Assignment 6 (S-670)

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Note:- Question 1 and 2 , done at last

Solution 3

```
library(aplpack)

## Loading required package: tcltk

# a)
x = c(576, 635, 558, 578, 666, 580, 555, 661, 651, 605, 653, 575, 545, 572,
594)
y = c(339, 330, 281, 303, 344, 307, 300, 343, 336, 313, 312, 274, 276, 288,
296)
Data = data.frame(x,y)
colnames(Data) <- list("LSAT","GPA")
mean = 0.5*log(1.77637/(1-0.77637))
n = 15
var = 1/(n -3)
se = sqrt(var)/sqrt(n)
CI= mean + c(-1,1)*1.96*se
CI

## [1] 0.8900774 1.1822569

# b)

calculatePV = function(data) {
  n = length(data[[1]])
  rho = cor(data, method="pearson")[1,2]
  yall = 0.5*log((1+rho)/(1-rho))
  PV = numeric(n)
  for( i in 1:n) {
    rhominusi = cor(data[-i,], method="pearson")[1,2]
    yminusi = 0.5*log((1+rhominusi)/(1- rhominusi))
    PV[i] = n*yall - (n-1)*yminusi
  }
  PV
}
PVAll = calculatePV(Data)
JKEstimate = mean(PVAll)
JKEstimate

## [1] 0.9170373
```

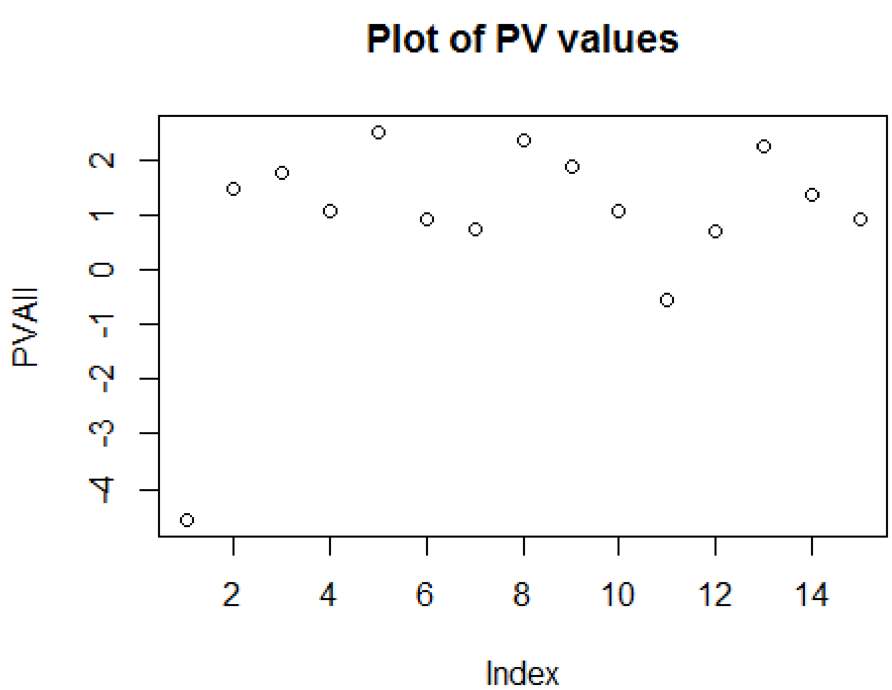
```
varJK = sum((PVAll - JKEstimate)^2)/15*14
CI = JKEstimate + c(-1,1)*qt(0.975,df=nrow(Data)-1)*sqrt(varJK)
CI
```

```
## [1] -12.31516  14.14923
```

```
# c)
stem.leaf(PVAll)
```

```
## 1 | 2: represents 1.2
##  leaf unit: 0.1
##             n: 15
## LO: -4.56529576001235
##    2    -0. | 5
##         -0* |
##          0* |
##    6     0. | 7799
##   (4)    1* | 0034
##    5     1. | 78
##    3     2* | 23
##    1     2. | 5
```

```
plot(PVAll,main="Plot of PV values")
```




```
PVre = calculatePV(Data[-1,])
JM = mean(PVre)
JM
```

```
## [1] 1.359238
```

```r
varJKRe = sum((PVre - JM)^2)/(14*(14-1))
varJKRe
```

```
## [1] 0.1096831
```

```r
#seJKRecalc = sqrt(varJK) #0.33
CI = JM + c(-1,1)*qt(0.975,df=13)*sqrt(varJKRe)
CI
```

```
## [1] 0.6437573 2.0747180
```

```r
# d)
bootstrap = function(data, nsim) {
  theta = numeric(nsim)
  varTheta = numeric(nsim)

  n = length(data[[1]])
  index = 1:n
  for (i in 1:nsim){
    sampleindex= sample(index,n,replace=TRUE)
    PViter = calculatePV(data[sampleindex, ])
    theta[i] = mean(PViter)
    varTheta[i] = sum((PViter - theta[i])^2)/(n*(n-1))
  }

  ciLower = mean(theta) - 1.96*mean(varTheta)
  ciUpper = mean(theta) + 1.96*mean(varTheta)

  output = list(thetaBS = mean(theta), varBS = mean(varTheta),
                theta = theta, varTheta = varTheta,
                ciLower = ciLower, ciUpper = ciUpper)
  output
}
Results = bootstrap(Data, 10)
Results
```

```
## $thetaBS
## [1] 1.050227
##
## $varBS
## [1] 0.1946566
##
## $theta
##  [1] 0.7980123 1.4864029 1.1025587 1.1333013 0.7835986 0.5434859 0.9603396
##  [8] 0.8628983 1.9851469 0.8465256
##
## $varTheta
##  [1] 0.77476139 0.06224270 0.05472431 0.04101526 0.10988584 0.07823188
##  [7] 0.20508282 0.22140352 0.09468920 0.30452915
```

```
##
## $ciLower
## [1] 0.6687001
##
## $ciUpper
## [1] 1.431754
```

** e) Effect of Outliers on Confidence Interval is being reduced by Bootstrapping and Jacknifing. We got a) 0.89, 1.182 b)-12.31 and 14.14 first and then 0.643,2.074 after removing outlier d) 0.408 and 1.551. Effect of Bootstrapping is reduced greatly by bootstrapping.

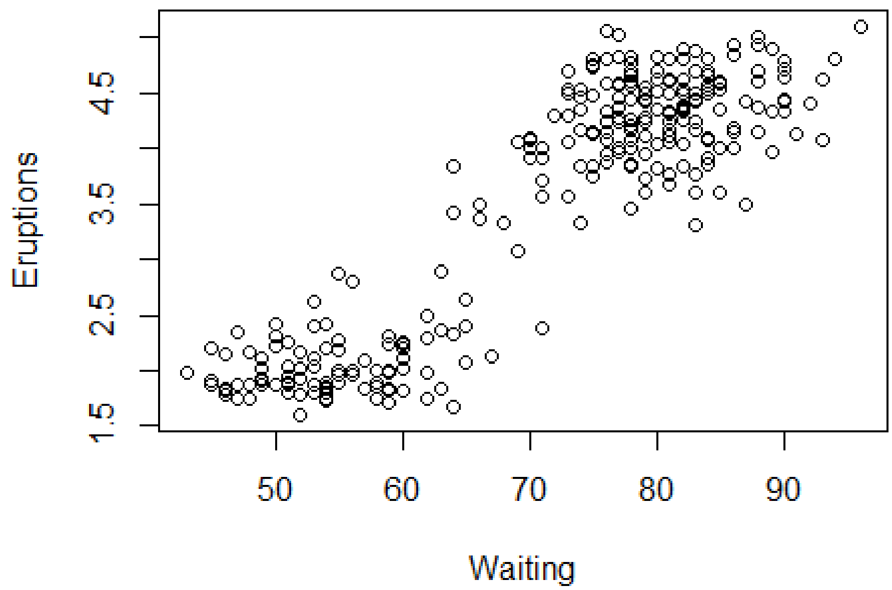
Solution 4

```
rrline1 <- function(x,y) {
  n3 <- floor((length(x)+1.99)/3)
  x.order <- order(x)
  medxL <- median(x[x.order][1:n3])
  medxR <- median(rev(x[x.order])[1:n3])
  medyL <- median(y[x.order][1:n3])
  medyR <- median(rev(y[x.order])[1:n3])
  slope1 <- (medyR - medyL)/(medxR - medxL)
  int1 <- median(y - slope1 * x)
  # print(c(paste("Intercept = ", format(round(int1,5))),
  #   paste("Slope = ",format(round(slope1,5)))))
  newy <- y - slope1*x - int1
  sumres <- sum(abs(newy))
  list(a=int1, b=slope1, sumres = sumres, res=newy)
}
#Code courtesy: Prof David King Lecture Notes
run.rrline <- function(x,y,iter=5) {
  out.coef <- matrix(0,iter,3)
  newy <- y
  for (i in 1:iter) {
    rr <- rrline1(x,newy)
    out.coef[i,] <- c(rr$a,rr$b,rr$sumres)
    newy <- rr$res
  }
  dimnames(out.coef) <- list(format(1:iter),c("a","b","|res|"))
  aa <- sum(out.coef[,1])
  bb <- sum(out.coef[,2])
  cc <- sum(abs(y - aa - bb*x))
  res <- y - aa - bb*x
  out.coef <- rbind(out.coef,c(aa,bb,cc))
  #print(round(out.coef,5))
  list(a = aa, b = bb, res = res, coef=out.coef)
}
bootprog = function (x,nsim)
{
  # This program is a silly program which will be used to estimate the
```

```
  # bootstap error of the sample median statistic
  #  the input data is a vector x of data.
  #  nsim is the number of bootstrap simulations
  n = length(x)
  index = 1:n
  m = median(x)
  stat = numeric(nsim)
  ooberr = numeric(nsim)
  for (i in 1:nsim){
    sampleindex= sample(index,n,replace=TRUE)
    stat[i] = median(x[sampleindex])
    oobindex = setdiff(index,unique(sampleindex))
    oobdat = x[oobindex]
    ooberr[i] = sum((oobdat-stat[i])^2)/length(oobindex)
  }
  bias = m - mean(stat)
  variance = var(stat)
  se = sqrt(variance)
  avgooberr = mean(ooberr)
  output = list(bias=bias,var=variance,se=se,avgooberr=avgooberr)
  output
}

cvprog = function (x,nfold)
{
  # This program is a silly program which will be used to estimate the
  # crossvalidation error of the sample median statistic
  # the input data is a vector x of data.
  #  nfold is the number of folds you want to divide your data up into
  n = length(x)
  m = floor(n/nfold)
  # Generally speaking n/nfold would be an integer, however if it is not
  # and the remainder of n/nfold is k then we will take the extra k
datapoints
  # and give them to the first k folds.
  folds = rep(1:nfold,m)
  k = n - length(folds)
  if(k>0){folds = c(folds,1:k)}
  # now folds is of length n and we can randomly permute the indicies
  foldindicies = sample(folds,n,replace=FALSE)
  m = median(x)
  stat = numeric(nfold)
  cverr = numeric(nfold)
  for (i in 1:nfold){
    b = foldindicies == i
    stat[i] = median(x[!b])
    cverr[i] = sum((x[b]-stat[i])^2)/length(x[b])
  }
  bias = m - mean(stat)
  variance = var(stat)
```

```
  se = sqrt(variance)
  avgcverr = mean(cverr)
  output = list(bias=bias,var=variance,se=se,avgcverr=avgcverr)
  output
}

getOOBforBootstrap = function(oobdata) {
  # Based on the class slides and hints from the professor,
  # this function calculates rrline for
  # the oob data to get a and b, shuffles the residuals,
  # adds them to the original data
  # calculate rr line again to get new a and b, this is repeated till we have
  # n estimates of a and b where n is the number of oob samples
  # oobdata
  originalData = oobdata
  n = length(originalData[[1]])
  aOutofBag = numeric(n)
  bOutofBag = numeric(n)
  for (q in 1: n) {
    results = run.rrline(originalData[[1]], originalData[[2]])
    residuals = results$res
    aOutofBag[q] = results$a
    bOutofBag[q] = results$b
    shuffledResiduals = sample(residuals)
    originalData[[2]] = oobdata[[2]] + shuffledResiduals
  }
  list(a0 = aOutofBag, b0 = bOutofBag)
}



rrlineWithBootstrap = function(data, nsim) {
  # This function runs rrline with bootstrapping
  n = length(data[[1]])
  index = 1:n

  # We maintain 2 different stats and oob for each a and b
  statA = numeric(nsim)
  statB = numeric(nsim)
  ooberrA = numeric(nsim)
  ooberrB = numeric(nsim)

  # Run rrline to get initial statistic on entire data,
  # for confirmatory purposes only
  results = run.rrline(data[[1]], data[[2]])
  a = results$a
  b = results$b

  #Run nsim times
```

```r
  for (i in 1:nsim){
    sampleindex= sample(index,n,replace=TRUE)
    results = run.rrline(data[[1]][sampleindex], data[[2]][sampleindex])
    statA[i] = results$a
    statB[i] = results$b

    oobindex = setdiff(index,unique(sampleindex))
    oobResults = getOOBforBootstrap(data[oobindex,])

    ooberrA[i] = sum((oobResults$a0-statA[i])^2)/length(oobindex)
    ooberrB[i] = sum((oobResults$b0-statB[i])^2)/length(oobindex)
  }

  # Calculate bias and variance
  biasA = a - mean(statA)
  varianceA = var(statA)
  biasB = b - mean(statB)
  varianceB = var(statB)

  # Calculate standard error and average oob error
  seA = sqrt(varianceA)
  seB = sqrt(varianceB)
  avgooberrA = mean(ooberrA)
  avgooberrB = mean(ooberrB)

  output = list(a= mean(statA), b = mean(statB),
                biasA=biasA,varA=varianceA, biasB = biasB, varB = varianceB,
                seA=seA,seB = seB, ooberrA = ooberrA, ooberrB = ooberrB,
                avgooberrA=avgooberrA, avgooberrB = avgooberrB)
  output
}

q4Data = data.frame(faithful$waiting, faithful$eruptions)
colnames(q4Data) <- c("Waiting", "Eruptions")
plot(q4Data)
```



Solution 5

```
library(DAAG)

## Loading required package: lattice

getOOBforCV = function(oobdata) {
originalData = oobdata
 n = length(originalData[[1]])
 aOutofBag = numeric(n)
 bOutofBag = numeric(n)
 for (q in 1: n) {
 results = run.rrline(originalData[[1]], originalData[[2]])
 residuals = results$res
 aOutofBag[q] = results$a
 bOutofBag[q] = results$b
 shuffledResiduals = sample(residuals)
 originalData[[2]] = oobdata[[2]] + shuffledResiduals
 }
 print(aOutofBag)
 print(bOutofBag)
 list(a0 = aOutofBag, b0 = bOutofBag)
}
newcvprog = function (x,nfold)
{
 n = length(x)
 m = floor(n/nfold)
```

```
folds = rep(1:nfold,m)
k = n - length(folds)
if(k>0){folds = c(folds,1:k)}
foldindicies = sample(folds,n,replace=FALSE)
statA = numeric(nfold)
statB = numeric(nfold)

ooberrA = numeric(nfold)
ooberrB = numeric(nfold)
results = run.rrline(x[[1]], x[[2]])
a = results$a
b = results$b

statA = numeric(nfold)
statB = numeric(nfold)

cverrA = numeric(nfold)
cverrB = numeric(nfold)

for (i in 1:nfold){
print(paste("Fold ", i))
p = foldindicies == i
resultiter = run.rrline(x[[1]][!p], x[[2]][!p])
statA[i] = resultiter$a
statB[i] = resultiter$b

print(statA)
print(statB)
oobResults = getOOBforCV(x[p,])
cverrA[i] = sum((oobResults$a0-statA[i])^2)/length(p)
cverrB[i] = sum((oobResults$b0-statB[i])^2)/length(p)

}
biasA = a - mean(statA)
varianceA = var(statA)


biasB = b - mean(statB)
varianceB = var(statB)

seA = sqrt(varianceA)/sqrt(nfold)
seB = sqrt(varianceB)/sqrt(nfold)

avgcverrA = mean(cverrA)
avgcverrB = mean(cverrB)

output = list(biasA=biasA, varA=varianceA,
biasB = biasB, varB = varianceB,
```

```r
 seA=seA, seB=seB,
 avgcverrA=avgcverrA, avgcverrB = avgcverrB)
 output
}
q5Data = data.frame(faithful$waiting, faithful$eruptions)
colnames(q5Data) <- c("Waiting", "Eruptions")
resultsq5 = newcvprog(q5Data, 3)
```

```
## [1] "Fold  1"
## [1] -1.383236  0.000000  0.000000
## [1] 0.06937156 0.00000000 0.00000000
##   [1] -1.98705398 -1.85157440 -1.02493458 -2.27895288 -1.26071143
##   [6] -1.17486614 -1.52658021 -2.08683503 -1.55093277 -1.26634000
##  [11] -1.69391234 -1.44354647 -0.86776925 -4.19342070  0.35967328
##  [16] -3.48963733 -1.31501007 -2.16262319 -3.08048847 -2.17034854
##  [21] -1.33264022 -2.67857198 -0.83823602 -3.42049430 -1.58324913
##  [26] -3.63507794 -0.58396768 -2.42314875 -3.63182220 -0.88549330
##  [31] -0.63405830 -3.56451877 -0.20388043 -2.55674944 -1.99389760
##  [36] -1.03677598 -4.03389934 -3.19522654 -2.88946506 -2.76369166
##  [41] -2.21186786 -0.60829479 -2.14590194 -4.87380944 -0.99857402
##  [46] -2.75953024 -0.38431723  1.00586106  3.03877663  0.45077412
##  [51] -4.50248327 -2.84051424 -1.81118058 -2.54900028 -1.36671207
##  [56]  1.27303516 -5.08573957 -6.01235675 -0.18317027 -3.40823471
##  [61] -1.22853528  2.40684736 -2.91652824 -4.91242480  0.45222949
##  [66] -3.30884238 -2.60840335 -1.48820333 -5.73990232 -2.53561204
##  [71]  0.03255303 -1.52410631 -0.89172743 -0.13440970 -0.52863047
##  [76] -3.56646707 -1.46956238 -2.93470033  2.00085475 -0.14130132
##  [81] -1.86389050 -0.79868506 -2.78823369  1.16606372 -2.58596474
##  [86] -4.78062957 -1.92351949 -1.31113979 -3.50209008 -4.74256242
##  [91]  0.06150328 -1.99895934  1.36813174 -0.72035368 -0.19388860
##  [96] -0.63155035  0.65590714 -4.53798082 -2.65799314 -3.29100572
## [101] -2.58516096 -2.04362627 -3.07030494 -4.36995475  1.46278865
## [106] -0.68663850  2.38835129 -2.71760236  0.66506486 -2.88764928
## [111] -1.23868645 -3.78419263  0.68422411  0.71175988 -1.53346970
## [116]  2.44173824 -1.91590589  3.66923558 -6.12056850 -2.76974213
## [121] -3.77306082 -2.87731398 -0.86157740  0.37329925 -0.07388773
## [126] -2.74165658  3.48002719 -2.91504609  1.21915073 -4.95309295
## [131] -3.13320360 -3.23023708  0.14631105 -3.23112256 -0.63034855
## [136] -0.19132357
##   [1]  0.077441358  0.075708781  0.063360607  0.081069676  0.067257771
##   [6]  0.064461660  0.071831073  0.076162308  0.071437745  0.064936277
##  [11]  0.073156473  0.070403670  0.062591839  0.105912324  0.045726206
##  [16]  0.098561268  0.067234620  0.079526686  0.094481332  0.078898058
##  [21]  0.069850737  0.086244181  0.060842739  0.098538046  0.071030986
##  [26]  0.102384926  0.059223611  0.084829704  0.098444145  0.061148999
##  [31]  0.060706616  0.098284696  0.052140321  0.086491575  0.074507688
##  [36]  0.063608024  0.106978869  0.096517302  0.090425695  0.087302123
##  [41]  0.081337717  0.056429958  0.077842141  0.122427571  0.063115345
##  [46]  0.086468319  0.054367009  0.037554568  0.002563788  0.041774139
##  [51]  0.114960540  0.089746293  0.074644160  0.084938265  0.067246243
```

```
##  [56]  0.026168877  0.123408699  0.136930649  0.050782276  0.096682743
##  [61]  0.065062811  0.015077274  0.091180976  0.118347776  0.044375087
##  [66]  0.094896666  0.087896927  0.071672270  0.130405528  0.087125318
##  [71]  0.043039622  0.068940093  0.061471175  0.048515411  0.059257687
##  [76]  0.098801723  0.069745300  0.094331207  0.020461766  0.051755971
##  [81]  0.075082033  0.057163096  0.091277477  0.030707131  0.086451338
##  [86]  0.117709337  0.076557321  0.064936961  0.100365935  0.113929957
##  [91]  0.050401057  0.076176495  0.033334300  0.056294676  0.053521481
##  [96]  0.059200494  0.035662808  0.117274419  0.088330073  0.096431852
## [101]  0.085075637  0.082834241  0.092981934  0.107752041  0.025054446
## [106]  0.055766536  0.015876670  0.088555273  0.038202720  0.092173542
## [111]  0.062096828  0.105519902  0.038602423  0.033638100  0.069519577
## [116]  0.016006264  0.075821126 -0.005306682  0.134865285  0.088596138
## [121]  0.105273943  0.090190557  0.057797156  0.043962166  0.048871664
## [126]  0.092012537 -0.004896847  0.090037018  0.033506156  0.118190533
## [131]  0.096074394  0.098219861  0.045316818  0.096036829  0.052127872
## [136]  0.049181097
## [1] "Fold  2"
## [1] -1.383236 -1.987054  0.000000
## [1] 0.06937156 0.07744136 0.00000000
##   [1] -1.38323613 -1.50441408 -1.35794621 -2.21841013 -2.07457476
##   [6] -1.63005288 -3.00377925 -2.01935974 -0.58032496 -2.21824706
##  [11] -2.21891659 -2.42186388 -1.58113389 -1.55255441 -0.01608162
##  [16] -2.15056020  1.23154443  0.22281423  0.46259140 -2.67539120
##  [21]  0.19703414 -1.57828043 -3.32392576 -0.55331357  0.24572479
##  [26] -0.23890730 -0.75002349 -3.13536047 -3.95263865 -1.96145723
##  [31]  0.96353767 -0.60045992 -0.57903756 -1.99926910  0.23627231
##  [36] -3.45121836 -3.30370125 -0.35890940 -3.29976133  0.23275710
##  [41] -1.97581487 -0.12286225 -2.03834015 -1.39639808 -1.15037185
##  [46]  2.71909225 -3.31339472 -1.73526002  0.46135041 -3.47777648
##  [51] -2.71169975 -2.89369638  2.32048608 -1.69614847 -2.12978391
##  [56] -0.63672833 -3.60801335 -3.84103404  1.13849938  0.67992916
##  [61] -1.51750096 -2.55014370 -0.82362056 -1.35784763 -4.22844749
##  [66]  1.25837146 -2.94841252  3.06315034 -3.35457089 -1.89874957
##  [71] -0.69784678 -2.21702707 -3.42803141 -6.51011977 -6.46494022
##  [76] -0.21498298 -4.75283257 -0.54635044  4.58997527  0.11228264
##  [81] -0.84628756 -1.46901540 -7.43369738 -0.70419297 -0.53962408
##  [86]  3.13964278 -3.09573989 -4.27591199 -0.33223146 -4.20837043
##  [91]  0.93080203 -4.34404654 -3.45753890 -5.23601645 -2.20847433
##  [96] -1.73655225 -5.53488505 -3.85987859 -1.30546163  0.85884061
## [101] -2.57600772  0.89037901  0.01752309 -0.34003249  2.84147618
## [106] -1.88346683  5.26945351  3.81962424  0.51379925  1.86433787
## [111]  0.89307839  0.33684986  2.42964228  1.37270690  1.95257582
## [116] -5.00922827 -2.93720148 -7.59686349 -3.96800907 -5.46367601
## [121] -3.81016844  1.60107347 -3.84968114  2.93714987  0.53358601
## [126]  0.15866848 -3.28199507 -0.67427140 -1.13081869  3.14423288
## [131] -8.35129873 -3.01652882  1.15727320 -1.08989010 -0.22479737
## [136] -1.73578920
##   [1]  0.069371558  0.069688116  0.069557838  0.080146879  0.078356527
##   [6]  0.072053426  0.090380378  0.077543886  0.059857217  0.076977433
```

```
##  [11]  0.080194037  0.082590712  0.073226524  0.072201671  0.048594953
##  [16]  0.080171892  0.034501707  0.046289664  0.043144619  0.084814409
##  [21]  0.050863879  0.069910661  0.094781580  0.056621718  0.047681585
##  [26]  0.052129872  0.061304448  0.091451989  0.105280208  0.079340377
##  [31]  0.037321374  0.060866067  0.055107242  0.076880830  0.042239975
##  [36]  0.099175749  0.095799847  0.053970497  0.096342174  0.047363703
##  [41]  0.076667091  0.053156542  0.077178028  0.068939546  0.064374158
##  [46]  0.017116403  0.092867125  0.072357113  0.043189648  0.097316203
##  [51]  0.087899165  0.089080997  0.019439401  0.074372058  0.078274922
##  [56]  0.059706476  0.096731739  0.102752550  0.036325157  0.040774443
##  [61]  0.071132028  0.083157999  0.066477393  0.064004933  0.107530757
##  [66]  0.036554122  0.088987002  0.008524082  0.095817143  0.077187673
##  [71]  0.057214617  0.079961665  0.097632751  0.136840731  0.136194446
##  [76]  0.052075387  0.117731703  0.053753679 -0.009967500  0.051992733
##  [81]  0.057954902  0.071847996  0.149679805  0.057459615  0.054866651
##  [86]  0.009197247  0.093667977  0.110113629  0.055594636  0.106520524
##  [91]  0.038115447  0.106943536  0.099037573  0.124784422  0.080386146
##  [96]  0.070074940  0.127888921  0.101686647  0.070174227  0.037415951
## [101]  0.084551702  0.039054755  0.048849028  0.054017511  0.016436912
## [106]  0.069233009 -0.020003696 -0.003111683  0.041821704  0.032544783
## [111]  0.035613378  0.045141275  0.018331564  0.030893348  0.025597871
## [116]  0.115918107  0.088604931  0.150484775  0.103779143  0.123356714
## [121]  0.102231271  0.027458988  0.101351973  0.010381686  0.044538314
## [126]  0.049198206  0.093581632  0.060411514  0.064607009  0.013105688
## [131]  0.163656646  0.091494498  0.037330740  0.065417397  0.050550936
## [136]  0.075723387
## [1] "Fold  3"
## [1] -1.383236 -1.987054 -2.007400
## [1] 0.06937156 0.07744136 0.07780000
## [1] NA
## [1] NA
```

```r
test = c(1,2,3,4,5,6,7)
sd(test)/sqrt(length(test))
```

```
## [1] 0.8164966
```

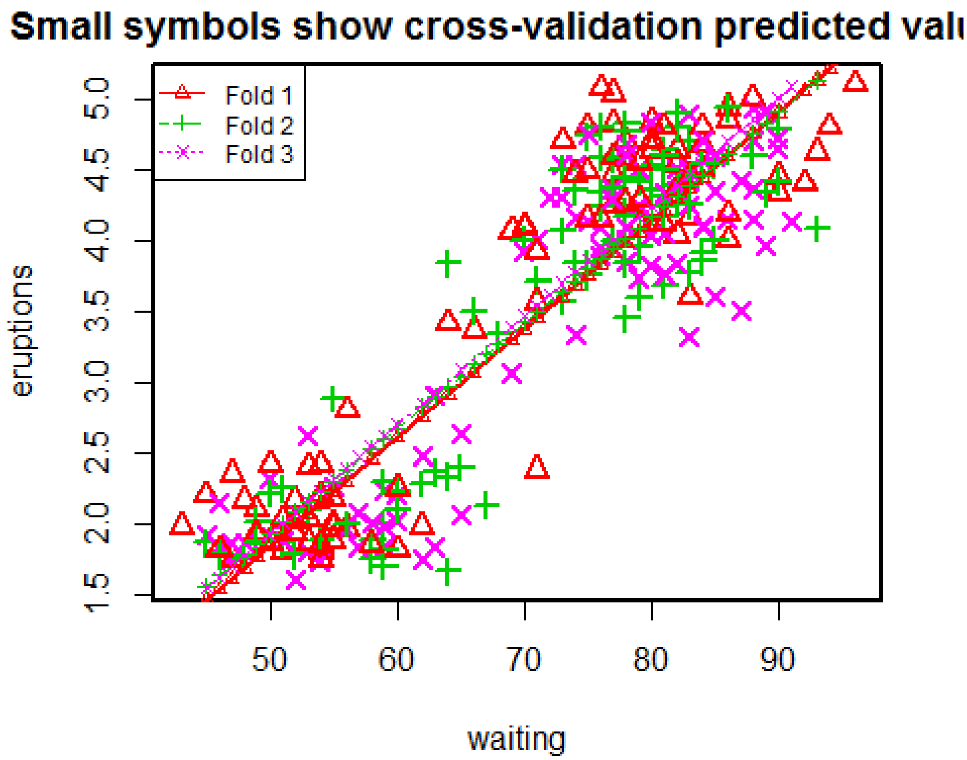
```r
sqrt(var(test))/sqrt(length(test))
```

```
## [1] 0.8164966
```

```r
mydata<-as.data.frame(faithful)
fit<-lm(eruptions~waiting,mydata)
cv.lm(mydata, fit, m=3)
```

```
## Analysis of Variance Table
##
## Response: eruptions
##            Df Sum Sq Mean Sq F value Pr(>F)
## waiting     1  286.5   286.5    1162 <2e-16 ***
## Residuals 270   66.6     0.2
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```




```
##
## fold 1
## Observations in test set: 90
##                  2     11    16     18    21     22     26     27     32
## waiting      54.00 54.000 52.00 84.000 51.00 47.000 83.000 55.000 77.000
## cvpred        2.15  2.150  2.00  4.443  1.92  1.615  4.367  2.226  3.908
## eruptions     1.80  1.833  2.17  4.800  1.80  1.750  3.600  1.967  4.467
## CV residual -0.35 -0.317  0.17  0.357 -0.12  0.135 -0.767 -0.259  0.559
##                 33      36     38     49    51     53      62     65     66
## waiting      66.00 52.0000 80.000 82.000 75.00 54.000 84.0000 60.000 92.000
## cvpred        3.07  1.9968  4.137  4.290  3.76  2.150  4.4432  2.608  5.055
## eruptions     3.37  2.0170  4.833  4.633  4.80  1.833  4.5000  1.817  4.400
## CV residual   0.30  0.0202  0.696  0.343  1.04 -0.317  0.0568 -0.791 -0.655
##                  68   70     71     72     75    76     78     80     83
## waiting      78.000 73.0 82.000 56.000 62.000 76.00 78.000 83.000 70.000
## cvpred        3.984  3.6  4.290  2.303  2.761  3.83  3.984  4.367  3.373
## eruptions     4.700  4.7  4.033  1.967  1.983  5.07  4.567  3.600  4.100
## CV residual   0.716  1.1 -0.257 -0.336 -0.778  1.24  0.583 -0.767  0.727
##                  89     90     92      93     97    103    109    122
## waiting      48.000 86.000 90.000 50.0000 84.000 49.000 86.000 69.000
## cvpred        1.691  4.596  4.902  1.8439  4.443  1.767  4.596  3.296
## eruptions     2.167  4.000  4.333  1.8670  4.667  2.100  4.850  4.067
## CV residual   0.476 -0.596 -0.569  0.0231  0.224  0.333  0.254  0.771
##                123    132    133    138     139    143    144    147    149
```

```
## waiting     77.000 83.00 56.000 86.000 53.0000 82.000 77.000 80.000 96.000
## cvpred       3.908  4.37  2.303  4.596  2.0732  4.290  3.908  4.137  5.361
## eruptions    4.250  4.17  2.800  4.933  2.0330  4.533  4.817  4.633  5.100
## CV residual  0.342 -0.20  0.497  0.337 -0.0402  0.243  0.909  0.496 -0.261
##                151    155    161    168     169    170    171    178    181
## waiting     77.00 71.000 45.000 88.000 52.0000 93.000 49.00 50.000 55.000
## cvpred       3.91  3.449  1.462  4.749  1.9968  5.131  1.77  1.844  2.226
## eruptions    5.03  3.567  2.200  5.000  1.9330  4.617  1.92  2.417  1.883
## CV residual  1.12  0.118  0.738  0.251 -0.0638 -0.514  0.15  0.573 -0.343
##                 182    185     190    191    204    209   211    212    215
## waiting     77.000 51.000 55.0000 81.000 53.000 49.000 71.00 80.000 64.000
## cvpred       3.908  1.920  2.2261  4.214  2.073  1.767  3.45  4.137  2.914
## eruptions    4.583  2.033  2.1830  4.800  1.867  1.933  2.38  4.700  3.417
## CV residual  0.675  0.113 -0.0431  0.586 -0.206  0.166 -1.07  0.563  0.503
##                 217    218    219    220     221   223    224     226
## waiting     53.000 94.000 55.000 76.000 50.0000 54.00 75.000 79.0000
## cvpred       2.073  5.208  2.226  3.832  1.8439  2.15  3.755  4.0609
## eruptions    2.400  4.800  2.000  4.150  1.8670  1.75  4.483  4.1170
## CV residual  0.327 -0.408 -0.226  0.318  0.0231 -0.40  0.728  0.0561
##                 228    230   231    232    233    235   237    241    242
## waiting     78.000 79.000 70.00 54.000 86.000 90.000 54.00 75.000 47.000
## cvpred       3.984  4.061  3.37  2.150  4.596  4.902  2.15  3.755  1.615
## eruptions    4.267  4.550  4.08  2.417  4.183  4.450  1.85  4.150  2.350
## CV residual  0.283  0.489  0.71  0.267 -0.413 -0.452 -0.30  0.395  0.735
##                  248     251    257     258    260    263    265    266
## waiting     82.0000 54.0000 71.000 83.0000 79.000 58.000 43.000 60.000
## cvpred       4.2903  2.1497  3.449  4.3667  4.061  2.455  1.309  2.608
## eruptions    4.3670  2.2000  3.917  4.4500  4.283  1.850  1.983  2.250
## CV residual  0.0767  0.0503  0.468  0.0833  0.222 -0.605  0.674 -0.358
##                  268    271    272
## waiting     81.0000 46.000 74.000
## cvpred       4.2138  1.538  3.679
## eruptions    4.1170  1.817  4.467
## CV residual -0.0968  0.279  0.788
## 
## Sum of squares = 23.6    Mean square = 0.26    n = 90 
## 
## fold 2 
## Observations in test set: 91 
##                  1      4        5      6     12     13     15     23
## waiting     79.000 62.000 8.50e+01 55.000 84.000 78.000 83.000 78.000
## cvpred       4.087  2.825 4.53e+00  2.305  4.458  4.012  4.384  4.012
## eruptions    3.600  2.283 4.53e+00  2.883  3.917  4.200  4.700  3.450
## CV residual -0.487 -0.542 9.73e-04  0.578 -0.541  0.188  0.316 -0.562
##                 30     35    40     41     42     44    47      48     50
## waiting     79.000 74.000 90.00 80.000 58.000 58.000 64.00 53.0000 59.000
## cvpred       4.087  3.716  4.90  4.161  2.528  2.528  2.97  2.1567  2.602
## eruptions    4.433  3.833  4.78  4.350  1.883  1.750  3.83  2.1000  2.000
## CV residual  0.346  0.117 -0.12  0.189 -0.645 -0.778  0.86 -0.0567 -0.602
##                  57     58     59      60      63     64     67      82
```

```
## waiting     71.000 64.00 77.000 81.0000 48.0000 82.000 78.000 82.0000
## cvpred       3.493  2.97  3.938  4.2351  1.7856  4.309  4.012  4.3093
## eruptions    3.717  1.67  4.567  4.3170  1.7500  4.800  4.167  4.3330
## CV residual  0.224 -1.31  0.629  0.0819 -0.0356  0.491  0.155  0.0237
##                 85     88     94      98     99    100    108    110
## waiting     73.000 80.000 78.000 75.0000 51.000 82.000 52.000 81.000
## cvpred       3.641  4.161  4.012  3.7897  2.008  4.309  2.082  4.235
## eruptions    4.067  4.517  4.817  3.7500  1.867  4.900  1.783  3.683
## CV residual  0.426  0.356  0.805 -0.0397 -0.141  0.591 -0.299 -0.552
##                111    112    114    115    116    119    125    131    134
## waiting     75.000 59.000 79.00 59.000 81.000 59.000 88.000 45.000 89.000
## cvpred       3.790  2.602  4.09  2.602  4.235  2.602  4.755  1.563  4.829
## eruptions    4.733  2.300  4.42  1.700  4.633  1.817  4.600  1.867  4.333
## CV residual  0.943 -0.302  0.33 -0.902  0.398 -0.785 -0.155  0.304 -0.496
##                 136      141    142    145    148     152    153    154
## waiting     82.0000 81.00000 60.000 76.000 49.000 77.0000 65.000 81.000
## cvpred       4.3093  4.23511  2.676  3.864  1.860  3.9382  3.047  4.235
## eruptions    4.3830  4.23300  2.233  4.333  2.017  4.0000  2.400  4.600
## CV residual  0.0737 -0.00211 -0.443  0.469  0.157  0.0618 -0.647  0.365
##                156    157   158    164    165    166    167    173     174
## waiting     70.000 81.000 93.00 78.000 66.000 76.000 63.000 77.000 68.0000
## cvpred       3.419  4.235  5.13  4.012  3.122  3.864  2.899  3.938  3.2701
## eruptions    4.000  4.500  4.08  3.833  3.500  4.583  2.367  4.583  3.3330
## CV residual  0.581  0.265 -1.04 -0.179  0.378  0.719 -0.532  0.645  0.0629
##                 175    177    179    184    186    193     195      196
## waiting     81.0000 73.000 85.000 83.000 78.000 76.000 77.0000 81.00000
## cvpred       4.2351  3.641  4.532  4.384  4.012  3.864  3.9382  4.23511
## eruptions    4.1670  4.500  4.000  3.767  4.433  4.800  3.9660  4.23300
## CV residual -0.0681  0.859 -0.532 -0.617  0.421  0.936  0.0278 -0.00211
##                199    201    206    207    208      213     214    216
## waiting     51.000 60.000 46.000 77.000 84.000 49.00000 75.0000 76.000
## cvpred       2.008  2.676  1.637  3.938  4.458  1.85979  3.7897  3.864
## eruptions    2.250  2.100  1.783  4.367  3.850  1.86700  3.8330  4.233
## CV residual  0.242 -0.576  0.146  0.429 -0.608  0.00721  0.0433  0.369
##                 222    234    236    239   240    243   249    250     252
## waiting     82.0000 50.000 54.000 79.000 64.00 86.000 67.00 74.000 83.0000
## cvpred       4.3093  1.934  2.231  4.087  2.97  4.606  3.20  3.716  4.3836
## eruptions    4.2670  2.217  1.883  3.950  2.33  4.933  2.13  4.350  4.4500
## CV residual -0.0423  0.283 -0.348 -0.137 -0.64  0.327 -1.06  0.634  0.0664
##                 253    254    259    261     262    264    270
## waiting     73.0000 73.000 56.000 78.000 84.0000 83.000 90.000
## cvpred       3.6413  3.641  2.379  4.012  4.4578  4.384  4.903
## eruptions    3.5670  4.500  2.000  4.767  4.5330  4.250  4.417
## CV residual -0.0743  0.859 -0.379  0.755  0.0752 -0.134 -0.486
##
## Sum of squares = 22.6    Mean square = 0.25    n = 91
##
## fold 3
## Observations in test set: 91
##                   3     7     8       9     10      14    17     19      20
```

```
## waiting     74.000 88.00 85.00 51.0000 85.000 47.0000 62.00 52.000 79.0000
## cvpred       3.774  4.85  4.62  2.0067  4.619  1.6993  2.85  2.084  4.1582
## eruptions    3.333  4.70  3.60  1.9500  4.350  1.7500  1.75  1.600  4.2500
## CV residual -0.441 -0.15 -1.02 -0.0567 -0.269  0.0507 -1.10 -0.484  0.0918
##                 24     25     28     29     31     34      37     39
## waiting     69.000 74.000 76.000 78.000 73.000 80.000 48.0000 59.000
## cvpred       3.390  3.774  3.928  4.081  3.697  4.235  1.7762  2.621
## eruptions    3.067  4.533  4.083  3.850  4.300  4.033  1.8670  1.833
## CV residual -0.323  0.759  0.155 -0.231  0.603 -0.202  0.0908 -0.788
##                  43     45    46     52     54     55     56     61    69
## waiting     84.0000 73.000 83.00 90.000 80.000 54.000 83.000 59.000 65.00
## cvpred       4.5424  3.697  4.47  5.003  4.235  2.237  4.466  2.621  3.08
## eruptions    4.5670  4.533  3.32  4.716  4.833  1.733  4.883  2.233  2.07
## CV residual  0.0246  0.836 -1.15 -0.287  0.598 -0.504  0.417 -0.388 -1.02
##                 73     74     77      79     81     84      86      87
## waiting     79.000 71.000 60.000 76.0000 75.000 65.000 88.0000 76.0000
## cvpred       4.158  3.543  2.698  3.9277  3.851  3.082  4.8498  3.9277
## eruptions    4.500  4.000  2.017  3.8830  4.133  2.633  4.9330  3.9500
## CV residual  0.342  0.457 -0.681 -0.0447  0.282 -0.449  0.0832  0.0223
##                 91    95    96    101    102     104    105    106    107
## waiting     60.000 63.00 72.00 62.000 88.000 83.0000 81.000 47.000 84.000
## cvpred       2.698  2.93  3.62  2.852  4.850  4.4656  4.312  1.699  4.542
## eruptions    2.200  1.83  4.30  2.483  4.367  4.5000  4.050  1.867  4.700
## CV residual -0.498 -1.10  0.68 -0.369 -0.483  0.0344 -0.262  0.168  0.158
##                 113    117     118    120    121    124    126    127
## waiting     89.0000 50.000 85.0000 87.000 53.000 56.000 81.000 45.000
## cvpred       4.9266  1.930  4.6193  4.773  2.160  2.391  4.312  1.546
## eruptions    4.9000  2.317  4.6000  4.417  2.617  1.967  3.767  1.917
## CV residual -0.0266  0.387 -0.0193 -0.356  0.457 -0.424 -0.545  0.371
##                128     129    130    135    137    140    146   150   159
## waiting     82.000 55.0000 90.000 46.000 51.000 79.000 59.000 53.00 53.00
## cvpred       4.389  2.3141  5.003  1.622  2.007  4.158  2.621  2.16  2.16
## eruptions    4.500  2.2670  4.650  1.833  1.883  3.733  1.983  1.80  1.80
## CV residual  0.111 -0.0471 -0.353  0.211 -0.124 -0.425 -0.638 -0.36 -0.36
##               160    162    163    172     176    180    183    187    188
## waiting     89.00 86.000 58.000 57.000 81.0000 74.000 83.000 84.000 46.000
## cvpred       4.93  4.696  2.545  2.468  4.3119  3.774  4.466  4.542  1.622
## eruptions    3.97  4.150  2.000  2.083  4.3330  4.167  4.250  4.083  1.833
## CV residual -0.96 -0.546 -0.545 -0.385  0.0211  0.393 -0.216 -0.459  0.211
##                 189    192    194   197    198    200     202    203
## waiting     83.0000 57.000 84.000 87.00 77.000 78.000 82.0000 91.000
## cvpred       4.4656  2.468  4.542  4.77  4.005  4.081  4.3887  5.080
## eruptions    4.4170  1.833  4.100  3.50  4.366  4.667  4.3500  4.133
## CV residual -0.0486 -0.635 -0.442 -1.27  0.361  0.586 -0.0387 -0.947
##                205     210     225      227   229    238     244     245
## waiting     78.000 83.0000 78.0000 78.00000 70.00 77.000 63.0000 85.0000
## cvpred       4.081  4.4656  4.0814  4.08137  3.47  4.005  2.9288  4.6193
## eruptions    4.600  4.5000  4.0000  4.08300  3.92  4.283  2.9000  4.5830
## CV residual  0.519  0.0344 -0.0814  0.00163  0.45  0.278 -0.0288 -0.0363
##                246    247   255    256    267    269
```

```
## waiting     82.000 57.000 88.00 80.000 75.000 46.000
## cvpred       4.389  2.468  4.85  4.235  3.851  1.622
## eruptions    3.833  2.083  4.15  3.817  4.750  2.150
## CV residual -0.556 -0.385 -0.70 -0.418  0.899  0.528
##
## Sum of squares = 22.8    Mean square = 0.25    n = 91
##
## Overall (Sum over all 91 folds)
##    ms
## 0.253
```

Assignment-6

1. $\hat{\theta} = \bar{x}$ (sample mean)

Jacknife estimate is similar to the bootstrap in that it involves resampling, but instead of sampling with replacement, the method samples without replacement.

Definition:- The i^{th} jacknife replication of $\hat{\theta}_{(i)}$ of the statistic $\hat{\theta} = s(x)$ is $\hat{\theta}_{(i)} = s(x_i)$ $\forall i = 1, \ldots n$

Jacknife estimation of mean

$$s(x_{(i)}) = \frac{1}{n-1} \sum_{j \neq i} x_j$$

$$= \frac{n\bar{x} - x_i}{n-1}$$

$$= \bar{x}_i$$

Now,

$$\bar{x}_{(i)} = \frac{1}{n} \sum_{i=1}^{n} \bar{x}_i = \bar{x}$$

Thus, Jacknife estimate of sample mean is same as $\bar{x}$.

Jacknife estimate of the standard error of the mean

For $\hat{\theta} = \bar{x}$, it is easy to show that

$$x_i = \frac{1}{n-1} \sum_{j \neq i} x_j$$

$$= \frac{n\bar{x} - x_i}{n-1} = \bar{x}_i \ \&$$

$$\bar{x}_{(.)} = \frac{1}{n} \sum_{i=1}^{n} \bar{x}_i = \bar{x}$$

$$\therefore \; se_{jack}(\bar{x}) = \left(\sum_{i=1}^{n} \frac{x_i - \bar{x}^2}{(n-1)n} \right)^{1/2}$$

$$= \frac{\bar{\sigma}}{\sqrt{(n)}}$$

$$= \frac{s}{\sqrt{(n)}}$$

(2) To Prove: $SE(PV's) = \frac{(n-1)}{\sqrt{n}} \cdot SD(y_{(-i)})$

where $y_{(-i)} = \text{mean}(x \text{ without } x_i) = \sum_{k \neq i} x_k / 19$

$PV's = n \cdot \text{mean}(y) - (n-1) y_{(-i)}$

so,

$var(PV's) = var(n.\, mean(y)) - (n-1)\, y_{(-i)})$

$var(PV's) = var(n.\, mean(y)) + var((n-1)\, y_{(-i)})$

$var(PV's) = n^2\, var(mean(y)) + (n-1)^2\, var(y_{(-i)})$

$Sd(PV's) = n * sd(mean(y)) + (n-1) * sd(y_{(-i)})$

(Taking square root on both sides)

$SE(PV's) = \frac{sd(mean(y))}{\sqrt{n}} + \frac{(n-1)\, sd\, y_{(-i)})}{\sqrt{n}}$

(i.e dividing by n/2 on both sides)

Now, $sd \frac{(mean(y))}{\sqrt{n}} = 0$ as mean(y) is true population ~~mean and~~

mean and it wont have any variance.

$$SE(PV's) = \frac{n-1}{\sqrt{n}} sd(y_{(-i)})$$

Alternate

Since $y_{-i} = \sum_{k \neq x_i} \frac{x_k}{n-1}$

and $\bar{Y}_{-i} = \frac{1}{n} \sum_{i=1}^{n} \sum_{k \neq x_i} \frac{x_k}{n-1} = \bar{x}$

$$SD(Y_{-i}) = \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} (Y_i - \bar{x})^2}$$

$$= \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \left(\sum_{k \neq x_i} \frac{x_k}{n-1} - \bar{x} \right)^2}$$

$$= \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \left(\sum_{k \neq x_i} \frac{x_k}{n-1} + \frac{x_i}{n-1} - \bar{x} - \frac{x_i}{n-1} \right)^2}$$

$$= \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \left(\frac{n\bar{x}}{n-1} - \bar{x} - \frac{x_i}{n-1} \right)^2}$$

$$= \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \left(\frac{\bar{x}}{n-1} - \frac{x_i}{n-1} \right)^2}$$

$$= \left[\frac{1}{(n-1).\sqrt{n}} \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right] = \frac{1}{n-1} SD(PV$$

So $SDC(PV) = (n-1) SD(y_{-i})$

Thus $\frac{SD(PV)}{\sqrt{n}} = \frac{(n-1)}{\sqrt{n}} SD(y_{-i}) = SE(PV$