Supplementary Material for "A Personalized Screening Strategy to Monitor the Development of Chronic Allograft Failure in Renal Rransplant Recipients"

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Appendix A Joint Modeling Framework

We start with the definition of the joint modeling framework that will be used to fit a model to the kidney transplant dataset, and then to plan biomarker measurements for future patients. Let T_i^* denote the true graft failure time, and C_i denote the censoring time for the *i*-th patient. Let $T_i = \min(T_i^*, C_i)$ denote the observed graft failure time and $\delta_i = I(T_i^* < C_i)$ the event indicator for the *i*-th patient. The indicator function $I(\cdot)$ takes the value 1 when $T_i^* < C_i$ and 0 otherwise. Let y_{i1} and y_{i2} denote the log transformed $n_{i1} \times 1$ and $n_{i2} \times 1$ vectors of protein creatinine ratio (PCR) and serum creatinine (SCr) levels, respectively, for the *i*-th patient. For a sample of n patients the observed data is denoted by $\mathcal{D}_n = \{T_i, \delta_i, y_{i1}, y_{i2}; i = 1, \dots, n\}$.

The two outcomes, PCR and SCr are continuous in nature and thus to model them the joint model utilizes a multivariate linear mixed effects model (LMM). The PCR outcome is modeled as (model for SCr is same):

$$y_{i1}(t) = m_{i1}(t) + \varepsilon_{i1}(t)$$

= $\boldsymbol{x}_i^T(t)\boldsymbol{\beta}_1 + \boldsymbol{z}_i^T(t)\boldsymbol{b}_{i1} + \varepsilon_{i1}(t),$

where $x_i(t)$ denotes the row vector of the common design matrix for fixed effects of both PCR and SCr, respectively, and $z_i(t)$ denotes the same for random effects. The corresponding fixed effects are denoted by β_1 , β_2 and the complete vector of random effects by $\mathbf{b}_i = (\mathbf{b}_{i1}, \mathbf{b}_{i2})^T$. The complete vector of random effects are assumed to be normally distributed with mean zero and variance-covariance matrix \mathbf{D} . The true and unobserved PCR and SCr levels at time t are denoted by $m_{i1}(t)$ and $m_{i2}(t)$, respectively. Unlike $y_{i1}(t), y_{i2}(t)$, the former are not contaminated with the measurement errors $\varepsilon_{i1}(t)$ and $\varepsilon_{i2}(t)$, respectively. The errors are assumed to be normally distributed with mean zero and variance σ_1^2 and σ_2^2 , respectively, and are independent of the random effects $\mathbf{b}_{i1}, \mathbf{b}_{i2}$.

To model the effect of the two longitudinal outcomes on hazard of graft failure, joint models utilize a relative risk sub-model. The hazard of graft failure for patient i at any time point t, denoted by $h_i(t)$, depends on a function of subject specific linear predictors $m_{i1}(t)$, m_{i2} and/or the random effects:

$$h_{i}(t \mid \mathcal{M}_{i1}(t), \mathcal{M}_{i2}(t), \boldsymbol{w}_{i}) = \lim_{\Delta t \to 0} \frac{\Pr\{t \leq T_{i}^{*} < t + \Delta t \mid T_{i}^{*} \geq t, \mathcal{M}_{i1}(t), \mathcal{M}_{i2}(t), \boldsymbol{w}_{i}\}}{\Delta t}$$

$$= h_{0}(t) \exp\left[\gamma^{T} \boldsymbol{w}_{i} + f_{1}\{\mathcal{M}_{i1}(t), \boldsymbol{b}_{i1}, \boldsymbol{\alpha}_{1}\} + f_{2}\{\mathcal{M}_{i1}(t), \boldsymbol{b}_{i2}, \boldsymbol{\alpha}_{2}\}\right], \quad t > 0,$$

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where $\mathcal{M}_{i1}(t) = \{m_{i1}(v), 0 \leq v \leq t\}$, and $\mathcal{M}_{i2}(t) = \{m_{i2}(v), 0 \leq v \leq t\}$ denote the history of the underlying PCR and SCr levels, respectively, up to time t. The vector of baseline covariates is denoted by \boldsymbol{w}_i , and $\boldsymbol{\gamma}$ are the corresponding parameters. The function $f_1(\cdot), f_2(\cdot)$ parametrized by vectors $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2$ specify the functional form of PCR and SCr levels (Brown, 2009; Rizopoulos, 2012; Rizopoulos et al., 2014; Taylor et al., 2013) that are used in the linear predictor of the relative risk model. Some functional forms of PCR, relevant to the problem at hand are the following (functional forms for SCr are similar):

$$\begin{cases} f_1\{M_{i1}(t), \boldsymbol{b}_{i1}, \boldsymbol{\alpha}_1\} = \alpha_1 m_{i1}(t), \\ f_1\{M_{i1}(t), \boldsymbol{b}_{i1}, \boldsymbol{\alpha}_1\} = \alpha_{11} m_{i1}(t) + \alpha_{12} m'_{i1}(t), & \text{with } m'_{i1}(t) = \frac{\mathrm{d} m_{i1}(t)}{\mathrm{d} t}. \end{cases}$$

These formulations of $f_1(\cdot)$ postulate that the hazard of graft failure at time t may be associated with the underlying level $m_{i1}(t)$ of PCR (and/or SCr) at t, or with both the level and velocity $m'_{i1}(t)$ of PCR (and/or SCr) at t. Lastly, $h_0(t)$ is the baseline hazard at time t, and is modeled flexibly using P-splines. More specifically:

$$\log h_0(t) = \gamma_{h_0,0} + \sum_{q=1}^{Q} \gamma_{h_0,q} B_q(t, \boldsymbol{v}),$$

where $B_q(t, \mathbf{v})$ denotes the q-th basis function of a B-spline with knots $\mathbf{v} = v_1, \dots, v_Q$ and vector of spline coefficients γ_{h_0} . To avoid choosing the number and position of knots in the spline, a relatively high number of knots (e.g., 15 to 20) are chosen and the corresponding B-spline regression coefficients γ_{h_0} are penalized using a differences penalty (Eilers and Marx, 1996).

Appendix A.1 Parameter Estimation

We estimate parameters of the joint model using Markov chain Monte Carlo (MCMC) methods under the Bayesian framework. Let θ denote the vector of the parameters of the joint model. The joint model postulates that given the random effects, time to graft failure and longitudinal responses taken over time are all mutually independent. Under this assumption the posterior distribution of the parameters is given by:

$$p(\boldsymbol{\theta}, \boldsymbol{b} \mid \mathcal{D}_n) \propto \prod_{i=1}^n p(T_i, \delta_i, \boldsymbol{y}_{i1}, \boldsymbol{y}_{i2} \mid \boldsymbol{b}_i, \boldsymbol{\theta}) p(\boldsymbol{b}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})$$

$$\propto \prod_{i=1}^n p(T_i, \delta_i \mid \boldsymbol{b}_i, \boldsymbol{\theta}) p(\boldsymbol{y}_{i1} \mid \boldsymbol{b}_{i1}, \boldsymbol{\theta}) p(\boldsymbol{y}_{i2} \mid \boldsymbol{b}_{i2}, \boldsymbol{\theta}) p(\boldsymbol{b}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}),$$

$$p(\boldsymbol{b}_i \mid \boldsymbol{\theta}) = \frac{1}{\sqrt{(2\pi)^q \det(\boldsymbol{D})}} \exp(\boldsymbol{b}_i^T \boldsymbol{D}^{-1} \boldsymbol{b}_i),$$

where the likelihood contribution of PCR conditional on random effects is (contribution of SCr can be derived similarly):

$$p(\mathbf{y}_{i1} \mid \mathbf{b}_{i1}, \boldsymbol{\theta}) = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)^{n_i}} \exp\left(-\frac{\|\mathbf{y}_{i1} - \mathbf{X}_{i1}\boldsymbol{\beta}_1 - \mathbf{Z}_{i1}\mathbf{b}_{i1}\|^2}{\sigma_1^2}\right),$$

$$\mathbf{X}_{i1} = \{\mathbf{x}_{i1}(t_{i11})^T, \dots, \mathbf{x}_{i1}(t_{i1n_i})^T\}^T,$$

$$\mathbf{Z}_{i1} = \{\mathbf{z}_{i1}(t_{i11})^T, \dots, \mathbf{z}_{i1}(t_{i1n_i})^T\}^T.$$

The likelihood contribution of the time to graft failure outcome is given by:

$$p(T_i, \delta_i \mid \boldsymbol{b}_i, \boldsymbol{\theta}) = h_i(T_i \mid \mathcal{M}_{i1}(s), \mathcal{M}_{i2}(s), \boldsymbol{w}_i)^{\delta_i} \exp\Big\{ - \int_0^{T_i} h_i(s \mid \mathcal{M}_{i1}(s), \mathcal{M}_{i2}(s), \boldsymbol{w}_i) ds \Big\}.$$
(1)

The integral in (1) does not have a closed-form solution, and therefore we use a 15-point Gauss-Kronrod quadrature rule to approximate it.

We use independent normal priors with zero mean and variance 100 for the fixed effects β_1, β_2 , and inverse Gamma prior with shape and rate both equal to 0.01 for the parameters σ_1^2, σ_2^2 . For the variance-covariance matrix D of the random effects we take inverse Wishart prior with an identity

scale matrix and degrees of freedom equal to q (number of random effects). For the relative risk model's parameters γ and the association parameters α , we use a global-local ridge-type shrinkage prior. For example, for the s-th element of α we assume (similarly for γ):

$$\alpha_s \sim \mathcal{N}(0, \tau \psi_s), \quad \tau^{-1} \sim \text{Gamma}(0.1, 0.1), \quad \psi_s^{-1} \sim \text{Gamma}(1, 0.01).$$

The global smoothing parameter τ has sufficiently mass near zero to ensure shrinkage, while the local smoothing parameter ψ_s allows individual coefficients to attain large values. For the penalized version of the B-spline approximation to the baseline hazard, we use the following prior for parameters γ_{h_0} (Lang and Brezger, 2004):

$$p(\gamma_{h_0} \mid \tau_h) \propto \tau_h^{\rho(\mathbf{K})/2} \exp\left(-\frac{\tau_h}{2} \gamma_{h_0}^T \mathbf{K} \gamma_{h_0}\right),$$

where τ_h is the smoothing parameter that takes a Gamma(1, 0.005) hyper-prior in order to ensure a proper posterior for γ_{h_0} , $\boldsymbol{K} = \Delta_r^T \Delta_r + 10^{-6} \boldsymbol{I}$, where Δ_r denotes the r-th difference penalty matrix, and $\rho(\boldsymbol{K})$ denotes the rank of \boldsymbol{K} .

Appendix B Joint Model for the Kidney Transplant Dataset

A total of 239 kidney transplant patients were included in the data set. The transplantation characteristics of these patients is presented in Table 1. The data set also includes periodical measurements of SCr and PCR. Our goal is to check if SCr and PCR both, are useful to predict graft failure. To this end, we model the two longitudinal outcomes and graft failure together using the joint modeling framework described in Appendix B. In this model we use log transformed values of both SCr and PCR (Fournier et al., 2016). More specifically, we model the impact of log(SCr) value and log(SCr) velocity, log(PCR) value and log(PCR) velocity, and transplantation characteristics on the hazard of graft failure. In this regard, the JM consists of a multivariate longitudinal sub-model to model the evolution of SCr and PCR and a relative risk sub-model to model the impact of transplantation characteristics and biomarkers on the hazard of graft failure. The longitudinal evolution of the two outcomes over time is modeled flexibly using B-splines. The model formulation for PCR outcome is as follows (for SCr outcome it is same):

$$\log PCR(t) = \beta_{0} + \beta_{1}rec_age + \beta_{2}d_age + \beta_{3}d_bmi + \beta_{4}rec_bmi + \beta_{5}cit + \beta_{6}hla + \beta_{7}pra + \beta_{8}dial_days + \beta_{9}ah_nr + \beta_{10}rec_gender + \beta_{11}d_gender + \beta_{12}dgf + \beta_{13}prev_tp + \beta_{14}dm + \beta_{15}hvdis + \beta_{16}d_cadaveric + \sum_{k=1}^{4} \beta_{k+16}B_{k}(t, \mathcal{K}) + b_{i0} + \sum_{k=1}^{4} b_{ik}B_{k}(t, \mathcal{K}) + \varepsilon_{i}(t),$$

$$(2)$$

where $B_k(t,\mathcal{K})$ denotes the k-th basis function of a B-spline with three internal knots at $\mathcal{K} = \{0.082, 0.219, 1\}$ (30 and 80 days recommended by the clinicians) years, and boundary knots at 0.039 and 6 years (minimum and 0.95 quantile of the time of measurements two outcomes). The quantitative transplantation characteristics are standardized to avoid convergence issues in parameter estimation. For the relative risk sub-model the hazard function we fit is given by:

$$h_i(t) = h_0(t) \exp \left\{ \gamma_1 \text{prev_tp} + \gamma_2 \text{hla} + \gamma_3 \text{cit} + \gamma_4 \text{dial_days} \right. \\ + \alpha_{11} m_{i1}(t) + \alpha_{12} m'_{i1}(t) + \alpha_{21} m_{i2}(t) + \alpha_{22} m'_{i2}(t) \right\},$$
(3)

where $\alpha_{11}, \alpha_{12}, \alpha_{21}$ and α_{22} are measures of strength of the association between hazard of graft failure and log PCR value $m_{i1}(t)$, log PCR velocity $m'_{i2}(t)$, log SCr value $m_{i1}(t)$ and log SCr velocity $m'_{i2}(t)$ respectively.

The parameters of the JM are estimated using the R package JMbayes (Rizopoulos, 2016), which uses the Bayesian methodology to estimate the model parameters. The parameter estimates for the relative risk sub-model are provided in Table 2. Since the quantitative variables are standardized, the effect sizes correspond to one standard deviation increase in the corresponding variable. The parameter estimates for the longitudinal sub-model for SCr and PCR are provided in Table 3 and Table 4, respectively. To avoid the tricky interpretation of variables corresponding to evolution over time, instead the evolution of SCr and PCR over time is depicted in Figure 1, and Figure 2, respectively.

Table 1: Observed transplantation characteristics of the studied population (n=239).

Quantitative characteristics					
Name	Abbreviation	Mean (SD)			
Receiver age (at baseline)	rec_age	50.70 (13.09)			
Donor age	d_age	$49.73\ (12.66)$			
Donor BMI	d_{bmi}	25.10(4.43)			
Receiver BMI	rec_bmi	25.43(4.31)			
Cold ischemia time (minutes)	cit	887.25 (522.95)			
#HLA A, B and DR mismatches	hla	$2.81\ (1.57)$			
Panel reactive antibody (%)	pra	4.81 (14.20)			
#Days on dialysis before transplant	$dial_days$	$1334.91\ (1283.93)$			
#Anti-hypertensive medicaments	ah_nr	1.58 (0.96)			

Categorical characteristics

Name	Abbreviation	Category (%)
Receiver gender	rec_gender	Female (42.68 %)
Donor gender	d_{gender}	Female (56.49 %)
Delayed graft function after transplant	dgf	No (67.78 %)
Previous transplantation	$prev_tp$	No (84.45 %)
Diabetes mellitus	dm	No (84.52 %)
Known cardiovascular events before transplant	hvdis	No (61.92 %)
Deceased donor	$d_{cadaveric}$	No (25.94 %)

Table 2: Relative risk sub-model estimates for mean and 95% credible interval.

Variable	Mean	Std. Dev	2.5%	97.5%	P
Previous transplant: Yes	0.305	0.339	-0.099	0.986	0.352
#HLA mismatches between donor and recipient	0.048	0.093	-0.114	0.269	0.620
Cold ischemia time	-0.051	0.105	-0.277	0.133	0.644
#Days on dialysis before transplant	-0.013	0.102	-0.251	0.178	0.934
$\log \mathrm{PCR}$	0.145	0.125	-0.056	0.431	0.188
Slope(log PCR)	0.021	0.058	-0.076	0.145	0.828
$\log { m SCr}$	1.599	0.241	1.067	2.063	< 0.000
Slope(log SCr)	0.203	0.123	-0.017	0.443	0.082

Table 3: Parameter estimates for the longitudinal sub-model for SCr. $\,$

Variable	Mean	Std. Dev	2.5%	97.5%	P
Intercept	5.226	0.080	5.064	5.378	< 0.000
Receiver age	-0.063	0.022	-0.107	-0.019	0.010
Donor age	0.083	0.020	0.045	0.119	< 0.000
Donor BMI	-0.011	0.021	-0.054	0.028	0.612
Receiver BMI	0.018	0.023	-0.025	0.060	0.420
#HLA mismatches between donor and recipient	0.020	0.022	-0.022	0.065	0.342
Panel reactive antibody percentage	0.048	0.027	-0.008	0.100	0.082
#Anti-hypertensive medicaments	0.040	0.020	0.001	0.080	0.048
Cold ischemia time	0.029	0.035	-0.039	0.102	0.390
#Days on dialysis before transplant	0.015	0.029	-0.042	0.071	0.580
Receiver gender: Male	0.197	0.042	0.111	0.276	< 0.000
Previous transplant: Yes	0.016	0.064	-0.115	0.141	0.786
Donor gender: Male	0.053	0.042	-0.027	0.136	0.198
Delayed graft function: Yes	0.118	0.049	0.025	0.216	0.006
Diabetes Mellitus: Yes	-0.103	0.059	-0.217	0.012	0.076
Cardiovascular events before transplantation: Yes	-0.047	0.043	-0.129	0.044	0.272
Deceased donor: Yes	0.163	0.082	0.004	0.313	0.044
Spline: visit time [0.039, 0.082] years	-0.440	0.041	-0.517	-0.358	< 0.000
Spline: visit time [0.082, 0.219] years	-0.182	0.053	-0.284	-0.081	< 0.000
Spline: visit time [0.219, 1] years	-0.545	0.081	-0.712	-0.395	< 0.000
Spline: visit time [1, 6] years	0.007	0.083	-0.155	0.176	0.946
σ	0.190	0.001	0.187	0.192	

Table 4: Parameter estimates for the longitudinal sub-model for PCR. $\,$

Variable	Mean	Std. Dev	2.5%	97.5%	P
Intercept	3.731	0.179	3.398	4.083	< 0.000
Receiver age	0.030	0.052	-0.066	0.138	0.604
Donor age	0.209	0.047	0.118	0.301	< 0.000
Donor BMI	-0.019	0.051	-0.121	0.084	0.716
Receiver BMI	-0.116	0.050	-0.219	-0.021	0.014
#HLA mismatches between donor and recipient	-0.013	0.049	-0.112	0.086	0.776
Panel reactive antibody percentage	0.047	0.061	-0.066	0.166	0.446
#Anti-hypertensive medicaments	0.056	0.047	-0.03	0.147	0.208
Cold ischemia time	0.062	0.082	-0.097	0.211	0.468
#Days on dialysis before transplant	0.006	0.066	-0.120	0.134	0.952
Receiver gender: Male	-0.026	0.094	-0.207	0.166	0.798
Previous transplant: Yes	0.035	0.149	-0.241	0.332	0.816
Donor gender: Male	0.114	0.096	-0.079	0.303	0.228
Delayed graft function: Yes	0.043	0.118	-0.174	0.275	0.740
Diabetes Mellitus: Yes	0.153	0.135	-0.124	0.396	0.256
Cardiovascular events before transplantation: Yes	-0.016	0.106	-0.221	0.199	0.890
Deceased donor: Yes	0.144	0.193	-0.246	0.509	0.462
Spline: visit time [0.039, 0.082] years	-0.821	0.090	-0.989	-0.638	< 0.000
Spline: visit time [0.082, 0.219] years	-0.578	0.131	-0.838	-0.304	< 0.000
Spline: visit time [0.219, 1] years	-0.898	0.160	-1.218	-0.587	< 0.000
Spline: visit time [1, 6] years	0.460	0.234	0.015	0.927	0.036
σ	0.479	0.004	0.472	0.486	

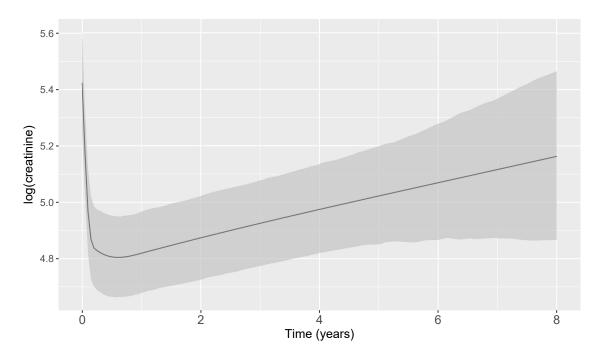


Figure 1: Fitted longitudinal evolution of SCr and 95% credible interval for a patient with the transplantation characteristics described in Table 1.

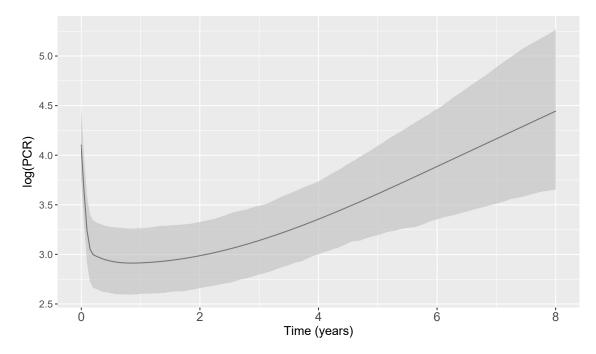


Figure 2: Fitted longitudinal evolution of PCR and 95% credible interval for a patient with the transplantation characteristics described in Table 1.

Appendix C Personalized Schedules for Measurement of SCr

Currently, the schedule for measurement of SCr levels and fixed and common for all patients. SCr levels are measured 20 times in the first year after transplantation and every three months thereafter. Instead of a common fixed schedule for all patients, we propose using a different schedule for every patient. More specifically, we propose using personalized schedules based on joint models Rizopoulos et al. (2016). Since the SCr measurements are already taken for the kidney transplant patients, in order to demonstrate the efficacy of the personalized schedules we conduct a small simulation. To this end, we first assume a population of kidney transplant patients, whose SCr and hazard of graft failure follow a JM of the form described in Appendix B, with parameters equal to the posterior mean of parameters estimated from the joint model fitted to the kidney transplant dataset. From this population we sample 625 patients, which are further split into a training (575 patients) and test (50 patients) part. For the training patients we generate a graft failure time T_i^* as well as a random and non-informative censoring time C_i . For the test patients the graft failure time T_i^* and an intervention time T_i^I is generated. The intervention time is the time at which the 6 month dynamic risk of graft failure of the patient becomes larger than a certain threshold κ . The choice of κ dictates the amount of time at hand between intervention and graft failure. In this simulation we evaluate two κ values, namely 0.05 and 0.025. While the results for $\kappa = 0.05$ are presented in the main manuscript, here we present results for $\kappa = 0.025$. We fit a joint model of the specification described in Section Appendix B to the training data set and obtain a MCMC sample from the posterior distribution of the parameters of the JM. Using the fitted JM, we then iteratively schedule SCr measurements for the test patients, until the dynamic risk of graft failure (Rizopoulos, 2011) of the patients becomes larger than the threshold κ . Let N_i^I denote the number of SCr measurements conducted for the j-th test patient. The time difference between the observed intervention time due to the schedule (T_j^S) and the true intervention time, that is, the intervention offset is denoted by $O_j^I = T_j^S - T_j^I$. Lastly, the failure offset $O_j^* = T_j^S - T^*j$ is the time at hand between the observed intervention time and the time of graft failure. Using the test patients, we calculate these measures for both personalized and fixed schedules. It is to be noted that in the ideal scenario, N_j^I will be one, and offset O_j^I will be zero.

A boxplot of the observed values of the number of SCr measurements N_j^I , intervention offset O_j^I and failure offset O_j^* are presented in Figure 3, Figure 4 and Figure 5, respectively. An advantage of using 2.5% risk over 5% risk is that the risk of overshooting the true graft failure time is less due to less risk being taken. In this scenario, although the personalized schedule conducts less SCr measurements, it also exceeds the true intervention time more often than the fixed schedule.

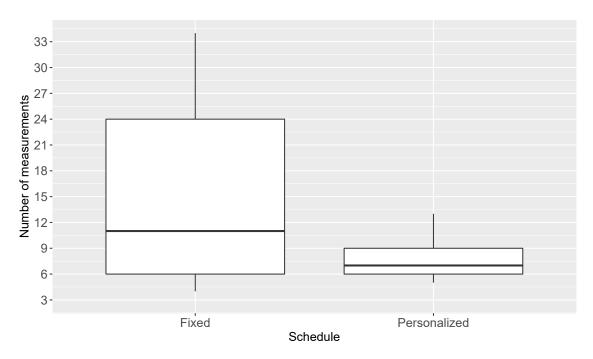


Figure 3: Boxplot of the number of SCr measurements N_i^I for the test patients, for $\kappa = 0.025$.

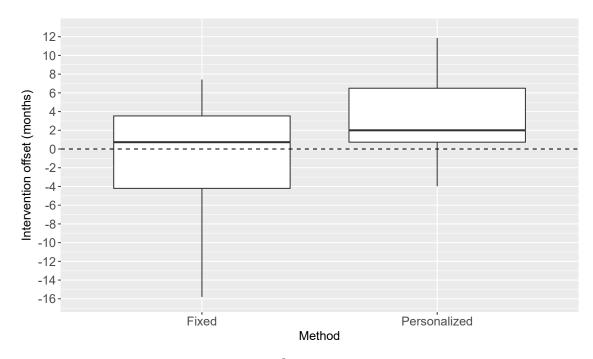


Figure 4: Boxplot of the intervention offset O_j^I for the test patients, for $\kappa=0.025$. The zero offset mark is displayed with the dashed line.

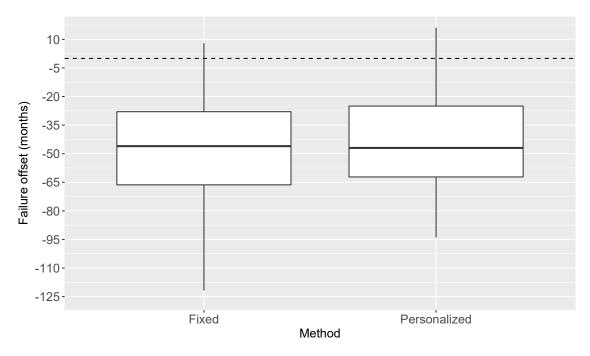


Figure 5: Boxplot of the failure offset O_j^* for the test patients, for $\kappa=0.025$. The zero offset mark is displayed with the dashed line.

Appendix D Source Code

The source code for the joint model fitted to the kidney transplant data set can be found at:

https://goo.gl/phQkxG

The source code for the simulation study can be found at:

https://goo.gl/TpLTM8

A README file explaining the usage of the code can be found at:

 ${\tt https://goo.gl/h2pAep.}$

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