Introduction to Probability and Statistics for Engineers and Scientists

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Notes

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Chapter 14

Life Testing

"Ch14 Quote here."

14.1 Introduction

The general problem being considered here is a population of independent items having some common underlying distribution of lifetimes, which is known but for a single parameter.

The concept of a hazard rate is used in engineering to analyze this problem and the most common choice of *a-priori* distribution assumed is an exponential RV.

14.2 Hazard rate functions

Consider a positive RV X with some CDF (F) and PDF (f). The *failure rate* (also called *hazard rate*) function of F is now defined as

$$\lambda(t) = \frac{f(t)}{1 - F(t)} \tag{14.1}$$

$\lambda(t)$ represents the conditional probability that an item of age t will fail imminently.

$$P\{X \in (t,\ t + \mathrm{d}t) \mid X > t\} \approx \frac{f(t)}{1 - F(t)}\ \mathrm{d}t \tag{14.2}$$

For an underlying exponential distribution, which is memoryless, $\lambda(t) = \lambda$ constant equal to its rate. The rate function is uniquely able to determine an the underlying CDF for a positive continuous RV.

$$\lambda(s) = \frac{\mathrm{d}\ F(s)}{\mathrm{d}s}\ \frac{1}{1 - F(s)} = \frac{\mathrm{d}}{\mathrm{d}s}\ - \log\left[1 - F(s)\right][1ex] \tag{14.3}$$

$$1 - F(t) = \exp\left[-\int_0^t\ \lambda(s)\ \mathrm{d}s\right] \tag{14.4}$$

A special case of $\lambda(t) = bt$ is called the *Rayleigh* density function. A linear relationship between death rates between two conditions leads to a power law relationship between the probability of both conditions surviving to the same age.

$$\begin{aligned} \lambda_y &= n\lambda_x \\ \implies P\{Y > b \mid Y > a\} &= P\{X > b \mid X > a\}^n \end{aligned} \tag{14.5}$$

Where $b > a$ are some ages and X, Y are two categories being compared.

14.3 Exponential distributions in life testing

Stopping at the r^{th} failure : Consider a population of n items which are all IID using an exponential distribution with unknown mean θ. A problem of great interest is to attempt to estimate θ using observations about the time taken for r out of n simultaneously initialized items to fail.

The observed data takes the form of an ascending ordered set of failure times $\{x_1,\ \ldots,\ x_r\}$ along with a set of indices for the failing items $\{i_1\ \ldots\ i_r\}$. If the lifetime of component i_j is denoted by X_{i_j},