bioelectric HW1

Anirudh Manika

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BME 471: Bioelectric Phenomena
Fall 2024
Homework #1

Given:

Na+ max conducatance	=	120 mS/cm^2
K+ max conductance	=	36 mS/ cm^2
Leak conducation	=	0.3 mS/ cm^2
Extracellular Na concentration	=	490 mmol/L
Extracellular K concentration	=	20 mmol/L
Intracellular Na concentration	=	50 mmol/L
Intracellular K concentration	=	400 mmol/L
Leak Nernst potential	=	-50 mV
Membrane capacitance	=	1.0 uF/cm^2

1) Using the Hodgkin-Huxley equations and the parameters above, develop a Matlab program (e.g. m-file) that will solve for the $V_m(t)$, n(t), m(t), h(t), $I_{Na}(t)$, $I_K(t)$ for a given stimulus $I_s(t)$. You will be solving a system of four, coupled first order differential equations. Use 'ODE45' to integrate your model.

2) Using a depolarizing square pulse with duration of 0.35 ms, increase the magnitude of the pulse until you initiate an action potential. Plot $I_s(t)$, $V_m(t)$, n(t), m(t), h(t), $I_{Na}(t)$, $I_K(t)$ for a typical action potential.

3) A) How soon after an action potential can another action potential be initiated by the same depolarizing square pulse as Step 2. B) If you were to double the magnitude, how long do you have to wait? (plot your results)

4) Using the same magnitude stimulus found in part 2 but in a hyperpolarizing way, increase the duration of the stimulus until you get an anode break initiation. Plot your results.

1 Problem 1

```
%% Initialize parameters

g_na = 120;
g_k = 36;
g_l = 0.3;


E_leak = -50;
E_na = 57;
E_k = -75;

Cm = 1.0;

Vm_init = -65;
tspan = [0 20];

% Initial conditions for ss gating variables
alpha_n = 0.01 * (Vm_init + 55) / (1 - exp(-0.1 * (Vm_init +
     55)));
beta_n = 0.125 * exp(-(Vm_init) / 80);
alpha_m = 0.1 * (Vm_init + 40) / (1 - exp(-0.1 * (Vm_init +
    40)));
beta_m = 4 * exp(-(Vm_init) / 18);
alpha_h = 0.07 * exp(-(Vm_init) / 20);
beta_h = 1 / (1 + exp(-0.1 * (Vm_init + 30)));

n_init = alpha_n / (alpha_n + beta_n);
m_init = alpha_m / (alpha_m + beta_m);
h_init = alpha_h / (alpha_h + beta_h);


y0 = [Vm_init; n_init; m_init; h_init];

options = odeset('RelTol',1e-4,'AbsTol',[1e-8 1e-8 1e-8 1e
    -8],'MaxStep',0.01);


[t, y] = ode45(@hh_ode, tspan, y0, options);

Vm = y(:, 1);
n = y(:, 2);
m = y(:, 3);
h = y(:, 4);

% ionic currents
I_Na = g_na * m.^3 .* h .* (Vm - E_na);
I_K = g_k * n.^4 .* (Vm - E_k);
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```matlab
I_L = g_l * (Vm - E_l);

%% Plots

% Plot membrane potential Vm(t)
figure;
plot(t, Vm, 'LineWidth', 2);
xlabel('Time (ms)');
ylabel('Membrane Potential (mV)');
title('Membrane Potential (Vm) Over Time');
grid on;

% Plot gating variables n(t), m(t), h(t)
figure;
plot(t, n, t, m, t, h, 'LineWidth', 2);
legend('n(t)', 'm(t)', 'h(t)');
xlabel('Time (ms)');
ylabel('Gating Variables');
title('Gating Variables Over Time');
grid on;

% Plot sodium and potassium currents I_Na(t), I_K(t)
figure;
plot(t, I_Na, t, I_K, 'LineWidth', 2);
legend('I_{Na}(t)', 'I_{K}(t)');
xlabel('Time (ms)');
ylabel('Ionic Currents (\muA/cm^2)');
title('Ionic Currents Over Time');
grid on;

%% Function
function dy = hh_ode(t, y)


    V = y(1);
    n = y(2);
    m = y(3);
    h = y(4);
    g_na = 120;
    g_k = 36;
    g_l = 0.3;
    E_na = 115;
    E_k = -12;
    E_l = 10.6;
    Cm = 1.0;

    % Alpha and beta rate equations
    alpha_n = 0.01 * (V + 55) / (1 - exp(-0.1 * (V + 55)));
    beta_n = 0.125 * exp(-(V) / 80);
    alpha_m = 0.1 * (V + 40) / (1 - exp(-0.1 * (V + 40)));
```

```
    beta_m = 4 * exp(-(V) / 18);
    alpha_h = 0.07 * exp(-(V) / 20);
    beta_h = 1 / (exp((30-Vm)/10) + 1);

    % Gating variable differential equations
    dn = alpha_n * (1 - n) - beta_n * n;
    dm = alpha_m * (1 - m) - beta_m * m;
    dh = alpha_h * (1 - h) - beta_h * h;

    % (Is) square pulse for depolarization
    if t >= 1 && t <= 1.35  % Pulse duration is 0.35 ms
        I_ext = 10;  %   A
    else
        I_ext = 0;
    end

    I_Na = g_na * m^3 * h * (V - E_na);
    I_K = g_k * n^4 * (V - E_k);
    I_L = g_l * (V - E_l);


    dV = (I_ext - I_Na - I_K - I_L) / Cm;

    dy = [dV; dn; dm; dh];
end
```

2 Problem 2

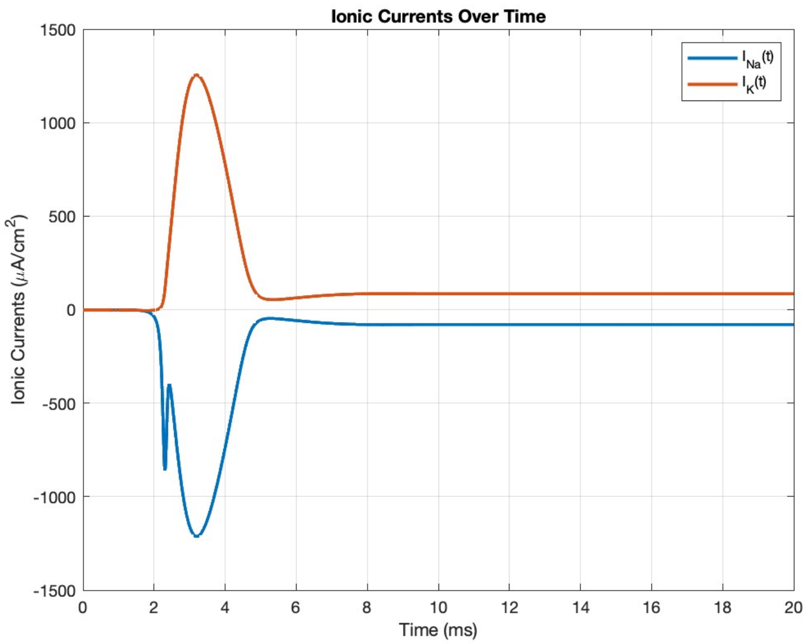

Figure 1: Ionic Currents I(t)

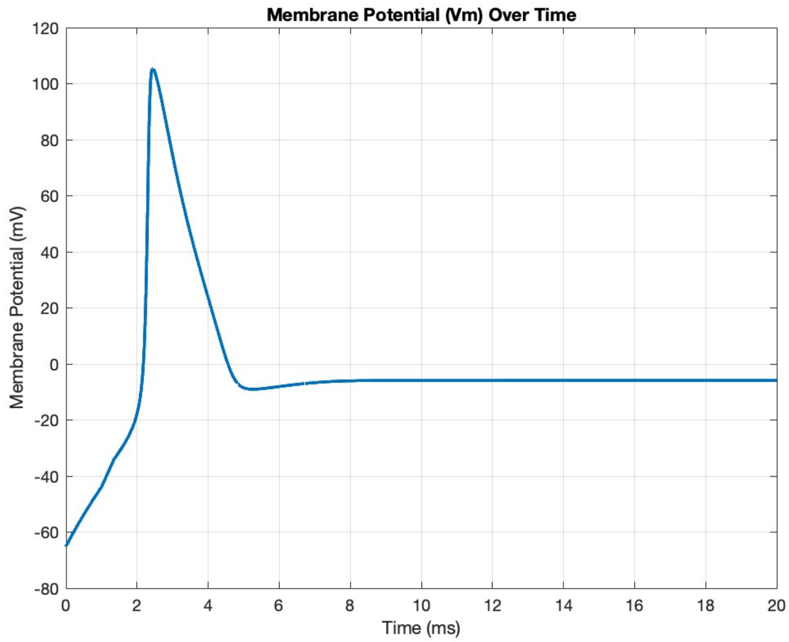

Figure 2: Membrane Potential Vm

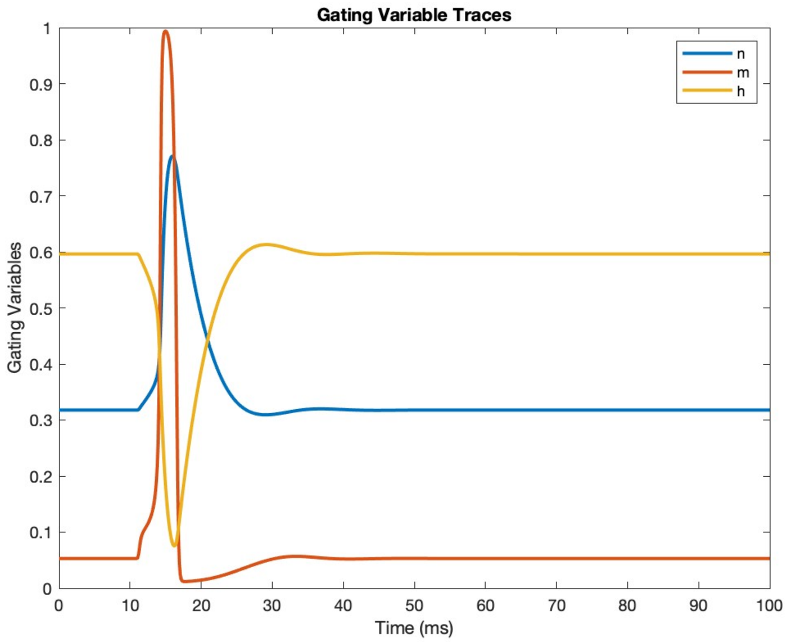

Figure 3: Gating Variables m(t), h(t), n(t)

3 Problem 3

3.1 A

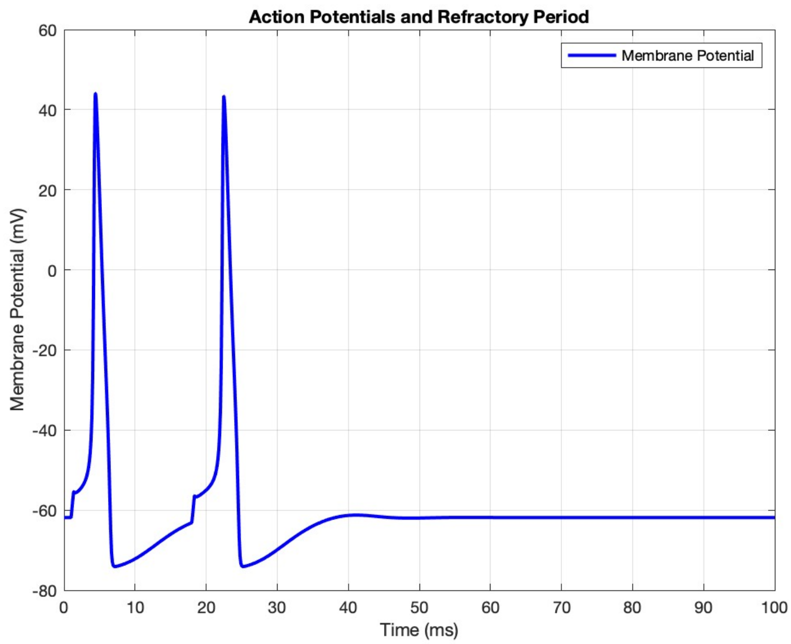

Figure 4: Plot showing Second Action Potential Initiated 17 ms After the First

After an action potential is initiated, the neuron enters a refractory period, during which it is difficult or impossible to trigger another action potential. The refractory period has two phases:

Absolute Refractory Period: No second action potential can be triggered, no matter the stimulus strength, because the sodium channels are inactivated. Relative Refractory Period: A second action potential can be initiated, but only with a stronger-than-normal stimulus because some of the sodium channels are recovering, and the potassium channels are still open, making the membrane potential more negative (hyperpolarized). By 17 ms, enough sodium channels had recovered, and the potassium channels had closed sufficiently for the neuron to be able to fire another action potential in response to the second pulse.

3.2 B

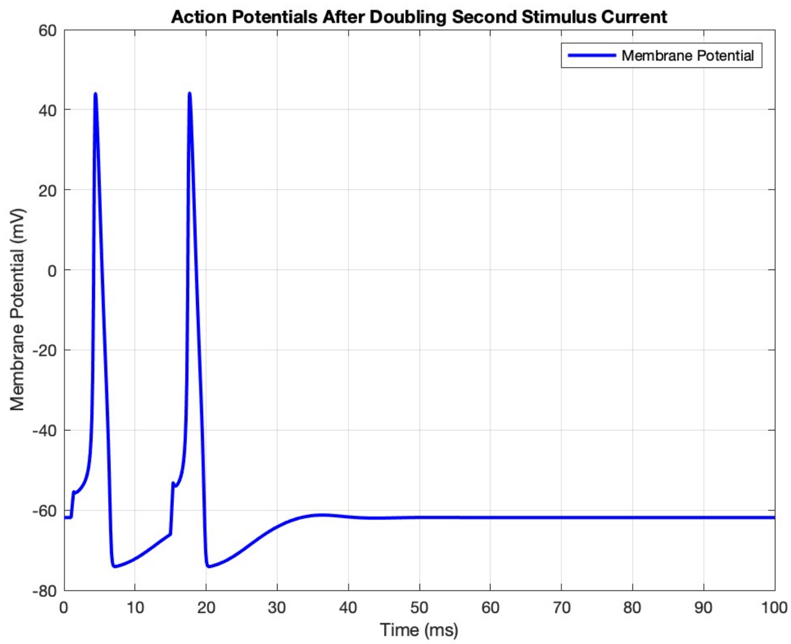

Figure 5: Plot showing Second Action Potential Initiated 14 ms After the First

After I doubled the magnitude of the depolarizing square pulse, the second action potential could be initiated after 15 ms. This shorter interval indicates that the increased stimulus strength allowed the neuron to overcome the refractory period faster. By applying a stronger depolarizing current, the membrane potential recovered more quickly, and the neuron was able to reach the threshold for firing another action potential earlier.

4 Problem 4

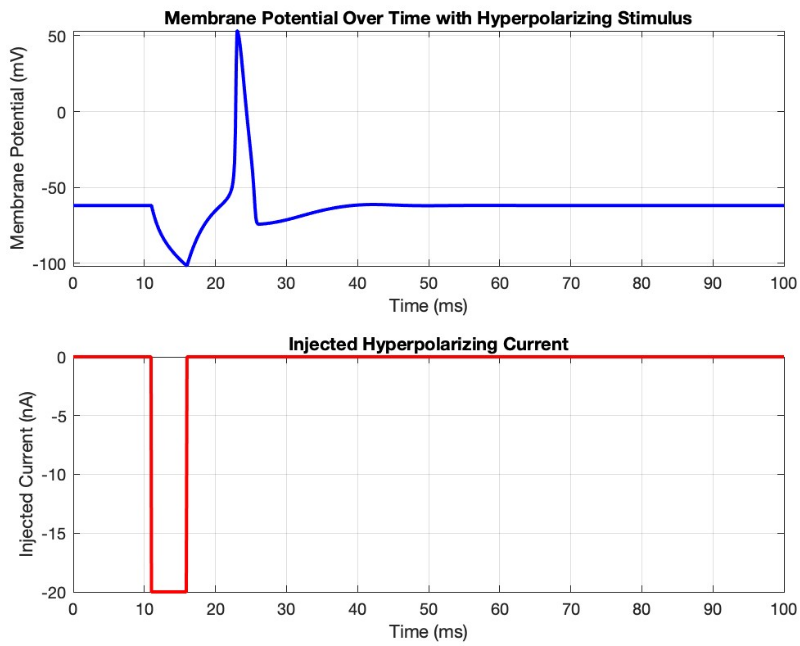

Figure 6: Effects of A Hyperpolarized Current on Action Potential

The hyperpolarizing stimulus causes the membrane potential to drop significantly below the resting potential, but as soon as the stimulus ends (at 16 ms), the membrane rebounds and generates an action potential. This is anode break excitation, where hyperpolarization makes the neuron more likely to fire upon removal of the stimulus.