Sine

Cosine

Wavelet

Wavelet Transform

Morlet   Daubechies   Coiflets

Biorthogonal   Mexican Hat   Symlets

Wavelet Transform

Scaling

Time

Frequency

10Hz sine wave

Scaling the 10Hz sine wave by 2

5Hz sine wave
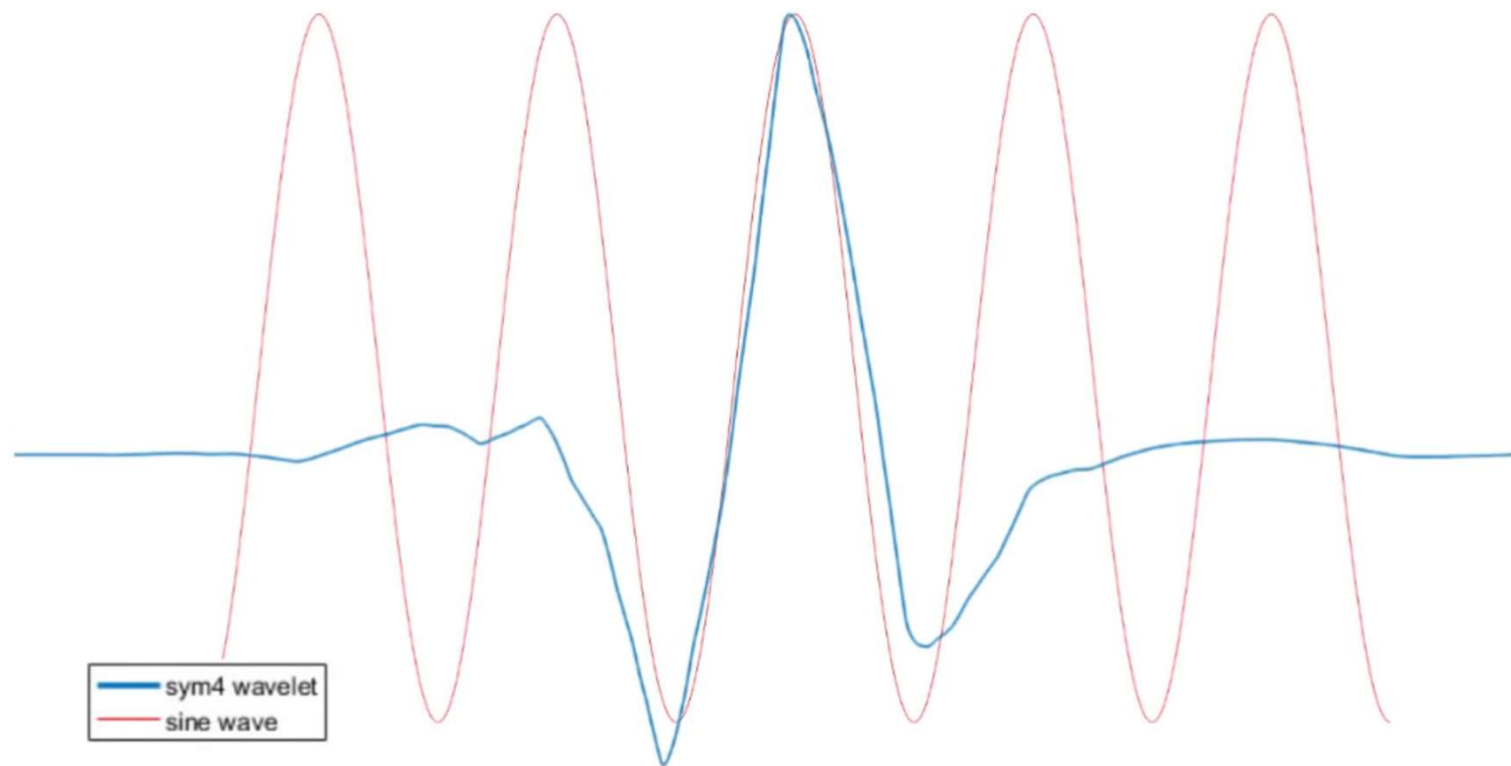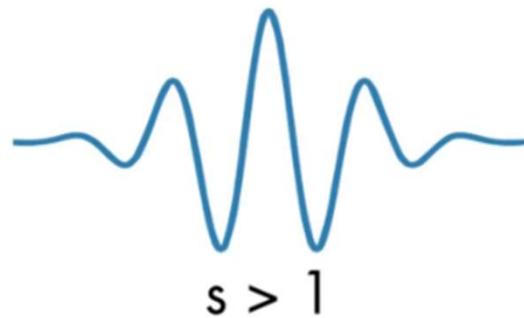
Wavelet Transform

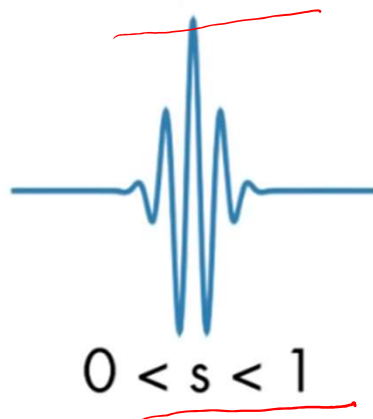**Comparison of a Wavelet with a Sine Wave of Same Frequency**

Wavelet Transform

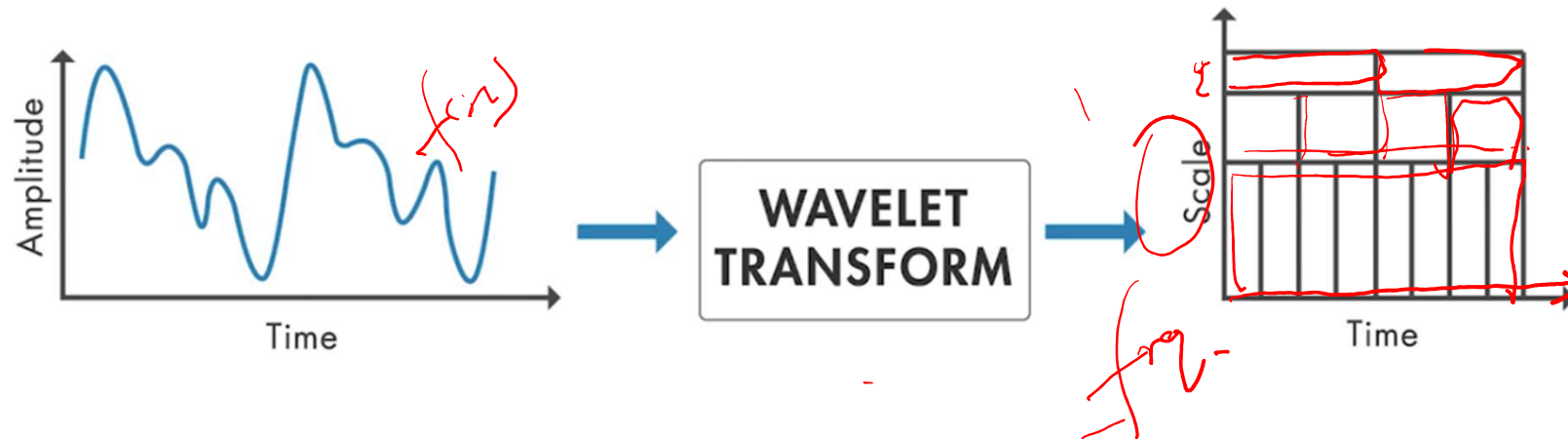$$\Psi\left(\frac{t}{s}\right)$$

$$\Psi(t) = \frac{1}{\sqrt{a}}$$

large scale factor
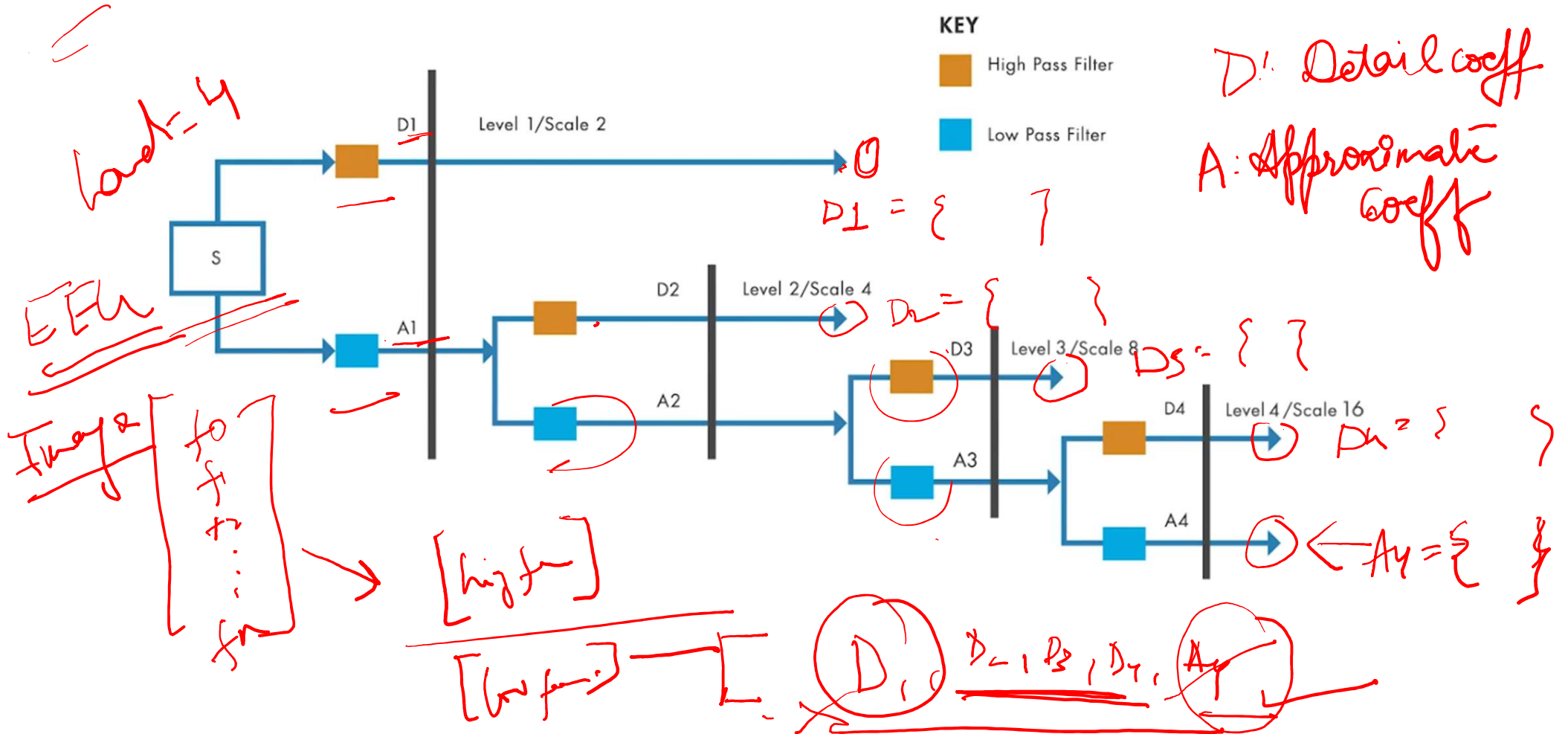low frequency

s > 1

small scale factor
high frequency

0 < s < 1

Continuous Wavelet Transform

Discrete Wavelet Transform

← EEG

FFT/DWT

KEY

■ High Pass Filter

■ Low Pass Filter

D: Detail coeff

A: Approximate coeff

band = 4

EEG

Image & [f0 f1 f2 ... fn]

→ [high freq]

[low freq]

S

D1 — Level 1/Scale 2 → $\mathcal{O}$

$D1 = \{ \quad \}$

A1

D2 — Level 2/Scale 4 → $D_2 = \{ \quad \}$

A2

D3 — Level 3/Scale 8 → $D_3 = \{ \quad \}$

A3

D4 — Level 4/Scale 16 → $D_4 = \{ \quad \}$

A4 → $A_4 = \{ \quad \}$

$I = D_1 \quad D_2, D_3, D_4, A_4$

Original Signal

Level 1 Details

D1

Level 2 Details

D2

Level 3 Details

D3

Level 4 Details

D4

Level 5 Details
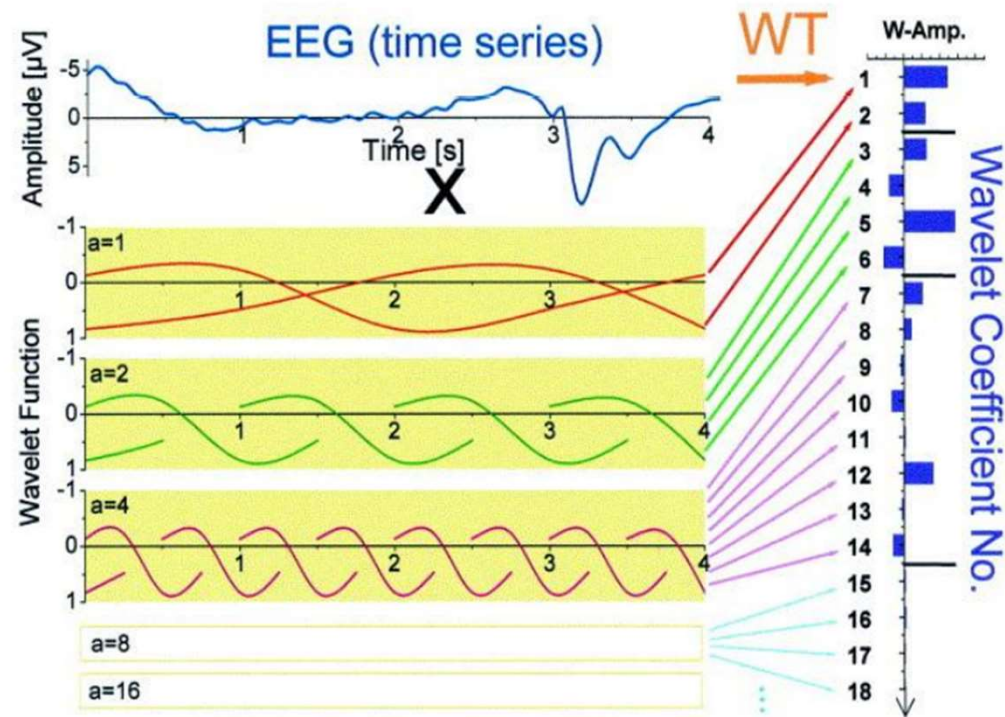
D5

DWT
Levels
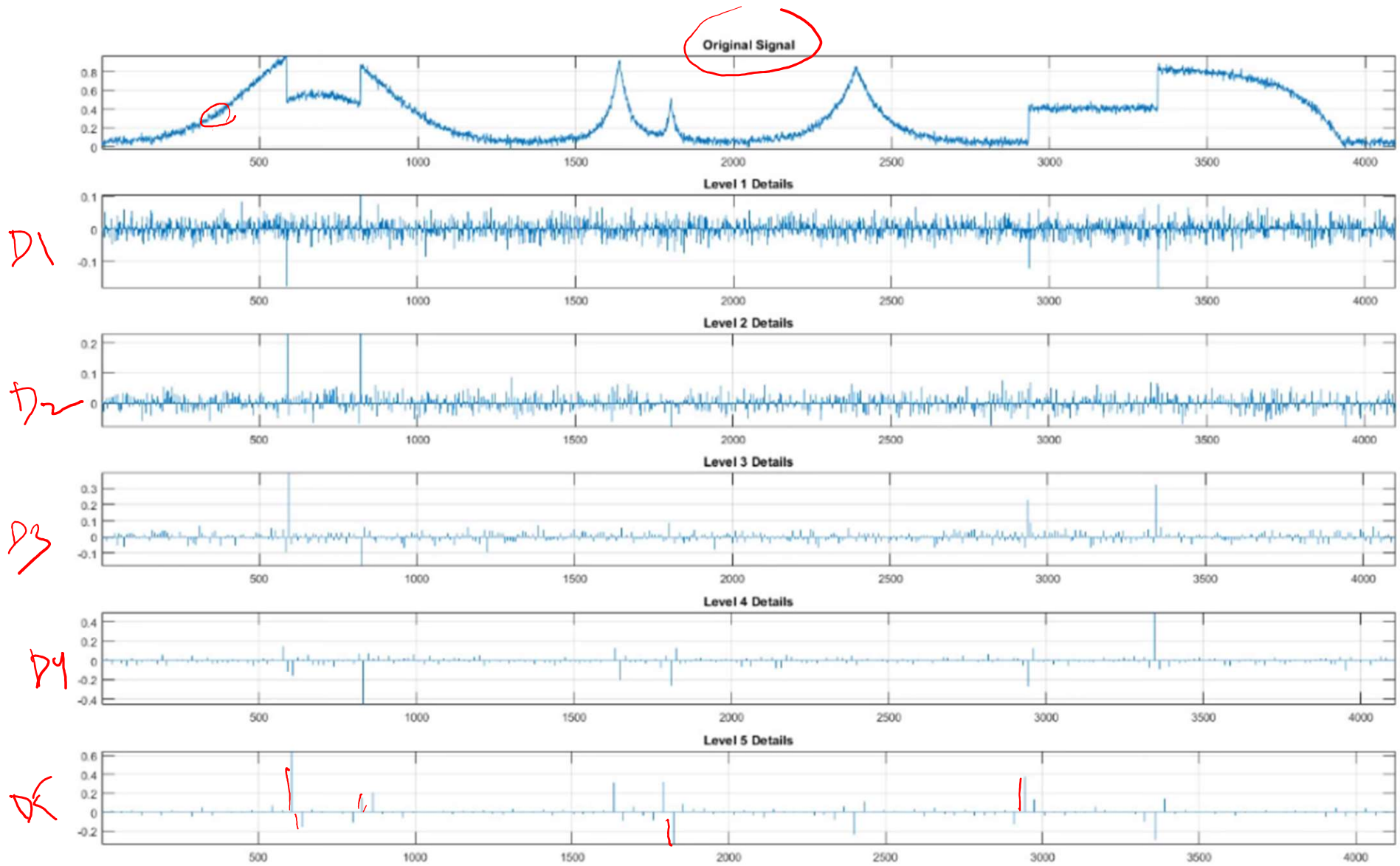
# Example: Wavelet Decomposition of EEG

Credits: (Hinterberger et al., 2003)

Filters

- High Pass Filter:
  - Blocks dc offset in high gain amplifiers or single supply circuits. Filters can be used to separate signals, passing those of interest, and attenuating the unwanted frequencies.

- Low Pass Filter:
  - Stabilizes amplifiers by rolling off the gain at higher frequencies where excessive phase shift may cause oscillations

Filters

- Band Pass Filter:
  - If a high-pass filter and a low-pass filter are cascaded, a band pass filter is created. The band pass filter passes a band of frequencies between a lower cutoff frequency, f_L, and an upper cutoff frequency, f_h

- Notch (Band Reject Filter):
  - The pass bands include frequencies below f_L and above f_h. The band from f_L to f_h is in the stop band.
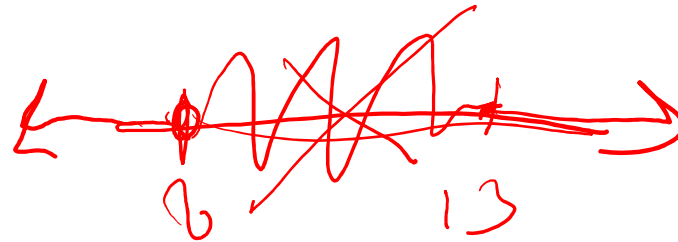
TIME DOMAIN ANALYSIS

(i) Hjorth Parameters [1970s] ← (EEG) $S(t)$

      ↳ Mean Power

      ↳ RMS freq.

      ↳ RMS freq. spread

(a) Activity

$$A = a_0$$

$a_0:$ variance of signal in the epoch under measurement

(b) Mobility

$$M = \sqrt{\frac{a_2}{a_0}}$$

$a_2 =$ variance of $\frac{dS(t)}{dt}$

(3) Complexity

$$C = \sqrt{\frac{a_4}{a_0}}$$

$a_4 =$ variance of $\frac{d^2 S(t)}{dt^2}$

② Auto Regressive Modeling (AR)

$$x_t = \sum_{i=1}^{p} \underline{a_i} \, \underline{x_{t-i}} + \varepsilon_t$$

ε_t - Some white noise

p - Order of AR model

Adaptive Auto Regressive (AAR) Model?

$$x_t = \sum_{i=1}^{p} \underline{a_{i,t}} \, x_{t-i} + \varepsilon_t$$

Statistical structure over time