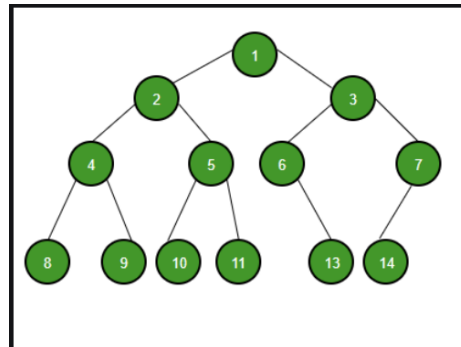


<b>Module:4</b>	<b>Trees</b>	<b>6 hours</b>
Introduction - Binary Tree: Definition and Properties - Tree Traversals- Expression Trees:- Binary Search Trees - Operations in BST: insertion, deletion, finding min and max, finding the k <sup>th</sup> minimum element.		

### Binary Tree:



- 1) The top most node is called the root node, and it contains both left and right pointers to indicate left and right child.
- 2) Binary Tree is something in the case where it can have at most two childrens.
- 3) They can be very well used for network routing , game AI , as well as expression evaluation.
- 4) APPLICATION : apply priority queue in order to find maximum / minimum in O(1) complexity.
- 5) Used to even store cache in a given system.

### TRAVERSALS IN BINARY TREE CAN BE DONE BY:

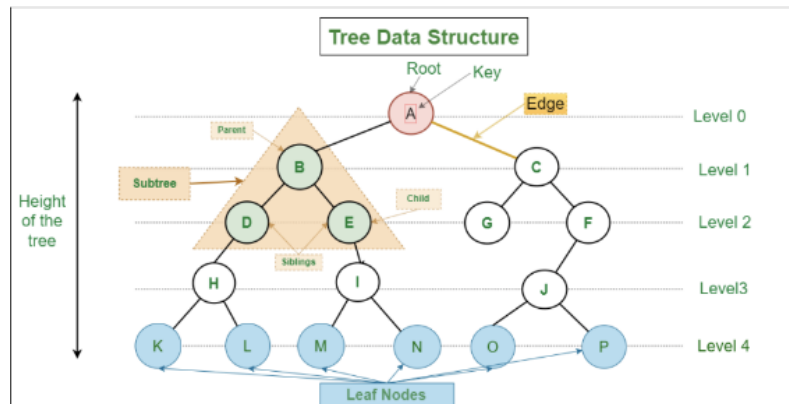
<b>PREORDER</b> (prefix)	ROOT	LEFT	RIGHT
<b>INORDER</b> (infix)	LEFT	ROOT	RIGHT
<b>POSTORDER</b> (postfix)	LEFT	RIGHT	ROOT

### Tree Terminologies:

- 1) Root : The first node is called the root node and every tree must have a root node.
- 2) Edge : The link between two nodes is called an edge.
- 3) Leaf Node : The node which does not have a child is known as the leaf node.They are also called as external /terminal nodes.
- 4) Internal Node : The node at least has one child , and the nodes other than the leaf nodes.

- 5) Height : The total number of edges from **leaf node to particular node** is called height and height of a tree is known as leaf node to root node.
- 6) Depth : The total number of edges from **root node to a particular node** is called depth of a tree and from root node to a leaf node is called depth of the tree

Representation of all the properties:



### Types of binary trees:

#### STRICT BINARY TREE :

If a binary tree has exactly two children or no children

#### FULL BINARY TREE:

A binary tree which has exactly two children for each given parent node.

#### COMPLETE BINARY TREE:

A binary tree , which is left skewed.

### \*Creating Binary Tree \*

#### And Applying DFS algorithms in them (namely Inorder, PostOrder , PreOrder)

```
#include <bits/stdc++.h>
using namespace std;
```

```
class Node {
public:
    int data;
    Node* left;
    Node* right;
```

```
Node(int val) {  
    this->data = val;  
    this->left = NULL;  
    this->right = NULL;  
}
```

```
Node(int val, Node* lchild, Node* rchild) {  
    this->data = val;  
    this->left = lchild;  
    this->right = rchild;  
}  
};
```

```
void inorderTraversal(Node* root) {  
    if (root == NULL) {  
        return;  
    }  
  
    inorderTraversal(root->left);  
    cout << root->data << " ";  
    inorderTraversal(root->right);  
}
```

```
void preorderTraversal(Node* root) {  
    if (root == NULL) {  
        return;  
    }  
  
    cout << root->data << " ";  
    preorderTraversal(root->left);  
    preorderTraversal(root->right);  
}
```

```
void postorderTraversal(Node* root) {  
    if (root == NULL) {  
        return;  
    }  
  
    postorderTraversal(root->left);  
    postorderTraversal(root->right);
```

```

    cout << root->data << " ";
}

int main() {

    Node* root = new Node(0,new Node(1,new Node(3,NULL,NULL),new
Node(4,NULL,NULL)),new Node(2,new Node(5,NULL,NULL),new Node(6,NULL,NULL)));

    cout << "In-order traversal of the tree: ";
    inorderTraversal(root);
    cout << endl;

    cout << "Pre-order traversal of the tree: ";
    preorderTraversal(root);
    cout << endl;

    cout << "Post-order traversal of the tree: ";
    postorderTraversal(root);
    cout << endl;

    return 0;
}

```

**\*\* Or we can even use a buildtree function for a specific tree as shown in diagram \*\***

```

#include <bits/stdc++.h>
using namespace std;

class Node {
public:
    int data;
    Node* left;
    Node* right;

    Node(int val) : data(val), left(nullptr), right(nullptr) {}
};

```

```

Node* buildTree() {
    // Create and connect nodes recursively
    Node* root = new Node(0);
    root->left = new Node(1);
    root->right = new Node(2);
    root->left->left = new Node(3);
    root->left->right = new Node(4);
    root->right->left = new Node(5);
}

```

```

    root->right->right = new Node(6);

    return root;
}

void inorderTraversal(Node* root) {
    if (root == nullptr) {
        return;
    }
    inorderTraversal(root->left);
    cout << root->data << " ";
    inorderTraversal(root->right);
}

int main() {
    Node* root = buildTree();

    cout << "In-order traversal of the tree: ";
    inorderTraversal(root); //pass the root as we use recursion to find out the
    cout << endl;

    return 0;
}

```

## **\*\*Binary Search Tree \*\***

### **BASIC IMPLEMENTATION:**

```

#include <bits/stdc++.h>
using namespace std;

```

```

class Node{
public:
    int val;
    Node* left;
    Node* right;

    Node(int val){
        this->val = val;
        this->left = NULL;
        this->right = NULL;
    }
};

```

```

Node* insertBst(Node* root, int val){
    if(root==NULL) return new Node(val);

    if(root->val > val){
        root->left = insertBst(root->left,val);
    }else {
        root-> right = insertBst(root->right,val);
    }
    return root;
}

```

```

void inorderTraversal(Node* root){
    if(root == NULL){
        return ;
    }
    inorderTraversal(root->left);
    cout << root->val << " ";
    inorderTraversal(root->right);
}

```

```

void preorderTraversal(Node* root){
    if(root == NULL) return;

    cout << root->val << " ";
    preorderTraversal(root->left);
    preorderTraversal(root->right);

}

```

```

int main(){
    Node* root = NULL;
    int n;
    cin >> n;
    vector<int> v(n);

    for(int i=0;i<n;i++){
        cin >> v[i];
        root = insertBst(root,v[i]);
    }
}

```

```
    cout << "InorderTraversal for the given BST" << endl; //GIVES US SORTED ARRAY FROM  
THE TREE.
```

```
    inorderTraversal(root);  
    cout << endl;
```

```
    cout << "Preorder " << endl;  
    preorderTraversal(root);  
}
```

**\*\* The operations such as finding max , min and kth max and kth min in a given BST\*\***

```
#include<iostream>  
#include<vector>  
using namespace std;  
class Node{  
    public:  
        int val;  
        Node* left;  
        Node* right;  
  
        Node(int val){  
            this->val = val;  
            this->left = NULL;  
            this->right = NULL;  
        }  
};
```

```
Node* insertBst(Node* root , int val){  
    if(root == NULL) return new Node(val);  
    if(root->val > val){  
        root->left = insertBst(root->left,val);  
    }else{  
        root->right = insertBst(root->right , val);  
    }  
    return root;  
}
```

```
void InorderTraversal(Node* root,vector<int>& elements){  
    if(root==NULL) return;  
    InorderTraversal(root->left,elements);  
    elements.push_back(root->val);
```

```

        InorderTraversal(root->right,elements);
    }

```

```

int maxe(Node* root){
    if(root->right == NULL){
        return root->val;
    }else{
        maxe(root->right);
    }
}

```

```

int mine(Node* root){
    if(root->left == NULL){
        return root->val;
    }
    else{
        mine(root->left);
    }
}

```

```

int kthmaximum(vector<int>& el , int k){
    if(k>0 && k<=el.size()){
        return el[el.size()-k];
    }
    return -1;
}

```

```

int kthminimum(vector<int>& el,int k){
    if(k>0 && k<=el.size()){
        return el[k-1];
    }
    return -1;
}

```

```

int main(){
    Node* root = NULL;
    int n , k , k2;
    cin >> n;
    vector<int> v(n);
    vector<int> elements;
    for(int i=0;i<v.size();i++){
        cin >> v[i];
        root = insertBst(root,v[i]);
    }
}

```



```

    cout << "Inorder Traversal of an BST : " << endl;
    InorderTraversal(root,elements);
    cout << endl;

    cout << "The maximum element in BST is : " << endl;
    cout << maxe(root) << endl;

    cout << "The minimum element in BST is : " << endl;
    cout << mine(root) << endl;

    cout << "The kth max ele:" << endl;
    cin >> k;
    cout << "The kth min ele : " << endl;
    cin >> k2;

    cout << "The kthmaximum element in BST is : " << endl;
    cout << kthmaximum(elements, k) << endl;

    cout << "The kth minimum element in BST is : " << endl;
    cout << kthminimum(elements , k2) << endl;

    return 0;
}

```

### **\*\* To delete the Node in a given BST \*\***

```

#include <iostream>
#include <vector>
using namespace std;

class Node{
public:
    int val;
    Node* left;
    Node* right;

    Node(int val){
        this->val = val;
        this->left = NULL;
        this->right = NULL;
    }
}

```

```
};
```

```
Node* insertBst(Node* root,int val){  
    if(root == NULL){  
        return new Node(val);  
    }  
  
    if(root->val>val){  
        root->left = insertBst(root->left,val);  
    }  
    else{  
        root->right = insertBst(root->right , val);  
    }  
    return root;  
}
```

```
Node* findMin(Node* node){  
    while(node->left != NULL){  
        node = node -> left;  
    }  
    return node;  
}
```

```
Node* deleteNode(Node* root,int val){  
    if(root == NULL){  
        return root;  
    }  
  
    if(val > root->val){  
        root->right = deleteNode(root->right , val);  
    }else if(val < root->val){  
        root->left = deleteNode(root->left , val);  
    }else{  
  
        if(root->left == NULL){  
            Node* temp = root->right;  
            delete root;  
            return temp;  
        }else if(root->right == NULL){  
            Node* temp = root->left;  
            delete root;  
            return temp;  
        }  
    }  
}
```

```

        //if the node has two children.
        Node* temp = findMin(root->right);
        root->val = temp->val;
        root->right = deleteNode(root->right , temp->val);
    }
    return root;
}

```

```

void inordertraversal(Node* root){
    if (root == NULL){return ;}

    inordertraversal(root->left);
    cout << root->val << " ";
    inordertraversal(root->right);
}

```

```

int main(){
    Node* root = NULL;
    int n , del;
    cin >> n;
    vector<int> v(n);
    for(int i=0;i<n;i++){
        cin >> v[i];
        root = insertBst(root,v[i]);
    }

    cout << "Inorder Traversal : " << endl;
    inordertraversal(root);
    cout << endl;

    cout << "Enter the element to be deleted : " << endl;
    cin >> del;
    deleteNode(root,del);

    cout << "Inorder Traversal : " << endl;
    inordertraversal(root);
    cout << endl;

    return 0;
}

```

## **Binary Tree Based Problems:**

### **1. Finding Height or Depth of a given Tree:**

```
int findheight(Node* root){
    if(root == NULL) return 0;
    int lefth = findheight(root->left);
    int righth = findheight(root->right);

    return 1+max(lefth , righth);
}

int findNodeDepth(Node* root , int target , int depth){
    if(root == NULL){
        return -1;
    }
    if(root->data == target){
        return depth;
    }

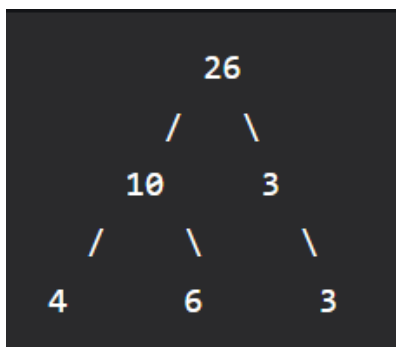
    int leftd = findNodeDepth(root->left , target , depth +1);
    int rightd = findNodeDepth(root->right , target , depth+1);

    if(leftd == -1 && rightd == -1){
        return -1;
    }

    return (leftd != -1)?leftd : rightd;
}
```

### **2) Check if its a Sum Tree:**

A Sum tree is basically a tree where the nodes , are either equal to sum of left subtree or right subtree.



```

int sum(TreeNode* root) {
    if (root == nullptr)
        return 0;

    return sum(root->left) + root->data + sum(root->right);
}

bool isSumTree(TreeNode* node) {
    int ls, rs;

    if (node == nullptr || (node->left == nullptr && node->right == nullptr))
        return true;

    ls = sum(node->left);
    rs = sum(node->right);

    if ((node->data == ls + rs) && isSumTree(node->left) && isSumTree(node->right))
        return true;

    return false;
}

```

### **3) Check if they are cousins (i.e in the same level).**

**//in this case we find out the level , if its sibling or not and the final condition**

// Recursive function to check if two Nodes are siblings

```

int isSibling(Node* root, Node* a,
              Node* b)
{
    if (root == NULL) //in case of an empty tree.
        return 0;

    return ((root->left == a && root->right == b)
            || (root->left == b && root->right == a)
            || isSibling(root->left, a, b)
            || isSibling(root->right, a, b));
}

```

```

int level(Node* root, Node* ptr, int lev)
{
    // base cases
    if (root == NULL)
        return 0;
}

```

```

    if (root == ptr)
        return lev;

    int l = level(root->left, ptr, lev + 1);
    if (l != 0)
        return l;

    return level(root->right, ptr, lev + 1);
}

int isCousin(Node* root, Node* a, Node* b)
{
    // 1. The two Nodes should be on the same level in the
    // binary tree.
    // 2. The two Nodes should not be siblings (means that
    // they should
    // not have the same parent Node).
    if ((level(root, a, 1) == level(root, b, 1)) && !(isSibling(root, a, b)))
        return 1;
    else
        return 0;
}

```

#### **4)Count Number of Given Nodes :**

```

int count(Node* root){
    if(root == NULL){
        return 0;
    }
    return count(root->left) + count(root->right) + 1;
}

```

#### **5)Both the subtree's are mirror: ie left of one tree is equal to the right of one tree.**

```

bool areMirror(Node* a,Node* b){
    if(a==NULL && b==NULL) return true;
    if(a==NULL || b==NULL) return false;
    return a->data == b->data && areMirror(a->left,b->left) && areMirror(a->right,b->right);
}

```

#### **6)Two trees to be identical:**

```

int identicalTrees(Node* a,Node* b){
    if(a==NULL && b ==NULL){
        return 1;
    }
}

```

```

    if(a!=NULL && b!=NULL){
        return (a->data == b->data && identicalTrees(a->left , b->left) && identicalTrees(a->right ,
b->right));
    }
    return 0;
}

```

### **7) Number of leaves in a given Tree:**

```

unsigned int getLeafCount(Node* node) {
    if (node == nullptr)
        return 0;
    if (node->left == nullptr && node->right == nullptr)
        return 1;
    else
        return getLeafCount(node->left) + getLeafCount(node->right);
}

```

### **8)Creating a mirror of the given tree:**

```

Node* createMirrorTree(Node* root) {
    if (root == nullptr) {
        return nullptr;
    }

    // Swap the left and right subtrees
    Node* temp = root->left;
    root->left = root->right;
    root->right = temp;

    // Recursively create mirror trees for left and right subtrees
    createMirrorTree(root->left);
    createMirrorTree(root->right);

    return root;
}

```

### **Binary Expression tree:**

```

#include <iostream>
#include <stack>
#include <string>
#include <cctype>
using namespace std;

```

```

class Node {
public:
    char data;
    Node* left;
    Node* right;

    Node(char key) {
        data = key;
        left = right = nullptr;
    }
};

bool isOperator(char c) {
    return (c == '+' || c == '-' || c == '*' || c == '/');
}

void inOrderTraversal(Node* root) {
    if (root) {
        inOrderTraversal(root->left);
        cout << root->data << " ";
        inOrderTraversal(root->right);
    }
}

void postOrderTraversal(Node* root) {
    if (root) {
        postOrderTraversal(root->left);
        postOrderTraversal(root->right);
        cout << root->data << " ";
    }
}

void preOrderTraversal(Node* root) {
    if (root) {
        cout << root->data << " ";
        preOrderTraversal(root->left);
        preOrderTraversal(root->right);
    }
}

Node* constructExpressionTree(const string& expression) {
    stack<Node*> stack;

    for (char c : expression) {

```



```

    if (isalnum(c)) {
        Node* operand = new Node(c);
        stack.push(operand);
    } else if (isOperator(c)) {
        Node* operatorNode = new Node(c);
        operatorNode->right = stack.top();
        stack.pop();
        operatorNode->left = stack.top();
        stack.pop();
        stack.push(operatorNode);
    }
}

return stack.top();
}

int main() {
    std::string expression = "a+b*c";

    Node* root = constructExpressionTree(expression);

    cout << "Infix Expression: ";
    inOrderTraversal(root);
    cout << endl;

    cout << "Postfix Expression: ";
    postOrderTraversal(root);
    cout << endl;

    cout << "Prefix Expression: ";
    preOrderTraversal(root);
    cout << endl;

    return 0;
}

```

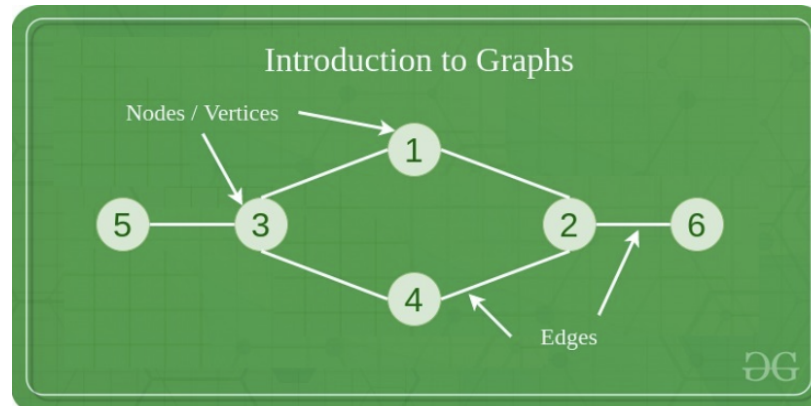
**Module:5 Graphs****6 hours**

Terminology – Representation of Graph – Graph Traversal: Breadth First Search (BFS), Depth First Search (DFS) - Minimum Spanning Tree: Prim's, Kruskal's - Single Source Shortest Path: Dijkstra's Algorithm.

**Graph:**

It is a non linear data structure which has vertices and edges.

Graph is represented as  $G(V,E)$ .

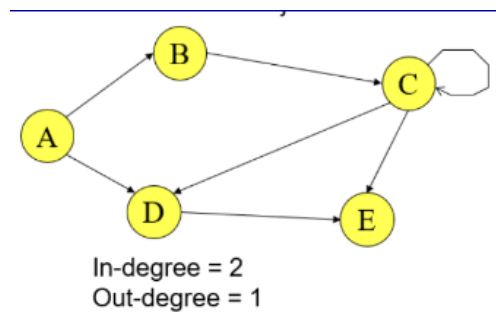
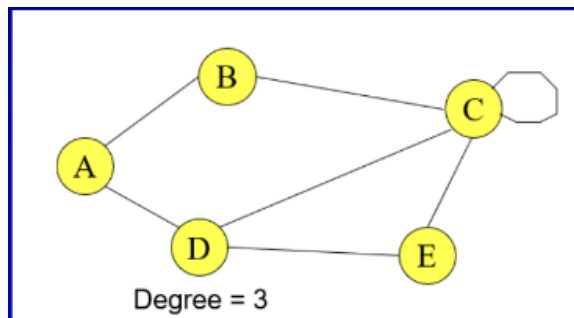


**No direction** : Undirected Graph.

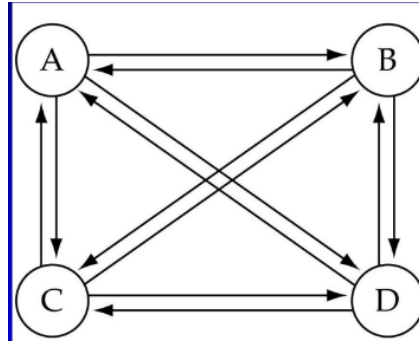
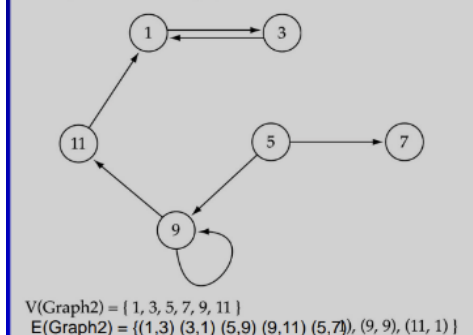
**Direction** : Directed Graph

**Degree** : The number of edges on the given node.

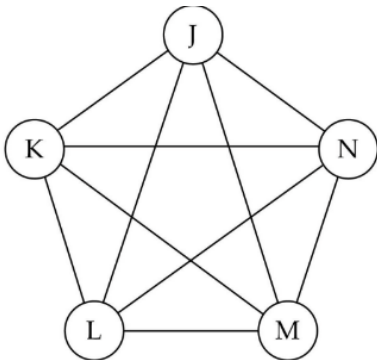
**Complete Graph** : Where every vertices of a graph is connected to the other vertices in the graph,



(b) Graph2 is a directed graph.



(a) Complete directed graph.



(b) Complete undirected graph.

Number of edges of the complete directed graph :  $N*(N-1)$

Number of edges of the complete undirected graph :  $N*(N-1)/2$

**Adjacency Matrix is used to represent a graph.**

**Undirected graph:**

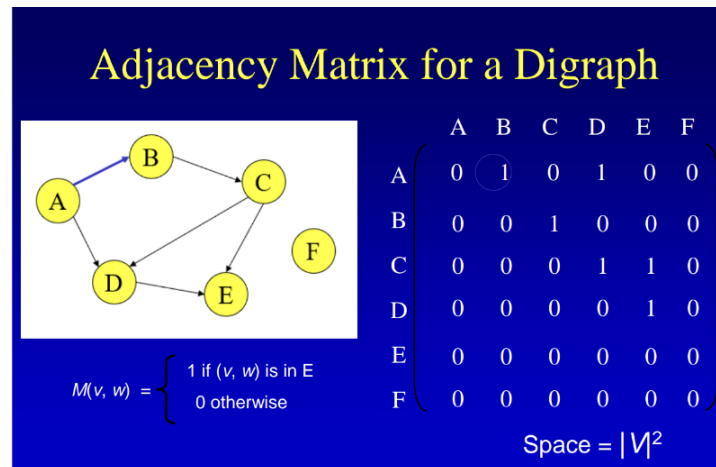
### Adjacency Matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	0	0
C	0	1	0	1	1	0
D	1	0	1	0	1	0
E	0	0	1	1	0	0
F	0	0	0	0	0	0

$$M(v, w) = \begin{cases} 1 & \text{if } [v, w] \text{ is in } E \\ 0 & \text{otherwise} \end{cases}$$

Space =  $|V|^2$

### Directed Graph:



### Graph Traversals :

**BFS (Breadth first search )** -> uses a queue ,and we can take any node / root node for the traversal.

**DFS (Depth first search )**-> uses a stack , and we can take any node / root node for traversal.

### BFS:

#### Code:

```
#include <iostream>
#include <map>
#include <list>
#include <vector>
#include <queue>
using namespace std;

class Graph{
public:
    map<int , bool> visited;
    map<int , list<int>> adj;

    void addEdge(int v , int w){
        adj[v].push_back(w);
    }

    void BFS(int start){
        queue<int> q;
        visited.clear();
```

```

        q.push(start);
        visited[start] = true;

        while(!q.empty()){
            int v = q.front();
            cout << v << " ";
            q.pop();

            for(int n : adj[v]){
                if(!visited[n]){
                    q.push(n);
                    visited[n]=true;
                }
            }
        }
    };

int main() {
    int v,e,n1,n2,node;
    cin >> v;
    cin >> e;
    Graph g;
    while(e>0) {
        cin >> n1 >> n2;
        g.addEdge(n1,n2);
        e--;
    }
    cin >> node;
    g.BFS(node);

}

```

### **DFS:**

```

#include <iostream>
#include <map>
#include <vector>
#include <list>

```

```

#include <stack>
using namespace std;

class Graph {
public:
    map<int, bool> visited;
    map<int, list<int>> adj;

    void addEdge(int v, int w) {
        adj[v].push_back(w);
    }

    void DFS(int start) {
        stack<int> s;
        s.push(start);
        visited[start] = true;

        while (!s.empty()) {
            int v = s.top();
            s.pop();
            cout << v << " ";

            for (const int& neighbor : adj[v]) {
                if (!visited[neighbor]) {
                    s.push(neighbor);
                    visited[neighbor] = true;
                }
            }
        }
    }
};

int main() {
    Graph g;
    int v, e, n1, n2, node;
    cin >> v;
    cin >> e;
    while (e > 0) {
        cin >> n1 >> n2;
        g.addEdge(n1, n2);
        e--;
    }
    cin >> node;
    g.DFS(node);
    return 0;
}

```

```
}
```

## **Minimum Spanning Tree : (PRIMS AND KRUSKALS ALGO FOR OBTAINING MST).**

### **PRIMS ALGORITHM:**

```
#include <bits/stdc++.h>
using namespace std;

class Graph {
public:
    vector<pair<int, int>> findMST(int V, vector<vector<pair<int,
int>>> adj[]) {
        priority_queue<pair<int, int>, vector<pair<int, int>>,
greater<pair<int, int>>> pq;
        vector<int> vis(V, 0);
        pq.push({0, 0});
        vector<pair<int, int>> mst;
        while (!pq.empty()) {
            auto it = pq.top();
            pq.pop();
            int node = it.second;
            int weight = it.first;

            if (vis[node] == 1) continue;
            vis[node] = 1;

            if (weight != 0) {
                mst.push_back({node, weight});
            }

            for (auto neighbor : adj[node]) {
                int adjNode = neighbor.first;
                int edgeWeight = neighbor.second;
                if (!vis[adjNode]) {
                    pq.push({edgeWeight, adjNode});
                }
            }
        }
        return mst;
    }
}
```

```

    int findMSTSum(int V, vector<vector<pair<int, int>>> adj[]) {
        priority_queue<pair<int, int>, vector<pair<int, int>>,
greater<pair<int, int>>> pq;
        vector<int> vis(V, 0);
        pq.push({0, 0});
        int sum = 0;

        while (!pq.empty()) {
            auto it = pq.top();
            pq.pop();
            int node = it.second;
            int weight = it.first;

            if (vis[node] == 1) continue;
            vis[node] = 1;

            sum += weight;

            for (auto neighbor : adj[node]) {
                int adjNode = neighbor.first;
                int edgeWeight = neighbor.second;
                if (!vis[adjNode]) {
                    pq.push({edgeWeight, adjNode});
                }
            }
        }

        return sum;
    }
};

```

```

int main() {
    int V = 5;
    vector<vector<pair<int, int>>> edges = {
        {{1, 2}, {2, 1}},
        {{0, 2}, {2, 1}, {3, 2}},
        {{0, 1}, {1, 1}, {4, 2}},
        {{1, 2}, {4, 1}},
        {{2, 2}, {3, 1}}
    };
    vector<vector<pair<int, int>>> adj[V];

    for (int i = 0; i < V; i++) {
        adj[i] = edges[i];
    }
}

```



```

    Graph g;
    vector<pair<int, int>> mst = g.findMST(V, adj);
    int sum = g.findMSTSum(V, adj);

    cout << "Minimum Spanning Tree Edges: " << endl;
    for (const auto& edge : mst) {
        cout << "Edge: " << edge.first << " - " << edge.second <<
endl;
    }

    cout << "The sum of all the edge weights in the MST: " << sum <<
endl;

    return 0;
}

```

### **KRUSKALS ALGORITHM:**

```

#include <bits/stdc++.h>
using namespace std;

class DisjointSet {
    vector<int> rank, parent, size;
public:
    DisjointSet(int n) {
        rank.resize(n + 1, 0);
        parent.resize(n + 1);
        size.resize(n + 1);
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
            size[i] = 1;
        }
    }

    //to find the representative parent.
    int findUPar(int node) {
        if (node == parent[node])
            return node;
        return parent[node] = findUPar(parent[node]);
    }

    //to ensure the height remains minimal.
    void unionByRank(int u, int v) {
        int ulp_u = findUPar(u);

```

```

        int ulp_v = findUPar(v);
        if (ulp_u == ulp_v) return;
        if (rank[ulp_u] < rank[ulp_v]) {
            parent[ulp_u] = ulp_v;
        }
        else if (rank[ulp_v] < rank[ulp_u]) {
            parent[ulp_v] = ulp_u;
        }
        else {
            parent[ulp_v] = ulp_u;
            rank[ulp_u]++;
        }
    }

//adding / unioning by the size.
void unionBySize(int u, int v) {
    int ulp_u = findUPar(u);
    int ulp_v = findUPar(v);
    if (ulp_u == ulp_v) return;
    if (size[ulp_u] < size[ulp_v]) {
        parent[ulp_u] = ulp_v;
        size[ulp_v] += size[ulp_u];
    }
    else {
        parent[ulp_v] = ulp_u;
        size[ulp_u] += size[ulp_v];
    }
}

};

//to find the weighted sum.
int findMSTSum(int V, vector<pair<int, int>> mstEdges,
vector<pair<int, int>> edgesWithWeights) {
    int sum = 0;
    for (const auto& edge : mstEdges) {
        int u = edge.first;
        int v = edge.second;

        for (const auto& ew : edgesWithWeights) {
            int wt = ew.first;
            int src = ew.second.first;
            int dest = ew.second.second;

            if ((u == src && v == dest) || (u == dest && v == src)) {
                sum += wt;
            }
        }
    }
}

```

```

        break;
    }
}
return sum;
}

int main() {
    int V = 5;
    vector<vector<int>> edges = {{0, 1, 2}, {0, 2, 1}, {1, 2, 1}, {2,
3, 2}, {3, 4, 1}, {4, 2, 2}};
    vector<pair<int, int>> mstEdges;
    vector<pair<int, pair<int, int>> edgesWithWeights;

    for (const auto& edge : edges) {
        edgesWithWeights.push_back({edge[2], {edge[0], edge[1]}});
    }

    DisjointSet ds(V);
    sort(edgesWithWeights.begin(), edgesWithWeights.end());

    for (auto it : edgesWithWeights) {
        int wt = it.first;
        int u = it.second.first;
        int v = it.second.second;

        if (ds.findUPar(u) != ds.findUPar(v)) {
            mstEdges.push_back({u, v});
            ds.unionBySize(u, v);
        }
    }

    cout << "Minimum Spanning Tree Edges:" << endl;
    for (const auto& edge : mstEdges) {
        cout << "Edge: " << edge.first << " - " << edge.second <<
endl;
    }

    int sum = findMSTSum(V, mstEdges, edgesWithWeights);
    cout << "Sum of MST edge weights: " << sum << endl;

    return 0;
}

```

**Module:3 | Searching and Sorting****7 hours**

Searching: Linear Search and binary search – Applications.

Sorting: Insertion sort, Selection sort, Bubble sort, Counting sort, Quick sort, Merge sort - Analysis of sorting algorithms.

**Linear Search:**

Algo:

#include &lt;bits/stdc++.h&gt;

using namespace std;

```
int linearsearch(int arr[],int N,int x){
    int a = -1;
    for(int i=0;i<N;i++){
        if(arr[i]==x){
            a=i;
        }
    }
    return a+1;
}

int main(){
    int n;
    cin>>n;
    int arr[n];
    for(int i=0;i<n;i++){
        cin >> arr[i];
    }
    cout << linearsearch(arr,n,6) << endl;
    return 0;
}
```

### **Binary Search:**

To implement binary search before hand , the array must be sorted to perform the search algorithm.

There are two ways to go through this iterative as well as recursive approach...

### **Algo:**

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
//for binary search to be applied the data must be sorted always.
```

```
//iterative approach.
```

```
int binarysearch(vector<int>& arr,int l,int r,int x){
    while(l<=r){ //ensures it reaches and doesnt overflow.
        int m = l+ (r-l)/2;
        if(arr[m] == x){
            return (m+1);
        }

        if(arr[m] < x){
            l=m+1; //ignore left side.
        }
        else{
            r= m-1; //ignore right side.
        }
    }
    return -1;
}
```

```
int main(){
    int n;
    cin >> n;
    vector<int> arr(n);
    for(int i=0;i<n;i++){
        cin >> arr[i];
    }
    sort(arr.begin(),arr.end());
    cout << binarysearch(arr,0,n-1,10) << endl;
    return 0;
}
```

## **RECURSIVE METHOD:**

### **Algo:**

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
//for binary search to be applied the data must be sorted always.
```

```
//recursive approach.
```

```
int binarysearch(vector<int>& arr,int l,int r,int x){
    while(r>=l){ //ensures it reaches and doesnt overflow.
        int m = l+ (r-l)/2;
        if(arr[m] == x){
            return (m+1);
        }

        if(arr[m] < x){
            return binarysearch(arr,m+1,r,x); //ignore left side.
        }
        else{
            return binarysearch(arr,l,m-1,x); //ignore right side.
        }
    }
    return -1;
}
```

```
int main(){
    int n;
    cin >> n;
    vector<int> arr(n);
    for(int i=0;i<n;i++){
        cin >> arr[i];
    }
    sort(arr.begin(),arr.end());
    cout << binarysearch(arr,0,n-1,10) << endl;
    return 0;
}
```

```
//practice leetcode questions based out of strivers..
```

### TIME COMPLEXITY:

Worst case :  $O(1)$

Average case :  $O(\log n)$

Best case :  $O(\log n)$

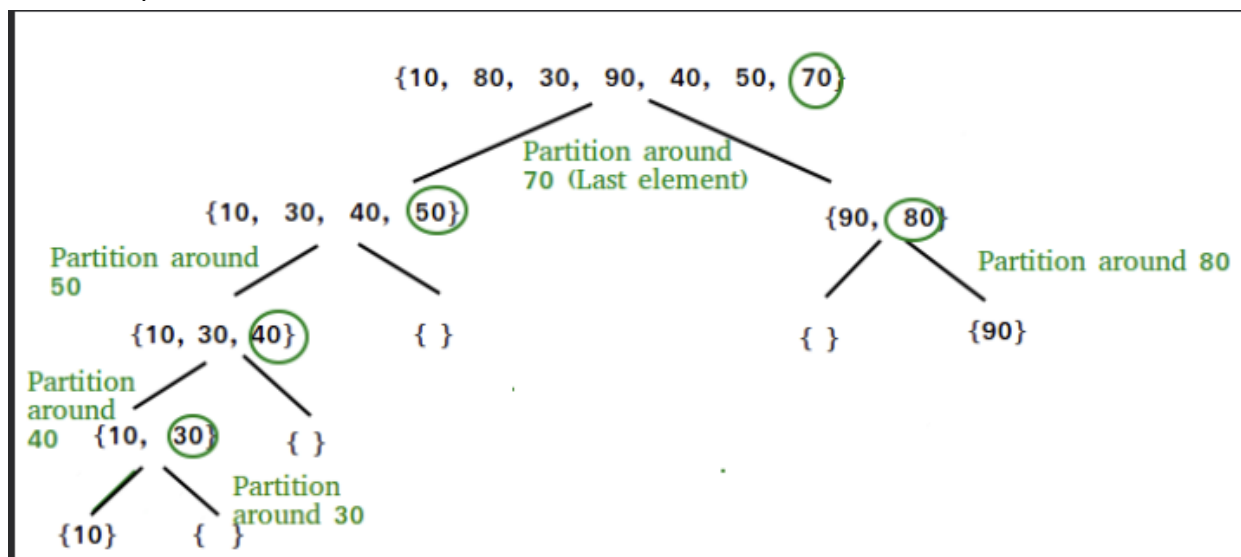
### Points:

- 1) Its faster than linear search , as it basically halves the searching rather than going through iteratively.
- 2) Its the most efficient searching algorithm , and is very well used in large datasets.
- 3) But the array must be sorted before applying binary search.

### SORTING:

#### Quick Sort:

Based out of divider and conquer and the main ideology is to find a partition and the left hand side are all lesser values and the right hand side are all larger values , and then implement recursive quick sort.



```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
int partition(vector<int>& arr,int l,int h){
    int pivot = arr[h];    //pivot is need for quick sort.
    int j = (l-1);
    for(int i=l;i<=h-1;i++){
        if(arr[i]<pivot){
            j++;
            swap(arr[j],arr[i]);
        }
    }
}
```

```

    }
    swap(arr[j+1],arr[h]);
    return(j+1);
}

void quicksort(vector<int>& arr, int l,int h){
    if(l<h){
        int pi = partition(arr,l,h);
        quicksort(arr,l,pi-1);
        quicksort(arr,pi+1,h);

    }
}

int main(){
    int n;
    cin >> n;
    vector<int> f(n);
    for(int i=0;i<n;i++){
        cin >> f[i];
    }
    quicksort(f,0,n-1);

    for(int j=0;j<n;j++){
        cout << f[j] << " ";
    }
    return 0;
}

```

### **TIME COMPLEXITY:**

Worst case :  $O(N^2)$  //occurs when pivot is chosen poorly.

Average case :  $O(n \log n)$

Best case :  $O(n \log n)$

### **Points:**

- 1) Since it uses divide and conquer its easier to solve the problems.
- 2) Efficient on large data sets.
- 3) Doesn't work well for small data sets.
- 4) Not a stable algorithm to sort any set of given data.



## **Merge Sort**

This type of sorting is based on the **divide and merge** concept and therefore the main part of this algo is find the middle and sort the left and right and then eventually merge:

### **CODE:**

```
#include <bits/stdc++.h>
using namespace std;

void Merge(vector<int>& arr,int low,int mid,int high){
    vector<int> temp;
    int left = low;
    int right = mid+1;
    while(left<=mid && right<=high){
        if(arr[left]<=arr[right]){
            temp.push_back(arr[left]);
            left++;
        }
        else{
            temp.push_back(arr[right]);
            right++;
        }
    }
    while(left<=mid){                //for remaining elements in the left
        temp.push_back(arr[left]);
        left++;
    }
    while(right<=high){              //for remaining elements in the right.
        temp.push_back(arr[right]);
        right++;
    }
    for(int i=low;i<=high;i++){
        arr[i] = temp[i-low];
    }
}

void mergesort(vector<int> &arr,int low , int high){
    if(low>=high) return ;
    int mid = (low+high)/2;
    mergesort(arr,low,mid);
    mergesort(arr,mid+1,high);
    Merge(arr,low,mid,high);
}
```

```

int main(){
    int n;
    cin>>n;
    vector<int> arr(n);
    for(int i=0;i<n;i++){
        cin >> arr[i];
    }
    mergesort(arr,0,n-1);

    for(int i=0;i<n;i++){
        cout << arr[i] << " ";
    }
    return 0;
}

```

### **TIME COMPLEXITY:**

Worst case :  $O(n \log n)$

Average case :  $O(n \log n)$

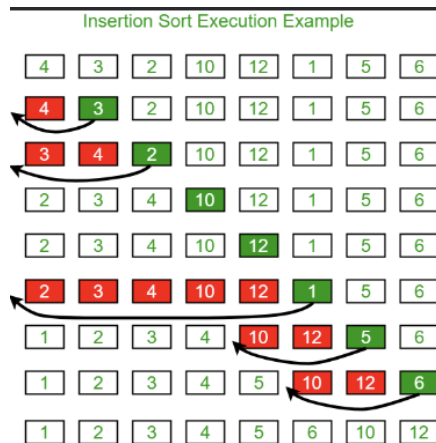
Best case :  $O(n \log n)$

### **Points:**

- 1) Used for sorting large datasets , due to its worst time complexity still being  $O(n \log n)$  .
- 2) Merge sort can handle various types such as partially sorted , nearly sorted , or completely unsorted data.
- 3) It is a stable algorithm , as well as parallelizable algorithm , which means it can be easily parallelised to take advantages of multiple processors and threads.
- 4) Major drawback is the space complexity (as it requires additional space ) and as well as its not optimal for small datasets.

### **Insertion Sort:**

The unsorted elements are inserted in the position in such a way that they are in a sorted manner.



### **CODE:**

```
#include <bits/stdc++.h>
using namespace std;
```

```
void insertionSort(vector<int> &arr, int n){
    int key , j;
    for(int i=0;i<n;i++){
        key = arr[i];
        j=i-1;
        while(j>=0 && arr[j]>key){
            arr[j+1] = arr[j];
            j--;
        }
        arr[j+1] = key;
    }
}
```

```
int main(){
    int n;
    cin >> n;
    vector<int> v(n);
    for(int i=0;i<n;i++){
        cin >> v[i];
    }
    insertionSort(v,n);
    for(int i=0;i<v.size();i++){
        cout << v[i] << " ";
    }
    return 0;
}
```

```
}
```

**TIME COMPLEXITY:**

Worst case :  $O(N^2)$

Average case :  $O(N^2)$

Best case :  $O(N)$

**Points:**

- 1) Efficient for small data values.
- 2) Adaptive ie, perfect for the partially sorted data.
- 3) Algorithmic Paradigm : It follows an incremental approach.
- 4) It is indeed a stable algorithm.
- 5) So we can use them when the dataset is small , or the data is partially sorted.

**Selection Sort:**

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
void selectionSort(vector<int> &arr){  
    int n = arr.size();  
    int min_idx;  
    for(int i=0;i<n-1;i++){  
        min_idx = i;  
        for(int j=i+1;j<n;j++){  
            if(arr[j] < arr[min_idx]){  
                min_idx = j;  
            }  
        }  
        if(min_idx != i){  
            swap(arr[min_idx] , arr[i]);  
        }  
    }  
}
```

```
int main(){  
    int n;  
    cin >> n;  
    vector<int> v(n);  
    for(int i=0;i<n;i++){  
        cin >> v[i];  
    }  
    selectionSort(v);  
}
```

```

    for(int i=0;i<v.size();i++){
        cout << v[i] << " ";
    }
    return 0;
}

```

### **TIME COMPLEXITY:**

Worst case :  $O(N^2)$

Average case :  $O(N^2)$

Best case :  $O(N)$

### **Points:**

- 1) It works well with small datasets.
- 2) Doesnt work well with if the data set has duplicate elements.
- 3) It is not stable for the given algorithm to sort it.
- 4) Its an in - place algorithm.

### **Counting Sort:**

Counting sort is a non - comparison based sorting algorithm , that works well when there is a limited range of input values.

It works on the principle of frequency counting.

### **Steps:**

- 1) Find out the maximum element in array.
- 2) Initialize a counter array of the size of the maximum element.
- 3) Counter array stores all the frequencies of the elements in it.
- 4) Calculate the prefix sum for the counter array.
- 5) Iterate from the end of input array , (making the algorithm stable) and when found negate it from counter array and it in the output array.

Update the output array , **outputArray[inputArray[ i ] - 1 ] =inputArray [ i ]**  
the counter array , **counterArray[inputArray [ i ] ] = countArray [ inputArray [ i ] ]**.

### **CODE:**

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
vector<int> countSort(vector<int>& arr){
```

```
    int n = arr.size();
```

```
    int m = 0;
```

```
    for(int i=0;i<n;i++){
```

```

        m = max(m,arr[i]);
    }

    vector<int> counterarr(m+1,0);

    for(int i=0;i<n;i++){
        counterarr[arr[i]]++;
    }

    for(int i=1 ; i<=m ; i++){
        counterarr[i] += counterarr[i-1];
    }

    vector<int> outputarr(n);
    for(int i=n-1;i>=0;i--){
        outputarr[counterarr[arr[i]]-1] = arr[i];
        counterarr[arr[i]]--;
    }
    return outputarr;
}

int main(){
    int n;
    cin >> n;
    vector<int> v(n);
    for(int i=0;i<n;i++){
        cin >> v[i];
    }
    vector<int> out = countSort(v);
    for(int i=0;i<out.size();i++){
        cout << out[i] << " ";
    }
    return 0;
}

```

### **TIME COMPLEXITY:**

Worst case :  $O(N+K)$  //k represents the counter array size.

Average case :  $O(N+K)$

Best case :  $O(N+K)$

### **Points:**

- 1) Counting sort is faster than any sorting algorithms out there , and as well proven to be a stable algorithm.
- 2) But it doesn't work on decimal values , and not on larger sets of data
- 3) It is not an in place algorithm , since it uses extra space.