Module - 3

Problems on processor performance:

1. If computer A runs a program in 10 seconds and computer B runs the same program in 15s, how much faster is A than B?

Soln:

$$\frac{\text{Execution time}_B}{\text{Execution time}_A} = \frac{15}{10} = 1.5$$

$\therefore$ A is 1.5 times faster than B.

2. If a computer has a clock period/clock cycle time of 5ns, what is the clock rate?

Soln:

$$\text{Clock rate} = \frac{1}{\text{Clock cycle time}}$$

$$= \frac{1}{5 \times 10^{-9}} = 0.2 \times 10^{9}$$

$$\text{clock rate} = 200\text{ MHz}.$$

3. Our favorite program runs in 10s on computer A, which has a 2 GHz clock. We are trying to help a comp. designer build a comp. B, which will run this prog. in 6s. The designer has determined that a substantial increase in the clock rate is possible, but this will cause comp. B to require 1.2 times as many clock cycles as comp. A. What clock rate should the designer target?

Soln :

$$\text{CPU time}_A = \frac{\text{CPU clock cycles}_A}{\text{Clock rate}_A}$$

$$10 = \frac{\text{CPU clock cycles}_A}{2 \times 10^9}$$

$$\therefore \text{CPU clock cycles}_A = 20 \times 10^9 \text{ cycles.}$$

Given: B requires $1.2 \times \text{CPU clock cycles}_A$
and $\text{CPU time}_B = 6s$.
To find: Clock rate_B.

$$\text{CPU time}_B = \frac{\text{CPU clock cycles}_B}{\text{Clock rate}_B}$$

$$6 = \frac{1.2 \times 20 \times 10^9}{\text{Clock rate}_B}$$

$$\text{Clock rate}_B = 4 \times 10^9 = 4 \text{ GHz.}$$

$\therefore$ B should have a clock rate of 4 GHz.

4. Suppose we have two implementations of the same instruction set architecture. Computer A has a clock cycle time of 250 ps and a CPI of 2 for some program, and computer B has a clock cycle time of 500 ps and a CPI of 1.2 for the same program. Which computer is faster and by how much?

Soln :

$$\text{CPU time}_A = \frac{\text{CPU clock cycle}_A}{\text{Clock rate}}$$

$$= \text{Instruction count (I)} \times \text{CPI}_A \times \text{Clock cycle time}_A$$

CamScanner

$CPU\ time_A = I \times 2 \times 250 \times 10^{-12} = 500\ I \times 10^{-12} s$

$CPU\ time_B = I \times CPI_B \times \text{Clock cycle time}_B$

$= I \times 1.2 \times 500 \times 10^{-12}$

$CPU\ time_B = 600\ I \times 10^{-12} s$

$\therefore$ Computer A is faster.

$$\frac{Performance_A}{Performance_B} = \frac{CPU\ time_B}{CPU\ time_A} = \frac{600\ I}{500\ I} = 1.2$$

$\therefore$ A is 1.2 times faster than B.

5. If the CPU clock rate is 1MHz and a program takes 45 million cycles to execute, what is the CPU time?

Soln.

$$CPU\ time = \frac{CPU\ clock\ cycles}{Clock\ rate}$$

$$= \frac{45,000,000}{1\,000\,000}$$

$CPU\ time = 45 s$

6. Assume that a benchmark has 100 instructions where 25 instructions are loads/stores (2 cycles)
50 instructions are related to addition (1 cycle)
25 instructions are related to square root (50 cycles).

What is the CPI for this benchmark?

Soln:

$$CPI = \sum_{i=1}^{n} CPI_i \times I_i$$

$$= (2 \times 0.25) + (1 \times 0.5) + (50 \times 0.25)$$

$$CPI = 13.5$$

7. The following table indicates the frequency of ~~all~~ occurrence of all instruction types executed in a typical program and the no. of cycles per instruction for each type. Calculate the CPI.

Instruction Type	Freq. of occurence (%)	Cycles
ALU instructions	50	4
Load instructions	30	5
Store instructions	5	4
Branch instructions	15	2

Soln:

$$CPI = \sum_{i=1}^{n} CPI_i \times I_i$$

$$= (4 \times 0.5) + (5 \times 0.3) + (4 \times 0.05) + (2 \times 0.15)$$

$$CPI = 4$$

8. A compiler designer is trying to decide between two code sequences for a particular computer. The hardware designers have supplied the following facts:

Instruction class	A	B	C
CPI	1	2	3

For a particular high-level language statement, the compiler writer is considering two code sequences that require the following instruction counts:

	Instⁿ counts for each class		
Code seq.	A	B	C
1	2	1	2
2	4	1	1

Which code sequence executes the most instructions? Which will be faster? What is the CPI for each sequence.

Soln:

Code seq. 1 executes $2+1+2=5$ instructions
Code seq. 2 executes $4+1+1=6$ instructions
$\therefore$ Code seq. 2 executes the most instructions

Instⁿ count

$$\text{CPU clock cycles}_1 = \sum_{i=1}^{n} (CPI_i \times C_i)$$

$$= (1\times 2) + (2\times 1) + (3\times 2)$$

$$\text{CPU clock cycles}_1 = 10 \text{ cycles}$$

$$\text{CPU clock cycles}_2 = (1\times 4) + (2\times 1) + (3\times 1)$$

$$= 9 \text{ cycles}$$

$\therefore$ Code sequence 2 is faster.

$$CPI = \frac{\text{CPU clock cycles}}{\text{Instruction count}}$$

$$CPI_1 = \frac{10}{5} = 2$$

$$CPI_2 = \frac{9}{6} = 1.5$$

9. Consider the following performance measurements for a program:

Measurement	Comp. A	Comp. B
Instruction count	10 billion	8 billion
Clock rate	4 GHz	4 GHz
CPI	1.0	1.1

Which computer has the higher MIPS rating and which computer is faster?

Soln:

$$MIPS_A = \frac{\text{clock rate A}}{CPI_A \times 10^6} = \frac{4 \times 10^9}{1 \times 10^6}$$

$$MIPS_A = 4000$$

$$MIPS_B = \frac{4 \times 10^9}{1.1 \times 10^6} = 3.63 \times 10^3$$

$$MIPS_B = 3636$$

Since $MIPS_A > MIPS_B$, computer A is faster.

10. You are on the design team for a new processor. The clock of the processor runs at 200 MHz. The following table gives instruction frequencies, as well as how many cycles the instructions take. Calculate the CPI and MIPS. (4)

Inst^n Type	Freq. of occurrence	Clock cycles
Load + store	30%	6
Arithmetic	50%	4
Others	20%	3

Soln:

$$CPI = \sum_{i=1}^{n} CPI_i \times I_i$$

$$= (6 \times 0.3) + (4 \times 0.5) + (3 \times 0.2)$$

$CPI = 4.4$ cycles per instruction.

$$MIPS = \frac{\text{Clock rate}}{CPI \times 10^6} = \frac{200 \times 10^6}{4.4 \times 10^6}$$

$$MIPS = 45.45$$

11. Calculate CPI and MIPS for a CPU with 200 MHz frequency which is executing a program with the following instruction mix

Instruction Category	Percentage of Occurrence	No. of cycles/ instruction
ALU	38	1
Load + Store	15	3
Branch	42	4
Others	5	5

Soln:

$$CPI = \sum_{i=1}^{n} CPI_i \times I_i$$

$$= (1 \times 0.38) + (3 \times 0.15) + (4 \times 0.42) + (5 \times 0.05)$$

$$CPI = 2.76$$

$$MIPS = \frac{\text{Clock rate}}{CPI \times 10^6}$$

$$= \frac{200 \times 10^6}{2.76 \times 10^6}$$

$$MIPS = 72.46$$

12. Consider three different processors, P1, P2, and P3 executing the same instruction set. P1 has a 3GHz clock rate and a CPI of 1.5. P2 has a 2.5GHz clock rate and a CPI of 1.03. P3 has a 4GHz clock rate and a CPI of 2.2.

(a) Which processor has the highest performance expressed in instructions per second.

(b) If the processors each execute a program in 10s, find the no. of instructions.

(c) We are trying to reduce the execution time by 30% but this leads to an increase of 20% in CPI. What clock rate should we have to get this time reduction?

Soln:

(a) To find IPS.

$$IPS = \frac{\text{Clock rate}}{CPI}$$

$$IPS_1 = \frac{3 \times 10^9}{1.5} = 2 \times 10^9$$

$$IPS_2 = \frac{2.5 \times 10^9}{1.03} = 2.4 \times 10^9$$

$$IPS_3 = \frac{4 \times 10^9}{2.2} = 1.82 \times 10^9$$

$\therefore$ Processor, P2 has the highest performance in terms of IPS.

(b) CPU time = 10 s.

$$IPS = \frac{\text{Instructions count}}{\text{CPU Time}}$$

$$I_{C1} = IPS_1 \times \text{CPU Time}_1$$

$$I_{C_1} = 2 \times 10^9 \times 10 = 2 \times 10^{10}$$

$$I_{C_2} = 2.4 \times 10^{10}$$

$$I_{C_3} = 1.82 \times 10^{10}$$

(c) Given: Execution time is to be reduced by 30%

$$\Rightarrow \text{Exec. time}_{old} - \frac{30}{100} \cdot \text{Exec. time}_{old} = \text{Exec. time}_{new}$$

$$\therefore \text{Exec. time}_{new} = 70\% \text{ of Exec. time}_{old}$$

Also, $CPI_{new} = CPI_{old} + \frac{20}{100} CPI_{old}$

$$CPI_{new} = 1.2\ CPI_{old}$$

To find clock rate new.

$$\text{Exec. time} = \frac{\text{Inst}^n \text{ count} \times CPI}{\text{Clock rate}}$$

$$\frac{\text{Inst}^n_{new} \times CPI_{new}}{\text{clock rate}_{new}} = 0.7 \, \frac{\text{Inst}^n \times CPI_{old}}{\text{clock rate}_{old}}$$

$$\frac{1.2\, CPI_{old}}{\text{Clock rate}_{new}} = \frac{0.7\, CPI_{old}}{\text{Clock rate}_{old}}$$

$$\text{Clock rate}_{new} = \frac{1.2}{0.7} \times \text{clock rate}_{old}$$

$$\text{Clock rate}_{new} = 1.71 \times \text{clock rate}_{old}$$

$\therefore$ The clock rate must be increased by about 71%.

$\left[\because 1.71\, \text{clock rate}_{old} = \text{Clock rate}_{old} + \frac{71}{100}\, \text{clock rate}_{old}\right]$.