SC1.310 Assignment 2

Course Instructor: Samyadeb Bhattarcharya (samyadeb.b@iiit.ac.in)

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Question 1.

A spin $\frac{1}{2}$ system is known to be an eigenstate of $\vec{S} \cdot \hat{n}$ with eigenvalue $\frac{\hbar}{2}$, where $\hat{n}$ is a unit vector lying in xz plane that makes an angle γ with the positive z axis.
(a) Suppose $\hat{S}_x$ is measured. What is the probability of getting $+\frac{\hbar}{2}$?
(b) Evaluate the dispersion in $\hat{S}_x$, that is, $\langle (S_x - \langle S_x \rangle)^2 \rangle$.

Question 2.

Consider the spin-precession problem with Hamiltonian $H = -(\frac{eB}{mc})S_z \implies H = \omega S_z$. Write down the Heisenberg equation of motion for S_x, S_y, S_z. Solve those equations to find the $S_x(t), S_y(t), S_z(t)$ as a function of time.

Question 3.

Consider a particle in three dimensions whose Hamiltonian is given by

$$H = \frac{p^2}{2m} + V(\hat{x}) \tag{1}$$

By calculating $[\vec{x} \cdot \vec{p}, H]$, obtain

$$\frac{d}{dt} \langle \vec{x} \cdot \vec{p} \rangle = \langle \frac{p^2}{2m} \rangle - \langle \vec{x} \cdot \vec{\nabla} V \rangle \tag{2}$$

$\vec{x} = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}, \vec{p} = p_1 \hat{i} + p_2 \hat{j} + p_3 \hat{k}$,
$\vec{\nabla} V = \frac{\partial V}{\partial x_1} \hat{i} + \frac{\partial V}{\partial y_1} \hat{j} + \frac{\partial V}{\partial z_1} \hat{k}$

Question 4.

Consider an one dimensional Harmonic oscillator.
Then consider a normalized state $|\alpha\rangle = C_0 |0\rangle + C_1 |1\rangle$ where $|n\rangle$ is number state.
(a) Find the expectation value of $\langle x \rangle_\alpha$ to be

$$\langle x \rangle_\alpha = 2\sqrt{\frac{\hbar}{2m\omega}} \cos(\delta_1 - \delta_0) |C_0| \sqrt{1 - |C_0|^2} \tag{3}$$

where $C_0 = |C_0| e^{i\delta_0}$, $C_1 = |C_1| e^{i\delta_1}$.
(b) Now find the dispersion $\langle \Delta x^2 \rangle$.

Question 5.

Let $x(t)$ be the co-ordinate operator for a free particle with Hamiltonian $H = \frac{p^2}{2m}$.
Find the Commutator $[x(t), x(0)]$.