ANIRUDH MALIK

Determining the details about the path of Comet as asked.

Solution

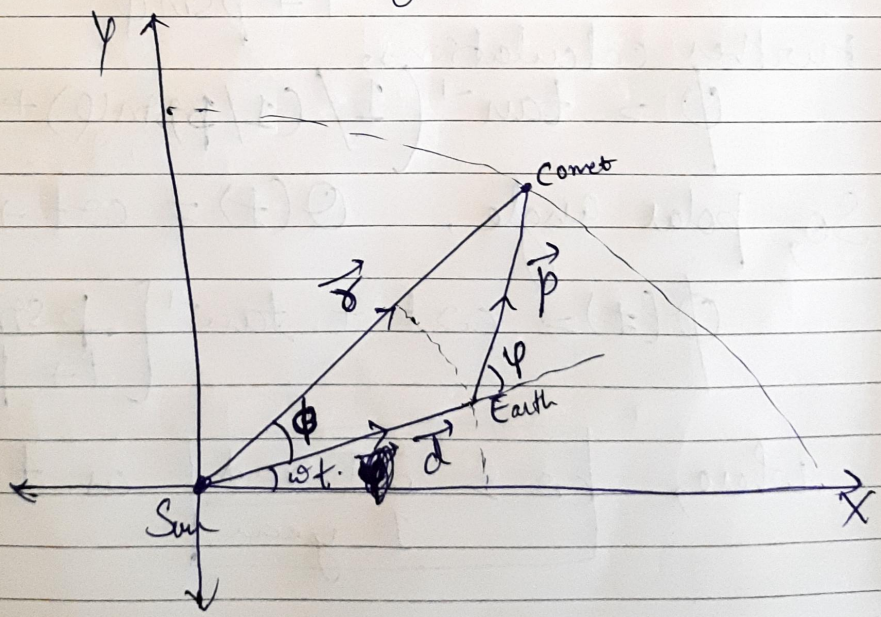
firstly, Let the three given quantities:-

$\rho(t)$ ⇒ Distance between observer (earth) and comet. (In A.U.)

$\varphi(t)$ = The complementary angle b/w the line joining earth and sun, i.e. the supplementary angle given in dataset.

time(t) = times given (in years)

first I make a coordinate system with sun at origin:-



Denoting Earth → E
Sun → S
Comet → C

$|\vec{d}| = 1$ (d/w Earth & Sun 1 A.U.)

$\vec{p}$ & $\vec{r}$ are functions of time.

Let the angular speed of earth around the Sun be 'ω'.

I calculated $\phi(t)$ and $\vec{r}(t)$.

$$\Rightarrow \vec{d} + \vec{p} = \vec{r} \quad , \quad \vec{d} \cdot \hat{r} = \cos\phi$$

$$\Rightarrow r(t) = \left[1 + 2p(t)\cos(\varphi(t)) + (p(t))^2\right]^{1/2}$$

To find $\phi(t)$,

$$r\cos\phi = 1 + p\cos\varphi$$

$$r\sin\phi = p\sin\varphi$$

$$\Rightarrow \tan\phi = \frac{p\sin\varphi}{1 + p\sin\varphi}$$ {Divide both}

Further calculations,

$$\phi = \tan^{-1}\left(1/(1/p\sin(\varphi) + 1)\right)$$

So, polar angle, $\Theta(t) = \omega t + \phi(t)$

$$\phi(t) = \omega t + \tan^{-1}\left[\frac{p\sin\varphi}{1 + p\sin\varphi}\right]$$

where, $\boxed{\omega = \frac{2\pi \text{ rad}}{1 \text{ year}}}$ as t is in time.

ALGORITHM USED IN PROGRAM

Firstly, let's see what are standing with before starting,

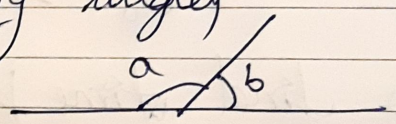
We have, $r(t)$, $\theta(t)$ and time. using which we will calculate r and θ and do the curve fitting.

Programming ⇓

Step I Read csv file using pandas.

Step II Change the given angle into its supplementary angles

for eg:



$$a = \pi - b$$ (In Radians)

Step III Calculate the polar coordinate (θ, r) using the maths done before.

Step IV for Curve fitting : Define a function such that it takes input

theta (variable), e, r_0, θ_0 and

(constants)

returns the value of r. Using formula

$$r = \frac{r_0}{1 + e\cos(\theta - \theta_0)}$$

Step V. Now, use scipy.optimize curve fitting function to find the parameters e, r_0 and θ_0.

e → eccentricity

r_0 → $a(1-e^2)$ → semi-major axes.

which will be saved as popt.

So, e = popt[0]

r_0 = popt[1]

θ_0 = popt[2]

Step VI To find time period of $0 < e < 1$.

Using Kepler's third law,

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2} \approx 7.496 \times 10^{-6} \left(\frac{AU^3}{days^2}\right)$$

$$\approx 0.9986546 \left(\frac{AU^3}{year^2}\right)$$

to find a; used e and r_0 as,

$$\boxed{a = \frac{r_0}{1-e^2}} \Rightarrow T = \sqrt{\frac{4\pi^2}{GM} \times a^3} \text{ years}$$

Step VII Plotting the path.

(Using matplotlib.pyplot.figure().add_subplot (projection='polar').scatter.

to plot scattering path of comet.

FUNCTIONING :-

1. Input → Given through datafile (csv).
2. Output →

* Terminal Window

eccentricity = 0.257811
semimajor axis (a) = 11.0986692 (A.U)
Time period = 36.99934 years

* A plot of comet.