# CONTENTS

- Transformation
- Types of transformation
- 3-D Transformation
    1) Translation
    2) Rotation
    3) Scaling
    4) Reflection
    5) Shearing

# OBJECTIVE

To understand basic conventions for object transformations in 3D

To understand other transformations like Reflection, Shear

To understand basic transformations in 3D including Translation, Rotation, Scaling

# TRANSFORMATION

- Transformations are a fundamental part of the computer graphics. Transformations are the movement of the object in Cartesian plane .
- TYPES OF TRANSFORMATION
- There are two types of transformation in computer graphics.

  1) 2D transformation

  2) 3D transformation
- Types of 2D and 3D transformation

  1) Translation 2) Rotation 3) Scaling 4) Shearing 5) reflection

# 3D Transformation

- When the transformation takes place on a 3D plane, it is called 3D transformation
- The translation, scaling and rotation transformations used for 2D can be extended to three dimensions.
- In 3D, each transformation is represented by a 4x4 matrix.
- Using homogeneous coordinates it is possible to represent each type of transformation in a matrix form and integrate transformations into one matrix
- To apply transformations, simply multiply matrices, also easier in hardware and software implementation
- Homogeneous coordinates can represent directions

- Homogeneous coordinates: 4 components
- Transformation matrices: 4×4 elements

$$\begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & h & i & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- WHY WE USE TRANSFORMATION

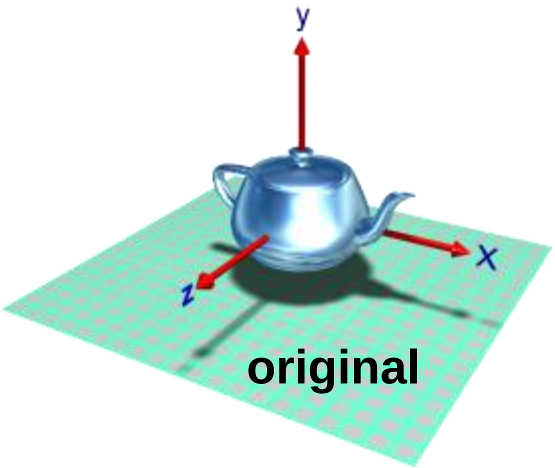
Transformation are used to position objects, to shape object, to change viewing positions, and even how something is viewed.

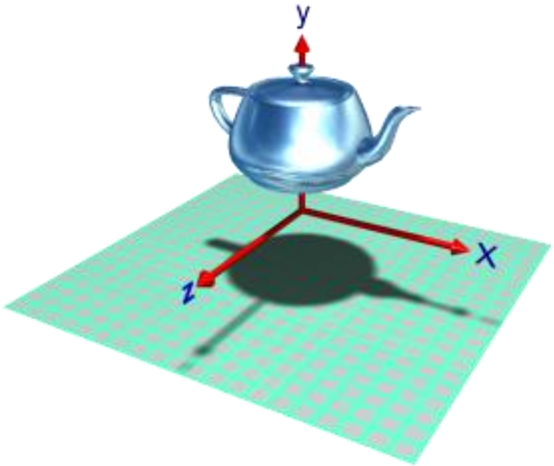
# 3D TRANSLATION

- Moving of object is called translation.
- In 3 dimensional homogeneous coordinate representation , a point is transformed from position P = ( x, y , z) to P'=( x', y', z')

This can be written as: Using P'= T . P where

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

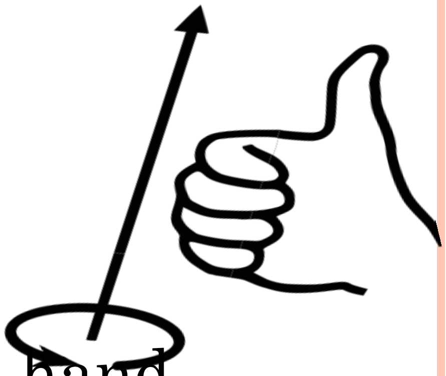


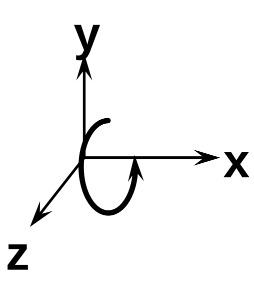

**translation along y,**
**or V = (0, k, 0)**

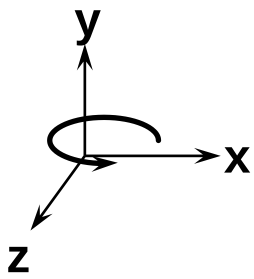
# TRANSLATION

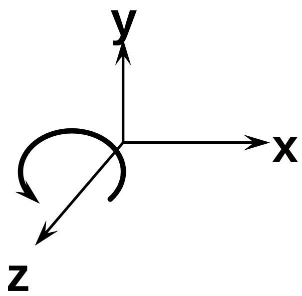
# 3-D Rotations



- Rotation needs an angle and an axis.
- Rotation is defined according to the right-hand rule (our convention)
- In 3D, rotation is about a vector, which can be done through rotations about x, y or z axes.
- Positive rotations are anti-clockwise, negative rotations are clockwise, when looking down a positive axis towards the origin



# 3-D ROTATION TRANSFORMATION MATRIX

Rotations are orthogonal matrices, preserving distances and angles.

1. Rotation about X-axis

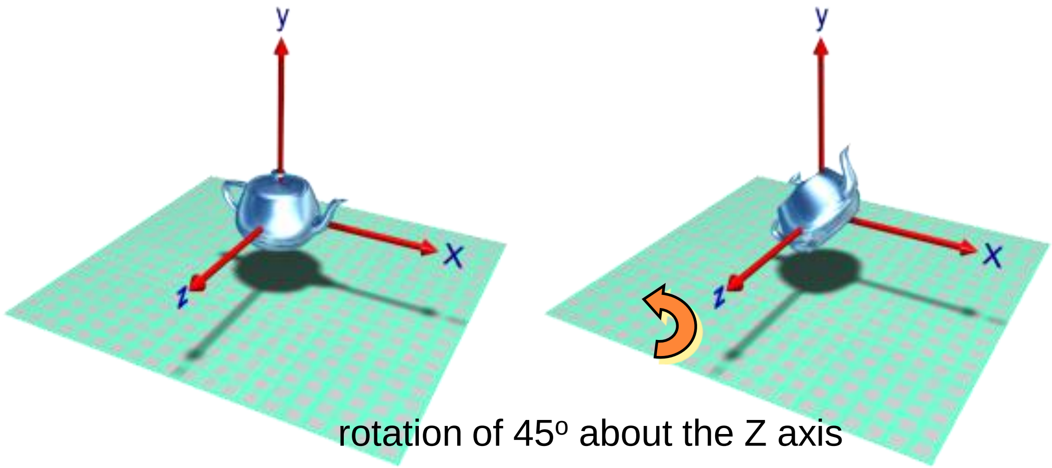
$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Rotation about Y-axis

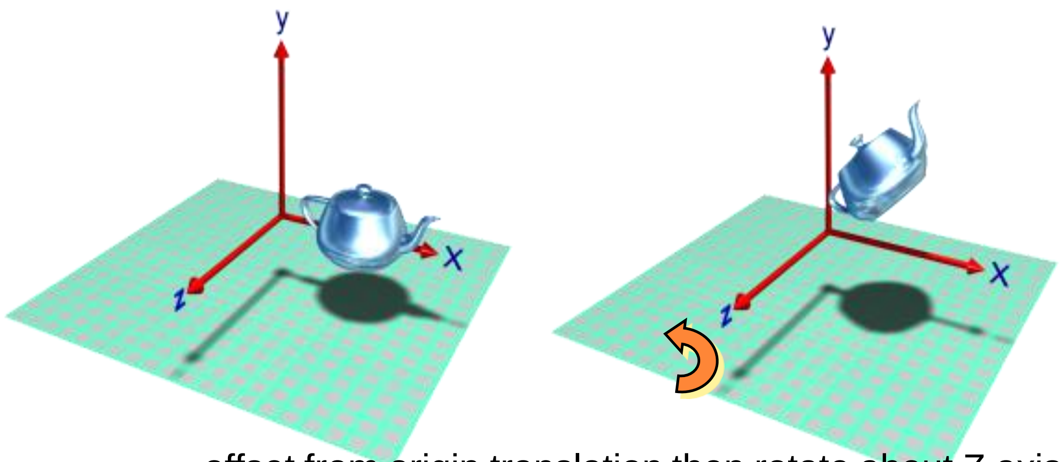
$$\mathbf{R}_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Rotation about Z-axis

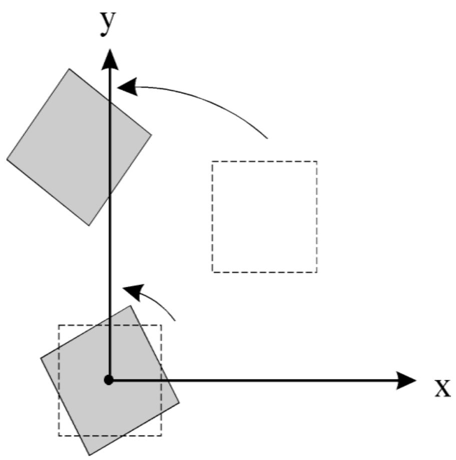
$$\mathbf{R}_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




rotation of $45^{o}$ about the Z axis




offset from origin translation then rotate about Z axis




# Rotation

# 3-D SCALING

- You can change the size of an object using scaling transformation .
- In the scaling process , you either expand or compress the dimensions of the object .
- Scaling can be achieved by multiplying the original coordinates of the object with scaling factor to get the desired result
- The general transformation matrix for scaling is

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} t_x & 0 & 0 & 0 \\ 0 & t_y & 0 & 0 \\ 0 & 0 & t_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# 3-D SCALING

- Scaling in x-direction by factor a

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Scaling in y-direction by factor e

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Scaling in z-direction by factor j

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3-D SCALING

- Scaling in x,y,z-direction by factor a,e, j-units respectively.

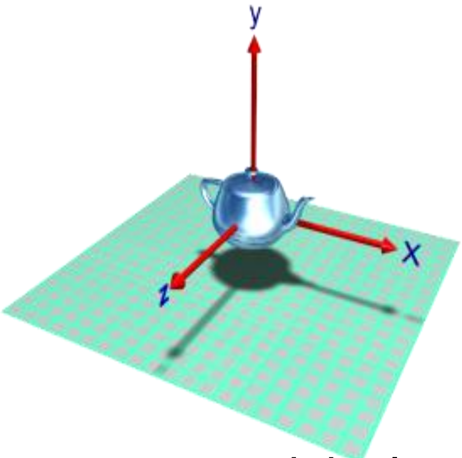
$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Uniform scaling in x-direction by factor a, if a>1 then expansion occurs
  if 0<a<1 contraction occurs

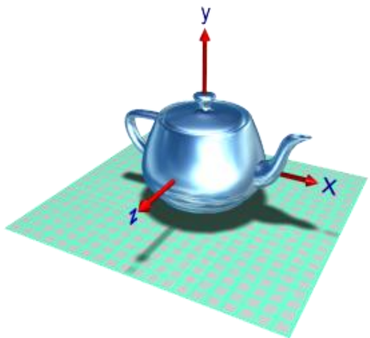
$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Overall scaling by s units
  Here, a=1/s
  if 0<s<1 then expansion occurs
  if s>1 then contraction occurs

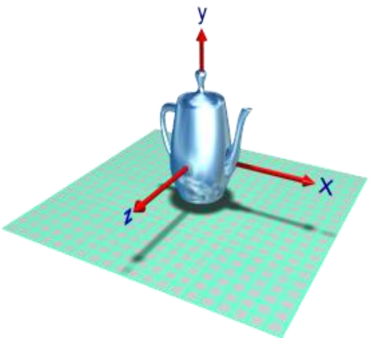
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}$$



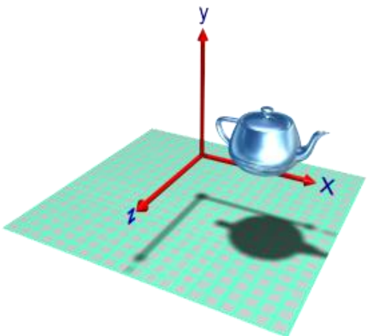

Original




scale all axes




scale Y axis




offset from origin

# Scaling

# 3D REFLECTION

- Reflection in computer graphics is used to emulate reflective objects like mirrors and shiny surfaces
- Reflection may be an x-axis y-axis , z-axis. and also in the planes xy-plane,yz-plane , and zx-plane.
- **Note:** Reflection relative to a given Axis are equivalent to 180 Degree rotations

# 3D REFLECTION

- Reflection about xy-plane(or z=0plane)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Reflection about yz-plane(or x=0plane)

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D REFLECTION

- Reflection about zx-plane(or y=0plane)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D Shearing

- Shearing:

  The change in each coordinate is a linear combination of all three
- The general Transformation matrix for shearing in x, y and z-direction is given by,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & b & 0 \\ c & 1 & d & 0 \\ e & f & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# 3D SHEARING

- Shearing in x-direction  proportional to y and z-axis by c-units and e-units respectively is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ c & 1 & 0 & 0 \\ e & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

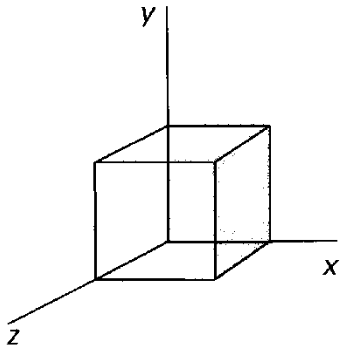
- Shearing in y-direction  proportional to x and z-axis by a-units and f-units respectively is

$$\begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & f & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

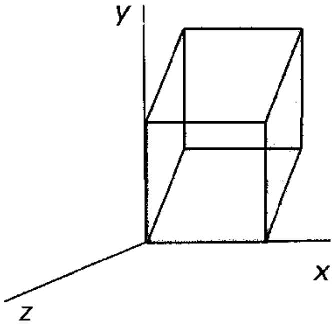
# 3D SHEARING

- Shearing in z-direction  proportional to x and y-axis by b-units and d-units respectively is

$$\begin{bmatrix} 1 & 0 & b & 0 \\ 0 & 1 & d & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




**Original object**




**Shearing in z-direction**

# Summary

- Methods for object modelling transformation in three dimensions are extended from two dimensional methods by including consideration for the z coordinate.
- We saw transformation in 3-D using which we can transformed the required object in required form.