

End Semester

Consider the following commutation relation between two operators

$$X \text{ and } P \quad [X, P] = i\mathbf{I} (\mathbf{I} \rightarrow \text{identity operator}).$$

Further consider

$$H = X^2 + P^2; S = \frac{XP + PX}{2}; K = H^2.$$

Then the following commutator $[P, [K, S]]$ amounts to be,

- ☐ a. XP
- ☐ b. P^2
- ☐ c. X^2
- ☒ d. X^3

The classical information capacity of a bipartite state ρ_{AB} of $N * N$ dimension is given by $C(\rho) = \log_2 N + S(\rho^B) - S(\rho_{AB})$ where $S(\rho) = -\text{Tr}[\rho \log_2 \rho]$ and $\rho_B = \text{Tr}_A(\rho_{AB})$. Therefore the classical capacity of the bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is

Consider the following master equation

$$\frac{d\rho}{dt} = \gamma(\sigma_- \rho \sigma_+ - \frac{1}{2} \sigma_+ \sigma_- \rho - \frac{1}{2} \rho \sigma_+ \sigma_-);$$

where

$$\sigma_- = |0\rangle\langle 1|; \sigma_+ = (\sigma_-)^\dagger;$$

$$|0\rangle = [0, 1]^T; |1\rangle = [1, 0]^T; |\pm\rangle = \frac{|1\rangle \pm |0\rangle}{\sqrt{2}};$$

The solution of this master equation is dynamical map Λ if ρ_s is the state, such that $\Lambda(\rho_s) = \rho_s$; then for the given operation ρ_s is

Consider the operation

$$\Lambda(\rho) = \sum_i A_i \rho A_i^\dagger$$

where

$$A_1 = \sqrt{1-N}(|0\rangle\langle 0| - \sqrt{1-\gamma}|1\rangle\langle 1|)$$

$$A_2 = \sqrt{\gamma(1-N)}|0\rangle\langle 1|$$

$$A_3 = \sqrt{N}(\sqrt{1-\gamma}|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$A_4 = \sqrt{\gamma N}|1\rangle\langle 0|$$

Then which of the following statements are true

- ☐ a. The operation is not completely positive if $N=1$
- ☐ b. The operation is completely positive under only when γ is strictly less than 1.
- ☒ c. The operation is positive
- ☐ d. The operation is complete positive under the condition $\gamma, N \in [0, 1]$