AC CIRCUITS

P SASIPRIYA

VIT CHENNAI

AC SUPPLY

Historically, DC sources were the main means of providing electric power up until the late 1800s. At the end of that century, the battle of direct current versus alternating current began. Because AC is more efficient and economical to transmit over long distances, AC systems ended up the winner.

The analysis of circuits in which the source voltage or current is time-varying particularly interested in sinusoidally time-varying excitation, or simply, excitation by a sinusoid.

SINUSOIDAL SIGNAL

A sinusoid is a signal that has the form of the sine or cosine function. A sinusoidal current is usually referred to as alternating current (ac). Circuits driven by sinusoidal current or voltage sources are called **ac circuits**.

Sinusoids are used in AC circuits for a number of reasons

- First, nature itself is characteristically sinusoidal. For Eg. motion of a pendulum, the vibration of a string, the ripples on the ocean surface, and the natural response of underdamped second order systems.

- Second, a sinusoidal signal is easy to generate and transmit. It is the form of voltage generated throughout the world and supplied to homes, factories, laboratories, and so on.

- Third, through Fourier analysis, any practical periodic signal can be represented by a sum of sinusoids. Sinusoids, therefore, play an important role in the analysis of periodic signals.

- Lastly, a sinusoid is easy to handle mathematically. The derivative and integral of a sinusoid are themselves sinusoids. For these and other reasons, the sinusoid is an extremely important function in circuit analysis.

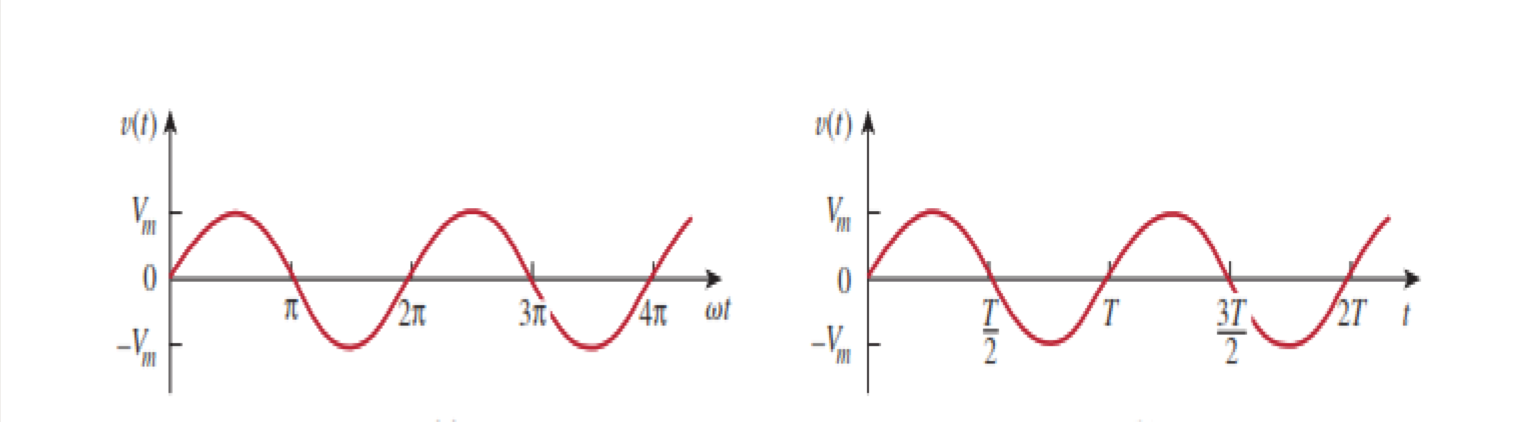
SINUSOIDAL SIGNAL

Consider the sinusoidal voltage

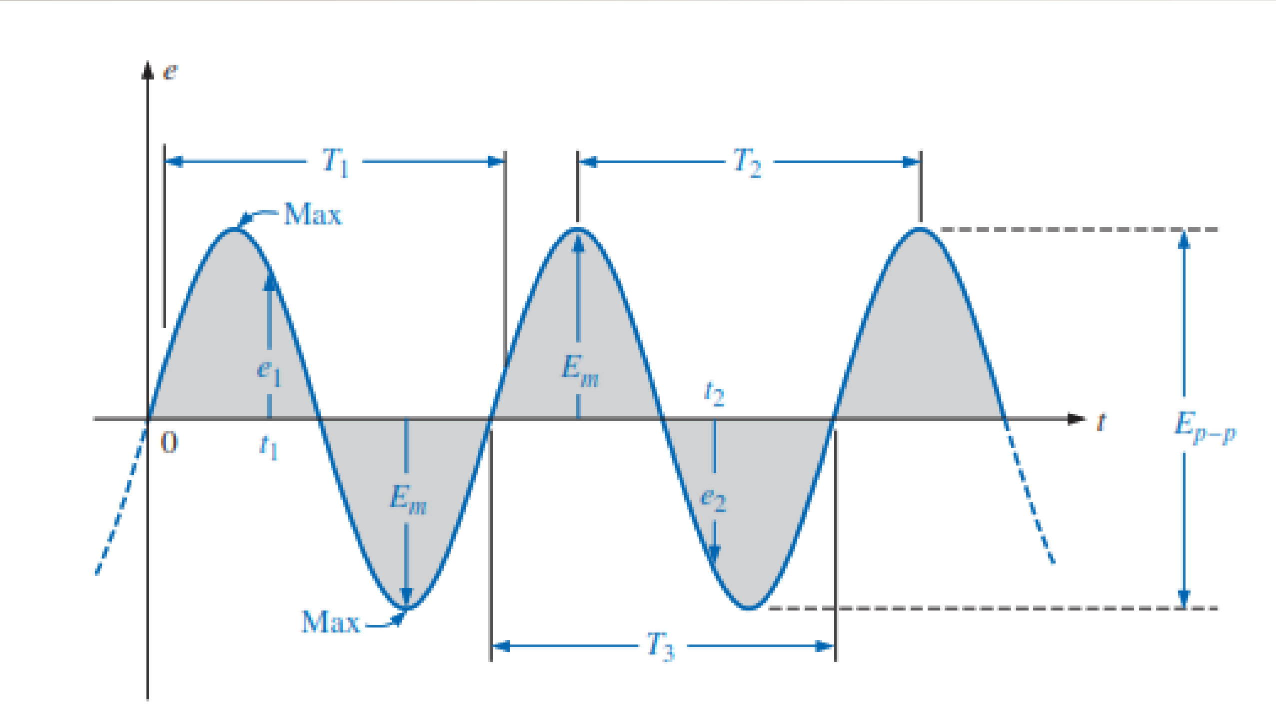
$$v(t) = V_m \sin wt$$

where V_m the amplitude of the sinusoid w is the angular frequency in radians/s
wt the argument of the sinusoid. It is evident that the sinusoid repeats itself every T seconds; thus, T is called the period of the sinusoid. From the two plots in Fig. , we observe that

$T = 2\pi/w$



IMPORTANT PARAMETERS FOR A SINUSOIDAL VOLTAGE SIGNAL

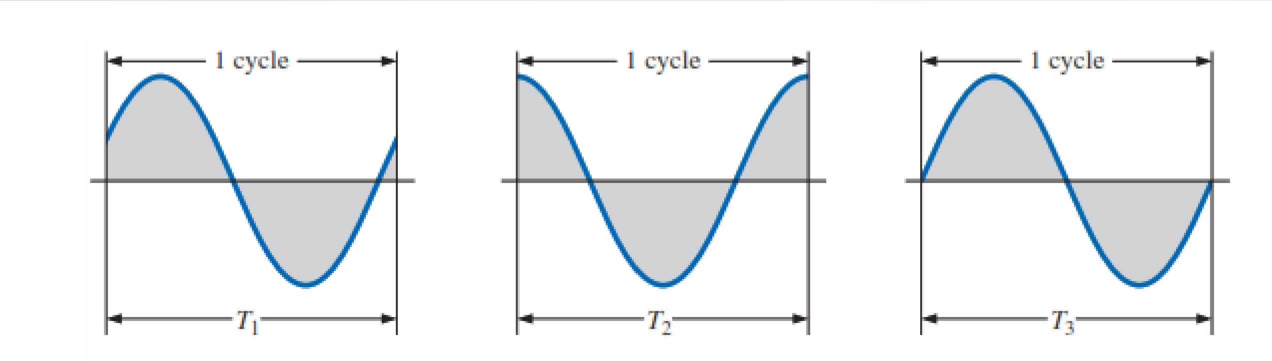


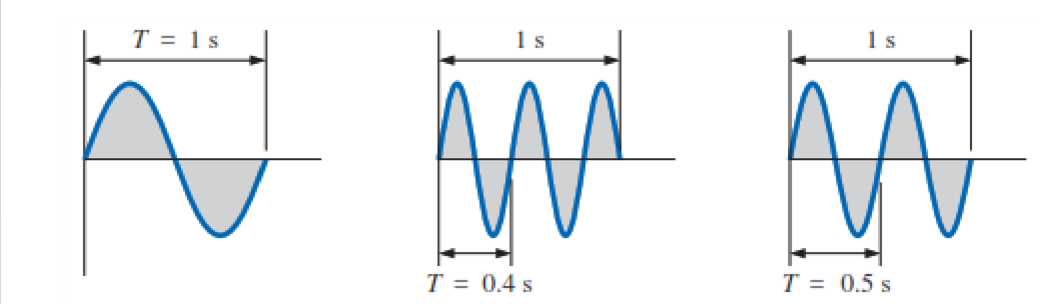
Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e1, e2).

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level.

Peak-to-peak value: Denoted by Ep-p or Vp-p, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. is a periodic waveform.





Cycle: The portion of a waveform contained in one period of time. The cycles within T1, T2, and T3 of Fig. are the definition of a cycle.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig. (a) is 1 cycle per second, and for (b), 2 cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13.5(c)], the frequency would be 2 cycles per second. The unit of measure for frequency is the hertz (Hz), where 1 hertz (Hz) 1 cycle per second (c/s)

Since the frequency is inversely related to the period, the two can be related by the following equation:

$f= 1/T$ Hz

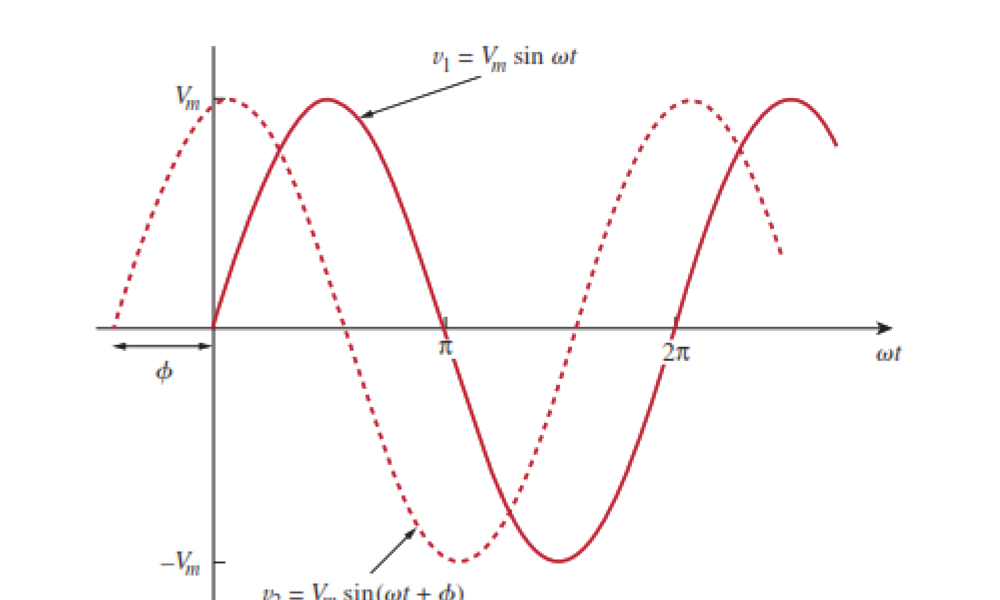
The fact that v(t) repeats itself every T seconds is shown by replacing t by t + T and we get

$$v(t+T) = V_m \sin\omega(t+T) = V_m \sin\omega\left(t + \frac{2\pi}{\omega}\right)$$
$$= V_m \sin(\omega t + 2\pi) = V_m \sin\omega t = v(t)$$

Hence , v(t+T) = v(t)

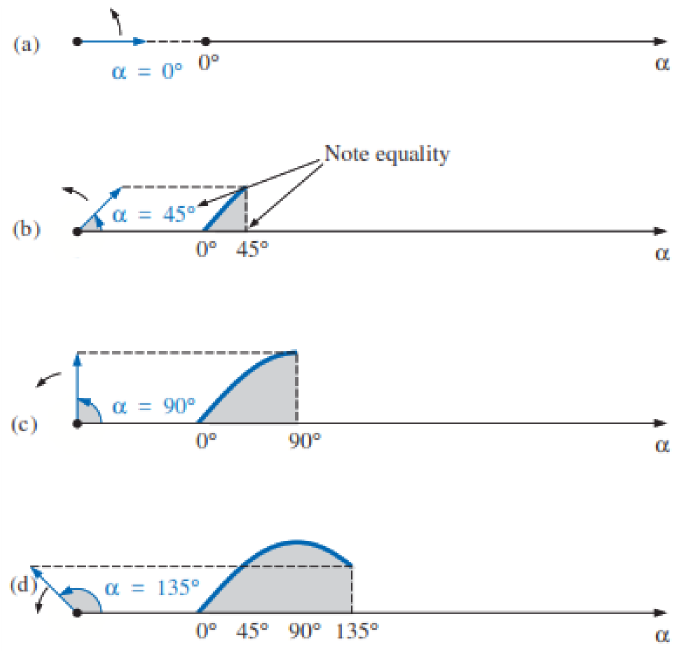
Let us now consider a more general expression for the sinusoid,
v(t) = Vm sin(wt + ϕ)
where wt is the argument and ϕ is the phase. Both argument and phase can be in radians or degrees.
Let us examine the two sinusoids

$$v_1(t) = V_m \sin\omega t \qquad \text{and} \qquad v_2(t) = V_m \sin(\omega t + \phi)$$



PHASORS

Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions. A phasor is a complex number that represents the amplitude and phase of a sinusoid.



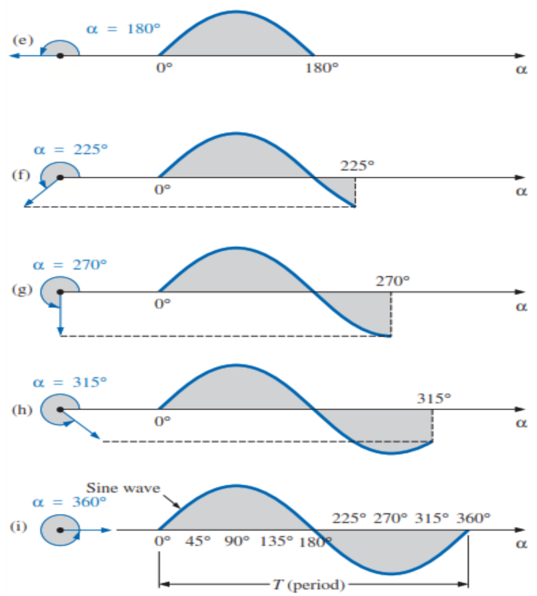


FIG. 13.16
Generating a sinusoidal waveform through the vertical projection of a

$$z = x + jy \quad \text{Rectangular form}$$
$$z = r\angle\phi \quad \text{Polar form}$$
$$z = re^{j\phi} \quad \text{Exponential form}$$

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$\boxed{z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)}$$

AC WAVEFORMS AND AVERAGE VALUE

While we can describe ac quantities in terms of frequency, period, instantaneous value, etc., we do not yet have any way to give a meaningful value to an ac current or voltage in the same sense that we can say of a car battery that it has a voltage of 12 volts.

This is because ac quantities constantly change and thus there is no one single numerical value that truly represents a waveform over its complete cycle. For this reason, ac quantities are generally described by a group of characteristics, including instantaneous, peak, average, and effective values.

AVERAGE VALUE

To find the average of a waveform, you can sum the instantaneous values over a full cycle, then divide by the number of points used. The trouble with this approach is that waveforms do not consist of discrete values.

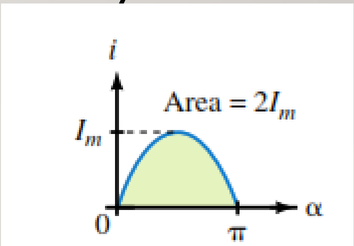
An approach more suitable for use with waveforms is to find the area under the curve, then divide by the baseline of the curve. Thus, to find the average value of a waveform, divide the area under the waveform by the length of its base. Areas above the axis are counted as positive, while areas below the axis are counted as negative.

This approach is valid regardless of waveshape. Average values are also called dc values, because dc meters indicate average values rather than instantaneous values. Thus, if you measure a non dc quantity with a dc meter, the meter will read the average of the waveform,

Sine Wave Averages

Because a sine wave is symmetrical, its area below the horizontal axis the same as its area above the axis; thus, over a full cycle its net area zero, independent of frequency and phase angle. Thus, the average of sin ωt, sin(ωt+ϕ), sin 2ωt, cos ωt, cos(ωt+ϕ), cos 2ωt, and so on are each zero.

The average of half a sine wave, however, is not zero. Consider Figure The area under the half-cycle may be found using calculus as area. Similarly, the area under a half-cycle of voltage is 2Vm.



$$\text{area} = \int_0^{\pi} I_m \sin\alpha \, d\alpha = -I_m \cos\alpha \Big|_0^{\pi} = 2I_m$$

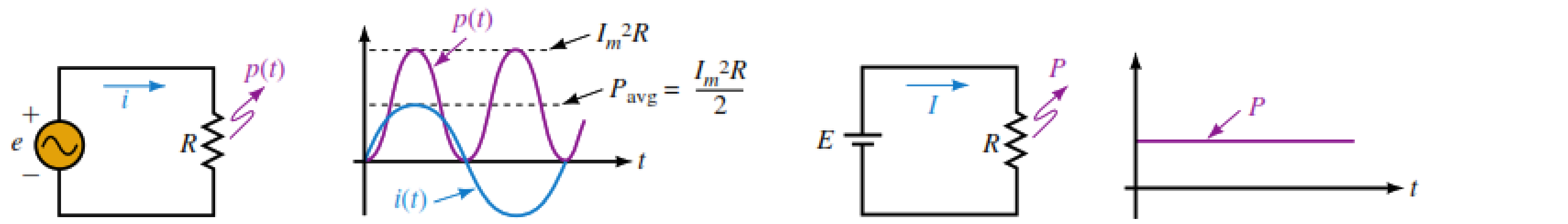
$$I_{avg} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi} = 0.318 I_m$$

EFFECTIVE VALUE

While instantaneous, peak, and average values provide useful information about a waveform, none of them truly represents the ability of the waveform to do useful work. In practice, most ac voltages and currents are expressed as effective values. Effective values are also called rms value.

An effective value is an equivalent dc value: it tells you how many volts or amps of dc that a time-varying waveform is equal to in terms of its ability to produce average power. Effective values depend on the waveform. A familiar example of such a value is the value of the voltage at the wall outlet in your home. In North America its value is 120Vac. This means that the sinusoidal voltage at the wall outlets of your home is capable of producing the same average power as 120 volts of steady dc.

The effective value of a waveform can be determined using the circuits of Figure. Consider a sinusoidally varying current, i(t). By definition, the effective value of i is that value of dc current that produces the same average power. Consider (b). Let the dc source be adjusted until its average power is the same as the average power in (a). The resulting dc current is then the effective value of the current of (a). To determine this value, determine the average power for both cases, then equate them.



$p(t) = i^2R$. Therefore, $p(t)$ varies cyclically.

$P = I^2R$. Therefore, P is constant.

In dc case,

$$P_{avg} = P = I^2R$$

$$\begin{aligned} p(t) &= i^2R \\ &= (I_m \sin \omega t)^2 R = I_m^2 R \sin^2 \omega t \\ &= I_m^2 R\left[\frac{1}{2}(1 - \cos 2\omega t)\right] \end{aligned}$$

$$p(t) = \frac{I_m^2R}{2} - \frac{I_m^2R}{2}\cos 2\omega t$$

$$P_{avg} = \text{average of } p(t) = \frac{I_m^2R}{2}$$

$$I^2 = \frac{I_m^2}{2}$$

$$I = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$