

Assignment 12 (S-520)

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Solution 1

Successful Flight Maneuver should depend on skill and chance, so for a particular candidate, performance on next flight maneuver will be less than his previous performance because we can't expect chance factor (random) to be equally good on his next maneuver even though skills are the same. This can be attributed to Regression to Mean.

Solution 2

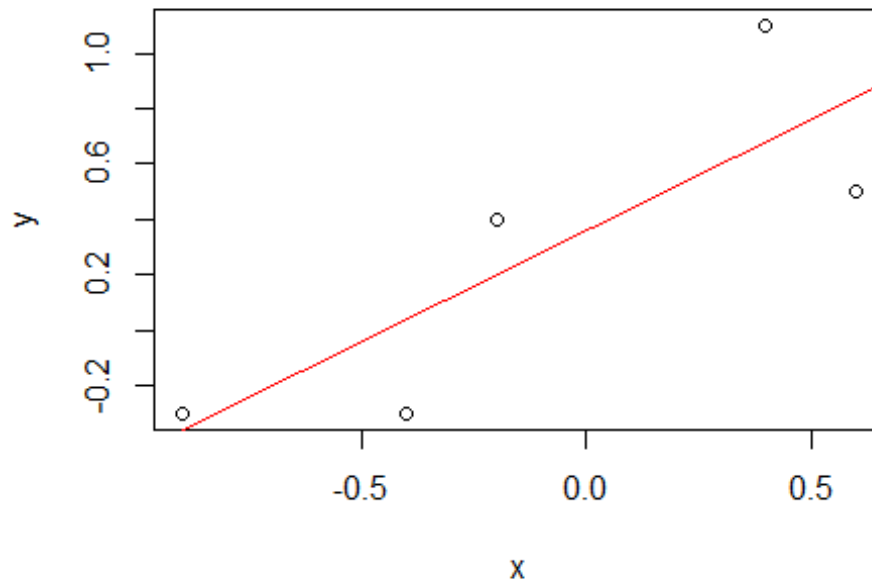
```
x<- c(-0.2,-0.9,-0.4,0.6,0.4)
y<- c(0.4,-0.3,-0.3,0.5,1.1)
# a) Pearson's Correlation Coefficient is given by
print("Pearson's Correlation Coefficient is =")

## [1] "Pearson's Correlation Coefficient is ="

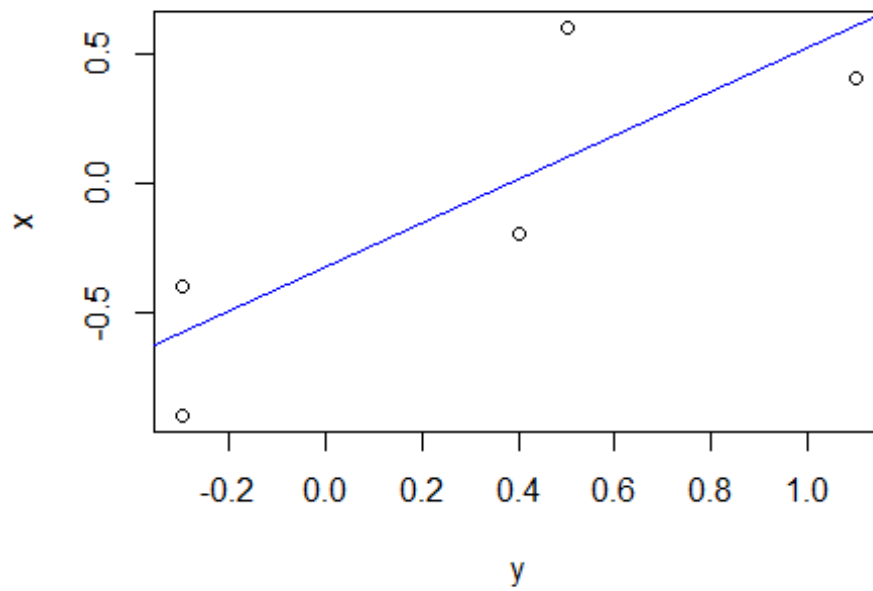
r=cor(x,y)
r

## [1] 0.8243559

# b)
plot(x,y)
# Predict y from x
slope = r *(sd(y) / sd(x))
intercept = mean(y) - (slope * mean(x))
predictions = intercept + slope*x
SSE = sum((y - predictions)^2)
abline(intercept, slope, col="red")
```



```
# Equation of line y=intercept + slope*x
# In this case, Equation is y=0.36 + 0.80x
# c)
# Predict x from y
plot(y,x)
slope2 = r *(sd(x) / sd(y))
intercept2 = mean(x) - (slope * mean(y))
predictions2 = intercept + slope*y
SSE2 = sum((x - predictions)^2)
abline(intercept2, slope2, col="blue")
```



```
# Equation of line here is  $x = \text{intercept1} + \text{slope2} * y$ 
```

```
#  $x = -0.32 + 0.84y$ 
```

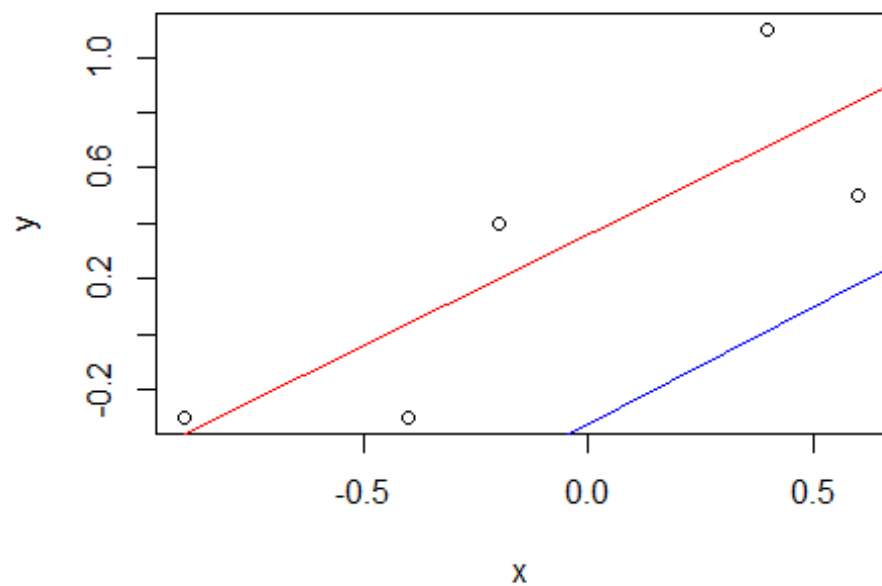
```
# d) Scatter Plot
```

```
source('C:/Stats/Assignment 11/binorm.R')
```

```
plot(x,y)
```

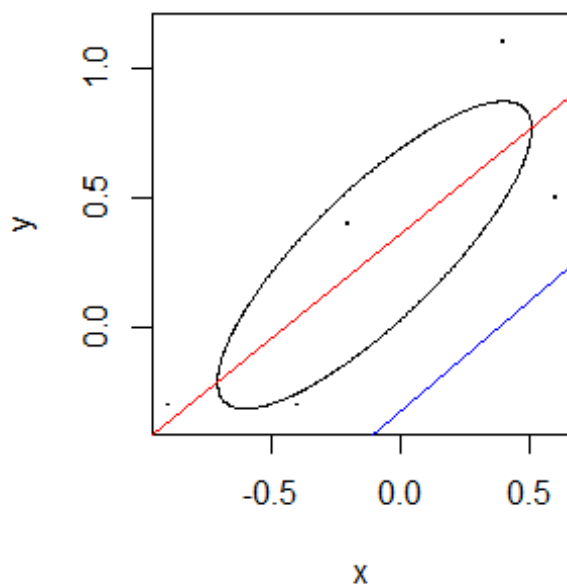
```
abline(intercept, slope, col="red")
```

```
abline(intercept2, slope2, col="blue")
```



```
binorm.scatter(cbind(x, y))
abline(intercept, slope, col="red")
abline(intercept2, slope2, col="blue")
```

Scatter Diagram



Solution 3

```
sister<- c(69,64,65,63,65,62,65,64,66,59,62)
brother<- c(71,68,66,67,70,71,70,73,72,65,66)
all<- c(sister,brother)
mb=mean(brother)
ms=mean(sister)
mall=mean(all)
n=length(sister)
sx=sum((sister-ms)^2)/(n-1)
sy=sum((brother-mb)^2)/(n-1)
r=cor(sister,brother)
r2=r^2
# a) The sample coefficient of determination, the proportion of
# variables "explained" by simple linear regression is square of
# correlation coefficient
r2

## [1] 0.3114251

SST = sy*(n-1)
SSR = r2 *SST
SSE=(1-r2)*SST
df1=1
df2=9
msr=r2*SST
mse=SSE/df2
f=(n-2)*r2/(1-r2)
1-pf(f,df1,df2)

## [1] 0.07441681

# b) Ho:beta>0
# Since p-value>0.05, we fail to reject Null Hypothesis hence we
# cannot say that knowing sister's height helps one predict her
# brother's height.
# c)
syy=sqrt(sy)
sxx=sqrt(sx)
beta1=r*(syy/sxx)
gt=qt(0.95,df2)
se=((1-r2)*sy)/((n-2)*sx)
print("Confidence Interval is=")

## [1] "Confidence Interval is="

lower=beta1- (gt*sqrt(se))
upper=beta1+ (gt*sqrt(se))
lower

## [1] 0.05401643
```

upper

```
## [1] 1.127802
```

d) Given $L=0.1$

$$L^2=0.1$$

$$L = \beta + q_t \sqrt{MSe/txx} - \beta + q_t \sqrt{MSe/txx} = 2 \sqrt{MSe/txx}$$

Which can be written as

$$\frac{(1-r^2) \cdot s_y^2}{(n-2) \cdot s_x^2} = \left(\frac{0.1}{2 \cdot q_t} \right)^2$$

$$n-2 = \frac{4 \cdot q_t^2 \cdot (1-r^2) \cdot s_y^2}{(0.1)^2 \cdot s_x^2}$$

For 95% Confidence Interval

$$q_t = qt(0.975, df=9) = 2.262157$$

$$n-2 = \frac{4 \cdot (2.262157)^2 \cdot (1 - 0.311425) \cdot 7.4}{(0.1)^2 \cdot 6.6}$$
$$= 1580$$

We should plan to observe close to 1580 pair of sister-brother.

Solution 4

a) $b = \text{cor}(\text{test2}, \text{test1}) \cdot \text{sd}(\text{test1})$
 $a = \text{mean}(\text{test2}) - b \cdot \text{mean}(\text{test1})$

test2= a+ b*test1

So,

test2= 19 + 0.6*80 = 67

Jill should get 67 in Test2 if she scored 80 in first and missed the second one.

b) $b = \text{cor}(\text{test2}, \text{test1}) * \text{sd}(\text{test1}) / \text{sd}(\text{test2})$

$a = \text{mean}(\text{test1}) - b * \text{mean}(\text{test2})$

test1= a+btest2

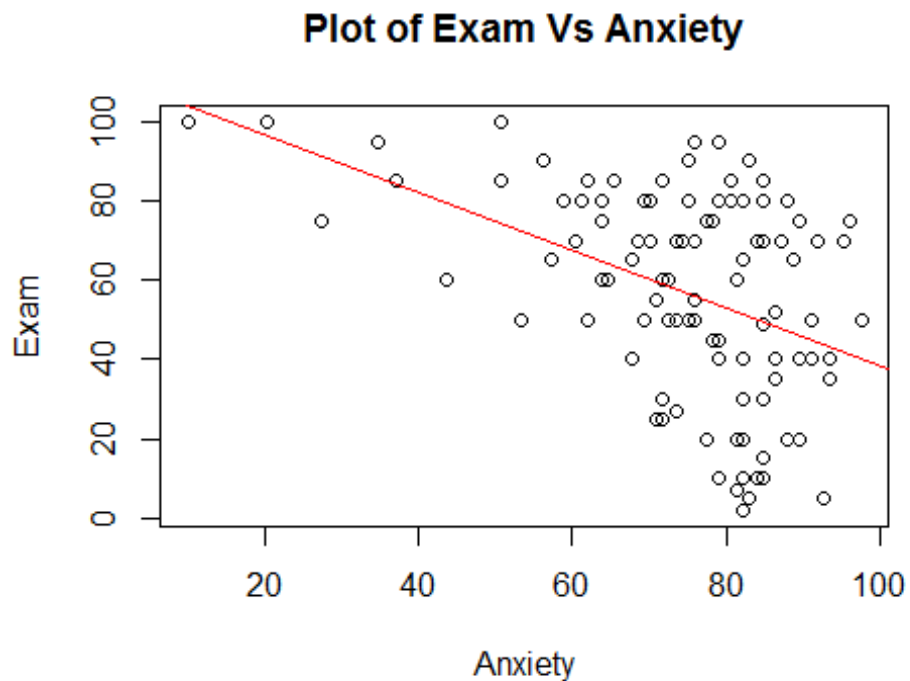
So,

test1= 48.76 + 0.41*76=79.92

Jack should get close to 80 in test 1 if he scored 76 in test 2 and missed the first one.

Solution 5

```
ext=read.table('C:/Stats/Assignment 12/examanxiety.txt',header = TRUE)
exam<-ext$Exam
anxiety<-ext$Anxiety
b = cor(exam, anxiety) * sd(exam) / sd(anxiety)
a = mean(exam) - b * mean(anxiety)
plot(anxiety, exam,main="Plot of Exam Vs Anxiety",xlab="Anxiety",ylab="Exam")
abline(a, b, col="red")
```



```
# Equation of line is  $y=a+bx$ 
# Exam=111-0.73* Anxiety
```

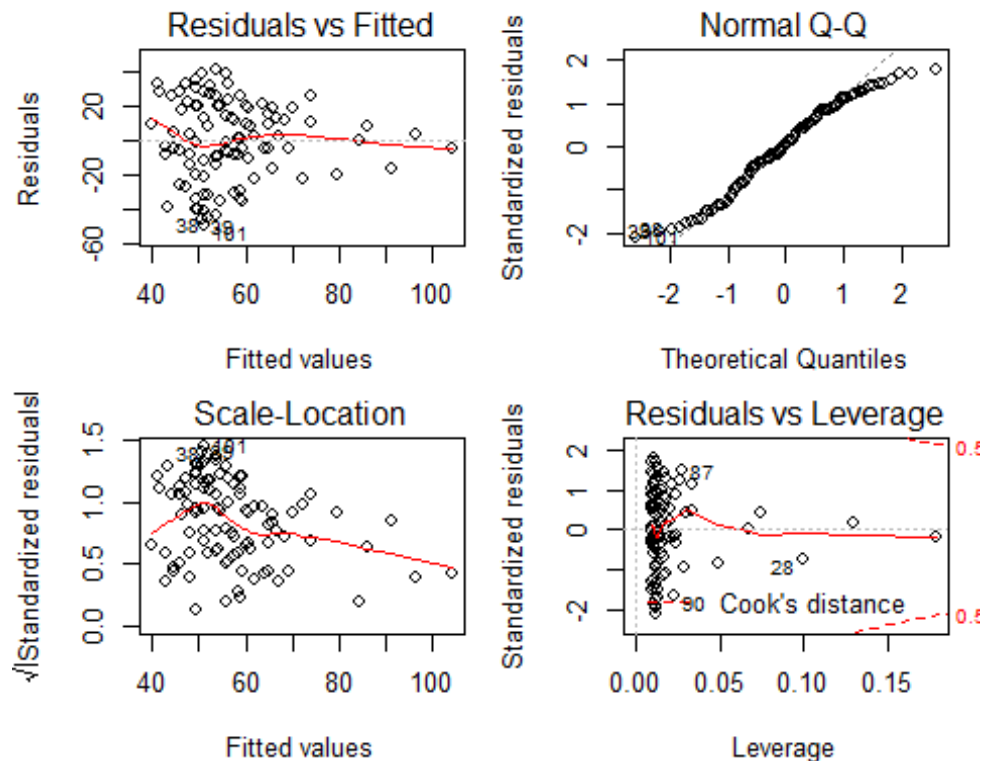
```
# b)
```

```
fit<-lm(Exam~Anxiety,data = ext)
```

```
par(mfrow=c(2,2),mar=c(4,4,2,1))
```

```
# Plot to check for Homoskedasticity, Normality of errors, Independence  
# of errors
```

```
plot(fit)
```



```
# From Plot, we conclude that out of all assumptions only assumption  
# about normality of error seems true.
```

```
# c)
```

```
# Suppose X and Y are bivariate normal. Then given a specific value  
# of X, Y follows normal distribution.
```

```
# cor(anxiety,exam)=-0.439 correlation is negative hence we should  
# not use bivariate normal and instead fit an ellipse in scatter plot  
# ellipse won't fit density but will be cross like shape.
```

Solution 6

```
fit1<-lm(Exam~Anxiety+Revise+Gender,data=ext)
```

```
fit2<-lm(Exam~Anxiety+Revise,data=ext)
```

```
summary(fit1)
```



```
##
## Call:
## lm(formula = Exam ~ Anxiety + Revise + Gender, data = ext)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -45.834 -15.029  -1.121   21.845   40.243
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   89.7932    17.8090   5.042 2.08e-06 ***
## Anxiety       -0.5202     0.1973  -2.636 0.00974 **
## Revise         0.2746     0.1703   1.612 0.11009
## GenderMale     0.5587     4.6497   0.120 0.90460
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23.34 on 99 degrees of freedom
## Multiple R-squared:  0.214, Adjusted R-squared:  0.1902
## F-statistic: 8.987 on 3 and 99 DF, p-value: 2.544e-05
```

`summary(fit2)`

```
##
## Call:
## lm(formula = Exam ~ Anxiety + Revise, data = ext)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -45.591 -14.763  -1.397   21.831   40.505
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   90.3422    17.1279   5.275 7.72e-07 ***
## Anxiety       -0.5230     0.1950  -2.682 0.00857 **
## Revise         0.2717     0.1678   1.619 0.10849
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23.23 on 100 degrees of freedom
## Multiple R-squared:  0.2139, Adjusted R-squared:  0.1982
## F-statistic: 13.61 on 2 and 100 DF, p-value: 5.931e-06
```

From above we can say that model should be Exam~ Anxiety+Revise since exam scores depend on Revision and Anxiety before the exam, Exam score has nothing to do with gender of person. R^2 value decreases when Gender is included and Residual Standard error increases hence model should be Exam~Anxiety+Revise.