Magnetic Circuits

Module:3 Magnetic Circuits

Magnetic field; Toroidal core: Flux density, Flux linkage; Magnetic circuit with airgap; Reluctance in series and parallel circuits; Self and mutual inductance; Transformer: turn ratio determination

3.10 Magnetic Effect of an Electric Current (Electromagnets)

When a coil or a conductor carries a current, it produces the magnetic flux around it. Then it starts behaving as a magnet. Such a current carrying coil or conductor is called an **electromagnet.** This is due to magnetic effect of an electric current.

If such a coil is wound around a piece of magnetic material like iron or steel and carries current then piece of material around which the coil is wound, starts behaving as a magnet, which is called an electromagnet.

The flux produced and the flux density can be controlled by controlling the magnitude the current.

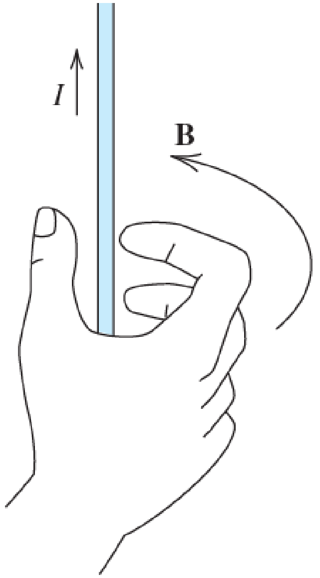
The direction and shape of the magnetic field around the coil or conductor depends on the direction of current and shape of the conductor through which it is passing. The magnetic field produced can be experienced with the help of iron fillings or compass needle.

Let us study two different types of electromagnets,

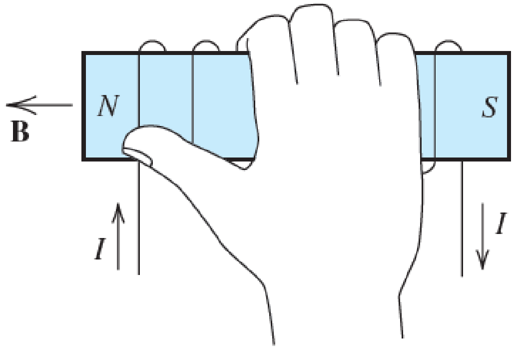
1) Electromagnet due to straight current carrying conductor
2) Electromagnet due to circular current carrying coil

Right-Hand Rule

The direction of the magnetic field produced by a current can be determined by the right-hand rule. There are several interpretations of this rule.



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field



(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

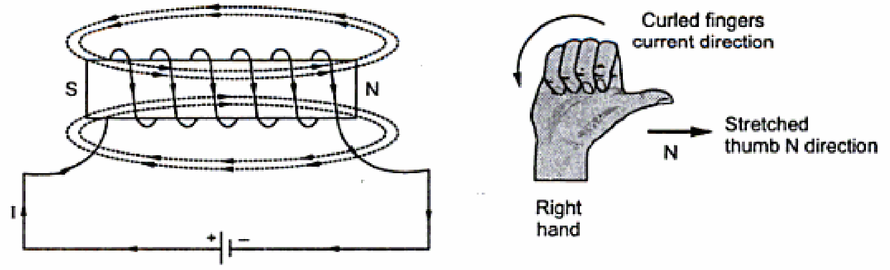


Fig. 3.19 (a) Direction of flux around a solenoid

This is shown in Fig. 3.19 (a) and (b).

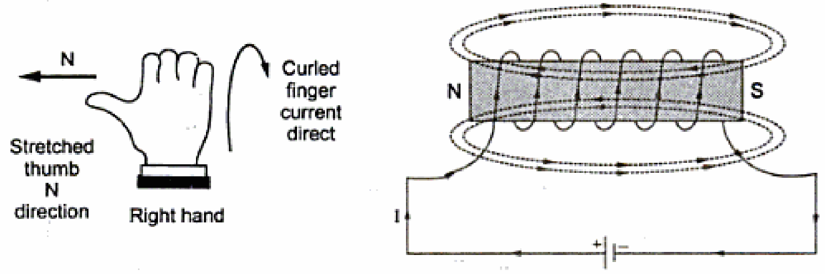
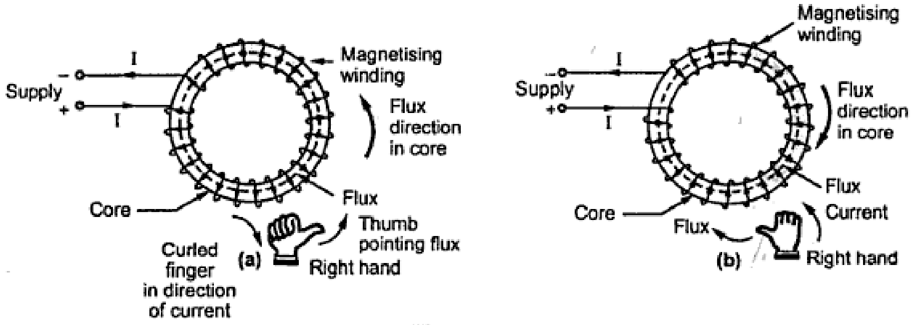
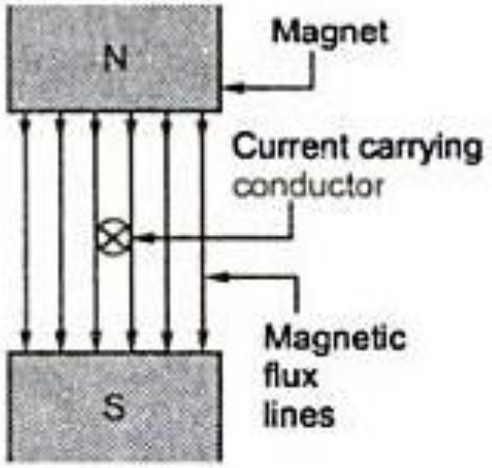


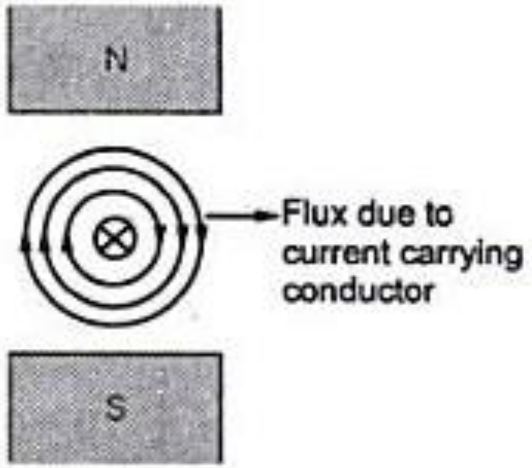
Fig. 3.19 (b) Direction of flux around a solenoid



Force on a Current carrying Conductor



(a) Flux due to magnet



(b) Flux due to current carrying conductor

There is a presence of two magnetic fields namely, i) due to permanent magnet and ii) due to current carrying conductor.
These two fluxes interact with each other. The flux lines gathered will exert a force on the conductor. So, conductor experiences a mechanical force from high flux lines area towards low flux lines area.

Forces on Current-Carrying Wires

Current flowing in a conductor consists of charge (usually, electrons) in motion. Thus, forces appear on a current-carrying wire immersed in a magnetic field. The force on an incremental length of the wire is given by

df = i dl * B

in which the direction of dl and the reference direction for the current are the same. For a straight wire of length l and a constant magnetic field, we have

f = ilB sin(Θ)

in which Θ is the angle between the wire and the field. Notice that the force is maximized if the direction of the field is perpendicular to the wire.

Flux Linkage

Magnetic flux passing through a surface area A is given by,

$$\phi = BA$$

The flux passing through the surface bounded by a coil **links** the coil. If the coil has *N* turns, then the total flux linkages are given by

$$\lambda = N\phi$$

According to **Faraday's law of magnetic induction**, a voltage is induced in a coil whenever its flux linkages are changing. This can occur either because the magnetic field is changing with time or because the coil is moving relative to a magnetic field

$$e = \frac{d\lambda}{dt}$$

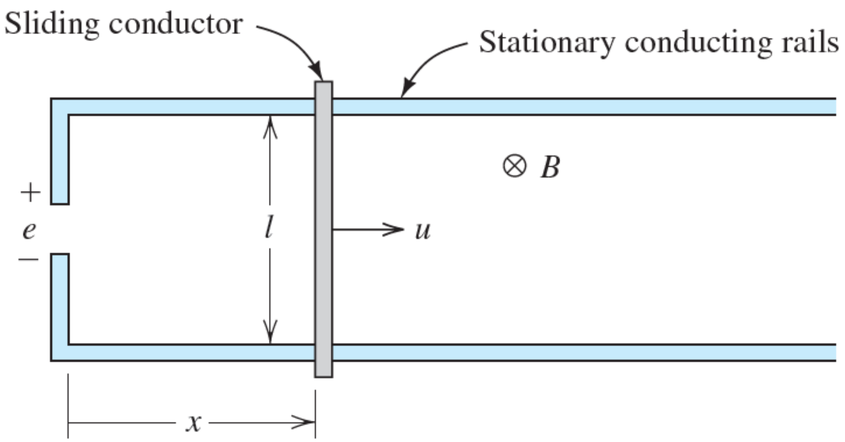
Voltages Induced in Field-Cutting Conductors

Voltage is also induced in a conductor moving through a magnetic field in a direction such that the conductor cuts through magnetic lines of flux. For example, consider Figure. A uniform magnetic field is directed into the page. The sliding conductor and the stationary rails form a loop having an area of $A = lx$. The flux linkages of the coil are

$$\lambda = BA = Blx$$

$$e = \frac{d\lambda}{dt} = Bl\frac{dx}{dt}$$

$$e = Blu$$



Magnetic Field Intensity

In general, magnetic fields are set up by charges in motion. In most of the applications that we consider, the magnetic fields are established by currents flowing in coils.

We will see that **H** is determined by the currents and the configuration of the coils.
The resulting flux density **B** depends on **H**, as well as the properties of the material filling the space around the coils.

The magnetic field intensity **H** and magnetic flux density **B** are related by

$\mathbf{B} = \mu\mathbf{H}$

in which μ is the magnetic permeability of the material.
The units of **H** are amperes/meter (A/m),
units of μ are webers/ampere-meter (Wb/Am).
For free space, we have

$\mu = \mu_0 = 4\prod * 10^{-7}$ Wb/Am

Some materials, most notably iron and certain rare-earth alloys, have a much higher magnetic permeability than free space. The relative permeability of a material is the ratio of its permeability to that of free space

Magnetic Flux Density

Consider the toroidal coil shown in Figure. Find an expression for the magnetic flux density *B* on the center line of the core in terms of the number of coil turns *N*, the current *I*, the permeability μ of the core, and the physical dimensions. Then, assuming that the flux density is constant throughout the core (this is approximately true if *R* >> *r*), find expressions for the total flux and the flux linkages.

- The magnetic field intensity produced in the core is ***H*** and from ampere circuital law,

$$\oint H.dl \;=\; N\,I$$

$$H.l = N\,I$$

$$H = \frac{N\,I}{l}\;\; {}^{AT}\!/_{m} \;\; ------ -1$$

- Magnetic field intensity ***H*** causes a flux density ***B*** to be set up in the magnetic core. It is given by,

$$B = \;\mu\,H \;\; ------ -2$$

$$B = \;\mu_0\mu_r H$$

Magnetic Circuit

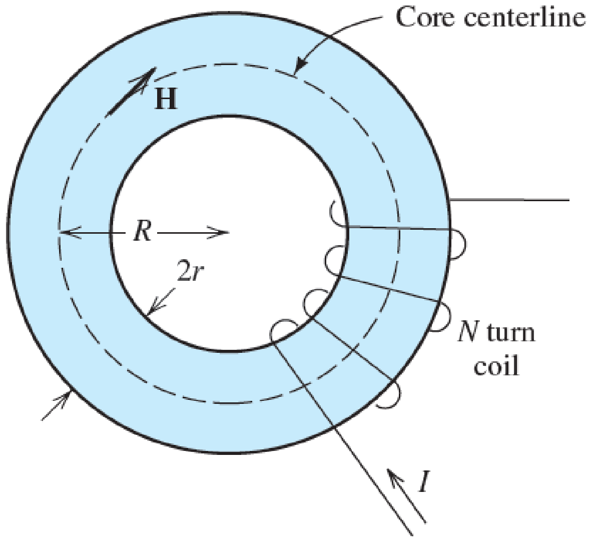
- Sub equation 1 in equation 2,

$$B = \mu \frac{N\,I}{l} \;-----3$$

- Flux flowing through the core is given by,

$$\phi = B\,.A \;-----4$$

- Where ***B*** is the average flux density and ***A*** is the area of cross section of the core.
- Substituting equation 3 in equation 4, we get,



$$\phi = BA = \frac{\mu NI}{2\pi R}\pi r^2 = \frac{\mu NI r^2}{2R}$$

Suppose that we have a toroidal core with $\mu r = 5000$, $R = 10$ cm, $r = 2$ cm, and $N = 100$. The current is
$i(t) = 2 \sin(200\prod t)$
Compute the flux and the flux linkages. Then, use Faraday's law of induction to determine the voltage induced in the coil.

Magnetic Circuit Concepts

The **magnetomotive force** (mmf) of an *N*-turn current-carrying coil is given by

$$\mathcal{F} = Ni$$

A current-carrying coil is the magnetic-circuit analog of a voltage source in an electrical circuit. Magnetomotive force is analogous to source voltage. The **reluctance** of a path for magnetic flux, is given by

$$\mathcal{R} = \frac{l}{\mu A}$$

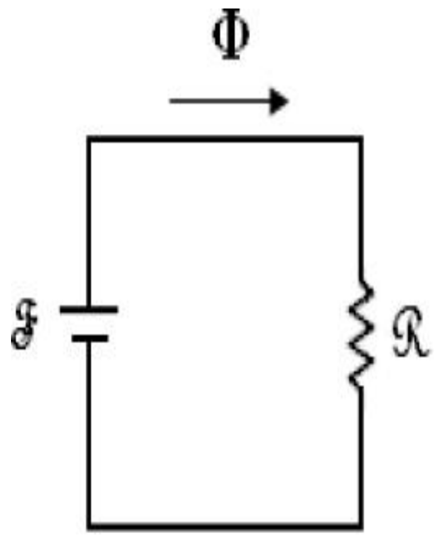
in which l is the length of the path (in the direction of the magnetic flux), A is the cross sectional area, and μ is the permeability of the material. Reluctance is analogous to resistance in an electrical circuit.

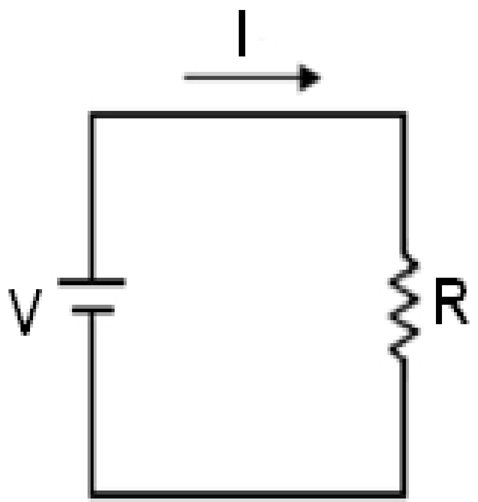
When the bar is not straight, the length of the path is somewhat ambiguous, and then we estimate its value as the length of the center line. Thus, l is sometimes called the **mean length** of the path.

Magnetic flux ϕ in a magnetic circuit is analogous to current in an electrical circuit. Magnetic flux, reluctance, and magnetomotive force are related by

$$\mathcal{F} = \mathcal{R}\phi$$

Magnetic Circuit and Electric Circuit



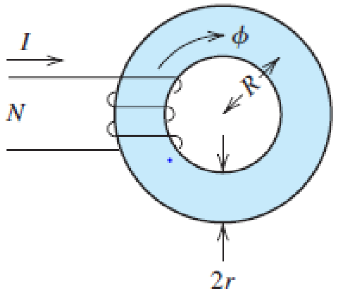


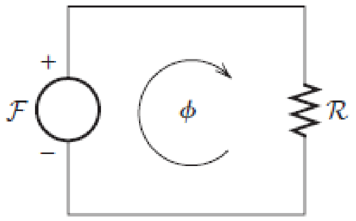
Comparison of Magnetic and Electric Circuits

Magnetic Circuit	Electric Circuit
Hopkinson's Law $\left(\phi = \frac{\mathcal{F}}{\mathcal{R}}\right)$	Ohm's Law $\left(I = \frac{V}{R}\right)$
Reluctance, $\mathcal{R} = \frac{\ell}{\mu . A}$	Resistance, $\mathrm{R} = \frac{\ell}{\sigma . A}$
Flux (ϕ)	Current (I)
MMF ($\mathcal{F}$)	EMF (V)
Permeability (μ)	Conductivity (σ)
Permeance ($\mathcal{P}$)	Conductance (G)

Toroidal coil as magnetic circuit

The magnetic circuit of the toroidal coil is analogous to a simple electrical circuit with a resistance connected across a voltage source.





(b) Magnetic circuit

The mean length of the magnetic path is

$$l = 2\pi R$$

The cross section of the core is circular with radius r. Thus, the area of the cross section is

$$A = \pi r^2$$

$$\mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2}$$

The magnetomotive force is

$$\mathcal{F} = NI$$

$$\phi = \frac{\mu N r^2 I}{2R}$$