Dual state and parameter estimation of a four wheel vehicle

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***Abstract*—Knowledge of a vehicle's state and parameters is crucial for automatic control of the vehicle. This work presents a methodology to estimate the internal parameters of the vehicle like mass, the distance of the C.O.G (Centre of Gravity) from the wheel axle, inertia and tyre coefficients along with the estimation of vehicle dynamic states like lateral, longitudinal velocity and yaw rates. The kinematic and dynamic model of vehicle is employed for estimation to ensure that the parameter estimates converges consistently. Further standard measurements are utilized keeping in mind the practical constraints and relevance to online estimation. Dual state and parameter are estimated using Extended Kalman Filter, Unscented Kalman Filter and combination of UKF and particle filter. The performance of the estimators is compared based on convergence time, initial uncertainty and accuracy.**

***Index Terms*—State and parameter estimation, vehicle model, EKF, UKF, Particle Filter**

I. Introduction

There has been a tremendous advancement in technology in the automotive industry in the past decade. Various active and passive controller based driver assistance methods have been developed to improve the stability, safety and controllability of the vehicles. Controllers like an autonomous emergency braking system, cruise control, active steering are few such examples. Adding to this, with the advent of self-driving cars and autonomous vehicles, the scope of such a controller has increased manifold. The operation of such controllers depends considerably on the knowledge of the vehicle's internal state and parameters. The accuracy of the state and parameter plays a key role in the performance of such controllers in order to prevent unintended vehicle behaviour.

With the advancement of novel semiconductor technology, various sensors and measurement devices are being utilized with aim of estimating the internal states of the vehicle like optical wheel encoders, GPS, MEMs accelerometers and gyroscopes etc. However, very few can directly measure the internal parameters of the vehicle like mass, inertia, the distance of the C.O.G from wheel axle, tyre co-efficient etc. Though these internal parameters of the vehicle are known after manufacturing, its value changes over time and may vary significantly from one journey condition to others. This can have a drastic effect on the efficiency of the controller in a vehicle. An illustrative example would be - for an autonomous emergency braking system, it is crucial to know the current mass and the position of the C.O.G of the vehicle for effective braking.

To overcome this limitation, the state and the internal parameters need to be estimated from the available sensors. In the literature, several vehicle state estimation methods have been presented like Nonlinear filters, Bayesian approaches [1] etc. However, most of the methods proposed assume that the vehicle parameters remain constant. Combined state and parameter estimation for a vehicle is a challenging problem and only a few approaches are presented in the literature. For combined state and parameter estimation, various model-based estimation tools are utilized. Erik et al. [2] employed system identification tools of Grey Box Model to estimate the tyre coefficients as well as vehicle states. The method could only estimate in an offline fashion, which may have less significance in real-time control. For online estimation, Bayesian and Maximum likelihood-based approaches have also been proposed. Among Bayesian based approaches, primarily two kinds of strategies exist - One is to augment the state variables by adding the parameter. Best and Gordan et al. [3] employed the Extended Kalman filter as a joint filter for this purpose. Similarly, You et al. [4] utilised joint Unscented Kalman filter technique. The second approach is realized by running two filters in parallel, thus dividing state and parameter estimation. The filters are not completely separated due to the interdependency but are connected intuitively. This concept of dual state and parameter estimation was presented by the author of Unscented Kalman Filter Wan et al. [5]. Wenzel et.al [6] employed the second approach for state and parameter estimation in a vehicle and showed how it has better performance than the augmenting the state vectors. Few other authors like Wielitzka et al. [8] and Sun et al. [7] have employed a various version of this approach, however, their approach is unable to estimate all the crucial internal parameters.

Recently nonparametric Bayesian-based estimation methods coined as Sequential Monte Carlo has also gained significant importance. Nemeth et al. [9] presented a few of such methods in simultaneous state and parameter estimation as general. However, an application of nonparametric Bayesian estimation in the vehicle has not been reported to the best of authors knowledge. In this work, a combination of parametric and nonparametric Bayesian-based estimation framework has been explored for specifically a four-wheel vehicle which becomes a significant contribution in the field and thus a novelty.

As the estimation framework depends on the model of the vehicle, the selection of a decent model plays a key role in the accuracy. Numerous models of a four-wheel vehicle are available in the literature based on different complexity. Wielitzka et et al. [8] utilized a complete 11 degree of freedom (DOF) vehicle model, however, was able to estimate only three parameters. You et al. [4] provided a comprehensive comparison and estimation of 3 models of increasing complexity mentioning the major parameters of the vehicle as mass, the distance of the C.O.G and tyre coefficients. Erik et al. [2] showed that how the tyre model also plays a crucial role in the vehicle dynamics and presented estimation of tyre coefficients based on different tyre models. In Erik's thesis, it was reported that the linear tyre model was able to give a decent result in the estimation of vehicle states, therefore without increasing the complexity, linear tyre model was used in this work. Based on the previous work, it was observed that a single model of the vehicle was not able to estimate both states and parameters satisfactorily, facing divergence issue in many cases. Therefore, a combination of two models were utilized to overcome this limitation. This ensured that the parameters estimated converges consistently and there was significantly less chance of divergence.

A kinematic model was employed to estimate the distance of the C.O.G from the front and rear axles, while mass and tyre coefficients were estimated using a simplified dual track dynamic bicycle model. States from the dynamic model were used in comparison as it plays a much important role than states of the kinematic model. Through this work, three promising estimation techniques have been evaluated. Comparison between Dual Extended Kalman Filter (DEKF) and Dual Unscented Kalman Filter (DUKF) in state and parameter estimation in vehicles had not been reported to a large extent. Further, a novel UKF coupled with Liu-West particle filter [10] based approach is presented, which has the potential to outperform existing estimation frameworks. The frameworks are tested on the simulation model of the vehicle and comparison among the methods are presented.

Outline: In the following paragraphs contain the detailed methodology of the work. The kinematic and dynamic model of the vehicle is discussed in section *Vehicle Model*. The detailed estimation frameworks of the techniques are presented in section *Estimation Framework*. The results of the estimation and its effects due to various input, uncertainty etc and comparison of both the estimators with respect to accuracy, stability and convergence is presented in the *Results and Discussion* section.

II. METHODOLOGY

A. Vehicle Model

In this work, two models have been utilized for efficient estimation. As stated in the work of [4], even simplified dynamic models like the bicycle model has its parameters coupled together. This leads to inconsistency in the estimation framework and often results in incorrect estimation. Therefore, after careful consideration, two separate models were utilized based on the work of Kong et al. in vehicel modelling for autonoumous control [11]. The estimation framework couples both these models together and utilized the parameters estimated from the kinematic model in the dynamic one. Two parameters are estimated using the kinematic model, and three parameters are estimated from the dynamic model. The models are coupled in the sense that estimated parameters from the kinematic model are used in the dynamic model. The kinematic model is much simpler and the parameters are not coupled in a complex manner. Further, estimating these two kinematic parameters reduces the complexity of the dynamic model and thus reduces the possibility divergence of the entire framework.

1) Kinematic Model: The kinematic model is a simplified point mass model of vehicle dealing with the kinematics of the vehicle. The states in this model are position and velocity and the equations are governed by the geometric relation as shown in the Fig.1. This model is based on the paper [11].

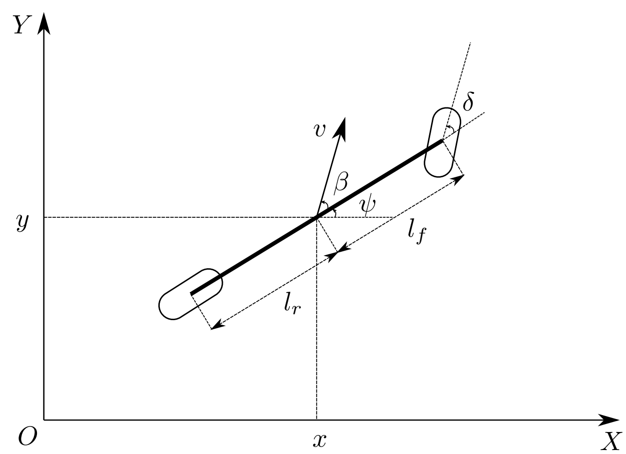

Fig. 1. Kinematic Model from [11]

The equations in the inertial frame XOY are as follows -

$$\beta = tan^{-1}(\frac{l_r}{l_r + l_f})tan(\delta) \tag{1}$$

$$\dot{x} = v\cos(\psi + \beta) \tag{2}$$

$$\dot{y} = v\sin(\psi + \beta) \tag{3}$$

$$\dot{\psi} = \frac{v}{l_r}sin(\beta) \tag{4}$$

$$\dot{v} = a \tag{5}$$

where, x and y are the position of vehicle along x and y coordinates with respect to the inertial frame XOY. v is the vehicle speed in the heading direction. ψ is the heading angle and δ is the input steering angle. β is the angle of the current velocity of the C.O.G of the vehicle with respect to the longitudinal axis of the car. l_f and l_r represents the distance of the C.O.G of the vehicle from the front and rear axles of the car respectively. These distance are crucial for control and may vary based on the distribution of load. Through the kinematic model, these distances are estimated which are further utilized in the dynamic model. Thus, the kinematic model consists of 2 inputs $[a, \delta]^T$, 4 states $[x, y, \psi, v]^T$ and 2 parameters $[l_f, l_r]^T$.

2) Dynamic Model: The dynamic model used in this work is the simplified bicycle model based on the one presented in [11]. The vehicle model is derived from standard rigid body mechanics, wherein only planar model of the vehicle is considered ignoring roll and pitch motions as shown in Fig. 2 resulting in 3 degrees of freedom. The assumption is valid in general cases where vehicle motions with sudden jerks are not observed.

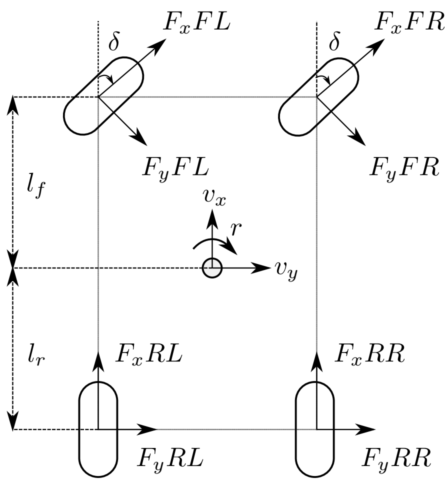

Fig. 2. 4-wheel Dynamic Model from [11]

$$\begin{aligned}\dot{v_x} &= v_y r + \frac{1}{m}(F_{xFR} + F_{xFL})cos(\delta) - \\ &(F_{yFR} + F_{xFL})sin(\delta) + F_{xRR} + F_{xRL} - C_a {v_x}^2\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{v_y} &= -v_x r + \frac{1}{m}(F_{xFR} + F_{xFL})sin(\delta) \\ &(F_{yFR} + F_{xFL})cos(\delta) + F_{yRR} + F_{yRL}\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{r} &= \frac{1}{J}(a((F_{xFR} + F_{xFL})sin(\delta) + \\ &(F_{yFR} + F_{xFL})cos(\delta)) - b(F_{yRR} + F_{yRL})\end{aligned} \tag{8}$$

Here, v_x and v_y are the vehicle's longitudinal and lateral velocities respectively at the vehicle's C.O.G. r is the yaw rate.

The forces F_{xXX} and F_{yXX} depends on the tyre model. In this work, piecewise continuous linear tyre models are considered both in longitudinal and lateral direction based on [11].

The equation of the forces are determined by the following equations

$$\begin{aligned} F_{yFX} &= C_y \alpha_{FX} \\ F_{yRX} &= C_y \alpha_{RX} \\ F_{xFX} &= C_x s_{FX} \\ F_{xRX} &= C_x s_{RX} \end{aligned} \tag{9}$$

Where C_x and C_y are the tyre coefficients in x and y direction respectively which have been assumed to be same for all 4 tyres. s_{FX}, and s_{RX} are the slip inputs on the front and rear tyres respectively, for either left or right tyres. In this case, front-wheel driven vehicle is considered and therefore s_{RR}, $s_{RL} = 0$.

α_{FX} and α_{RX} are the slip angles for front and rear tyres respectively and calculated as below.

$$\begin{aligned} \alpha_{FX} &= \delta - \arctan \frac{v_y + ar}{v_x} \\ \alpha_{RX} &= -\arctan \frac{v_y - br}{v_x} \end{aligned} \tag{10}$$

, where l_f and l_r are the distance of C.O.G from front and rear axle, similar to the kinematic model

So, the dynamic model consists of 5 inputs $[s_{FL}, s_{FR}, s_{RL}, s_{RR}, \delta]^T$, 3 states $[v_x\ v_y\ r]^T$ and 3 parameters $[m, C_x, C_y]^T$.

B. Estimation Framework

Estimation framework presented takes the form of parametric and non-parametric Bayesian filter. For the parametric estimation framework, the Kalman filter becomes an ideal choice. In this work, the Extended and Unscented version of Kalman Filter was utilised as both the models of the vehicle are nonlinear in nature. The basic flow of these two dual estimators is based on the method proposed by Wan and Nelson [12]. Fig. 3 illustrates the working scheme of the dual estimators, wherein, two separate filters for state and parameter run in parallel while passing interdependent parameters. In each iteration, parameters are estimated initially followed by state. After incorporating the measurement, the state is corrected followed by the parameter correction. Since parameter correction is solely based on the measurement of the state, and the estimated states are based on these parameters, the performance of the filters cannot be predicted in advance. Further, there is no standard rule for determining the controllability and observability of nonlinear vehicle models, so no prior judgments could be made about the viability of the framework.

For the non-parametric based approach, a general framework similar to Fig. 3 is not present in any literature as the working principle is significantly different. A particle filter is one of the promising non-parametric Bayesian-based approaches. Previous attempts on estimating combined state and parameter estimation using joint particle filter based approach involved selecting a prior distribution of the parameter and augmenting it with the state. However, naively adding noise for the diffusion step leads to overly diffuse parameters. This in turn drastically effect the state estimation. This is due to fact that parameters do not evolve with time. Further dual particle filter will created multiple particle peaks corresponding to multiple peaks in the state distribution due to prediction inaccuracy in the parameter distribution. Various solution for these issues are presented in the paper [9]. After going through previous solutions, in [9], a combination of nonparametric and parametric Bayesian filter was decided to combine the advantage of particle filter and simplicity of Kalman filter. In this work, a modified particle filter named as Liu-West Particle Filter [10] is used for parameter estimation along with Unscented Kalman Filter for state estimation is presented. The proposed combination outperforms other estimators for combined state and parameter estimation of the vehicle.

To correct the state, the measurement model is simplified and all the states are supposed to be measured with decent accuracy with some measurement noise. This assumption holds valid as modern vehicles have a wide range of sensors capable of measuring most of the states utilized in this work. In the current section, the details of each estimation framework are presented. Throughout the section the variables with subscript s deals with state and p deals with parameters.

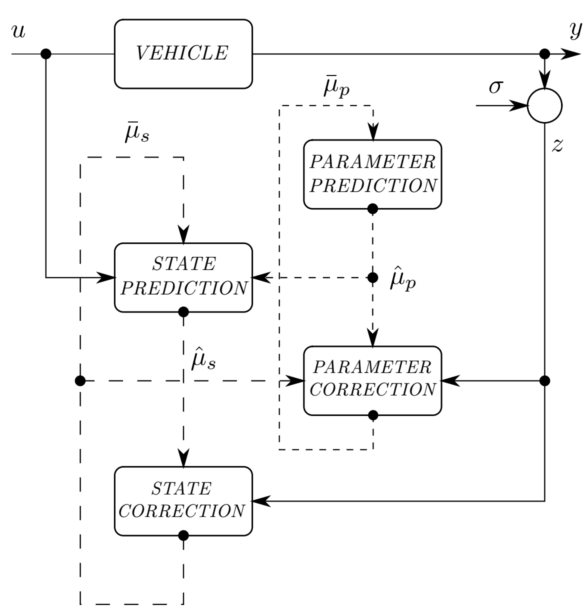

Fig. 3. Scheme of DEKF and DUKF.

1) Dual EKF: Extended Kalman Filter (EKF) is an extension of Kalman filter originally proposed by Kalman for non-linear estimation [13]. In the EKF, the distributions are assumed to Gaussian in nature. As discussed in the *Introduction* section, two primary variants of EKF is utilised in simultaneous state and parameter estimation - Joint EKF and Dual EKF. For this work, both Joint EKF and Dual EKF were explored and it was found that dual EKF outperformed joint EKF. This might be due to the separation of the covariance matrices for state and parameter. Dual EKF takes the form of the general formulation of EKF with two separate filters running in parallel for state and parameter. In each iteration, a parameter prediction takes place which only adds parameter process noise to the covariance of the parameter from the previous iteration. Thereafter, this predicted parameter is utilised to predict the state, as state evolution depends on parameters. After incorporating the measurement of the states, states are corrected and finally, parameters are corrected. The crucial part of dual EKF is the Jacobian utilised in the state prediction - G_s. This is given by -

$$\boldsymbol{G_s} = \frac{\partial f(\boldsymbol{\hat{x}_s}, \boldsymbol{\hat{x}_p})}{\partial \boldsymbol{\hat{x}_s}} \tag{11}$$

where f denotes the discrete version of the kinematic and dynamic model, $\hat{x}_s$ denotes state vector and $\hat{x}_p$ denotes parameter vector. Along with state Jacobian, measurement Jacobian of the state is also required. Since the measurement of the state is straightforward as all the states are assumed to be directly measurable, the measurement Jacobian for the state is simply identity matrix I.

$$\boldsymbol{H_s} = I(4) \tag{12}$$

The measurement jacobian for parameter is crucial as it relates the innovation in measurement to the parameter correction term. This is defined as -

$$\boldsymbol{H_p} = \boldsymbol{H_s} \frac{\partial f(\boldsymbol{\hat{x}_s}, \boldsymbol{\hat{x}_p})}{\partial \boldsymbol{\hat{x}_p}} \tag{13}$$

Since parameter correction takes place indirectly, the parameter covariance does not seem to decrease considerably after correction, in spite of the fact that by successive iteration, the state and parameter are getting closer to the true value. This can be taken into care by reducing the process noise of the parameter which is based on [15]. The complete algorithm of dual EKF is stated in Algorithm 1. The notations of covariance matrixes, process noise, measurement noise follow the convention set by [13].

2) Dual UKF: Dual Unscented Kalman for this work was inspired by the paper [8]. The framework of dual UKF is quite similar to that of dual EKF. Unlike EKF, UKF does not compute the Jacobian matrices, thus avoiding the approximation by linearization. In UKF, the unscented transform is utilized instead. This involves deterministic extraction of sigma points from the Gaussian distribution and using these sigma points as

Algorithm 1 Dual EKF algorithm

procedure DUALEKF($\mu_{s,p_{t-1}}, \Sigma_{s,p_{t-1}}, z_t, u_t, R_{p_{t-1}}, \gamma$)
  %Parameter Prediction%
  $\hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}} = \boldsymbol{\mu}_{\boldsymbol{p_{t-1}}}$
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}} = \boldsymbol{\Sigma}_{\boldsymbol{p_{t-1}}} + \boldsymbol{R}_{\boldsymbol{p_{t-1}}}$

  %State Prediction%
  $\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}} = nextState(\boldsymbol{\mu_{s_{t-1}}}, \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}}, \boldsymbol{u_t})$ ▷ Eqns [1-10]
  $\boldsymbol{G_s} = getStateJacobian(\boldsymbol{\mu_{s_{t-1}}}, \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}})$ ▷ Eqn [11]
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}} = \boldsymbol{G_s}\boldsymbol{\Sigma}_{\boldsymbol{s_{t-1}}}\boldsymbol{G_s^T} + \boldsymbol{R_s}$

  %State Correction%
  $\boldsymbol{K_{s_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}}\boldsymbol{H_s^T}[\boldsymbol{H_s}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}}\boldsymbol{H_s^T} + \boldsymbol{Q_t}]^{-1}$
  $\boldsymbol{\mu_{s_t}} = \hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}} + \boldsymbol{K_{s_t}}(\boldsymbol{z_t} - \boldsymbol{H_s}\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}})$

  %Parameter Correction%
  $\boldsymbol{H_p} = getParameterJacobian(\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}}, \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}})$ ▷ Eqn [13]
  $\boldsymbol{K_{p_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}}\boldsymbol{H_p^T}[\boldsymbol{H_p}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}}\boldsymbol{H_p^T} + \boldsymbol{Q_t}]^{-1}$
  $\boldsymbol{\mu_{p_t}} = \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}} + \boldsymbol{K_{p_t}}(\boldsymbol{z_t} - \boldsymbol{H_s}\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}})$
  $\boldsymbol{R_{p_t}} = \gamma\boldsymbol{R_{p_{t-1}}}$

  return $\mu_{p_t}, \mu_{p_t}, \Sigma_{p_t}, \Sigma_{s_t}, R_{p_t}$

an input to the nonlinear function [5]. The resultant Gaussian is also approximated from these transformed sigma points, thus the entire Kalman filter formulation remains the same.

For the state and parameter, different sets of sigma points were selected based on -

$$\chi^{[0]} = \mu \tag{14}$$

$$\chi^{[i]} = \mu + (\sqrt{(n+\lambda)\Sigma})_i \; for \; i = 1, ..., n \tag{15}$$

$$\chi^{[i]} = \mu - (\sqrt{(n+\lambda)\Sigma})_{i-n} \; for \; i = n+1, ..., 2n \tag{16}$$

where n is the dimension the either state and parameter vector. The term λ is given by $\lambda = \alpha^2(n+\kappa) - n$, with α and κ are scaling parameter and reflect where the sigma points are selected in the gaussian distribution. Adding to this, each sigma points have an associated weight as well which helps to regenerate the Gaussian distribution from sigma points.

$$w_m^{[0]} = \frac{\lambda}{(\lambda+n)} \tag{17}$$

$$w_c^{[0]} = \frac{\lambda}{(\lambda+n)} + (1 - \alpha^2 + \beta) \tag{18}$$

$$w_m^{[i]} = w_c^{[i]} = \frac{1}{2(n+\lambda)} \; for \; i = 1, .., 2n \tag{19}$$

the sigma points are passed to the nonlinear functions of the vehicle model for update step. The complete algorithm for Dual UKF is presented in Algorithm 2. The formulation and notation of the Dual UKF are based on [13].

Algorithm 2 Dual UKF algorithm

procedure DUALUKF($\mu_{s,p_{t-1}}, \Sigma_{s,p_{t-1}}, z_t, u_t, R_{p_{t-1}}, [\alpha, \beta, \gamma]$)
  %Parameter Prediction%
  $\boldsymbol{w_{mp}}, \boldsymbol{w_{cp}} = generateWeights(\boldsymbol{\alpha_p}, \boldsymbol{\beta_p}, \boldsymbol{\gamma_p})$ ▷ Eqn [17-19]
  $\boldsymbol{\chi_{p_{t-1}}} = generateSigmaPoints(\boldsymbol{\mu_{p_{t-1}}}, \boldsymbol{\Sigma_{p_{t-1}}})$ ▷ Eqn [14-16]
  $\boldsymbol{\chi_{p_t}} = \boldsymbol{\chi_{p_{t-1}}}$
  $\hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2m}} \boldsymbol{w_{mp}}[\boldsymbol{i}]\boldsymbol{\chi_{p_t}}[\boldsymbol{i}]$
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2m}} \boldsymbol{w_{cp}}[\boldsymbol{i}](\boldsymbol{\chi_{p_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}})(\boldsymbol{\chi_{p_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}})^{\boldsymbol{T}}$
  $+\boldsymbol{R_{p_{t-1}}}$

  %State Prediction%
  $\boldsymbol{w_{ms}}, \boldsymbol{w_{cs}} = generateWeights(\boldsymbol{\alpha_s}, \boldsymbol{\beta_s}, \boldsymbol{\gamma_s})$
  $\boldsymbol{\chi_{s_{t-1}}} = generateSigmaPoints(\boldsymbol{\mu_{s_{t-1}}}, \boldsymbol{\Sigma_{s_{t-1}}})$
  $\boldsymbol{\chi_{s_t}} = nextState(\boldsymbol{\chi_{s_{t-1}}}, \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}}, \boldsymbol{u_t})$ ▷ Eqn [1-10]
  $\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2n}} \boldsymbol{w_{ms}}[\boldsymbol{i}]\boldsymbol{\chi_{s_t}}[\boldsymbol{i}]$
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2n}} \boldsymbol{w_{cs}}[\boldsymbol{i}](\boldsymbol{\chi_{s_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}})(\boldsymbol{\chi_{s_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}})^{\boldsymbol{T}}$
  $+\boldsymbol{R_s}$

  %State Correction%
  $\hat{\boldsymbol{\chi}}_{\boldsymbol{s_t}} = generateSigmaPoints(\hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}}, \hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}})$
  $\boldsymbol{\chi_{z_{st}}} = \hat{\boldsymbol{\chi}}_{\boldsymbol{s_t}}$
  $\hat{\boldsymbol{z}}_{\boldsymbol{st}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2n}} \boldsymbol{w_{ms}}[\boldsymbol{i}]\boldsymbol{\chi_{z_{st}}}[\boldsymbol{i}]$
  $\boldsymbol{S_z} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2n}} \boldsymbol{w_{cs}}[\boldsymbol{i}](\boldsymbol{\chi_{z_{st}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{st}})(\boldsymbol{\chi_{z_{st}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{st}})^{\boldsymbol{T}}$
  $+\boldsymbol{Q_t}$
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{sz}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2n}} \boldsymbol{w_{cs}}[\boldsymbol{i}](\hat{\boldsymbol{\chi}}_{\boldsymbol{s_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}})(\boldsymbol{\chi_{z_{st}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{st}})^{\boldsymbol{T}}$

  $\boldsymbol{K_{s_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{sz}}\boldsymbol{S_z^{-1}}$
  $\boldsymbol{\mu_{s_t}} = \hat{\boldsymbol{\mu}}_{\boldsymbol{s_t}} + \boldsymbol{K_{s_t}}(\boldsymbol{z_t} - \hat{\boldsymbol{z}}_{\boldsymbol{st}})$
  $\boldsymbol{\Sigma_{s_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{s_t}} - \boldsymbol{K_{s_t}}\boldsymbol{S_z}\boldsymbol{K_{s_t}^T}$

  %Parameter Correction%
  $\hat{\boldsymbol{\chi}}_{\boldsymbol{p_t}} = generateSigmaPoints(\hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}}, \hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}})$
  $\boldsymbol{Z_{p_t}} = nextState(\hat{\boldsymbol{\chi}}_{\boldsymbol{p_t}}, \boldsymbol{\mu_{s_{t-1}}}, \boldsymbol{u_t})$
  $\hat{\boldsymbol{z}}_{\boldsymbol{pt}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2m}} \boldsymbol{w_{mp}}[\boldsymbol{i}]\boldsymbol{\chi_{z_{pt}}}[\boldsymbol{i}]$
  $\boldsymbol{P_z} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2m}} \boldsymbol{w_{cp}}[\boldsymbol{i}](\boldsymbol{\chi_{z_{pt}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{pt}})(\boldsymbol{\chi_{z_{pt}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{pt}})^{\boldsymbol{T}}$
  $+\boldsymbol{Q_t}$
  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{pz}} = \sum_{\boldsymbol{i=0}}^{\boldsymbol{2m}} \boldsymbol{w_{cp}}[\boldsymbol{i}](\hat{\boldsymbol{\chi}}_{\boldsymbol{p_t}}[\boldsymbol{i}] - \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}})(\boldsymbol{\chi_{z_{pt}}}[\boldsymbol{i}] - \hat{\boldsymbol{z}}_{\boldsymbol{pt}})^{\boldsymbol{T}}$

  $\boldsymbol{K_{p_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{pz}}\boldsymbol{P_s^{-1}}$
  $\boldsymbol{\mu_{p_t}} = \hat{\boldsymbol{\mu}}_{\boldsymbol{p_t}} + \boldsymbol{K_{p_t}}(\boldsymbol{z_t} - \hat{\boldsymbol{z}}_{\boldsymbol{pt}})$
  $\boldsymbol{\Sigma_{p_t}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{p_t}} - \boldsymbol{K_{p_t}}\boldsymbol{P_z}\boldsymbol{K_{p_t}^T}$
  $\boldsymbol{R_{p_t}} = \gamma\boldsymbol{R_{p_{t-1}}}$

  return $\mu_{p_t}, \mu_{s_t}, \Sigma_{p_t}, \Sigma_{s_t}, R_{p_t}$

3) Particle Filter+UKF: In this work, a novel method of a combined particle filter and Unscented Kalman Filter (UKF) has been implemented for vehicle state and parameter estimation. The UKF filter is utilised for state estimation which eases the density extraction concept and removes possibility of multiple peaks. For parameter estimation, a modified Liu West particle filter is proposed. This avoids the particle deprivation while taking advantage of the superior estimation capability of the particle filter. Liu West [10] proposed a method to avoid particle deprivation in cases of parameter estimation by approximating the parameter posterior distribution through a mixture of multivariate Gaussian distributions.

Liu West filter can be interpreted as a systematic diffusion process in the general particle filter. Here, a short description of the filter is presented. The joint state and parameter posterior can be represented as

$$\begin{aligned} p(x_s, x_p|z_{1:t}) &\propto p(z_t|x_s, x_p)p(x_s, x_p|z_{1:t-1}) \\ &\propto p(z_t|x_s, x_p)p(x_s|z_{1:t-1}, x_p)p(x_p|z_{1:t-1}) \end{aligned}$$

The parameter posterior approximated as a mixture of multivariate gaussian, given by -

$$p(x_p|z_{1:t-1}) \approx \sum_{i=0}^{M} w_{t-1}^{[i]} \mathcal{N}(x_p|m_{t-1}, h^2 V)$$

The term m is calculated by -

$$m = a x_p + (1-a)\hat{x}_{p_t}$$

where $\hat{x}_{p_t}$ is the mean and V is the variance of the parameter distribution. It includes a smoothening parameter h and shrinkage parameter $a = \sqrt{(1-h^2)}$. The method provided by Liu and West corrects the over-dispersion of the parameter by pushing the particles towards the mean of the distribution and also corrects the distribution by the recomputing the variance.

In addition to the method proposed by Liu West, an important modification is proposed by the term γ, the residual weight factor. Since the algorithm is probabilistic it is quite possible that poor hypothesis is weighted higher than all other good hypotheses thus eliminating most or all good particle. To prevent this, a fraction, γ, of particle's current weight is propagated to the future steps. Thus preserving good particles that might accidentally perform poorly in future.

The combined algorithm is presented in Algorithm 3. In the algorithm parameter distribution is represented by θ, M denotes the number of particles. Rest of the notations follows [13]. Weight is updated based on the Gaussian likelihood of measurement.

Algorithm 3 Particle filter + UKF

procedure LIU_WEST PF($\bar{\mu}_{s_{t-1}}, z_t, u_t, [a, \gamma]$)
  $\hat{\theta}_t = \bar{\theta}_{t-1}$
  for $i = 1$ to M **do**
    $\hat{\mu}_{s_t} = nextState(\bar{\mu}_{s_{t-1}}, \hat{\theta}_t^{[i]}, u)$
    $w_t^{[i]} = w_{t-1}^{[i]} e^{(-\frac{1}{2}(z_t - H_s\hat{\mu}_{s_t})Q^{-1}(z_t - H_s\hat{\mu}_{s_t})^T)}$
  $w_t = \frac{w_t}{\sum_{i=0}^{M} w_t^{[i]}}$
  $\hat{\mu}_{p_t} = \sum_{i=0}^{M} w_t^{[i]} \hat{\theta}_t^{[i]}$
  $m = a\theta + (1-a)\hat{\mu}_{p_t}$
  $V = \sum_{i=0}^{M} w_t^{[i]} \hat{\theta}_t^{[i]} - \hat{\mu}_{p_t}$
  $\bar{\theta} \sim \mathcal{N}(m, h^2 V)$
  $\bar{\mu}_{p_t} = \sum_{i=0}^{M} w_t^{[i]} \bar{\theta}_t^{[i]}$
  $w_t = \gamma w_t$
  return $\bar{\mu}_{p_t}$

procedure UKF($\bar{\mu}_{p_t}, \bar{\mu}_{s_{t-1}}, \Sigma_{s_{t-1}}, z_t, u_t, [\alpha, \beta, \gamma]$)

  $w_{ms}, w_{cs} = generateWeights(\alpha_s, \beta_s, \gamma_s)$
  $\chi_{s_{t-1}} = generateSigmaPoints(\mu_{s_{t-1}}, \Sigma_{s_{t-1}})$
  $\chi_{s_t} = nextState(\chi_{s_{t-1}}, \bar{\mu}_{p_t}, u_t)$
  $\hat{\mu}_{s_t} = \sum_{i=0}^{2n} w_{ms}[i]\chi_{s_t}[i]$
  $\hat{\Sigma}_{s_t} = \sum_{i=0}^{2n} w_{cs}[i](\chi_{s_t}[i] - \hat{\mu}_{s_t})(\chi_{s_t}[i] - \hat{\mu}_{s_t})^T$
  $+R_s$

  $\hat{\chi}_{s_t} = generateSigmaPoints(\hat{\mu}_{s_t}, \hat{\Sigma}_{s_t})$
  $\chi_{z_{st}} = \hat{\chi}_{s_t}$
  $\hat{z}_{st} = \sum_{i=0}^{2n} w_{ms}[i]\chi_{z_{st}}[i]$
  $S_z = \sum_{i=0}^{2n} w_{cs}[i](\chi_{z_{st}}[i] - \hat{z}_{st})(\chi_{z_{st}}[i] - \hat{z}_{st})^T$
  $+Q_t$
  $\hat{\Sigma}_{sz} = \sum_{i=0}^{2n} w_{cs}[i](\hat{\chi}_{s_t}[i] - \hat{\mu}_{s_t})(\chi_{z_{st}}[i] - \hat{z}_{st})^T$

  $K_{s_t} = \hat{\Sigma}_{sz} S_z^{-1}$
  $\mu_{s_t} = \hat{\mu}_{s_t} + K_{s_t}(z_t - \hat{z}_{st})$
  $\Sigma_{s_t} = \hat{\Sigma}_{s_t} - K_{s_t} S_z K_{s_t}^T$

  return μ_{s_t}, Σ_{s_t}

III. Results & Discussion

In this section, the results of the state and parameter estimation of the simulation model of vehicle is presented. All the three estimation frameworks discussed above has been demonstrated.

A. Parameter estimation from Kinematic model

The kinematic model was not complex in terms of nonlinear function and all the three estimation framework was able to estimate the C.O.G parameters (l_f, l_r) consistently. The states from the kinematic model were not of focus for this work, therefore the results for states from the kinematic model is not presented. The Gaussian noise was kept similar for all the three estimation framework and following were the initial values taken. These values were selected based on the range of estimation and repeated experimentation.

$$\Sigma_s = diag([1, 1, 0.2, 2])$$
$$\Sigma_p = diag([3, 3])$$
$$R_s = diag([0.5^2, 0.5^2, 0.2^2, 1^2])$$
$$R_p = diag([1.25^2, 1.25^2])$$
$$Q = diag([1^2, 1^2, 1.5^2, 0.1^2])$$

where, Σ_s and Σ_p denotes the initial uncertainty, R_s, R_p is the process noise in state and parameter respectively and Q is the measurement noise of the state. The simulation noise of $\mathcal{N}(0, 0.1)$ was added to the model to make the simulation closer to the actual scenario. For the particle + UKF, instead of Σ_p a uniform distribution of range for both the parameters was taken as the initial distribution and R_p was also not utilized. The number of particles M was taken to be 1000. Table. I shows the result of the estimation. Fig 4 demonstrates the convergence of the parameter l_f and Fig 4 for l_r. It can be seen that the proposed particle + UKF converges promptly, with low error standard deviation compared to Dual UKF and Dual EKF, however, a constant error is accumulated. The constant error may be due to incorrect estimation of the noise by the particle filter. Further, Dual EKF converges with lower error standard deviation, compared to Dual UKF which is due to the linearization of the nonlinear function, making it smoother.

Estimation	$l_f(m)$	$l_r(m)$
True Value	1.50	1.50
Dual EKF	1.56	1.43
Dual UKF	1.58	1.39
Particle+UKF	1.67	1.36

TABLE I
COMPARISON OF PARAMETER ESTIMATION FROM KINEMATIC MODEL

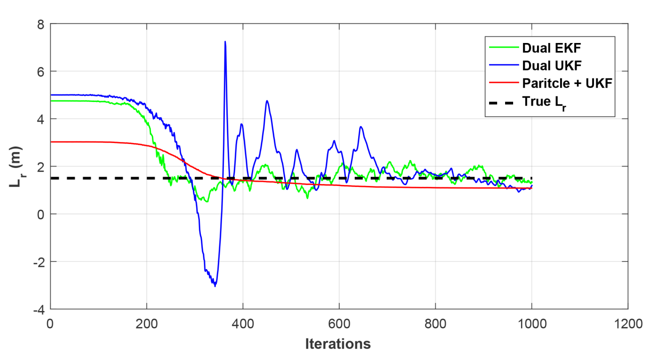

Fig. 4. Comparison of parameter l_f from kinematic model

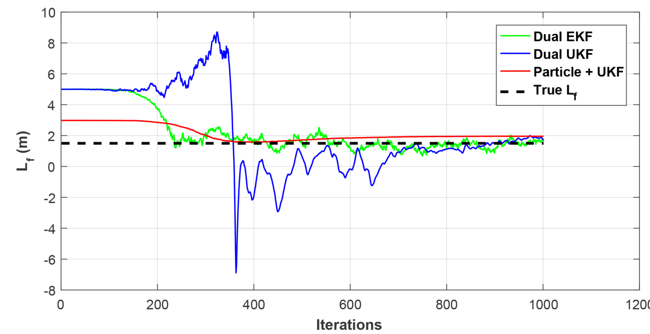

Fig. 5. Comparison of parameter l_r from kinematic model

Effect of input and parameter estimation: It was also observed that the frequency and magnitude of the input also affected the parameter estimation substantially. This is might be related to the order of persistence of excitation [14], which is relevant in grey box modelling. Since this work deals on the estimation framework, an in-depth analysis was not done related to the effect of input on parameter excitation. Fig.6 shows one such result on parameter estimation with varying input on Dual UKF. Increasing the frequency results in faster convergence, however, may lead to an unrealistic scenario which may not be possible in real experiments.

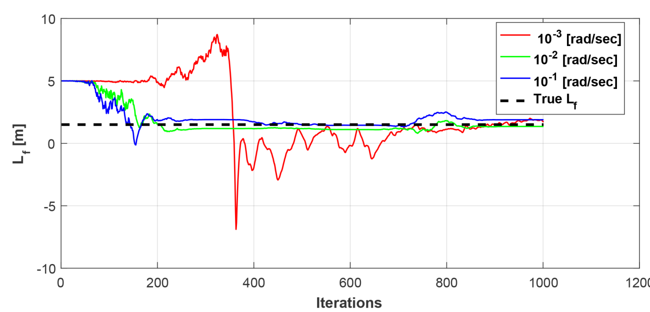

Fig. 6. Effect of input frequency on parameter estimation in Dual UKF

B. State estimation from the Dynamic model

In the dynamic model, the parameters and state were coupled in a complex manner. Dual EKF was unable to converge even with substantially low noise values due to linearization, therefore the results of only Dual UKF and Particle + UKF are presented. The input and noise values were kept similar to the one presented by Erik et al.[2], and available standard MATLAB [16] dataset was utilized. Similar to the kinematic model, following were the noise and initial uncertainty matrices. These values were selected based on the range of estimation and repeated experimentation.

$$\Sigma_s = diag([1, 1, 0.2])$$
$$\Sigma_p = diag([200, 2 * 10^4, 1.5 * 10^4])$$
$$R_s = diag([1.2^2, 1.2^2, 0.45^2])$$
$$R_p = diag([20^2, 175^2, 150^2])$$
$$Q = diag([0.2^2, 0.2^2, 0.15^2])$$
$$M = 1000.$$

Figs. [7, 8, 9] shows the state estimation over time. Compared to Erik's work the results presented in this is substantially better. Table II. shows the mean square error of the state estimates. The proposed particle + UKF estimation framework outperformed the Dual UKF method. In the initial iteration, the state estimation is worse than Erik's method as the presented estimation framework is online and the initial uncertainty in parameters is large.

RMS Error	$v_x(m/sec)$	$v_y(m/sec)$	$r(rad/sec)$
Dual UKF	1.3197	0.636	0.1277
Particle+UKF	0.1727	0.1566	0.0630

TABLE II
COMPARISON OF STATE ESTIMATION FROM DYNAMIC MODEL

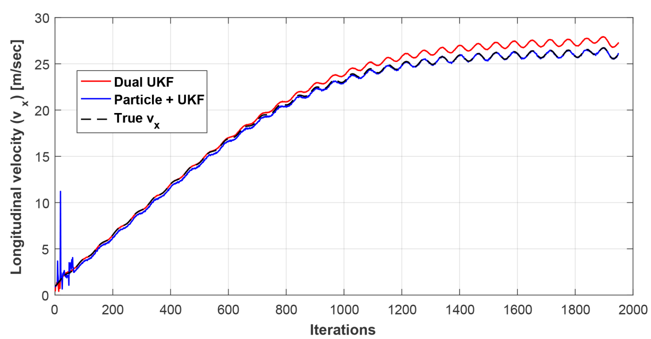

Fig. 7. Comparison of state v_x from dynamic model

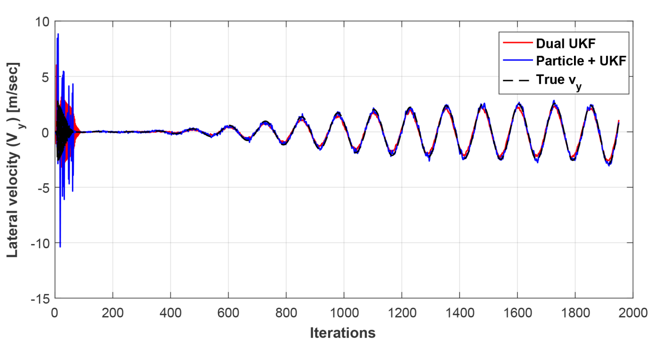

Fig. 8. Comparison of state v_y from dynamic model

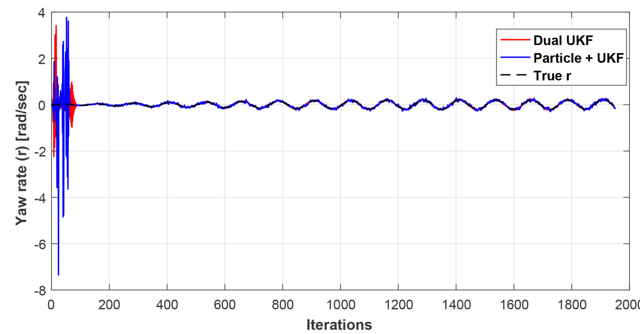

Fig. 9. Comparison of state r from dynamic model

C. Parameter estimation from the Dynamic model

The parameter estimation from the dynamic model was performed by Dual UKF and particle+UKF. Particle+UKF outperformed the dual UKF in both convergence time and the final estimated value. Table III shows the result of the final value of the estimated parameter compared to true value. Figs. [10, 11, 12] demonstrates the convergence of the parameter estimation with iteration. The final error in the Dual UKF for tyre coefficients is significantly high compared to the proposed particle+UKF.

Compared to the Wenzel et.al [6] and Wielitzka et al. [8], the presented approach is able to estimate 3 parameters from significantly lower measurement in the dynamic model and the estimated values are better in the case of particle+UKF compared to the one presented in Wenzel's estimate [6].

Unlike in the kinematic model, a different input from that of Erik's were leading to inconsistency, therefore analysis of the effect of input was not performed in the dynamic model. It was also observed that compared to the kinematic model, Dual UKF was more sensitive to noise and uncertainty values and taking large values lead to inconsistency like Dual EKF. However, the parameter+UKF was better at handling noisy measurements and model due to the advantage of the particle filter. This adds to the credibility of the proposed method.

Estimation	$m(kg)$	$C_x(N/rad)$	$C_y(N/rad)$
True Value	1700	$15 * 10^4$	$4 * 10^4$
Dual UKF	1620	$13.2 * 10^4$	$4.75 * 10^4$
Particle+UKF	1715	$14.6 * 10^4$	$3.85 * 10^4$

TABLE III
COMPARISON OF PARAMETER ESTIMATION FROM DYNAMIC MODEL

IV. CONCLUSION

In this work, the problem of simultaneous dual state and parameter estimation of a four-wheel vehicle is attempted. Two models of the vehicle were utilized to reduce the complexity and order of estimation parameters from a single model, thereby ensuring consistency. Two parametric Bayesian-based approach - Dual Extended Kalman Filter

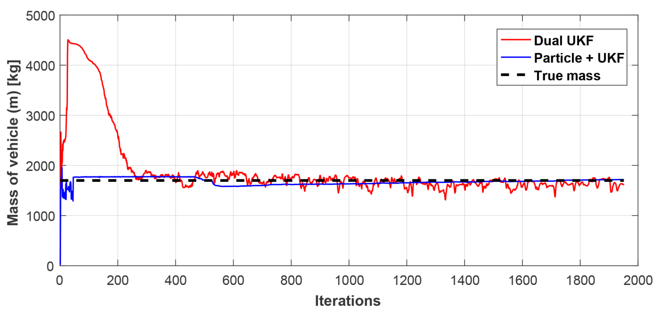

Fig. 10. Comparison of parameter m from dynamic model

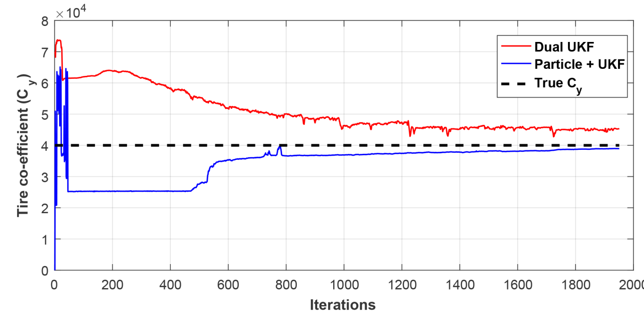

Fig. 12. Comparison of parameter C_y from dynamic model

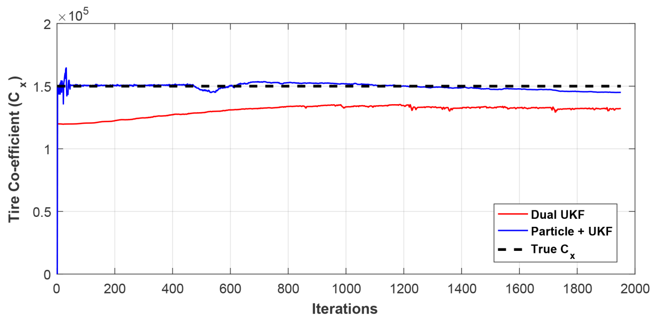

Fig. 11. Comparison of parameter C_x from dynamic model

and Dual Unscented Kalman Filter were compared. Though Dual EKF seems to perform better in the simpler kinematic model case, it failed to converge for the complex dynamic model compared to Dual UKF. Thus, Dual UKF has an advantage over Dual EKF in state and parameter estimation. Adding to this a modified Liu-West particle filter along with a combination of Unscented Kalman Filter was proposes which has the potential to outperform existing estimation framework. Compared to Dual UKF, the particle+UKF was able to handle more rigorous noise values and had reduced chances of inconsistency. Comment on the detailed accuracy is not made as the results may vary due to the probabilistic nature of the estimation framework. The work has a scope of improvement especially in the area of the measurement model. A more general measurement system could be utilized which may or may not measure all the states. Further, the covariance of the parameter from the estimation framework could be studied in more detail such that in real life scenario, once the parameters are predicted sufficiently accurate, the parameter filter could be switched off, thus complying to the intention of separating the filter for state and parameter. More rigorous simulation tests with varying input can be performed to ensure practical feasibility and applicability. However, the proposed work is exhaustive in nature and detailed framework is presented to ensure repeatability.

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