

HOMEWORK-3

Course: DSAA, Monsoon 2017 @ IIITS

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Problem 1.

- Consider the signal

$$x[n] = (\alpha^n) * u[n]$$

– Sketch the signal $g[n] = x[n] - \alpha x[n - 1]$

– Note: Need not use matlab for this part of the exercise. Assume $\alpha = 1/2$

. Use the result above in conjunction with properties of the linear convolution to determine a sequence $h[n]$ such that

$$x[n] * h[n] = (1/2)^n (u[n + 2] - u[n - 2])$$

OBSERVATIONS:-

The sketch of the signal along with the calculation of $h[n]$ is as shown below in the figure. The discussion and explanation are given below the figure.

HOME ASSIGNMENT- 03

$$x[n] = 2^n u[n]$$

⇒ sketch the signal $g[n] = x[n] - 2x[n-1]$

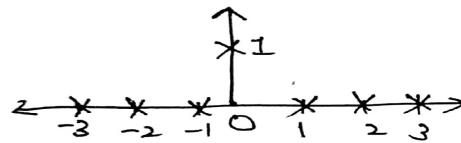
$$2 = \frac{1}{2}$$

Sketch the signal ⇒ $g(n) = 2^n u[n] - 2 \cdot 2^{n-1} u[n-1]$
 $= 2^n [u[n] - u[n-1]]$
 $= \left(\frac{1}{2}\right)^n \times \delta(n)$

⇒ Set $n = -1$
 $n = 1$
 $n = 0$

⇒ $g(0) = 1$ ⇒ else $g(n) = 0$

GRAPH:-



$g(n) = \left(\frac{1}{2}\right)^0$
 ⇒ as in eq

$$\begin{aligned} (2) \quad x[n] * h[n] &= \left(\frac{1}{2}\right)^n \{u[n+2] - u[n-2]\} \\ &= 2^n u[n+2] - 2^n u[n-2] \\ &= 4 \cdot 2^{n+2} u[n+2] - \frac{1}{4} 2^{n-2} \end{aligned}$$

where $2 = \frac{1}{2}$. (as in eq)

$$y(n) = 4x(n+2) - \frac{1}{4}x(n-2)$$

$$\Rightarrow h(n) = 4\delta(n+2) - \frac{1}{4}\delta(n-2)$$

The signal $g[n]$ is plotted as shown from the equation of $g[n]$ simplified as shown.

Next the output convoluted signal $x[n] * h[n]$ is as shown and $h[n]$ is calculated as shown in the picture. Thus the result in conjunction with properties of linear convolution are used to

determine the sequence $h[n]$.

Problem 2.

Solve the following problem using matlab and theoretical approaches and compare the results. Consider the two LTI systems with the unit sample responses $h_1[n]$ and $h_2[n]$ given below.

These two systems are

cascaded. Let the input be $x[n] = ((-1)^{(n+1)} * \{u[n] - u[n - 6]\})$.

$$h_1[n] = (1/2)^n * \{u[n] - u[n - 5]\}$$

$$h_2[n] = u[n + 4] - u[n]$$

– Compute $y[n]$ by first computing $w[n] = x[n] * h_1[n]$ and then computing $y[n] = w[n] * h_2[n]$.

– Again compute $y[n]$ by first convolving $h_1[n]$ with $h_2[n]$ to obtain $g[n] = h_1[n] * h_2[n]$ and then convolving $x[n]$ with $g[n]$.

Compare the two results.

- For this exercise, in order to verify the coincidence of the results, please plot the two results in the same plot, one with a smaller marker size and the other with a larger marker size.

OBSERVATIONS:-

The two different types of convolution are as shown in the working. Ultimately the two convolutions should yield the same result irrespective of in which way we do the convolution. Thus by computing $w[n] = x[n] * h_1[n]$ and then computing $y[n] = w[n] * h_2[n]$.

and $y[n]$ by first convolving $h_1[n]$ with $h_2[n]$ to obtain $g[n] = h_1[n] * h_2[n]$ and then convolving $x[n]$ with $g[n]$ we get the same result as shown below.

$$(2) \quad h_1[n] = \left(-\frac{1}{2}\right)^n \{u[n] - u[n-5]\}$$

$$h_2[n] = u[n+4] - u[n]$$

$$y[n] = w[n] * h[n] \quad \text{where}$$

$$w[n] = x[n] * h_1[n]$$

$$x[n] = (-1)^{n+1} \begin{cases} u[n] \\ u[n-1] \end{cases}$$

$\Rightarrow 2$ sig
ans ca

$$x[n] = [-1, 1, -1, 1, -1, 1]$$

$$h_1[n] = [1, -\frac{1}{2}, +\frac{1}{4}, -\frac{1}{8}, \frac{1}{16}]$$

	1	$-\frac{1}{2}$	$+\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
-1	-1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{16}$
1	-1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
-1	-1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{16}$
1	-1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
-1	-1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{16}$
1	-1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$

$$w[n] = [-1, 1.5, -1.75, 1.875, -1.9375, 1.9375, -0.9375, 0.4375, -0.1875, 0.0625]$$

$$y[n] = w[n] * h_2[n]$$

~~0.0625, 0.4375, -0.1875, 0.0625, -0.1875, 0.0625~~

~~0.0625, 0.4375, -0.1875, 0.0625, -0.1875, 0.0625~~

	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875
1	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875
1	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875
1	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875
1	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875
1	-1	1.5	-1.75	1.875	-1.9375	1.9375	-0.9375	0.4375	-0.1875

$$y[n] = [-1, 0.5, -1.25, 0.625, -0.3125, 0.1875, -0.0625, 0.0625]$$

Now in Method (2) solve

$h_1[n]$ and $h_2[n]$ to obtain $g[n]$

$$g[n] = h_1[n] * h_2[n]$$

$$y[n] = x[n] * g[n]$$

$$g[n] = h_1[n] * h_2[n]$$

$$h_1[n] = [1, -\frac{1}{2}, +\frac{1}{4}, -\frac{1}{8}, \frac{1}{16}]$$

$$h_2[n] = [1, 1, 1, 1]$$

$$g[n] = h_1[n] * h_2[n]$$

	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
1	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
1	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
1	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$
1	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{16}$

$$\therefore g[n] = [1, 0.5, 0.75, 0.625, 0.3125, 0.1875, -0.0625, 0.0625]$$

$$y[n] = x[n] * g[n]$$


```

n=-10:10;

%DECLARING y
y=zeros(1,length(n));
y(n>=0)=1;

%DECLARING U(n-6)
u6=zeros(1,length(n));
u6(n-6>=0)=1;

%DECLARING U(n-5)
u5=zeros(1,length(n));
u5(n-5>=0)=1;

%DECLARING U(n-4)
u4=zeros(1,length(n));
u4(n-4>=0)=1;

x=power(-1,n+1).*(y-u6);

h1=power(-0.5,n).*(y-u5);

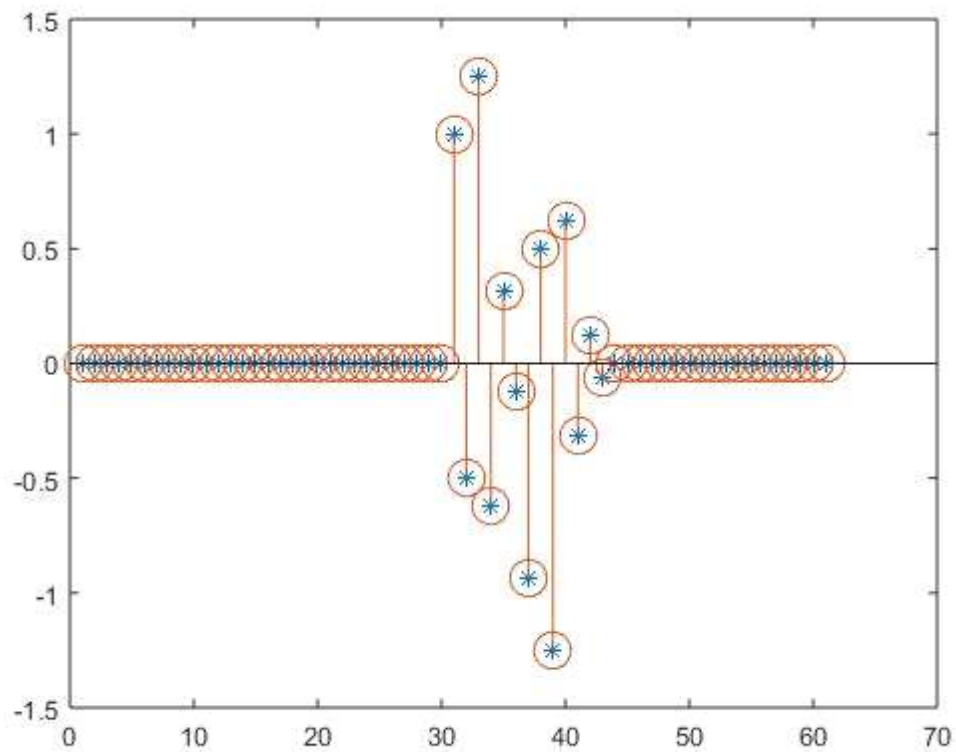
h2=u4-y;

z1=conv(x,conv(h1,h2));
z2=conv(conv(x,h1),h2);

stem(z1,'*');
hold on
stem(z2,'o','MarkerSize',14);

```

OUTPUT:-



The convolution is done using two methods. First y vector is declared as shown which is unit step function by the following code :-

```
%DECLARING y
y=zeros(1,length(n));
y(n>=0)=1;
```

Next the delayed impulses are plotted as follows. $u(n-6)$, $u(n-5)$ and $u(n-4)$ are declared as shown as below.

```
%DECLARING U(n-6)
u6=zeros(1,length(n));
u6(n-6>=0)=1;
```

```
%DECLARING U(n-5)
u5=zeros(1,length(n));
u5(n-5>=0)=1;
```

```
%DECLARING U(n-4)
u4=zeros(1,length(n));
u4(n-4>=0)=1;
```


Next the three functions are declared as shown below.

```
x=power(-1,n+1).*(y-u6);  
h1=power(-0.5,n).*(y-u5);  
h2=u4-y;  
z1=conv(x,conv(h1,h2));  
z2=conv(conv(x,h1),h2);  
  
stem(z1,'*');  
hold on  
stem(z2,'o','MarkerSize',14);
```

Now convolution is first performed between the first two signals and stored in z1 then convolution is performed between the other two signals and stored in a different variable z2. Next the convoluted signal is plotted using stem function. To plot in the same graph hold on function is used. The first function is plotted using a different marker and for the second function is plotted using a different marker and different marker size. As seen from the graph the convolution of the signals in both ways produce the same output as the markers are being marked at the same place. Thus verified the coincidence of the results by plotting the two results in the same plot one with a smaller marker size and the other with a larger marker size.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX-**END OF ASSIGNMENT**-XXXXXXXXXXXXXXXXXXXXXXXXXXXX