POWER IN AC CIRCUITS

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Power in AC Circuits

The power to the AC circuit is varying with respect to time, If we consider v and i of general case

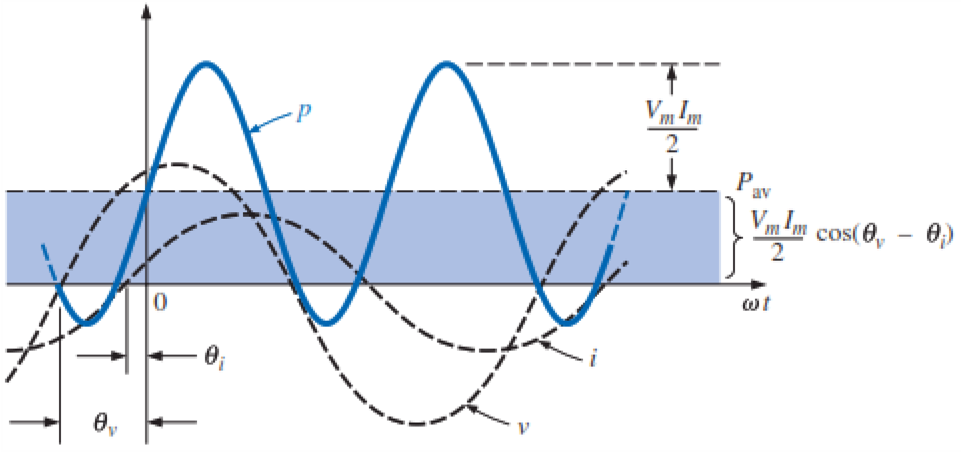
$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

Then power is defined by,

$$\begin{aligned} p = vi &= V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i) \\ &= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \end{aligned}$$

$$p = \overbrace{\left[\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t\text{)}}$$



Note that the second factor in the preceding equation is a cosine wave with an amplitude of VmIm /2 and with a frequency twice that of the voltage or current. The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction.

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the average power.

The average power, or real power as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks.

The angle (ɵv - ɵi) is the phase angle between v and i. Since cos(-a) = cos a, the magnitude of average power delivered is independent of whether v leads i or i leads v.

Defining ɵ as equal to |ɵv - ɵi|, where | |indicates that only the magnitude is important and the sign is immaterial, we have

P= VmIm/2 Cosɵ

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{\text{eff}} I_{\text{eff}} \cos\theta}$$

Power in R, L and C

In a purely resistive circuit, since v and i are in phase, |ɵv - ɵi| 0°, and cos 0° 1, so that

$$\boxed{P = \frac{V_m I_m}{2} = V_{\text{eff}} I_{\text{eff}}} \qquad \text{(W)}$$

$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R}$$

$$\boxed{P = \frac{V_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R} \qquad \text{(W)}$$

In a purely inductive circuit, since v leads i by 90°, |ɵv - ɵi| is |-90°| = 90°. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

In a purely capacitive circuit, since i leads v by 90°, $|\theta v - \theta i|$ is $|90°|=$ 90°. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

Power Factor

In the equation $P \ (V_m I_m /2)\cos\Theta_v$, the factor that has significant control over the delivered power level is the $\cos \theta$. No matter how large the voltage or current, if $\cos \theta$ is 0, the power is zero; if $\cos \theta$ is 1, the power delivered is a maximum. Since it has such control, the expression was given the name power factor and is defined by Power factor = F_p = $\cos \theta$

For a purely resistive load, the phase angle between v and i is 0° and cos 0° = 1. The power delivered is a maximum of $(V_m I_m/2)$

For a purely reactive load (inductive or capacitive), the phase angle between v and i is 90° and cos 90° = 0. The power delivered is then the minimum value of zero watts, even though the current has the same peak value as that encountered by resistor.

For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0.

In terms of the average power and the terminal voltage and current,

$$\boxed{F_p = \cos\theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}}}$$

The terms leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load.

If the current leads the voltage across a load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor.

In other words, capacitive networks have leading power factors, and inductive networks have lagging power factors.

Complex Power

Complex power is important in power analysis because it contains all the information pertaining to the power absorbed by a given load. Given the phasor forı $V_m\angle\theta_v$ a and $I_m\angle\theta_i$ of voltage and current, the complex power S absorbed by the ac load is the product of the voltage and the complex conjugate of the current,

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$\begin{aligned}\mathbf{S} &= V_{\text{rms}}I_{\text{rms}}\angle\theta_v - \theta_i \\ &= V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i)\end{aligned}$$

The magnitude of the complex power is the apparent power; hence, the complex power is measured in volt-amperes (VA). Also, the angle of the complex power is the power factor angle.

The complex power may be expressed in terms of the load impedance Z. The load impedance Z may be written as

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}}\angle\theta_v - \theta_i \qquad \mathbf{S} = I_{\text{rms}}^2\mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^*$$

$$\mathbf{S} = I_{\text{rms}}^2(R + jX) = P + jQ$$

where P and Q are the real and imaginary parts of the complex power; that is,

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

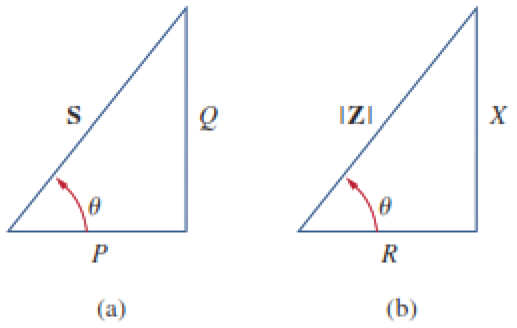
P is the average or real power and it depends on the load's resistance R.
Q depends on the load's reactance X and is called the reactive (or quadrature) power.

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

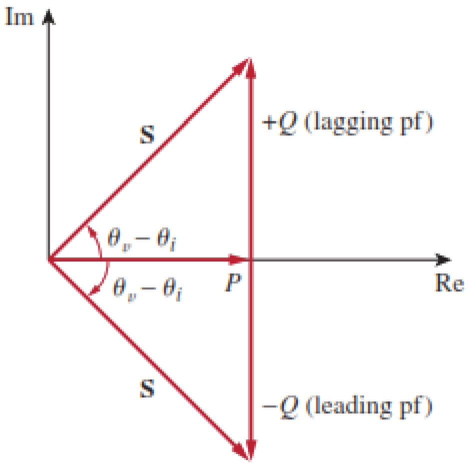
The real power P is the average power in watts delivered to a load; it is the only useful power. It is the actual power dissipated by the load.
The reactive power Q is a measure of the energy exchange between the source and the reactive part of the load.
The unit of Q is the volt-ampere reactive (VAR) to distinguish it from the real power, whose unit is the watt.
The energy storage elements neither dissipate nor supply power, but exchange power back and forth with the rest of the network. In the same way, the reactive power is being transferred back and forth between the load and the source. It represents a lossless interchange between the load and the source.

1. Q = 0 for resistive loads (unity pf).
2. Q < 0 for capacitive loads (leading pf).
3. Q > 0 for inductive loads (lagging pf).

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$
$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$



(a) Power triangle, (b) impedance triangle.



Power triangle.

v(t) = 120 cos(377t+ 45) V and i(t)= 10 cos(377t -10) A find the instantaneous power and the average power absorbed by the passive linear network.

$$p = \overbrace{\left[\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t\text{)}}$$

$$p = \frac{1200}{2}(\cos 55) - 600 \cos(754t + 35)$$

$$P_{avg} = \frac{V_m I_m}{2} \cos\theta$$

$$= 600 \cos 55$$

$$= 344 \text{ Watts}$$

Calculate the average power absorbed by an impedance Z = 30 + j70 when a voltage V= 120∠0   is applied across it.

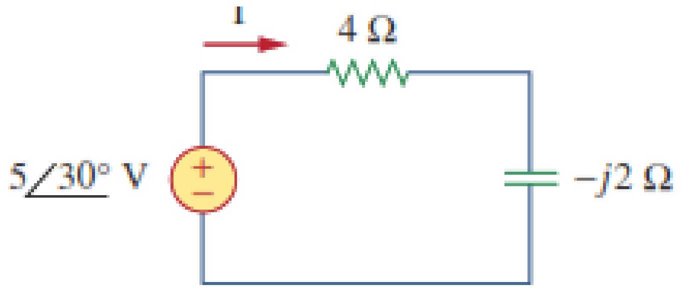
$$Z = 30 + j70 = 76.15 \angle 66.8$$

$$I = \frac{V}{Z} = \frac{120}{76.15} = 1.58\ A$$

$$P_{avg} = VI\cos\theta$$

$$= 74.5 \text{ Watts}$$

For the circuit shown in Fig., find the average power supplied bythe source and the average power absorbed by the resistor.



$$P_{total} = \frac{V_m I_m}{2}\cos\theta = \frac{5 \times 1.12}{2}\cos(-26.57) = 2.5W$$

$$Z = 4 - j2$$

$$I = \frac{5\angle 30^\circ}{4 - j2} = 1.12\angle 56.57^\circ A$$

$$P_R = \frac{V_R I_R}{2}\cos\theta = 2.5W$$

$$V_R = 1.12 \times 4 = 4.48$$

The voltage across a load is v(t) = 60 cos(wt - 10) V and the current through the element in the direction of the voltage drop is i(t) = 1.5 cos(wt+ 50) A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

$$S = \frac{V_m I_m}{2} \angle \theta_v - \theta_i$$

$$S = 45 \angle -60^\circ$$

Apparent = 45 VA

$$P = \frac{V_m I_m}{2} \cos\theta$$

$$= 45 \times 0.5 = 22.5 \text{ VA}$$

$$Q = \frac{V_m I_m}{2} \sin\theta$$

$$= -38.97 \text{ VAR}$$

$$Z = \frac{V}{I} = 40\,\Omega \quad \theta = -60^\circ$$

$$PF = \cos\theta = 0.5 \text{ (leading)}$$

A series-connected load draws a current i(t)= 4 cos(100 πt +10) A when the applied voltage is v(t) = 120 cos(100πt- 20) V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

$V = 120\angle -20^\circ$

$I = 4\angle 10^\circ$

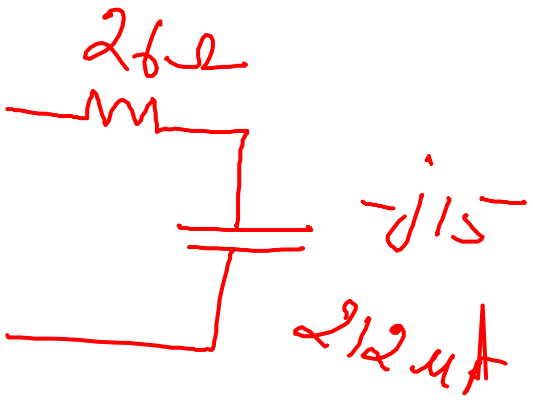
$Z = 30\angle -30^\circ$

$= 26 - j15$

$\omega = 100\pi$

$2\pi f = 100\pi$

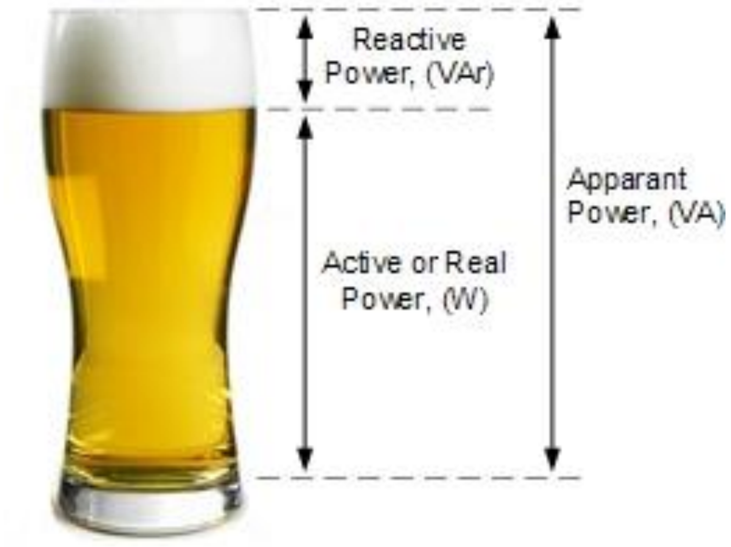
$f = 50$



Reactive power - Analogy

The main advantage of using AC electrical power distribution is that the supply voltage level can be changed using transformers, but transformers and induction motors of household appliances, air conditioners and industrial equipment all consume reactive power which takes up space on the transmission lines since larger conductors and transformers are required to handle the larger currents which you need to pay for.

Pay for a full glass of beer but drink only the actual liquid beer itself which, on many occasions, is always less than a full glass.
This is because the head (or froth) of the beer takes up additional wasted space in the glass leaving less room for the real liquid beer that you consume, and the same idea is true in many ways for reactive power.



But for many industrial power applications, reactive power is often useful for an electrical circuit to have. While the real or active power is the energy supplied to run a motor, heat a home, or illuminate an electric light bulb, reactive power provides the important function of regulating the voltage thereby helping to move power effectively through the utility grid and transmission lines to where it is required by the load.

While reducing reactive power to help improve the power factor and system efficiency is a good thing, one of the disadvantages of reactive power is that a sufficient quantity of it is required to control the voltage and overcome the losses in a transmission network. This is because if the electrical network voltage is not high enough, active power cannot be supplied. But having too much reactive power flowing around in the network can cause excess heating (I^2*R losses) and undesirable voltage drops and loss of power along the transmission lines.