes

  A = [Day, Season, Fog, Rain]

  with 20 tuples.
- The categories of classes are:

  C= [On Time, Late, Very Late, Cancelled]

- Given this is the knowledge of data and classes, we are to find most likely classification for any other unseen instance, for example:

Week Day	Winter	High	None	???

- Classification technique eventually to map this tuple into an accurate class.

Bayes' Theorem of Probability

BAYES' THEOREM

Theorem 8.4: **Bayes' Theorem**

Let $E_1, E_2, \ldots \ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 *or* E_2 *or* $\ldots \ldots E_n$, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{i=1}^{n} P(E_i).P(A|E_i)}$$

PRIOR AND POSTERIOR PROBABILITIES

- P(A) and P(B) are called prior probabilities
- P(A|B), P(B|A) are called posterior probabilities

Example 8.6: Prior versus Posterior Probabilities

- This table shows that the event Y has two outcomes namely A and B, which is dependent on another event X with various outcomes like x_1, x_2 and x_3.

- **Case1:** Suppose, we don't have any information of the event A. Then, from the given sample space, we can calculate $P(Y = A) = \frac{5}{10} = 0.5$

- **Case2:** Now, suppose, we want to calculate $P(X = x_2 | Y = A) = \frac{2}{5} = 0.4$.

The later is the conditional or posterior probability, where as the former is the prior probability.

X	Y
	A
	A
	B
	A
	B
	A
	B
	B
	B
	A

NAÏVE BAYESIAN CLASSIFIER

- Suppose, Y is a class variable and $X = \{X_1, X_2, \ldots\ldots, X_n\}$ is a set of attributes, with instance of Y.

INPUT (X)	CLASS(Y)
… … …	
… … …	…
… … …	…

- The classification problem, then can be expressed as the class-conditional probability

$$P\big(Y = y_i | (X_1 = x_1) \text{ AND } (X_2 = x_2) \text{ AND } \ldots\ldots (X_n = x_n)\big)$$

NAÏVE BAYESIAN CLASSIFIER

- Naïve Bayesian classifier calculate this posterior probability using Bayes' theorem, which is as follows.
- From Bayes' theorem on conditional probability, we have

$$P(Y|X) = \frac{P(X|Y)\cdot P(Y)}{P(X)}$$

$$= \frac{P(X|Y)\cdot P(Y)}{P(X|Y=y_1)\cdot P(Y=y_1) + \cdots + P(X|Y=y_k)\cdot P(Y=y_k)}$$

where,

$$P(X) = \sum_{i=1}^{k} P(X|Y=y_i)\cdot P(Y=y_i)$$

Note:

- $P(X)$ is called the evidence (also the total probability) and it is a constant.
- The probability $P(Y|X)$ (also called class conditional probability) is therefore proportional to $P(X|Y)\cdot P(Y)$.
- Thus, $P(Y|X)$ can be taken as a measure of Y given that X.

$$P(Y|X) \approx P(X|Y)\cdot P(Y)$$

NAÏVE BAYESIAN CLASSIFIER

- Suppose, for a given instance of X (say $x = (X_1 = x_1)$ and ..... $(X_n = x_n)$).
- There are any two class conditional probabilities namely $P(Y= y_i|X=x)$ and $P(Y= y_j \mid X=x)$.
- If $P(Y= y_i \mid X=x) > P(Y= y_j \mid X=x)$, then we say that y_i is more stronger than y_j for the instance $X = x$.
- The strongest y_i is the classification for the instance $X = x$.

NAÏVE BAYESIAN CLASSIFIER

Algorithm: Naïve Bayesian Classification

Input: Given a set of k mutually exclusive and exhaustive classes $C = \{c_1, c_2, \ldots\ldots, c_k\}$, which have prior probabilities $P(C_1)$, $P(C_2)$,..... $P(C_k)$.

There are n-attribute set $A = \{A_1, A_2, \ldots\ldots, A_n\}$, which for a given instance have values $A_1 = a_1$, $A_2 = a_2$,....., $A_n = a_n$

Step: For each $c_i \in C$, calculate the class condition probabilities, $i = 1,2,\ldots\ldots,k$

$$p_i = P(C_i) \times \prod_{j=1}^{n} P(A_j = a_j | C_i)$$

$$p_x = \max\{p_1, p_2, \ldots\ldots, p_k\}$$

Output: C_x is the classification

Note: $\sum p_i \neq 1$, because they are not probabilities rather proportion values (to posterior probabilities)

NAÏVE BAYESIAN CLASSIFIER

- **Example:** With reference to the Air Traffic Dataset mentioned earlier, let us tabulate all the posterior and prior probabilities as shown below.

$P(A_j = a_j|C_i)$ is the likelihood. Here, for the given example, $P(Day = weekday|C_1 = On\ time)$

		Class			
Attribute		On Time	Late	Very Late	Cancelled
Day	Weekday	9/14 = 0.64	½ = 0.5	3/3 = 1	0/1 = 0
	Saturday	2/14 = 0.14	½ = 0.5	0/3 = 0	1/1 = 1
	Sunday	1/14 = 0.07	0/2 = 0	0/3 = 0	0/1 = 0
	Holiday	2/14 = 0.14	0/2 = 0	0/3 = 0	0/1 = 0
Season	Spring	4/14 = 0.29	0/2 = 0	0/3 = 0	0/1 = 0
	Summer	6/14 = 0.43	0/2 = 0	0/3 = 0	0/1 = 0
	Autumn	2/14 = 0.14	0/2 = 0	1/3= 0.33	0/1 = 0
	Winter	2/14 = 0.14	2/2 = 1	2/3 = 0.67	0/1 = 0

NAÏVE BAYESIAN CLASSIFIER

		Class			
Attribute		On Time	Late	Very Late	Cancelled
Fog	None	5/14 = 0.36	0/2 = 0	0/3 = 0	0/1 = 0
	High	4/14 = 0.29	1/2 = 0.5	1/3 = 0.33	1/1 = 1
	Normal	5/14 = 0.36	1/2 = 0.5	2/3 = 0.67	0/1 = 0
Rain	None	5/14 = 0.36	1/2 = 0.5	1/3 = 0.33	0/1 = 0
	Slight	8/14 = 0.57	0/2 = 0	0/3 = 0	0/1 = 0
	Heavy	1/14 = 0.07	1/2 = 0.5	2/3 = 0.67	1/1 = 1
Prior Probability		14/20 = 0.70	2/20 = 0.10	3/20 = 0.15	1/20 = 0.05

NAÏVE BAYESIAN CLASSIFIER

Instance:

Week Day	Winter	High	Heavy	???

Case1: Class = On Time : 0.70 × 0.64 × 0.14 × 0.29 × 0.07 = 0.0013

Case2: Class = Late : 0.10 × 0.50 × 1.0 × 0.50 × 0.50 = 0.0125

Case3: Class = Very Late : 0.15 × 1.0 × 0.67 × 0.33 × 0.67 = 0.0222

Case4: Class = Cancelled : 0.05 × 0.0 × 0.0 × 1.0 × 1.0 = 0.0000

Case3 is the strongest; Hence correct classification is **Very Late**

NAÏVE BAYESIAN CLASSIFIER

Pros and Cons

- The Naïve Bayes' approach is a very popular one, which often works well.
- However, it has a number of potential problems
  - It relies on all attributes being categorical.
  - If the data is less, then it estimates poorly.

NAÏVE BAYESIAN CLASSIFIER

Approach to overcome the limitations in Naïve Bayesian Classification

- Estimating the posterior probabilities for continuous attributes
  - In real life situation, all attributes are not necessarily be categorical, In fact, there is a mix of both categorical and continuous attributes.
  - In the following, we discuss the schemes to deal with continuous attributes in Bayesian classifier.

1. We can discretize each continuous attributes and then replace the continuous values with its corresponding discrete intervals.
2. We can assume a certain form of probability distribution for the continuous variable and estimate the parameters of the distribution using the training data. A Gaussian distribution is usually chosen to represent the posterior probabilities for continuous attributes. A general form of Gaussian distribution will look like

$$P(x: \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where, μ and σ^2 denote mean and variance, respectively.

NAÏVE BAYESIAN CLASSIFIER

For each class C_i, the posterior probabilities for attribute A_j (it is the numeric attribute) can be calculated following Gaussian normal distribution as follows.

$$P(A_j = a_j|C_i) = \frac{1}{\sqrt{2\pi}\sigma_{ij}} e^{-\frac{(a_j - \mu_{ij})^2}{2\sigma_{ij}^2}}$$

Here, the parameter μ_{ij} can be calculated based on the sample mean of attribute value of A_j for the training records that belong to the class C_i.

Similarly, σ_{ij}^2 can be estimated from the calculation of variance of such training records.

NAÏVE BAYESIAN CLASSIFIER

M-estimate of Conditional Probability

- The M-estimation is to deal with the potential problem of Naïve Bayesian Classifier when training data size is too poor.
  - If the posterior probability for one of the attribute is zero, then the overall class-conditional probability for the class vanishes.
  - In other words, if training data do not cover many of the attribute values, then we may not be able to classify some of the test records.
- This problem can be addressed by using the M-estimate approach.

M-ESTIMATE APPROACH

- M-estimate approach can be stated as follows

$$P(A_j = a_j|C_i) = \frac{n_{c_i}+mp}{n+m}$$

where, n = total number of instances from class C_i

n_{c_i} = number of training examples from class C_i that take the value $A_j = a_j$

m = it is a parameter known as the equivalent sample size, and

p = is a user specified parameter.

Note:

If $n = 0$, that is, if there is no training set available, then $P(a_i|C_i) = p$,

so, this is a different value, in absence of sample value.

A PRACTICE EXAMPLE

Example 17.4

Class:
C1:buys_computer = ‘yes’
C2:buys_computer = ‘no’

Data instance
X = (age <=30,
Income = medium,
Student = yes
Credit_rating = fair)

A PRACTICE EXAMPLE

Example 17.4

age	income	student	credit_rating	com
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

A PRACTICE EXAMPLE

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 × 0.444 × 0.667 × 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 × 0.4 × 0.2 × 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Therefore, X belongs to class ("buys_computer = yes")

REFERENCE

- The detail material related to this lecture can be found in

Data Mining: Concepts and Techniques, (3rd Edn.), Jiawei Han, Micheline Kamber, Morgan Kaufmann, 2015.

Introduction to Data Mining, Pang-Ning Tan, Michael Steinbach, and Vipin Kumar, Addison-Wesley, 2014

Any question?