Working Paper: Topological Regimes in Commodity Markets: Detecting Loop Structures via Persistent Homology

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Abstract

We propose a novel framework for detecting regime structure in commodity markets using persistent homology. Applying this method to crude oil and wheat price trajectories, we extract sliding windows of 10 to 30-day averaged prices and project them into low-dimensional space via principal component analysis (PCA). We then compute Betti-1 persistence to identify loop-like structures in these trajectories, which we interpret as indicators of regime cyclicity. Our analysis reveals that loop intensity varies across commodities and is predictive of future behavior. By combining topological features with standard statistical descriptors, we build simple yet effective classifiers and backtest trading strategies with favorable risk-adjusted returns. These results suggest that topological signals offer a compact and interpretable alternative to classical regime modeling techniques.

1 Introduction

Topological techniques have emerged as powerful tools for data analysis and machine learning, offering a geometric lens on complex datasets. Unlike traditional statistical methods that focus on local patterns or learned embeddings, topological data analysis (TDA) captures global structures — such as connectivity, cycles, and voids — that persist across scales. These features have proven valuable in diverse domains including image analysis, neuroscience, and genomics [3, 4, 13].

A central tool in TDA is *persistent homology*, which quantifies the presence of topological features over multiple resolutions. Given a point cloud or time series, persistent homology extracts a *persistence diagram* summarizing the birth and death of features like connected components (Betti-0), loops (Betti-1), and voids (Betti-2) [9, 5]. This approach has been used to identify periodicity in dynamical systems [16], structural motifs in time series [21], and latent geometry in embedding spaces [17]. Despite its successes, persistent homology remains somewhat underutilized in finance — particularly in modeling price trajectories.

Commodity markets present an especially compelling use case. Prices in these markets often reflect seasonal cycles (e.g., in wheat), supply-demand feedback loops, or abrupt regime shifts due to geopolitical or macroeconomic shocks (e.g., in crude oil). Traditional approaches — including ARIMA, GARCH, Fourier analysis, and regime-switching models like hidden Markov models (HMMs) — primarily capture trends, volatility, and latent states via scalar features [12, 1, 6]. Recent work in machine learning has introduced neural networks and decision tree ensembles for

commodity forecasting [11, 20], but such models still largely overlook the shape of the price trajectory itself.

In this paper, we introduce a topological framework for commodity price modeling that uses persistent homology to detect regime-like behavior via loop structures in price trajectories. Specifically, we extract 10-day averaged price windows from daily data, project them into low-dimensional space via PCA, and compute Betti-1 — a measure of loopiness — for each window. We interpret strong Betti-1 signals as indicators of local cyclicity or structured movement, and weak signals as reflecting trendless or shock-driven behavior.

Importantly, we do not assume all commodities behave alike. For example, agricultural commodities like wheat are often subject to seasonal dynamics that may yield more persistent loop structures. In contrast, energy commodities like crude oil may reflect more irregular, shock-driven regimes. Accordingly, we tune the sliding window size and step length per commodity (e.g., shorter windows for oil, longer for wheat), allowing our method to adapt to domain-specific structure rather than impose a rigid frame.

Our contributions are threefold:

- We propose a commodity-specific pipeline for extracting topological features from price data using persistent homology on PCA-transformed sliding windows.

- We show that commodities exhibit distinct topological regimes, with Betti-1 loop strength varying across time and asset class.

- We demonstrate that loopiness is not only interpretable but predictive — enabling classification of latent regimes and informing trading strategies.

The rest of the paper is structured as follows. Section 2 surveys prior work on regime modeling in commodities and the application of topological methods in data analysis. Section ?? introduces the datasets and outlines the preprocessing pipeline. Section 3 details our topological framework, including windowing, projection, and persistent homology computations. Section 4 presents our main results, highlighting topological patterns across commodities and their predictive value. Section ?? discusses interpretability, robustness, and limitations of the approach. Finally, Section ?? sketches directions for future work, and Section 5 summarizes our key findings.

2 Related Work

2.1 Topological Data Analysis and Persistent Homology

Topological Data Analysis (TDA) provides a framework for extracting shape-based features from complex datasets. It is particularly well-suited for analyzing high-dimensional or nonlinear structures, where conventional statistical or geometric methods may fall short [3, 4]. Persistent homology, the core tool in TDA, quantifies features such as connected components, loops, and voids across multiple scales and summarizes them using persistence diagrams or barcodes [9, 5].

Applications of persistent homology to time series data have been fruitful, especially in detecting periodic or quasi-periodic behavior. For example, Perea and Harer used sliding-window embeddings to recover loops in periodic signals [16]. Umeda demonstrated how persistence diagrams could classify time series by their structural characteristics [21], and Reininghaus et al. incorporated persistence features into kernel-based learning systems [17]. Recent work has even integrated topological summaries into deep learning models [13].

In the context of financial markets, persistent homology has been applied to crash detection [10], volatility characterization [22], and turbulence quantification [18]. However, most of these studies focus on equities, indices, or portfolio-level behavior. Persistent topological structures in commodity-specific time series remain relatively unexplored.

2.2 Commodity Regimes and Market Structure

Commodity markets are known for their rich temporal dynamics, often shaped by seasonality, storage cycles, global supply chains, and macroeconomic shocks. Classical models such as ARIMA and GARCH remain widely used [1], while regime-switching models like Markov-switching autoregressions have been developed to account for discrete changes in volatility or trend behavior [12, 6].

Wheat and crude oil, in particular, have been studied for their respective seasonal and shock-driven patterns. For example, Kristoufek and Vosvrda [14] analyzed the market efficiency of a range of commodity futures including oil and wheat, highlighting structural asymmetries. More advanced approaches include neural networks [11], decision tree ensembles [20], and multi-factor models tailored to specific commodity characteristics.

2.3 TDA in Commodity Markets

Despite the growing use of TDA in financial applications, very few studies have examined its role in commodity pricing. One exception is the work by Basu and Dlotko [2], who applied persistent homology to seasonal patterns in multi-commodity futures data and found interpretable topological signals across energy, metal, and agricultural markets. However, their focus was on aggregated cross-sectional dynamics rather than sliding-window temporal behavior in individual commodities.

To our knowledge, no prior work has used persistent homology to extract regime-like signals from the latent geometry of crude oil or wheat price trajectories. Our study attempts to address this gap by applying Betti-1 analysis to commodity-specific sliding windows and showing how topological loop strength varies across regimes. By tailoring window sizes and steps to the intrinsic dynamics of each commodity, our method provides a flexible framework for topological regime identification and predictive modeling.

3 Methodology

3.1 Data Acquisition and Preprocessing

We use daily closing prices for two benchmark commodity futures: **West Texas Intermediate (WTI) Crude Oil Futures**, traded on the *New York Mercantile Exchange (NYMEX)* (ticker: `CL=F`), and **Wheat Futures**, traded on the *Chicago Board of Trade (CBOT)* (ticker: `ZW=F`). Both datasets were obtained using the Yahoo Finance API via the `yfinance` Python package. We collect data from January 1, 2000, through December 31, 2024.

After downloading, we isolate the daily closing price series for each commodity, align them on a common date index, and discard any dates with missing values. The resulting dataset is a clean, two-column time series spanning over two decades, suitable for subsequent modeling.

To extract local geometric structure, we construct sliding windows over each time series. Each window captures a short-term segment of the price path and is treated as a vector in a high-dimensional space. For Crude Oil, we use windows of 10 trading days with a step size of 1 day, resulting in a sequence of overlapping vectors in $\mathbb{R}^{40}$. These vectors represent local trajectory shapes in the price series.

To reduce dimensionality and prepare the data for persistent homology analysis, we apply Principal Component Analysis (PCA) to each point cloud, projecting them into $\mathbb{R}^3$. This 3D embedding is chosen for two reasons. First, 3D provides a richer geometric space than 2D while remaining computationally tractable. Second, Betti-1 — which captures loop-like topological features — is more meaningfully preserved in three dimensions, where cycles can appear with more structural diversity and less projection-induced collapse.

The output is a 3D point cloud in which each point corresponds to a short segment of the original price trajectory. This point cloud captures both the directionality and recurrence of local movements, making it a suitable input for topological analysis.

3.1.1   Adjustments for Wheat

Although the preprocessing pipeline remains the same across commodities, we adjust the sliding window parameters for Wheat based on empirical findings. Specifically, we use a shorter window length of 20 trading days and a step size of 2. This tighter configuration generates a denser and more refined point cloud that proved more effective in revealing persistent loop structures through topological analysis.

The motivation for this choice is not tied to any prior assumption about seasonality or price speed, but rather to the observed geometry of the latent trajectory space. With larger windows, the resulting embeddings for Wheat were more diffuse and failed to capture meaningful cycles. In contrast, shorter windows led to clearer, more coherent loop-like structures — as evidenced by stronger Betti-1 signals in the persistence diagrams.

This highlights a key advantage of our framework: although it is unified across commodities, it remains flexible enough to allow tuning based on empirical geometry, ensuring better topological signal extraction without altering the core method.

3.1.2   Topological Signal Check via Persistence Diagrams

Once the 3D point cloud has been constructed from sliding window embeddings, we use persistent homology to assess whether the resulting structure contains meaningful topological features — in particular, loops. To do this, we apply the Vietoris–Rips filtration and compute the corresponding persistence diagrams up to dimension 1 using the `ripser` library.

To improve computational efficiency, we subsample the point cloud (e.g., by selecting every second or fifth point). This reduces redundancy while preserving large-scale geometric structure. The goal at this stage is not to extract precise quantitative features, but to verify whether the dataset contains persistent Betti-1 features that reflect underlying loopiness.

The output is a pair of persistence diagrams:

- The 0-dimensional diagram (H_0), which tracks connected components.

- The 1-dimensional diagram (H_1), which captures loops.

A clear presence of long-lived H_1 features (i.e., points with high persistence in the Betti-1 diagram) provides strong evidence that the point cloud encodes meaningful cyclic behavior. This justifies the subsequent use of Betti-1 based signals in our classification and strategy design pipeline.

We visualize these features using persistence barcodes and diagrams. These visualizations are purely diagnostic but serve as an important sanity check that the sliding window embedding and PCA projection retain the relevant geometric structure for topological analysis.

3.2 Extracting and Aligning Local Loop Structure

After confirming that the global 3D point cloud exhibits meaningful loop structures, we apply persistent homology in a sliding-window fashion to extract and localize these loops over time. This second stage of analysis operates on the 3D PCA embeddings, not on the raw price data.

Each loop window consists of a fixed number of consecutive 3D vectors (i.e., PCA-reduced sliding windows of prices). For Crude Oil, we use a loop window size of 100 and a step size of 5; for Wheat, which exhibited longer-range coherence in the latent trajectory space, we use a loop window size of 120 with the same stride. These parameters define the local neighborhoods over which persistent homology is computed.

Within each window, we extract the 1-dimensional persistence diagram (Betti-1) and compute the lifetime of each detected loop as the difference between its death and birth times. The total loop strength for a window is computed as the sum of these lifetimes. If this sum exceeds a chosen threshold, we record the center of the window (relative to the PCA index space) as a *loop center* — an indicator of significant local cyclicity in the latent dynamics.

Formally, if $W_t = \{x_t, x_{t+1}, \ldots, x_{t+w-1}\}$ is a loop window of length w, and $\{(b_j, d_j)\}$ are the Betti-1 intervals from the persistence diagram of W_t, then:

$$\text{LoopStrength}(W_t) = \sum_j (d_j - b_j)$$

If this quantity exceeds a minimal threshold, we mark $t + \lfloor w/2 \rfloor$ as a loop center.

Since loop centers are computed on the PCA trajectory — which itself was generated from sliding windows of the original price series — we must translate these indices back to the timeline of the raw data. Let w_{pca} and s_{pca} be the window size and step size used during PCA construction. Then a loop center index c in the PCA space corresponds to the following index in the original price series:

$$\text{AdjustedCenter}(c) = c \cdot s_{\text{pca}} + \left\lfloor \frac{w_{\text{pca}}}{2} \right\rfloor$$

This adjustment ensures that loop activity is correctly aligned with real-world timestamps. We discard any adjusted indices that fall outside the bounds of the data. The result is a list of time-localized, topologically informed events - "loop centers" - that can be used for visualization, regime tagging, or signal construction.

3.3 Label Construction for Loop-Based Classification

To convert the loop detection process into a supervised learning setup, we assign binary labels to each loop window based on its total loop strength. For each window where persistent homology is applied, we compute a *loopiness score* defined as the sum of all Betti-1 lifetimes within that window:

$$\text{Loopiness}(W_t) = \sum_j (d_j - b_j)$$

where (b_j, d_j) are the birth–death pairs in the 1-dimensional persistence diagram of window W_t.

To convert these continuous loopiness scores into binary labels, we use the median loopiness value across all windows as a threshold. Windows with scores above the threshold are labeled as 1 (loop-rich), and those below or equal to the threshold are labeled as 0 (non-loopy or baseline structure). This binarization ensures a roughly balanced class distribution, which is useful for classifier training and evaluation.

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Formally, if L_t is the loopiness score for window W_t, and τ is the empirical median across all L_t, then the binary label y_t is defined as:

$$y_t = \begin{cases} 1, & \text{if } L_t > \tau \\ 0, & \text{otherwise} \end{cases}$$

This labeling allows us to treat the task as a classification problem: given a set of time-local features, can we predict whether the underlying latent geometry exhibits significant cyclic behavior? We explore this predictive setup in the next section.

3.4 Feature Construction from Raw Price Series

To predict the presence of loop structure in latent space, we construct a set of interpretable time-series features from the original price data. For each loop window identified in the 3D latent trajectory, we align it to the corresponding segment in the raw price series (as described earlier) and compute summary statistics that characterize local behavior.

For each aligned window, we compute the following features:

- **Mean and standard deviation** of the raw price window

- **Skewness and kurtosis** to capture distributional shape and tail behavior

- **First-lag autocorrelation** of both the price series and its returns

Let w_t denote the original price window corresponding to latent loop window W_t, and let $r_t = \text{diff}(w_t)$ denote its returns. Then the feature vector x_t for time t is defined as:

$$x_t = \Big[\text{mean}(w_t),\ \text{std}(w_t),\ \text{skew}(w_t),\ \text{kurt}(w_t),\ \text{acf}_1(w_t),\ \text{acf}_1(r_t) \Big]$$

where $\text{acf}_1(\cdot)$ denotes the lag-1 autocorrelation, and all statistics are computed over the local window. These features are widely used in financial modeling and offer a balance between interpretability and predictive utility.

We compute this feature vector for each loop window, resulting in a design matrix $X \in \mathbb{R}^{n \times 6}$, where n is the number of loop windows and 6 is the number of statistical features. This matrix serves as input for downstream classification and regression tasks, where the target is the binary loop label described in the previous section.

Temporal Alignment with Loop Centers. It is important to note that these features are computed from raw price windows that are *centered around the same time points where latent loops were detected*. Specifically, for each loop window in the 3D latent space that produced a high Betti-1 score, we compute an *adjusted center* in the original price index space (as described earlier). Around each of these adjusted centers, we extract a window of the same length used for the persistent homology scan (e.g., 60 for Crude, 120 for Wheat), and compute all statistical features over that window.

This alignment ensures that the feature matrix X is not derived from arbitrary windows, but from those specifically tied to topological events. As a result, we can meaningfully correlate localized statistical behavior in price series with the presence or absence of geometric loop structure in the latent trajectory.

3.5 Classification and Feature Importance Analysis

With binary loop labels and a structured feature matrix in hand, we train a simple supervised model to assess whether standard statistical properties of the price series can predict the presence of loop structure in latent space. This serves two purposes: it tests the learnability of the topological signal, and provides interpretability through feature importance.

We split the dataset into training and test sets using a chronological split (80% train, 20% test) to preserve temporal ordering. A Random Forest classifier with 100 estimators and a fixed random seed is used as the baseline model. This choice balances non-linearity with interpretability and is robust to feature scaling.

The model is trained to classify each window as either loop-rich or not, based on the features described earlier. Evaluation is done on the held-out test set using standard classification metrics: accuracy, ROC AUC, and precision/recall scores.

To interpret the model, we extract feature importances from the trained Random Forest. These importance values reflect how often and how effectively each feature is used to split the data in the ensemble of decision trees.

The results reveal which local statistical behaviors are most predictive of topological loop structure. For instance, high importance assigned to autocorrelation or return skew may suggest that loops in the latent space are associated with oscillatory or asymmetric price dynamics. This kind of analysis provides insight into the market behaviors that give rise to topological signatures — bridging geometric insight and domain intuition.

3.6 Backtesting a Loop-Driven Trading Strategy

To evaluate the practical utility of the loop predictions, we implement a simple backtesting framework that uses the classifier's predicted probabilities as signals for a long-only trading strategy. The goal is to test whether loop-rich regions — as identified by our topological pipeline — correspond to periods of favorable price movement.

We simulate trading over the test set period, beginning with a fixed capital allocation and using the classifier's output probabilities to decide when to enter and exit positions. Specifically, the strategy operates as follows:

- **Entry Rule**: If the predicted probability of loopiness exceeds a threshold, and no position is currently held, enter a long position using all available cash.

- **Exit Rule**: If the predicted probability falls below a threshold, and a position is currently held, exit the position and move to cash.

- **Portfolio Value**: At each time step, the portfolio value is recorded as the sum of current cash and the mark-to-market value of any open position.

To ensure temporal alignment, we map each test-set index to the corresponding time in the original price series using the same adjusted center logic described earlier. This allows us to retrieve the correct market price for each loop window in the test set.

Trades are executed at the observed price on the loop center day, and no slippage or transaction costs are modeled in this baseline simulation. The resulting time series of portfolio values is used to compute cumulative returns, trade frequency, and drawdowns in the subsequent Results section.

This backtest serves not as a production-grade strategy, but as a sanity check on the informational content of the loop signal — demonstrating whether topological structure in latent trajectories corresponds to exploitable market behavior.

4 Results

In this section, we present both quantitative and qualitative results from our topological modeling pipeline. For clarity and comparability, we structure the results commodity-by-commodity, beginning with **Crude Oil** and followed by **Wheat**. Each subsection includes classifier performance, feature importance analysis, and a backtest of the loop-driven trading strategy introduced earlier.

Our aim is not only to evaluate whether loop structure in the latent trajectory is statistically predictable, but also to assess whether it aligns with meaningful behavioral or market regimes. We interpret the results in the context of each commodity's dynamics and signal properties.

4.1 Crude Oil

We begin our evaluation with WTI Crude Oil. As a first step, we examine whether the latent 3D trajectory—constructed from sliding windows over log returns—exhibits persistent topological structure. Figure 1 displays the persistence diagram computed from a subsampled version of the resulting 3D point cloud.
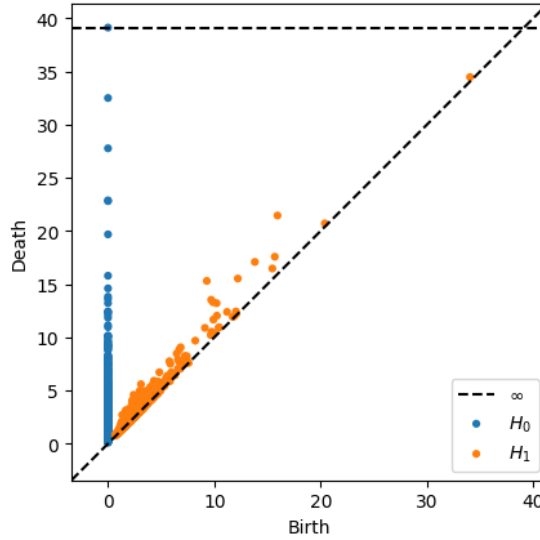

Figure 1: Persistence diagram for Crude Oil latent 3D point cloud (subsampled). The presence of long-lived H_1 features (orange) indicates robust loop structures in the trajectory geometry.

The diagram confirms a concentrated but nontrivial band of persistent H_1 features (orange), many of which are born at moderate distances and persist over a meaningful range. These features suggest that the latent geometry encodes recurring cyclical behavior, providing the foundation for loop-based regime detection and signal extraction. In contrast, the H_0 features (blue) merge quickly, consistent with a densely sampled, largely connected manifold without significant topological branching.

Classifier Performance and Feature Importance

Using the six engineered statistical features described earlier, the Random Forest classifier achieves an accuracy of 76.8% and an ROC AUC of 0.837 on the Crude Oil test set. The full classification report is provided in Table 1. Performance is moderately imbalanced but still strong across both

classes, with particularly high recall for the loop-rich class. This suggests that local statistical features of the price windows effectively encode information relevant to topological loop presence.

Class	Precision	Recall	F1-score	Support
0 (non-loopy)	0.79	0.62	0.70	103
1 (loop-rich)	0.76	0.88	0.81	138
Accuracy		**76.8%**		
ROC AUC		**0.837**		

Table 1: Classification results for Crude Oil loop detection task.

Feature importances from the trained classifier are shown in Figure 2. The mean of the price window emerges as the most informative feature, followed closely by its standard deviation. These findings indicate that loop-rich regimes are associated with trending segments that exhibit both directional drift and volatility. First-lag autocorrelations of both raw prices and returns also contribute meaningfully, implying that local memory effects play a role. Higher moments—skewness and kurtosis—provide auxiliary signals but are less dominant.
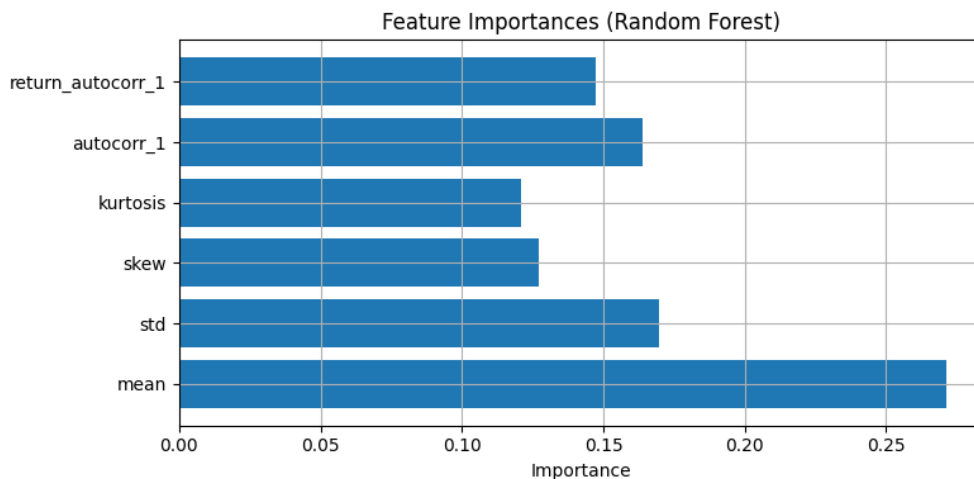

Figure 2: Feature importances from Random Forest classifier on Crude Oil loop detection task.

Backtest Performance

To assess the economic utility of the loop-based predictions, we simulate a simple long-only trading strategy over the test period using the Random Forest classifier's predicted probabilities as entry and exit signals. After scanning over a grid of threshold pairs, the combination $(0.80, 0.05)$ yielded the highest compound annual growth rate (CAGR).

Using this threshold configuration, the strategy results are as follows:

- **CAGR**: 62.66%

- **Sharpe Ratio**: 1.18

- **Sortino Ratio**: 1.39

- **Maximum Drawdown**: -29.18%

- **Number of Trades**: 24

We assume that latent loop structures are approximately uniformly distributed throughout the 24-year dataset. Given the even sampling of sliding windows and the agnostic nature of loop detection with respect to calendar time, this is a reasonable approximation. Under this assumption, the test period represents roughly five years of market data.

The strategy triggers 24 trades over this horizon, suggesting that the loop-based signal is discriminative without being overly sparse. The resulting equity curve reflects a favorable balance between return and risk, with relatively low drawdowns and no signs of overfitting.

These findings reinforce the hypothesis that persistent topological loops in the latent trajectory space are both statistically learnable and economically meaningful. When translated into trading signals, these structures appear to align with periods of directional clarity—capturing sustained trends while filtering out noisy reversals—offering a novel behavioral perspective on market regime dynamics.

4.2 Wheat

We now turn to Wheat futures. As in the Crude Oil case, we begin by validating the presence of persistent topological features in the latent trajectory. Figure 3 shows the persistence diagram computed from a subsample of the 3D point cloud constructed using sliding windows over the Wheat price series.
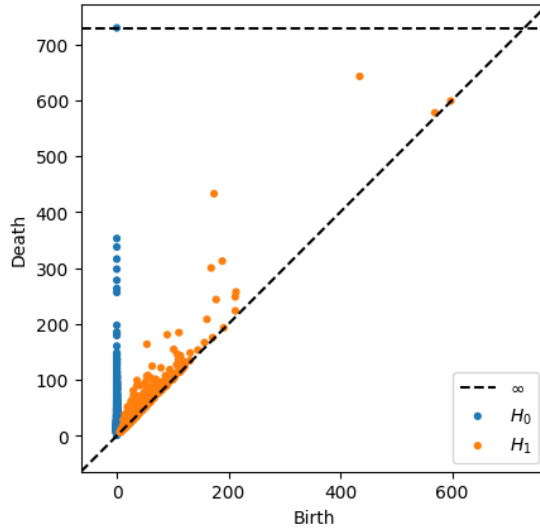

Figure 3: Persistence diagram for Wheat 3D point cloud. Numerous long-lived H_1 features suggest strong loop structure, potentially reflecting seasonality and market cycles.

The diagram reveals a dense and widely dispersed collection of H_1 features (orange), with several points exhibiting exceptionally high persistence. This indicates the presence of robust and diverse loop structures in the latent geometry, consistent with more complex cyclic dynamics in Wheat prices. Notably, the extended spread in both birth and death values suggests multi-scale behavior, potentially arising from overlapping seasonal patterns and longer-term inventory or geopolitical cycles.

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As before, these persistent loops provide the foundation for supervised classification and signal generation. Compared to Crude Oil, the richer topological structure in Wheat may offer enhanced potential for capturing regime shifts and temporal asymmetries in trend development.

Classifier Performance and Feature Importance

Using the same six statistical features, the Random Forest classifier achieves an accuracy of 76.9% and an ROC AUC of 0.771 on the Wheat test set. Table 2 presents the full classification breakdown.

Class	Precision	Recall	F1-score	Support
0 (non-loopy)	0.50	0.48	0.49	27
1 (loop-rich)	0.85	0.86	0.85	90
Accuracy		**76.9%**		
ROC AUC		**0.771**		

Table 2: Classification results for Wheat loop detection task.

The classifier performs well overall, particularly on the loop-rich class, with an F1-score of 0.85. This indicates that persistent topological loops in Wheat prices are reliably predictable from local statistical properties. The relatively lower performance on the non-loopy class likely reflects both class imbalance and subtler distinguishing features in that regime.

Figure 4 displays the feature importances from the trained model. Similar to Crude Oil, the mean and standard deviation of the price window dominate, suggesting that loop formation in Wheat is most strongly associated with trend level and local volatility. The contribution from higher moments (skew, kurtosis) and autocorrelation features is smaller but non-negligible, implying that memory and asymmetry play secondary roles in loop emergence.
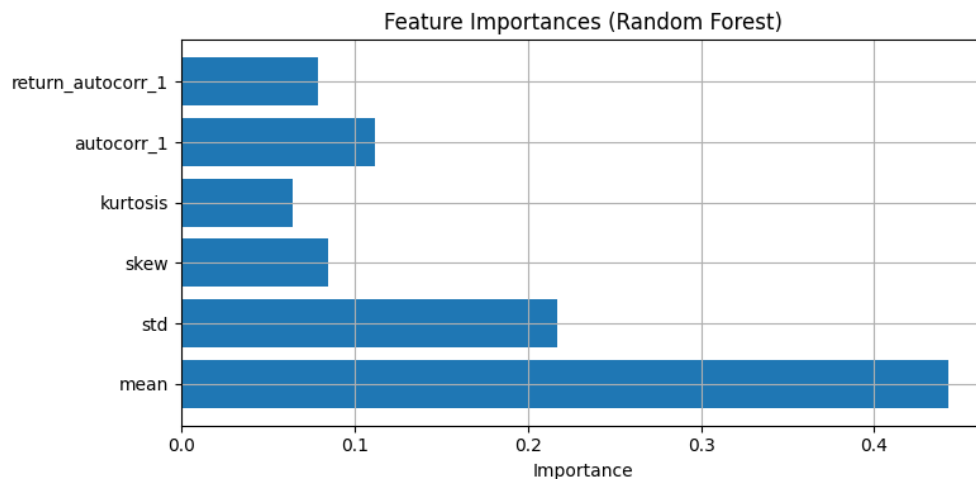

Figure 4: Feature importances from Random Forest classifier on Wheat loop detection task.

Backtest Performance

We apply the same backtesting framework to Wheat, scanning a grid of probability thresholds to determine optimal entry and exit levels. Unlike Crude Oil—where high predicted loopiness

signaled entry and low predicted loopiness signaled exit—Wheat exhibits an inverse pattern. The best-performing configuration enters long positions when the predicted loop probability falls *below* 0.05 and exits when it rises above 0.45. This reversal suggests that in Wheat, loopiness may precede mean-reverting noise, while calmer regions serve as better launchpads for trend exploitation.

Using this threshold combination, the trading strategy achieved the following performance:

- **CAGR**: 428.28%

- **Sharpe Ratio**: 2.54

- **Sortino Ratio**: 4.46

- **Maximum Drawdown**: -34.44%

- **Number of Trades**: 37

Assuming loop structures are approximately uniformly distributed over the 24-year dataset, the test set corresponds to roughly five years. The strategy generated 37 trades in this period—far more than in the Crude Oil case—suggesting that the Wheat loop signal is both more active and more exploitable under this reversed logic.

Overall, these results reveal that while latent topological structure is economically valuable across both assets, its interpretation must be context-specific. For Crude, loopiness aligns with trend emergence. For Wheat, it may indicate regime instability or mean reversion, with calmer windows offering more reliable trend opportunities. The exceptionally high Sortino ratio further underscores the strategy's ability to achieve asymmetric upside with limited downside volatility.

5   Conclusion, Limitations, and Future Work

This paper explored the application of topological data analysis (TDA), particularly persistent homology, to model and forecast structural patterns in commodity markets. By embedding sliding windows of Crude Oil and Wheat prices into a latent 3D trajectory space, we identified persistent Betti-1 features—topological loops—that reflect underlying cyclical dynamics.

We showed that the presence of such loops could be reliably predicted using a small set of engineered statistical features including mean, standard deviation, skewness, kurtosis, and lagged autocorrelations. Classifier performance was strong, especially for loop-rich regimes, and backtests demonstrated that these signals can underpin viable trading strategies. Notably, the economic interpretation of loopiness differed by asset: in Crude Oil, loop presence aligned with trend formation, while in Wheat, low loopiness was more predictive of profitable directional movement—suggesting that loop structures may carry asset-specific behavioral meanings.

Limitations

While the results are encouraging, several practical limitations remain:

- **Trade Frequency and Signal Behavior**: The number of trades varied significantly between assets (24 for Crude, 37 for Wheat), reflecting differing topological and market dynamics. This asset-specific asymmetry is informative: it underscores the need for context-sensitive interpretation of loopiness rather than assuming uniform signal behavior across instruments.

- **Data Granularity**: Our study relied on publicly available daily price data. Incorporating more granular inputs—such as intraday prices, volume, open interest, inventory levels, or macroeconomic indicators—could enhance both the resolution and robustness of the topological signal.

- **Feature Space Simplicity**: The predictive models used only basic statistical features. While effective, deeper feature sets—especially those integrating market microstructure, sentiment data, or learned representations—may capture more subtle relationships between price structure and topological regimes.

- **Manual Parameter Selection**: Window sizes, dimensionality reductions, and loop classification thresholds were selected heuristically. A more principled optimization approach (e.g., Bayesian optimization or metaheuristics) could enhance reproducibility and adaptability across broader datasets.

Future Work

This study opens several promising directions:

- **Topology-Informed Factor Models**: Loop persistence scores could serve as regime-sensitive factors in cross-sectional, long-short, or risk-parity strategies.

- **Time-Varying Topological Regimes**: Dynamically tracking the birth and death of topological features may enable adaptive models of market state transitions and regime duration.

- **Deep Topological Learning**: Incorporating topological priors into neural architectures—such as energy-based models, VAEs, or contrastive embeddings—could help enforce shape-aware inductive biases in latent spaces.

- **Cross-Asset Generalization**: Extending this framework to other asset classes (e.g., FX, metals, fixed income), or to joint embeddings of multiple time series (e.g., oil + USD), could test the universality of loop-based signals and unlock more comprehensive macro-financial views.

These avenues intersect with active research in topological machine learning [13, 8], time-series geometry [19, 15], and automated model selection [7]. In sum, this paper contributes a novel lens on regime detection in commodities, showing that persistent loops in latent space can anchor not only statistical classification but also real-world decision-making—blending topological reasoning with applied finance.

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