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Tutorial-1

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Clique 1 — Clique 2

(FW, SW) —— (SW, TW)

Clique tree

(i)

$$\delta_{i \to j} = \sum_{C_i - S_{i,j}} \psi_i \cdot \prod_{k \in (Nb_i - \{j\})} \delta_{k \to i}$$

$$\delta_{1 \to 2}(S_{12}) = \text{SP-Message}(1,2)$$

SP-Message :

$$\psi(C_1) = \psi_1 \cdot \prod_{k \in Nb_1 - \{2\}} \delta_{k \to 1}$$

$$\tau(S_{1,2}) = \sum_{C_1 - S_{12}} \psi(C_1)$$

$\psi(C_1) =$

ψ (SW \ FW)	N	V	O
N	1	2	3
V	10	1	3
O	3	5	2

$$\delta_{1 \to 2}(S_{1,2}) = \sum_{FW} \psi(C_1)$$

$$= \begin{array}{cc} & \\ N & \\ V & \\ O & \end{array} \begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

SW

SW

$$\beta_2 = \text{TW} \begin{array}{c} \\ N \\ V \\ O \end{array} \begin{array}{c} \begin{array}{ccc} N & V & O \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 10 & 1 & 3 \\ 3 & 5 & 2 \end{bmatrix} \\ \text{SW} \end{array} \times \ \text{SW} \begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

$$= \text{TW} \begin{bmatrix} 6 & 28 & 30 \\ 60 & 14 & 30 \\ 18 & 70 & 20 \end{bmatrix}$$

(iii)

$$\delta_{2\to 1}(s_{2,1}) = \sum_{TW} \ \text{TW} \begin{array}{c|c|c|c} & N & V & O \\ \hline N & 1 & 2 & 3 \\ \hline V & 10 & 1 & 3 \\ \hline O & 3 & 5 & 2 \end{array} \quad \text{SW}$$

$$= \text{SW} \begin{array}{c} \begin{array}{ccc} N & V & O \end{array} \\ \begin{bmatrix} 14 & 8 & 8 \end{bmatrix} \end{array}$$

$$\beta_1 = \text{SW} \begin{array}{c|c|c|c} & N & V & O \\ \hline N & 1 & 2 & 3 \\ \hline V & 10 & 1 & 3 \\ \hline O & 3 & 5 & 2 \end{array}^{\ \text{FW}} \times \ \text{SW} \begin{array}{c|c|c} N & V & O \\ \hline 14 & 8 & 8 \end{array}$$

$$\beta_1 = \text{SW} \begin{array}{c|c|c|c} & N & V & O \\ \hline N & 14 & 28 & 42 \\ \hline V & 80 & 8 & 24 \\ \hline O & 24 & 40 & 16 \end{array}^{\ \text{FW}}$$

(iii) For adjacent cliques to be callibrated.

$$\mu_{12} = \mu_{21}$$

ie $\sum_{TW} \beta_2(C_2) = \sum_{FW} \beta_1(C_1)$

$\sum_{TW} \beta_2(C_2) =$

N	V	O
84	112	80

SW

$\sum_{FW} \beta_1(C_1) =$

SW	
N	84
V	112
O	80

$\therefore$ resulting beliefs are callibrated

2

a.) $\sum_{SW} \beta_2$

$=$

N	64
V	104
O	108

b.) Partition function $= Z = 64 + 104 + 108$
$= 276$

Normalized marginal distribution over TW $=$

$$\begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 64/276 \\ 104/276 \\ 108/276 \end{bmatrix} = \begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 0.2318 \\ 0.3768 \\ 0.3913 \end{bmatrix}$$

3

a:)

$$\tilde{P}_\Phi(X) = I(Z=z) \cdot \frac{\prod_{i\in V_T}(\beta_i(C_i))}{\prod_{(i-j)\in \epsilon_T} \mu_{ij}(S_{i,j})}$$

$$= \frac{\beta_1(FW=N, SW)\ \beta_2}{\mu_{12}}$$

$$= \frac{\begin{array}{cc} & SW \\ \begin{array}{c} N \\ V \\ O \end{array} & \begin{bmatrix} 14 \\ 80 \\ 24 \end{bmatrix} \end{array} \quad \begin{array}{cc} & SW \\ TW & \begin{bmatrix} 6 & 28 & 30 \\ 60 & 14 & 30 \\ 18 & 70 & 20 \end{bmatrix} \end{array}}{\begin{array}{c} SW \\ \begin{bmatrix} 84 & 112 & 80 \end{bmatrix} \\ \begin{array}{ccc} N & V & O \end{array} \end{array}}$$

$$= \frac{\begin{array}{cc} & SW \\ TW & \begin{bmatrix} 6\times 14 & 28\times 80 & 30\times 24 \\ 60\times 14 & 14\times 80 & 30\times 24 \\ 18\times 14 & 70\times 80 & 20\times 24 \end{bmatrix} \end{array}}{\begin{array}{c} SW \\ \begin{bmatrix} 84 & 112 & 80 \end{bmatrix} \end{array}}$$

$$= \begin{array}{cc} & SW \\ TW & \begin{bmatrix} \frac{6\times 14}{84} & \frac{28\times 80}{112} & \frac{30\times 24}{80} \\ \frac{60\times 14}{84} & \frac{14\times 80}{112} & \frac{30\times 24}{80} \\ \frac{18\times 14}{84} & \frac{70\times 80}{112} & \frac{20\times 24}{80} \end{bmatrix} \end{array} \quad \ldots\ldots (1)$$

for marginal over TW, $\sum_{SW}$

$$TW \quad \begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 1 + 20 + 9 \\ 10 + 10 \quad 9 \\ 3 + 50 \quad 6 \end{bmatrix} = \begin{bmatrix} 30 \\ 29 \\ 59 \end{bmatrix}$$

$$Z = 118$$

$\therefore$ normalized version is

$$\begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 30/118 \\ 29/118 \\ 59/118 \end{bmatrix} \Rightarrow \begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 0.254 \\ 0.245 \\ 0.5 \end{bmatrix}$$

b.

from ①

$$TW \quad \begin{array}{c} N \\ V \\ O \end{array} \begin{bmatrix} 20 \\ 10 \\ 50 \end{bmatrix} \Rightarrow \overset{\text{after Normalizing}}{\begin{bmatrix} 20/80 \\ 10/80 \\ 50/80 \end{bmatrix}} = \begin{bmatrix} 0.25 \\ 0.125 \\ 0.625 \end{bmatrix}$$

$$P(TW = O \mid SW = V, FW = N) = \frac{50}{50 + 10 + 20} = 0.625$$

<u>alternatively</u>

from factor table

$$\psi(FW = N, SW = V) = 10$$

since we only want marginal of
$P(TW = \text{Other} \mid SW = V, FW = N)$

we consider only 3rd row, 2nd column
(TW = O) , (SW = V)

and normalize it

i.e $$\frac{5 \times 10}{2 \times 10 + 1 \times 10 + 5 \times 10}$$

$$= \frac{50}{80} = 0.625$$

i.e same answer as before.