Capacitance & Inductance

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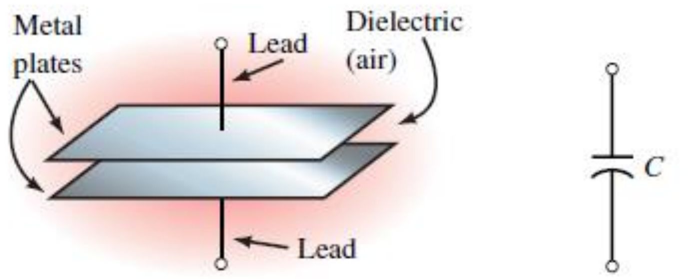
Capacitance

A capacitor consists of two conductors separated by an insulator. One of its basic forms is the parallel-plate capacitor shown in Fig. It consists of two metal plates separated by a non-conducting material (i.e., an insulator) called a **dielectric.** The dielectric may be air, oil, mica, plastic, ceramic, or other suitable insulating material



(a) Basic construction (b) Symbol

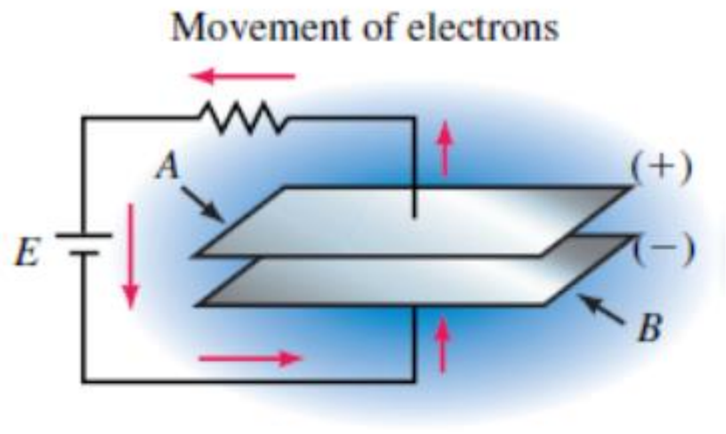


FIGURE Capacitor during charging. When the source is connected, electrons are removed from plate *A* and an equal number deposited on plate *B*. This leaves the top plate positively charged and the bottom plate negatively charged.

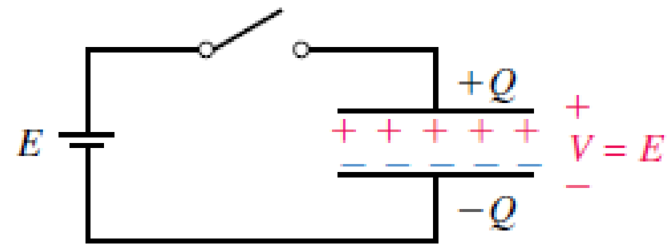


FIGURE Capacitor after charging. When the source is disconnected, electrons are trapped on the bottom plate. Thus, charge is stored.

The amount of charge Q that a capacitor can store depends on the applied voltage. Experiments show that for a given capacitor, Q is proportional to voltage. Let the constant of proportionality be C. Then,

$$Q = CV$$

$C = Q/V$ (farads, F)

The term C is defined as the capacitance of the capacitor and its unit is the farad. By definition, the capacitance of a capacitor is one farad if it stores one coulomb of charge when the voltage across its terminals is one volt.

The farad, however, is a very large unit. Most practical capacitors range in size from picofarads (pF or 10^{-12} F) to microfarads (uF or 10^{-6} F). The larger the value of C, the more charge that the capacitor can hold for a given voltage.

Capacitance is directly proportional to plate area, inversely proportional to plate separation, and dependent on the dielectric. In equation form,

$C = \epsilon A/d$ (F)

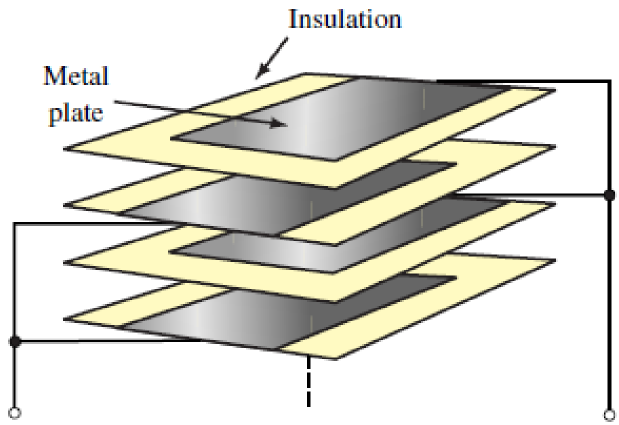
where area *A* is in square meters and spacing *d* is in meters . The constant in Equation is the **absolute dielectric constant** of the insulating material. Its units are farads per meter (F/m). For air or vacuum, it has a value of $\epsilon_0 = 8.85 \quad 10^{-12}$ F/m. For other materials, is expressed as the product of the relative dielectric constant, ϵ_r times ϵ_o. That is,

$$C = \epsilon A/d = \epsilon_r \epsilon_o A/d.$$

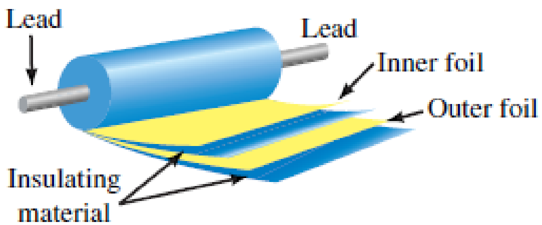
Types of Capacitors

Capacitors are made in a variety of types and sizes. There are fixed and variable types with differing dielectrics. Fixed capacitors are often identified by their dielectric. Common dielectric materials include ceramic, plastic, and mica, plus, for electrolytic capacitors, aluminum and tantalum oxide.

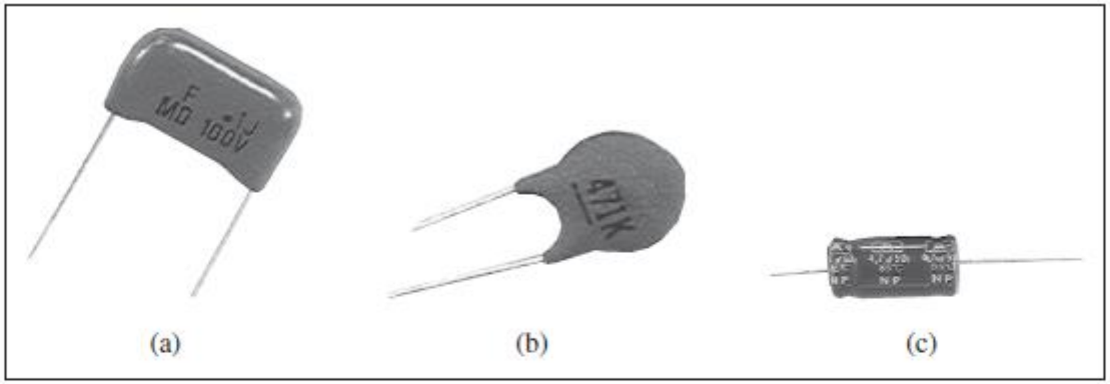
Design variations include tubular and interleaved plates. The interleave design uses multiple plates to increase effective plate area. A layer of insulation separates plates, and alternate plates are connected together. The tubular design uses sheets of metal foil separated by an insulator such as plastic film.



Stacked capacitor construction. The stack is compresse
attached, and the unit coated with epoxy resin or other insulating material.



Tubular capacitor with axial leads.



Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

Current and Voltage

To obtain the current-voltage relationship of the capacitor, we take the derivative of q= CV . Since i=dq/dt

Thus,
$$\boxed{i = C\frac{dv}{dt}}$$

This is the current-voltage relationship for a capacitor, assuming the passive sign convention.

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. We get

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

Where v(t) is the voltage across the capacitor at time

The instantaneous power delivered to the capacitor is

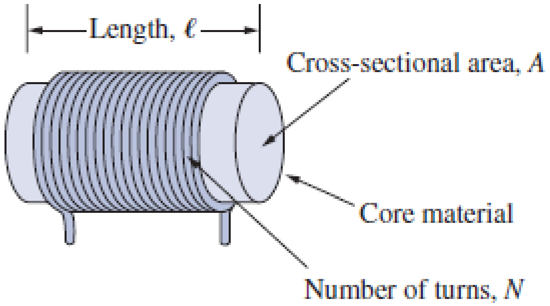
$$p = vi = Cv\frac{dv}{dt}$$

Inductance

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig.

An inductor consists of a coil of conducting wire.



Typical form of an inductor.

Inductance

By Faraday's law, the induced voltage is proportional to the rate of change of flux. In the SI system, the constant of proportionality is one and Faraday's law for this case may therefore be stated as

$$e = N \times \text{the rate of change of } \phi$$

$$e = N\frac{d\phi}{dt} \quad \text{(volts, V)}$$

where ϕ is in webers, t in seconds, and e in volts. Thus if the flux changes at the rate of 1 Wb/s in a 1 turn coil, the voltage induced is 1 volt.

Now consider an air-core inductor. Since not all flux lines pass through all windings, it is difficult to determine flux linkages as above. However, (since no ferromagnetic material is present) flux is directly proportional to current. In this case, then, since induced voltage is proportional to the rate of change of flux, and since flux is proportional to current, induced voltage will be proportional to the rate of change of current. Let the constant of proportionality be L. Thus,

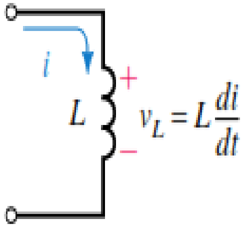
$$e = L \times \text{rate of change of current}$$

$$e = L\frac{di}{dt} \quad \text{(volts, V)}$$

The inductance of a coil is one henry if the voltage created by its changing current is one volt when its current changes at the rate of one ampere per second.

In practice, the voltage across an inductance is denoted by v_L rather than by e. Thus,

$$v_L = L\frac{di}{dt} \quad \text{(V)}$$



If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.
V=L di/dt

where *L* is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H), 1 henry equals 1 volt-second per ampere.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension and construction.

$$L = \frac{N^2 \mu A}{\ell}$$

where *N* is the number of turns, l is the length, *A* is the cross-sectional area, and u is the permeability of the core. We can see from Eq. that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

$$di = \frac{1}{L} v\, dt$$

$$i = \frac{1}{L} \int_{-\infty}^{t} v(\tau)\, d\tau$$

$$p = vi = \left(L \frac{di}{dt} \right) i$$

$$w = \int_{-\infty}^{t} p(\tau)\, d\tau = L \int_{-\infty}^{t} \frac{di}{d\tau}\, i d\tau$$

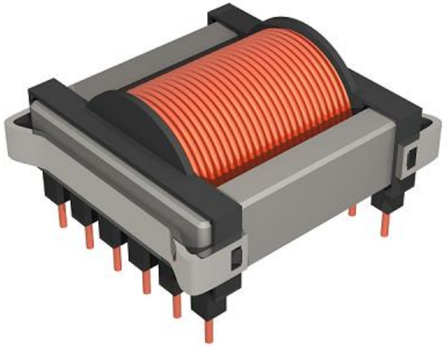
$$= L \int_{-\infty}^{t} i\, di = \frac{1}{2} L i^2(t) - \frac{1}{2} L i^2(-\infty)$$

Since $i(-\infty) = 0$,

$$\boxed{w = \frac{1}{2} L i^2}$$

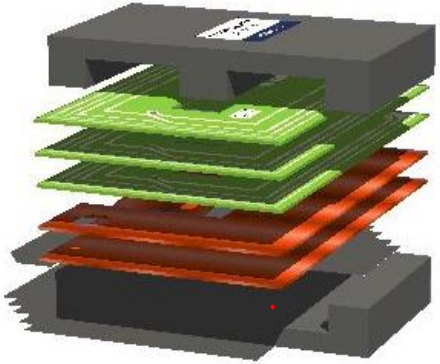
Types of Inductors

Air Core Inductor

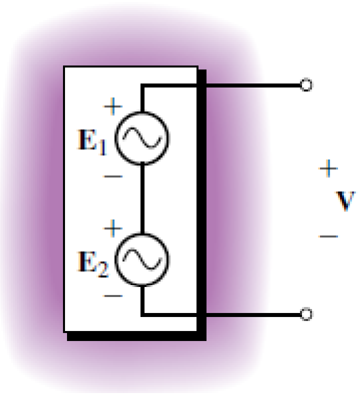


Iron core inductor

Toroidal inductor



Laminated core inductor



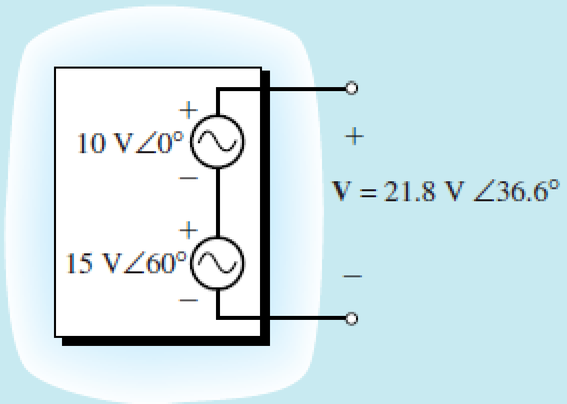
$e_1 = 10 \sin \omega t$ V. Thus, $\mathbf{E}_1 = 10\text{ V}\angle 0°$.

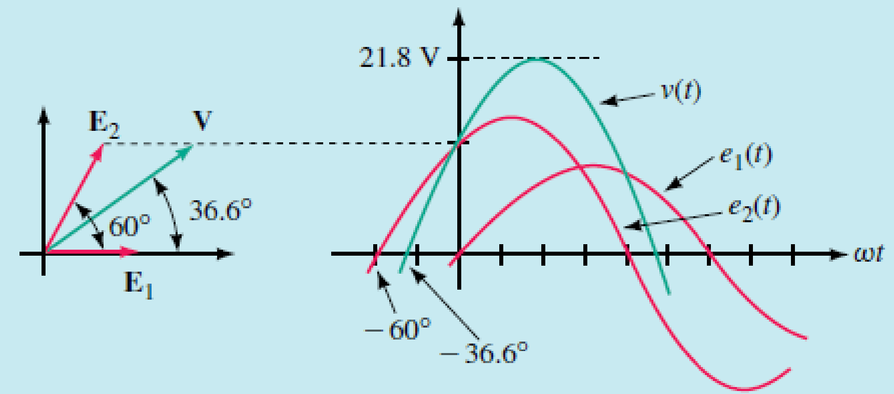
$e_2 = 15 \sin(\omega t + 60°)$ V. Thus, $\mathbf{E}_2 = 15\text{ V}\angle 60°$.

$$\mathbf{V} = \mathbf{E}_1 + \mathbf{E}_2 = 10\angle 0° + 15\angle 60° = (10 + j0) + (7.5 + j13)$$

$$= (17.5 + j13) = 21.8\text{ V}\angle 36.6°$$

$$v = 21.8 \sin(\omega t + 36.6°)\text{ V}$$





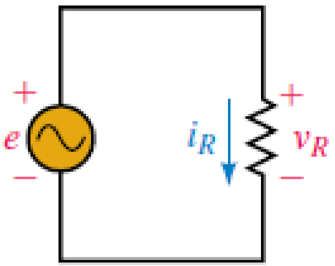
R, L and C Circuits with Sinusoidal Excitation

R, L, and *C* circuit elements each have quite different electrical properties. Resistance, for example, opposes current, while inductance opposes changes in current, and capacitance opposes changes in voltage. These differences result in quite different voltage-current relationships as you saw earlier.

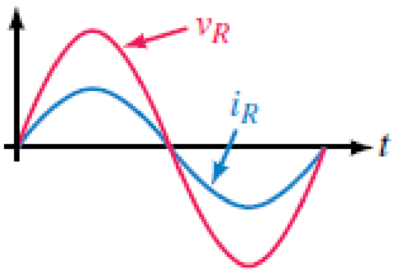
We now investigate these relationships for the case of sinusoidal ac. Sine waves have several important characteristics that you will discover from this investigation:

1. When a circuit consisting of linear circuit elements *R, L,* and *C* is connected to a sinusoidal source, all currents and voltages in the circuit will be sinusoidal.
2. These sine waves have the same frequency as the source and differ from it only in terms of their magnitudes and phase angles.

Resistance with Sine source



(a) Source voltage is a sine wave. Therefore, v_R is a sine wave.



(b) $i_R = v_R/R$. Therefore i_R is a sine wave.

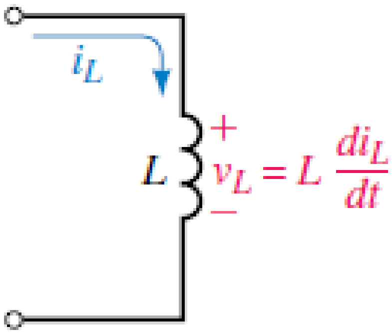
$$i_R = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R}\sin \omega t = I_m \sin \omega t$$

$$I_m = V_m/R$$

$$V_m = I_m R$$

The in-phase relationship is true regardless of reference. Thus, if vR = Vm sin(ωt + ϕ), then iR = Im sin(ωt + ϕ).

Inductance using Sine

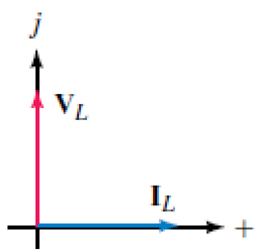


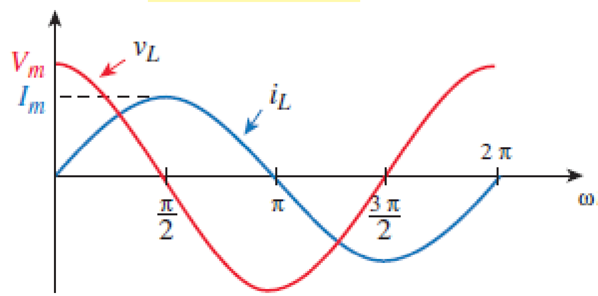
$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}(I_m \sin \omega t) = \omega L I_m \cos \omega t = V_m \cos \omega t$$

$$v_L = V_m \sin(\omega t + 90°)$$

$$\cos \omega t = \sin(\omega t + 90°),$$

$$V_m = \omega L I_m$$





Current always lags voltage by 90° regardless of the choice of reference

Inductive Reactance

$$\frac{V_m}{I_m} = \omega L$$

This ratio is defined as **inductive reactance** and is given the symbol *XL*. Since the ratio of volts to amps is ohms, reactance has units of ohms.

$$X_L = \frac{V_m}{I_m} \quad (\Omega)$$

$$X_L = \omega L \quad (\Omega)$$

Reactance XL represents the opposition that inductance presents to current for the sinusoidal ac case.

$$I_m = \frac{V_m}{X_L}$$

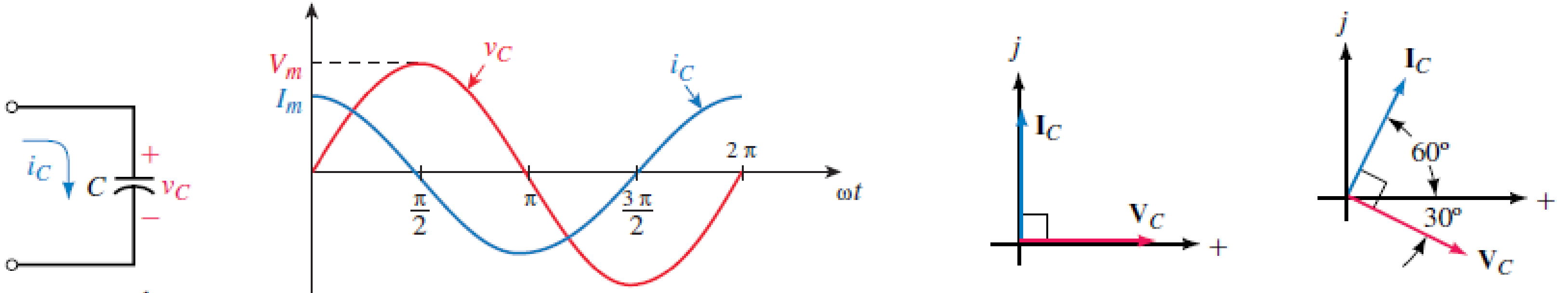
$$V_m = I_m X_L$$

The current through a 0.01-H inductance is $i_L = 20\sin(\omega t + 50°)$ A and $f = 60$ Hz. Determine v_L.

Capacitance and Sinusoidal AC

For capacitance, current is proportional to the rate of change of voltage, i.e., *iC = C dvC/dt* . Thus if *vC* is a sine wave,

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t = I_m \cos \omega t$$



(a) $i_C = C\frac{dv_C}{dt}$ (b) Waveforms with v_C as reference (c) $\mathbf{V}_e$ at 0° (d) $\mathbf{V}_C$ at −30°

$$i_C = I_m \sin(\omega t + 90°)$$

$$I_m = \omega C V_m$$

Capacitive Reactance

The ratio of Vm to Im is defined as capacitive reactance and is given the symbol XC. That is,

$$X_C = \frac{V_m}{I_m} \quad (\Omega)$$

$$X_C = \frac{1}{\omega C} \quad (\Omega)$$

Reactance XC represents the opposition that capacitance presents to current for the sinusoidal ac case.

$$I_m = \frac{V_m}{X_C}$$

$$V_m = I_m X_C$$

Impedance

The opposition that a circuit element presents to current in the phasor domain is defined as its **impedance.** The impedance of the element of is the ratio of its voltage phasor to its current phasor. Impedance is denoted by **Z.** Thus,

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{(ohms)}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V}{I}\angle\theta$$

$$\mathbf{Z} = Z\angle\theta$$

where V and I are the rms magnitudes of V and I respectively, and θ is the angle between them. Z = V/I. Since V = 0.707Vm and I = 0.707Im, Z can also be expressed as Vm/Im. Once the impedance of a circuit is known, the current and voltage can be determined using

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$

$$\mathbf{V} = \mathbf{IZ}$$

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of C, L, or R if sufficient data are provided (Fig. 14.18):

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 40°)$

b. $v = 1000 \sin(377t + 10°)$
   $i = 5 \sin(377t - 80°)$

c. $v = 500 \sin(157t + 30°)$
   $i = 1 \sin(157t + 120°)$

d. $v = 50 \cos(\omega t + 20°)$
   $i = 5 \sin(\omega t + 110°)$

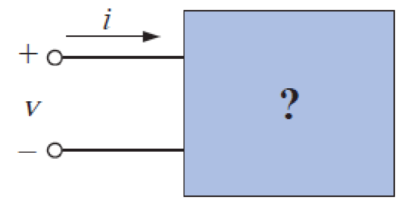
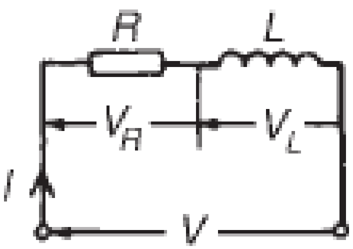


FIG. 14.18
Example 14.7.

Series RL Circuit

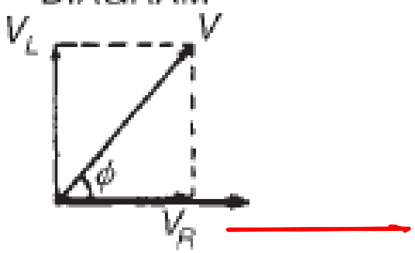


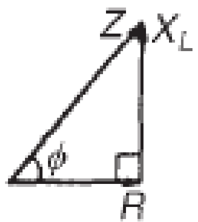
$$V = \sqrt{(V_R^2 + V_L^2)} \quad \text{(by Pythagoras' theorem)}$$

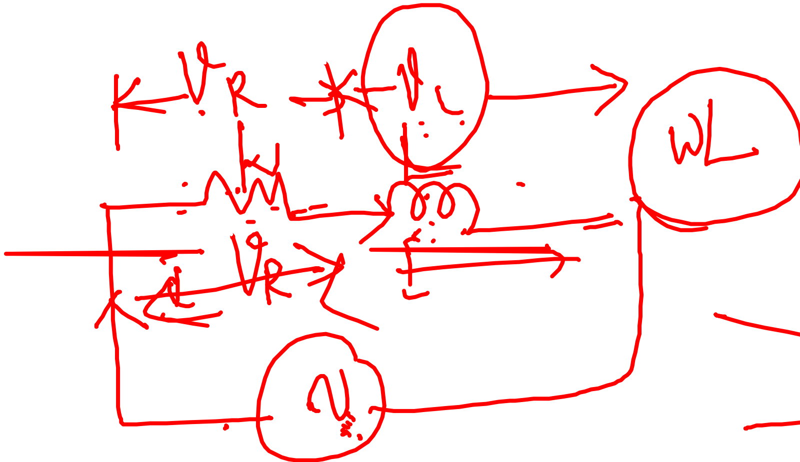
$$\tan\phi = \frac{V_L}{V_R}$$

$$Z = \sqrt{(R^2 + X_L^2)}$$

$$\tan\phi = \frac{X_L}{R}$$

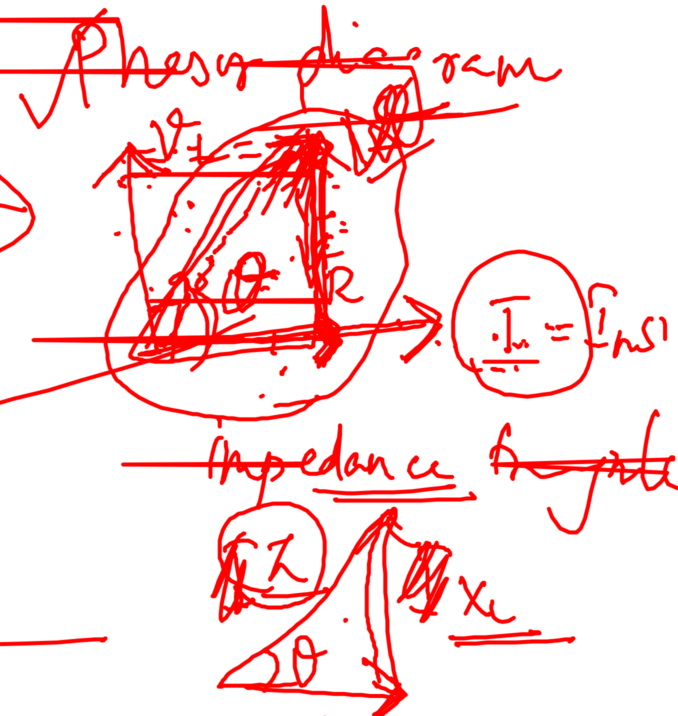






$v = v_R + v_L$

Phasor diagram



$V_m \sin \omega t = v$

$v_R + v_L$

$v = \sqrt{v_R^2 + v_L^2}$

IR

$Z = \sqrt{R^2 + X_L^2}$

$I = \frac{v}{Z}$

$\tan\theta = \frac{V_L}{V_R}$

$\theta = \tan^{-1} \frac{V_L}{V_R}$

$\theta = \tan^{-1}(X_L/R)$

$v = I.Z$

In a series *R–L* circuit the p.d. across the resistance *R* is 12V and the p.d. across the inductance *L* is 5V. Find the supply voltage and the phase angle between current and voltage.

$$V_R = 12V$$

$$V_L = 5V$$

$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{12^2 + 5^2}$$

$$= 144 + 25$$

$$= 13\ V$$

A coil has a resistance of 4 Ω and an inductance of 9.55mH. Calculate (a) the reactance, (b) the impedance, and (c) the current taken from a 240V, 50Hz supply. Determine also the phase angle between the supply voltage and current

$$X_L = 2\pi f L = \_\_\_\_\_\_ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \quad \Omega$$

$$I = V/Z = I_m \angle \theta \qquad v = V_m \sin \omega t$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = 40^\circ \qquad i = I_m (\sin \omega t$$

$$40^\circ$$

A pure inductance of 1.273mH is connected in series with a pure resistance of 30. If the frequency of the sinusoidal supply is 5kHz and the p.d. across the 30Ω resistor is 6V, determine the value of the supply voltage and the voltage across the 1.273mH inductance. Draw the phasor diagram.

$X_L = 2\pi f L \ \Omega$

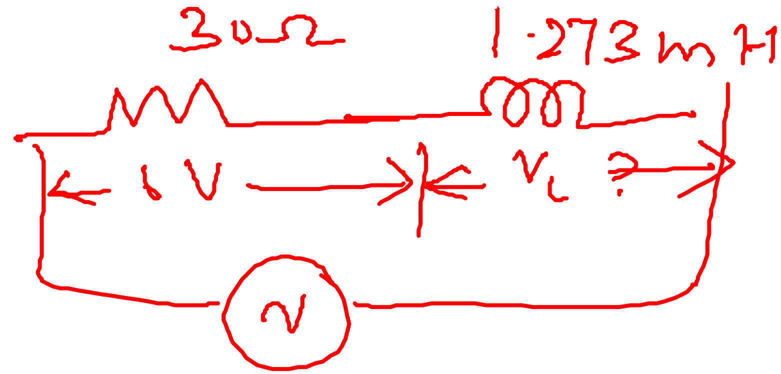
$R =$

$Z =$

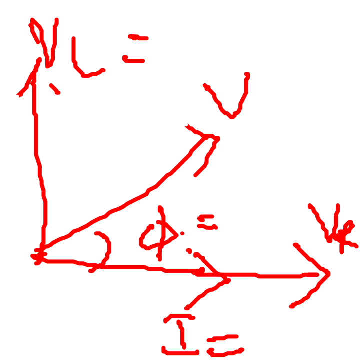
$\phi =$

$\tan\phi = V_L / V_R \rightarrow V_L = ?$

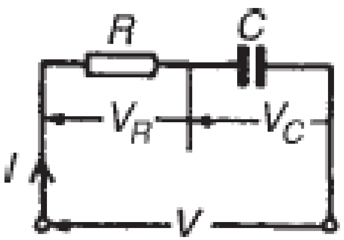
$V = ?$

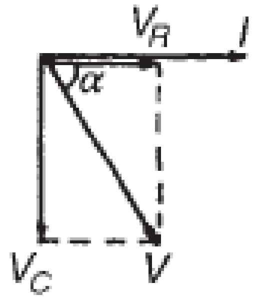


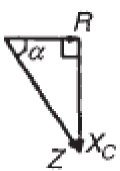
5KHZ



Series RC Circuit







$$V = \sqrt{(V_R^2 + V_C^2)}$$

$$\tan\alpha = \frac{V_C}{V_R}$$

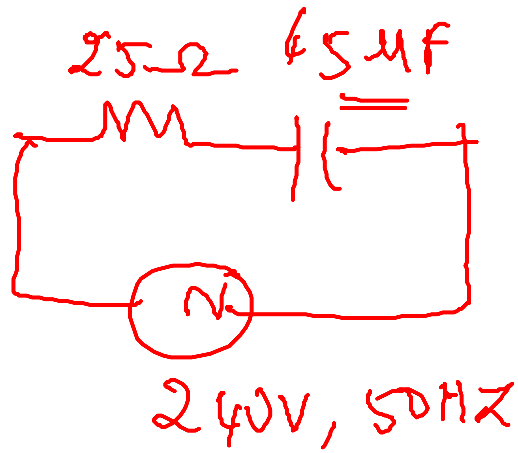
$$Z = \sqrt{(R^2 + X_C^2)}$$

$$\tan\alpha = \frac{X_C}{R}$$

A resistor of 25 is connected in series with a capacitor of 45µF. Calculate (a) the impedance, and (b) the current taken from a 240V, 50Hz supply. Find also the phase angle between the supply voltage and the current.

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{2\pi f C} =$$



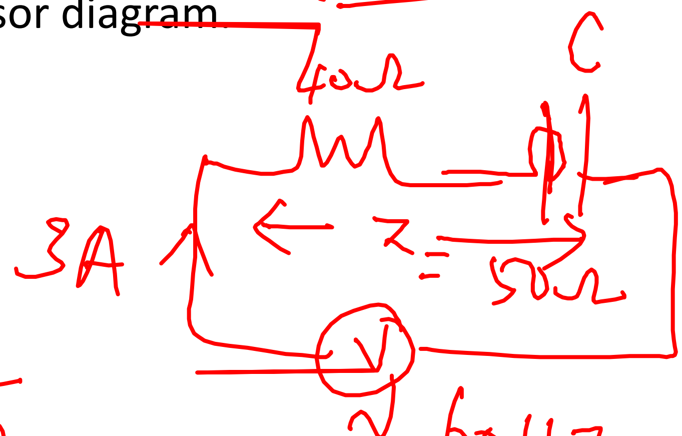
$$Z = 25^2 + X_C^2 = ?$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$I = V/Z = \frac{240}{\quad}$$

$$=$$

A capacitor C is connected in series with a 40Ω resistor across a supply of frequency 60Hz. A current of 3 A flows and the circuit impedance is 50 Ω. Calculate: (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.



60Hz = ?

$Z = \sqrt{R^2 + X_c^2}$

$X_c = ?$

$X_c = \frac{1}{\omega C}$

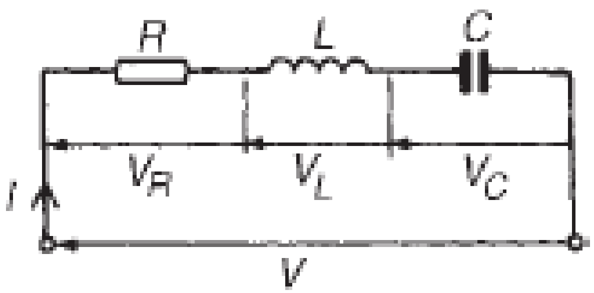
$V = I \cdot Z$

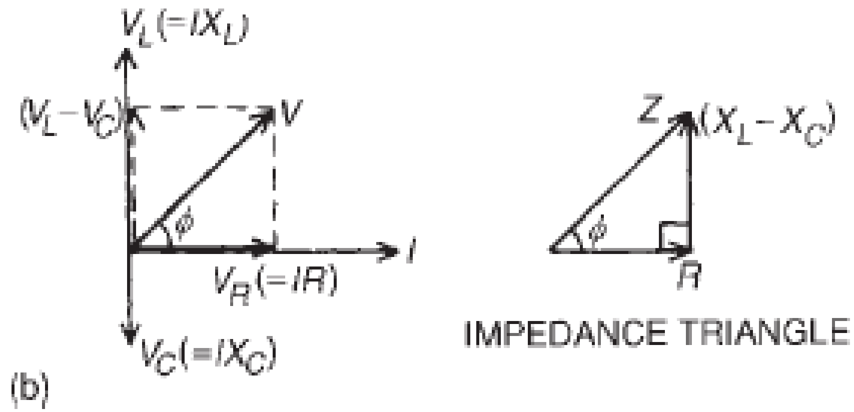
$V_c = I \cdot X_c$

$\theta = \tan^{-1}\left(\frac{X_c}{R}\right)$

$V_R \cancel{= I R}$

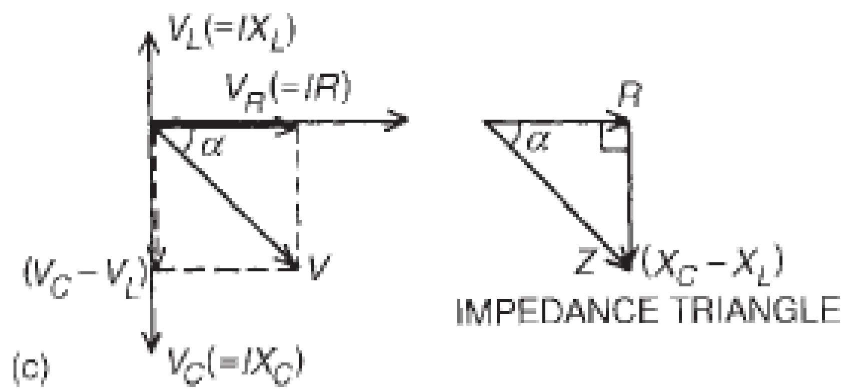
Series RLC Circuit





$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

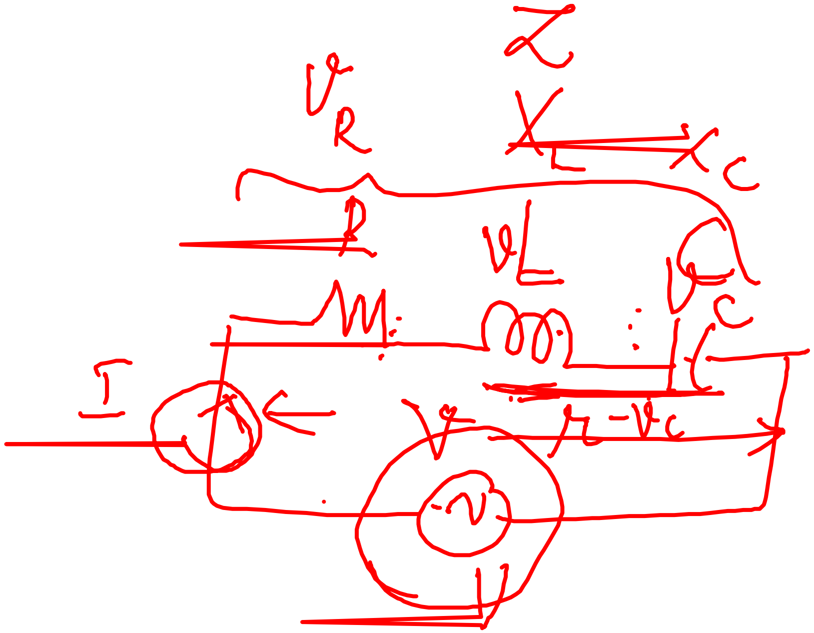
$$\tan\phi = \frac{(X_L - X_C)}{R}$$



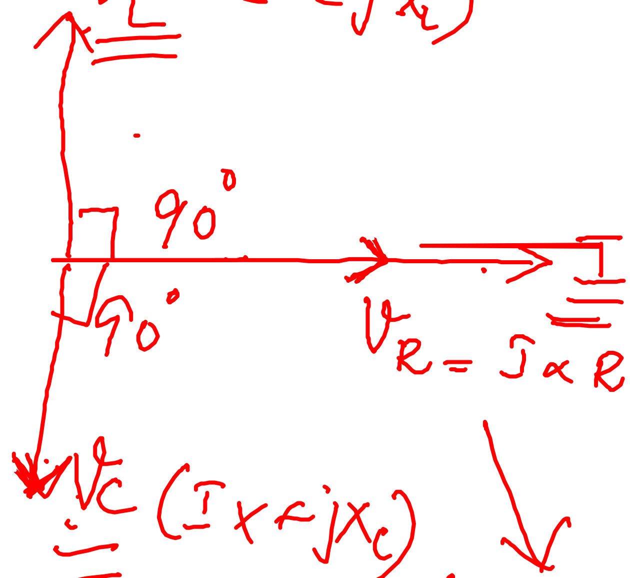
$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$\tan\alpha = \frac{(X_C - X_L)}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



$V_L = I(jX_L)$



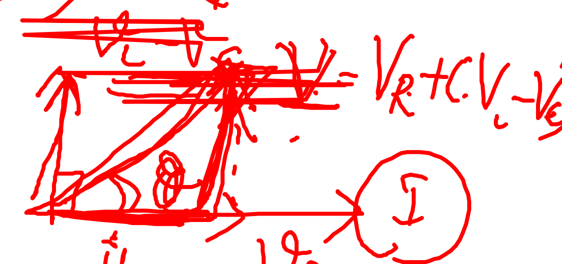
$V_R = I \times R$

$V_C (I \times -jX_C)$

$X_C > X_L$

$X_L > X_C$

$V_L > V_C$

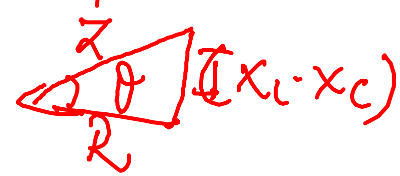


$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{\ }\right)$$

$$\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

imp



A coil of resistance 5Ω and inductance 120mH in series with a 100μF capacitor, is connected to a 300V, 50Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

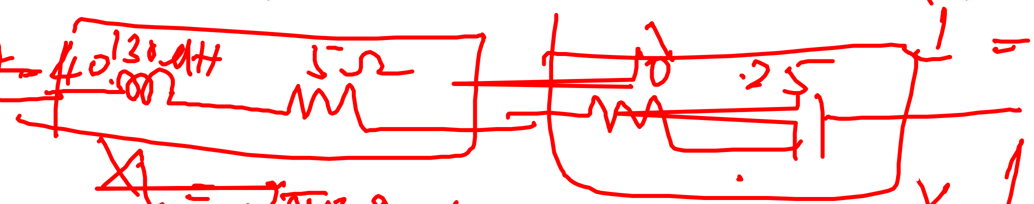
The following three impedances are connected in series across a 40V, 20kHz supply: (i) a resistance of 8Ω, (ii) a coil of inductance 130μH and 5Ω resistance, and (iii) a 10 Ω resistor in series with a 0.25μF capacitor. Calculate (a) the circuit current, (b) the circuit phase angle and (c) the voltage drop across each impedance.

$V_2 = I \cdot Z_2$

$V_3 = I \cdot Z_3$

$V_1 = I \cdot Z$ $\times 8$

$I = V/Z = \frac{40}{Z}$



$X_L = 2\pi \times 20k\,Hz \times 130\mu H$

$= 5200\pi$

$X_C = \frac{1}{2\pi f C}$

$= \frac{1}{2\pi \times 20k \times .25\times10^{-6}}$

$100\,\Omega$

40V 20kHz

$Z_1 = 8$

$Z_2 = 5 + j16.3\,\Omega$

$Z_3 = 10 - j10\,\Omega$

$Z = 23 + j6.3$

$Z = \sqrt{23^2 + 6.3^2}$

$\tan^{-1} \frac{6.3}{23}$

$\theta = \theta°$

Determine the p.d.'s V1 and V2 for the circuit shown in Figure if the frequency of the supply is 5kHz. Draw the phasor diagram and hence determine the supply voltage V and the circuit phase angle.