<u>*HOMEWORK-2*</u>

Course: DSAA, Monsoon 2017 @ IIITS

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Problem 1.

DSAA HOMEWORK-2

(1) given $x(t)$, a real valued signal and sampling rate $\Omega_s = 8000\pi$ rad/s, determine for what values of Ω does $X(\Omega)$ vanish?

⇒ According to sampling theorem

(1) Frequency of signal $< f_{max}$ (i.e it should be band limited)

(2) Sampling rate should be sufficiently enough

$f_{sampling} \geq 2f_{max}$

($f_s = 2f_{max}$) ↳ nyquist rate

If these two conditions are satisfied signal $x(t)$ can be recovered.

It is given that $\Omega_s = 8000\pi$ rad s^{-1}

Hence $\Omega <= 4000$ rad s^{-1}

<u>4000 → max</u>. Hence for Ω greater than 4000 rad s^{-1}, $X(\Omega)$ vanishes.

<u>OBSERVATIONS</u>:-

According to sampling theorem, frequency of sampling should be greater than 2* maximum frequency. Therefore for the condition to be satisfied omega should be less than 4000 rad second^-1.

Problem 2.

(2) Given $y(t)$ with Nyquist rate Ω_1, determine the Nyquist rates of following signals:

$$y_1(t) = y(t) + y(t-1) \quad — (1)$$

$$y_2(t) = \frac{d}{dt} x(t) \quad — (2)$$

$$y_3(t) = y(t) \exp(j\Omega_0 t) \quad (3$$

Problem 2.1

ANS :-

(1) $y_1(t) = y(t) + y(t-1)$

$y(t)$ is the given signal with Nyquist rate Ω_1)

we have to determine Nyquist rate of $y_1(t)$.

If Ω_2 is Nyquist rate of any signal $x(t)$ then

$X(\Omega) = 0$ for $|\omega| > \frac{\omega_2}{2}$

$|\Omega| > \frac{\Omega_2}{2}$

Apply fourier transform

$$Y_1(\Omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\Omega t} \; -① \\ + \int_{-\infty}^{\infty} y(t-1) e^{-j\Omega t} \; ②$$

$\Rightarrow$ Main eq $=$ ① $+$ ②

① $= \int_{-\infty}^{\infty} y(t) e^{-j\Omega t} = Y(\Omega)$

② $= \int_{-\infty}^{\infty} y(t-1) e^{-j\Omega t}$ — put $x_1 = t-1 \Rightarrow dx_1 = dt$, $t = x_1 + 1$

$\Rightarrow e^{-j\Omega} \int_{-\infty}^{\infty} y(x_1) e^{-j\Omega x_1} dx_1$

① part

$\Rightarrow$ [redacted] $Y_1(\Omega) = Y(\Omega) + e^{-j\Omega} \times Y(\Omega)$

$Y_1(\Omega) = 0$ for $|\Omega| > \frac{\Omega_0}{2}$

$\therefore$ Nyquist rate for $y_2(t)$ is also Ω_1.

OBSERVATIONS:-

The fourier transform is applied for the following signal. And the nquist rate for the signal is obtained as omega 1. The integration is done as in image. The nquist rate is 2* max frequency.

Problem 2.2

(2) $y_2(t) = \frac{d}{dt} y(t)$ (given wrong in PDF as $x(t)$)

$y(t)$ is the given signal with Nyquist rate Ω_1

If for any signal $x'(t)$; Ω_2 is Nyquist rate

then

$x'(\Omega) = 0$ for $|\omega| > \frac{\omega_2}{2}$ and $|\Omega| > \frac{\Omega_2}{2}$

Applying formula to given signal

$Y(\Omega) = 0$ for $|\Omega| > \frac{\Omega_1}{2}$

$y_2(t) = \frac{d}{dt} y(t)$ ⇒ Apply fourier transform

$\Rightarrow \text{Fourier}\,(y_2(t)) = F\left(\frac{d(y_2(t))}{dt}\right) = Y(\omega)$

$Y_2(\Omega) = \int_{-\infty}^{\infty} \frac{d}{dt}(y(t))\, e^{-j\Omega t}$ (by fourier Transform)

$\left(\because \int uv\, dx = u\int v\, dx - \int \left(u' \int v\, dx\right) dx\right)$

↳ By product rule of integration.

$Y_2(\Omega) = j\Omega \int_{-\infty}^{\infty} e^{-j\Omega t}\, y(t)\, dt$

$= \boxed{j\Omega\, Y(\Omega)}$

$Y_2(\Omega) = 0$ for $|\omega| > \frac{\omega_0}{2}$

∴ Nyquist rate for $y(t)$ is Ω_1.

OBSERVATIONS:-

The fourier transform is applied for the following signal. And the nquist rate for the signal is obtained as omega 1. The integration is done as in image. The nquist rate is 2* max frequency.

Problem 2.3

3) $y_3(t) = y(t)\exp(j\Omega_0 t)$

If $y(t)$ is the given signal with Nyquist rate Ω_1 in question.

For any signal $x'(t)$, Ω_2 is the nquist rate then

$x'(\Omega) = 0$ for $|\omega| > \frac{\omega_s}{2}$ and $|\Omega| > \frac{\Omega_2}{2}$

Applying formula to the given signal,

$y_3(t) = y(t)\exp(j\Omega_0 t)$

$\Rightarrow$ Fourier $(y_3(t))$ = Fourier $(y(t)\exp(j\Omega_0 t))$

$= \int_{-\infty}^{\infty} y(t)\, e^{(j\Omega_0 t)} \cdot e^{-j\Omega_1 t}\, dt$

$= \int_{-\infty}^{\infty} y(t)\, e^{j(\Omega_0 - \Omega_1)t}\, dt$

Assume $\Omega_0 - \Omega_1$ to be Ω_2.

$y_3(t) = Y(\Omega_2) = Y(\Omega_0 - \Omega_1)$

$\therefore$ The maximum frequency is $\Omega_0 - \Omega_1$

$\therefore$ Nquist rate for the given signal is $2(\Omega_0 - \Omega_1)$

OBSERVATIONS:-

The fourier transform is applied for the following signal. And the nquist rate for the signal is obtained as 2*(omega 0- omega 1). The integration is done as in image. The nquist rate is 2* max frequency.

Problem 3.

(3) Given 2 signals $x_1(t)$ and $x_2(t)$ with corresponding maximum frequencies Ω_1 and Ω_2 respectively. Determine the Nyquist rate for the product using matlab for some example signals

$$z(t) = x_1(t) \times (x_2(t))$$

HINT:- choose $x_1(t) = \cos(\Omega_1 t)$ and $x_2(t) = \cos(\Omega_2(t))$

The spectral frequencies that are max are given → & apply fourier transform.

⇒ Two signals $x_1(t)$ → (Ω_1) and $x_2(t)$ → (Ω_2)

$$z(t) = x_1(t) \; x_2(t)$$

choosing $x_1(t) = \cos(\Omega_1 t)$, $x_2(t) = \cos(\Omega_2(t))$

FORMULA: $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$\Rightarrow z(t) = \frac{1}{2}\left[\cos(\Omega_1+\Omega_2)t + \cos(\Omega_1-\Omega_2)t\right]$$

$$F_{\text{cosx}}\left(\frac{1}{2}\cos(\Omega_1+\Omega_2)t + \cos(\Omega_1-\Omega_2)t\right)$$

$$\int_{-\infty}^{\infty} e^{j\Omega_0 t} = \delta(\Omega-\Omega_0)$$

→ By applying euler's form we get

$$= \frac{1}{2}\left[\frac{e^{j(\Omega_1+\Omega_2)t} + e^{-j(\Omega_1+\Omega_2)t} + e^{j(\Omega_1-\Omega_2)} + e^{-j(\Omega_1-\Omega_2)t}}{2}\right]$$

we get n diff

$$= \delta(\Omega - (\Omega_1+\Omega_2)) + \delta(\Omega + (\Omega_1 - \Omega_2))$$
$$+ \delta(\Omega + (\Omega_2 - \Omega_1))$$
$$+ \delta(\Omega + (-\Omega_2 - \Omega_1))$$

as All Ω_2, Ω_1 are positive

$\therefore$ $\Omega_2 + \Omega_1$ is max frequency

$-\Omega_2 - \Omega_1$ is the min frequency

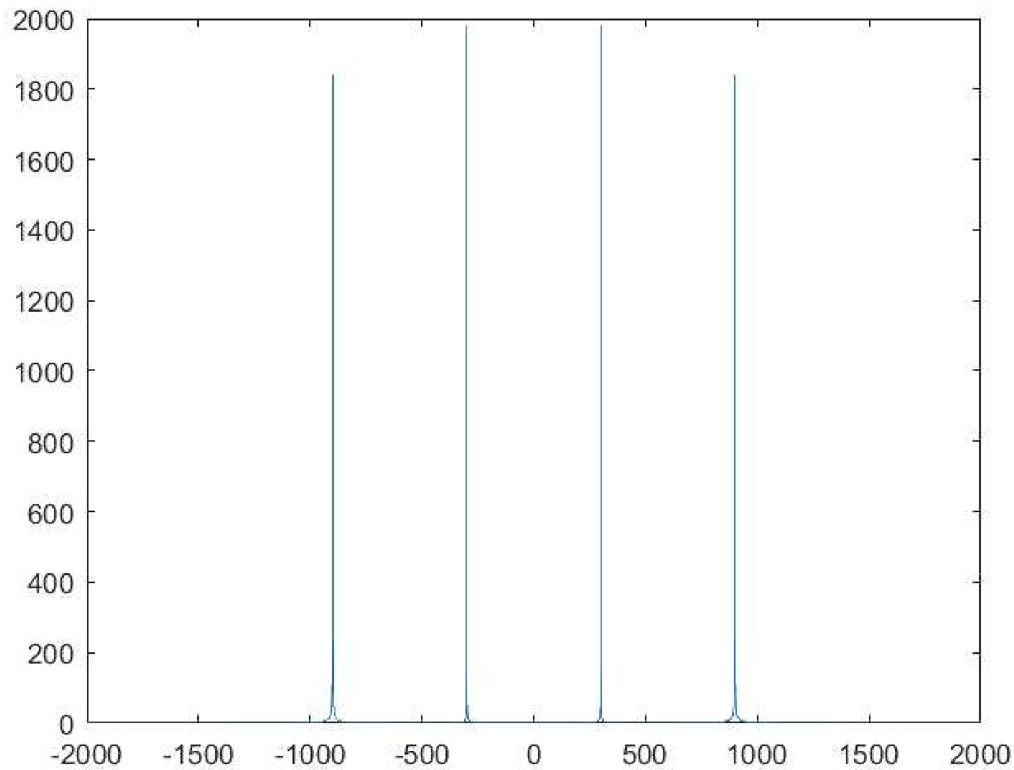
$\therefore$ Nyquist rate = 2(max freq)
$= 2(\Omega_2 + \Omega_1)$.

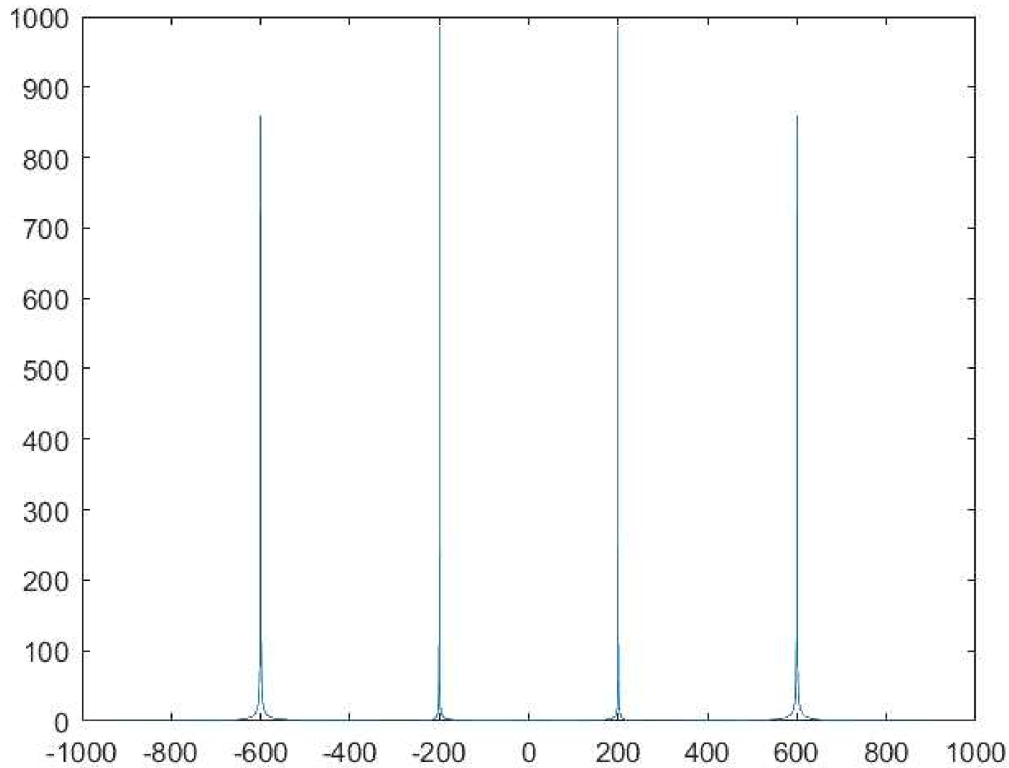
MATLAB CODE:-

```
%%HW2
%%ANIRUDH KANNAN V P
%%201601004
%%UG2 CSE

clc
clear all
close all

%%MAIN PROGRAM

N=2000;

om1=400;

om2=200;

Fs=N;

p=1/Fs;

time=(0:(2*N))*p;

sig1=cos(2*pi*om1*time);

sig2=cos(2*pi*om2*time);

sig=sig1.*sig2;

len=length(sig);
```

```
s=fft(sig,len);

s=fftshift(s);

fvec=Fs/2*linspace(-1,1,len);

b=abs(s);

plot(fvec,b);
```

OBSERVATIONS:-





This is got by checking it by setting 2 values of omega 1 and omega 2 for the signal. First is by setting omega 1 as 600 and omega 2 as 400 and second one is by setting omega 1 as 400 and omega 2 as 200. As evident from the graph after applying fourier transform for the product function we get spectral lines . The spectral lines give the corresponding frequencies.A time vector is being created. Signal 1 and signal 2 is defined as follows.

```
sig1=cos(2*pi*om1*time);

sig2=cos(2*pi*om2*time);

sig=sig1.*sig2;
```

Then The signal is calculated as the product of the two signals by using fft (FOURIER TRANSFORM FUNCTION). and it is shifted using fftshift and then it is plotted.The commands are given below

```
s=fft(sig,len);

s=fftshift(s);
```

```
fvec=Fs/2*linspace(-1,1,len);

b=abs(s);

plot(fvec,b);
```

The spectral points are obtained at four places. The maximum frequency is omega1+omega2

Therefore the answer is 2*maxfreq=nquist rate=2*(omega1+omega2)

$\Downarrow$

This is evident from matlab code

$x_1(t) = \cos(\Omega_1(t))$ and $x_2(t) = \cos(\Omega_2(t))$

$\Downarrow$

The spectral locations are the <u>maximum</u> frequencies

↳ as evident from Matlab graph