Homework Assignment-7 (S- 520)

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1. $H_0 : \mu \geq 800$ vs $\mu < 800$

We assume that the company would not want to waste unnecessary funds, and that they are going to counter advertise the alleged exaggerated claims of the light bulb manufacturer only if there is sufficient evidence to do so. In short, they want to minimize Type I error, which in this case would be that the bulbs actually work as advertised but that the test result support the opposite statement.

Let us assume $\alpha = 0.05$,
Then $p = P(T_n < t)$, where

$$t = \frac{\ddot{X} - \mu}{s / \sqrt{n}} = \frac{745.1 - 800}{23 / \sqrt{100}}$$

$$= \frac{-54.9}{23.8} = -2.3067$$

Thus, $P(T_n < -2.3067) =$ pnorm(-2.3067)
= 0.0105

Decision rule states that

$p = 0.0105 \leq 0.05 = \alpha \Longrightarrow$ reject H_0

2. In this case it is true that $1 - \alpha = 0.99 \Longrightarrow \alpha = 0.01$

Hence q= qnorm (1 – (0.01/2)) = 2.5758

$n = (2q\sigma/L)^2$

If L= 2 , $\sigma = 6$ and q= 2.5758 then

$n = ((2 * 2.5758 * 6)/ 2)^2$

n= 238.85

SAHC should take n= 239 (approximately)

3. We know that
$n = (2q\sigma/L)^2$

From the errata, we know that L = 0.001. q can be obtained as follows: $1 - \alpha = 0.95 \Longrightarrow \alpha = 0.05$

Thus, q_{1}- $\alpha/2$ = $q_1 - 0.05/2$ = $q_1 - 0.025$ = qnorm (0.975)
= 1.9600

At this point we are only missing σ. We need to estimate this quantity and we can model the situation like this: $X_1, X_2, ..., X_n \sim$ Bernoulli(p), where each $X_i = 1$ if the observed value lies between 170.5 and 199.5 and Xi = 0 otherwise. By properties of a Bernoulli trial, $EX_i = p$ and $Var\ X_i = p(1 - p)$.
The parameter p can be approximated using the CLT

$\mu = EV_i = 0.4 + 0.2 + 1 + 3 = 4.6$
Where V is a result of taking number out of urn,

$\sigma^2 = Var\ V_i = E\ V_i^2 - (EV_i)^2$
$= 0.4 + 0.4 + 5 + 30 - (4.6)^2$
$= 35.8 - 21.16$
$= 14.64$

From CLT

$Y \approx Normal(n\mu, n\sigma^2) = = Normal(184, \sqrt{585.6})$

$P(170.5 < Y < 199.5) = P(Y \leq 199.5) - P(Y \leq 170.6)$
$= pnorm(199.5, 184, \sqrt{585.6}) - pnorm(170.6, 184, \sqrt{585.6})$
$= 0.4506 = p$
Let's calculate Variance $X_i = p(1 - p) = 0.4506(1 - 0.4506)$
$= 0.24756$
$\sigma = \sqrt{0.24756} = 0.49755$

Finally,

$n = (2q\sigma/L)^2$

$= (2 * 1.96 * 0.49755/0.001)^2$

n = 3,804,044

4.
a) The probability of being right for each symbol if he does not have psychic power is p=0.2

Null Hypothesis: The true proportion of being right for each symbol if he does not have psychic powers is equal to 0.2

H_0: p= 0.2

Alternative Hypothesis: The true proportion of being right for each symbol if he does not psychic powers is not equal to 0.2

H_0: $p \neq 0.2$

b) Let X is the number of correctly identifying symbols. It follows binomial distribution

$P(X > 25) = 1 - P(X \leq 24)$

$= 1 - \text{pbinom}(24, 100, 0.2)$

$=$ **0.1313532**

c) No, result above showed us that a normal person with no psychic power can also guess 25 symbols co rrectly out 100 with 13% chance and with probability of identifying each symbol as 0.2.

5.

a) $\alpha = 1 - 0.95 = 0.05$
   $z = q1 - \alpha/2 = q1 - 0.05/2 = q1 - 0.025$
   $z = \text{qnorm}(0.975) = 1.96$

   Lower bound is

   $p - z\sqrt{p(i-p)/n}$

   $= 0.55 - 1.96 * \sqrt{0.55 * \frac{1-0.55}{1028}}$

   = **0.5195**

Upper bound is

   $= p + z\sqrt{p(i-p)/n}$

   $= 0.55 + 1.96 * \sqrt{0.55 * \frac{1-0.55}{1028}}$

   = **0.5804**

52 to 58% U.S adults support same sex marriage.

b) The discrepancy is due to different margin of error that we are using.