Updating Random Effects Necessary or Not

In joint model dynamic survival probability, a common scenario is dynamically changing time to event information while keeping the same longitudinal information. Since the dynamic survival probability in a joint model depends on both time to event and longitudinal parts, if one of them changes, we also need to update the random effects. This can create problems when we have to do this operation for thousands of patients in a simulation setting.

Consider a patient j, with the true time of event T_j^*. Let us assume that the last time we checked up on him if the event occurred was at time t_1. That is, $T_j^* > t_1$. Let $Y_j(s)$ be the history of longitudinal measurements available up to time $s \geq t_1$. Given a training dataset D_n, the posterior predictive survival probability for T_j^* at a time $u \geq t_1$ is given by:

$$p\{T_j^* \geq u \mid T_j^* > t_1, Y_j(s), D_n\} = \int\int p(T_j^* \geq u \mid T_j^* > t_1, b_j, \theta)p\{b_j \mid T_j^* > t_1, Y_j(s), \theta\}$$
$$\times\, p(b_j \mid \theta)p(\theta \mid D_n)db_j d\theta \qquad \text{-(1)}$$

Here $p\{b_j \mid T_j^* > t_1, Y_j(s), \theta\}$ is the random effects distribution. Now, suppose that we get a new information that event has not occurred until a new time $t_2 > t_1$. That is, $T_j^* > t_2$. Then the posterior predictive survival probability for T_j^* at a time v is given by:

$$p\{T_j^* \geq v \mid T_j^* > t_2, Y_j(s), D_n\}, \quad v \geq t_2 \qquad \text{- (2)}$$

Here we will require recalculation of the random effects distribution. The new distribution will be

$p\{b_j \mid T_j^* > t_2, Y_j(s), \theta\}$. However, there is a way to skip this calculation.

Since $T_j^* > t_2$, and $t_2 > t_1$ it means that $T_j^* > t1$. Thus the Equation (2) can be rewritten as

$$p\{T_j^* \geq v \mid T_j^* > t_2, T_j^* > t_1\ Y_j(s), D_n\}, \quad v \geq t_2 \qquad \text{- (3)}$$

By rules of conditional probability the Equation (3) can be rewritten as

$$\frac{p\{T_j^* \geq v \wedge T_j^* > t_2 | T_j^* > t_1\ Y_j(s), D_n\}}{p\{T_j^* > t_2 | T_j^* > t_1, Y_j(s), D_n\}}, \quad v \geq t_2 \qquad \text{- (4)}$$

Where $\wedge$ is the intersection or AND operator. Since $v \geq t_2$, the intersection $T_j^* \geq v \wedge T_j^* > t_2$ can simply be written as $T_j^* \geq v$. Thus, Equation (4) then becomes:

$$\frac{p\{T_j^* \geq v | T_j^* > t_1\ Y_j(s), D_n\}}{p\{T_j^* > t_2 | T_j^* > t_1, Y_j(s), D_n\}}, v \geq t_2 \qquad \text{- (5)}$$

Equation (2) and Equation (5) are just simple mathematical transformations of each other. However, in Equation (4), the conditional part is same as Equation (1). Thus if we know the random effects distribution from Equation (1), we can reuse it to calculate Equation (2), without updating the random effects.