Signal Processing

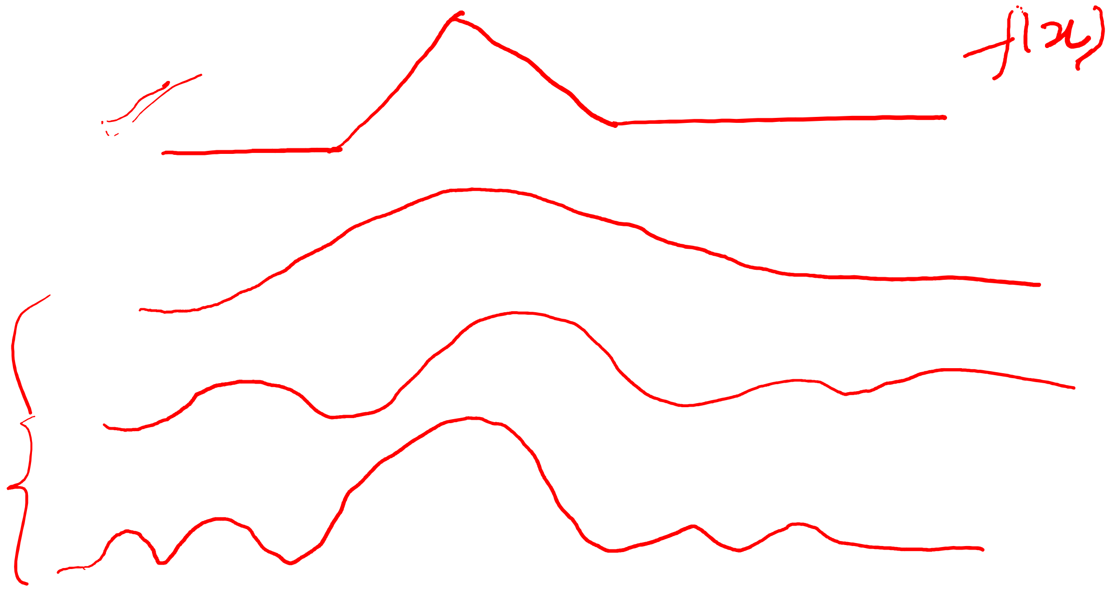
Frequency Domain Analysis

Frequency Domain Analysis

- EEG signals are recorded in the domain of TIME.
- Introduced by J.-B. Joseph Fourier in the early 1800s
- It's a coordinate transform system
- Fourier introduced the concept that sine and cosine functions of increasing frequency provide an orthogonal basis for the space of solution functions.

Fourier Series

- Any arbitrary function can be represented using sine and cosines.
- Sine and cosine form the basis of the function space.

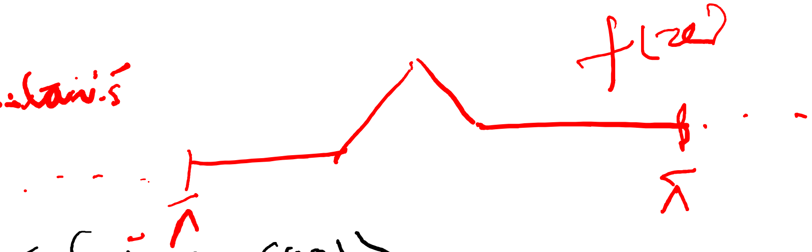


Sum ↓ inc. frequencies sine & cosine. approximate

Fourier Series

- Approximating arbitrary function f(x), as a infinite sum of sine and cosine of increasing high frequency.

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty}\left[A_k \cos(kx) + B_k \sin(kx)\right]$$



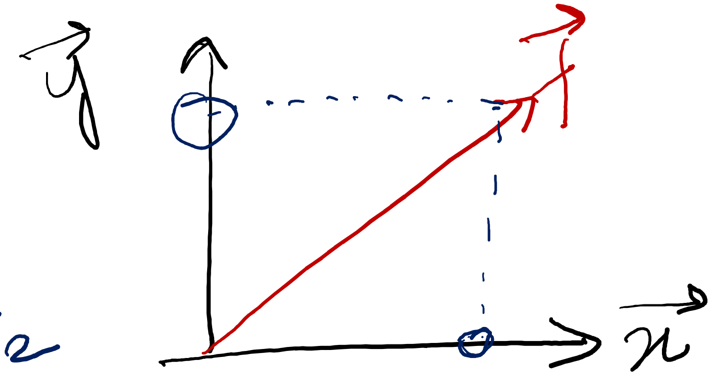
Ak & Bk are fourier constants

Where

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(k x)\,dx = \frac{1}{\|\cos(kx)\|^2}\langle f(x), \cos(kx)\rangle$$

$$B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(kx)\,dx = \frac{1}{\|\sin(kx)\|^2}\langle f(x), \sin(kx)\rangle$$

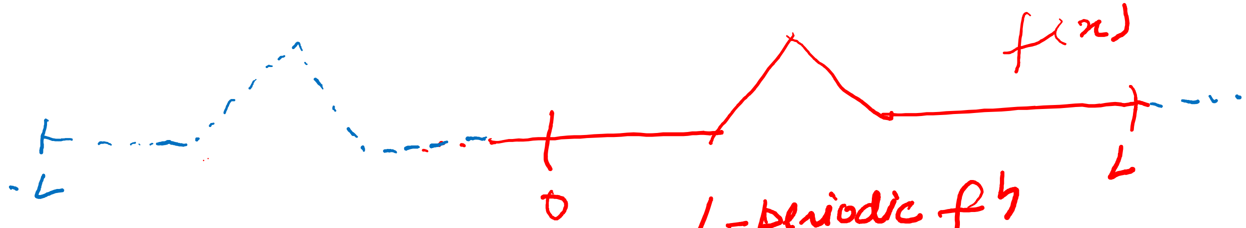
Fourier Series



$$\vec{f} = \langle \vec{f}, \vec{x} \rangle \cdot \frac{\vec{x}}{\|x\|^2} + \langle \vec{f}, \vec{y} \rangle \cdot \frac{\vec{y}}{\|y\|^2}$$

$$f(x) = \frac{f(x) \cdot \cos(kx) \cdot \cos}{\|\cos\|} + \frac{f(x) \cdot \sin(kx)\ \sin k}{\|\sin\|^2} \rightarrow f^n \text{ space}$$

Fourier Series



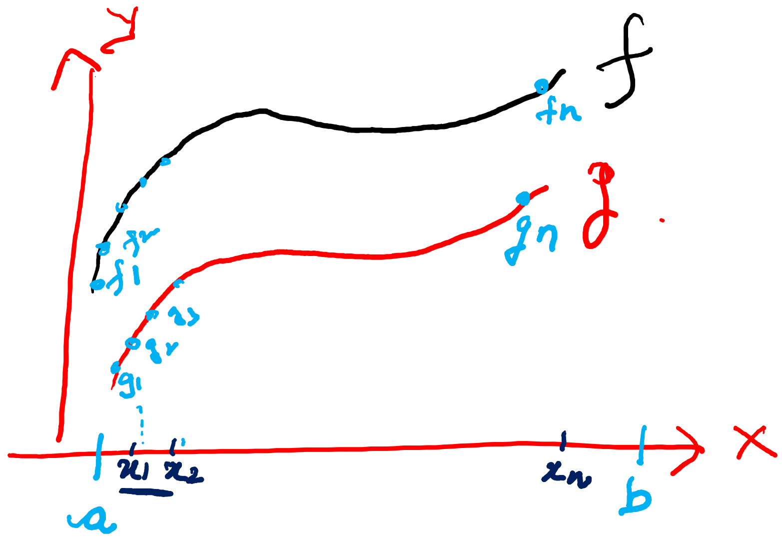
L-periodic fn

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left(A_k \cos\left(\frac{2\pi k x}{L}\right) + B_k \sin\left(\frac{2\pi}{L} k x\right) \right)$$

$$A_k = \frac{2}{L} \int_0^L f(x) \cdot \cos\left(\frac{2\pi}{L} k x\right) dx$$

$$B_k = \frac{2}{L} \int_0^L f(x) \cdot \sin\left(\frac{2\pi}{L} k x\right) dx$$

Inner Product of Functions



$$\Delta x = \frac{b-a}{n-1}$$

$f = [f_1 \; f_2 \; \cdots \; f_n]^T$

$g = [g_1 \; g_2 \; \cdots \; g_n]^T$

$$\langle f(x), g(x) \rangle = \int_a^b f(x)\, g(x)\, dx$$

When 'n' data points are increasing

$$\frac{b-a}{n-1} \langle f, g \rangle = \sum_{k=1}^{n} f(x_k)\, g(x_k) \cdot \Delta x$$

$$n \to \infty \equiv \Delta x \to 0$$

Fourier Series for Complex functions

$$f(x) = \sum_{k=-\infty}^{\infty} C_k \, e^{ikx}$$

$$\left[e^{ikx} = \cos(kx) + i \sin(kx) \right\}$$

Euler expansion

$i = \sqrt{-1}$

$$f(x) = \sum_{k=-\infty}^{\infty} \underline{(\alpha_k + i\beta_k)} \left[\cos(kx) + i \sin(kx) \right]$$

$$= (\alpha_0 + i\beta_0) + \sum_{k=1}^{\infty} \left[(\alpha_{-k} + \alpha_k) \cos(kx) + (\beta_{-k} - \beta_k) \sin(kx) \right]$$

$$+ i \sum_{k=1}^{\infty} \left[(\beta_{-k} + \beta_k) \cos(kx) - (\alpha_{-k} - \alpha_k) \sin(kx) \right]$$

Let $e^{ikx} = \cos(kx) + \sin(kx) = \psi_k$ $\quad k \in \mathbb{Z}$

$$\underline{\underline{\langle \psi_j, \psi_k \rangle}} = \int_{-\pi}^{\pi} e^{ijx} e^{-ikx} dx = \int_{-\pi}^{\pi} e^{i(j-k)x} dx$$

$$= \frac{1}{i(j-k)} \left[e^{i(j-k)x} \right]_{-\pi}^{\pi} = \begin{cases} 0, & \text{if } j \neq k \\ 2\pi, & \text{if } j = k \end{cases}$$

we can write as

$$\boxed{\underline{\underline{\frac{1}{2\pi}}} \sum_{k=-\infty}^{\infty} \underbrace{\langle f(x), \psi_k \rangle}_{c_k} \underbrace{\psi_k}_{e^{ikx}}}$$

$\|\psi_k\|^2 = 2\pi$