COLLAPSING DECISION BOUNDARIES

— Project Report —
Simulation Based Inference

Anirudh Parameswaran, 264787

Gautam Dilip Hariharan, 230237

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TU Dortmund University

1 Introduction

Speed Accuracy Tradeoff in cognitive tasks is central in understanding decision-making under time pressure. Prior work (Katsimpokis et al., 2020) found linear collapsing boundaries fit deadline conditions better than fixed boundaries. They use Evidence Accumulation Models to mimic the decision making process in the brain. Prior work (Katsimpokis et al., 2020) found linear collapsing boundaries fit deadline conditions better than fixed boundaries. Potential for non-linear collapsing boundaries to capture urgency effects more realistically. In this project report, we will be discussing the effect of using an exponentially collapsing decision boundary in explaining decision making, according to empirical data collected by (Katsimpokis et al., 2020).

In this report, we are trying to investigate whether an exponentially collapsing boundary improves fit to Speed-Accuracy-Tradeoff model data compared to linear / fixed models. We will be using Bayesflow to simulate trials and understand the challenges in recovering parameters like drift rate, initial boundary threshold and exponential decay rate.

2 Data

The dataset was obtained from the OSF repository linked in Katsimpokis et al., 2020. They conducted 3 experiments on speed-accuracy-tradeoffs (SAT) manipulations. There were 24 participants in Experiments 1 and 2, and a different set of 24 in Experiment 3 (ages 22–23 years, 60% female), rewarded with monetary compensation or research credits. All experiments followed a training phase designed to get the participants familiarized with the task and a test phase where SAT manipulations were applied and data was collected.

Experiment 1, referred to as the *Random Dot Motion* task, had participants decide the direction of motion of a set of dots on the screen, scattered along with other dots moving in random directions. Experiment 2, referred to as the *Flash* task, had participants decide which circle flashed faster out of two circles flashing at different rates.

The test phases of both Experiment 1 & 2 followed a 2x2 within-subjects design. The first factor was whether participants were cued to respond fast or accurately. The second factor was whether there was a response deadline (i.e., an upper limit on the available decision time) or not. This produced four conditions in the experiment: speed cue with deadline, speed cue without deadline, accuracy cue with deadline, and accuracy cue without deadline. The four conditions were presented in separate blocks with block order counterbalanced across participants. 1 shows the task instructions given at the beginning of each block for the four conditions. There were 200 trials in each of the 4 conditions/blocks for a total of 800 trials in the test phase.

Experiment 3 was included because for some participants, experiment 2 might have been too quick, so the same task was given with an early and a late deadline. Consequently, the experiment design was also modified to 2x3 within-subjects with two levels for cue-based SAT and 3 levels for deadline-based SAT (early, late, and no deadline).

Table 1: Experiment design for experiments 1 & 2

Factor	**Level**	**Instruction**	**Feedback**
Cue	Speed	"Focus on being as speedy as possible"	"Good time!" (if fast enough) or "Faster!" (if too slow)
	Accuracy	"Focus on giving as many accurate responses as possible"	"Correct!" (correct) or "Be more accurate!" (incorrect)
Deadline	Deadline	"Strict deadlines will apply"	"You missed the deadline!" (no response before deadline)
	No deadline	"There will be no deadline for your answer"	(nil)

3 Statistical Model

Since the trials in the dataset do not have any explicit time-series component, each trial can be considered an independent observation. Our modeling objective is to estimate decision-making parameters under different cue and deadline conditions. This allows us to treat each trial, across participants, as an independent data point.

3.1 The Drift Diffusion Model

We use the Drift Diffusion Model (DDM) Smith and Ratcliff, 2024 as the underlying evidence accumulation framework. The DDM assumes that decisions result from a noisy process of accumulating evidence over time until a decision threshold (boundary) is reached. As illustrated in Figure 1, this process involves continuous evidence accumulation with noise. Mathematically, the change in accumulated evidence x over a small time-step dt is given by:

$$dx = \nu \cdot dt + \sqrt{dt} \cdot \mathcal{N}(0,1), \tag{1}$$

where ν is the *drift rate*, representing the average rate of evidence accumulation. The noise term $\mathcal{N}(0,1)$ captures random fluctuations in evidence accumulation.

A decision is made when x reaches either the upper boundary (e.g., choice A) or the lower boundary (e.g., choice B). Higher drift rates lead to faster and more accurate decisions, whereas lower drift rates produce slower and less accurate decisions.

In our model, we consider a simplified DDM without inter-trial variability in drift rate or starting point. We restrict estimation to three parameters:

- ν: Drift rate (evidence quality).
- k: Exponential decay rate of the boundary over time.
- α_0: Initial boundary height.

3.2 Exponential Boundary Collapse

In the standard DDM, the decision boundaries are fixed. However, under time pressure, the effective boundary height may decrease over time — a phenomenon known as *collapsing boundaries*. Previous research has considered linear collapse (Katsimpokis et al., 2020), but human decision urgency often increases non-linearly under deadlines.

We implement an *exponential* boundary collapse function:

$$a(t) = \alpha_0 \cdot (1 - e^{k \cdot (t_{max} - t)}), \tag{2}$$

where:

- α_0 is the initial boundary height at $t = 0$.
- k is the decay constant controlling the rate of collapse.
- t is the elapsed decision time.
- t_{max} is the upper limit to the decision time.

This function allows for slower early declines in boundary height, followed by a sharper drop as time progresses, which may better reflect decision-making under deadlines.

3.3 Priors

Based on the results of Katsimpokis et al., 2020 and prior literature on the DDM Smith and Ratcliff, 2024, we define informative priors for the parameters as shown in Table 2. These priors are also based on experimentation with the DDM model such that response times and response accuracy match the data closely. Cutoffs and spread conditions are added to base Gaussian and Gamma distributions by multiplying and adding scalars.

Table 2: Prior distributions for model parameters

Parameter	**Description**	**Prior**
ν	Drift rate (evidence quality)	$\nu \sim 5 \times \mathcal{N}(\mu = 0.5, \sigma^2 = 0.5)$
k	Exponential decay rate	$k \sim -1 - 0.5 \times X,\ X \sim \Gamma(\text{shape} = 2, \text{scale} = 0.5)$
a_0	Initial boundary height	$\alpha_0 \sim 1.85 + 0.1 \times X,\ X \sim \Gamma(\text{shape} = 2, \text{scale} = 0.5)$

4 Approximator

The inference framework was implemented using the BayesFlow library, which separates the parameter inference process into two neural components: a summary network and an inference network. For the summary network, we employed a **DeepSet** architecture, which is well suited to handling trial-level data in an order-invariant manner, ensuring that the network remains insensitive to the permutation of trials within a condition. The

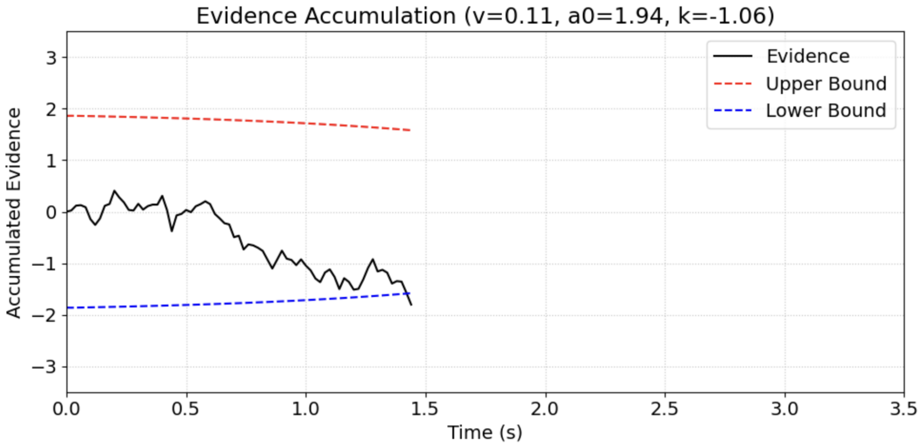

Figure 1: Evidence Accumulation - DDM Model graphic

DeepSet network consisted of 8 layers with a Gaussian base distribution. Dropout rate was set to 0.1 to mitigate overfitting.

The inference network was implemented as a **FlowMatching** normalizing flow model, which transforms the base Gaussian distribution in parameter space into the learned posterior distribution over parameters. The flow component comprised 8 coupling layers. Normalizing flows were chosen for their ability to approximate flexible, non-Gaussian posterior distributions, which is important in drift-diffusion models where parameter posteriors can be skewed.

The initial learning rate was set to 0.001 and all inputs, ν, k, a_0 as well as response time output was standardized using the default method.

5 Training

Training dataset consisted of 5000 samples where each sample had 200 trials. The model was trained for 50 epochs on the pre-generated dataset with batch size 100. Offline training was adopted, as generating simulations of the exponential-boundary DDM is computationally expensive; datasets were generated once and stored, enabling experimentation with different network architectures without repeated simulation. BayesFlow uses Adam as the default optimizer, which is how it was implemented for this project.

6 Diagnostics

The training process showed a consistent reduction in loss, (Figure 4), indicating stable convergence of the inference network. However, a closer inspection of the empirical cumulative distribution function (ECDF) plots (Figure 6) reveals that the drift rate parameter ν deviates slightly from the 95% confidence interval, suggesting mild overfitting.

Ground truth vs. estimate plots (Figure 5) show that while ν was recovered successfully, but the exponential decay rate k and the initial boundary height a_0 could not be estimated reliably. Posterior z-score contraction plots (Figure 3) reveal systematic bias for ν, though within acceptable limits. In contrast, k and a_0 show little to no learnable structure.

Exploratory data analysis (Figure 2) supports these findings. There exists a clear relationship between ν and response times, but such a dependency is not evident for k or a_0. This implies that while the model is sensitive to drift rate, it struggles to capture the dynamics of collapsing boundaries introduced via k and a_0.

7 Inference

The posterior distributions (Figure 7) highlight consistent trends across experimental conditions. The estimated ν is highest in the speed cue with deadline condition, followed by accuracy cue with deadline, and lowest in the no-deadline conditions. This aligns with the behavioral results reported in Katsimpokis et al., 2020, validating that drift rate reliably captures task-induced urgency effects.

In contrast, posterior estimates of k and a_0 remain unchanged across conditions, suggesting that the inference network failed to distinguish between the effects of high and low values of these parameters, maybe because a relationship does not exist. This limits the explanatory power of the exponential boundary collapse in its current implementation.

Posterior predictive checks (Figure 8) show that the model captures response time distributions well, but struggles to reproduce response accuracy correctly.

8 Limitations & Potential Improvements

Limitations

- **Weak signal in boundary parameters**: While drift rate (ν) was recoverable, both the exponential decay rate (k) and initial boundary height (α_0) showed poor identifiability. Simulations confirm that their effects appear only late in the trial, making them difficult to disentangle from noise.

- **Collusion of k and α_0**: The parameterization of the collapsing boundary leads to trade-offs between k and α_0, limiting interpretability.

- **Priors not fully informed by expertise**: Priors for k and α_0 were heuristically chosen. In the absence of strong expert or empirical constraints, inference is less stable for these parameters.

Potential Improvements

- **Hierarchical inference**: Extending SBI to a hierarchical framework across subjects × conditions could pool information more efficiently, improving recovery of

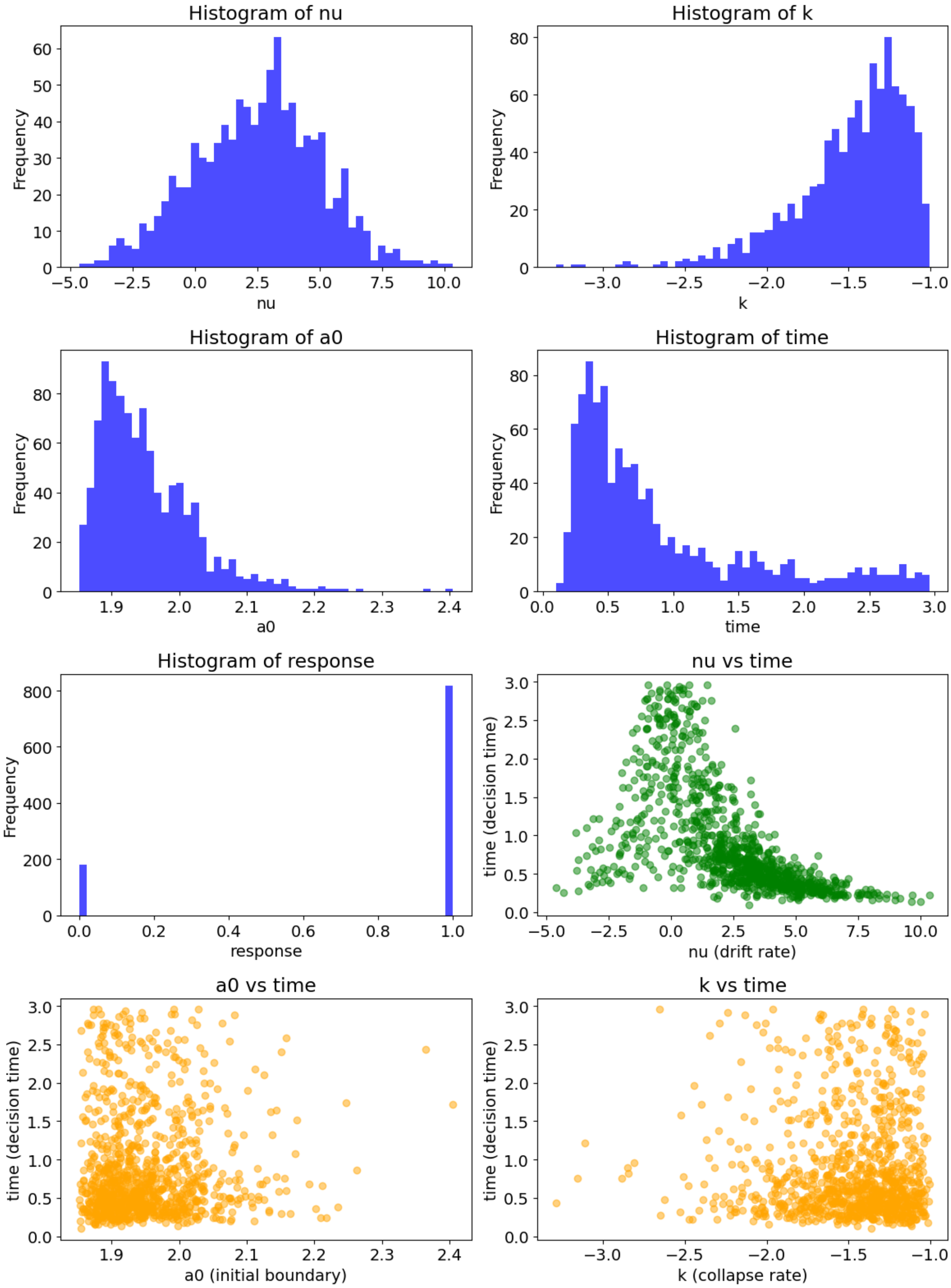

Figure 2: Exploratory Data Analysis Plots

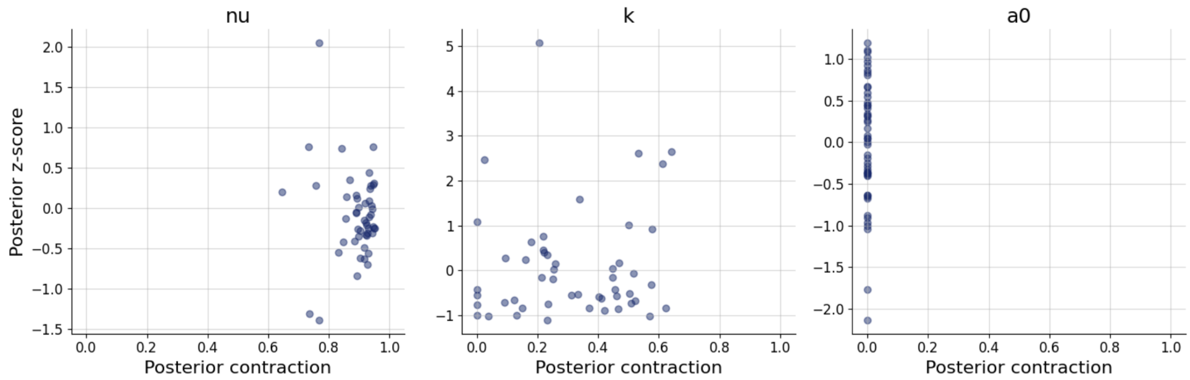

Figure 3: Posterior z-score

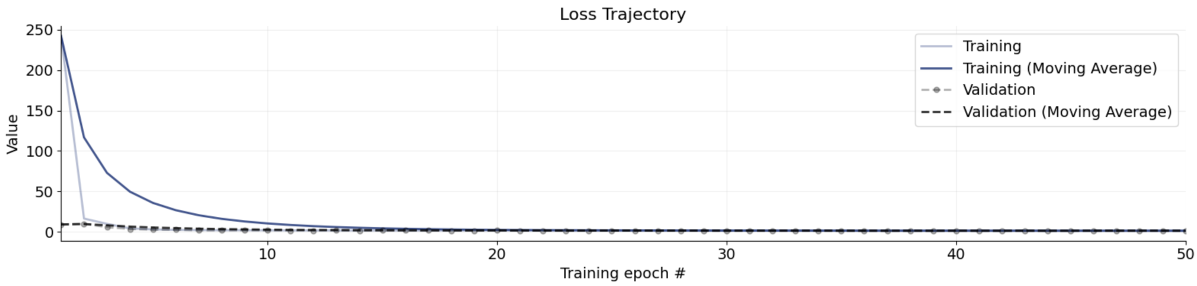

Figure 4: Loss curve

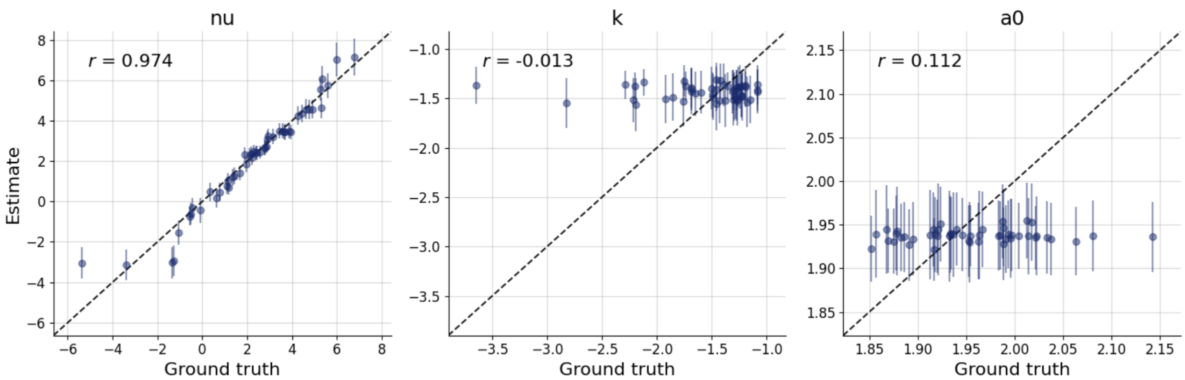

Figure 5: Ground Truth vs Estimate

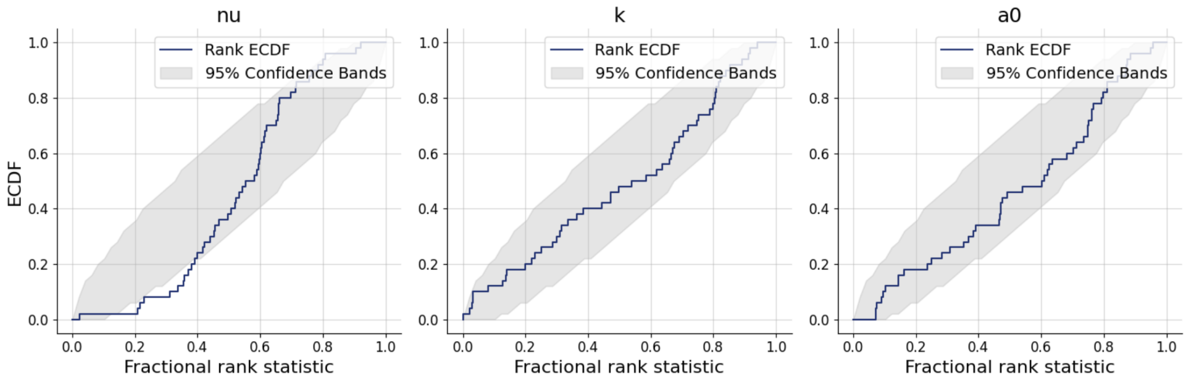

Figure 6: ECDF

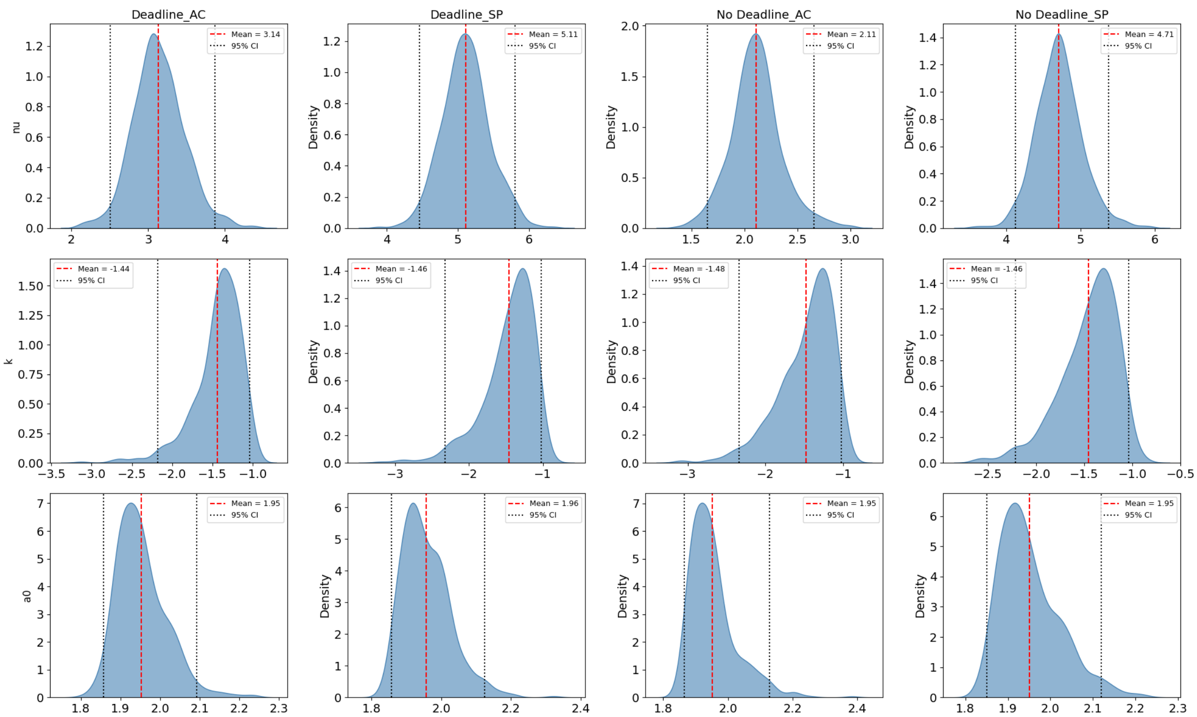

Figure 7: Full Inference

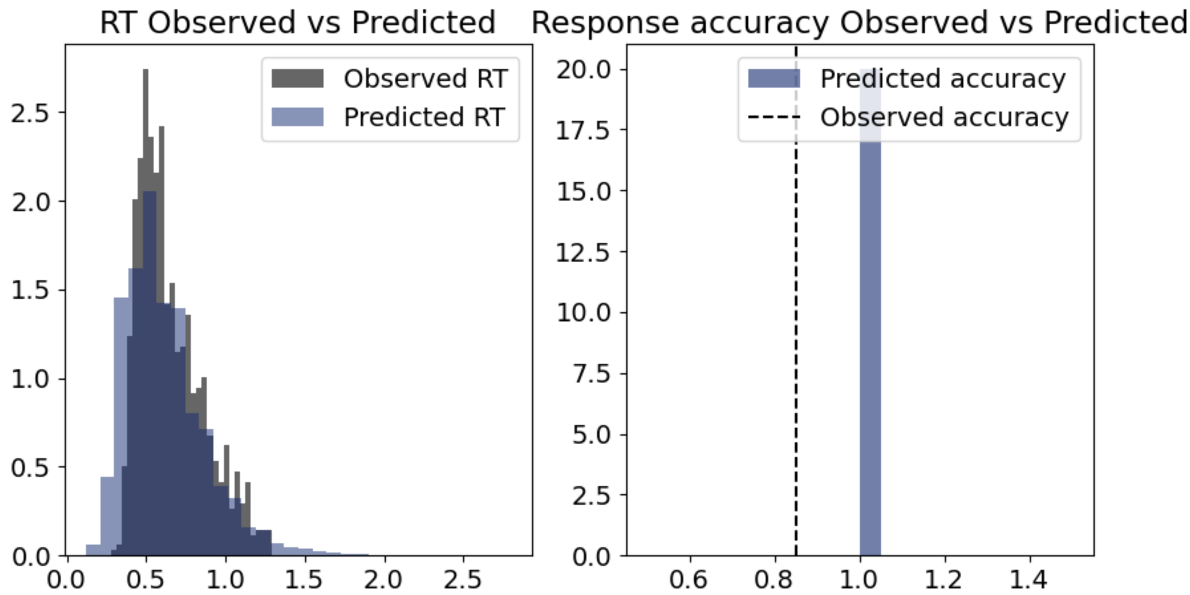

Figure 8: Posterior Predictive Check

weakly identified parameters.

- **Alternative boundary formulations**: Incorporating urgency gating or sigmoid-shaped collapse functions could yield more realistic urgency effects compared to the exponential form.

9 Conclusion

This project demonstrated how Simulation-Based Inference (SBI) can be applied to study decision-making models with collapsing boundaries. The method successfully recovered drift rate (ν), which aligned with behavioral manipulations across speed–accuracy trade-off conditions, confirming SBI's ability to capture dominant cognitive signals.

However, recovery of the exponential boundary parameters (k, α_0) proved unreliable. Simulation results reinforced that their behavioral signatures are weak and often confounded, which limits their explanatory power. This highlights a central challenge in SBI: while powerful in settings where likelihoods are intractable, inference depends critically on parameter identifiability and prior specification.

In summary, SBI provides strong potential for cognitive modeling, but collapsing boundary dynamics remain difficult to recover. Future work should explore hierarchical inference, alternative collapse formulations, and stronger prior integration to improve recovery beyond drift rate.

10 Reflection on own Learning

Through this project, we gained experience in applying **simulation-based inference** to cognitive modeling. Key takeaways include:

- The importance of parameter identifiability diagnostics, which revealed where the model succeeds (ν) and where it fails (k, a_0).
- There is a trade-off between theoretical model complexity and empirical recoverability. While exponential collapse adds psychological realism, it complicates inference.
- Practical insights into the strengths and limitations of BayesFlow, particularly its capacity for drift rate recovery and its struggles with more subtle boundary dynamics.
- Broader understanding of how independent vs. dependent data points impacts inference implications (DeepSet vs LSTM).

Overall, this project deepened our understanding of both decision-making models and the practical challenges of SBI, equipping us with tools to refine models and inference strategies in future work.

References

Katsimpokis, D., Hawkins, G. E., & van Maanen, L. (2020). Not all speed-accuracy trade-off manipulations have the same psychological effect. *Computational Brain & Behavior*, *3*(3), 252–268. https://doi.org/10.1007/s42113-020-00074-y

Smith, P. L., & Ratcliff, R. (2024). An introduction to the diffusion model of decision-making. In B. U. Forstmann & B. M. Turner (Eds.), *An introduction to model-based cognitive neuroscience* (pp. 67–100). Springer International Publishing. https://doi.org/10.1007/978-3-031-45271-0_4