Lab 1 Report
Longitudinal Dynamics – Slip Control

SD2231– Applied vehicle dynamics control

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Contents

1 Task 1 : Determine control parameters

1.1 Task 1.a

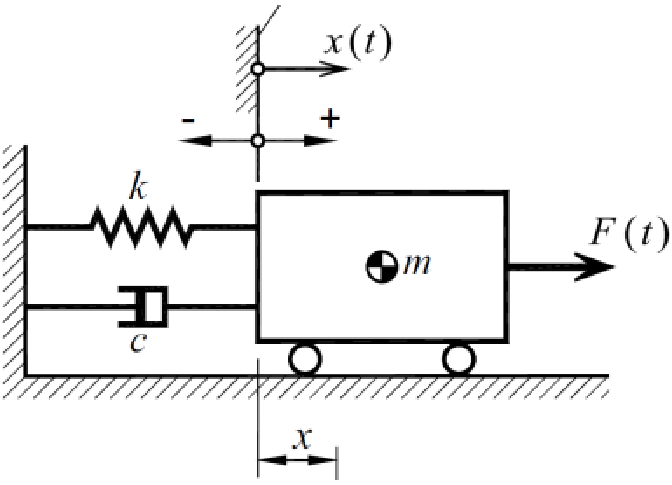


Figure 1: Single Mass System

The transfer function for the single mass system in Figure 1 is derived as follows :

$$m\ddot{x}(t) = F(t) - kx(t) - c\dot{x}(t)$$

Converting to the laplace domain, we get :

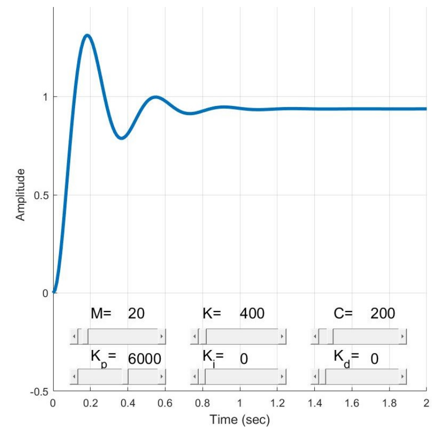
$$mX(s)s^2 = F(s) - kX(s) - cX(s)s$$
$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

1.2 Task 1.b

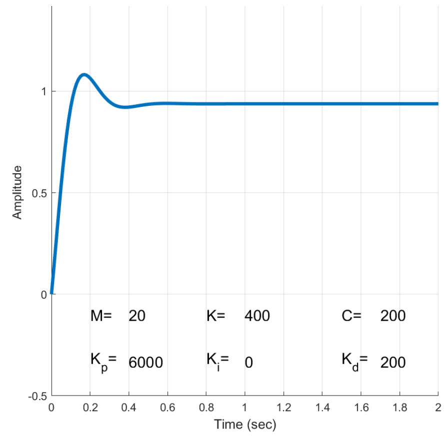
Using the model above, the following PID parameters achieve the performance as desired.

Table 1: PID parameters for different controllers

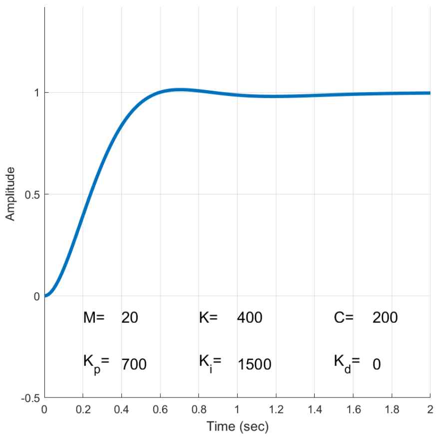
	Parameter	P	I	D
Controller				
P		6000	-	-
PD		6000	-	200
PI		700	1500	-
PID		5000	6000	600



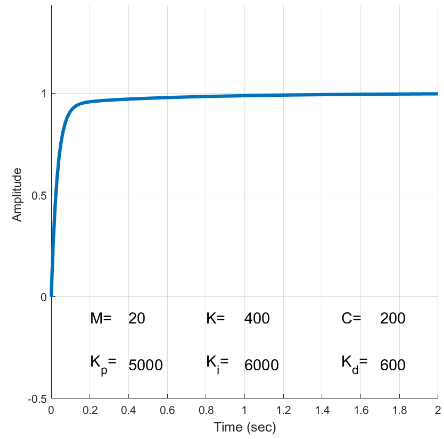
(a) P-Controller



(b) PD-Controller



(c) PI-Controller



(d) PID-Controller

Figure 2: Step response of different controllers

1.3 Task 1.c

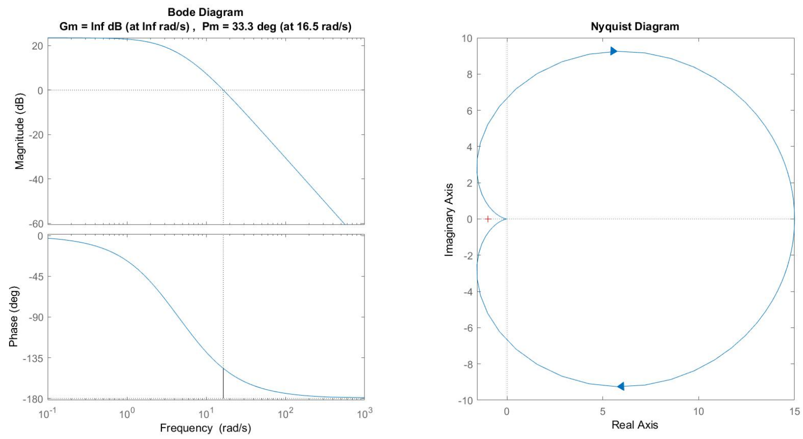
The stability of the open loop systems is analyzed by looking at their poles, in order to be stable they should all be non-positive. While the stability of closed loop systems is analyzed by looking at the bode/nyquist plot of their open loop system and checking their gain/phase margins.

The transfer function of the plant itself is stable, having only negative poles, i.e,

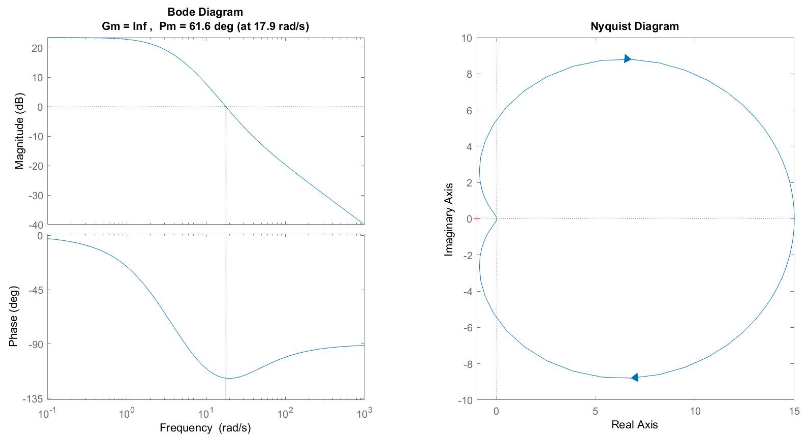
```
-7.2361
-2.7639
```

So adding a controller will not add any more positive poles to the open loop system, except the integrator which will introduce pole at zero.

Thus we only need to look at the nyquist and bode plot of the open loop systems to determine the closed loop stability of the systems.

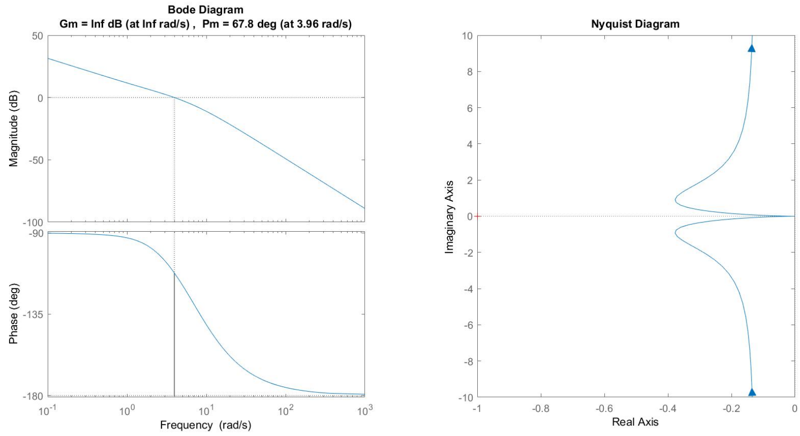


(a) P-Controller

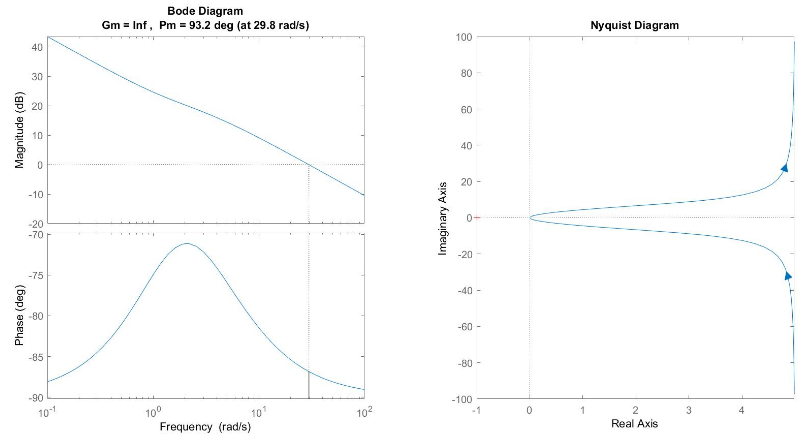


(b) PD-Controller

As can be seen, the phase margin is positive for all the systems and the nyquist plot do not encircle the -1. So all the closed loop systems are all stable. Although, the degree of stability of robustness of the system towards disturbances varies. For example, PI controller has the least bandwidth of all the 4 controllers.



(a) PI-Controller



(b) PID-Controller

Figure 4: Bode and Nyquist of different controllers

2 Task 2. Slip observer

2.1 Task 2.a

A basic observer for tractive and braking slip is implemented for all the 4 tyres using the formulation,

$$s_t = \frac{r\omega - v_x}{r\omega}$$
$$s_b = \frac{v_x - r\omega}{v_x}$$

The basic slip observer was implemented as shown in Figure.5 using only built in Simulink block. As, it was evident that tractive slip s_t and braking slip s_b was to be used separately by Traction Controller System and Anti-lock Braking System, both were computed simultaneously at all times by the slip observer.

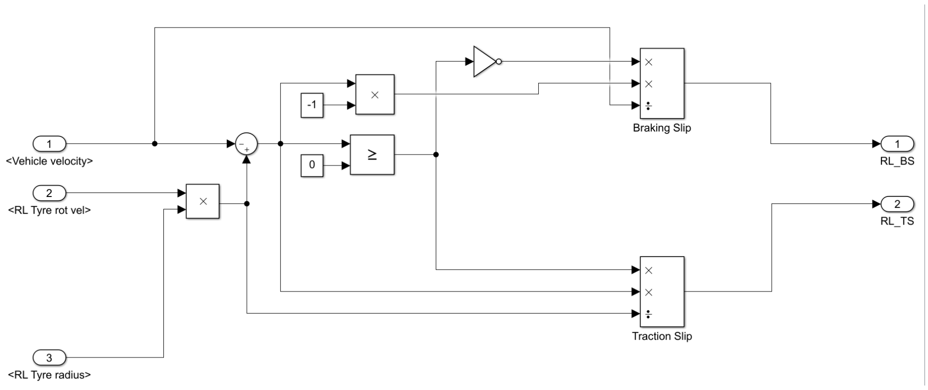
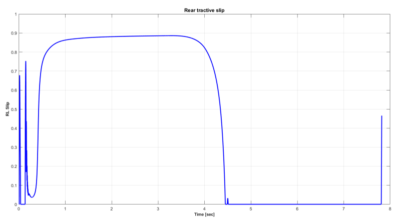
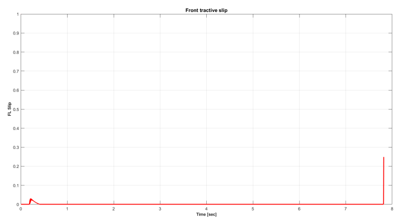


Figure 5: Basic Slip Observer

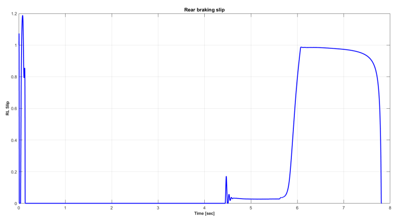
The slip values computed using the basic observer and are shown in Figure 6



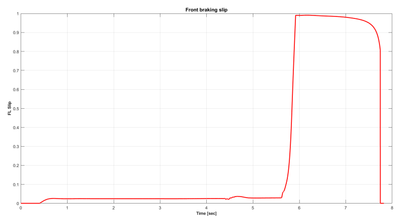
(a) Rear tractive slip vs Time



(b) Front tractive slip vs Time



(c) Rear braking slip vs Time



(d) Front braking slip vs Time

Figure 6: Slip values computed using basic slip observer using block shown in Figure.5

2.2 Task 2.b

The issues with basic slip observer were identified as follows:

- Division by zero - Before the vehicle gains any rotational or linear velocity, the slip cannot be calculated due to division by zero. This was raised as a warning by the Simulink error handler.
- The slips are only defined between zero and 1. Thus each of the tractive and braking slip is supposed to be calculated when their numerators are positive.
- There was significant oscillations during startup of the vehicle, due to numerical precision issues. Thus, the low values of rotational velocity and longitudinal velocities needed to be dealt with.

Keeping in mind these issues, an improved slip observer was implemented without using Function Block as shown in Figure.7 and a stable plots of slip was observed as shown in Figure.8.

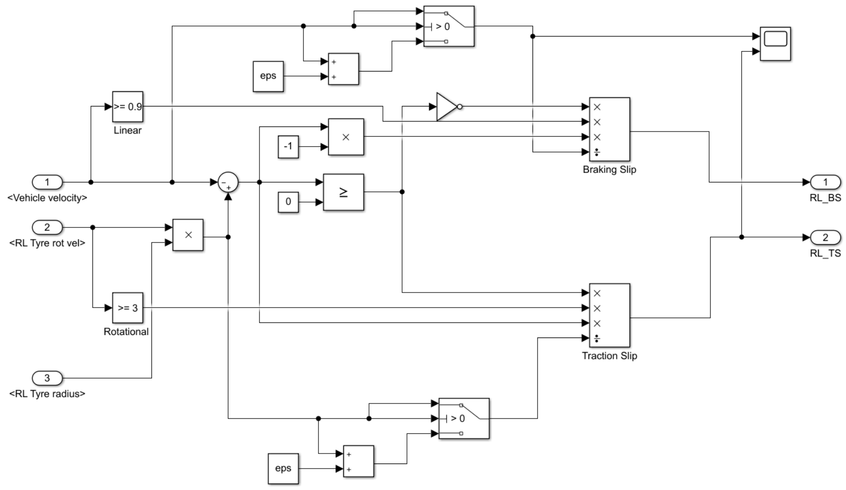
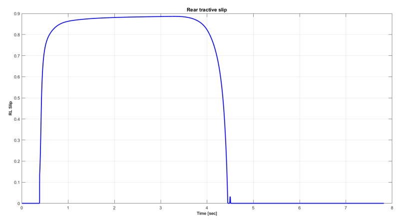
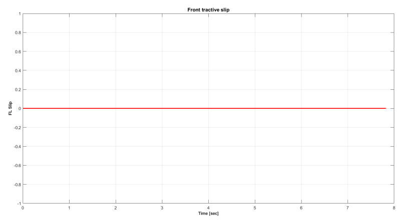
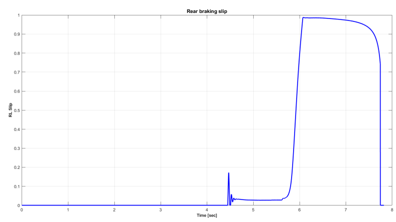


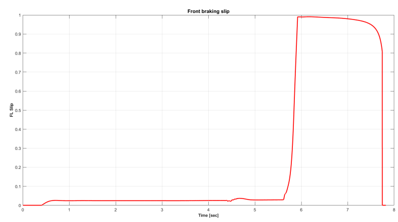
Figure 7: Improved slip observer handling division by zero and numerical inconsistency during low rotational/linear velocity





(a) Rear tractive slip vs Time (b) Front tractive slip vs Time





(c) Rear braking slip vs Time (d) Front braking slip vs Time

Figure 8: Slip values computed using improved slip observer using block shown in Figure.7

2.3 Task 2.c

In order to find the friction coefficient for the tyres, the friction force on each wheel needs to be calculated. Since the given vehicle model has only two rear wheels driven, so tractive slip and tractive friction is calculated only for these 2 wheels. Whereas braking friction and braking slip is calculated for all the four tyres.

The friction coefficient for the front tyres is calculated as :

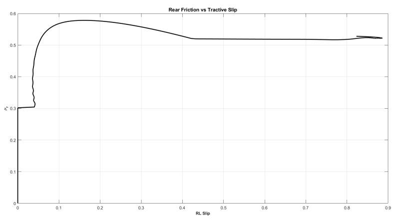
$$\mu_f = \frac{F_{xf}}{N_f} = \frac{F_{xFL}}{F_{zFL} + F_{zFR}}$$

And for the rear tyres,it is calculated as :

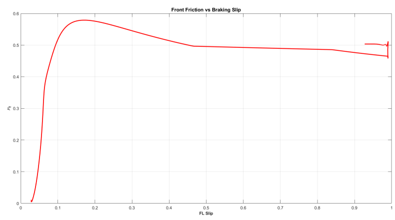
$$\mu_r = \frac{F_{xr}}{N_r} = \frac{F_{xRL}}{F_{zRL} + F_{zRR}}$$

Here, the friction forces F_{xf} and F_{xr} are the net forces from each driven wheel that accelerate or deaccelerate the vehicle. The $F = ma$ force is assumed to be equally divided among the driven wheels as per the slip being calculated, i.e divides between rear 2 wheels for tractive force and among all 4 wheels for braking force.

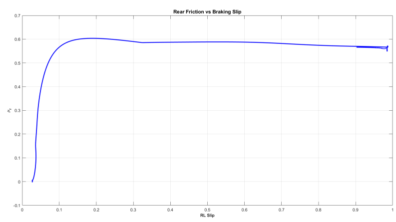
The used friction(μ) vs longitudinal slip(s) plot is provided in the Figure 9. The aim is to locate the slip for which maximum friction can be achieved from the road for both braking and acceleration. This corresponds to the highest point on the μ vs s plot. Based on this, the reference tractive slip from Figure.9a selected is **0.18** and the braking slip for rear was selected as **0.18** from Figure.9b and braking slip for front was selected as **0.19** from Figure.9c. Additionally, for selecting the reference braking slip (s_b) care was taken that minimum value of slip is selected in the highest μ range, such that longitudinal control is still available during braking making the vehicle turn.



(a) Friction vs rear tractive slip



(b) Friction vs rear braking slip



(c) Friction vs front braking slip

Figure 9: Friction μ vs clip curves

3 Task 3. Design PID Controller

3.1 Task 3.a

The TCS activation logics includes :

- As long as the throttle is pressed is above 4%.

- As long as the tractive slip is above 1%

The activation logic was formulated to ensure that the traction control is only active when vehicle's tractive slip was significant (1%) to utilise traction control and driver intends to accelerate (throttle above 4%). Adding to this, it was also ensured that at any given time, the torque provided by the traction controller does the exceed the driver's throttle value making sure that intended acceleration is achieved. The complete Traction Controller with activation is shown in Figure.10.

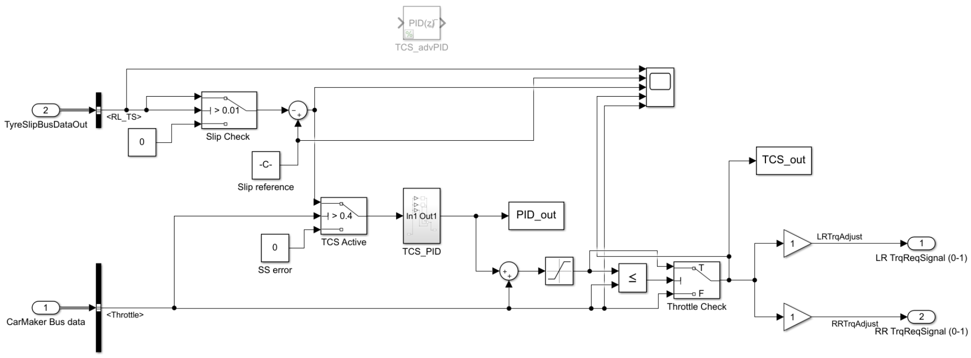


Figure 10: Implemented Traction Controller System with activation logic and PID controller

The ABS activation logic includes:
- As long as the brake is pressed above 5%.

Two identical ABS was implemented for both rear and front wheels as shown in Fig.11. Similar to traction controller, the ABS included a check to ensure that torque produced by the ABS controller is less than the brake pedal pressed by the driver to ensure desired de-acceleration.

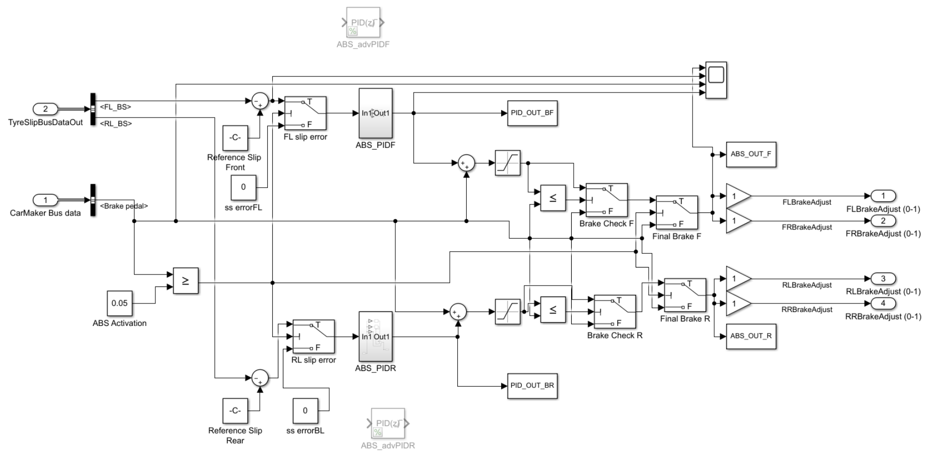


Figure 11: Implemented Antilock Braking System with activation logic and PID controller

3.2 Task 3.b and 3.c

The discrete time PID controller was implemented in both TCS and ABS system of compensation in nature of form -

$$\tau = I + (s_{ref} - s)\left[K_p + K_i \frac{\Delta_t z}{z-1} + K_d \frac{z-1}{\Delta_t z}\right] \tag{1}$$

The PID controller, as shown in figure 12, has been designed and used for controlling both TCS and ABS.

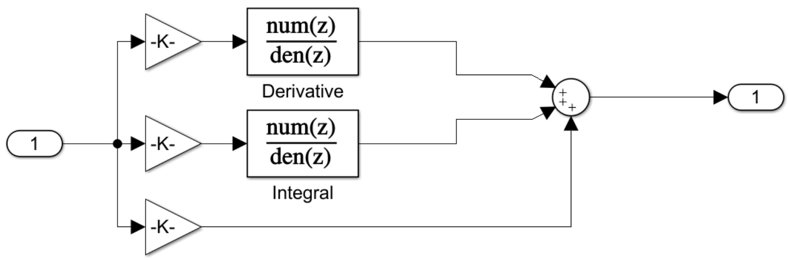
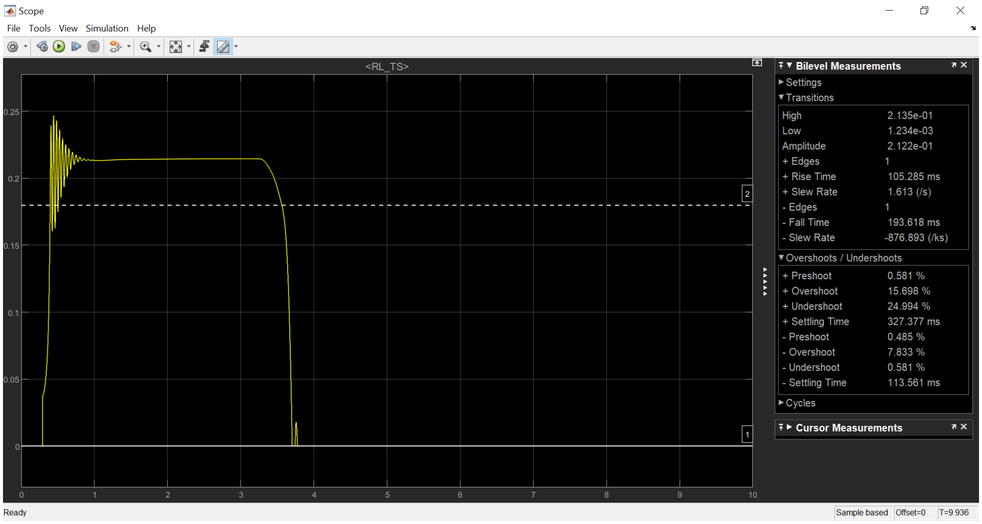


Figure 12: Basic Parallel PID Controller designed for TCS and ABS

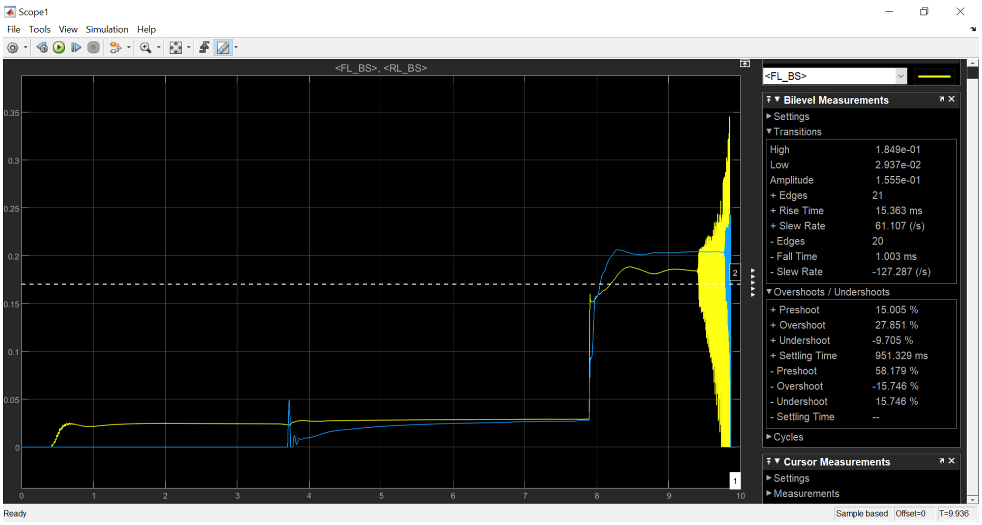
where, τ is torque output of, I is the driver input - throttle or brake signal. Δ_t is the sampling time, z is the Laplace operator. The continuous time integration and derivative has been expressed as the corresponding time discrete. K_p, K_i & K_d, are the co-efficient of the proportional, integral and derivative. The derivative and integral block was implemented as discrete transfer function Simulink blocks.

- Using P-Controller
  Only using P-controller does not lead to very good reference following. We have to compromise between steady state error on one side & overshoot and Settling time on the other.

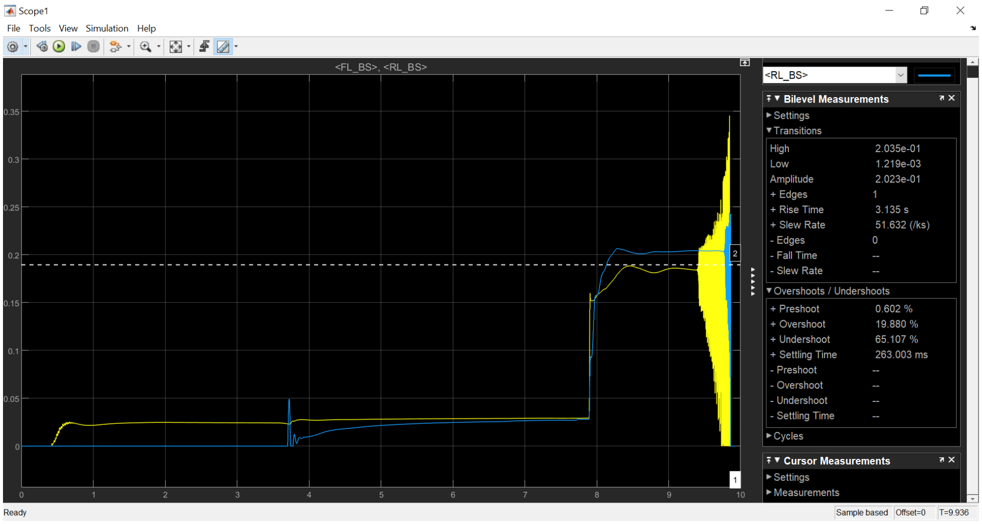
  So, the K_p values chosen are somewhere between these extreme ends. The Figure 14 shows the time response plot of the respective tractive and braking slips, and its transition time characteristics.



(a) Rear tractive slip vs Time



(b) Front braking slip vs Time

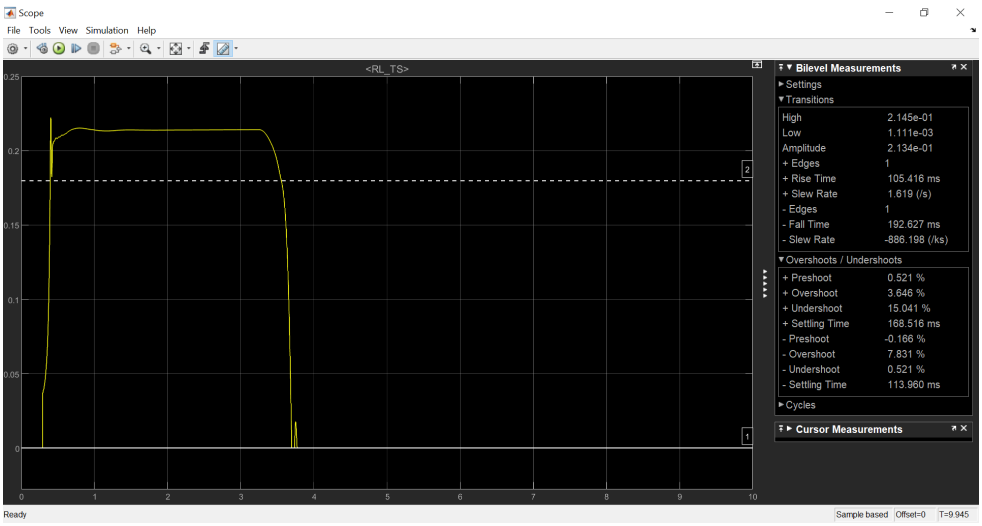


(a) Rear braking slip vs Time

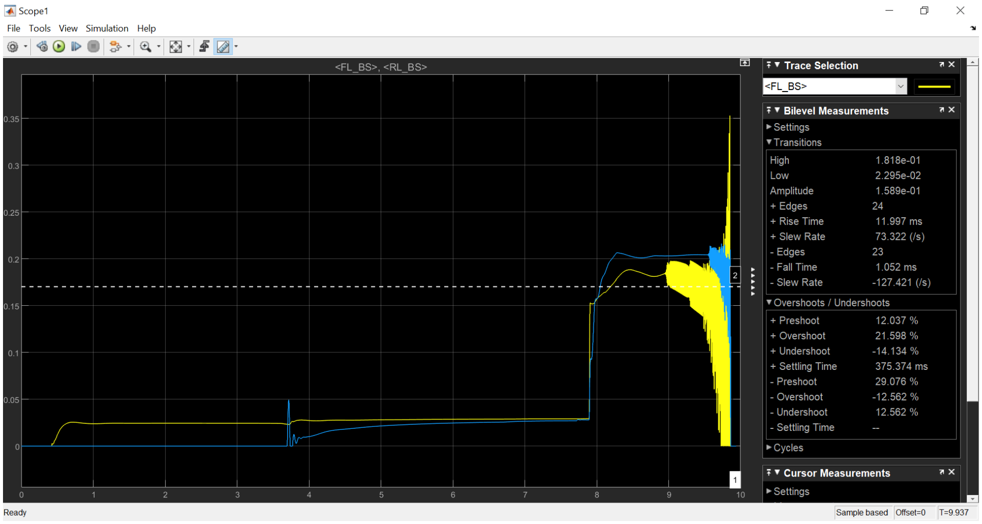
Figure 14: Time response and transition time characteristics of vehicle slip

- Using P-D Controller
  Same value of K_p as previous is used, in order to understand the effect of D-controller. While it reduces the oscillations and settling time for TCS, it does the opposite, and unintutively so, for the ABS controller.

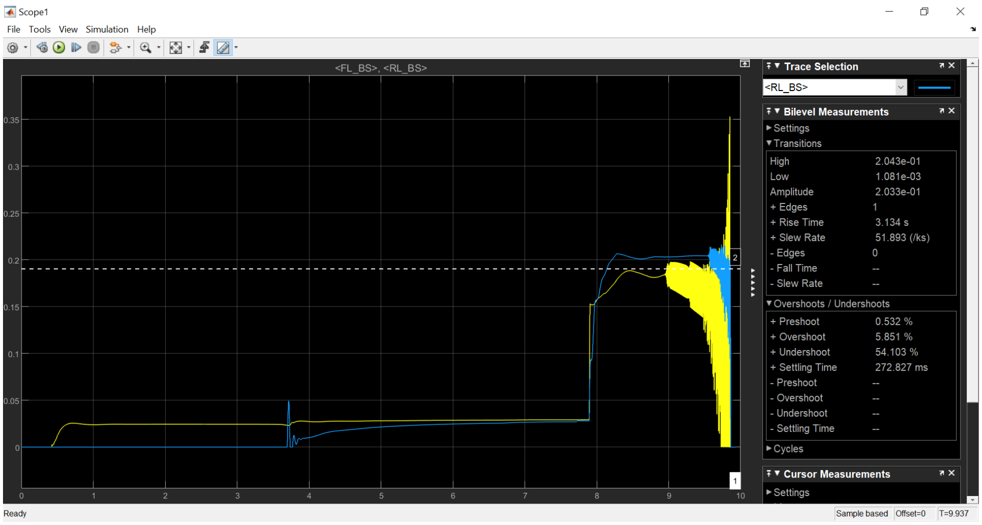
  The figure 16 shows the time response plot of the respective tractive and braking slips, and its transition time characteristics.



(a) Rear tractive slip vs Time



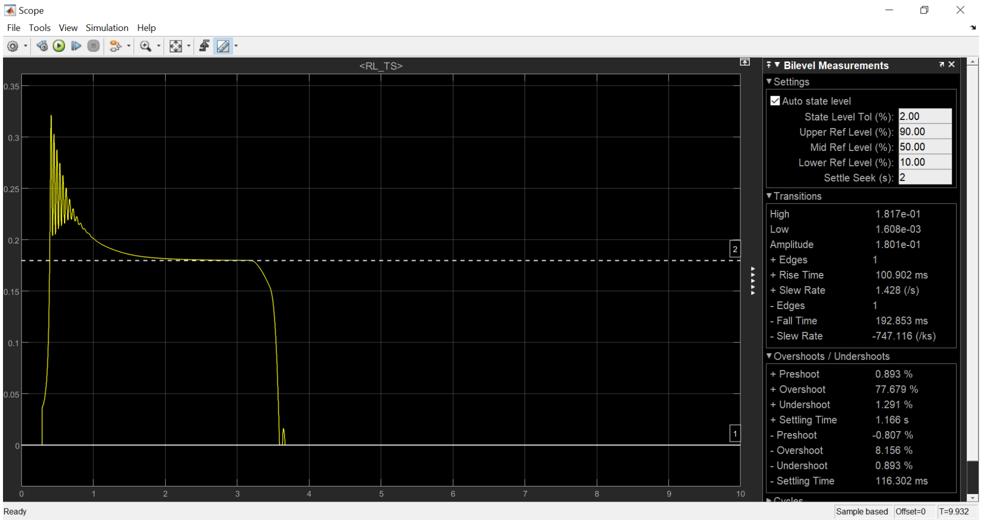
(a) Front braking slip vs Time



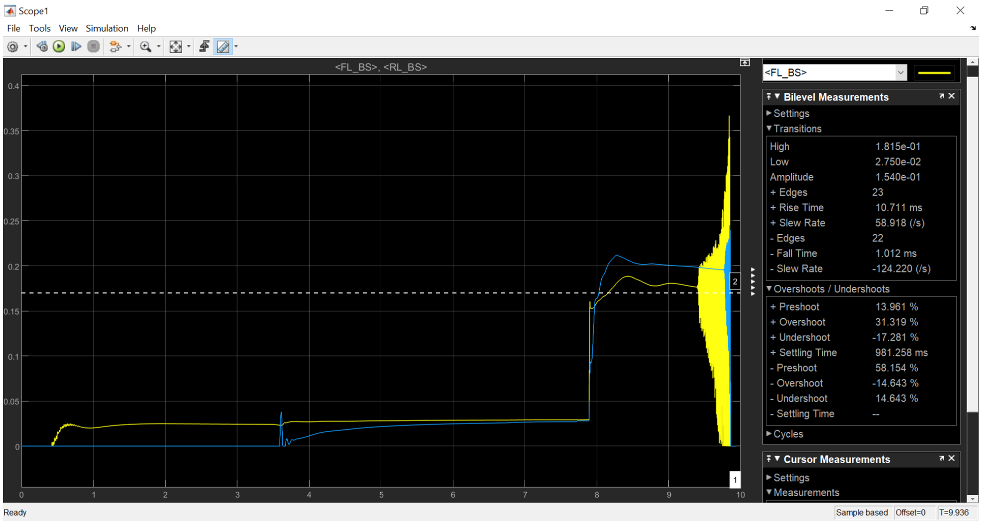
(b) Rear braking slip vs Time

Figure 16: Time response and transition time characteristics of vehicle slip

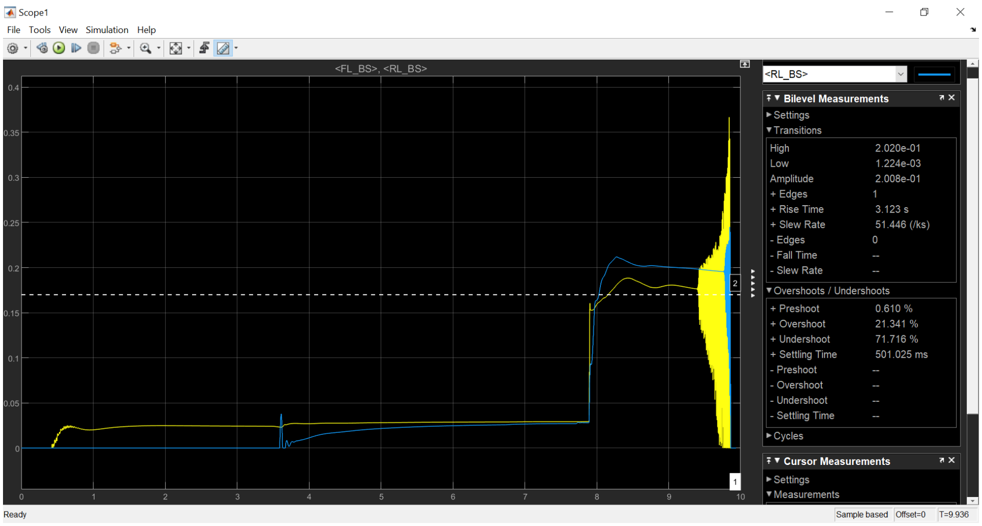
- Using P-I Controller
  Since the K_p value chosen was small to begin with, in order to keep oscillations low, the steady state error needed to be compensated by I-controller is quite large, hence the high K_i values. Nonetheless, we can see it trying to make the system reach the reference value, as can be seen in the figure 17 While the settling time is high, the system can be sped up by increasing the K_p values.



(a) Rear tractive slip vs Time



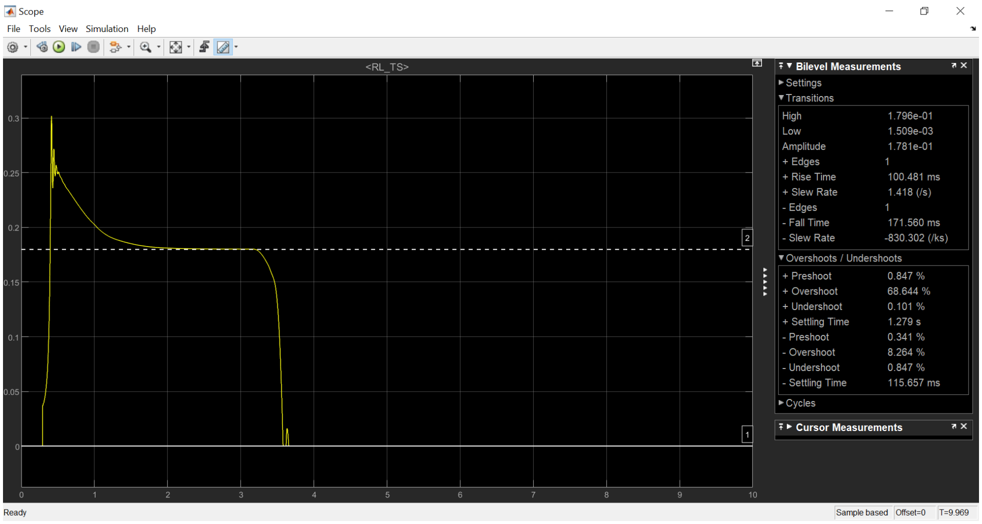
(b) Front braking slip vs Time



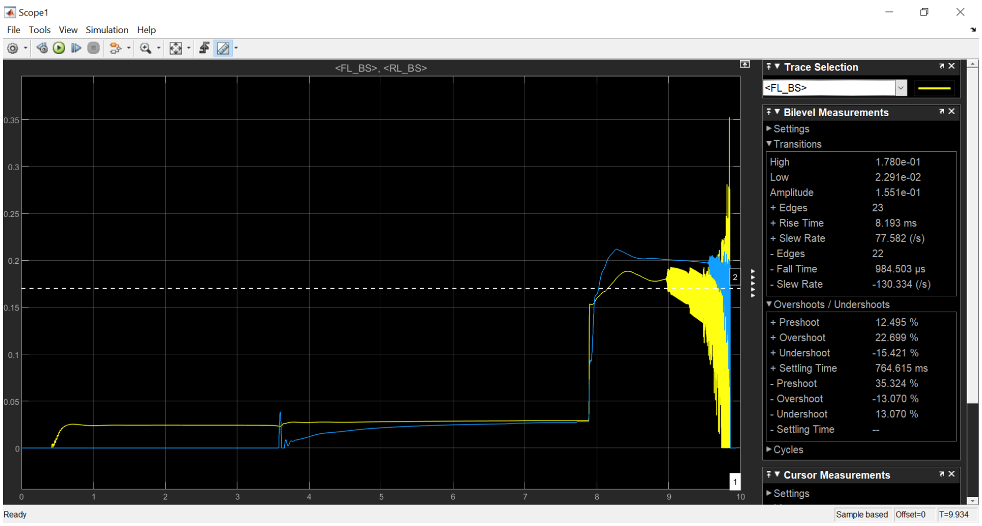
(c) Rear braking slip vs Time

Figure 17: Time response and transition time characteristics of vehicle slip

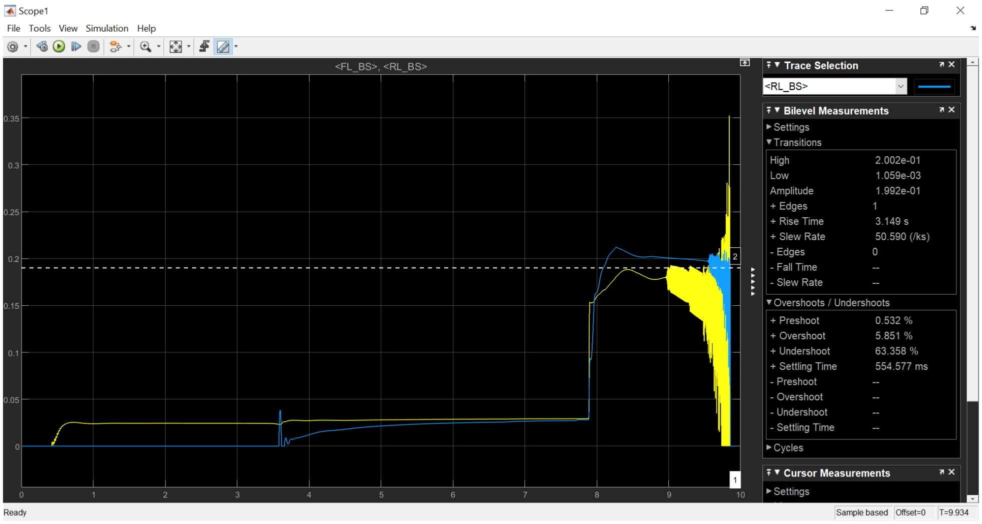
- Using PID Controller
  Using the values of K_p, K_i and K_d previously used in order to keep the variables to a minimum and show the working of PID controller on a real plant, the results and transition time characteristics obtained are shown in figure 19.



(a) Rear tractive slip vs Time



(b) Front braking slip vs Time



(a) Rear braking slip vs Time

Figure 19: Time response and transition time characteristics of vehicle slip

A comprehensive transition time characteristics are presented in tables 2, 3 and 4 below. Please note the kp, ki, kd values are not yet optimized for the system.

The TCS controller coefficients and tractive slip reference values used are :

```
Kp_em = 5.0;
Ki_em = 12.0;
Kd_em = 0.5;

TCS_slip_ref = 0.18;
```

Table 2: Transition time characteristics for tractive slip reference following

Controller \ Characteristic	Rise Time	Overshoot	Settling Time
P	105.29ms	15.7%	327.38ms
PD	105.42ms	3.65%	168.52ms
PI	100.9ms	77.68%	1.17s
PID	100.48ms	68.64%	1.28s

The ABS controller coefficients and braking slip reference values used for front tyre are :

```
Kp_brake = 20.0;
Ki_brake = 15;
Kd_brake = 0.02;

ABS_slip_refF = 0.17;
```

Table 3: Transition time characteristics for braking slip reference following for front tyre

Controller \ Characteristic	Rise Time	Overshoot	Settling Time
P	15.36ms	27.85%	951.33ms
PD	12ms	21.6%	375.37ms
PI	10.71ms	31.32%	981.26ms
PID	8.19ms	22.7%	764.615ms

The ABS controller coefficients and braking slip reference values for rear tyre are :

```
Kp_brake_r = 15.0;
Ki_brake_r = 12;
Kd_brake_r = 0.02;

ABS_slip_refR = 0.19;
```

Table 4: Transition time characteristics for braking slip reference following for rear tyre

Controller \ Characteristic	Rise Time	Overshoot	Settling Time
P	3.135s	19.88%	263ms
PD	3.134s	5.85%	272.83ms
PI	3.123s	21.34%	501ms
PID	3.149s	5.85%	554.58ms

3.3 Task 3.d

We choose to move forward with only P and I controller, because D controller is introducing more oscillations due to the noisy signals. The input can thus be pre-filtered before PID controller processes, and this will be included in the advances version of our controller.

4 Task 4. PID Controller Optimisation

4.1 Task 4.a

Here, we have presented our best controller, and as explained in section 3.d, we are only using PI controller. We have chosen these values keeping in mind that there should be minimum overshoot in case of TCS, even if it means reaching the steady state slowly. And for the ABS controller, we have aimed at reaching steady state as quickly as possible even with some overshoot. Total time taken by car to complete the maneuver is 10.0s and total distance covered during this time is 146.8m.

```
Kp_em = 7.00;
Ki_em = 12.5;
Kd_em = 0.0;
Kp_brake = 6.75;
Kp_brake_r = 7.25;
Ki_brake = 30.5;
Ki_brake_r = 35.0;
Kd_brake = 0.00;
Kd_brake_r = 0.00;
```

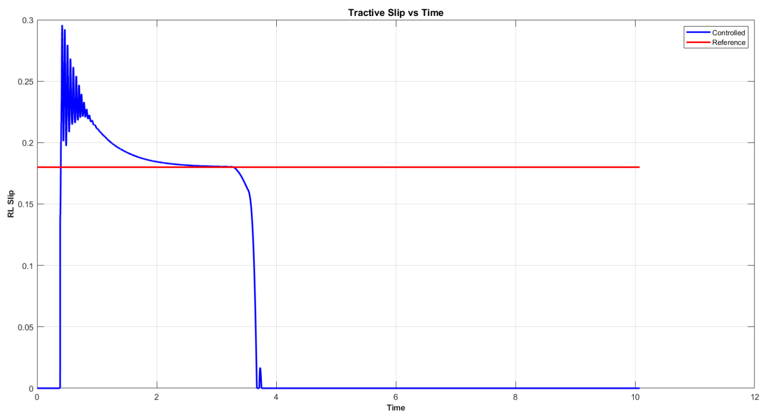


Figure 20: Tractive slip time response

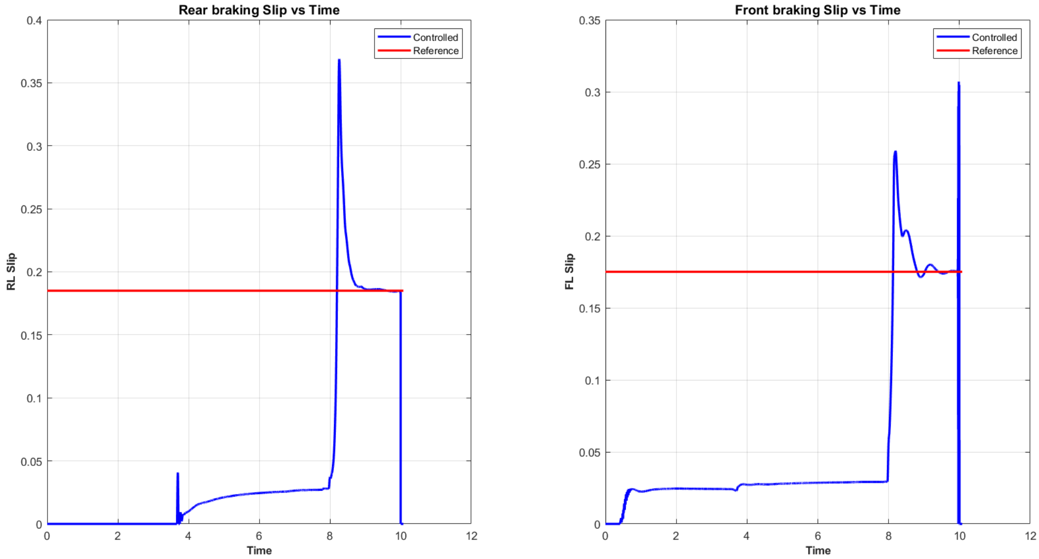


Figure 21: Braking Slip time response

The figure 22 presents the performance of the the system after TCS and ABS is implemented. The acceleration and deacceleration of the vehicle have improved slightly in value and response time.
Also, it can be seen that no unintended torque or braking is provided by the controller. The value of torque or braking signal delivered is never more than what the driver desires.

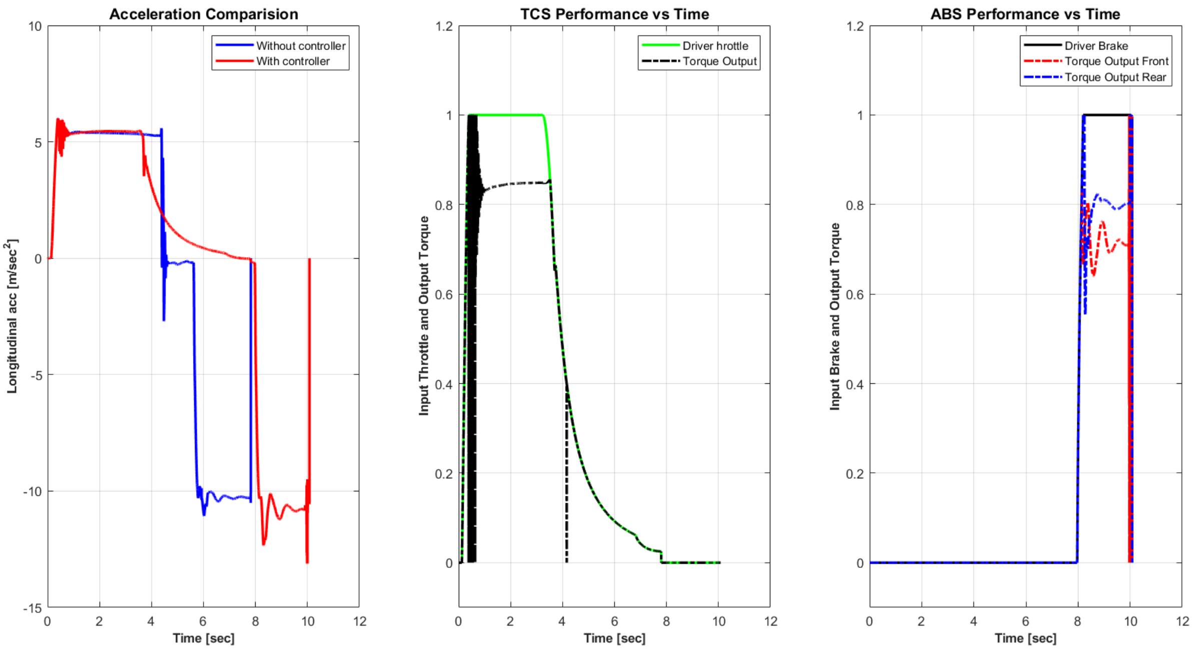


Figure 22: Performance and signals comparison

5 Task 5. More advanced PID

As discussed in Task 4, due to the noisy slip inputs, the basic derivative controller did not perform well. Additionally, the saturation of the torque signals between 0 and 1 lead to integral error accumulation, resulting in saturated torque outputs in the starting of the TCS. These limitations were improved in the advanced PID implementation. For the more advanced PID the inbuilt simulink block is used, and its additional functions of integral anti-wind up using clamping and derivative low pass filtering with co-efficient N are used. This leads to visible improvement in reference following as can be seen in Figure.23. Also it leads to a slight improvement in distance covered which is now 146.67m compared to 146.7m previously. It still takes 10.0s to complete the maneuver though.

```
Kp_em = 6.85;
Ki_em = 13.5;
Kd_em = 0.15;
Kp_brake = 6.75;
Kp_brake_r = 10.25;
Ki_brake = 30.5;
Ki_brake_r = 45.0;
Kd_brake = 0.00;
Kd_brake_r = 0.01;
N_TCS = 100
N_ABS = 50
```

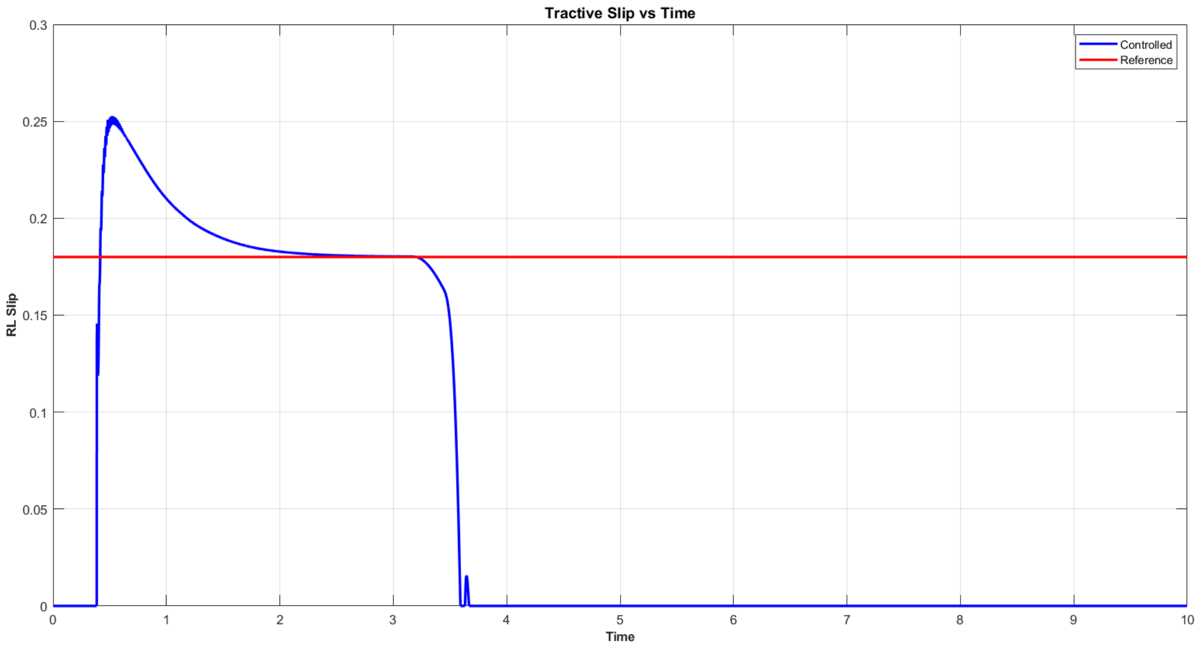


Figure 23: Tractive slip time response

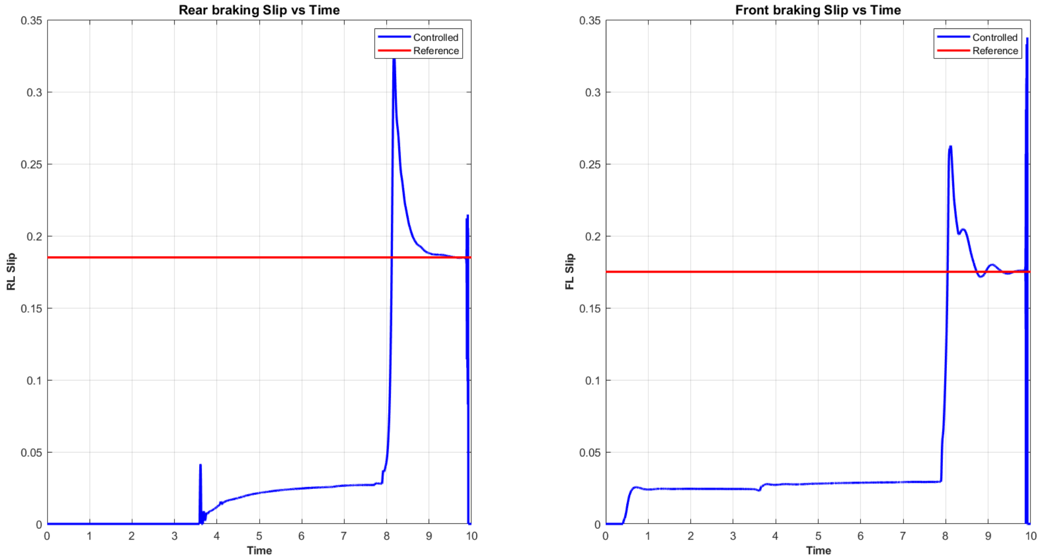


Figure 24: Braking slip time response

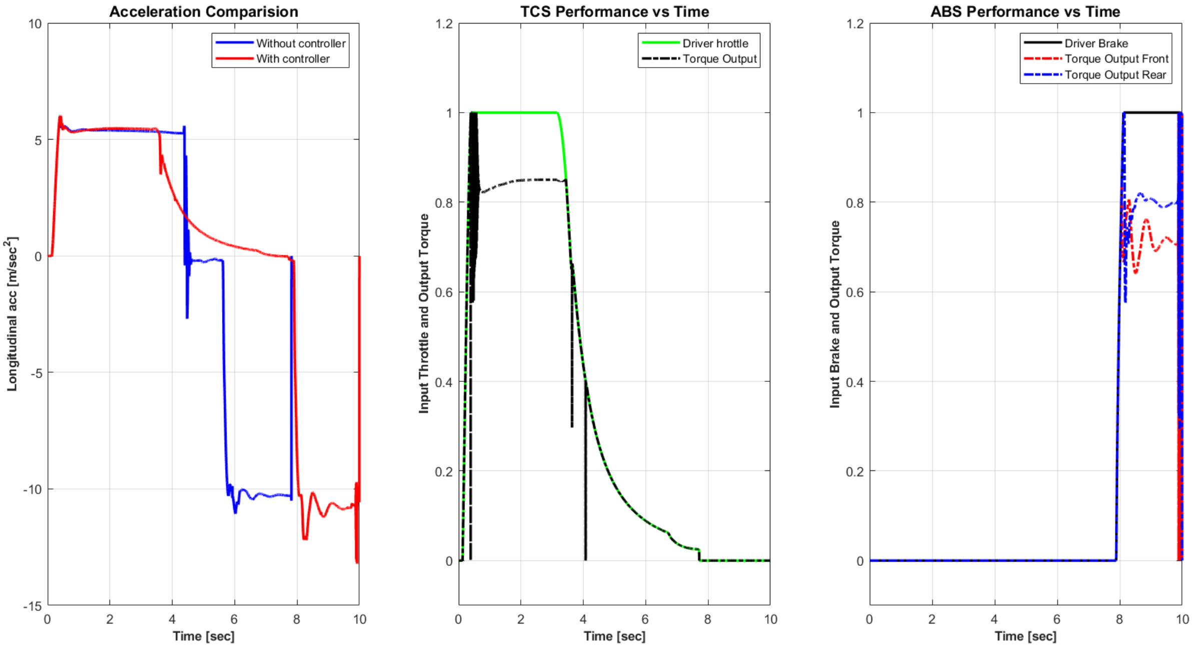


Figure 25: Performance and signals comparison